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Three nucleon effects in the microscopic optical potential.

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Nucleon Optical Potential in Brueckner Theory

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1. Introduction.
2. G-matrix (Effective Interaction).
3. Nucleon Optical Potential.
4. Spin-orbit force.
5. Three nucleon correction.
6. Conclusions.

1. Introduction:

First Order Microscopic description of the nuclear Collision:

$$U(k',k) = \int dp'dp \langle k', p' | G | k, p \rangle \hat{\rho}(p', p)$$

Nucleon scattering

Few tens of MeV – 400MeV

Comparison with Empirical Potentials.

Bethe-Brueckner-Goldtone …theory of Nuclear Matter

PR95,217(1954), Rev. Mod.Phys.30,745(1967), Proc. Roy. Soc. (Lon) A239,267(1937)

Brieva and Rook, NPA291,317(1977); 307,493(1978)

Jeukenne,Lejeune,Mahaux, Phys. Rep. 25,83(1976)

H.V.von Geramb, in The Interaction between Medium Energy nucleons in Nuclei (AIP,New York, 1983), Yamaguchi et al.

Recent developments:

Amos et al. Adv. In Nucl. Phys.25, 275 (2000)

Arelleno, Brieva love, Phys. Rev. Lett.63,605 (1989)…

Arlleno and BaugePRC76, 014613 (2007)

All the above approaches : **FOLDING of the generalised TWO-Body Infinite Nuclear Matter effective interaction over the Target ground state densities.**

We briefly discuss the basic formalism, and its successes and in its applications to finite nuclei.

Failures. Attempts to improve

1. Spin orbit (Direct+Exchange), Central Exchange.

2.Calculations of Three Nucleon effects in the Nucleon optical potential.

Conclusions

2. G-Matrix.

$$H_0 = \sum_i (T_i + U_i)$$
$$H_1 = \sum_{i<j} v_{ij} - \sum_i U_i$$

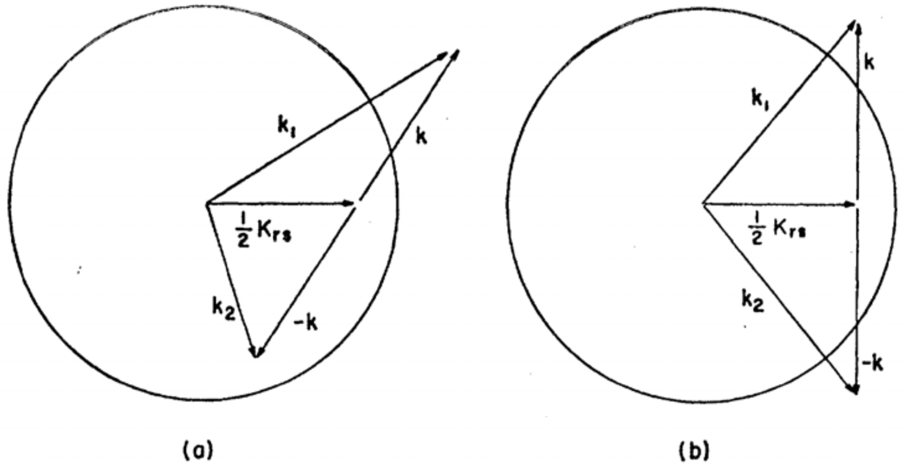
Goldstone Perturbation series. First order term:

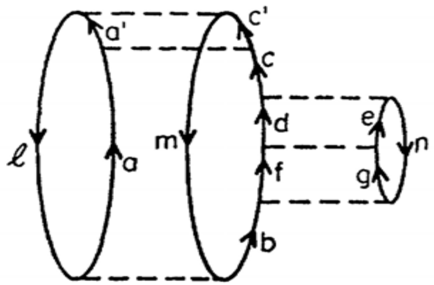
B.D. Day, Rev. Mod. Phys. 39, 719(1967)

$$E = \sum_m T_m + \frac{1}{2}\sum_{m,n}\left[\langle mn|v|mn\rangle - \langle mn|v|nm\rangle\right]$$

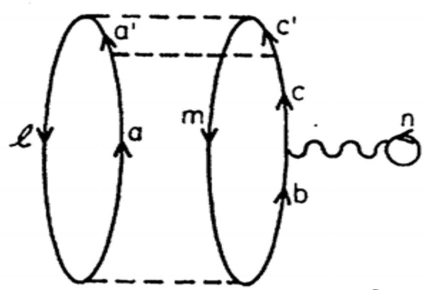
v is the realistic two-body inter-nucleon potential. v is replaced by the effective interaction, g-matrix:

$$v \to g(w) = v - v(Q/e)g(w)$$





v



g

Calculation of g is summing all the infinite ladder diagrams and it amounts to solving the Schrödinger equation between two particles in presence of all other nucleons.

$$\psi_{rs} = \phi_{rs} - (Q/e)g(w)\phi_{rs}$$

$$\phi_{rs}(r_1, r_2) = \phi_r(r_1)\phi_s(r_2) = |r, s\rangle$$

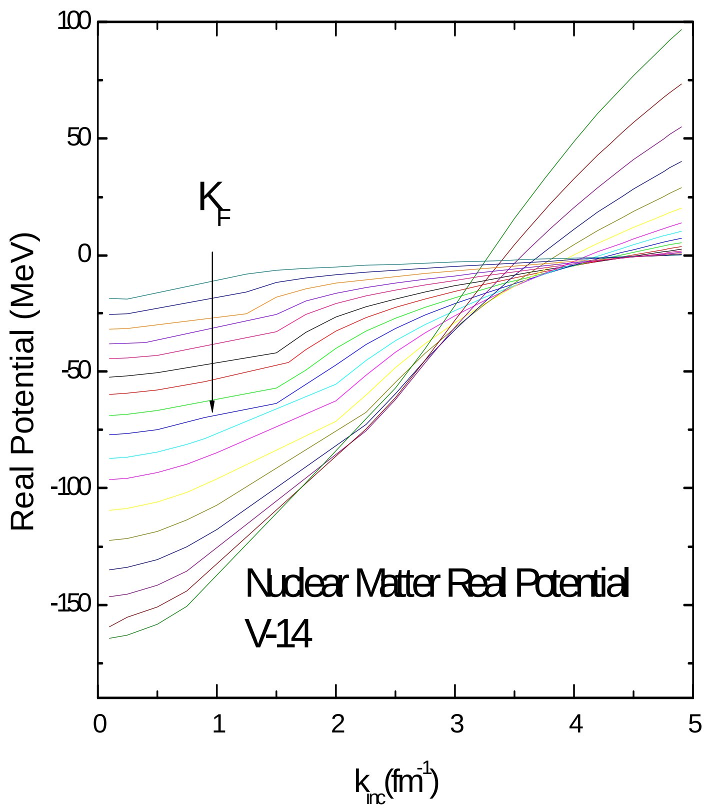
$$\psi_{rs} = \phi_{rs} - (Q/e)v\psi_{rs}$$

