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# Nucleon elastic and N-to-Δ transition form factors

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# Nucleon elastic and N-to-Δ transition form factors


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Trieste, May 12-16, 2008

# Outline

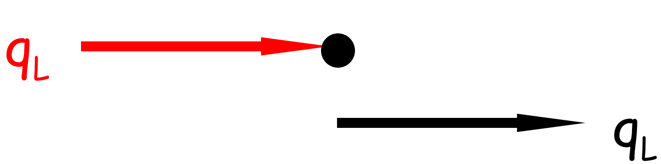
- introduction : nucleon densities and relativity
- transverse densities from nucleon elastic FFs and $N \rightarrow \Delta$ transition FFs
- transverse distance dependence and nucleon GPDs
- large $N_c$ relations
  between N elastic and $N \rightarrow \Delta$ FFs

work on densities in coll. with C.E. Carlson : PRL 100, 032004 (2008)

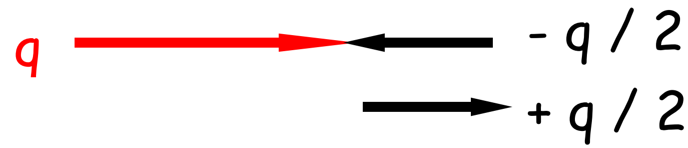
work on large $N_c$ in coll. with V. Pascalutsa : PRD 76, 111501 (R) (2007)

# Nucleon densities and relativity

**rest frame**




**Breit frame**



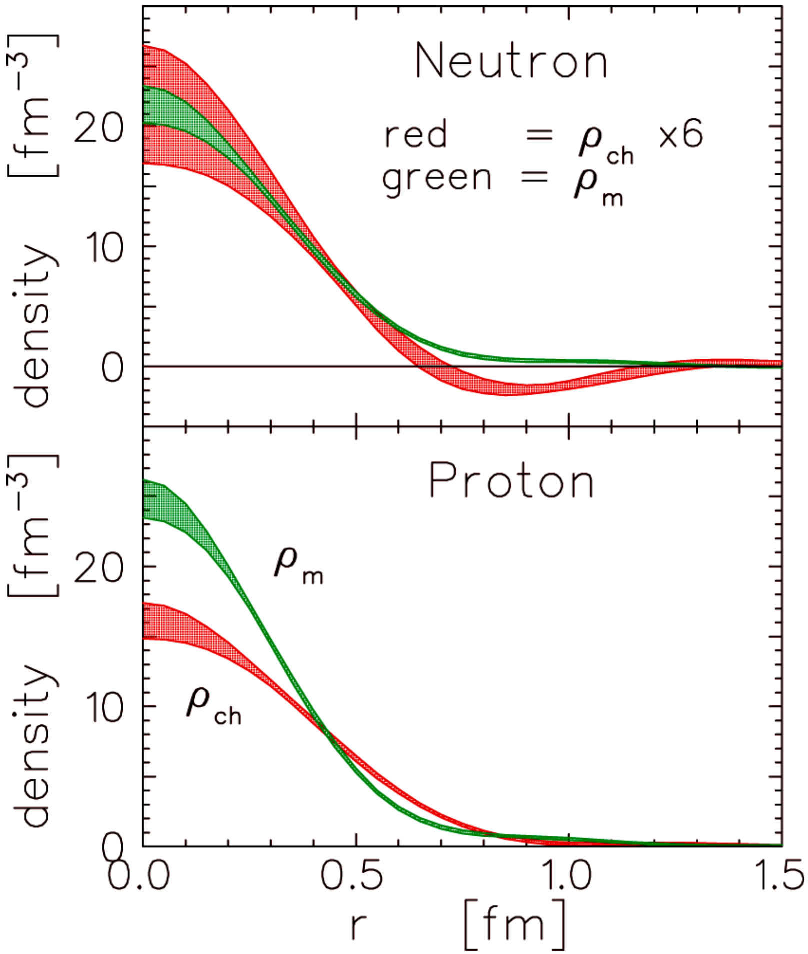

$$\vec{q}^{\,2} = Q^2$$

➡ relative velocity between frames :

$$\tau \equiv \frac{Q^2}{4M^2}$$

➡ Lorentz contraction factor : $\gamma = \sqrt{1+\tau}$

# Nucleon densities and relativity




Kelly (2000)

$$\rho(r) = \frac{2}{\pi}\int_0^\infty dk\, k^2\, j_0(k\,r)\, \tilde{\rho}(k)$$

rest frame density ⬆ intrinsic FF ⬆

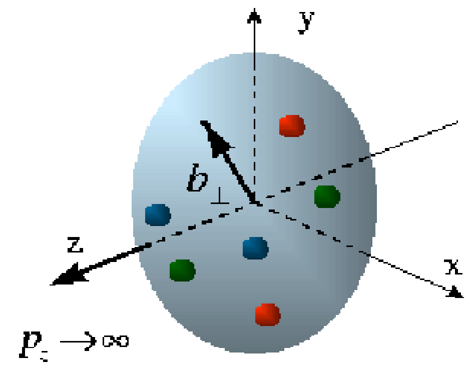
- non-rel : $\tilde{\rho}(k) = G(Q^2)$
- importance of relativity (with increasing Q^2) : Lorentz contraction of spatial distributions in Breit frame

$$k^2 = Q^2/(1+\tau) \qquad \tau = Q^2/(4M^2)$$

$$\tilde{\rho}_{E,M}(k) = G_{E,M}(Q^2)(1+\tau)^2$$

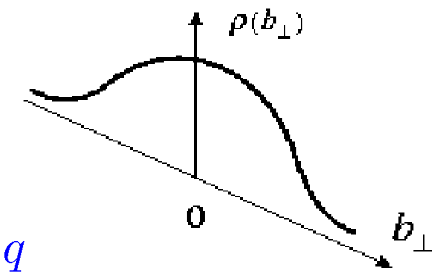
- limit : k = 2 M (Compton wavelength)

# quark transverse charge densities in nucleon (I)




- $q^+ = q^0 + q^3 = 0$ photon only couples to forward moving quarks

  $Q^2 \equiv \vec{q}_\perp^{\,2}$




- quark charge density operator

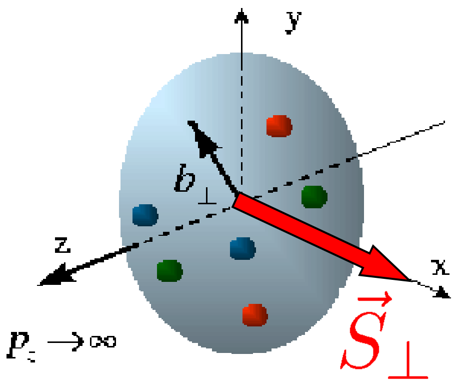
  $J^+ \equiv J^0 + J^3 = \bar{q}\gamma^+ q = 2\, q_+^\dagger\, q_+, \quad \text{with} \quad q_+ \equiv \frac{1}{4}\gamma^-\gamma^+ q$

☆ unpolarized nucleon

$$\rho_0^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda\,|\, J^+(0)\,|\, P^+, -\frac{\vec{q}_\perp}{2}, \lambda \rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q) F_1(Q^2)$$

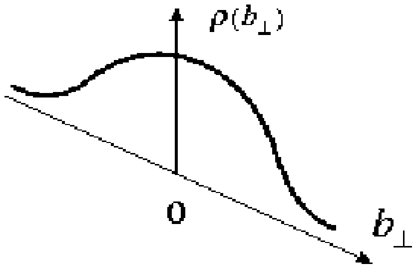
# quark transverse charge densities in nucleon (II)




