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Electromagnetic Form Factors of Hadrons in Quantum Field Theories.

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Electromagnetic Form Factors of Hadrons in Quantum Field Theories *

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* This talk draws on work done in collaboration with J.I. Jottar, M. Loewe, R. Röntsch, B. Willlers, Y.Zhang

ICTP 2008

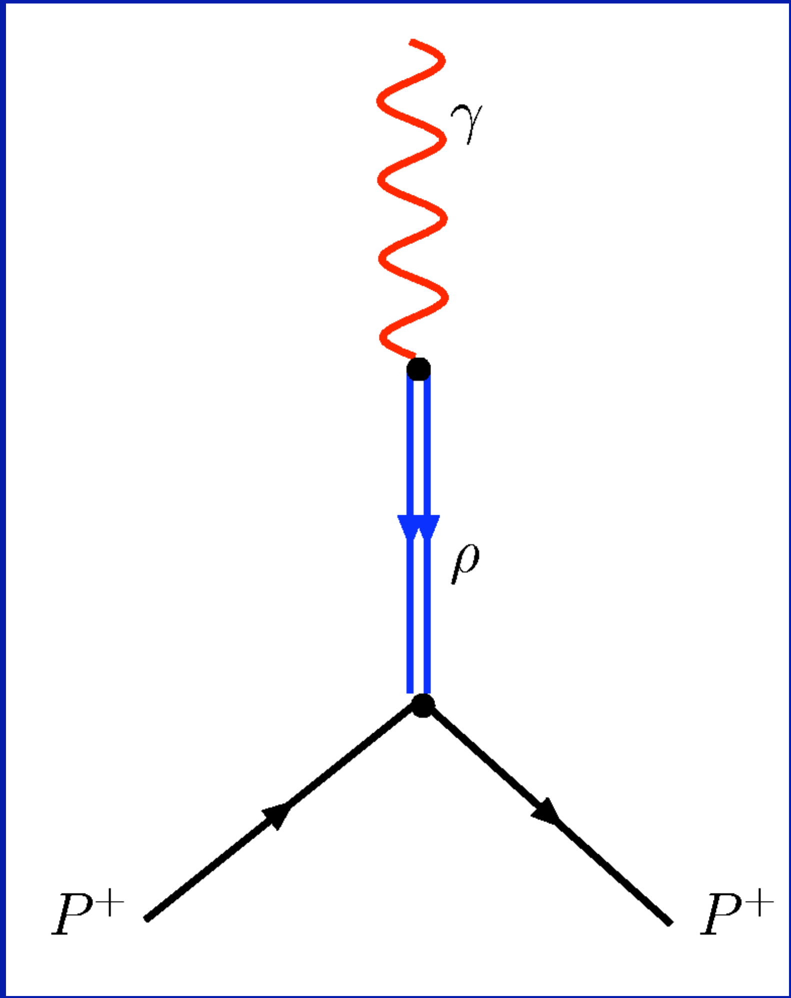
Two different Quantum Field Theory (QFT) Models

- **Kroll-Lee-Zumino Model**
- Abelian, Renormalizable QFT
- Platform to justify & extend beyond tree-level the well known Vector Meson Dominance (VMD) Model
- A viable alternative to non-renormalizable QFT (effective) models (e.g. Chiral Perturbation Theory)

- **Dual Large N_c QCD (QCD_∞)**
- Realization of QCD_∞ inspired in the Dual Resonance Model (Veneziano)
- NOT an expansion in N_c. $N_c = \infty$ *ab initio,* although finite-width corrections can be incorporated
- NOT the Veneziano model for hadronic scattering

VECTOR MESON DOMINANCE

- Abelian, TREE-LEVEL model
- No truly QFT platform
- Not subject to PERTURBATION THEORY improvement



$$F_\pi(q^2) = \frac{M_\rho^2}{M_\rho^2 - q^2} \quad \frac{g_{\rho\pi\pi}(q^2)}{f_\rho}$$

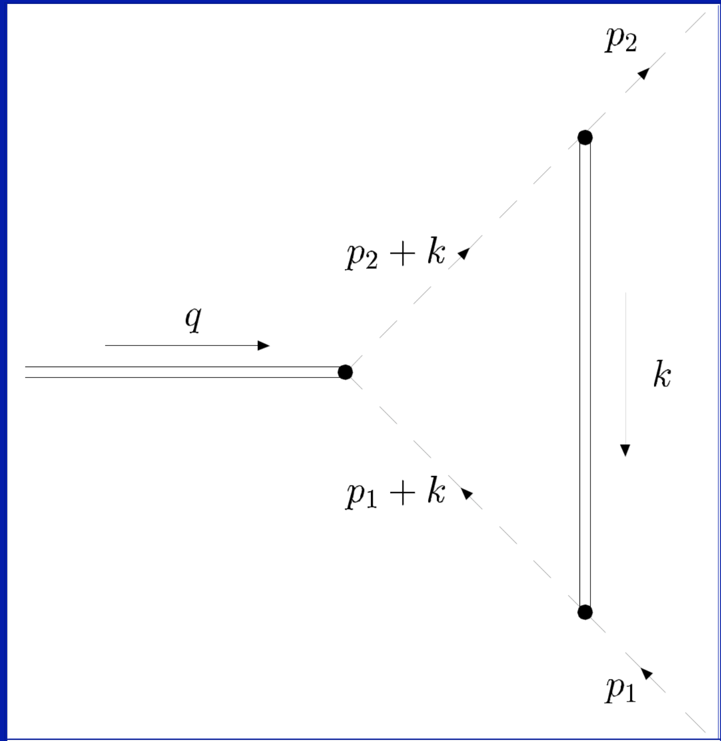
KROLL – LEE – ZUMINO (KLZ) QFT MODEL

$$
\begin{aligned}
\mathcal{L}_{KLZ} &= \partial_\mu \phi \, \partial^\mu \phi^* - m_\pi^2 \, \phi \, \phi^* \\
&- \tfrac{1}{4} \, \rho_{\mu\nu} \, \rho^{\mu\nu} + \tfrac{1}{2} \, m_\rho^2 \, \rho_\mu \, \rho^\mu \\
&+ \, g_{\rho\pi\pi} \, \rho_\mu \, J_\pi^\mu \\
&+ \, g_{\rho\pi\pi}^2 \, \rho_\mu \, \rho^\mu \, \phi \, \phi^*
\end{aligned}
$$

$$
\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu
$$

$$
J_\pi^\mu = i \phi^* \overleftrightarrow{\partial_\mu} \phi
$$

$$
\partial_\mu \rho^\mu = 0
$$



p_2
$p_2 + k$
q
k
$p_1 + k$
p_1

CALCULATING IN KLZ

- Regularization using **DIMENSIONAL REGULARIZATION**
- Renormalization (fields, masses, couplings)
- Renormalization subtraction point for vertex diagram: $q^2 = 0$
- Renormalization subtraction point for vacuum polarization diagram: $q^2 = M^2_\rho$

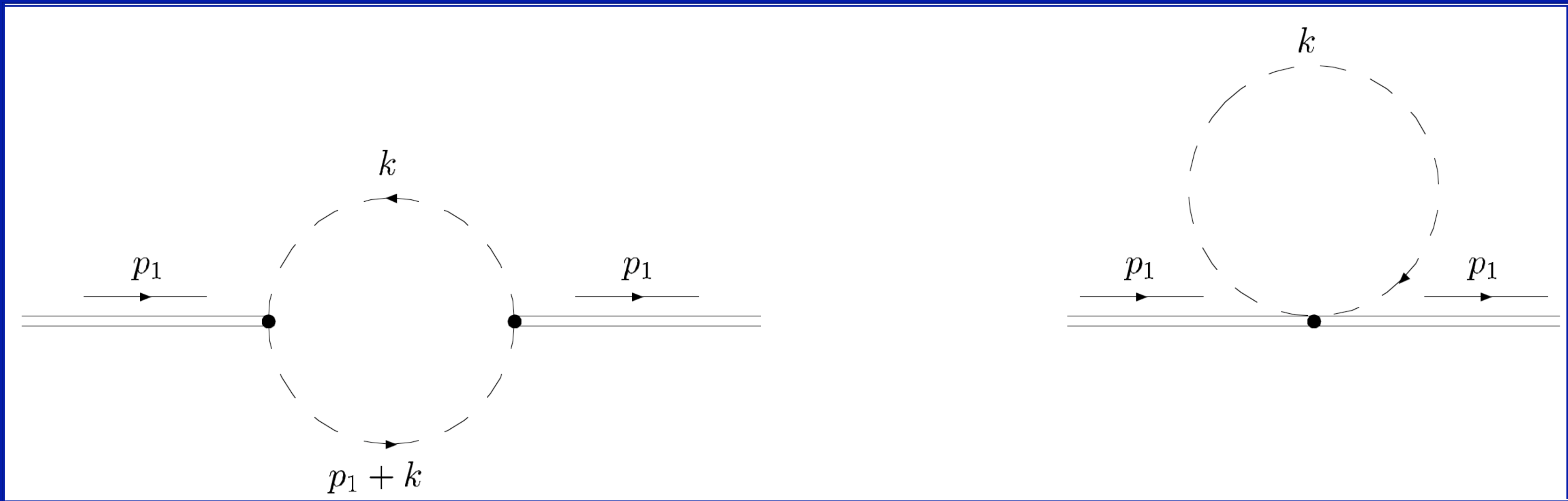
$$F_\pi(q^2)|_{\text{VMD}} = \frac{g_{\rho\pi\pi}}{f_\rho}\,\frac{M_\rho^2}{M_\rho^2 - q^2}$$

