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Generalized Parton Distributions, Analyticity and Formfactors.

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Generalized Parton Distributions, Analyticity and Formfactors

Perspectives in Hadronic Physics

Trieste, May 15 2008

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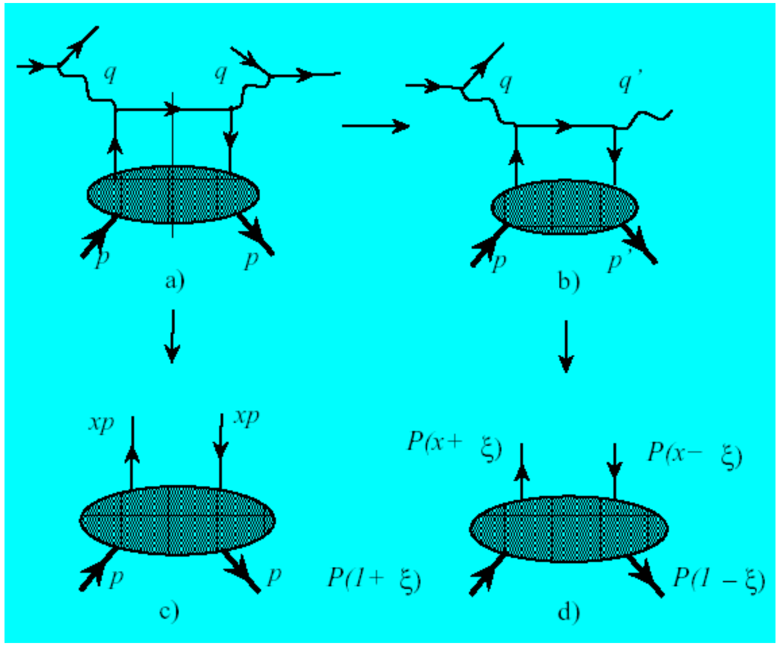
Main Topics

- Analyticity vs QCD factorization
- Crossing, Tomography, Holography and Duality of GPD’s
- GPDs and Hard Double Diffraction
- GPDs and (gravitational) formfactors: Equivalence Principle, its Extension and AdS/QCD

QCD Factorization for DIS and DVCS (AND VM production)



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}.$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Unphysical regions

- DIS : Analytical function – polynomial in $1/x_B$ if $1 \leq |x_B|$

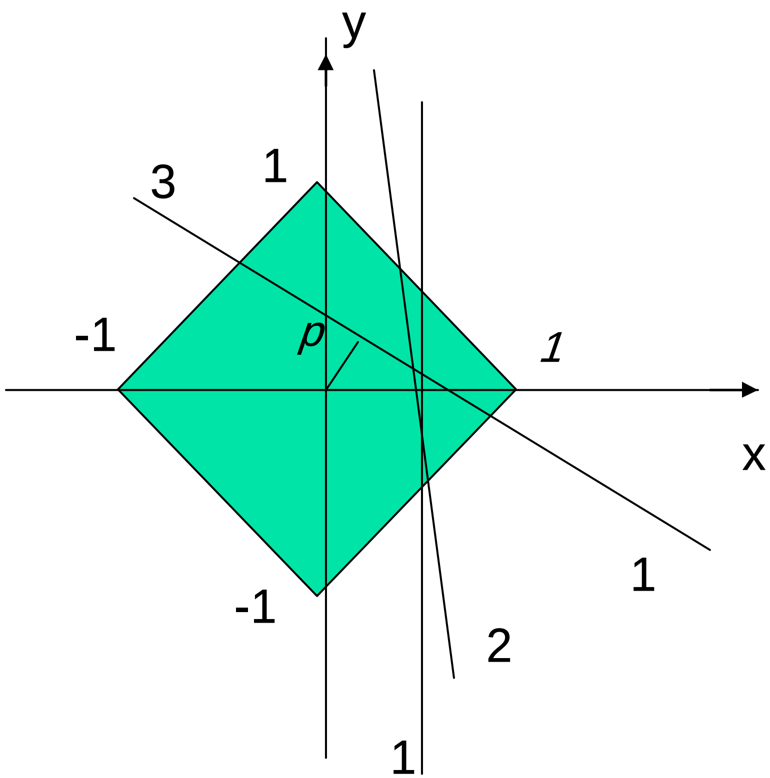
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x, \xi)$
- Solved by using of Double Distributions Radon transform

