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Structure and Dynamics of the Nucleon Spin on the Light-Cone.

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Structure and Dynamics of the Nucleon Spin on the Light-Cone

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Spin-Orbit Correlations and the Shape of the Nucleon

G.A. Miller, PRC76 (2007)

spin-dependent charge density operator
in non relativistic quantum mechanics

$$\hat{\rho}(\vec{r},\vec{n}) = \sum_i \delta(\vec{r}-\vec{r}_i)\frac{1}{2}(1+\vec{\sigma}_i\cdot\vec{n})$$

spin-dependent charge density operator
in quantum field theory

$$\hat{\rho}_{\rm rel}(\vec{k}_\perp,\vec{n}) = \int \frac{dz_\perp}{(2\pi)^2} e^{-i\vec{k}_\perp\cdot\vec{z}_\perp}\bar{\psi}(0)\gamma^0(1+\vec{\gamma}\cdot\vec{n}\gamma_5)\psi(\vec{z}_\perp)|_{z^\pm=0}$$

↓ nucleon state transversely polarized

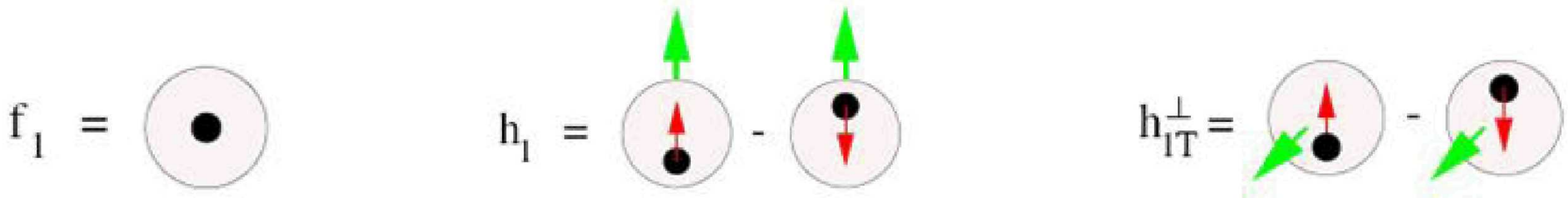
Probability for a quark to have
a momentum $\vec{k}_\perp$ and spin direction $\vec{n}$
in a nucleon polarized in the $\vec{S}_\perp$ direction

$$\rho^T_{\rm rel}(\vec{k}_\perp,\vec{n},\vec{S}_\perp) = \langle P,\vec{S}_\perp\,|\hat{\rho}_{\rm rel}(\vec{k}_\perp,\vec{n})|\,P,\vec{S}_\perp\rangle$$

↓

TMD parton distributions integrated over x

$$\frac{\rho^T_{\rm rel}(\vec{k}_\perp,\vec{n},\vec{S}_\perp)}{M} = \tilde{f}_1(k_\perp^2) + \tilde{h}_1(k_\perp^2)\,\vec{n}\cdot\vec{S}_\perp + \frac{(\vec{n}\cdot\vec{k}_\perp\,\vec{S}_\perp\cdot\vec{k}_\perp - \frac{1}{2}k_\perp^2\,\vec{n}\cdot\vec{S}_\perp)}{M^2}\,\tilde{h}^\perp_{1T}(k_\perp^2)$$



Spin-dependent densities

$$\frac{\rho^T_{\rm rel}(\vec{k}_\perp,\vec{n},\vec{S}_\perp)}{M\tilde{f}_1} = 1 + \frac{\tilde{h}_1(k_\perp^2)}{\tilde{f}_1(k_\perp^2)}\cos\phi_n + \frac{1}{2}\frac{k_\perp^2}{2M^2}\cos(2\phi-\phi_n)\frac{\tilde{h}^\perp_{1T}(k_\perp^2)}{\tilde{f}_1(k_\perp^2)}$$

$$\cos\phi_n = \vec{n}\cdot\vec{S}_\perp \qquad \cos\phi = \vec{n}\cdot\vec{k}_\perp$$

➢ Fix the directions of $\vec{S}_\perp$ and $\vec{n}$ ⇒ the spin-orbit correlations measured with $\tilde{h}^\perp_{1T}(k_\perp^2)$ is responsible for a non-spherical distribution with respect to the spin direction

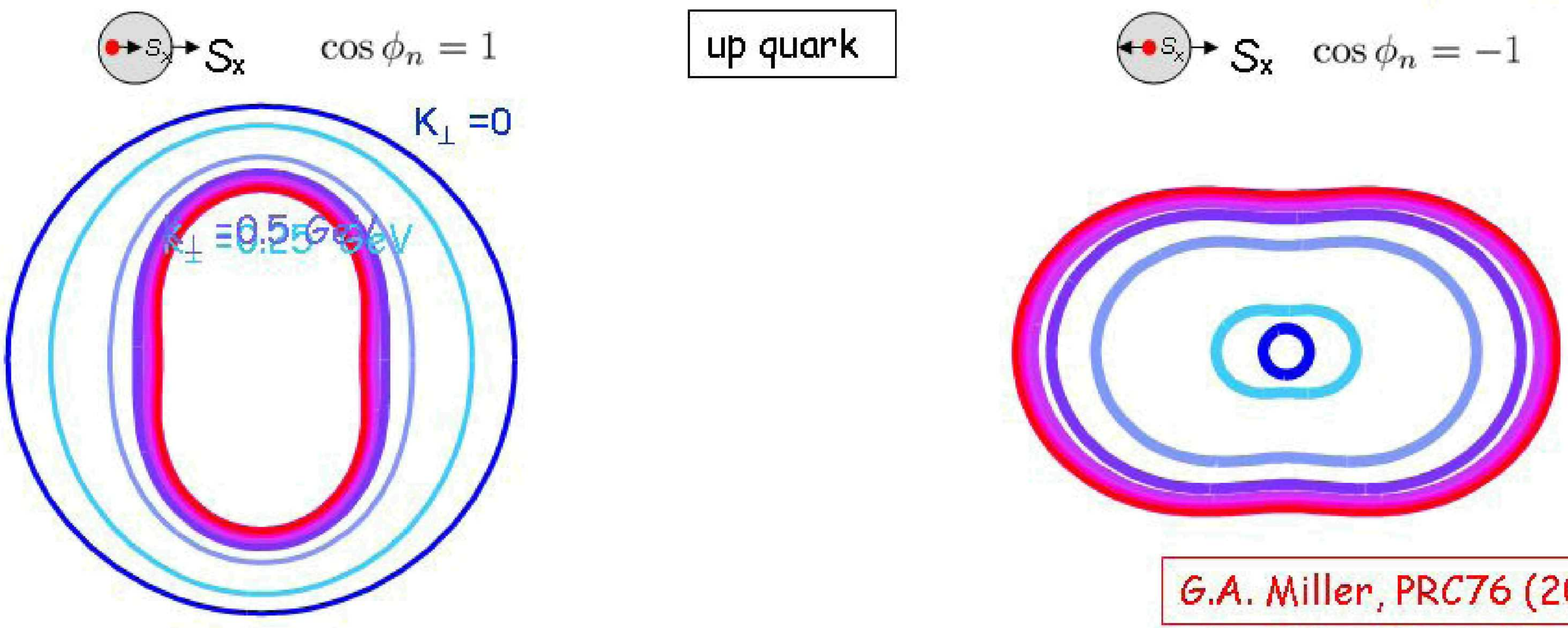
$\tilde{h}^\perp_{1T}(k_\perp^2)$: chirally odd tensor correlations

➡ matrix element from angular momentum components with $|L_z-L'_z|=2$

❖ Diquark spectator model: wave function with angular momentum components L_z = 0, +1, -1

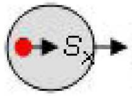
➡ deformation due only to L_z=1 and L_z=-1 components

Jakob, et al., (1997)

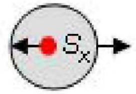


G.A. Miller, PRC76 (2007)

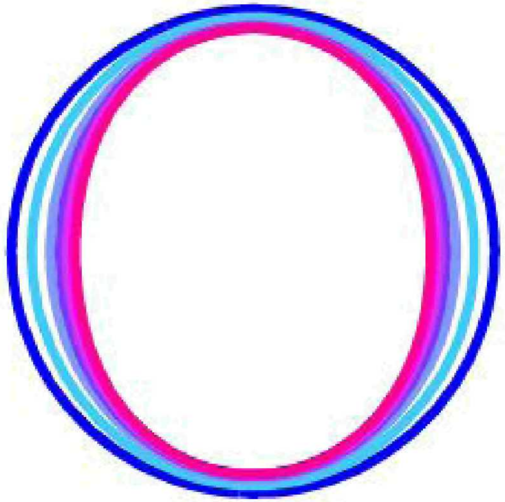
Light Cone Constituent Quark Model

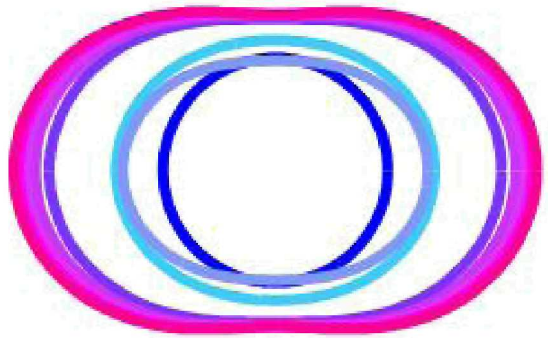

S_x $\cos\phi_n = 1$

up quark

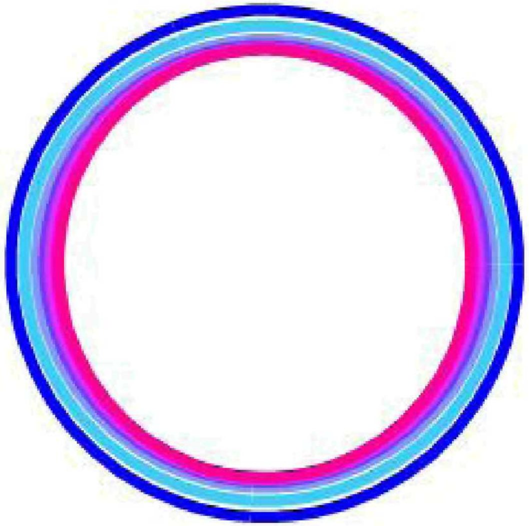

S_x $\cos\phi_n = -1$

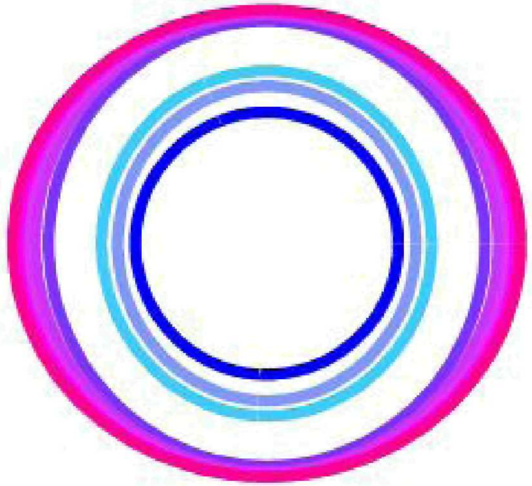
deformation induced from the L_z=+1 and L_z=-1 components