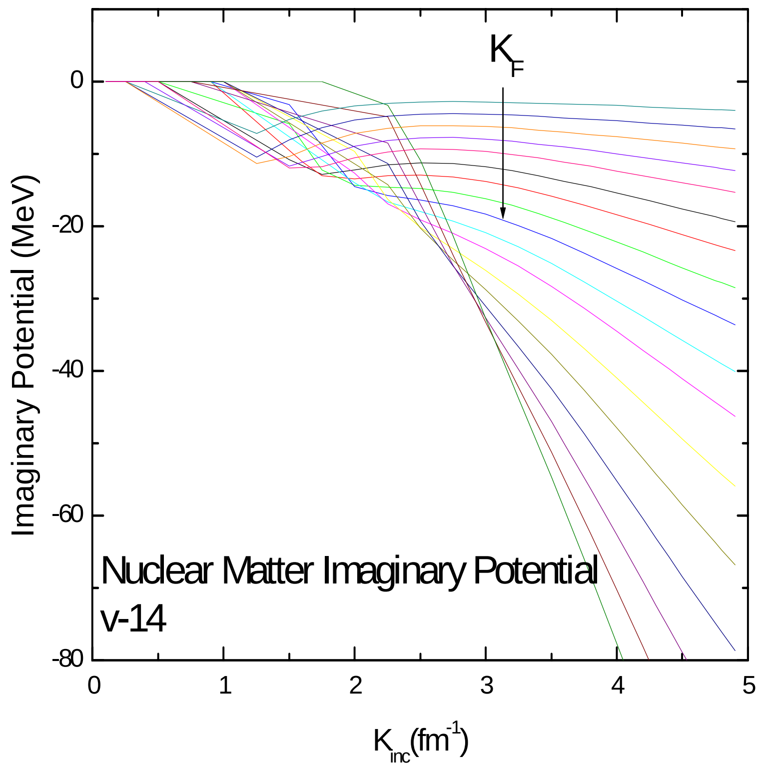
$$\psi_{rs}(\mathbf{r}) = \exp\,(i\mathbf{k}_{rs}\cdot\mathbf{r}) - \int K(\mathbf{r}, \mathbf{r}')v(r')\psi_{rs}(\mathbf{r}')d^3r'$$

$$k(r, r') = \int \frac{d^3k Q(k, K_{rs})}{(2\pi)^3 e(k, K_{rs})} e^{\left[i\vec{k}.(\vec{r}-\vec{r}')\right]}$$

K:0-6.0fm^{-1}; K$_F$:0.5-2.0fm^{-1}, L=0-6, Four Coupled states

$$U(k,k_F) = \sum_{j \le k_F} \langle kj | g(w) | kj \rangle$$





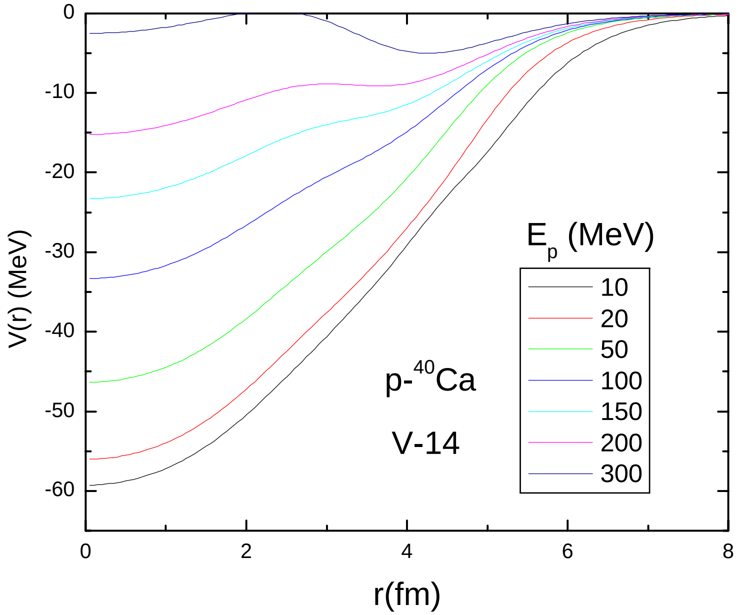
3. Nucleon Optical Potential:

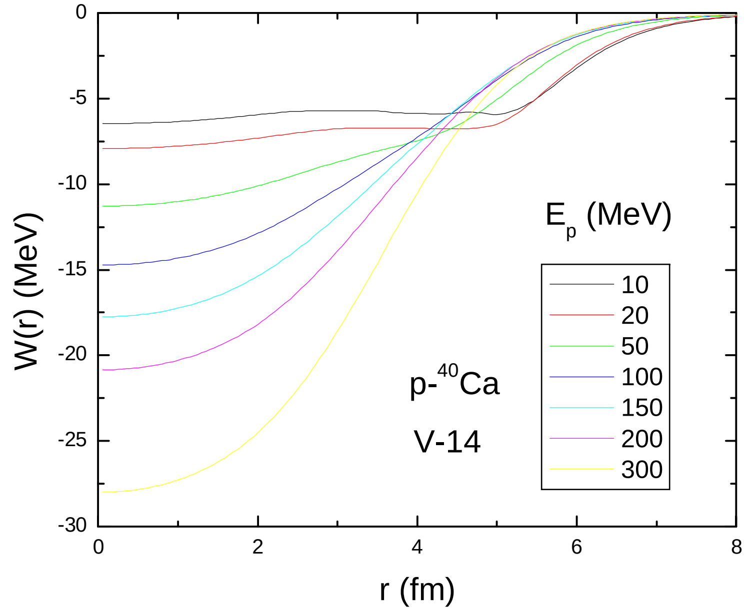
We define the radial dependence of the g-matrix such that the nuclear matter optical potential is reproduced ie:

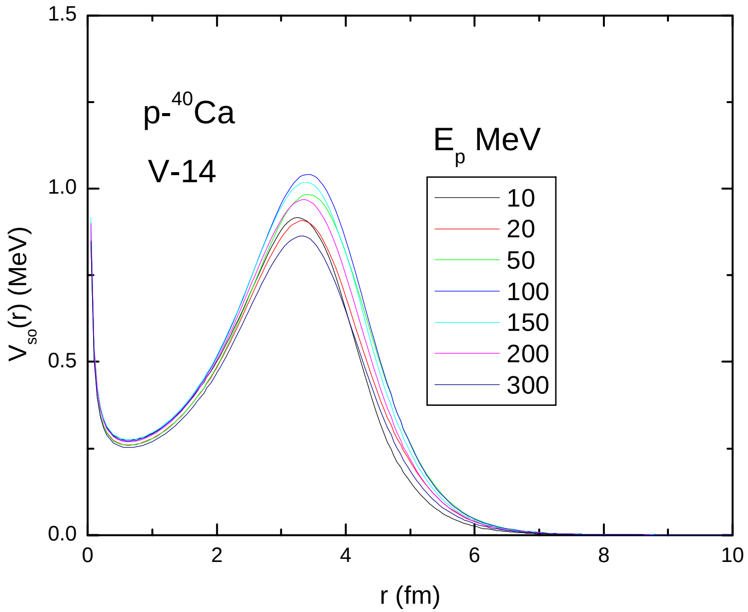
$$\left\langle \Phi_{rs} \middle| g(r) \middle| \Phi_{rs} \right\rangle = \left\langle \Phi_{rs} \middle| v \middle| \Psi_{rs} \right\rangle$$