$$F_\pi(q^2)|_{\text{vertex}} = \frac{g_{\rho\pi\pi}}{f_\rho}\,\frac{M_\rho^2}{M_\rho^2 - q^2}$$

$$\times\ [1 + G(q^2) - G(0)]$$

$$f_\rho = 4.97 \pm 0.07$$

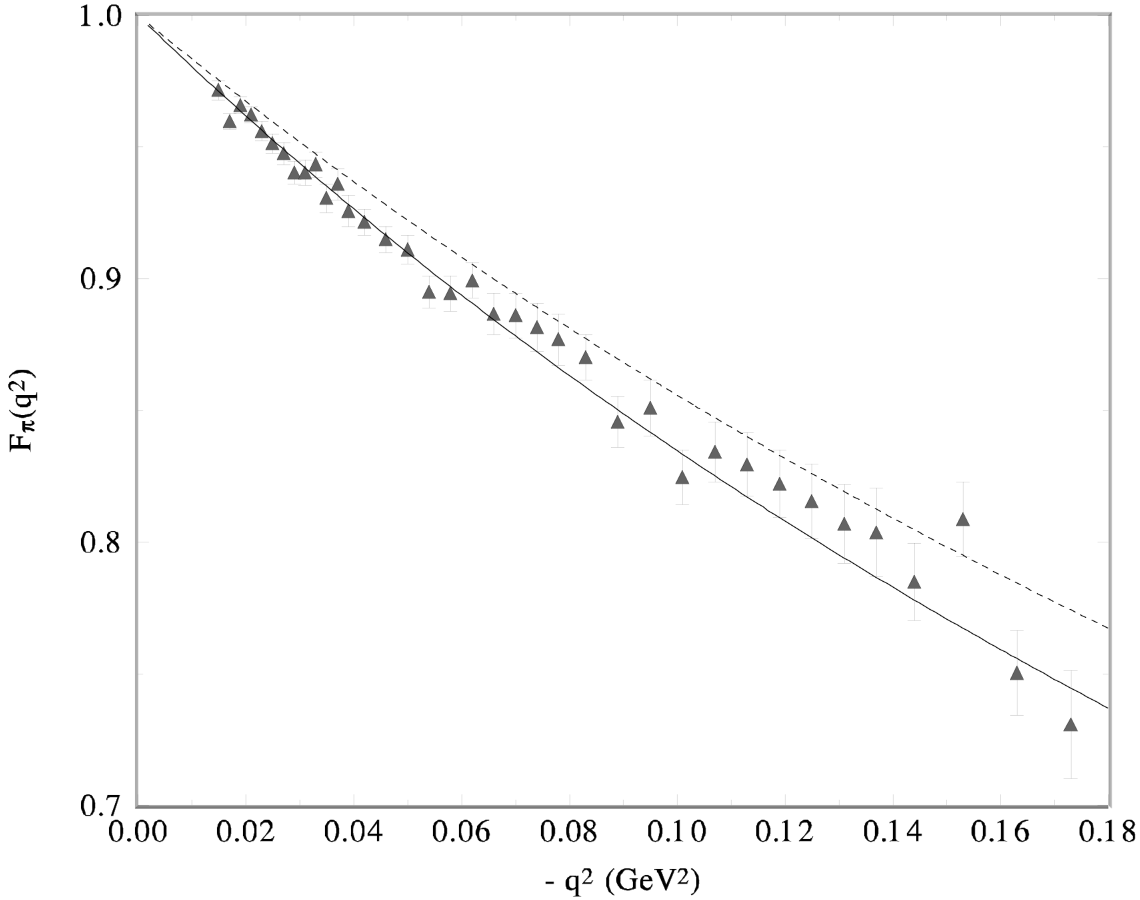
$$F_\pi(0) = 1 \longrightarrow g_{\rho\pi\pi}(0) = f_\rho$$



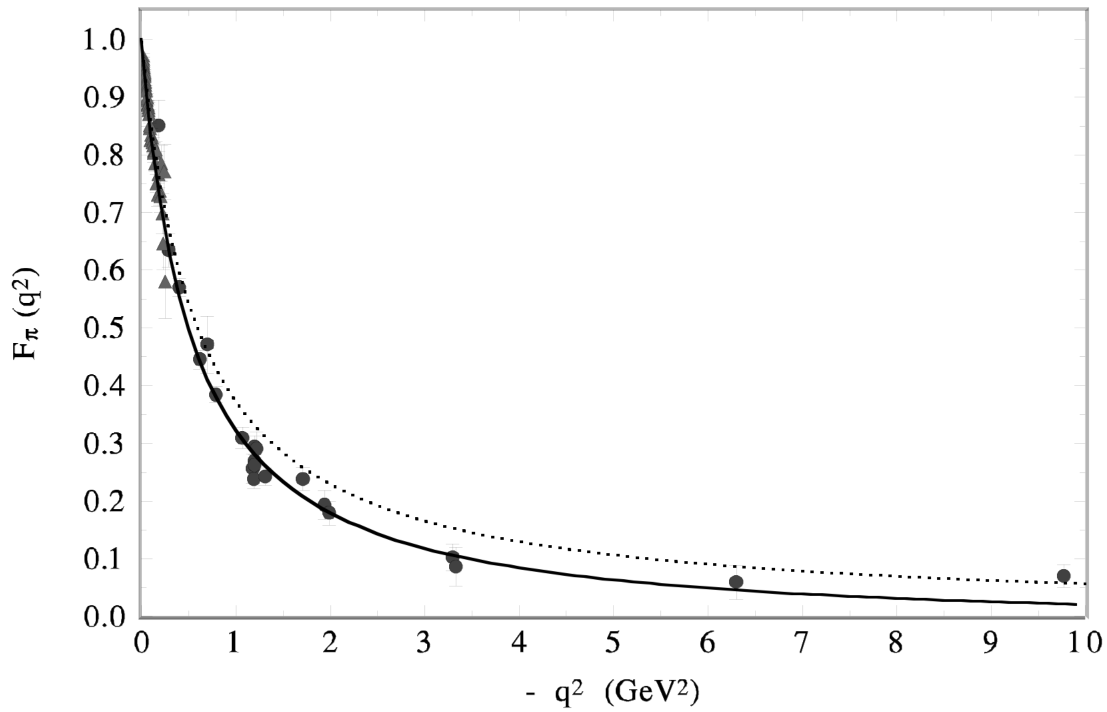
k
p_1
p_1
$p_1 + k$
k
p_1
p_1

$$F_\pi(q^2) = \frac{M_\rho^2 + \Pi(0)|_{\rm vac}}{M_\rho^2 - q^2 + \Pi(q^2)|_{\rm vac}} + \frac{M_\rho^2}{M_\rho^2 - q^2}$$

$$\times \left[G(q^2) - G(0) \right]$$



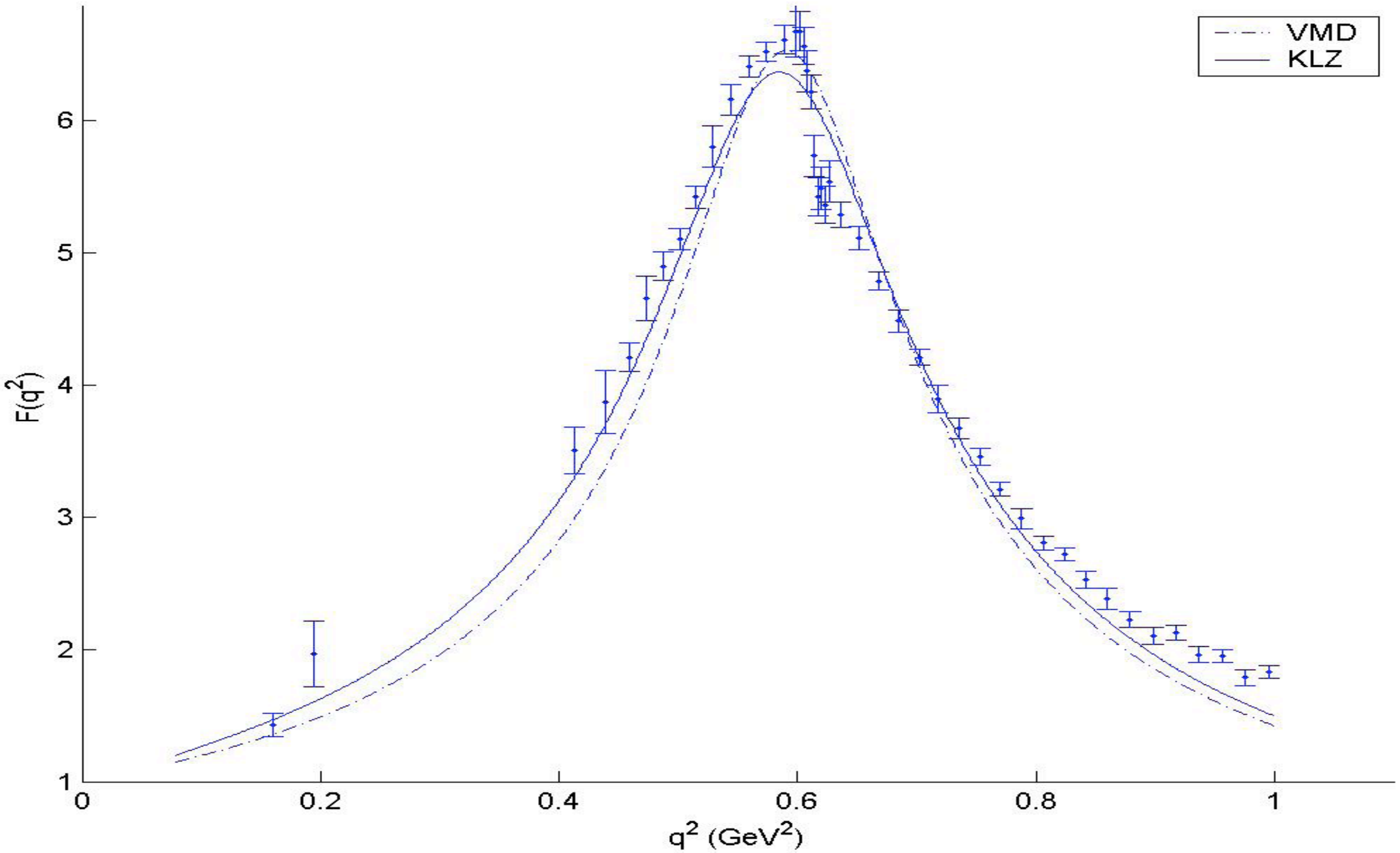
1.0
0.9
0.8
0.7
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
Fπ(q²)
- q² (GeV²)