adding the contribution from L_z=0 and L_z=2 components



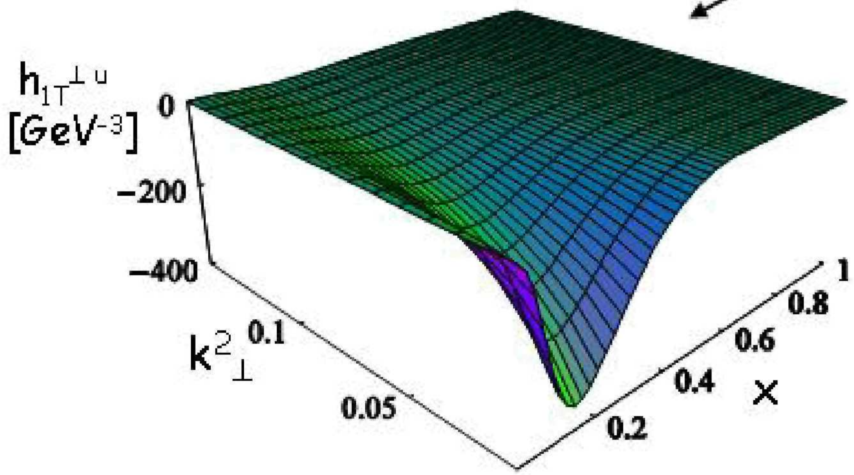


B.P., Cazzaniga, Boffi, in preparation

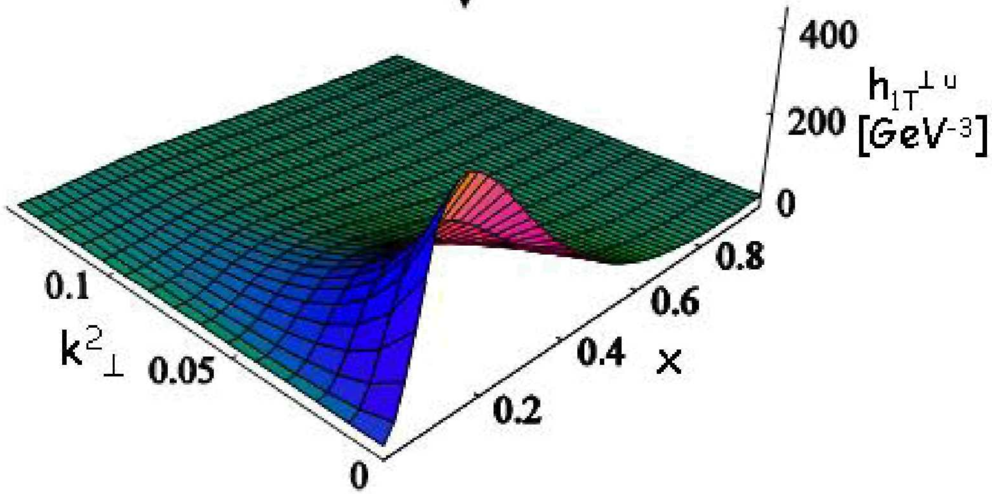
Angular Momentum Decomposition of $h_{1T}^{\perp u}$

$$\frac{k_y^2-k_x^2}{2M^2}\, h_{1T}^{\perp u}(x,k_\perp^2) = -\mathrm{Re}_{\frac{1}{2}}\langle P\downarrow|u_\uparrow^\dagger u_\downarrow|\,P\uparrow\rangle_{-\frac{1}{2}} - 2\mathrm{Re}_{\frac{3}{2}}\langle P\downarrow|u_\uparrow^\dagger u_\downarrow|\,P\uparrow\rangle_{\frac{1}{2}}$$

$L_z=+1$ and $L_z=-1$



$L_z=0$ and $L_z=2$

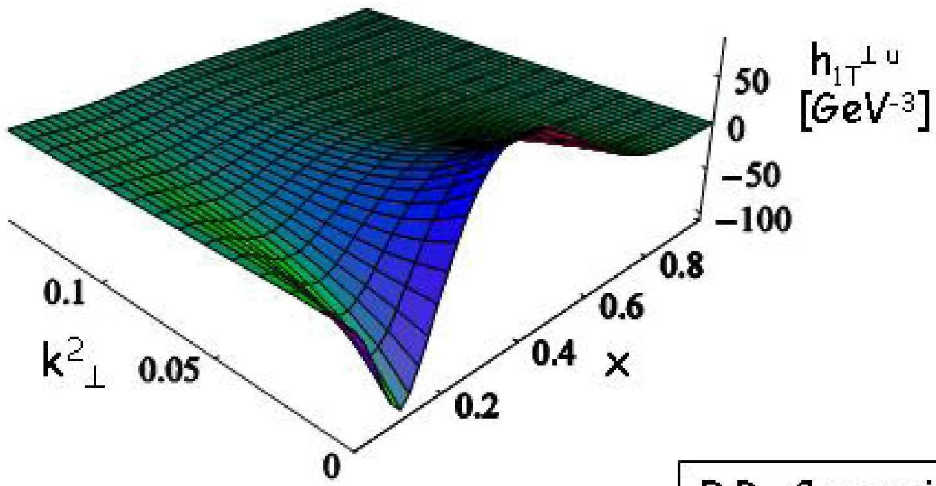


SUM

$$\tilde{h}_{1T}^{\perp u}(k_\perp^2) = \int \mathrm{d}x\, h_{1T}^{\perp u}(x,k_\perp^2)$$

compensation of opposite sign contributions

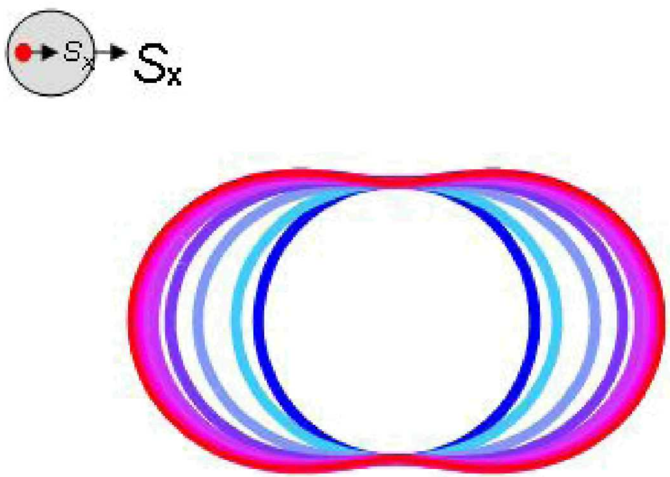
no deformation

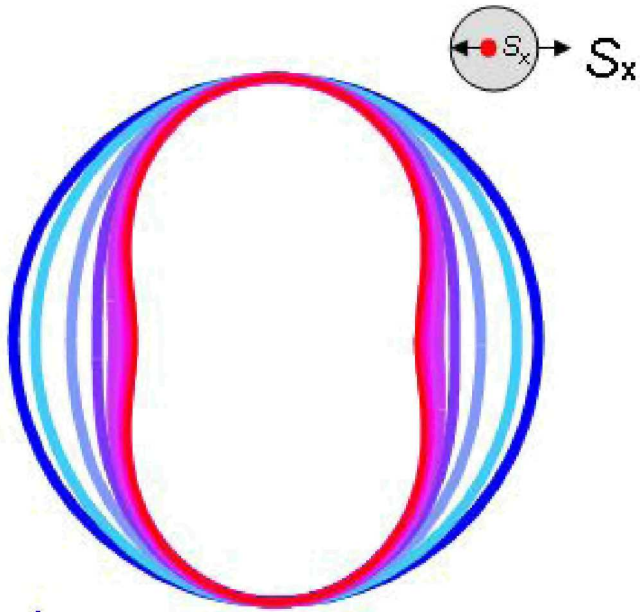


B.P., Cazzaniga, Boffi, in preparation

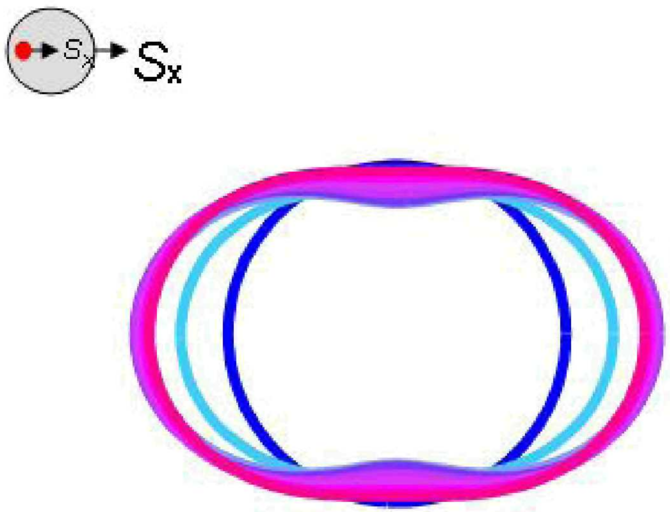
Spin dependent densities for down quark

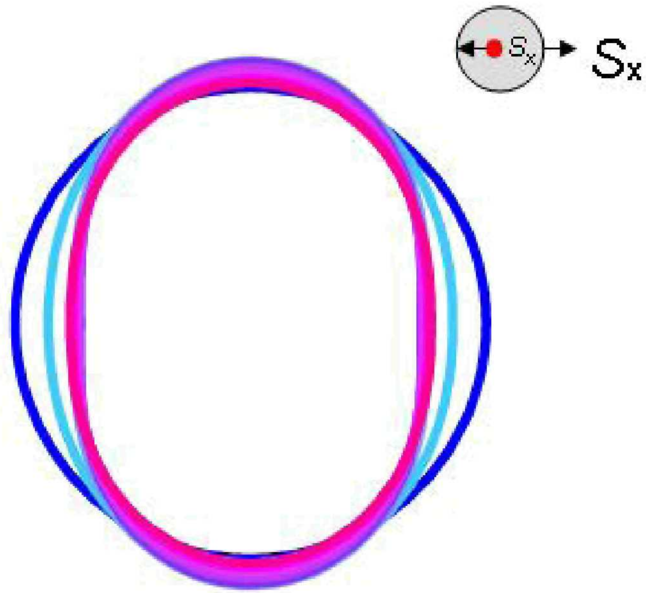
❖ Diquark spectator model: contribution of L_z=+1 and L_z=-1 components