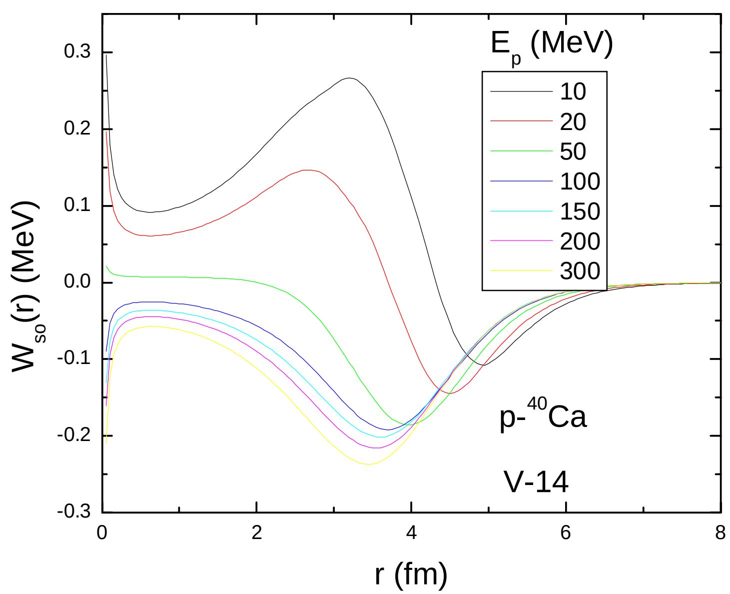
$$U^{c}_{opt}(r_1, E) = \int \rho(r_2) g^{c}_{D}(|r_1 - r_2|, \rho(\frac{1}{2}(r_1 - r_2)), E) d^3 r_2$$

$$+ \int \rho(r_1, r_2) g^{c}_{Ex}(|r_1 - r_2|, \rho(\frac{1}{2}(r_1 + r_2)), E) j_0(k|r_1 - r_2|) d^3 r_2$$


V(r) (MeV)
r(fm)
E_p (MeV)
10
20
50
100
150
200
300
p-40Ca
V-14


W(r) (MeV)
r (fm)
E_p (MeV)
10
20
50
100
150
200
300
p-40Ca
V-14


V_so(r) (MeV)
r (fm)
p-40Ca
V-14
E_p MeV
10
20
50
100
150
200
300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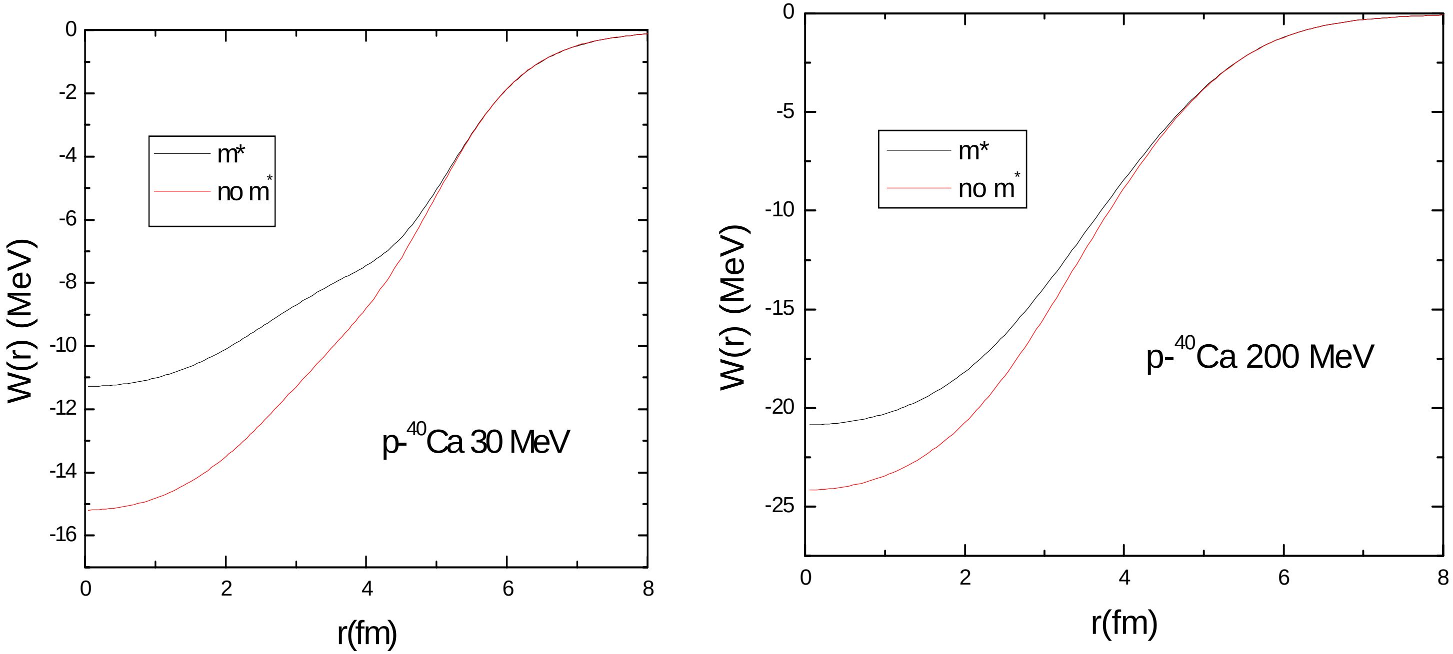
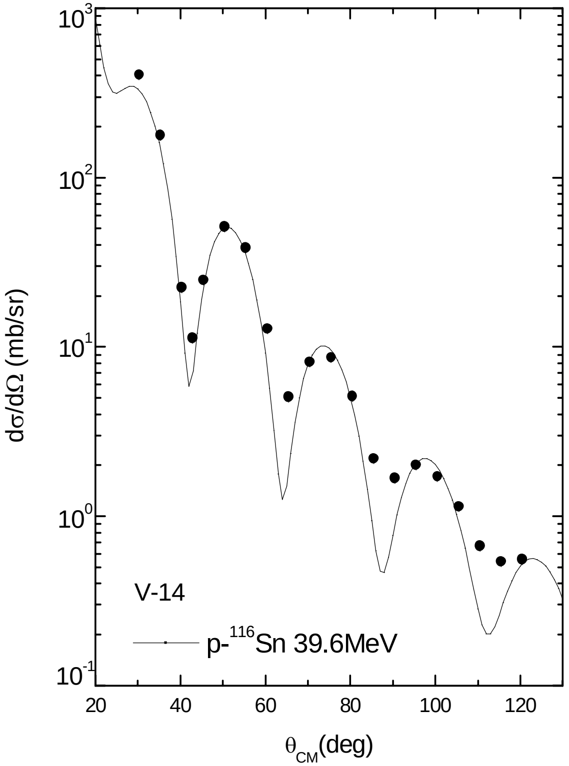
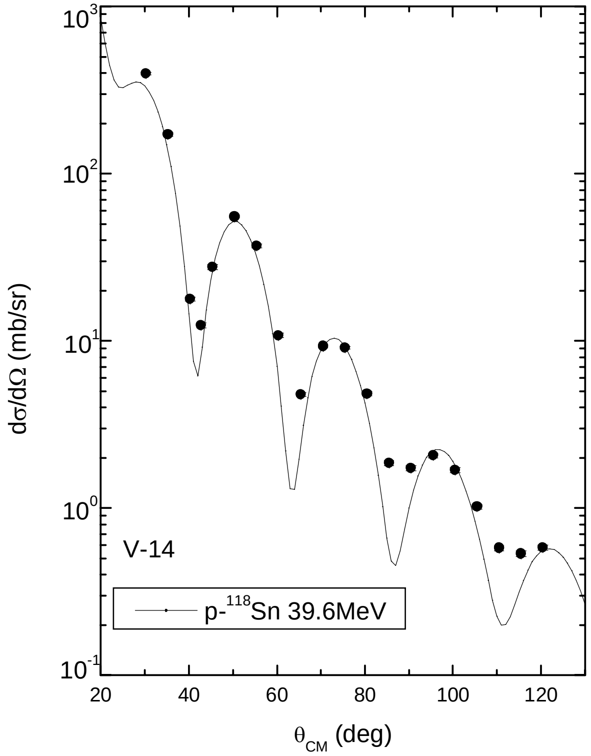
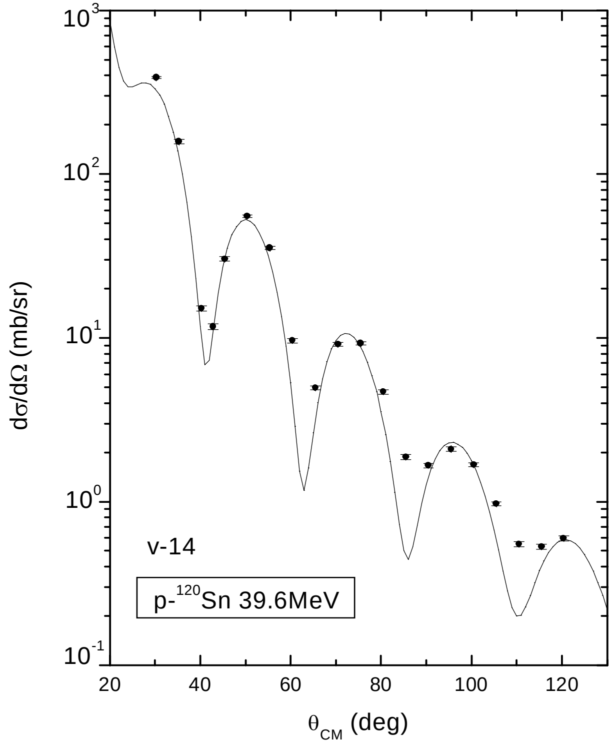
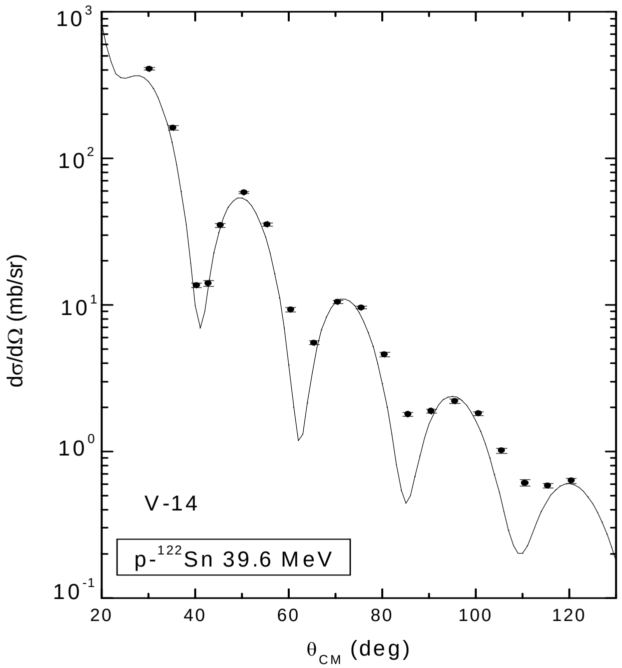