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
F_π (q^2)
0
1
2
3
4
5
6
7
8
9
10
- q^2 (GeV^2)

Gounaris-Sakurai empirical width

$$\Gamma_\rho(s)|_{KLZ} = \frac{M_\rho\,\Gamma_\rho}{\sqrt{s}}\left[\frac{s - 4\,\mu_\pi^2}{M_\rho^2 - 4\,\mu_\pi^2}\right]^{\frac{3}{2}}$$



VMD
KLZ
F(q²)
q² (GeV²)
1
2
3
4
5
6
0
0.2
0.4
0.6
0.8
1

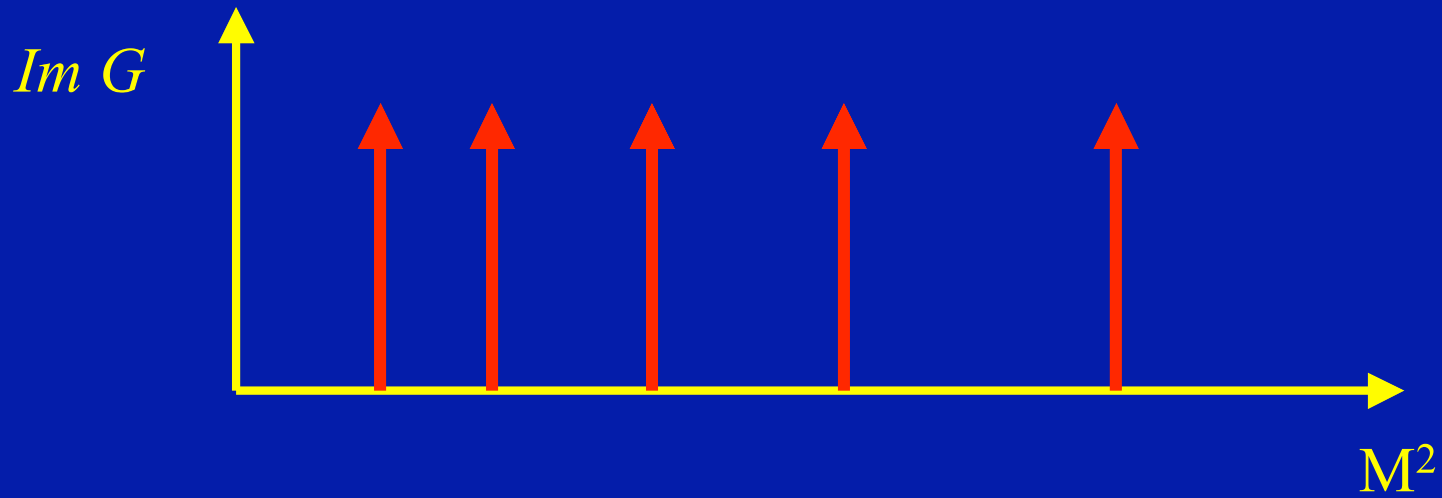
- KLZ: Strong coupling theory
- $g \approx 5$ & $1/(4\pi)^2$ per loop
- Hence: well defined (convergent) perturbative expansion

DUAL – LARGE N_c QCD

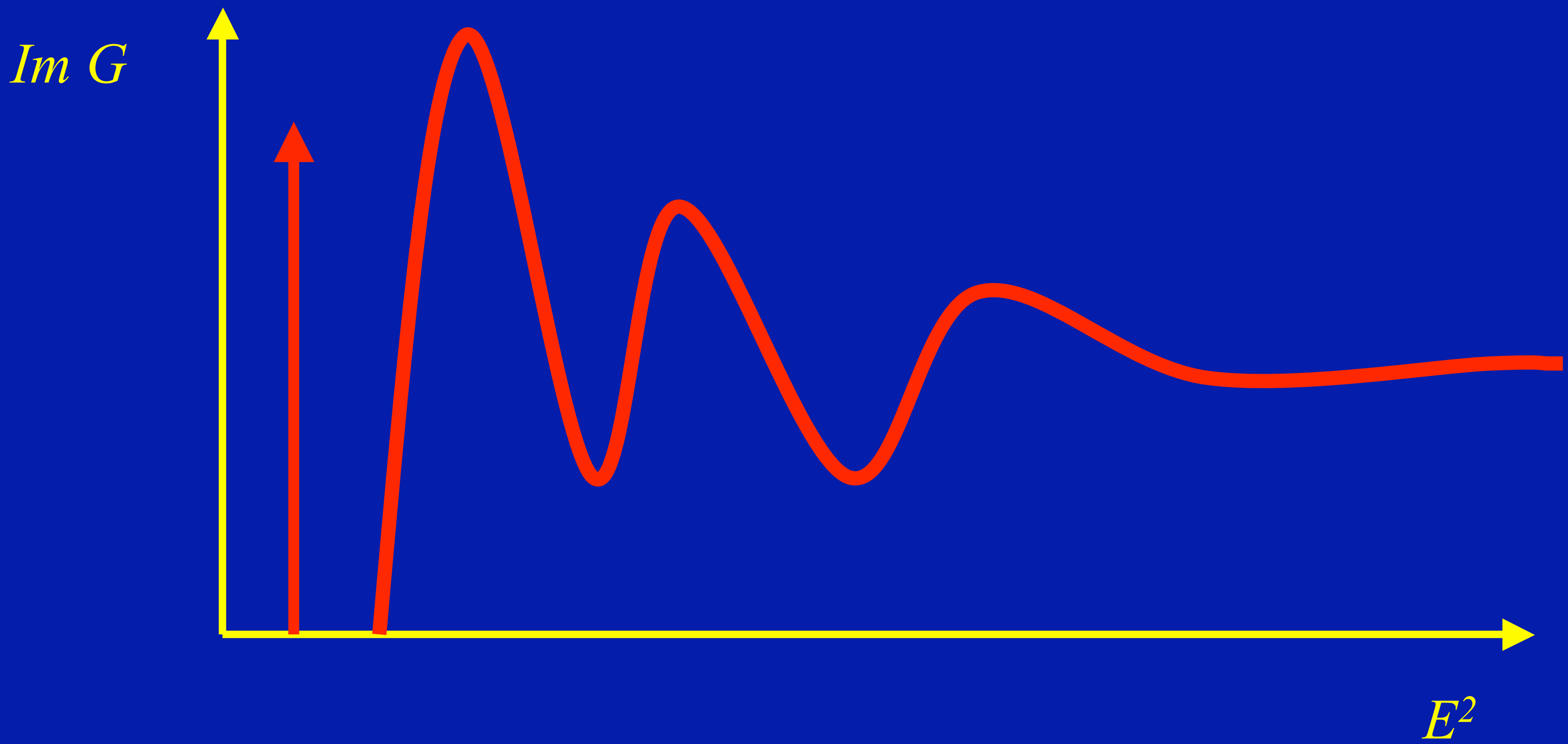
QCD ∞

QCD $_\infty$

- *Lim* $N_c \to \infty$ ($N_c = 3$) (t'Hooft '74 & Witten '79)
- Spectrum: ∞ number of zero width resonances