☆ transversely polarized nucleon

transverse spin $\vec{S}_\perp = \cos\phi_S\,\hat{e}_x + \sin\phi_S\,\hat{e}_y$

e.g. along x-axis : $\phi_S = 0$



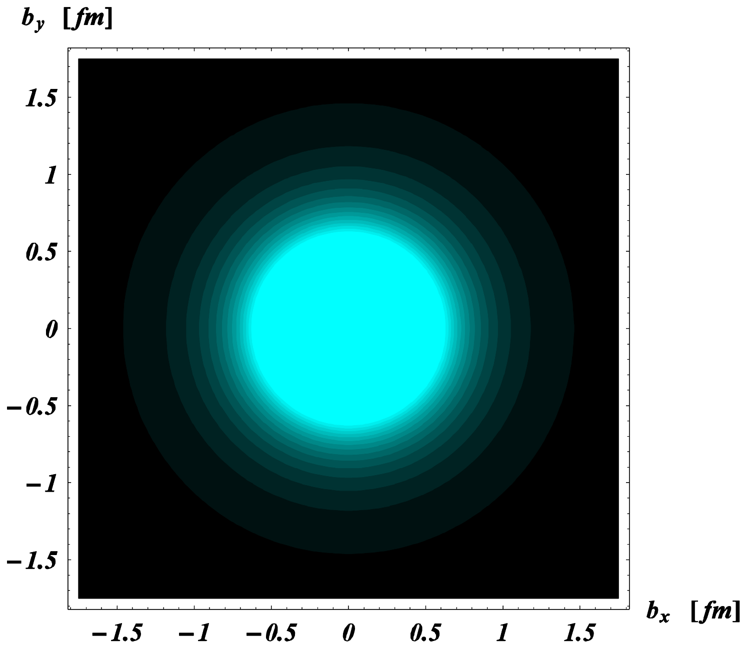

$$\vec{b} = b\,(\cos\phi_b\,\hat{e}_x + \sin\phi_b\,\hat{e}_y)$$

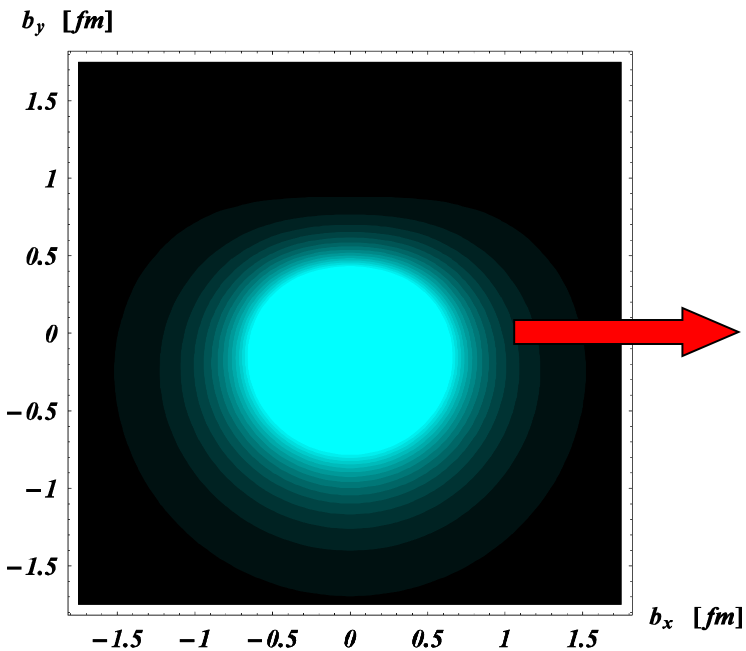
$$\rho_T^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2}\,|\,J^+(0)\,|\,P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2}\rangle$$

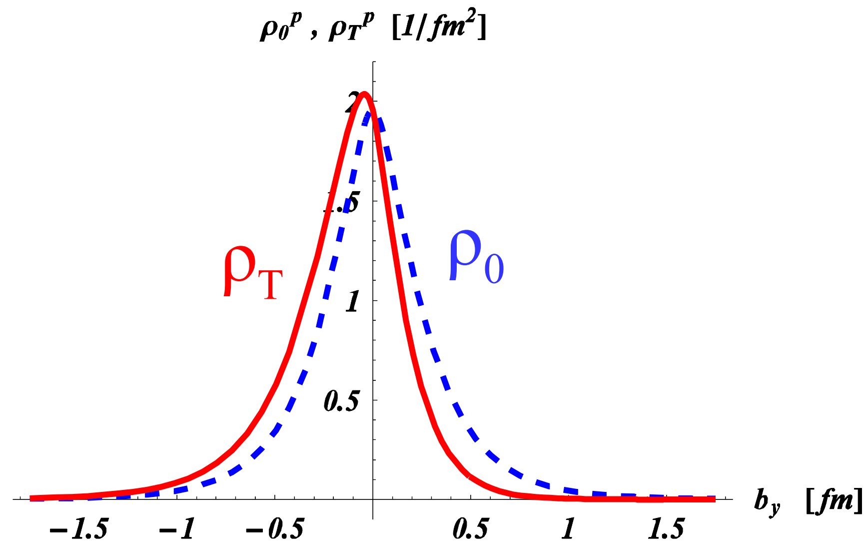
$$= \rho_0^N(b) - \sin(\phi_b - \phi_S)\int_0^\infty \frac{dQ}{2\pi}\,\frac{Q^2}{2M_N}\,J_1(b\,Q)F_2(Q^2)$$