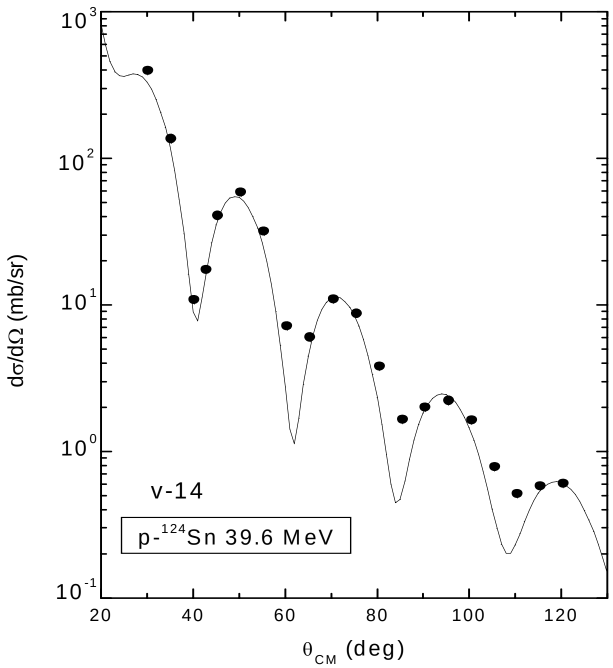
Effective mass correction in the calculated Imaginary part.