❖ Light-cone CQM: contribution of L_z=+1 and L_z=-1 components plus contribution of L_z=0 and L_z=2 components

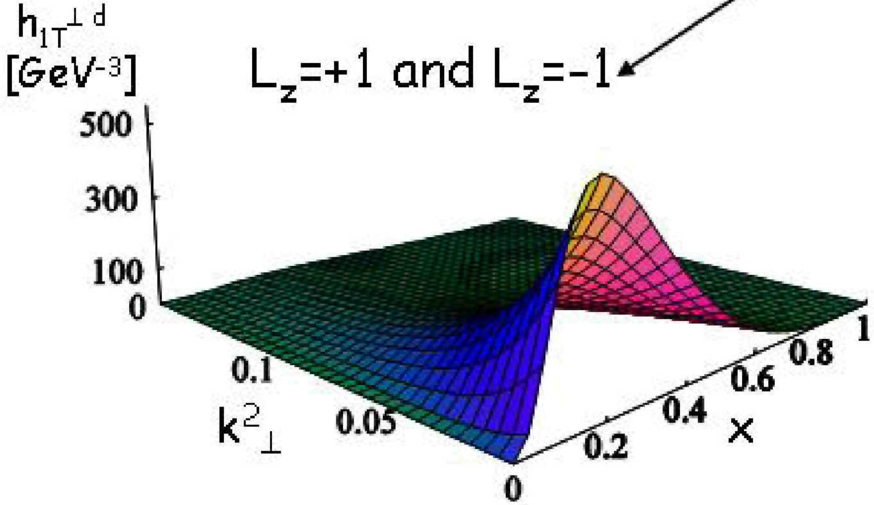




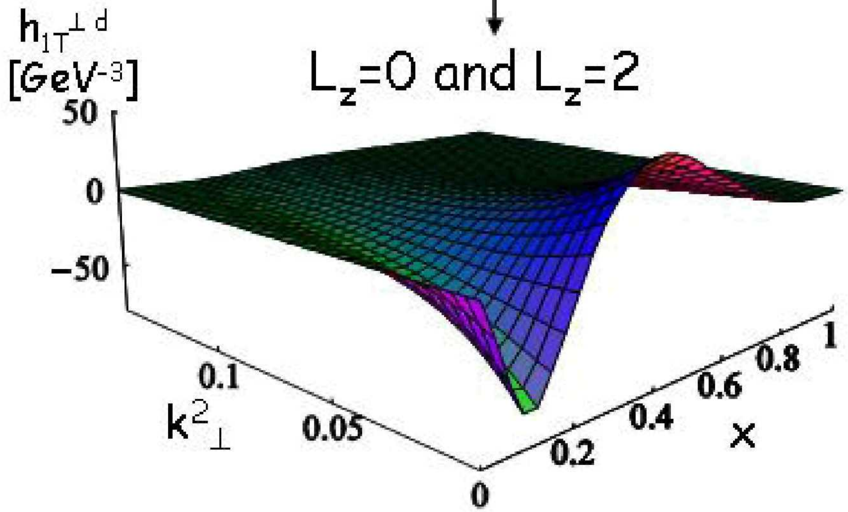
Angular Momentum Decomposition of $h_{1T}^{\perp d}$

$$\frac{k_y^2-k_x^2}{2M^2}\, h_{1T}^{\perp d}(x,k_\perp^2) = -\mathrm{Re}_{\frac{1}{2}}\langle P\downarrow|d_\uparrow^\dagger d_\downarrow|\,P\uparrow\rangle_{-\frac{1}{2}} - 2\mathrm{Re}_{\frac{3}{2}}\langle P\downarrow|d_\uparrow^\dagger d_\downarrow|\,P\uparrow\rangle_{\frac{1}{2}}$$

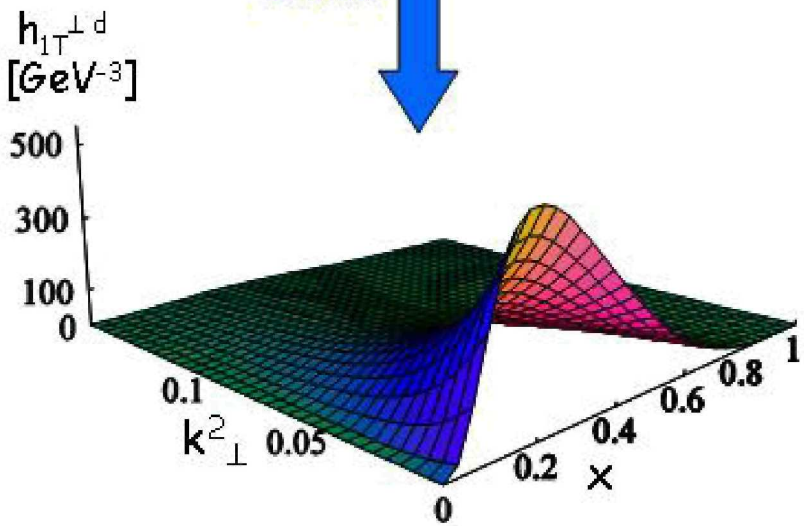
$L_z=+1$ and $L_z=-1$



$L_z=0$ and $L_z=2$



SUM



$$\tilde{h}_{1T}^{\perp d}(k_\perp^2) = \int \mathrm{d}x\, h_{1T}^{\perp d}(x,k_\perp^2)$$

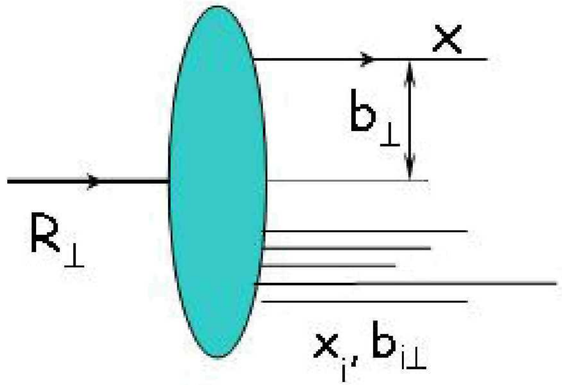
⬇

partial cancellation of different angular momentum components

⬇

non-spherical shape

Nucleon Spin densities



➢ average transverse position of the partons

$$R_\perp = \frac{\sum_i p_i^+ b_{i\perp}}{\sum_i p_i^+}, \qquad (i = q, \bar{q}, g)$$

➢ $b_\perp$: transverse distance between the struck parton and the centre of momentum of the hadron

❖ Nucleon state polarized in the X direction in IMF

$$|P^+, S_X\rangle \equiv \frac{1}{\sqrt{2}} \left(|P^+, R_\perp = 0_\perp, \uparrow\rangle + |P^+, R_\perp = 0_\perp, \downarrow\rangle \right)$$

❖ Impact parameter dependent GPD for the ⊥ pol. state

⟹ quark density in proton state ⊥ pol.

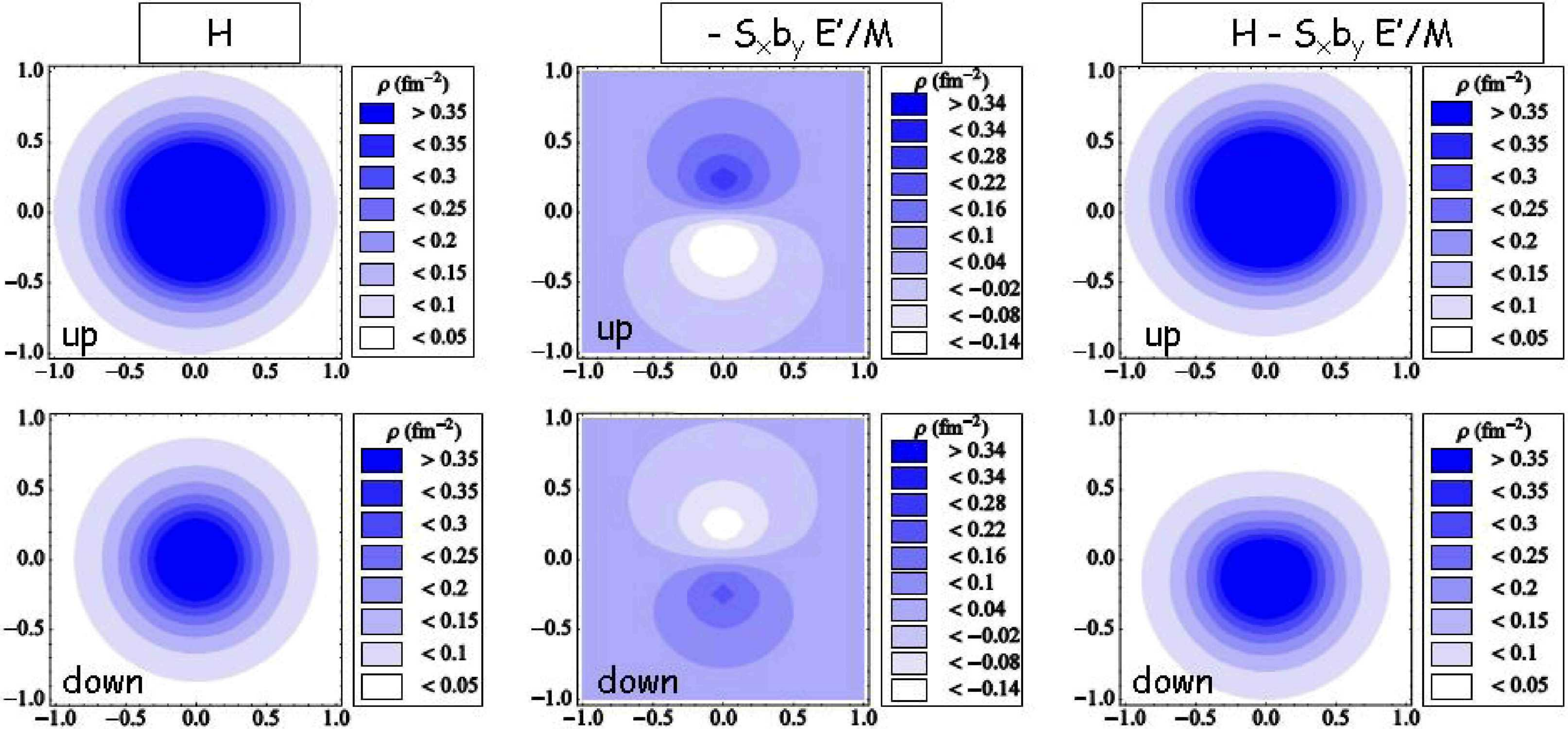
$$q_X(x, b_\perp) = \langle P^+, S_X | \int \frac{dx^-}{4\pi} e^{ixP^+x^-} \bar{q}(-\frac{x^-}{2}, b_\perp) \gamma^+ q(\frac{x^-}{2}, b_\perp) | P^+, S_X \rangle$$