dipole field pattern

# empirical quark transverse densities in proton



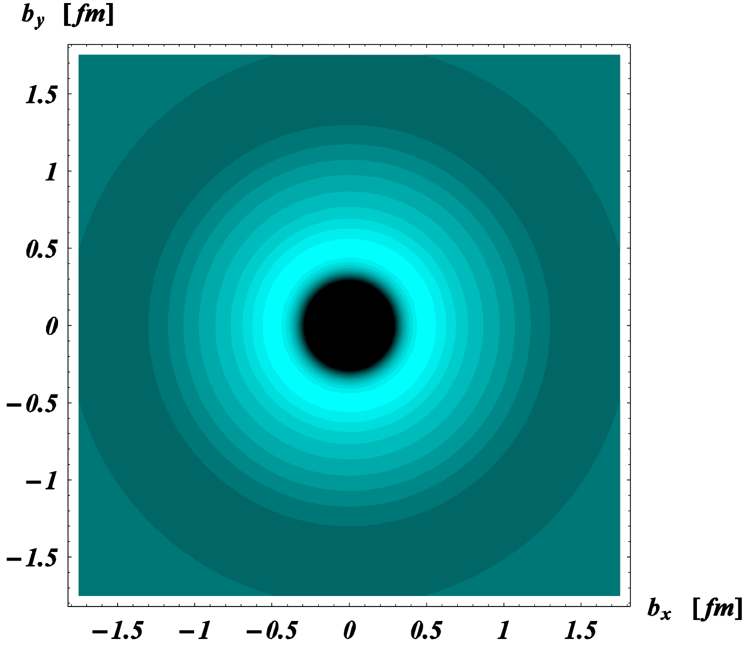



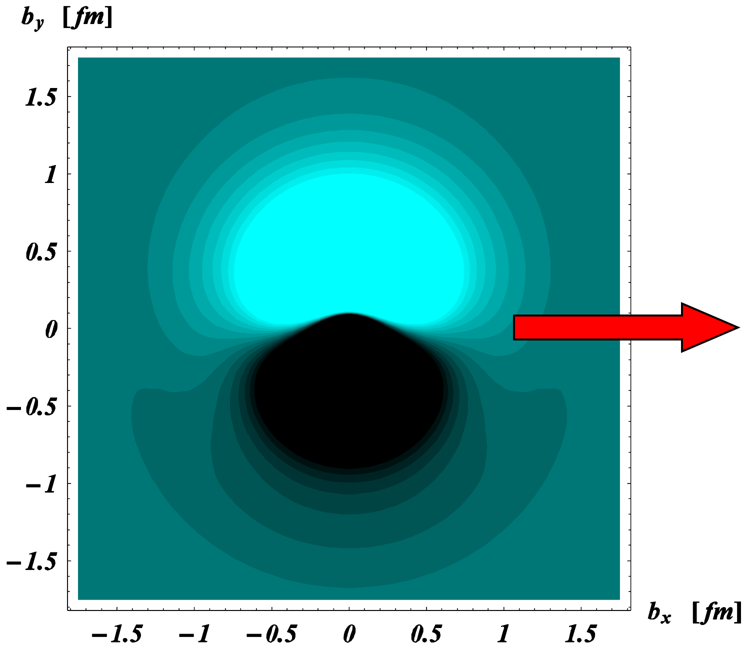

induced EDM : $d_y = - F_{2p}(0) . e / (2 M_N)$

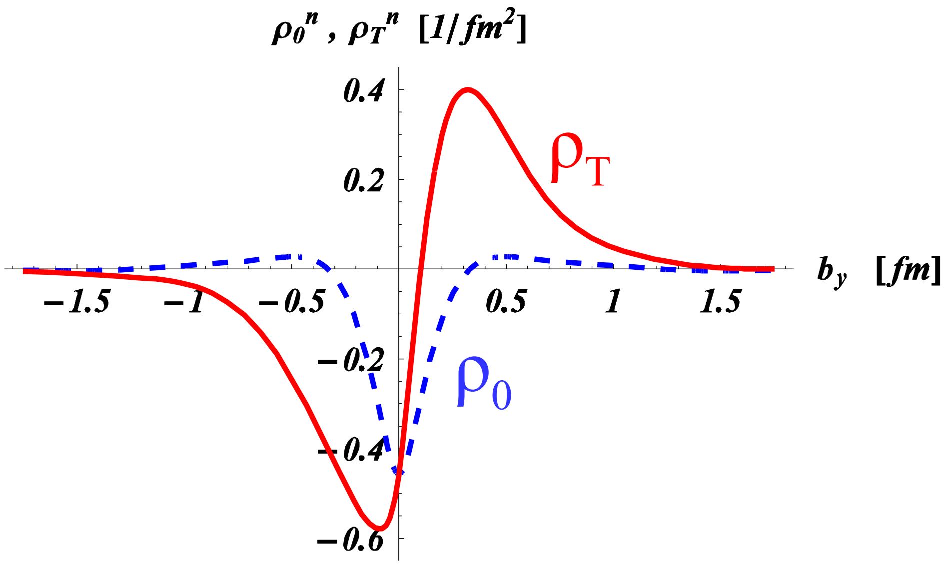
data : Arrington, Melnitchouk, Tjon (2007)

densities : Miller (2007); Carlson, Vdh (2007)

# empirical quark transverse densities in neutron



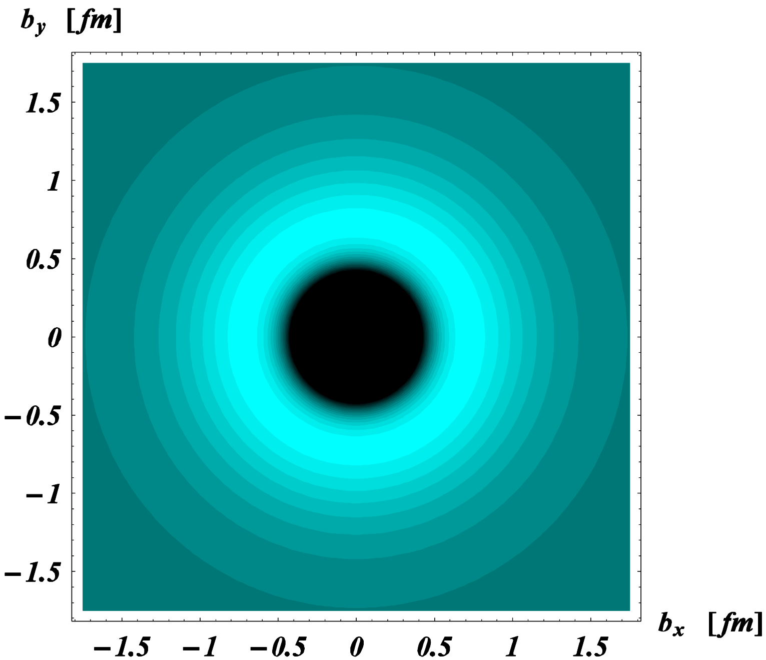




induced EDM : $d_y$ = - $F_{2n}$ (0) . e / (2 $M_N$)

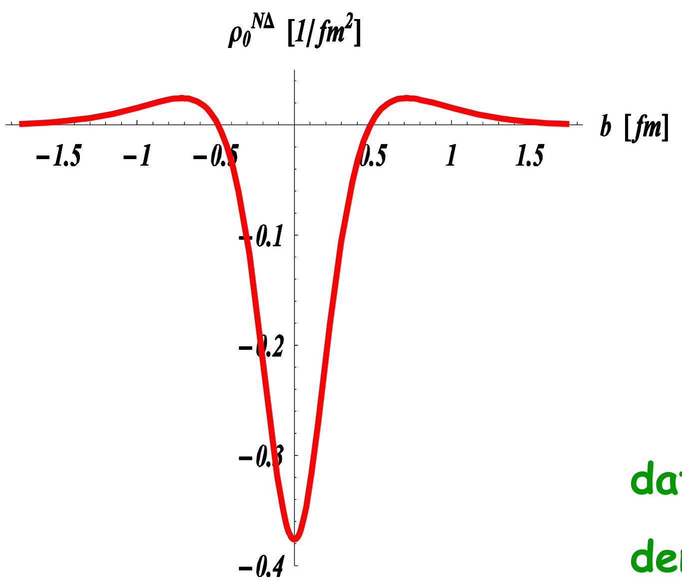
data : Bradford, Bodek, Budd, Arrington (2006)

densities : Miller (2007); Carlson, Vdh (2007)




# empirical transverse transition densities for N -> Δ excitation

$$\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda_\Delta | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda_N \rangle$$
$$= (2P^+) e^{i(\lambda_N - \lambda_\Delta)\phi_q} \, G^+_{\lambda_\Delta \, \lambda_N}(Q^2)$$




$$\rho_0^{N\Delta}(b) = \int_0^\infty \frac{dQ}{2\pi} Q \, J_0(b\,Q) \, G^+_{+\frac{1}{2}\,+\frac{1}{2}}(Q^2)$$

combination of **M1**, **E2**, **C2** FFs

data : MAID 2007 , Drechsel, Kamalov, Tiator (2007)