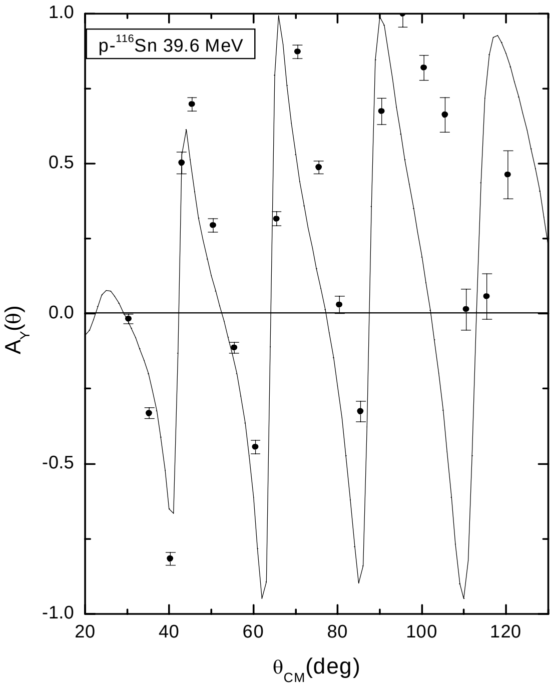


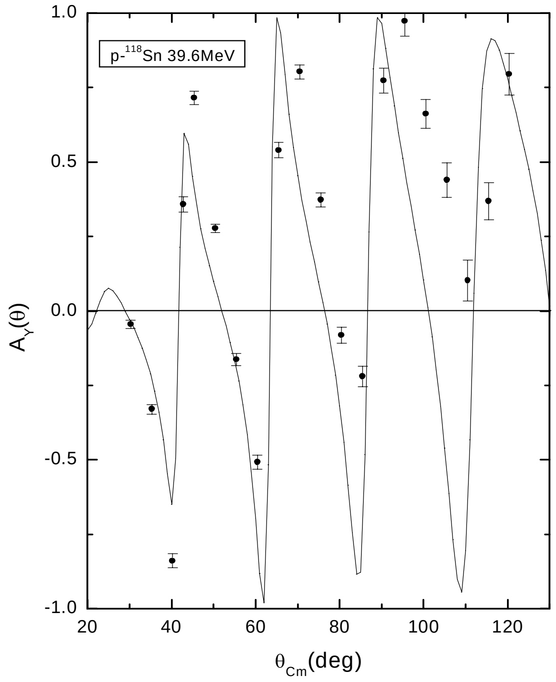


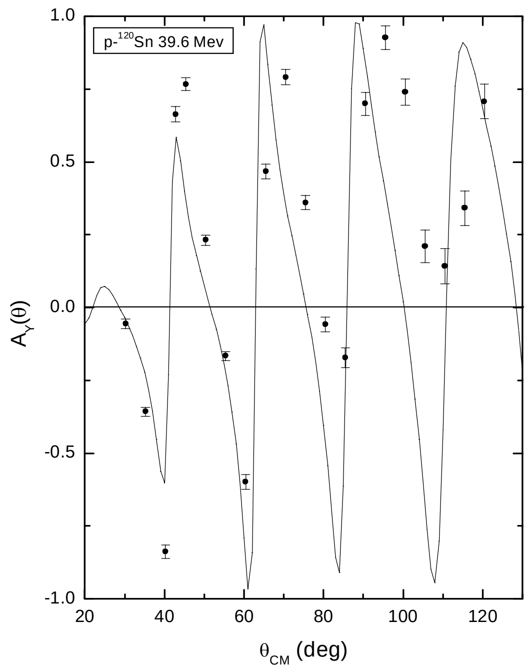
Differential cross-section: p-Sn Isotopes at 40 MeV.