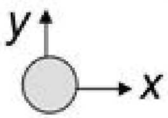
$$q_X(x, b_\perp) = H^q(x, b_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E^q(x, \Delta_\perp) e^{ib_\perp \cdot \Delta_\perp}$$

❖ First moments of q_X: probability density of unpolarized quark in a ⊥ pol. nucleon in impact parameter space

Burkardt, 2003

Unpolarized quarks in a transversely pol. nucleon





⊥ flavor dipole moment $d_y^q = \frac{\kappa^q}{2M}$

$\kappa_u^p = 1.86,\ \kappa_d^p = -1.57 \quad \Rightarrow \quad |d_y^q| \sim 0.1 - 0.2\,\text{fm}$

correlation κ^q and Sivers function

$f_{1T}^{\perp q}(x, k_\perp) \sim -\kappa^q$

$f_{1T}^{\perp u}(x, k_\perp) < 0$ and $f_{1T}^{\perp d}(x, k_\perp) > 0$ consistent with HERMES data, PRL 94 (2005)

Transverse Spin Densities

❖ Fourier transform of Tensor GPDs at $\xi = 0$: distributions in the transverse plane of transversely polarized quarks in a transversely polarized nucleon

Diehl, Haegler, 2005

transversity basis

$$| P^+, S_X\rangle = \frac{1}{\sqrt{2}}[| P^+, +\rangle + | P^+, -\rangle]$$

projector on the transverse quark spin $s_\perp$

$$\frac{1}{2}\bar{q}\,[\gamma^+ - s^j\, i\,\sigma^{+j}\gamma_5]\, q$$

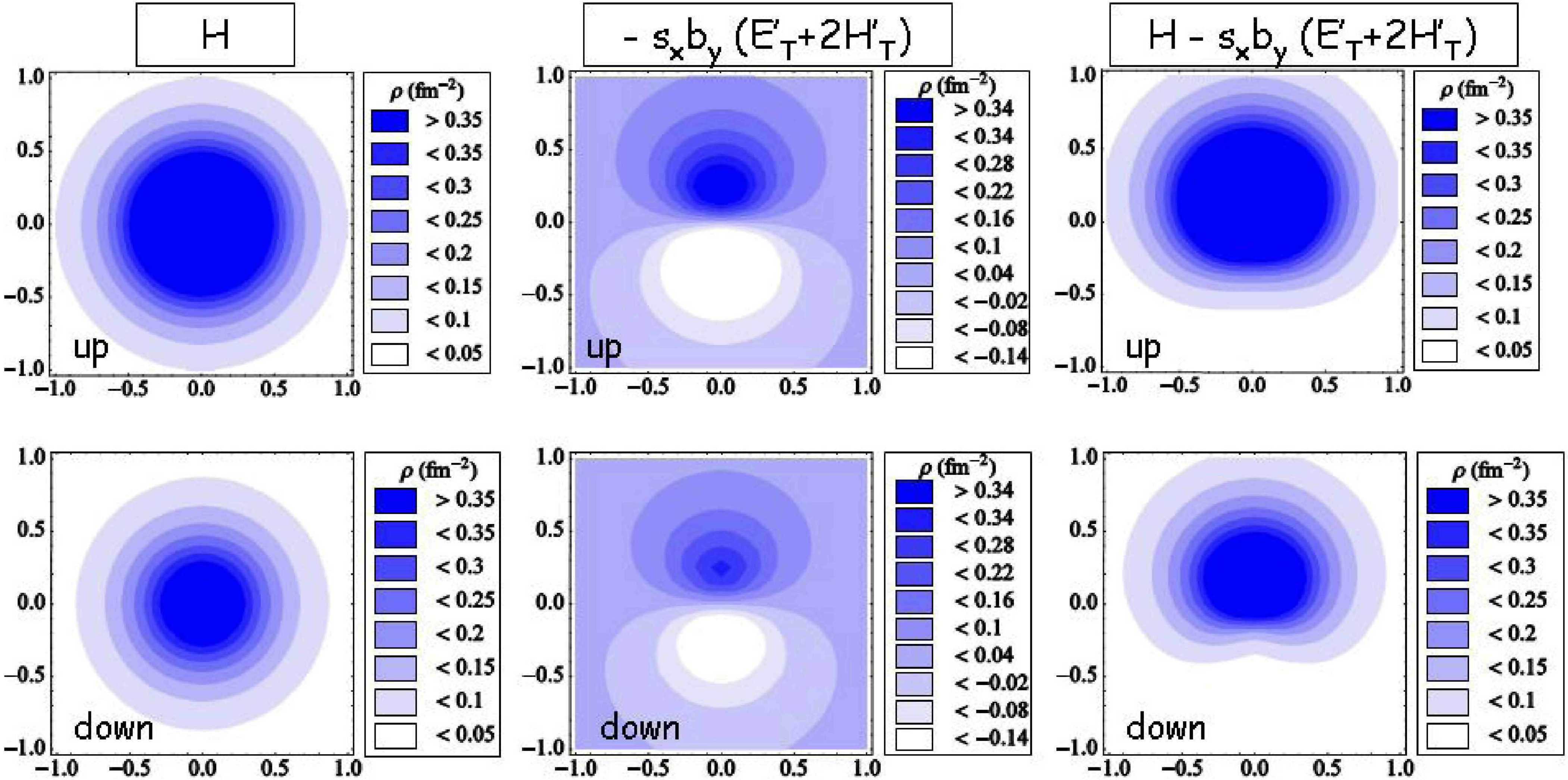
$$\rho(x, b_\perp, s_\perp, S_\perp) = \frac{1}{2}\Big[H(x, b_\perp^2) + s^i S^i \Big(H_T(x, b_\perp^2) - \frac{1}{M^2}\Delta_b \tilde{H}_T(x, b_\perp^2)\Big)$$ monopole

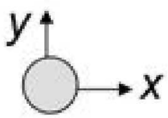
$$+ \frac{b^j \epsilon^{ji}}{M}\Big(S^i E'(x, b_\perp^2) + s^i \big(E'_T(x, b_\perp^2) + 2\tilde{H}'_T(x, b_\perp^2)\big)\Big)$$ dipole

$$+ s^i\,(2b^i b^j - b^2\delta_{ij})\, S^j \frac{1}{M^2}\,\tilde{H}''_T(x, b_\perp^2)\Big]$$ quadrupole

❖ First moments of ρ: transverse spin probability densities in impact parameter space

Transversely pol. quarks in a unpolarized nucleon





⊥ spin-flavor dipole moment: $\kappa_T^u = 3.98 \quad \kappa_T^d = 2.60$

correlation $\kappa_T{}^q$ and Boer-Mulders function $h_1^{\perp q}(x, k_\perp)$

$$h_1^{\perp q}(x, k_\perp) \sim -\kappa_T^q$$

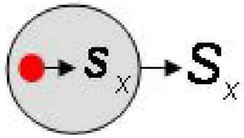
sizeable Boer-Mulders effect

consistent with lattice results (QCDSF/UKQCD Coll.,PRL98, 2007)