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line- skewness
- Kinematics of DIS: $\xi = 0$ ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$ - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

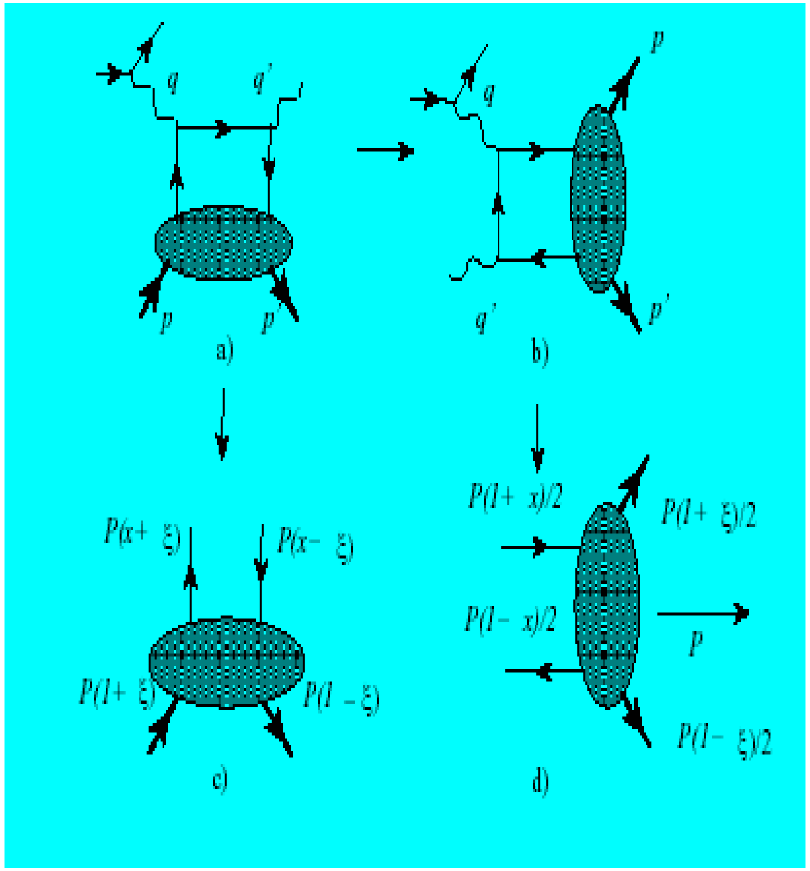


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi |cos\phi| (H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
- Duality between s and t channels (Polyakov,Shuvaev, Guzey, Vanderhaeghen)



Radon (OT'01) and Abel (Moiseeva, Polyakov'08) Transforms: even vs odd-dimensional spaces

- Even (integrals over lines in plane): integral (global) inversion formula
- Odd (integrals over planes in space) – differential (local) inversion formula – Huygens principle
- Triple distributions – THREE pions production (Pire, OT'01) or (deuteron) Decay PD. Relation to nuclei breakup in studies of SRC?!

GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in x weighted with x^n - are polynomials in $1/\xi$ of power $n+1$
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

- Analyticity -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

- "Holographic" equation (DVCS AND VM)

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

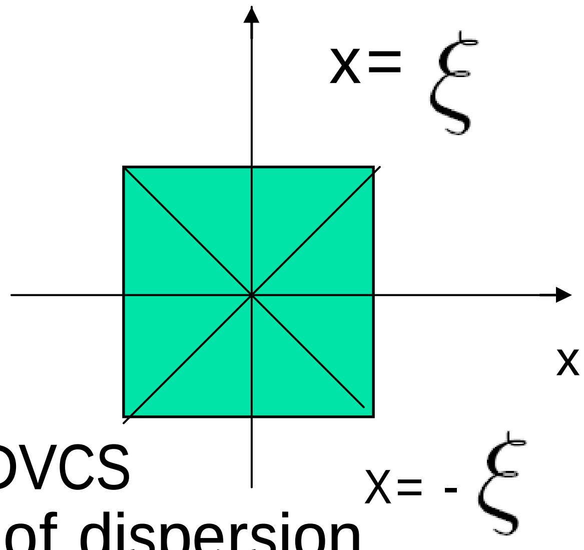
- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta \mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space (talk of S. Brodsky)?!



- New strategy of GPD's studies: start at diagonals (through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Angular distribution in hadron pairs production

- Back to GDA region

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- -> moments of H(x,x) - define the coefficients of powers of cosine!– $1/\xi$
- Higher powers of cosine in t-channel – threshold in s -channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1

Holographic property - IV

- Follows directly from DD -> preserved by (LO) evolution; NLO –Diehl, D.Ivanov'07
- Asymptotic GPD -> Pure real DVCS Amplitude (=subtraction term) growing like ξ^{-2}
- Direct consequence of finite asymptotic value of the quark momentum fraction

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\,\mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\,\mathcal{A}(\nu', Q^2)}{(\nu'^2-\nu^2)} + \Delta \qquad \Delta = 2\int_{-1}^{1} d\beta \frac{D(\beta)}{\beta - 1}$$

$$\Delta^p_{\mathrm{CQM}}(2) = \Delta^n_{\mathrm{CQM}}(2) \approx 4.4, \qquad \Delta^p_{\mathrm{latt}} \approx \Delta^n_{\mathrm{latt}} \approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!
- Stability of subtraction against NPQCD?twist -3 – related to twist 2 (Moiseeva, Polyakov'08)
- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

Is D-term independent?

- Fast enough decrease at large energy ->

$$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2-\nu^2} + \mathbf{C}_0$$

$$\mathbf{C}_0 = \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2} = \Delta + \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,x)}{x}.$$

- FORWARD limit of Holographic equation

$$\Delta = \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,0) - H^{(+)}(x,x)}{x} = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0) - H(x,x)}{x},$$

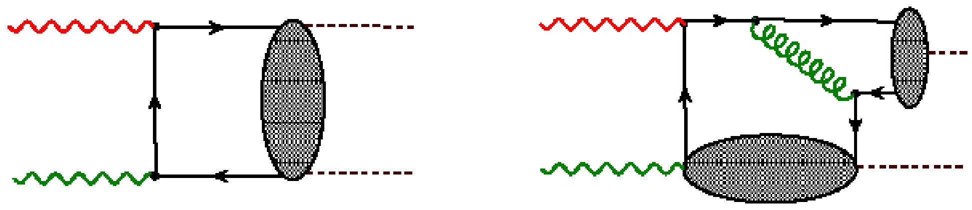
$$\mathbf{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0,t)}{x}$$

“D – term” 30 years before...

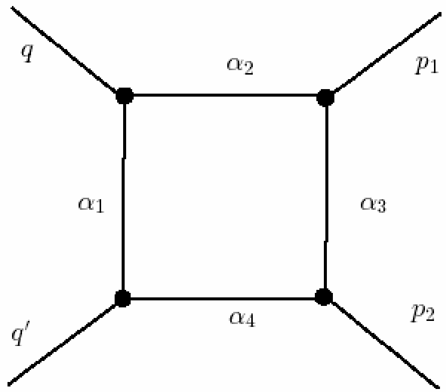
- Cf Brodsky, Close, Gunion’72
- D-term - typical renormalization constant
- May be calculated in effective theory if we know fundamental one
- OR
- Recover through special regularization procedure (D. Mueller)?