Real Spectral Function



CORRECTIONS to $1/N_c$

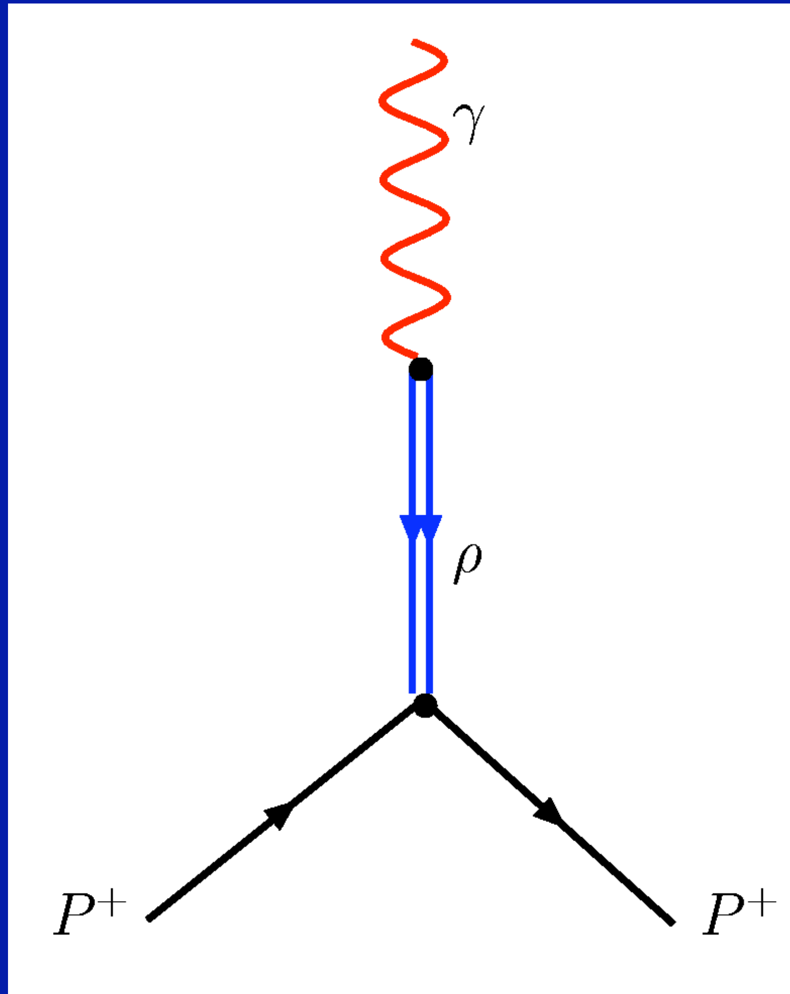
$$\Gamma / M \approx 10\,\%$$

RESONANCES

- γ - p^+ coupling :

  γ - ρ^0 - p^+

- ρ^0 : $J^{PC} = 1^{--}$
- **Mρ** $\approx$ 770 MeV
- **Mρ'** $\approx$ 1340 MeV
- **Mρ''** $\approx$ 1720 MeV
- **Mρ'''** $\approx$ 2034 MeV



Dual - QCD ∞

- Dual Resonance Model
  Veneziano (1968)
- ∞ number of zero width resonances, equally spaced
- Masses & couplings fixed to give an Euler Beta Function

$$M_n^2 = M_\rho^2(1+2n)$$

Mρ = 769 MeV

Mρ' ≈ 1340 MeV **[EXP.: 1465 ± 25 MeV]**

Mρ'' ≈ 1720 MeV **[EXP.: 1700 ± 20 MeV]**

Mρ''' ≈ 2034 MeV **[EXP.: 2149 ± 17 MeV]**

PION FORM FACTOR

Dual-Large N_c QCD

$$< \pi(p_2)|J_\mu^{EM}|\pi(p_1) >= (p_1+p_2)_\mu \;\; F_\pi(s)$$

$$F_\pi(s) = \sum_{n=0}^{\infty} \frac{C_n}{(M_n^2 - s)}$$

DUAL QCD$_\infty$

$$C_n = \frac{\Gamma(\beta - 1/2)}{\alpha' \sqrt{\pi}} \, \frac{(-1)^n}{\Gamma(n+1)} \, \frac{1}{\Gamma(\beta - 1 - n)}$$

$$\alpha' = 1/2{M_\rho}^2$$

$$\alpha_\rho(s) = 1 + \alpha'(s - M_\rho^2)$$

$$M_n^2 = M_\rho^2(1+2n)$$

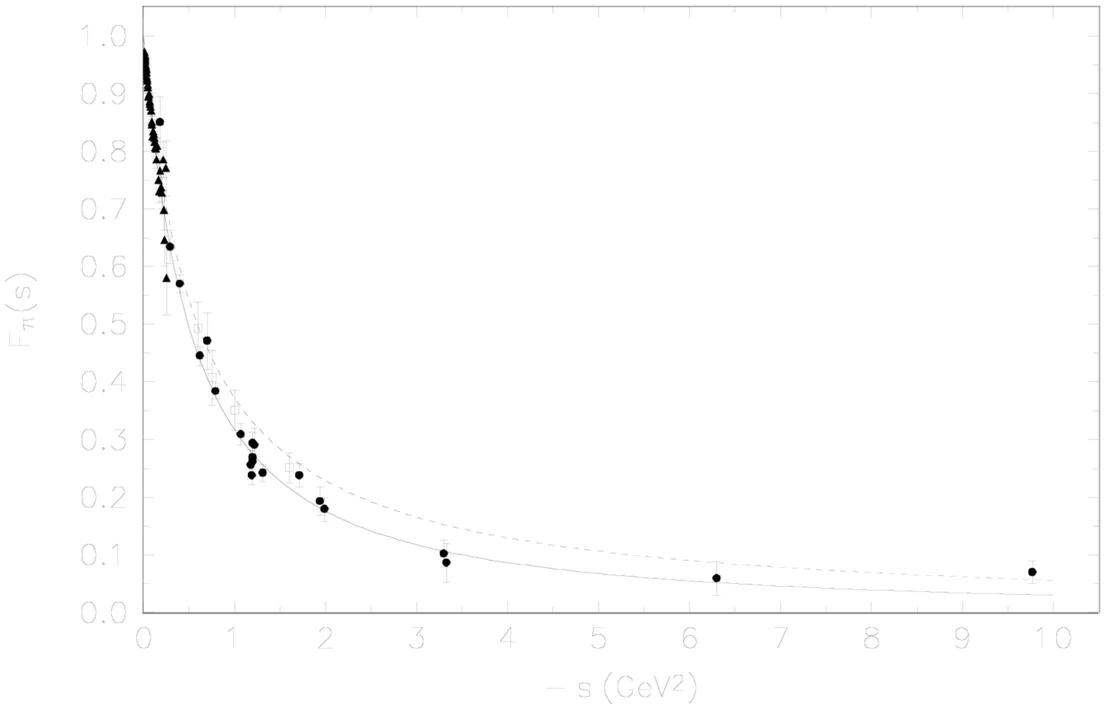
$$F_\pi(s) = \frac{\Gamma(\beta - 1/2)}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)} \frac{1}{\Gamma(\beta - 1 - n)}$$

$$\times \frac{1}{[n + 1 - \alpha_\rho(s)]}$$

$$= \frac{1}{\sqrt{\pi}} \frac{\Gamma(\beta - 1/2)}{\Gamma(\beta - 1)} \; B(\beta - 1,\; 1/2 - \alpha' s)$$

$$\lim_{s \to -\infty} F_\pi(s) = (-\alpha'\, s)^{(1-\beta)}$$

Figure 1



$$Im\ F_\pi(s)\ =\ \frac{\Gamma(\beta - 1/2)}{\alpha'\sqrt{\pi}}\ \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)}$$

$$\times\ \frac{1}{\Gamma(\beta - 1 - n)}\ \pi\ \delta(M_n^2 - s)$$

$$\pi\delta(M_n^2 - s) \to \frac{\Gamma_n M_n}{[(M_n^2 - s)^2 + \Gamma_n^2 M_n^2]}$$

$$Re\ F_\pi(s) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{Im\ F_\pi(s')}{(s' - s)}\, ds'$$