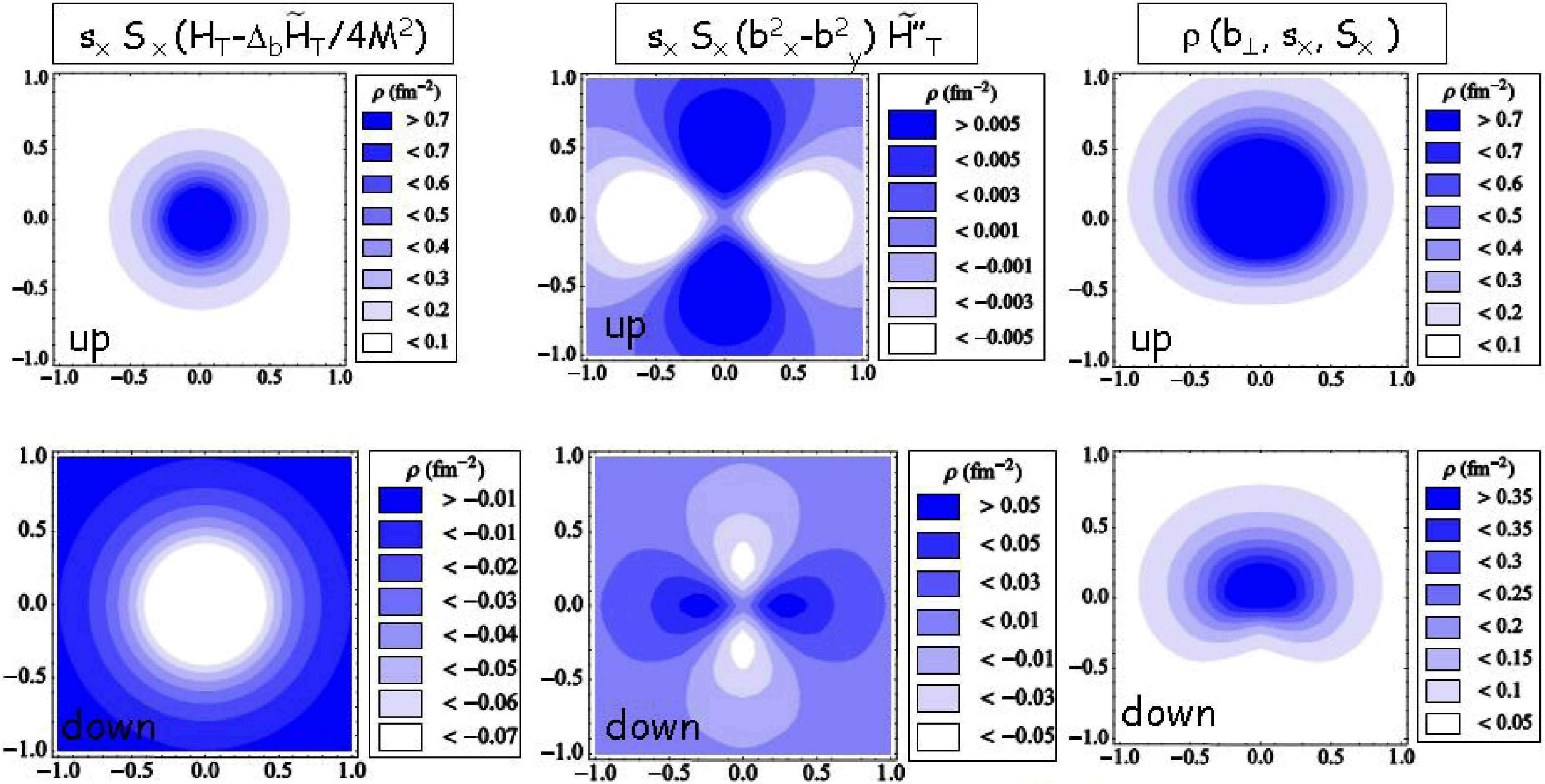
Outline

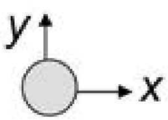
- Three-Quark Light-Cone Amplitudes of the Nucleon
- Spin-Spin and Spin-Orbit Correlations
  - Transverse Momentum Dependent Parton Distributions
    - ➡ shape of the nucleon
  - Generalized Parton Distributions
    - ➡ spin densities
  - Form Factors in the transverse plane
    - ➡ charge and helicities densities
  - Parton Distributions
    - ➡ kinematical relations of the light-cone spin
- Conclusions

Transversely pol. quarks in a transversely pol. nucleon



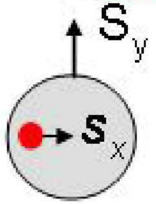
$$\rho(b_\perp, s_x, S_x) = \int \mathrm{d}x \, \frac{1}{2} \Big[H(x, b_\perp^2) + s_x S_x \Big(H_T(x, b_\perp^2) - \frac{1}{M^2} \Delta_b \tilde{H}_T(x, b_\perp^2) \Big) - \frac{b^y}{M} \Big(S_x E'(x, b_\perp^2) + s_x \big(E'_T(x, b_\perp^2) + 2\tilde{H}'_T(x, b_\perp^2) \big) \Big) + s_x (b_x^2 - b_y^2) S_x \frac{1}{M^2} \tilde{H}''_T(x, b_\perp^2) \Big]$$



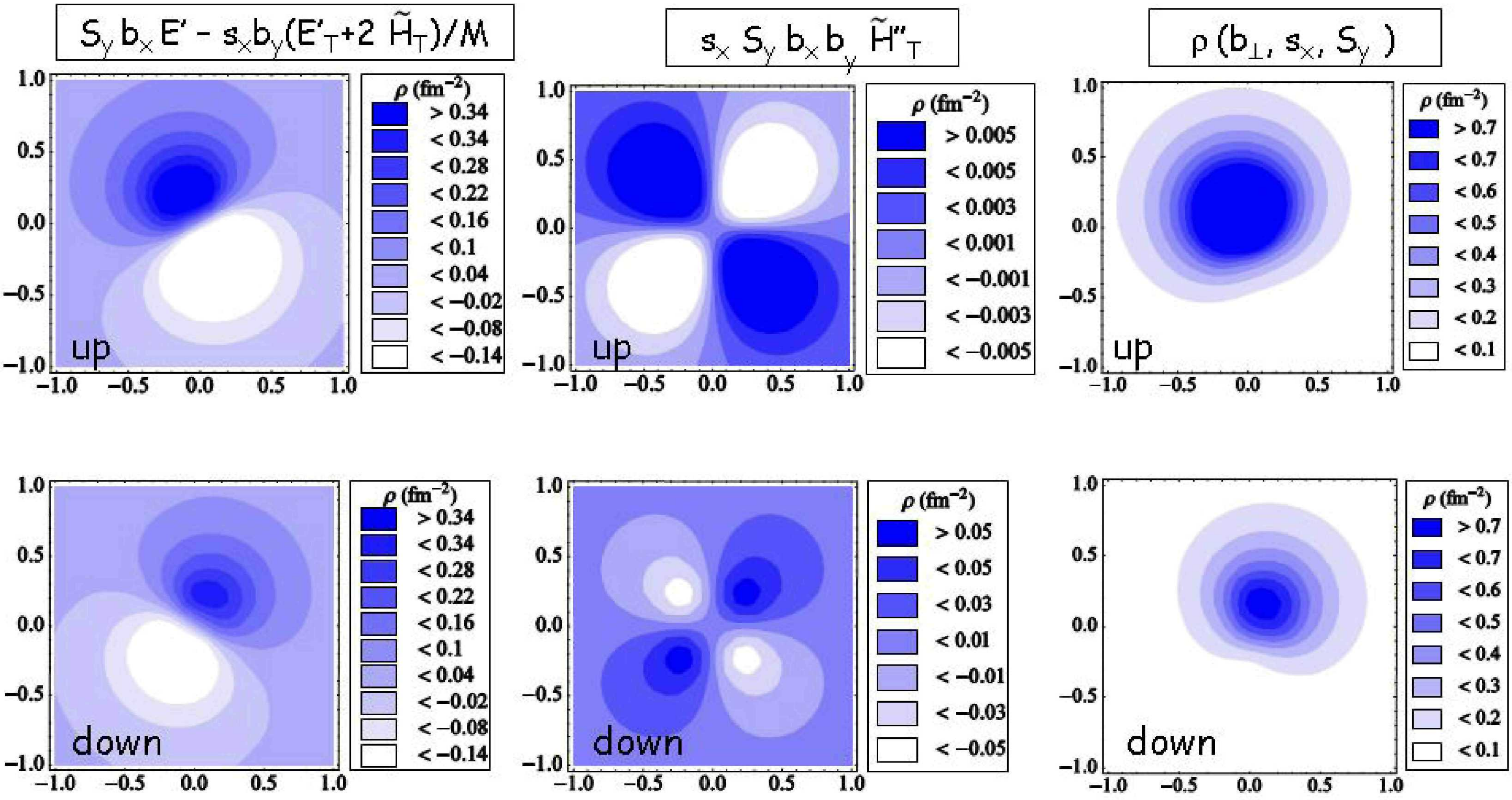


average quadrupole distortion { −0.04 up; 0.07 down

Transversely pol. quarks in a transversely pol. nucleon



$$\rho(b_\perp, s_x, S_y) = \int \mathrm{d}x \, \frac{1}{2} \left[H(x, b_\perp^2) + \frac{1}{M} \left(b_x \, S_y \, E'(x, b_\perp^2) - b_y \, s_x \left(E_T'(x, b_\perp^2) + 2\tilde{H}_T'(x, b_\perp^2) \right) \right) + 2 b_x b_y \, s_x \, S_y \frac{1}{M^2} \tilde{H}_T''(x, b_\perp^2) \right]$$



Charge density of partons in the transverse plane

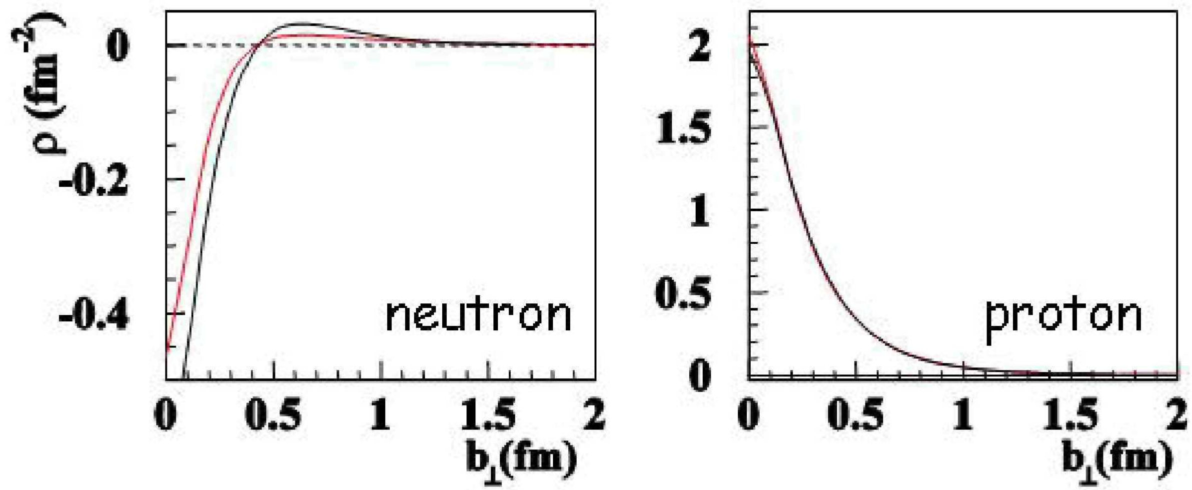
- Infinite-Momentum-Frame Parton charge density in the transverse plane
  - no relativistic corrections

$$\rho^q(b_\perp) = \int \mathrm{d}^2\Delta_\perp \, e^{i\Delta_\perp \cdot b_\perp} \int \mathrm{d}x \, H^q(x, \xi = 0, \Delta_\perp^2) = \int \mathrm{d}^2\Delta_\perp \, e^{i\Delta_\perp \cdot b_\perp} \, F_1^q(\Delta_\perp^2)$$