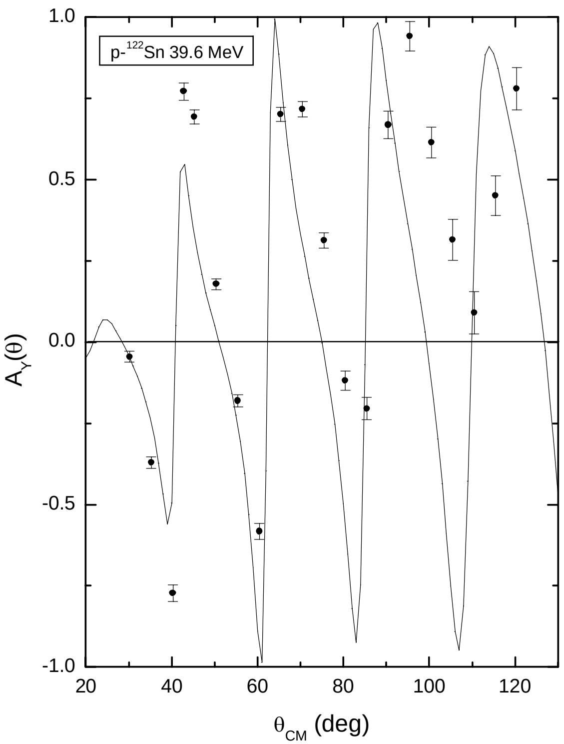
Predictions

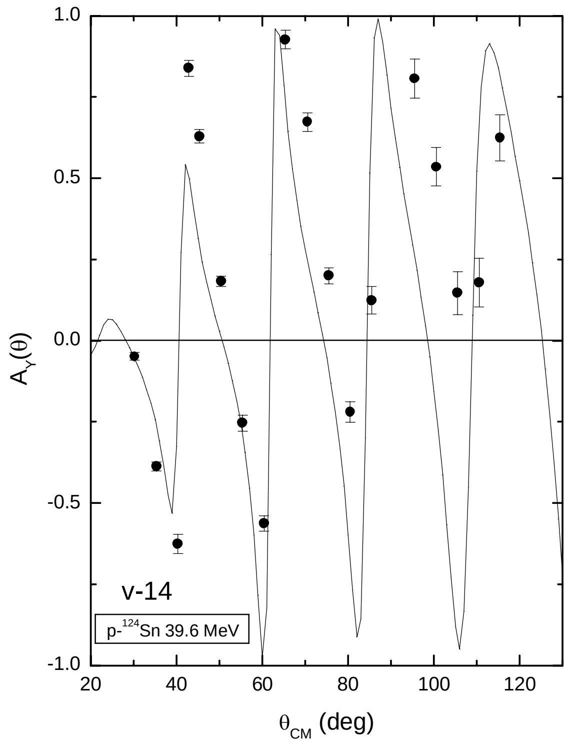
IIT, BARC, AMU collaboration





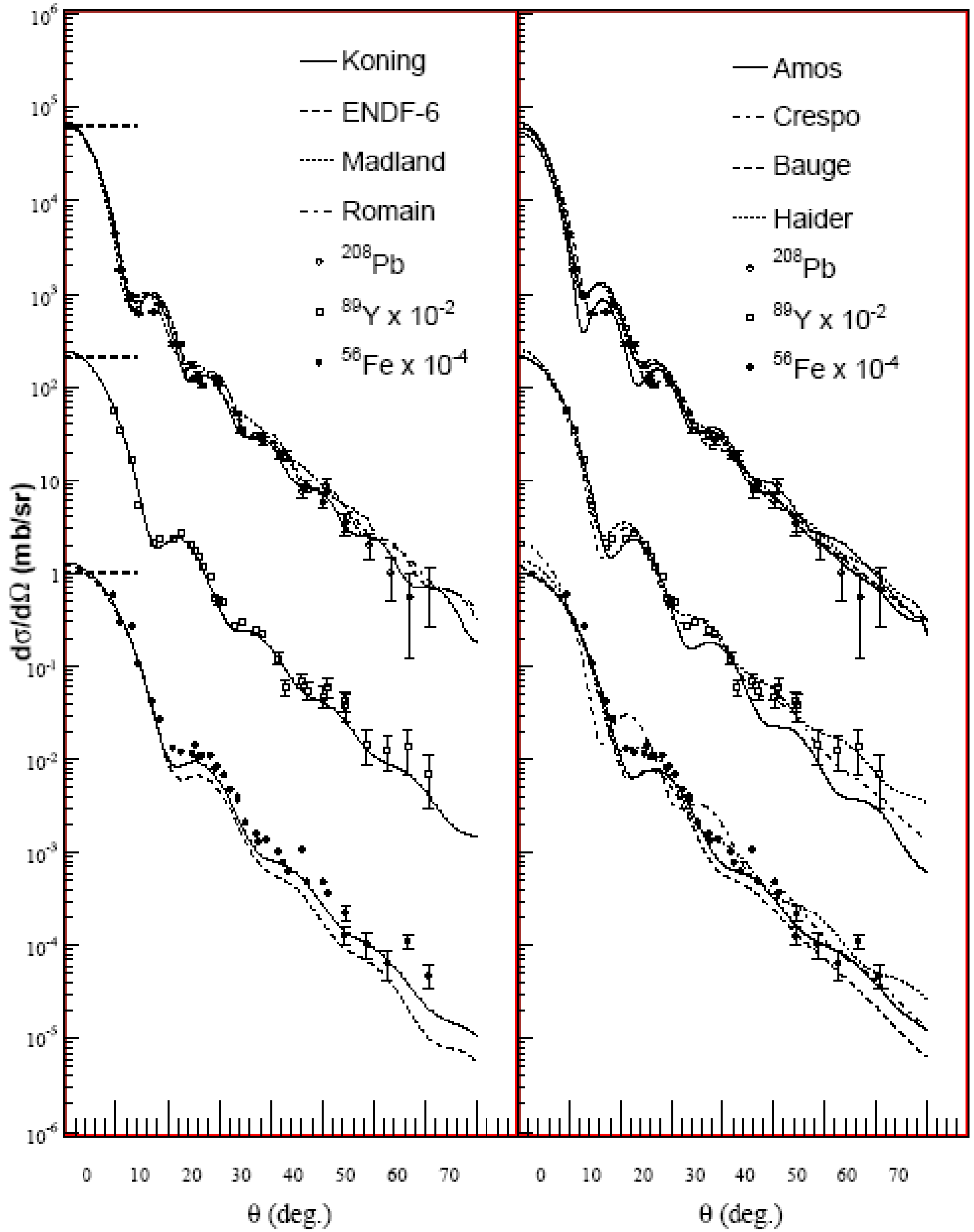






Polarisation Predictions for p-Sn Isotopes at 40 MeV

IIT, BARC, AMU Collaboration



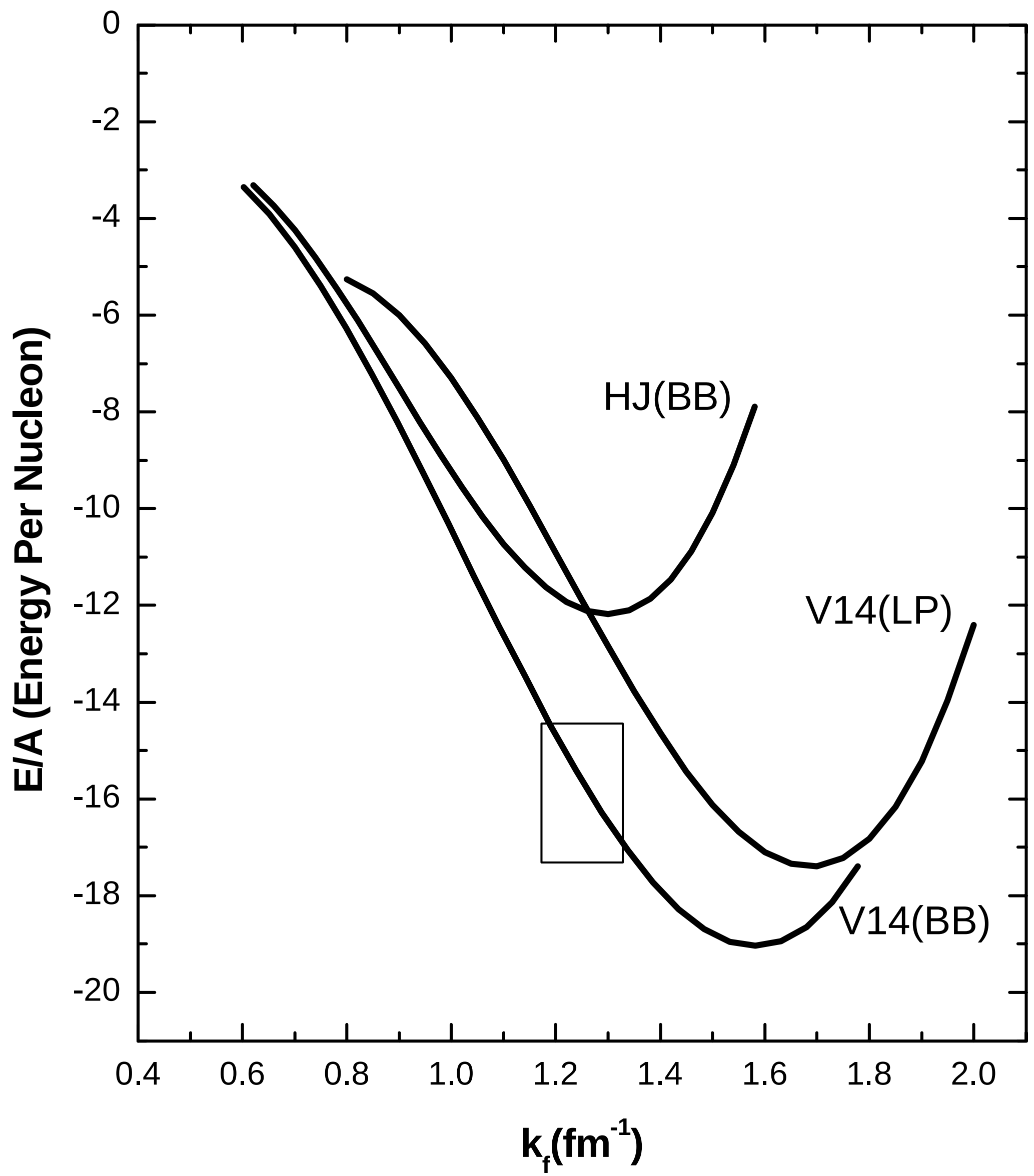
Neutron Elastic scattering from Fe^{56}, Y^{89} and Pb^{208} at 96 MeV

Uppsala, AMU Collaboration

PRC77, 024605 (2008)

Conclusions:

1. Neutron and proton scattering is nicely reproduces.
2. Calculated imaginary part is large.
3. Spin-orbit is not well determined.
4. Binding energy of Nuclear matter is not reproduced.