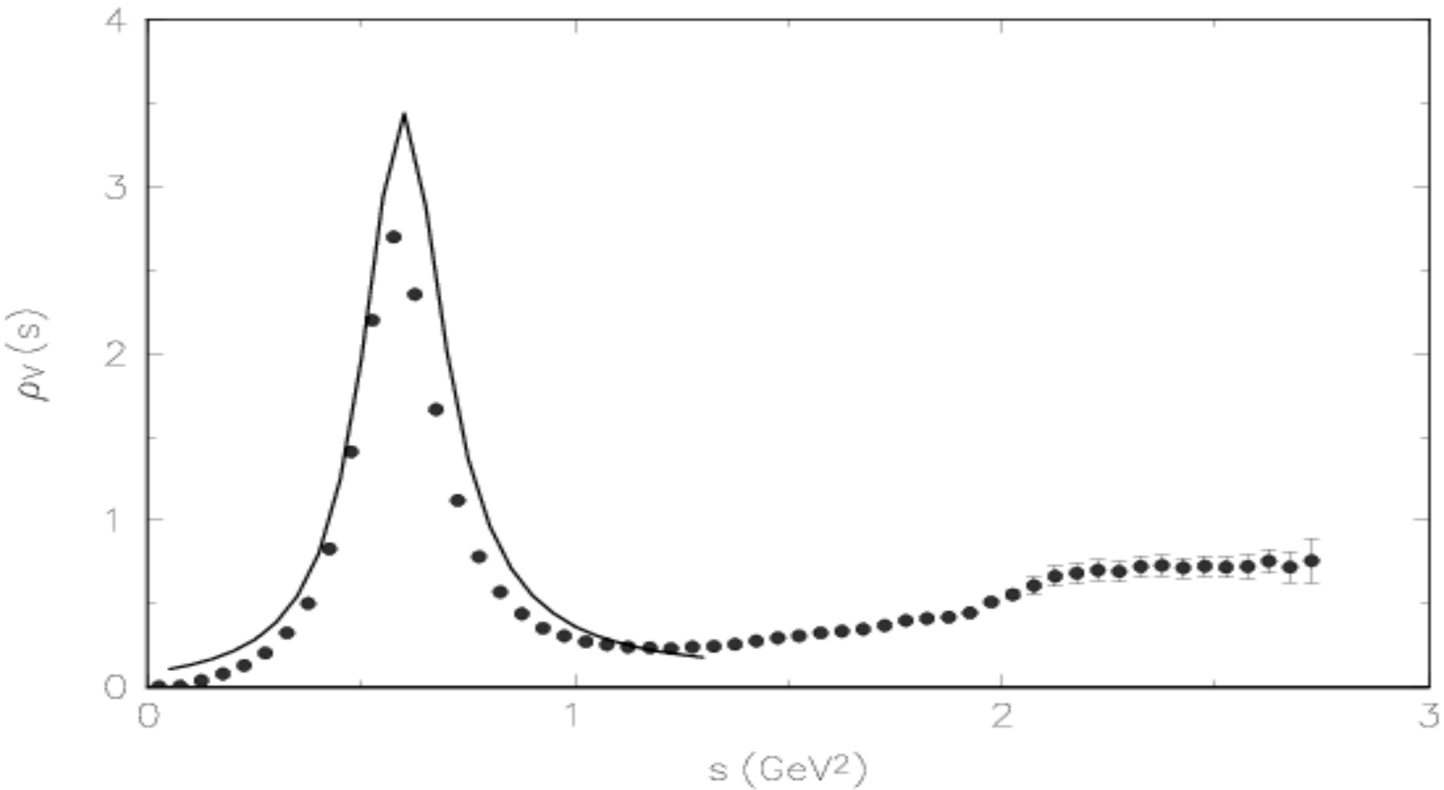
$$\Pi_{\mu\nu}^{VV}(q^2) = i\int d^4x e^{iqx} \; < 0|T(V_\mu(x)\; V_\nu^\dagger(0))|0 >$$
$$= \; (-g_{\mu\nu}\, q^2 + q_\mu q_\nu)\; \Pi_V(q^2)$$

$$|F_\pi(s)|^2 = 12\; \rho_V(s)$$

$$\int_0^{s_0} \rho_V(s)|_{QCD_\infty}\, ds \; = 1.1 \text{ GeV}^2$$

$$\int_0^{s_0} \rho_V(s)|_{EXP}\, ds \; = 0.9 \text{ GeV}^2$$

Figure 2



PROTON
ELECTRIC & MAGNETIC FORM FACTORS

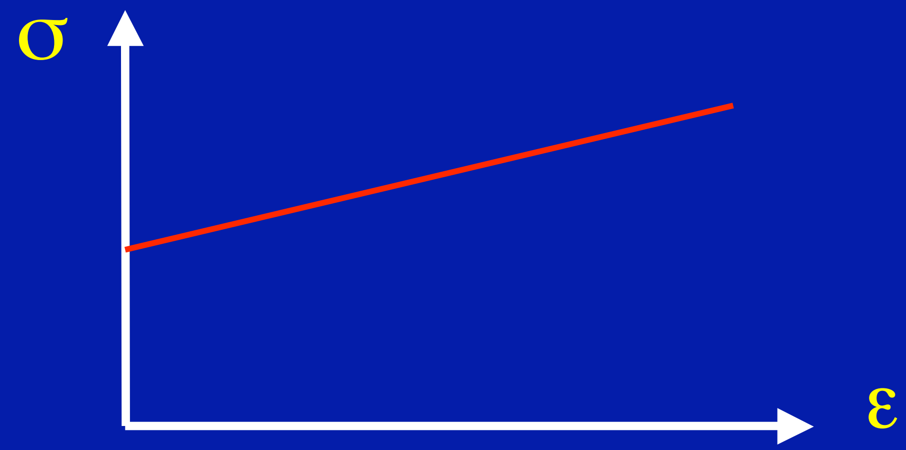
- $<P_f \mid J_\mu^{(p+)} \mid P_i> \Rightarrow <P_f \mid \gamma_\mu F_1(q^2)$
- $+ i \sigma_{\mu\nu} q^\nu a F_2(q^2)/M \mid P_i>$

$e^- p^+$ CROSS SECTION

- $G_E (q^2) = F_1(q^2) + (a\, q^2/4M^2)\, F_2(q^2)$

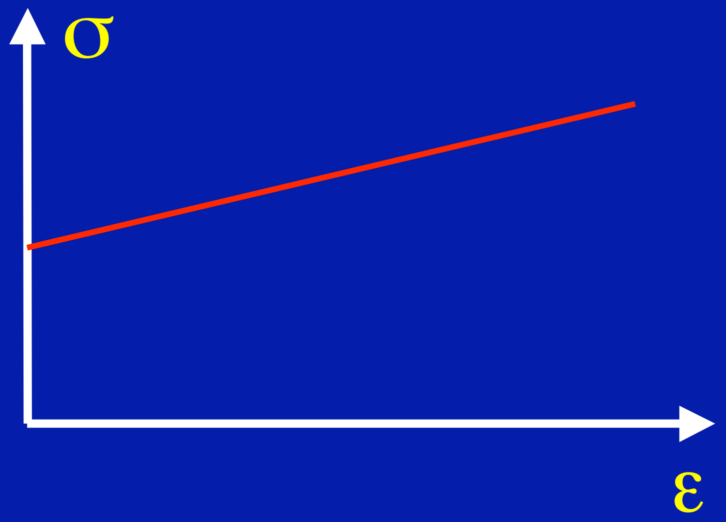
- $G_M (q^2) = F_1(q^2) + a\, F_2(q^2)$

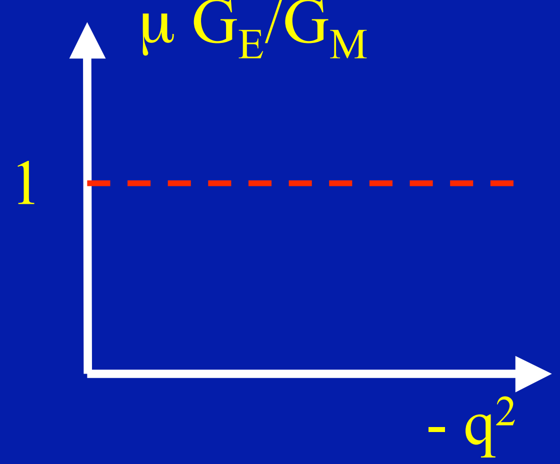
- $\sigma_R = (-\, q^2/4M^2)\, G^2_M (q^2) + \varepsilon\, G^2_E (q^2)$



ROSENBLUTH METHOD

- Unpolarized $e^- p^+$ scattering
- Measure σ_R for constant q^2 varying ε
- Determine $G_M(q^2)$ from intercept
- Determine $G_E(q^2)$ from slope
- Assume *Scaling Law* : $\mu G_E / G_M = 1$





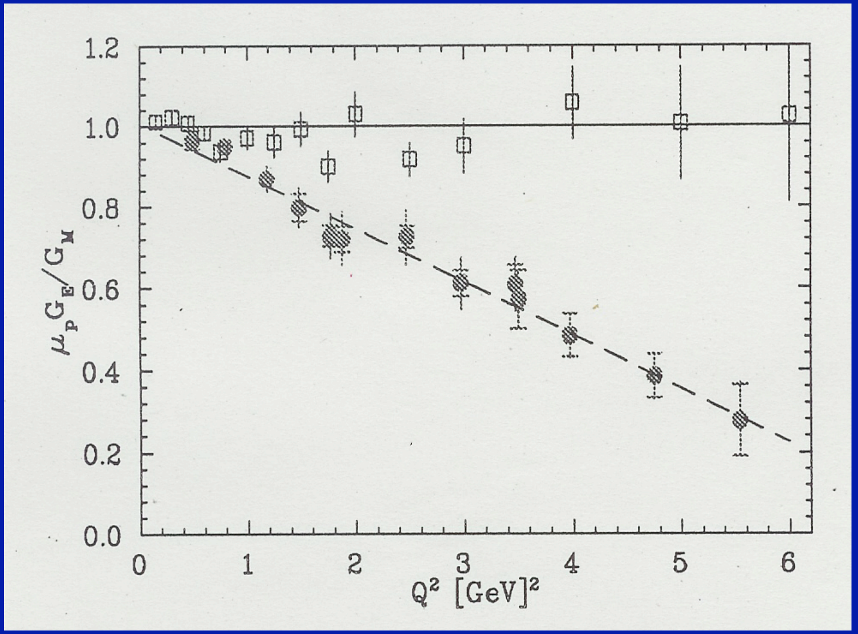
Polarized $e^- p^+$ Scattering
Jefferson Lab