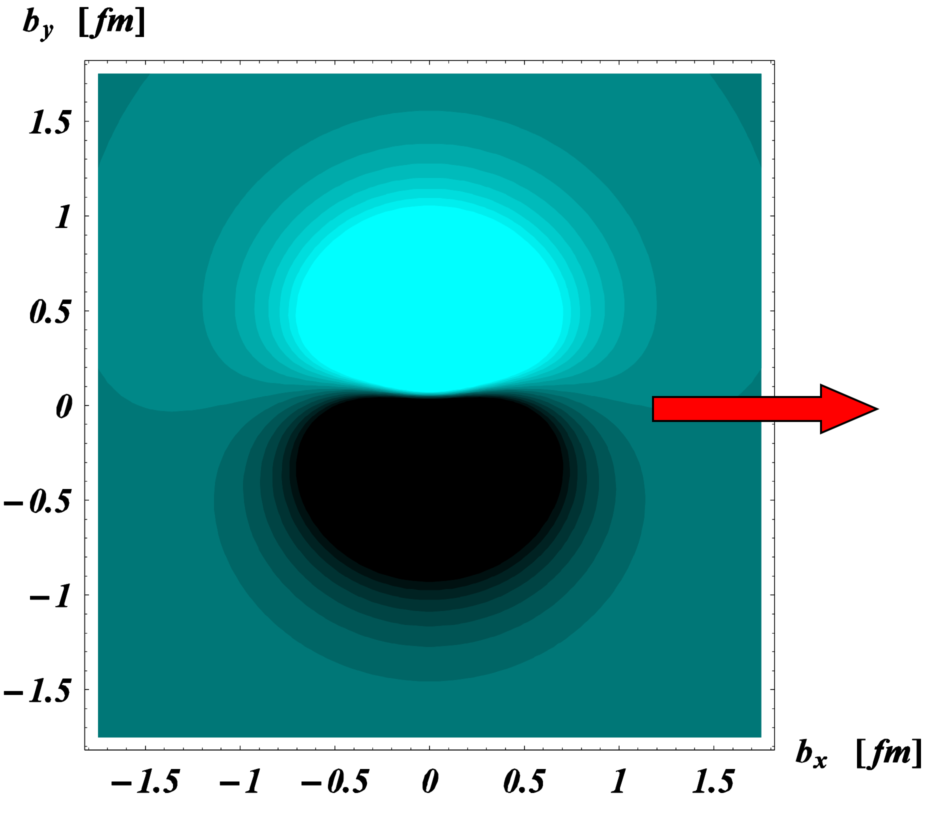
densities : Carlson, Vdh (2007)

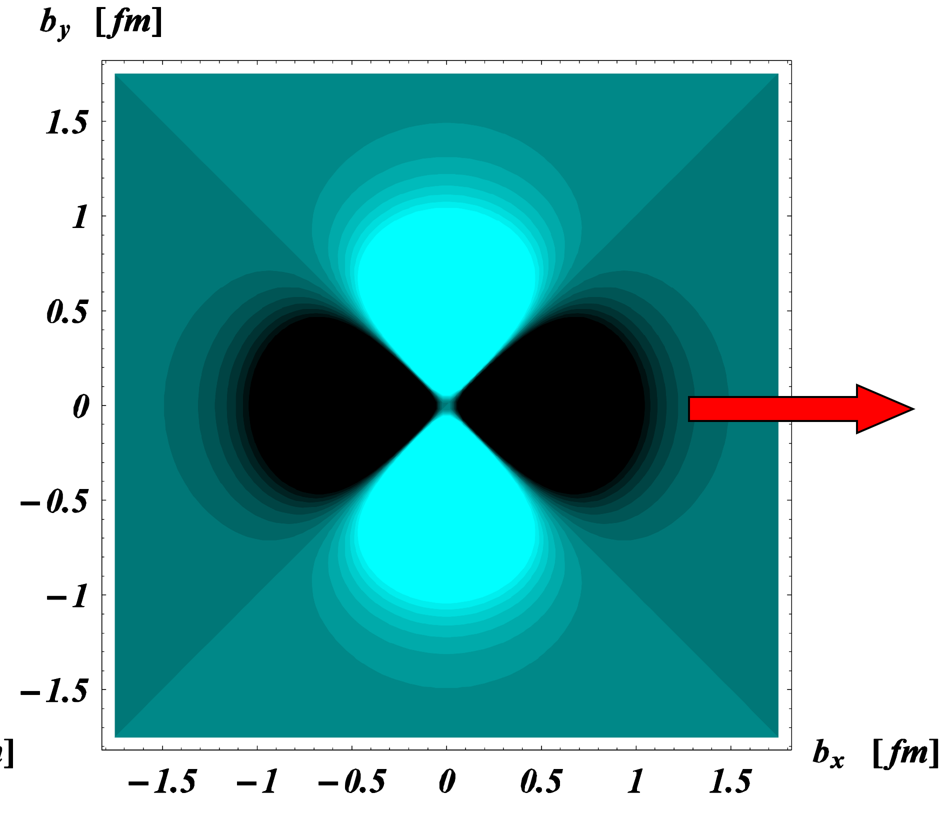
$$\rho_T^{N\Delta}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{i\,\vec{q}_\perp\cdot\vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2} \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2} \rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi} \frac{Q}{2} \left\{ J_0(b\,Q)\, G^+_{+\frac{1}{2}\,+\frac{1}{2}} \right.$$ → monopole

$$+ \sin(\phi_b - \phi_S)\, J_1(b\,Q) \left[ \sqrt{3} G^+_{+\frac{3}{2}\,+\frac{1}{2}} + G^+_{+\frac{1}{2}\,-\frac{1}{2}} \right]$$ → dipole

$$\left. - \cos 2(\phi_b - \phi_S)\, J_2(b\,Q) \sqrt{3}\, G^+_{+\frac{3}{2}\,-\frac{1}{2}} \right\}$$ → quadrupole

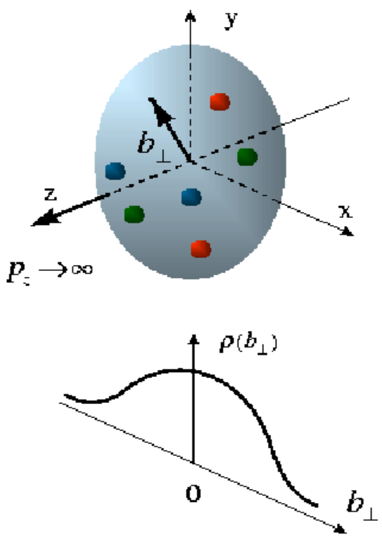


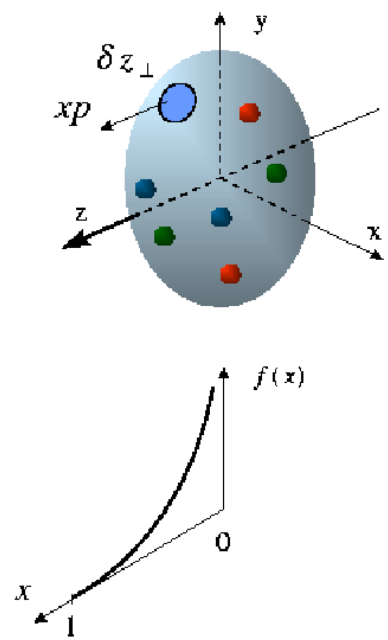



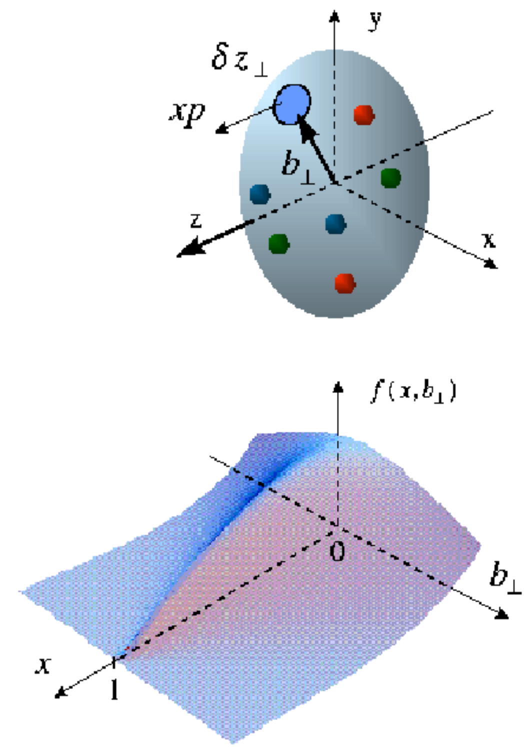
# GPDs yield 3-dim quark structure of nucleon