Remedial steps:

1. Spin-orbit part.
2. Three-body effects

4. Nucleon Spin – Orbit Potential:

The earliest formula is by Blin-Stoyle: $V_{s.o.}(r) = Const.\frac{1}{r}\frac{d\rho}{dr}$ where the constant is the first term of a series, Given by Greenlees et al. We show that the series is not rapidly convergent and we are able to calculate the Direct part exactly.

$$V^{D}_{s.}(r) = \sum_{N} \int \phi^{+}_{N}(\vec{r}_2) g^{D}_{s.o.} \vec{L}.\vec{S} \phi_N(\vec{r}_2) d\vec{r}_2$$

Using $\vec{L}.\vec{S} = \frac{1}{2}(\vec{r}_1 - \vec{r}_2) X (\vec{p}_1 - \vec{p}_2).(\vec{s}_1 + \vec{s}_2), \vec{x} = (\vec{r}_2 - \vec{r}_1)$

We get $$V^{D}_{s.o.}(r) = -\frac{1}{2}\int \rho(|\vec{r}_1 + \vec{x}|) g^{D}_{so} \vec{x} X \vec{p}_1 . \vec{s}_1 d\vec{x}$$

$$= -\frac{1}{2} A(r_1) \vec{l}_1 . \vec{s}_1 / r_1$$

where $$A(r_1)\vec{r}_1 / r_1 = \int \rho(|\vec{r}_1 + \vec{x}|) g^D_{s.o.} \vec{x} d\vec{x}$$

Greenlees makes a Taylor series expansion of $A(r_1)$:

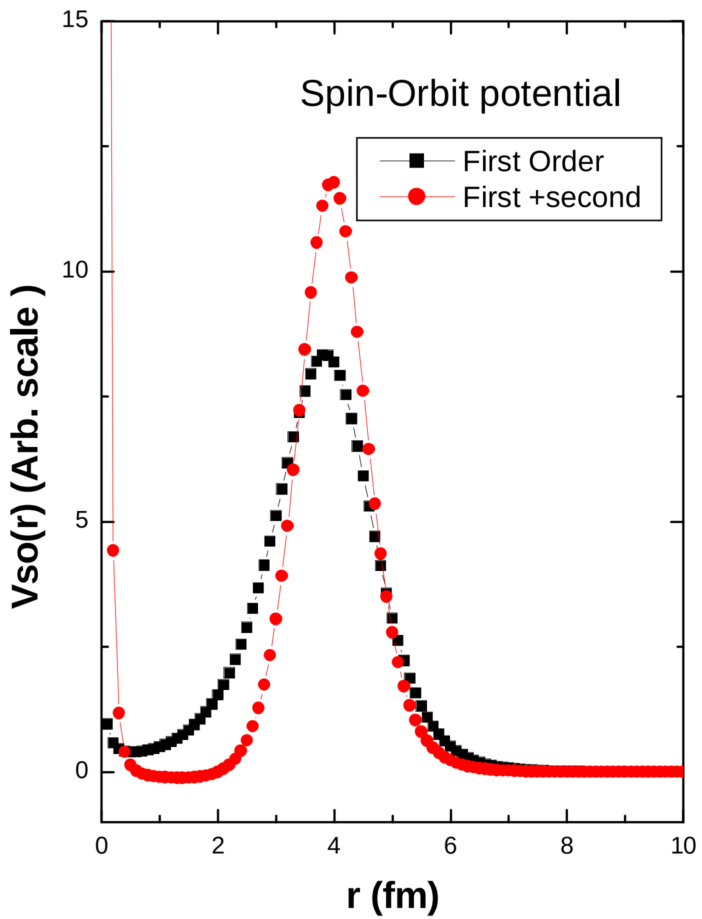
$$A(r) = 4\pi \sum_{v=0}^{\infty} \frac{(2v+2)}{(2v+3)!} I_{2v+4} \frac{d\nabla^{2v}}{dr} \rho(r)$$

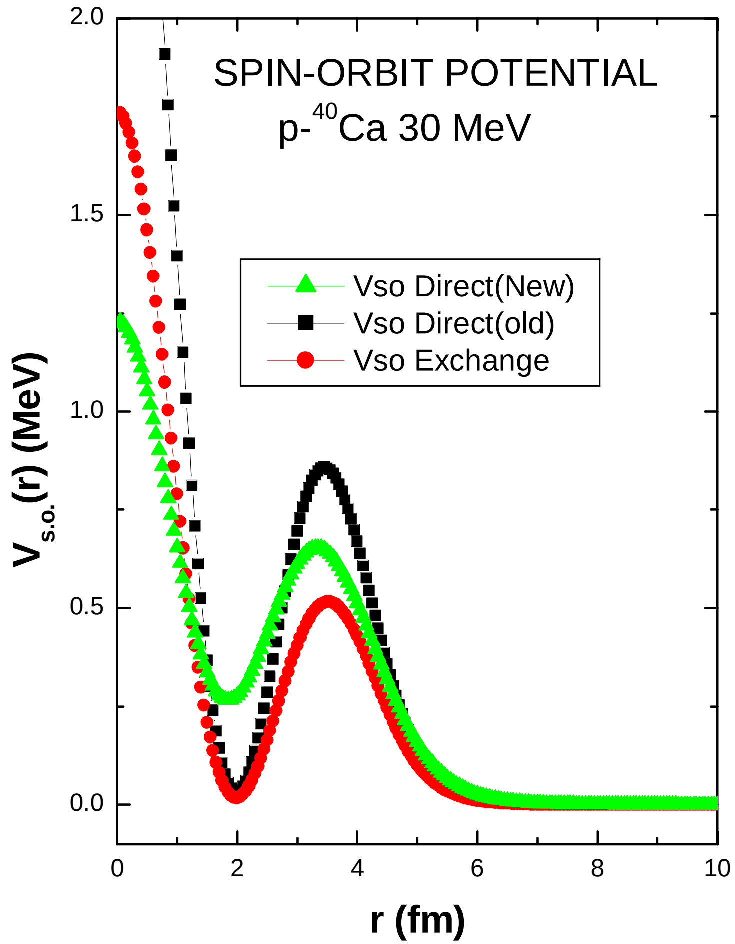
where

$$I_N = \int x^N g(x) dx, \qquad \nabla^{2v}\rho = \frac{d^{2v}\rho}{dr^{2v}} + \frac{2v}{r}\frac{d^{2v-1}\rho}{dr^{2v-1}}$$

The First Term of the series is $$A(r) = \frac{4\pi}{3} I_4 \frac{d\rho}{dr}$$

We have done model calculation of the first Two terms of the above series and find that the second term is quite large. Thus the first term alone is not enough. We calculate the whole series without making any approximation about the short range nature of the effective interaction





A slight rearrangement of the expansion gives the results of Scheerbaum

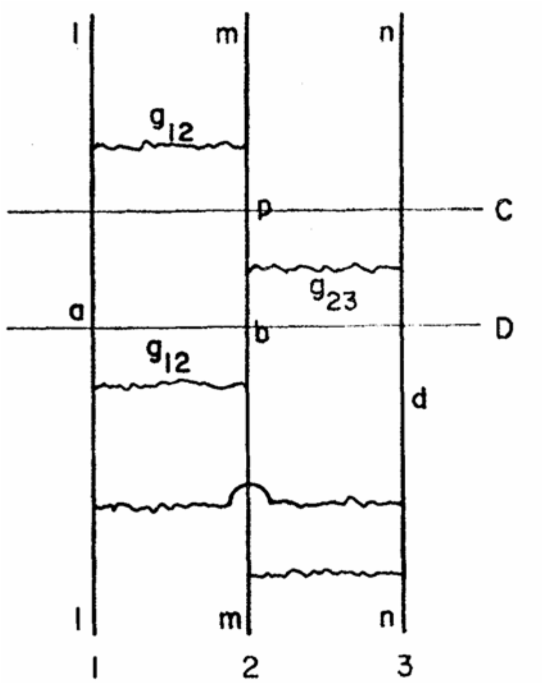
$$A(r) = 4\pi\{\int \frac{1}{k} j_1(kx)g(x)x^2dx\}\frac{d\rho}{dr}$$

Thus we are able to calculate the Direct part of the microscopic spin-orbit part exactly.