- Measure longitudinal & transverse polarizations of the recoil proton: P_l, P_t

. $\mu\, G_E / G_M \propto P_t \,/\, P_l$

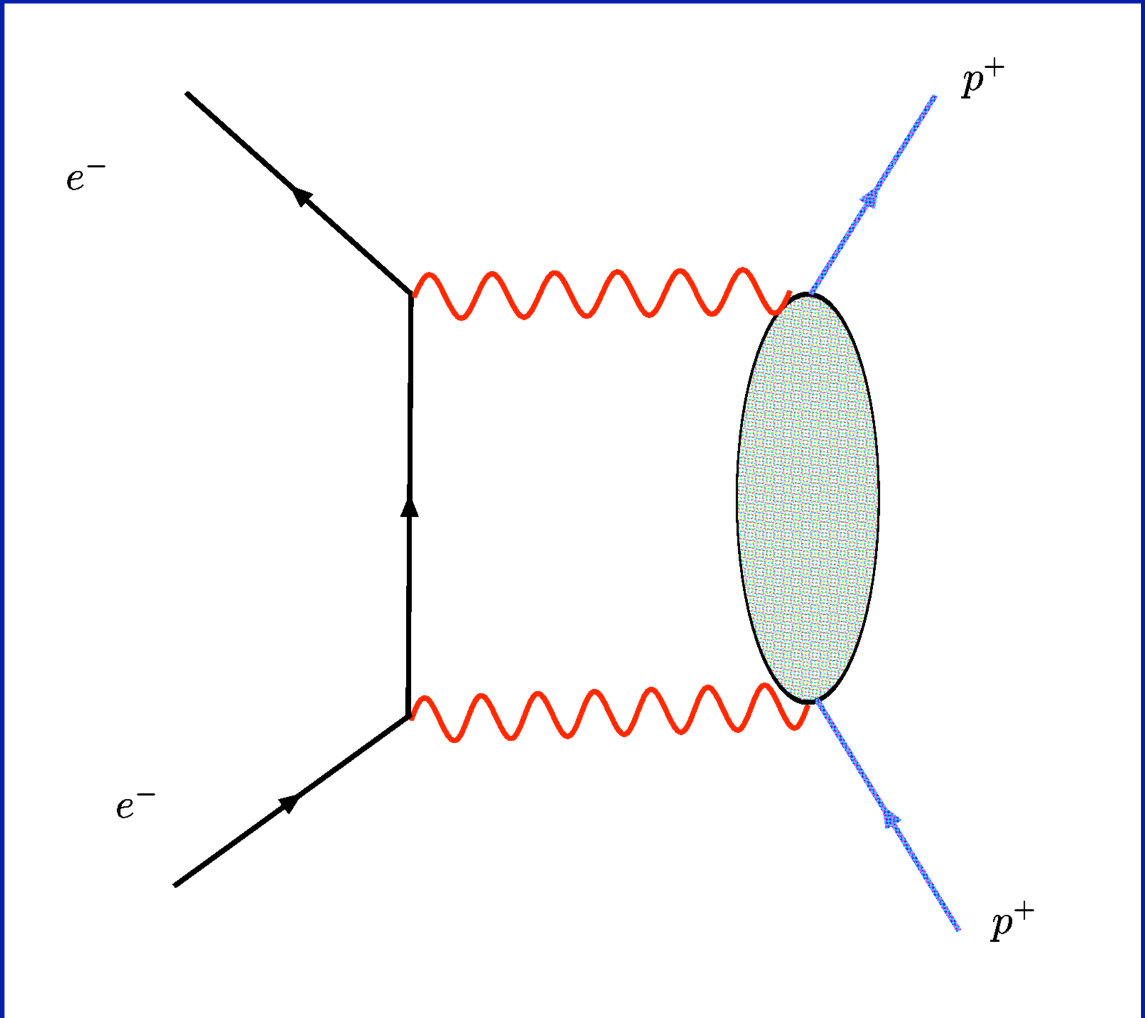
. $\mu\, G_E / G_M \approx 1 + 0.13\, q^2$