Burkardt (2000, 2003)

Belitsky, Ji, Yuan (2004)



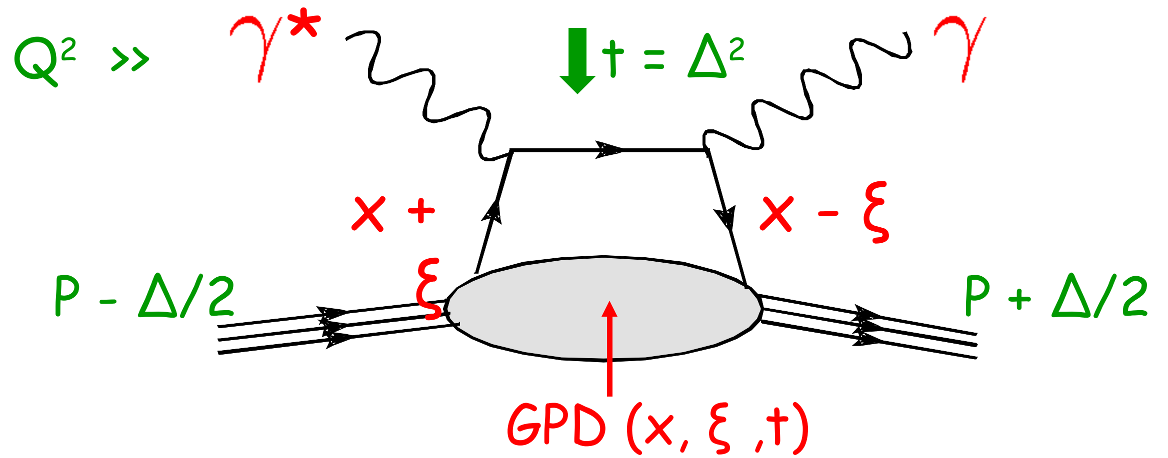




**Elastic Scattering**
transverse quark distribution in coordinate space

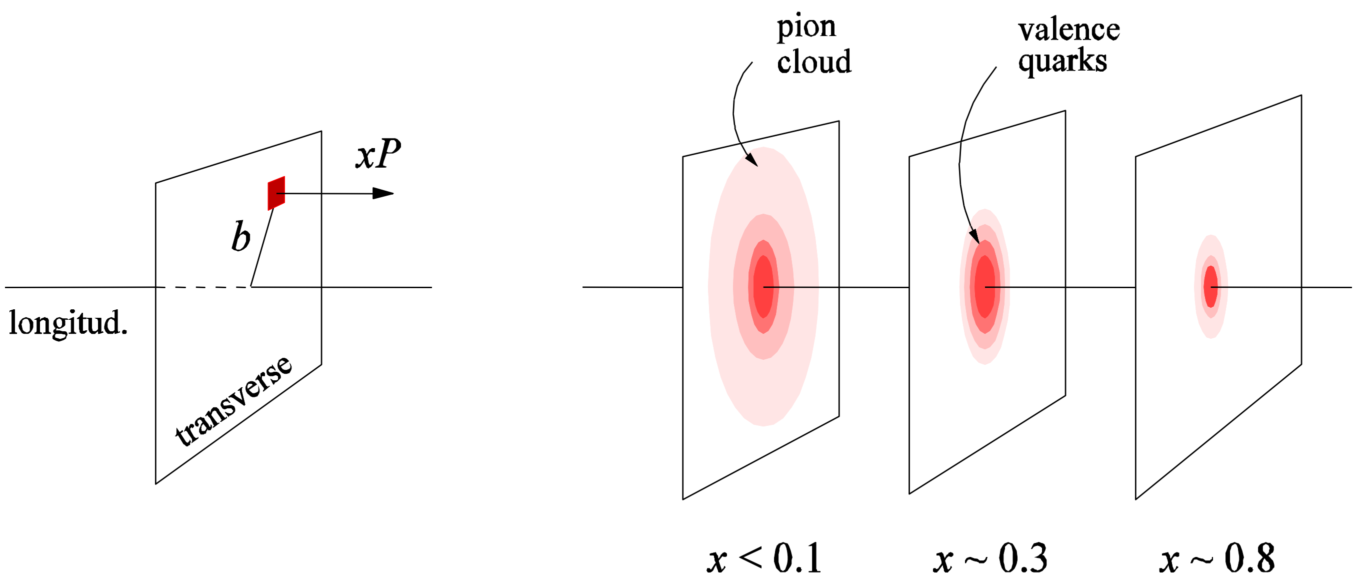
**DIS**
longitudinal quark distribution in momentum space