Duality between various QCD mechanisms (Anikin, Cherednikov, Stefanis, OT, in preparation)

- 2 pion production : GDA (small s) vs TDA+DA (small t)

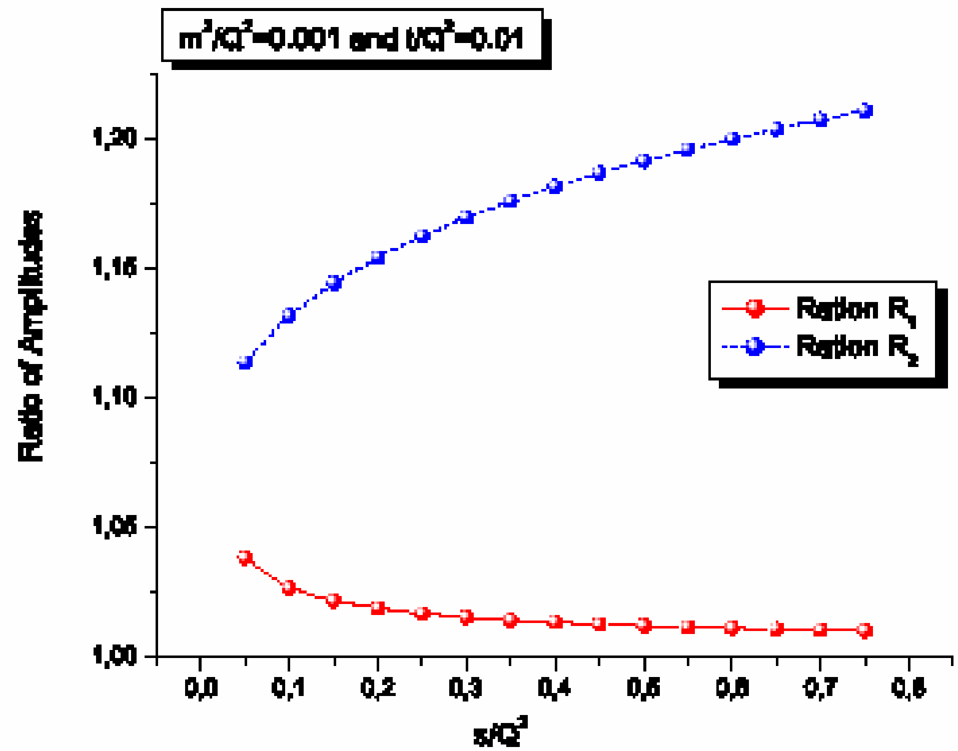


- Scalar model - asymptotics(Efremov, Ginzburg, Radyushkin...)



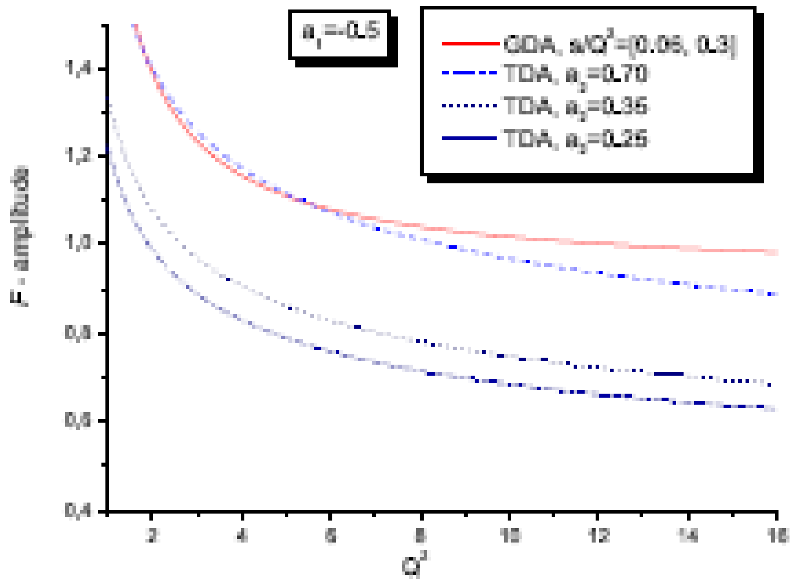
Duality in scalar model

- “Right” (TDA, red) and “wrong” (GDA, blue) asymptotics / exact result (>1- negative “Higher Twist”