- A zero at $- q^2 \approx 8\ \mathrm{GeV}^2$



Reconciliation between Rosenbluth & Polarization Measurements

Second order correction more important in Rosenbluth than in Polarization

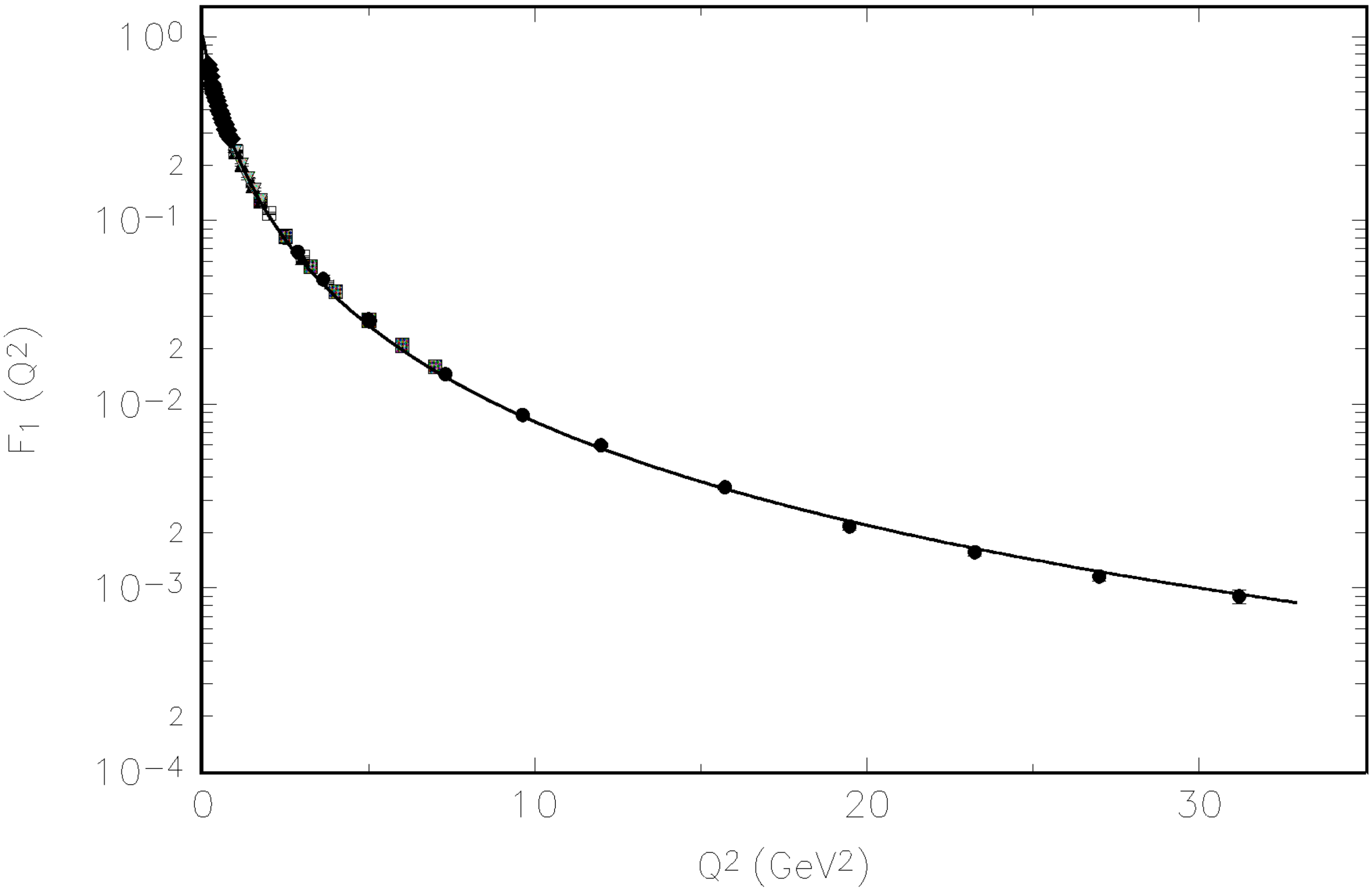


Nucleon Form Factors

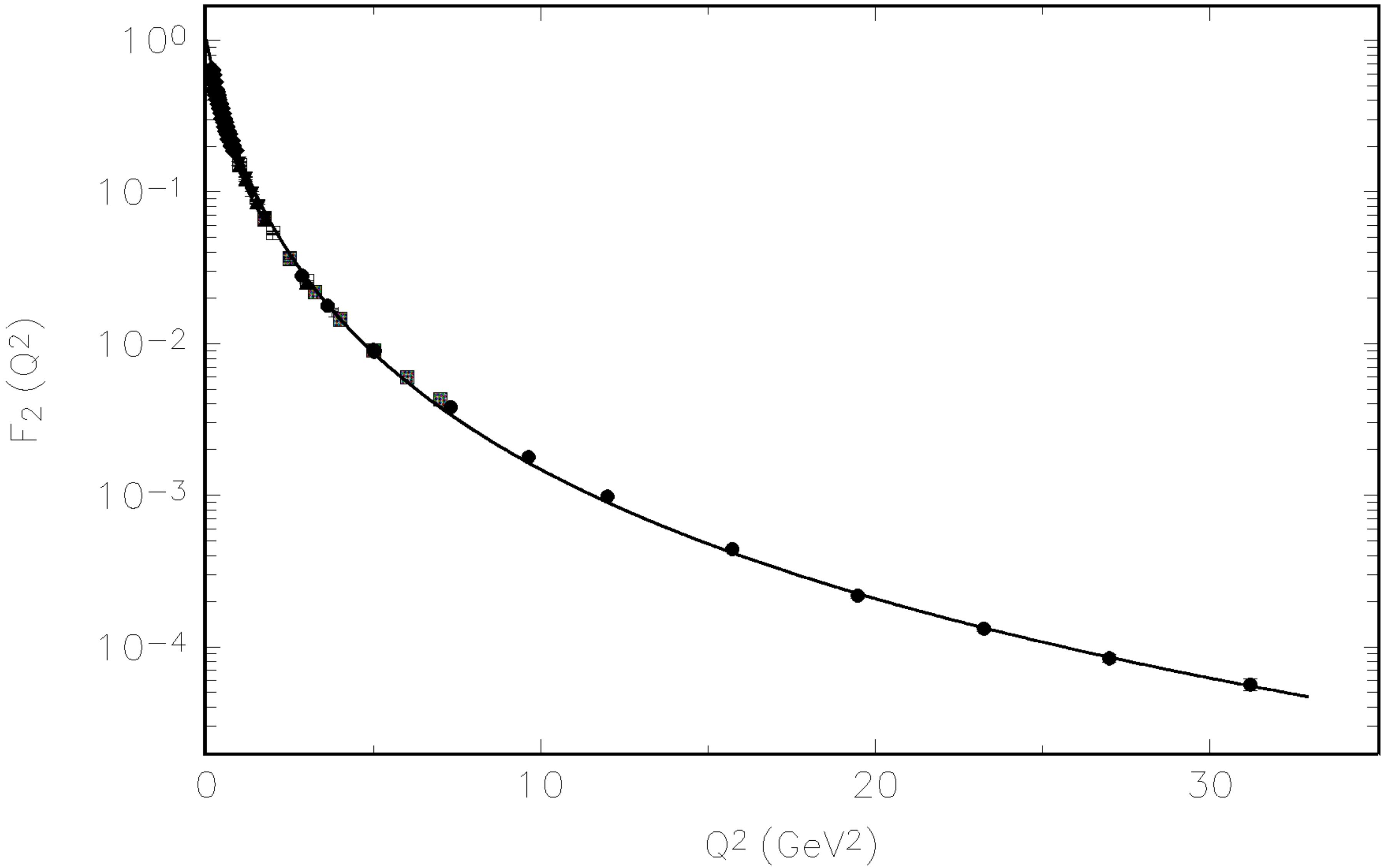
Dual-Large N_c QCD

- $F_1\,(q^2)$
- $F_2\,(q^2)$
- $G_M\,(q^2)$
- $G_E\,(q^2)$

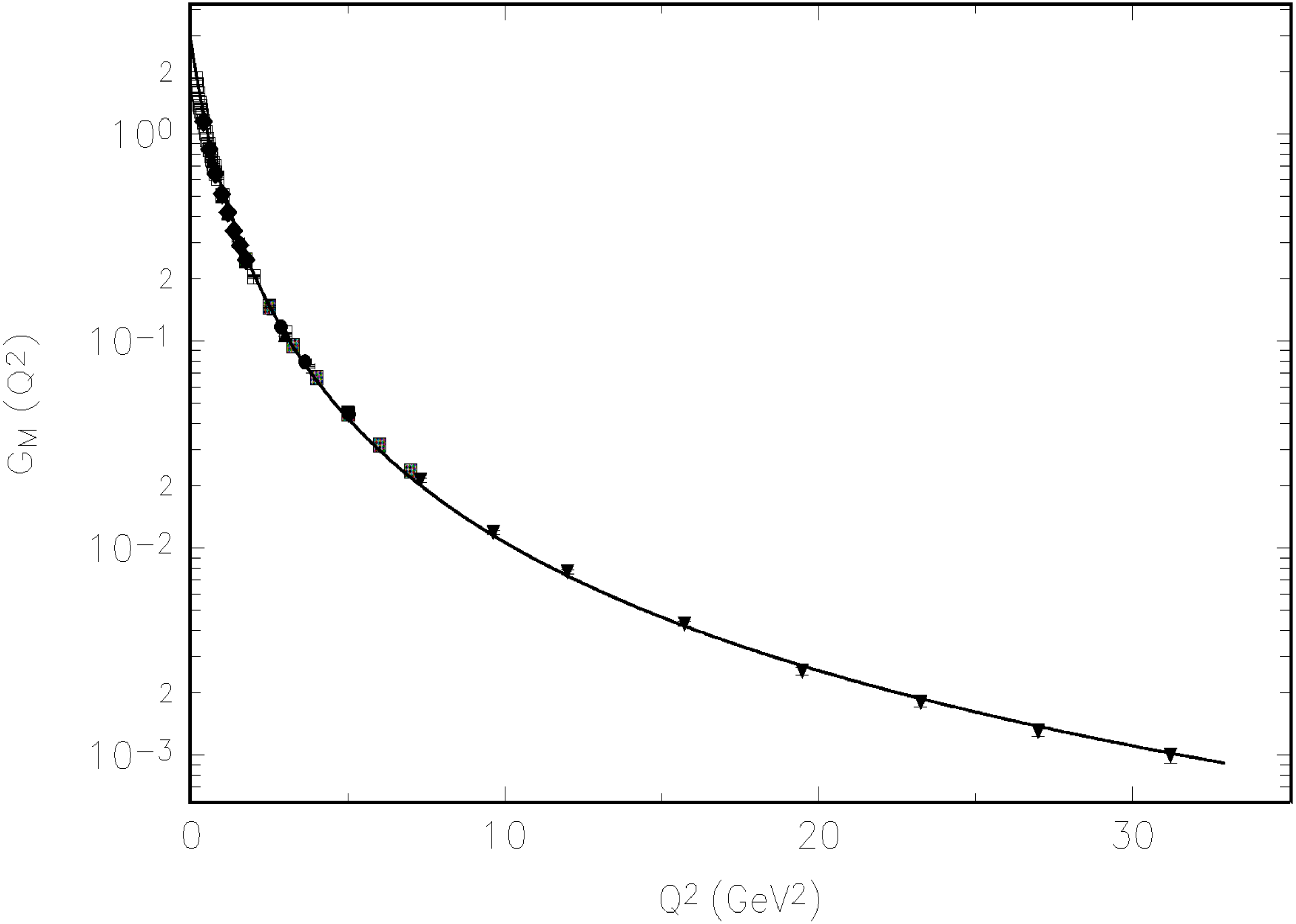
- $G_E\,(q^2)\,/\,G_M\,(q^2)$



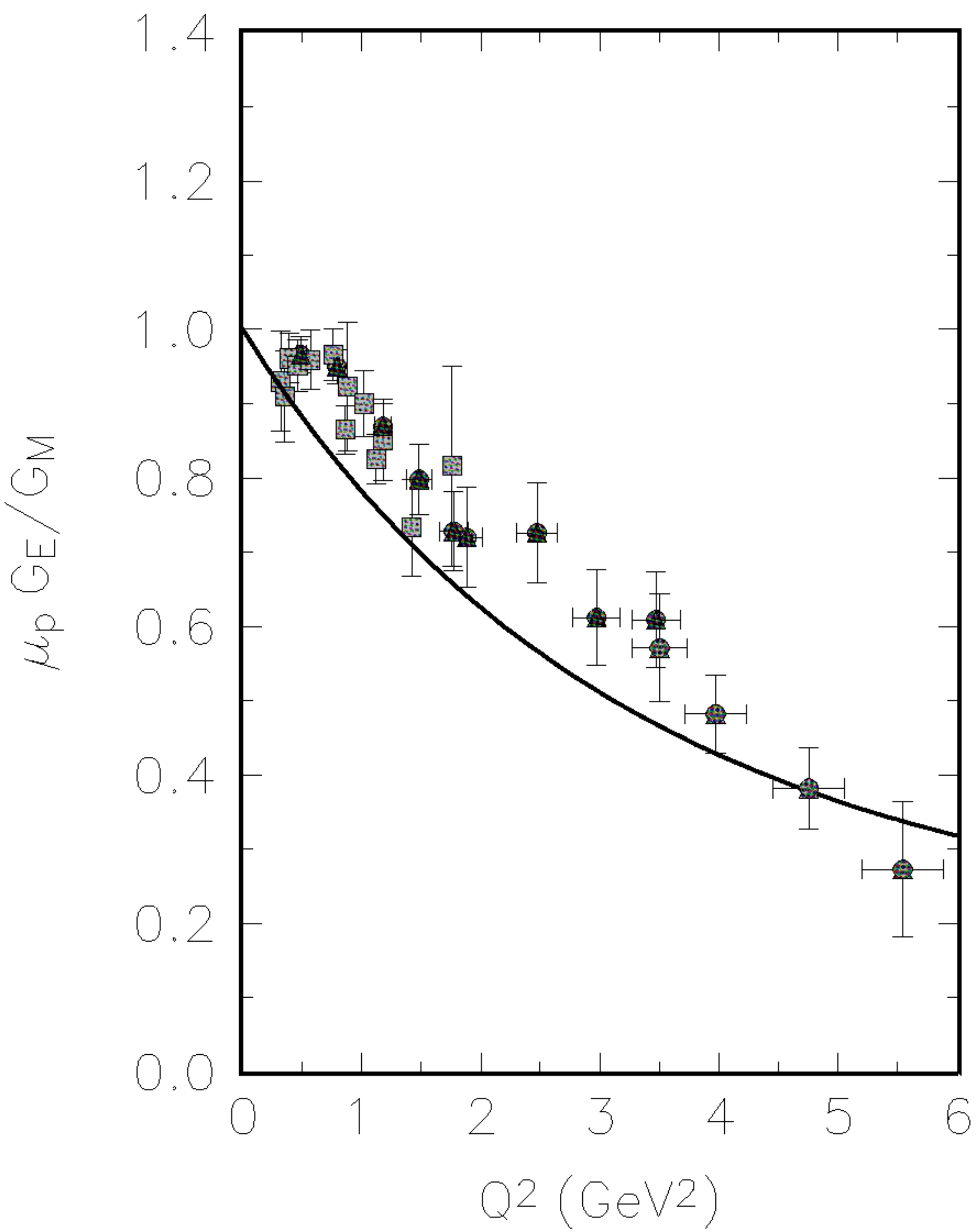
10^0
2
10^{-1}
2
10^{-2}
2
10^{-3}
2
10^{-4}
0
10
20
30
F_1 (Q^2)
Q^2 (GeV^2)