**DES (GPDs)**
fully-correlated quark distribution in both coordinate and momentum space

# GPDs :

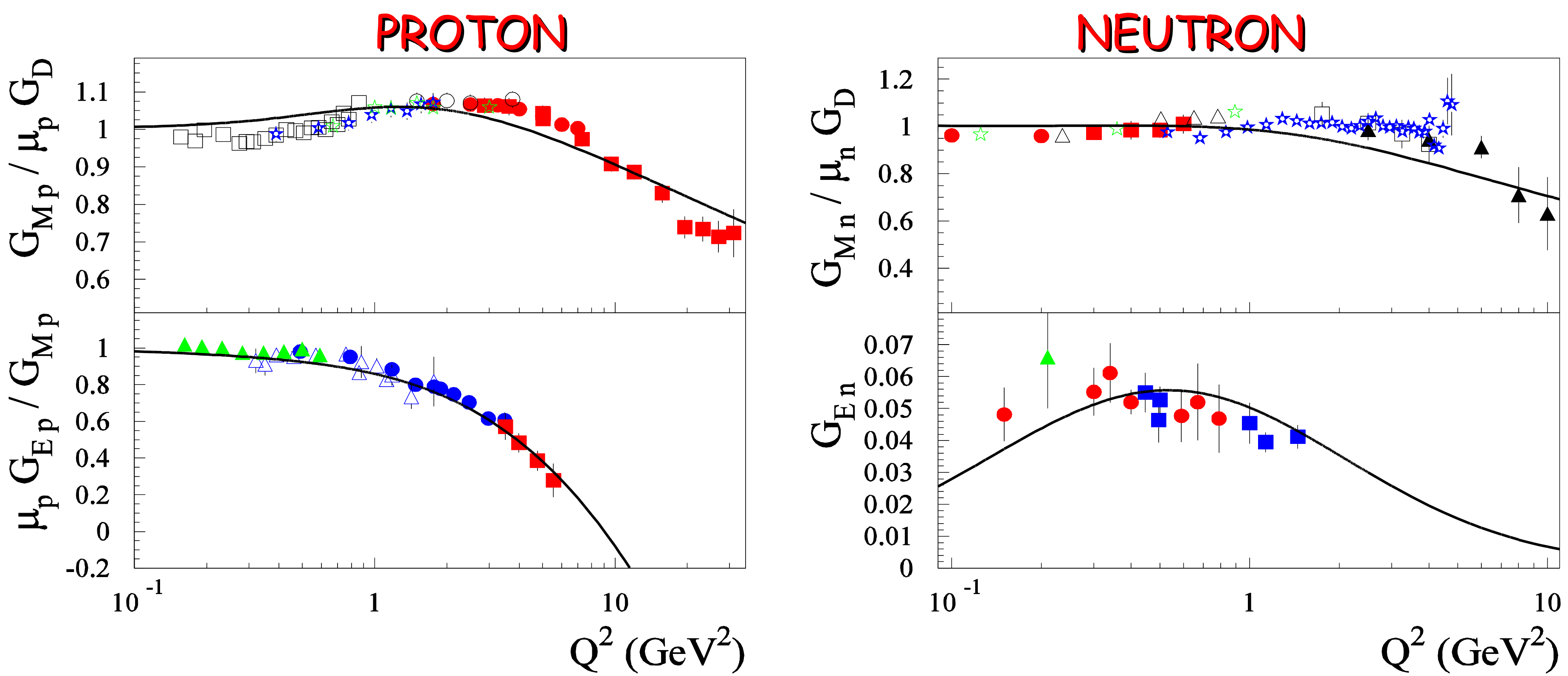



$\xi = 0$




Fourier transform of GPDs : simultaneous distributions of quarks w.r.t. longitudinal momentum x P and transverse position b

# electromagnetic form factors




→ modified Regge GPD parameterization

3-parameter fit
- 1 : Regge slope -> proton Dirac (Pauli) radius
- 2, 3 : large x behavior of GPD $E^u$, $E^d$ -> large $Q^2$ behavior of $F_{2p}$, $F_{2n}$

Guidal, Polyakov, Radyushkin, Vdh (2005)

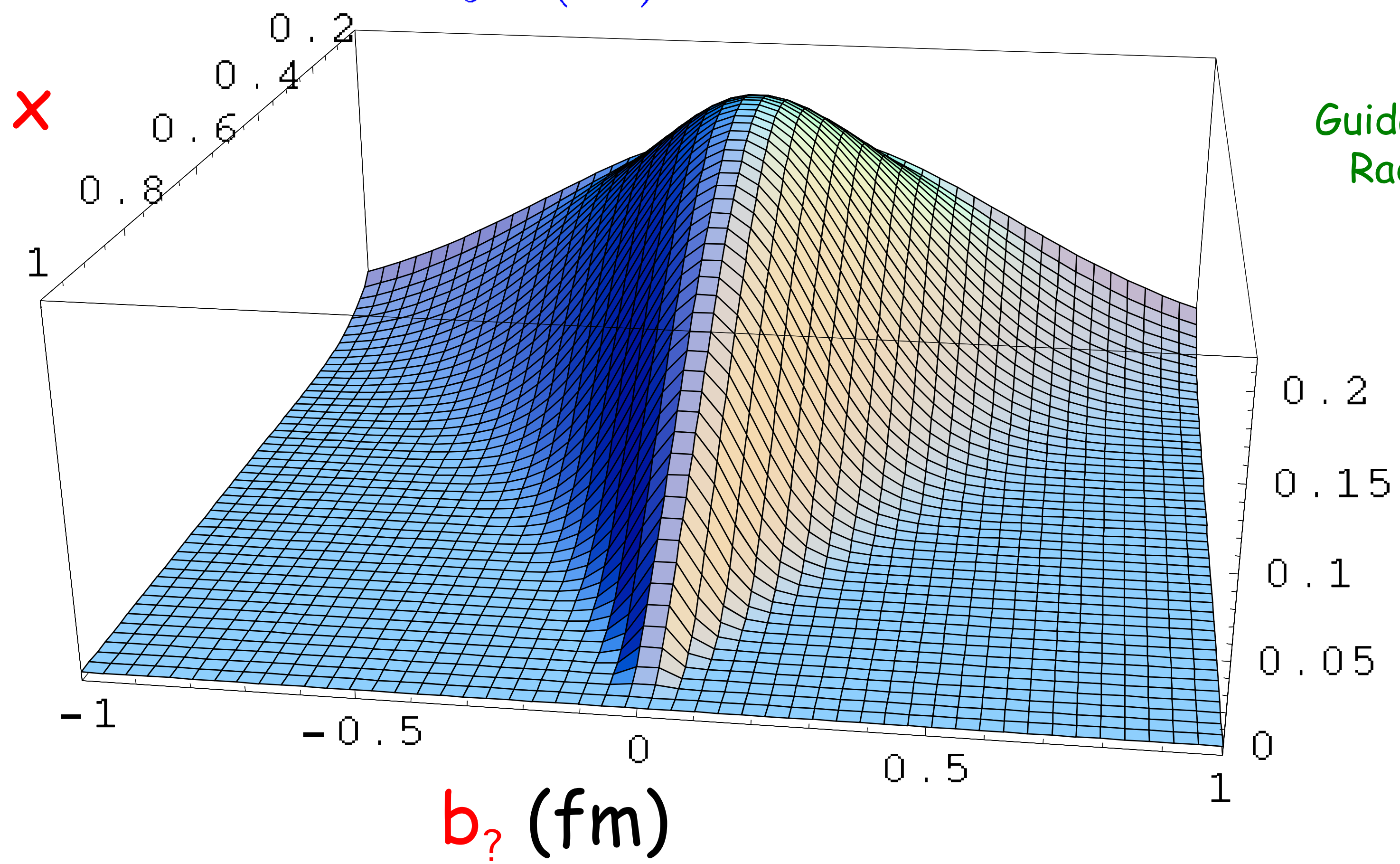
also Diehl, Feldmann, Jakob, Kroll (2005)

world data (2006)

# GPDs : transverse image of the nucleon (tomography)

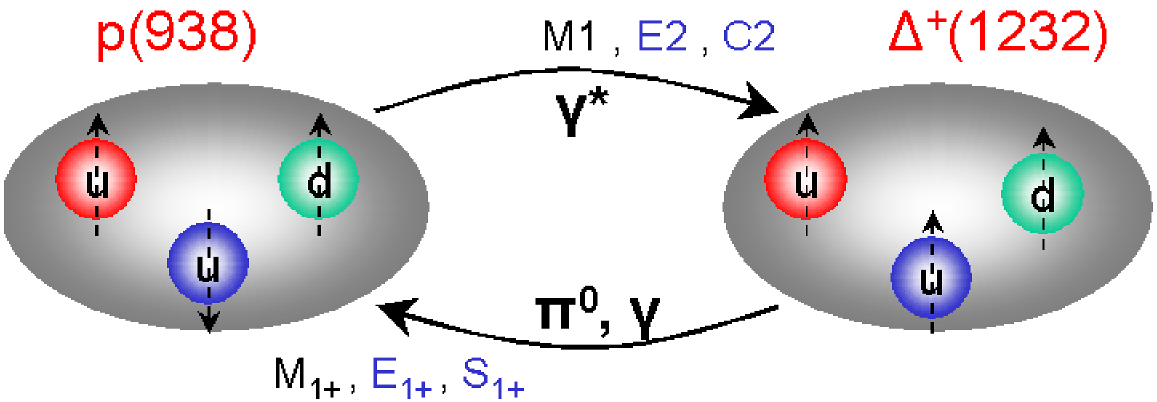
$H^u(x, b_?)$

$$H^q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} H^q(x, \xi = 0, -\mathbf{\Delta}_\perp{}^2)$$