5. Three-body terms:

Considerable efforts to Cal. The effect of Higher order terms in the Binding energy of Nuclear Matter: Bethe, Rajaraman, Day: (Three-body give: -5.0 MeV)

Only two efforts made for the Optical potential: Kidwai, WH.



Three hole-line Diagrams:

Faddeyev:

$T=T^{(1)}+T^{(2)}+T^{(3)},$

$T^{(3)}= g_{12} - g_{12}(Q/e)[T^{(1)} + T^{(2)}]$

We introduce a three-body wave function in Coordinate space: $T^{(3)}\Phi = g_{12}\Psi^{(3)}$

$$Z^{(3)} = (Q/e)g_{23}\Phi + (Q/e)g_{13}\Phi - (Q/e)g_{23}Z^{(1)} - (Q/e)g_{13}Z^{(2)}$$

Where $Z^{(i)} = \Phi - \Psi^{(i)}$; Main task is to Cal. Fns $Z^{(i)}$

Three-Body Functions: $(Q/e)\,g_{ij}$

Two types: (1) The 3rd Nucleon is in ground state, and

(2) The Third nucleon is also in the Excited State.

Accordingly we differentiate.

$$(Q/e)\,g_{ij}\,e^{\left(i\left[k.r_{ij}+2P.R_{ij}\right]\right)} = \eta_{k,P}(r_{ij})\,e^{\left(2iP.R_{ij}\right)}$$

and

$$(Q/e)\,g_{ij}\,e^{\left(i\left[k.r_{ij}+2P.R_{ij}\right]\right)} = \varsigma_{k,P}(r_{ij})\,e^{\left(2iP.R_{ij}\right)}$$

NPA 504, 323 (1989)

Two Approx. Methods:
Bethe, Day.

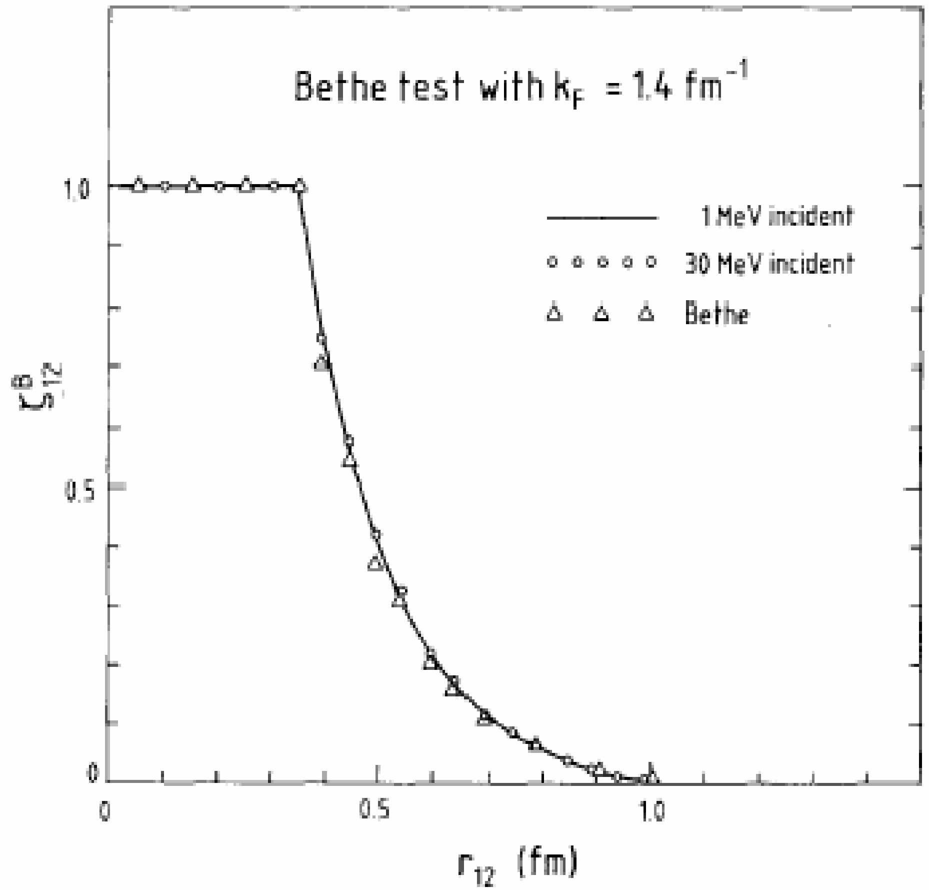
$$\varsigma_{ij}^{B}(r_{ij}) = \frac{\varsigma_{k,P}(r_{ij})}{j_0(kr_{ij})} \qquad \varsigma_{ij}^{D}(r_{ij}) = \frac{\varsigma_{k,P}(r_{ij})}{j_0(kr_c)}$$

The assumption is that the defect functions are independent of k,P.

Singlet s-sate.

We see that Bethe's approximation is justified.

Day's approximation gives similar results.



$$U_3 = 8\pi^2\rho^2\int \eta_{12} g_{23} Z^1(r_{12}, r_{23}, r_{13})\, dr_{12}\, dr_{23}\, dr_{13}$$

$$+8\pi^2\rho^2\int \eta_{12} g_{13} Z^2(r_{12}, r_{23}, r_{13})\, dr_{12}\, dr_{23}\, dr_{13}$$

Results:

KF**= 1.4fm^{-1}** (Nuclear Interior)

En (MeV	**Re U2**	**Im U2**	**Re U3**	**Im U3**	**Re U3ReU2**	**ImU3/ImU2**
30.0	-54.243	-1.566	-14.348	+0.793	0.132	-0.253
80.0	-58.076	-10.817	-11.535	+1.703	0.099	-0.079

KF = 0.90 fm-1 (Nuclear Surface)

30.0	-14.184	-0.661	-1.321	+0.164	0.047	-0.124
80.0	-14.015	-2.662	-1.001	+0.429	0.036	-0.081

The results using Day' approximation is very similar, and hence we do not quote them here.

- Conclusions:

1. Satisfactory agreement with Nucleon scattering data.
2. The exchange parts of the nucleon optical potential should be treated more carefully.
3. Calculation of three-body effects should be improved.
4. The calculated potentials depend sensitively on the point nucleon densities used. Hence the approach can be used to study neutron skin in nuclei.

Thank you.