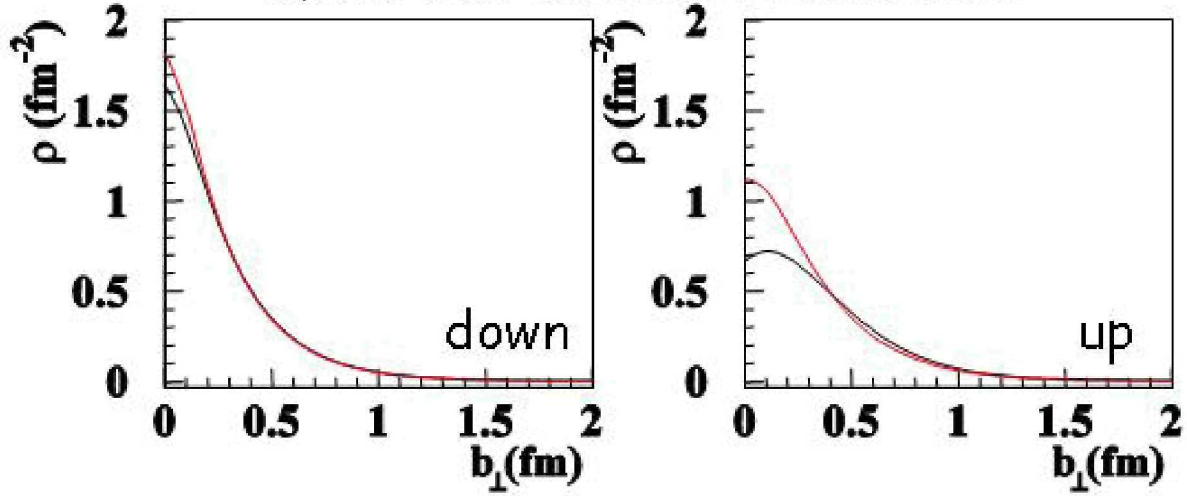
G.A. Miller, PRL99, 2007

fit to exp. form factor by Kelly, PRC70 (2004)

B.P., Boffi, PRD76 (2007) (meson cloud model)



Quark distributions in the neutron

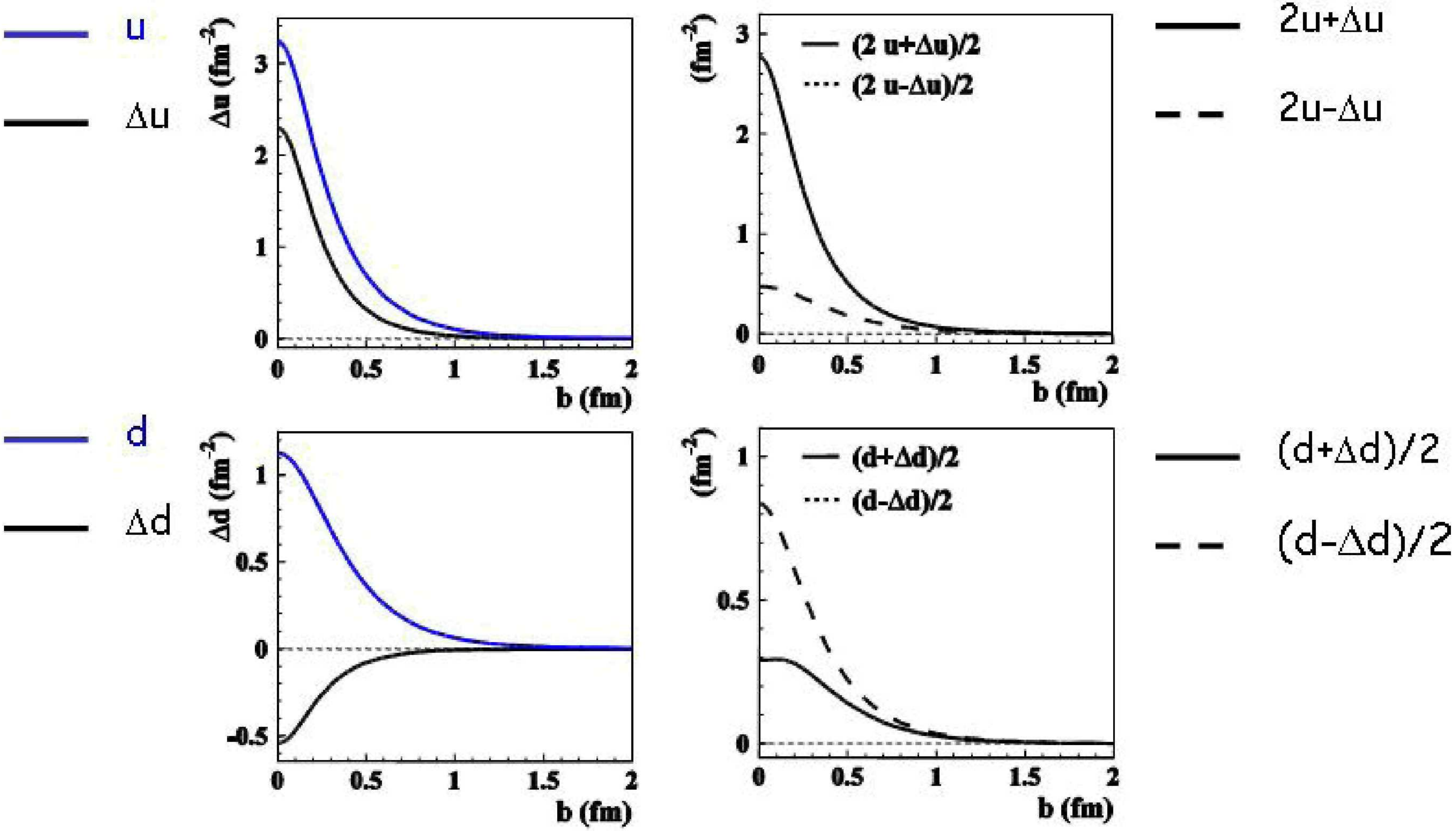


Helicity density in the transverse plane

❖ probability to find a quark with transverse position b and light-cone helicity λ in the nucleon with longitudinal polarization Λ

$$\rho^q(b_\perp, \lambda, \Lambda) = \int \mathrm{d}^2\Delta_\perp \, e^{i\Delta_\perp \cdot b_\perp} \int \mathrm{d}x \, \frac{1}{2}[H^q(x, \xi = 0, \Delta_\perp^2) + \lambda\Lambda \tilde{H}(x, \xi = 0, \Delta_\perp^2)]$$

$$= \int \mathrm{d}^2\Delta_\perp \, e^{i\Delta_\perp \cdot b_\perp} \, \frac{1}{2}[F_1^q(\Delta_\perp^2) + \lambda\,\Lambda\, G_A^q(\Delta_\perp^2)]$$



Light cone wave function overlap representation of Parton Distributions

$$f_1^q(x) = (2\delta_{\tau_q 1/2} + \delta_{\tau_q -1/2}) \int [dx]_3 [d\vec{k}_\perp]_3 \, \delta(x - x_3) |\tilde{\psi}_\uparrow(\{x_i\}, \{\vec{k}_{\perp,i}\}|^2$$

$$h_1^q(x) = \left(\frac{4}{3}\delta_{\tau_q 1/2} - \frac{1}{3}\delta_{\tau_q -1/2}\right) \int [dx]_3 [d\vec{k}_\perp]_3 \, \delta(x - x_3) |\tilde{\psi}_\uparrow(\{x_i\}, \{\vec{k}_{\perp,i}\}|^2 \mathcal{M}_T$$

$$g_1^q(x) = \left(\frac{4}{3}\delta_{\tau_q 1/2} - \frac{1}{3}\delta_{\tau_q -1/2}\right) \int [dx]_3 [d\vec{k}_\perp]_3 \, \delta(x - x_3) |\tilde{\psi}_\uparrow(\{x_i\}, \{\vec{k}_{\perp,i}\}|^2 \mathcal{M}$$

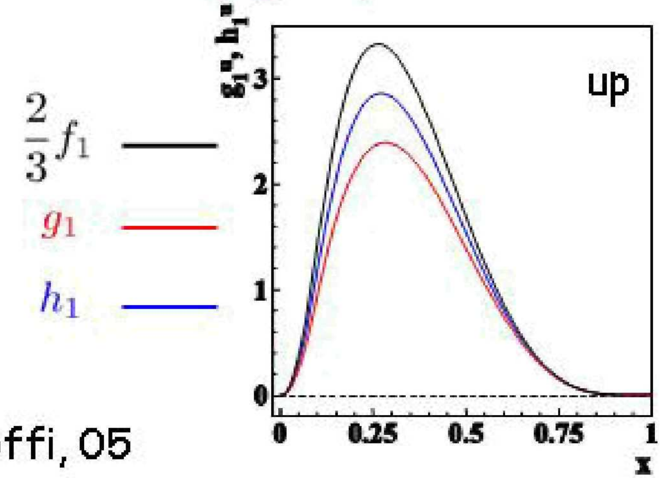
- Melosh rotations: relativistic effects due to the quark transverse motion

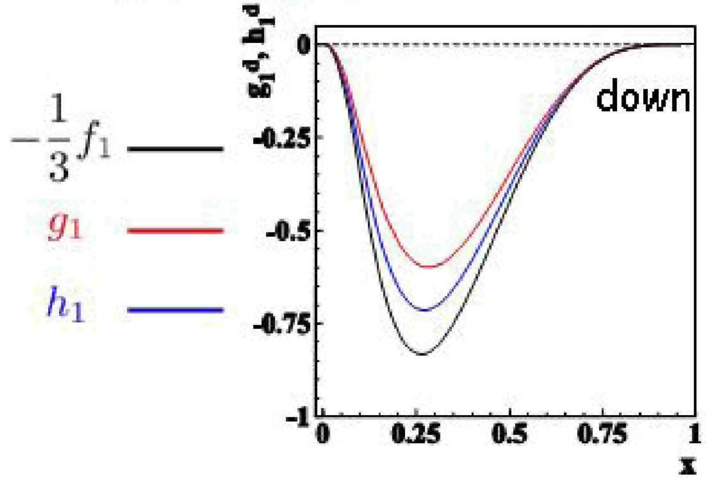
$$\mathcal{M}_T = \frac{(m + x_3 M_0)^2}{(m + x_3 M_0)^2 + \vec{k}_{\perp,3}^2} \qquad \mathcal{M} = \frac{(m + x_3 M_0)^2 - \vec{k}_{\perp,3}^2}{(m + x_3 M_0)^2 + \vec{k}_{\perp,3}^2}$$

$$2h_1^u(x) = g_1^u(x) + \frac{2}{3} f_1^u(x) \qquad 2h_1^d(x) = g_1^d(x) - \frac{1}{3} f_1^d(x)$$

consistent with Soffer bounds and $2\delta q = \Delta q + \Delta q_{\rm NR}$ (Ma, Schmidt, Soffer,'97)