Guidal, Polyakov, Radyushkin, Vdh (2005)

# electromagnetic N -> Δ(1232) transition




$J^P$=3/2+ (P33),

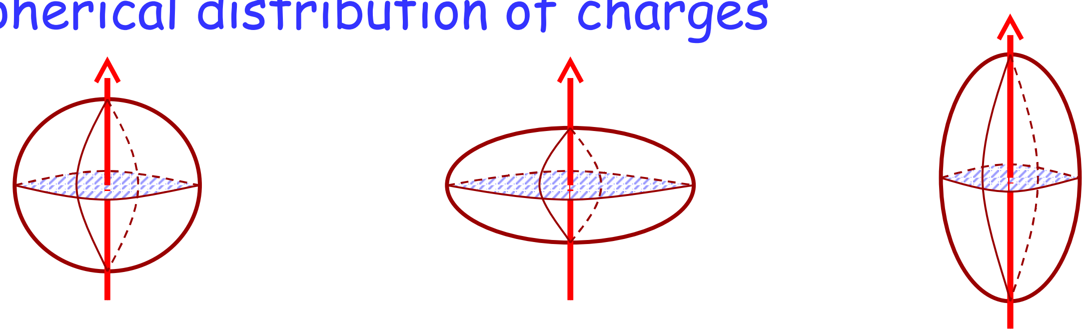
$M_\Delta$ ' 1232 MeV, $\Gamma_\Delta$ ' 115 MeV

N ! Δ transition:

π N ! Δ (99%), γ N ! Δ (<1%)

- non-zero values for E2 and C2 : measure of non-spherical distribution of charges

spin 3/2



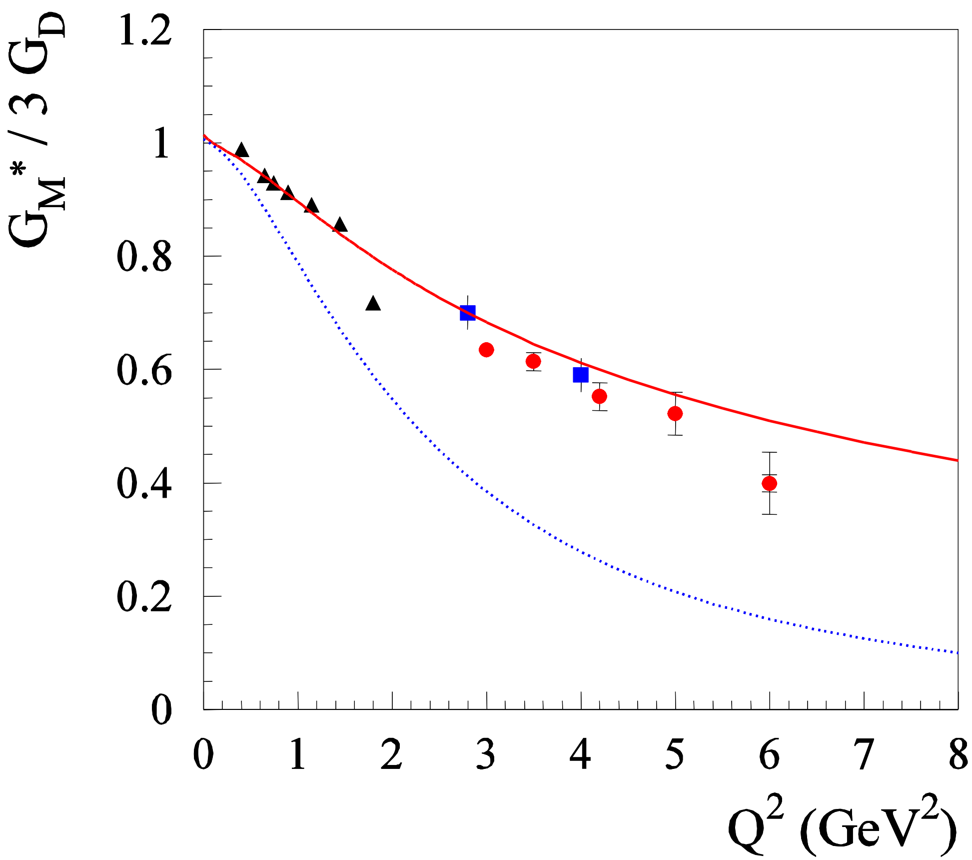
**Sphere: $Q_{20}$=0** **Oblate $Q_{20}/R^2 < 0$** **Prolate: $Q_{20}/R^2 > 0$**

- Role of quark core (quark spin flip) versus pion cloud

# N -> Δ magnetic dipole form factor

large $N_c$ limit

$$G^*_M(t) = \frac{G^*_M(0)}{\kappa_V} \int_{-1}^{+1} dx \left\{ E^u(x,\xi,t) - E^d(x,\xi,t) \right\} = \frac{G^*_M(0)}{\kappa_V} \left\{ F_2^p(t) - F_2^n(t) \right\}$$




large $N_c$ : $G^*_M(0) = \kappa_V / \sqrt{2} = 2.62$

EXP : $G^*_M(0) = 3.02$

modified Regge model

Regge model

Guidal, Polyakov, Radyushkin, Vdh (2005)

# N -> Δ E2 and C2 form factors

- large $N_c$ limit of QCD :

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}}\, r_n^2 \,\frac{N_c}{N_c+3}\sqrt{\frac{N_c+5}{N_c-1}}$$

Buchmann, Hester, Lebed (2002)

- $N_c = 3$

EXP : $r_n^2$ = - 0.113 (3) fm$^2$

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}}\, r_n^2$$

$$G_E^*(0) = -\frac{1}{6}\, r_n^2 \,\frac{1}{\sqrt{2}}\,\frac{(M_\Delta^2 - M_N^2)}{2}$$