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$F_2\ (Q^2)$
0
10
20
30
$Q^2\ (GeV^2)$



$G_M\ (Q^2)$
2
10^0
2
10^{-1}
2
10^{-2}
2
10^{-3}
0
10
20
30
$Q^2\ (GeV^2)$



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
$\mu_p G_E/G_M$
Q^2 (GeV^2)

FORM FACTORS OF Δ(1236)

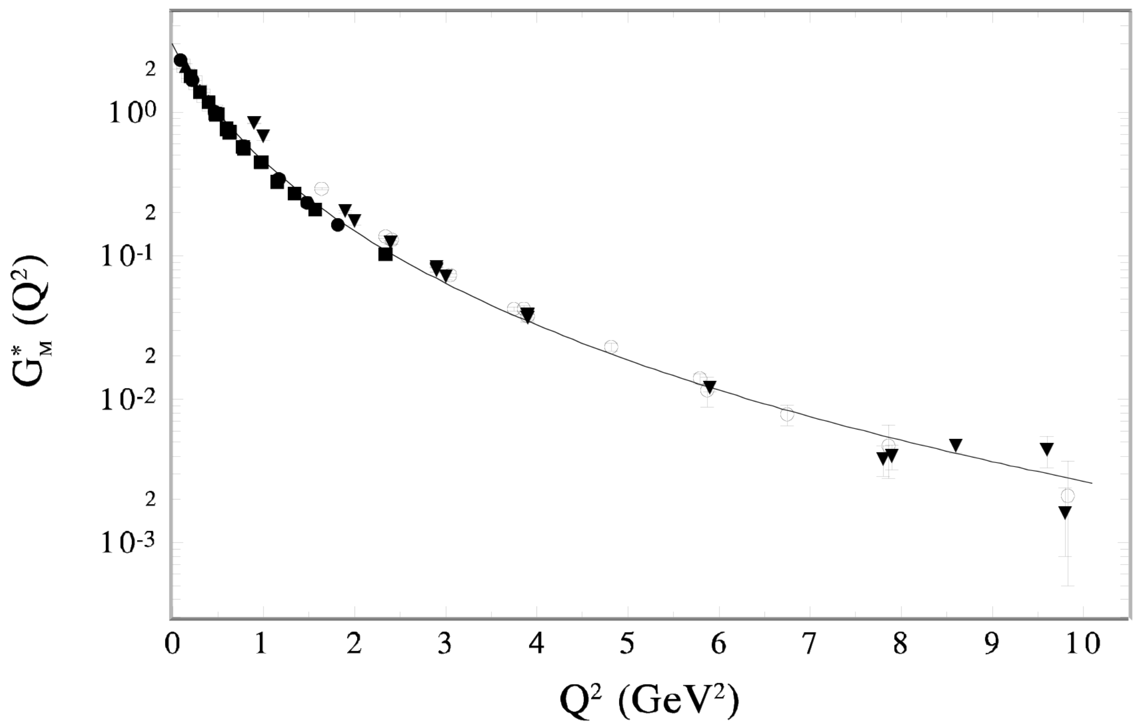
$$G^*_M(q^2)\ ,\ G^*_E(q^2)\ ,\ G^*_C(q^2)$$

$$G_M^*(q^2), \quad G_E^*(q^2), \quad G_C^*(q^2)$$

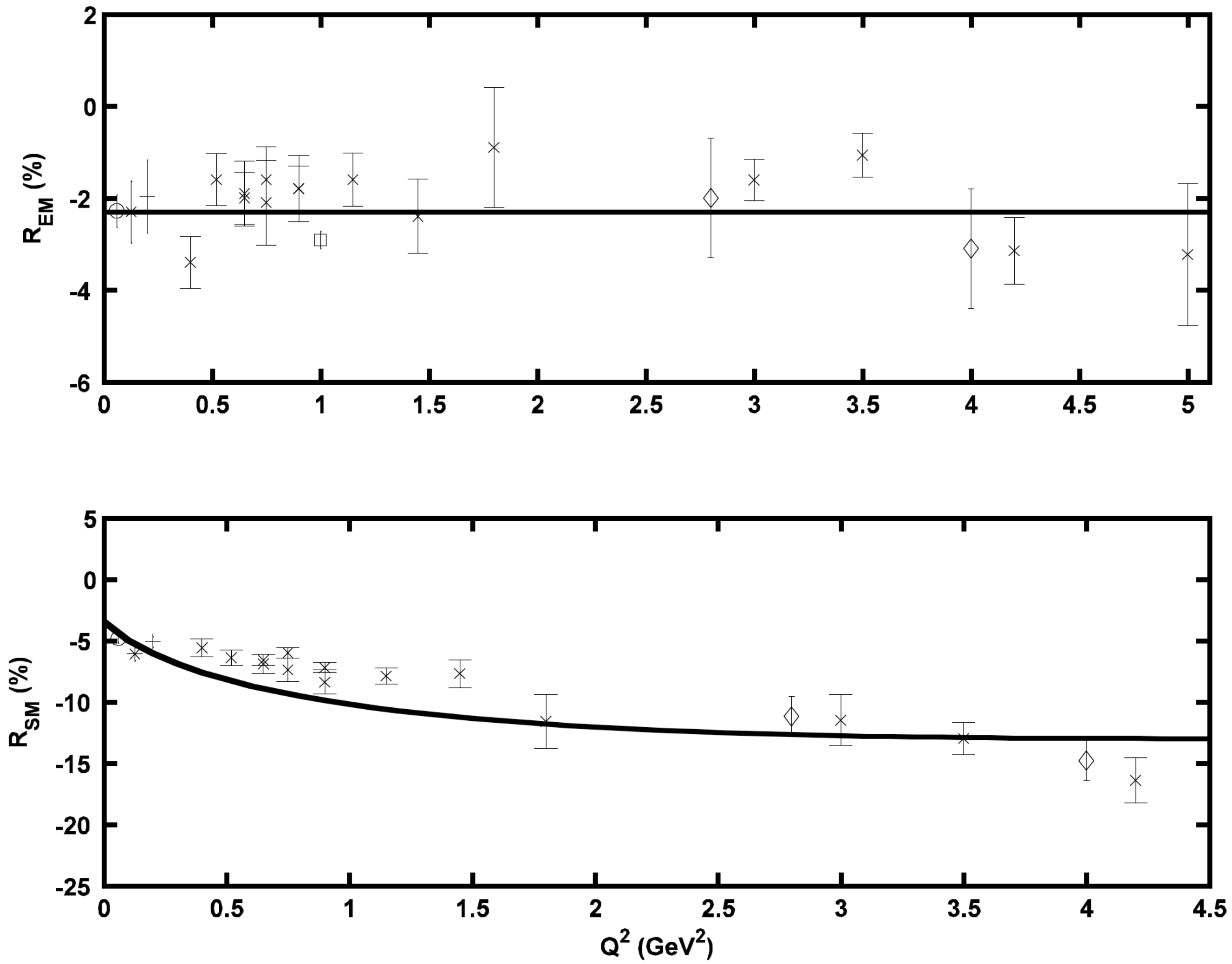
$$R_{EM} = -\,\frac{G_E^*(Q^2)}{G_M^*(Q^2)}\,,$$

$$R_{SM} = -\,\frac{Q_+Q_-}{4\,M_\Delta^2}\,\frac{G_C^*(Q^2)}{G_M^*(Q^2)}\,,$$

$$Q_\pm^2 = (M_\Delta \pm M_N)^2 + Q^2$$



$G^*_M\ (Q^2)$
10^0
10^{-1}
10^{-2}
10^{-3}
2
2
2
2
0
1
2
3
4
5
6
7
8
9
10
$Q^2\ (GeV^2)$



R_EM (%)
2
0
-2
-4
-6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
R_SM (%)
5
0
-5
-10
-15
-20
-25
Q^2 (GeV^2)

SUMMARY

- KLZ: F_π
- DUAL – $N_{c\,\infty}$: F_π , F_1 & F_2 + G_E / G_M ,
  G_M^* , G^*_E , G^*_C

PERFECT FITS