- Non relativistic limit ($k_\perp \to 0$) : $\mathcal{M}_T = \mathcal{M} = I$ ➡ $h_1^q(x) = g_1^q(x)$





B.P., Pincetti, Boffi, 05

Summary

- Relativistic effects due to Melosh rotations in LCWF introduce a non trivial spin structure and correlations between quark spin and quark orbital angular momentum

- Spin-Orbit Correlations in TMD Parton Distributions
  - ➡ shape of the nucleon
  - ➡ interplay of different angular momentum components

- Transverse Spin Densities in the Impact Parameter Space

  Unpolarized quarks in a transversely pol. nucleon
  ⇨ opposite sign for Sivers function of up and down quark as seen by HERMES

  Transversely pol. quarks in an unpolarized nucleon
  ⇨ sizeable Boer-Mulder function effect with the same sign for up and down quark, as seen by lattice results

- Charge and helicity densities of the nucleon show unexpected distributions for up and down quark distributions

- Parton Distributions
  - ➡ non-trivial relations for the valence quark contributions to f_1, g_1, h_1 at the hadronic scale

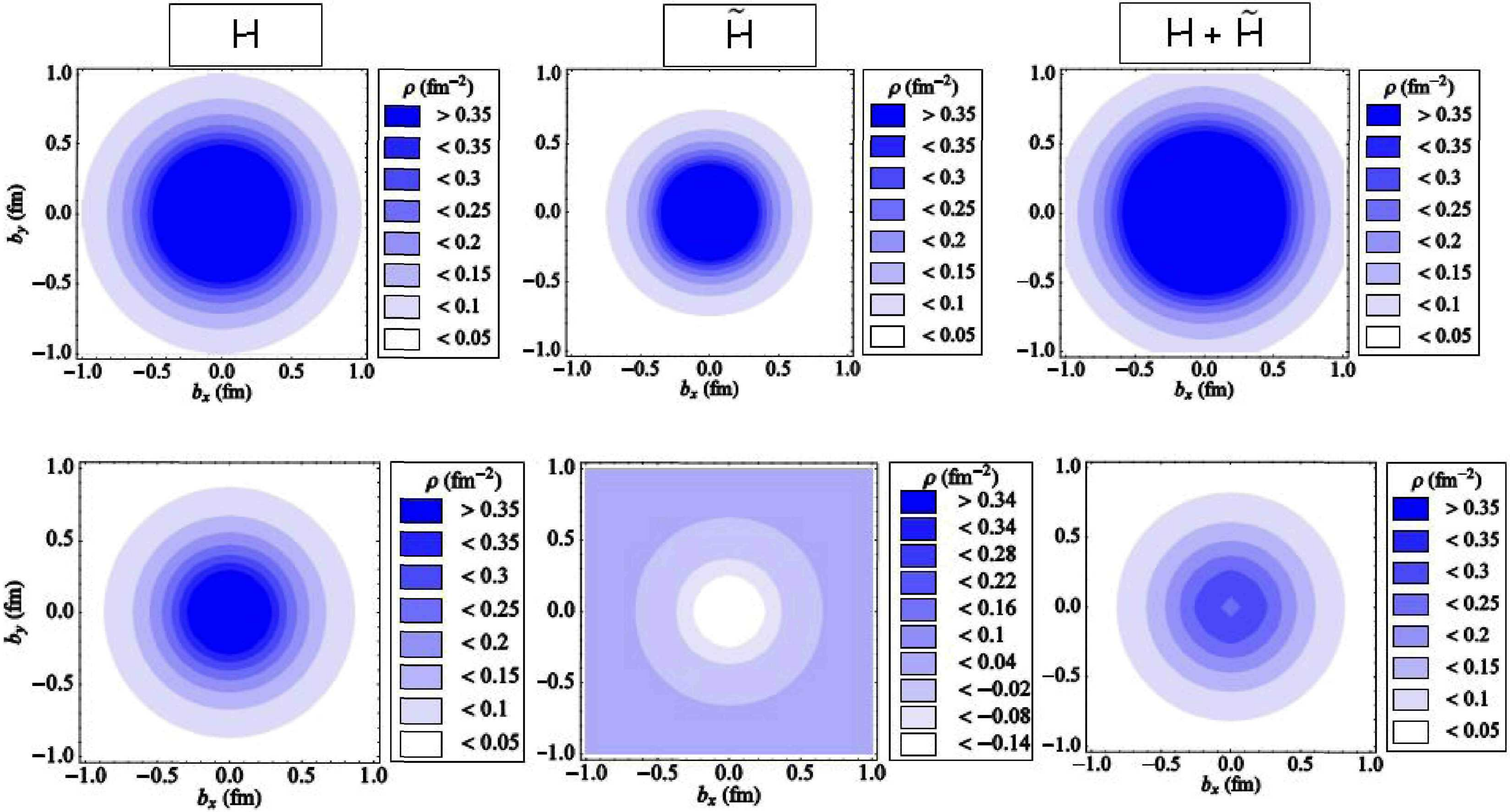
$$2h_1^u(x) = g_1^u(x) + \frac{2}{3} f_1^u(x) \qquad 2h_1^d(x) = g_1^d(x) - \frac{1}{3} f_1^d(x)$$

Longitudinally pol. quarks in a longitudinally pol. nucleon

- Probability density in impact parameter space of longitudinally polarized quarks in a longitudinally polarized nucleon

Burkardt, 2003

$$\int \mathrm{d}x\, \rho(x, b_\perp, \lambda, \Lambda) = \int \mathrm{d}x\, \frac{1}{2}\left[H(x, b^2) + \lambda\Lambda\tilde{H}(x, b^2)\right]$$



Momentum Space WF

- ✓ spin and isospin component: SU(6) symmetric
- ✓ momentum-space component: S wave

$$\tilde{\Psi} = \frac{N}{(M_0^2 + \beta^2)^\gamma} \text{ with } M_0 = \sum_i \sqrt{k_i^2 + m_q^2}$$

Three free parameters: m_q, β, γ fitted to reproduce the magnetic moment of the proton and the axial coupling constant g_A

m_q = 263 MeV β = 607 MeV γ = 3.5

Schlumpf, PhD thesis, hep-ph/9211255

❖ Light-cone wavefunction

- ✓ breaking of SU(6) symmetry
- ✓ non-zero quark orbital angular momentum

Melosh Rotations

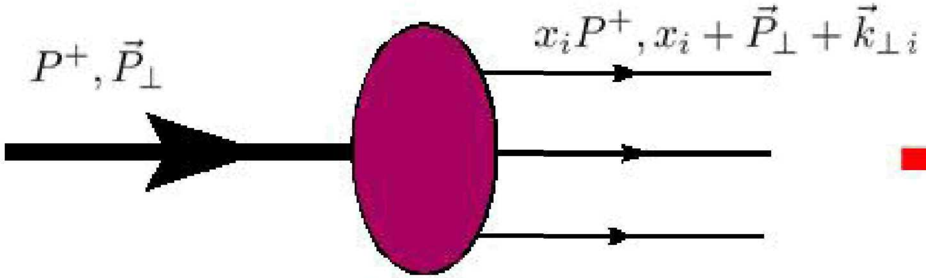
$$q_{LC}^{\uparrow} = w\,[(k^+ + m_q)\,q_I^{\uparrow} + (k_x + ik_y)\,q_I^{\downarrow}]$$

$$q_{LC}^{\downarrow} = w\,[-(k_x - ik_y)\,q_I^{\uparrow} + (k^+ + m_q)q_I^{\downarrow}]$$

$$(w = [(k^+ + m_q)^2 + k_\perp^2]^{-1/2})$$

The boost to infinite momentum frame (Melosh Rotations) introduces a non trivial spin structure and a correlation between quark spin and quark orbital angular momentum

Three Quark Light Cone Amplitudes



LCWF: $\Psi^f_{\lambda,\beta}(x_i, \vec{k}_{\perp,i})$

invariant under boost, independent of P^μ

internal variables: $\sum_{i=1}^{3} x_i = 1 \qquad \sum_{i=1}^{3} \vec{k}_{\perp i} = \vec{0}_\perp$

- 'uds' basis

$$| P, \uparrow\rangle = \frac{1}{\sqrt{3}} \left[| P, \uparrow\rangle^{(uud)} + | P, \uparrow\rangle^{(udu)} + | P, \uparrow\rangle^{(duu)} \right]$$

$$| P, \lambda\rangle^{(uud)} = \sum_\beta \int d[1]d[2]d[3] \, \Psi^f_{\lambda,uud}(x_i, \vec{k}_{\perp,i}) \frac{\varepsilon^{ijk}}{\sqrt{6}} u^\dagger_{i\lambda_1}(1) u^\dagger_{j\lambda_2}(2) d^\dagger_{k\lambda_3}(3) | 0\rangle$$

- classification of LCWFs in angular momentum components (X. Ji, J.-P. Ma, F. Yuan, 03)

$$| P, \uparrow\rangle = | P, \uparrow\rangle_{-\frac{3}{2}} + | P, \uparrow\rangle_{-\frac{1}{2}} + | P, \uparrow\rangle_{\frac{1}{2}} + | P, \uparrow\rangle_{\frac{3}{2}}$$