Duality in QCD

- Qualitatively- surprisingly good, quantitatively - model-dependent



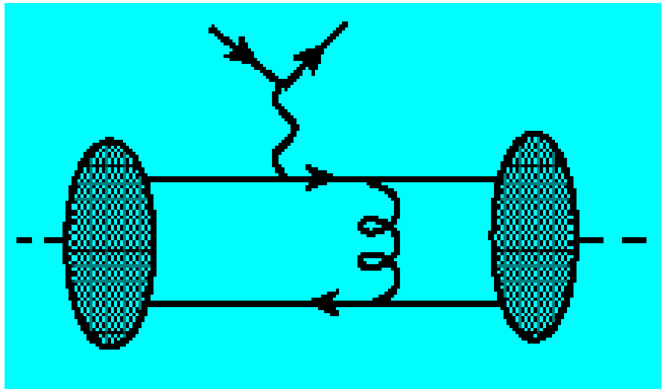
Applications of analyticity: complicated hard reactions

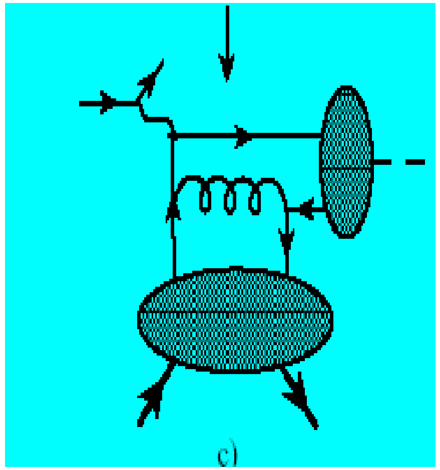
- Starting from (Pion) form factor- 2 DA's

$$F \sim \left(\int dx \frac{\phi(x)}{1-x}\right)^2$$

- 1 DA -> GPD :Exclusive mesons production (Frankfurt, Strikman): analytic continuation=factorization + D-subtraction

$$M \sim \int dx \frac{\phi(x)}{1-x} \int dx \frac{H(x,\xi)}{x-\xi+i\varepsilon}$$





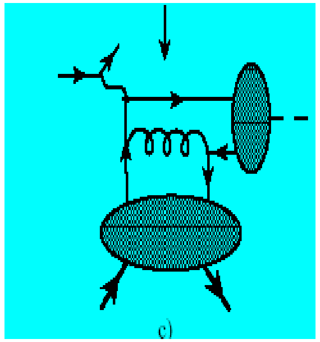
Next step: 2 DA’s -> 2 GPD’s- Double Diffraction

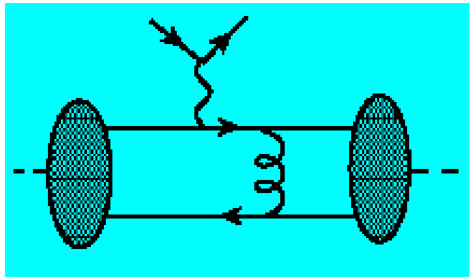
- Exclusive double diffractive DY process
- Analytic continuation:

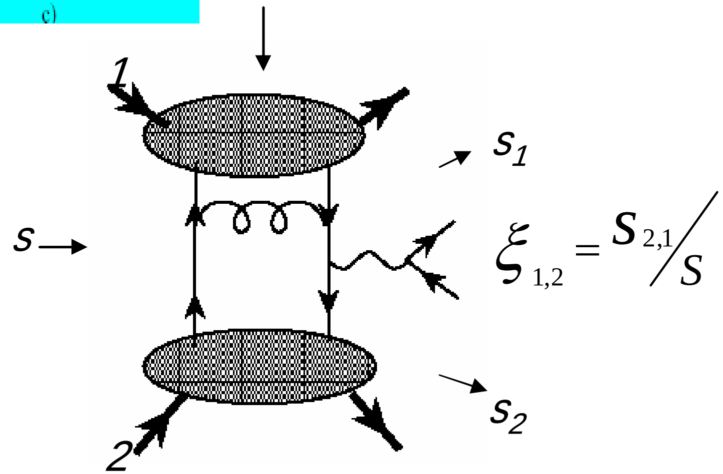
$$M \sim \int dx \frac{H(x,\xi_1)}{x-\xi_1 \pm i\varepsilon} \int dy \frac{H(y,\xi_2)}{y-\xi_2 \mp i\varepsilon}$$

- DIFFERS from direct calculation – NO factorization in physical region

$$M \sim \int\int dxdy \frac{H(x,\xi_1)H(y,\xi_2)}{(x-\xi_1)(y-\xi_2)+i\varepsilon}$$

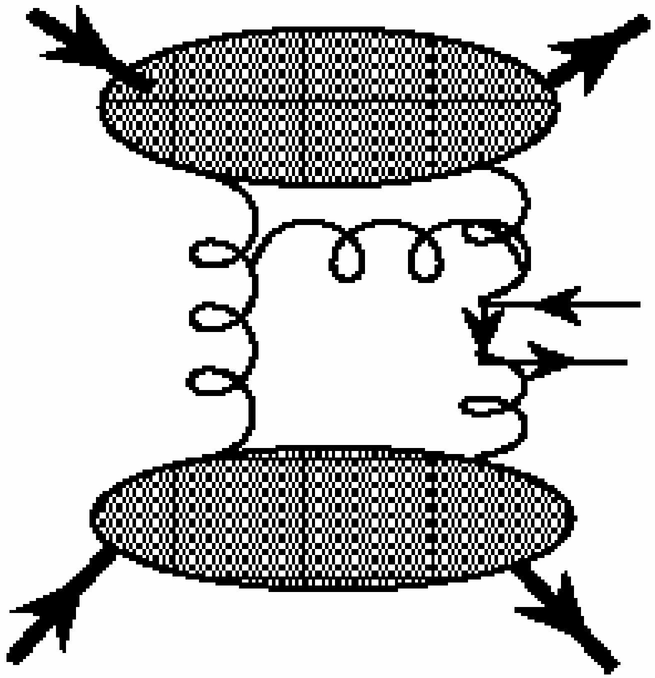






Double Diffraction: gluons

- One or both GPDs may be gluonic
- Complementary description of LHC DD (Higgs, Quarkonia, dijets)



Double Diffraction: properties and problems

- Holographic equation: DR contains double and single (linear in D-term) dispersion integrals as well as subtraction (quadratic in D-term)
- Analytic continuation in relation to

various cuts is still unclear...

GPDs and FF (talks of M. Vanderhaeghen, B.Pasquini)