large $N_c$ : $Q_{p\to\Delta^+}$ = - 0.080 fm$^2$

EXP : $Q_{p\to\Delta^+}$ = - 0.085 (3) fm$^2$

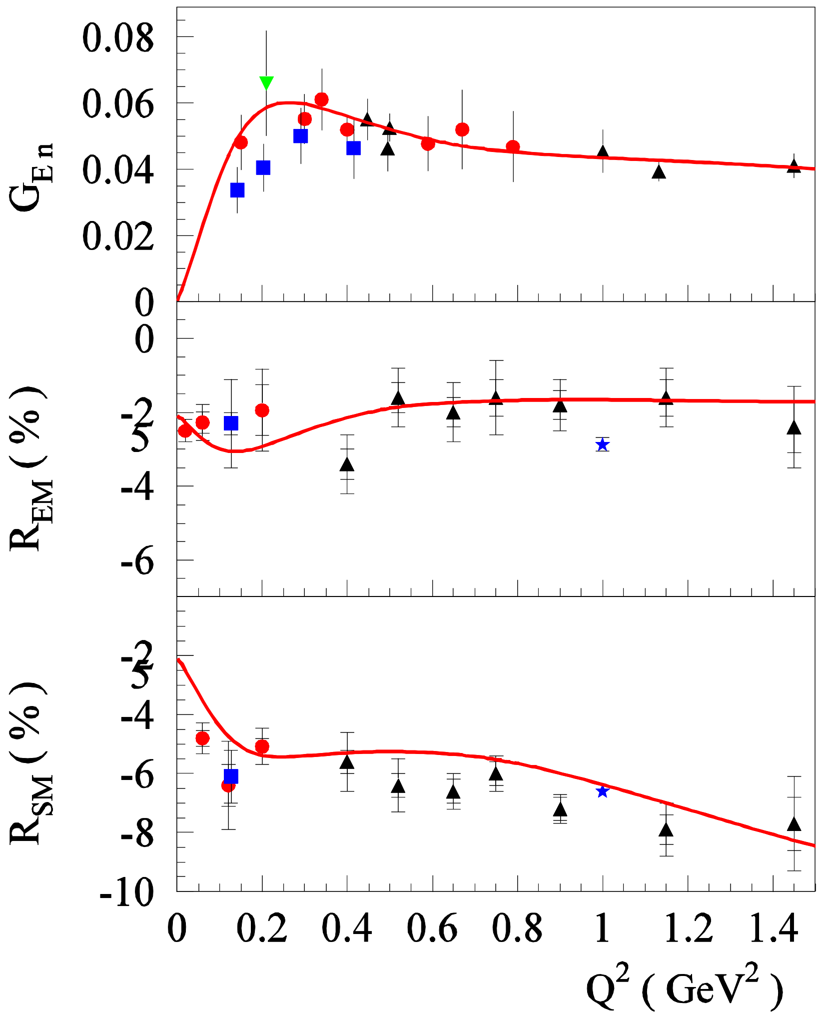
- finite (low) $Q^2$ : $G_E^n(Q^2) \approx -r_n^2\, Q^2/6$

$$G_E^*(Q^2) \simeq \frac{1}{\sqrt{2}}\,\frac{(M_\Delta^2 - M_N^2)}{2}\,\frac{G_E^n(Q^2)}{Q^2}$$

$$G_C^*(Q^2) \simeq \frac{4M_\Delta^2}{(M_\Delta^2 - M_N^2)}\, G_E^*(Q^2)$$

Pascalutsa, Vdh (2007)

# N -> Δ E2 and C2 form factors




$G_{En}$ fit : Bradford, Bodek, Budd, Arrington (2006)

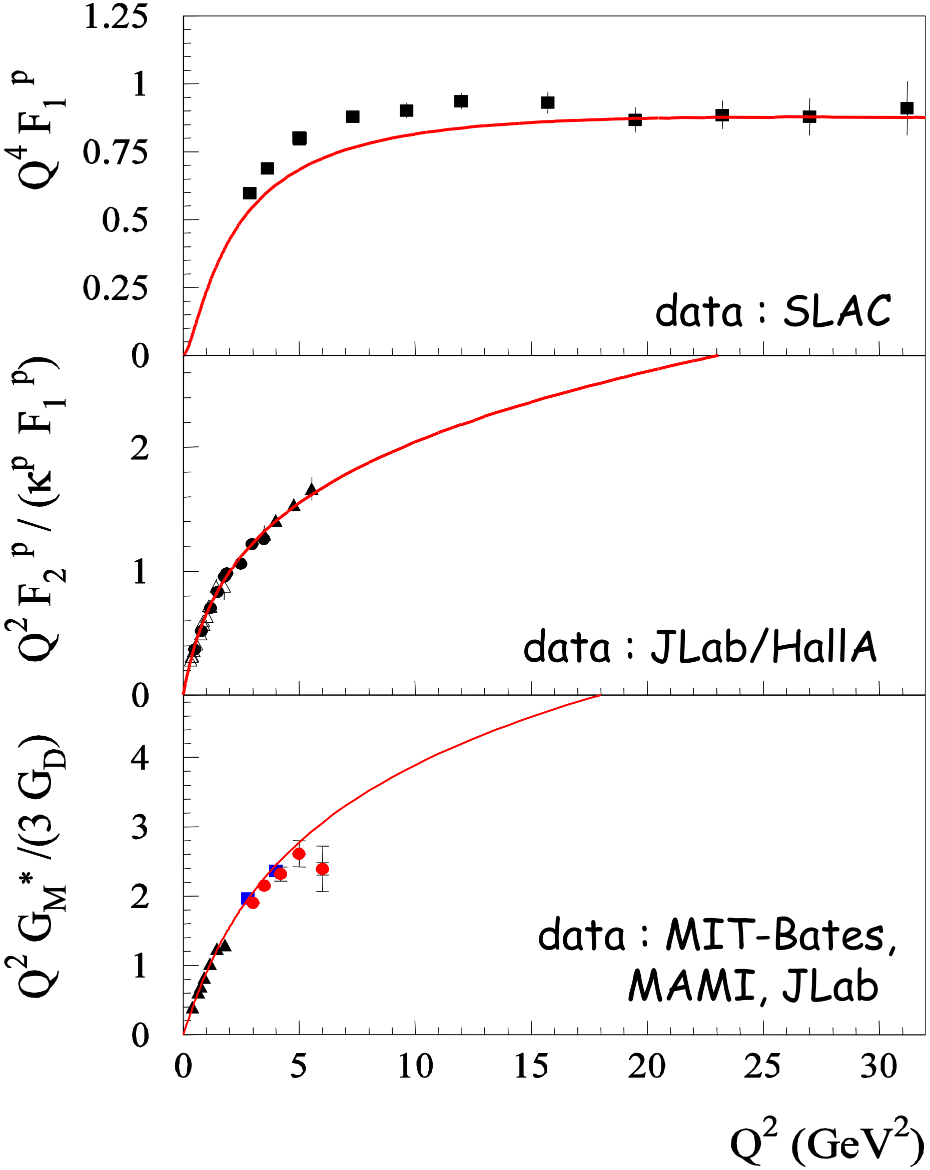
large $N_c$ limit

Pascalutsa, Vdh (2007)

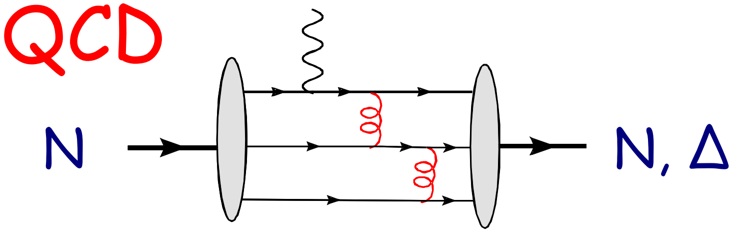
data :

MAMI, BATES, JLab/CLAS, JLab/Hall A

# scaling behavior of N and N -> Δ F.F.




PQCD




+ collinear quarks

$F_1{}^p \sim 1/Q^4$

$F_2{}^p / F_1{}^p \sim 1/Q^2$

$G_M{}^* \sim 1/Q^4$

GPD ——

modified Regge model

Guidal, Polyakov, Radyushkin, Vdh (2005)

# Summary

- for large momentum transfers : relation intrinsic densities and FFs ambiguous

- transverse densities (2D) : theoretically well defined
  relation with nucleon elastic FFs :
  $F_1$ : quark density in nucleon of definite helicity
  $F_2$ : quark density in transversely polarized nucleon

- transverse densities in the N -> Δ transition :
  monopole, dipole, quadrupole field pattern

- distance and GPDs : correlate information on transverse longitudinal quark momentum dependencies

- Large $N_c$ relations between N elastic and N -> Δ transition FFs