$J_z = J_z^q + L_z$

total quark helicity J^q

parity
time reversal
isospin symmetry

6 independent wave function amplitudes: $\psi^{(i)} \; i = 1, .., 6$

Three Quark Light Cone Amplitudes

$L_z = 0$

$$|P\uparrow\rangle_{\frac{1}{2}} = \int d[1]d[2]d[3] \left(\psi^{(1)}(1,2,3) + i(k_1^x k_2^y - k_1^y k_2^x)\psi^{(2)}(1,2,3)\right) \times \frac{\epsilon^{ijk}}{\sqrt{6}} u^\dagger_{i\uparrow}(1)\left(u^\dagger_{j\downarrow}(2)d^\dagger_{k\uparrow}(3) - d^\dagger_{j\downarrow}(2)u^\dagger_{k\uparrow}(3)\right)|0\rangle$$

$L_z = 1$

$$|P\uparrow\rangle_{-\frac{1}{2}} = \int d[1]d[2]d[3] \left((k_1^x + ik_1^y)\psi^{(3)}(1,2,3) + (k_2^x + ik_2^y)\psi^{(4)}(1,2,3)\right) \times \frac{\epsilon^{ijk}}{\sqrt{6}} \left(u^\dagger_{i\uparrow}(1)u^\dagger_{j\downarrow}(2)d^\dagger_{k\downarrow}(3) - d^\dagger_{i\uparrow}(1)u^\dagger_{j\downarrow}(2)u^\dagger_{k\downarrow}(3)\right)|0\rangle$$

$L_z = -1$

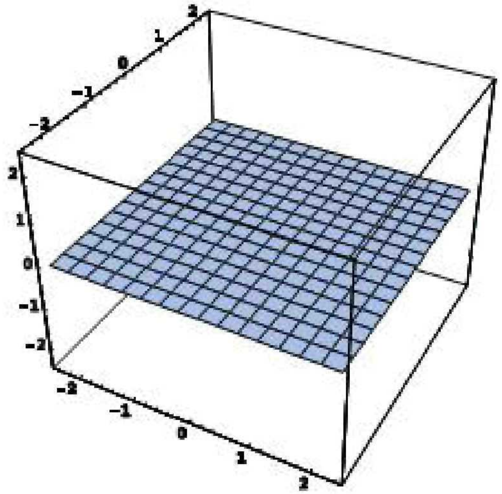
$$|P\uparrow\rangle_{\frac{3}{2}} = \int d[1]d[2]d[3]\ (k_2^x - ik_2^y)\psi^{(5)}(1,2,3) \times \frac{\epsilon^{ijk}}{\sqrt{6}} u^\dagger_{i\uparrow}(1)\left(u^\dagger_{j\uparrow}(2)d^\dagger_{k\uparrow}(3) - d^\dagger_{j\uparrow}(2)u^\dagger_{k\uparrow}(3)\right)|0\rangle$$

$L_z = 2$

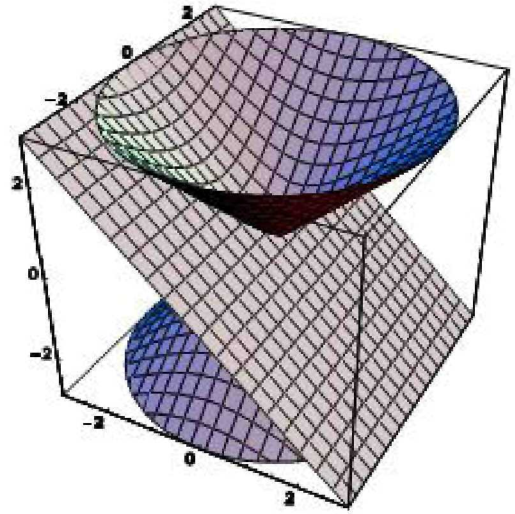
$$|P\uparrow\rangle_{-\frac{3}{2}} = \int d[1]d[2]d[3]\ (k_1^x + ik_1^y)(k_3^x + ik_3^y)\psi^{(6)}(1,2,3) \times \frac{\epsilon^{ijk}}{\sqrt{6}} u^\dagger_{i\downarrow}(1)\left(d^\dagger_{j\downarrow}(2)u^\dagger_{k\downarrow}(3) - u^\dagger_{j\downarrow}(2)d^\dagger_{k\downarrow}(3)\right)|0\rangle$$

Light Cone Constituent Quark Model

Instant form:
x^0 time;
x^1, x^2, x^3 space



Light-front form:
x^+ time;
x^-, $x_\perp$ space



- Instant Form (canonical) eigenvalue equation

$$M|\, M, j_c, \mu_c\rangle_c = [M_0 + V]|\, M, j_c, \mu_c\rangle_c$$

$M_0 = \sum_i^N \sqrt{m_i^2 + \vec{k}_i^2}$: free mass operator $\qquad V$: interaction operator

- Light-front eigenvalue equation

$$\mathcal{M}|\, M, j_f, \mu_f\rangle_f = [M_0 + \mathcal{V}]|\, M, j_f, \mu_f\rangle_f$$

$\mathcal{M} = \mathcal{R}^\dagger\, M\, \mathcal{R}$ ➡ $\mathcal{R} = \Pi_{i=1}^N\, R_M(\vec{k}_{\perp,i}, x_i, m_i)$: generalized Melosh rotations

$$\Psi_N^f = \langle \{x_i, \vec{k}_{\perp,i}, \lambda_i\}_N\, |M, j_f, \mu_f\rangle_f = \frac{2(2\pi)^3}{\sqrt{M_0}} \Pi_{i=1}^N \sqrt{\frac{E_i}{x_i}} \sum_{\{\lambda_i'\}} \langle \{\lambda_i\}\, |\mathcal{R}^\dagger|\, \{\lambda_i'\}\rangle \Psi_N^c$$

Light Cone Spin

❖ Instant-form wave function: $\Psi = \Phi^I \otimes \Phi^S \otimes \tilde{\Psi}(\{\vec{k}_i\})$

Schlumpf,
Ph.D. Thesis,
hep-ph/9211255

✓ momentum-space component: S wave ⟶ $\tilde{\Psi} = \frac{N}{(M_0^2 + \beta^2)^\gamma}$

✓ spin and isospin component: SU(6) symmetric

$$uud\,[\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow] \qquad J_z = J_z^q$$

Melosh Rotations

❖ Light-cone wavefunction

$$q_I^\uparrow = w\,[(k^+ + m_q)\, q_{LC}^\uparrow - (k_x + ik_y)\, q_{LC}^\downarrow]$$

$$q_I^\downarrow = w\,[(k_x - ik_y)\, q_{LC}^\uparrow + (k^+ + m_q) q_{LC}^\downarrow] \quad (w = [(k^+ + m_q)^2 + k_\perp^2]^{-1/2})$$

✓ breaking of SU(6) symmetry

✓ non-zero quark orbital angular momentum $\qquad J_z = J_z^q + L_z^q$

Six independent wave function amplitudes

$L_z^q = -1$	$L_z^q = 0$	$L_z^q = 1$	$L_z^q = 2$
$(\uparrow\uparrow\uparrow)_{LC}$	$(\uparrow\uparrow\downarrow)_{LC}$	$(\uparrow\downarrow\downarrow)_{LC}$	$(\downarrow\downarrow\downarrow)_{LC}$

The six independent wave function amplitudes obtained from the Melosh rotations satisfy the model independent classification scheme in four orbital angular momentum components

Relevance of OAM to Nucleon Structure

- Transverse Momentum Dependent Parton Distributions
- Generalized Parton Distributions
- Nucleon Spin Densities
- Anomalous Magnetic Moment of the Nucleon
- Helicity-Flip Pauli Form Factor
- ....
- ....

Quark-Quark Distribution Correlation Function

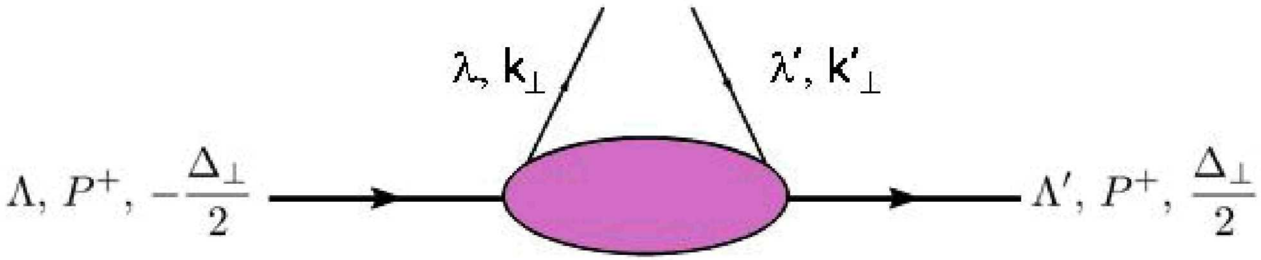
$$\mathcal{O}^q_{\lambda'\lambda} = \int \frac{dz^- dz_\perp}{(2\pi)^3} \bar{q}_{\lambda'}(-\frac{z}{2})\, \Gamma\, q_\lambda(\frac{z}{2})\, e^{i(k^+z^- - k_\perp \cdot z_\perp)}$$

$\Gamma =$

- $\gamma^+ \Rightarrow q^\dagger_\uparrow q_\uparrow + q^\dagger_\downarrow q_\downarrow$ quark-number density
- $\gamma^+\gamma^5 \Rightarrow q^\dagger_\uparrow q_\uparrow - q^\dagger_\downarrow q_\downarrow$ quark-helicity density
- $i\sigma^{x+}\gamma^5 \Rightarrow q^\dagger_\uparrow q_\downarrow + q^\dagger_\downarrow q_\uparrow$ transverse-spin density

❖ $k_\perp$ and $\Delta_\perp$ dependent correlator:

$$H(x, k_\perp, \Delta_\perp) = \langle (P^+, -\frac{\Delta_\perp}{2}), \Lambda' \,|\mathcal{O}^q_{\lambda'\lambda}|\, (P^+, \frac{\Delta_\perp}{2}), \Lambda \rangle$$



Spin-Spin and Spin-Orbit correlations of quarks in the nucleon

$$H(x, k_\perp, \Delta_\perp) = \langle (P^+, -\frac{\Delta_\perp}{2}), \Lambda' \mid \int \frac{dz^- dz_\perp}{(2\pi)^3} \bar{q}_{\lambda'}(-\frac{z}{2})\, \Gamma\, q_\lambda(\frac{z}{2})\, e^{i(k^+ z^- - k_\perp \cdot z_\perp)} \mid (P^+, \frac{\Delta_\perp}{2}), \Lambda \rangle$$