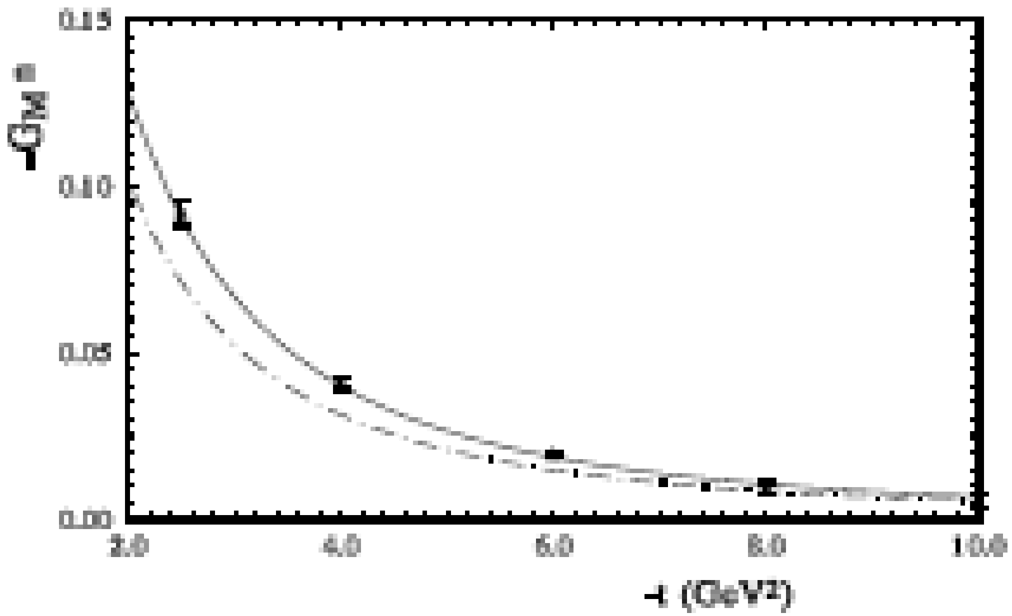
- Simple model for t-dependence (Selyugin, OT'07)

$$\mathcal{H}^q(x,t) = q(x)\ exp[a_+ \frac{(1-x)^2}{x^b}\ t]; \quad \mathcal{E}^q(x,t) = \mathcal{E}^q(x)\ exp[a_- \frac{(1-x)^2}{x^b}\ t]$$

- SAME t-dependence for u and d
- Difference only in t slope of E - description of Rosenbluth and polarisation data (talk of L. Pentchev)

Neutron vs Proton

- Neutron GE – (much?) better described by the slope obtained from proton polarisation data



1-st moments - EM, 2-nd - Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle \;=\; \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe ainteraction with both classical and TeV gravity

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \qquad M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2 e_q P^\mu \qquad \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2 e_q M \phi(q) \qquad M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’) – not checked on purpose but in fact checked in atomic spins experiments at % level (Silenko,OT’07)
- Anomalous gravitomagnetic moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Gravitomagnetism

- Gravitomagnetic field – action on spin – ½ from $M = \frac{1}{2}\sum_{q,G} \langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$ spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM $\vec{H}_L = rot\vec{g}$
- Orbital and Spin momenta dragging – the same - Equivalence principle $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L$

Equivalence principle for moving particles

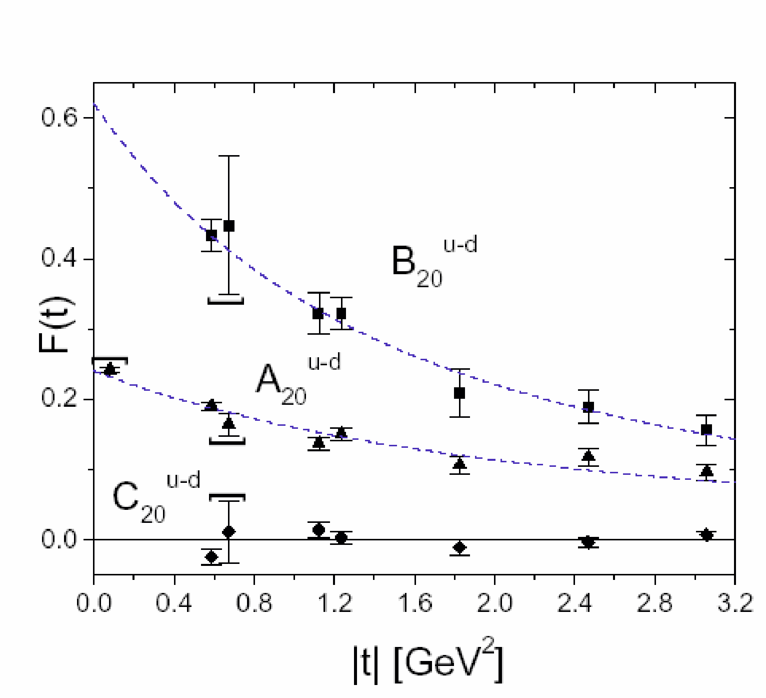
- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

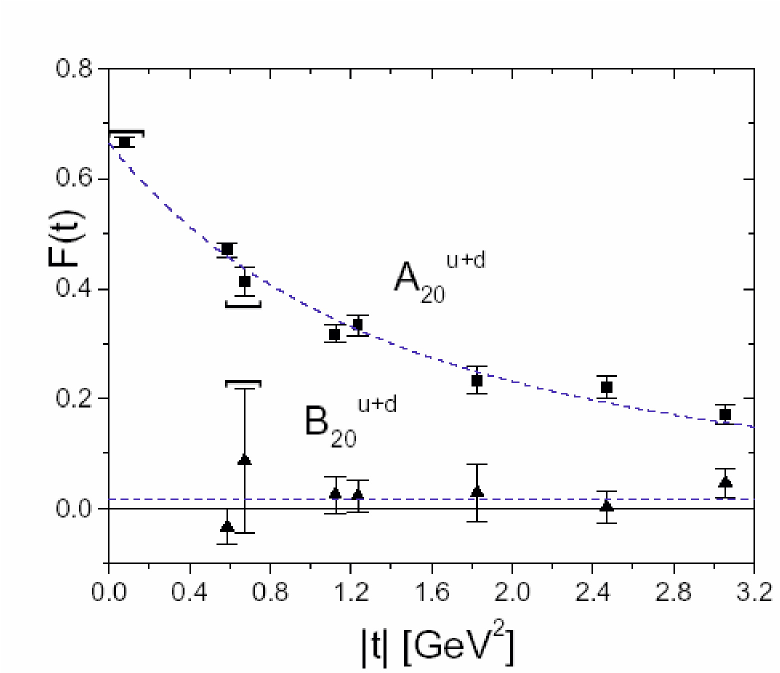
$$\mathcal{M}_g = (\epsilon^2 + p^2)h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, O.T.)

Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of EEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Supported by smallness of E (isoscalar AMM)
- Polyakov Vanderhaeghen: dual model with E=0

Vector mesons and EEP

- J=1/2 -> J=1. QCD SR calculation of Rho's AMM gives g close to 2.

- Maybe because of similarity of moments
- $g-2=\langle E(x)\rangle$; $B=\langle xE(x)\rangle$
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP: Gluons momentum fraction sizable. Direct calculation of AGM are in progress.

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

EEP and Sivers function (talks of U. d'Alesio, S. Scopetta)

- Sivers function – process dependent (effective) one
- T-odd effect in T-conserving theory- phase
- FSI – Brodsky-Hwang-Schmidt model
- Unsuppressed by M/Q twist 3
- Process dependence- colour factors
- After Extraction of phase – relation to universal (T-even) matrix elements

EEP and Sivers function -II

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**): $x f_{T}(x) \sim xE(x)$
- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_{T}(x) = \sum_{q,G} \int dx x E(x) = 0$$

EEP and Sivers function for deuteron

- EEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T^{\mu\rho}_B - x^\rho T^{\mu\nu}_B$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu}) \partial^\rho (J^\alpha_{5S} x^\nu) = 0$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha} \langle P|J^\alpha_{5S}|P+q\rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P|J^\gamma_{5S}|P+q\rangle$$

$$\langle P,S|J^5_\mu(0)|P+q,S\rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2,$$

$$q^2 G_2|_0 = 0$$

CONCLUSIONS

- Crossing analogs of GPD -> GDA
- Analyticity for DVCS – holographic property of GPD's: special role of sections $x = \pm\xi$
- Analytic continuation – QCD calculations of double diffractive processes: Drell-Yan, Higgs, Heavy flavours production.
- Gravitational formfactors – possible link between gravity and NP (and AdS) QCD