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Introductory School on Gauge Theory/Gravity Correspondence

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Non-AdS/non-CFT correspondence
Lecture 1

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The standard AdS/CFT correspondence relates $\mathcal{N}=4$ SYM with IIB string theory on $AdS_5 \times S^5$.

If we hope to have something to say about real physics, we need (at least) to:

- Break SUSY (down to $\mathcal{N}=1$ or $\mathcal{N}=0$)
- Break conformal invariance
- Be able to describe asymptotically free theories
- Obtain realistic matter content (fund. quarks)

Question: can we extend the Maldacena conjecture to less supersymmetric and non-conformal gauge theories?

The AdS/CFT duality stems from the DUAL nature of D-branes $\rightarrow$

OS p.o.v.: hypersurfaces on which OS can end $\rightarrow$ their dynamics is a $(p+1)$-dim gauge theory at $E \ll \frac{1}{\sqrt{\alpha'}}$

CS p.o.v. : non-perturbative states of CS spectrum ($T \sim 1/g_s$) $\rightarrow$ described by soliton-like solutions of effective SUGRA theory at $E \ll 1/\sqrt{\alpha'}$

Original duality from:

N D3-branes in $\mathbb{R}^{1,3} \times \mathbb{R}^6$

~~An~~ obvious idea in to replace $\mathbb{R}^6 \rightarrow CY$

However, AdS/CFT focuses on near brane region and since every smooth manifold is locally flat we do not find nothing new when the D3's are placed at a smooth point of the CY

N D3-branes in $\mathbb{R}^{1,3} \times CY$

$\hookrightarrow$ in the decoupling limit we get $AdS_5 \times S^5$, still dual to $N=4$ SYM at strong coupling

- Place the D3-branes at a SINGULAR point of the CY (the flatness of the ambient space, even near the brane, is lost).
- Consider higher dimensional branes whose WV is partially WRAPPED on topologically non-trivial cycles of a CY (both the flatness of the ambient space AND that of the brane's WV are lost).

Some general considerations:

1. Supergravity solutions dual to non-conformal gauge theories do NOT display AdS factors

$$\begin{array}{c}\text{Isometry group of } AdS_5 \\ \downarrow \\ SO(4,2) \\ \uparrow \\ \text{Conformal group of any 4d CFT}\end{array}$$

2.) Supergravity solutions dual to 4d gauge theories with $\mathcal{N}<4$ cannot display constant S^5 factors

$$\begin{array}{c}\text{Isometry of } S^5 \\ \downarrow \\ SO(6) \\ \uparrow \\ \text{R-symmetry group of } \mathcal{N}=4 \text{ SYM}\end{array}$$

3.) In the supergravity limit ($R\alpha' \ll 1$) all non conformal models have a <u>DECOUPLING PROBLEM</u>

- <u>Recall</u>: in the original AdS/CFT duality there exists a limit in the parameter space where the gauge theory dynamics is fully captured by <u>supergravity</u> alone

IIB SUGRA on $AdS_5 \times S^5$ $\simeq$ $\mathcal{N}=4$ SYM at $\lambda \gg 1$

with $\int_{S^5} F_5 = N$ $\qquad \lambda = g_{YM}^2 N$ 't-Hooft coupling

General arguments regarding QCD-like theories:

- In the AdS/CFT dictionary we have that

$$\mathcal{R}\alpha' = \frac{1}{\sqrt{g_{YM}^2 N}} = \frac{1}{\sqrt{\lambda}} \qquad (5.1)$$

$$\mathcal{R} \sim 1/R^2$$

AF theories are weakly coupled at high E and according to (5.1) this is related to large curvatures

- The spectrum of QCD-like theories (mesons, glueballs, ...) comes in Regge trajectories

$$M_J \sim \sqrt{J}\,\Lambda_{QCD}$$

SUGRA includes particles of spin ≤ 2: large mass gap between particles with $J \leq 2$ and $J > 2$

This problem manifests differently, depending on the way the gauge/gravity duality of the QCD-like theory is realized.

<u>Example</u> : wrapped branes scenario

Consider a FT defined on $\mathbb{R}^{1,3} \times S^1_R$ and a free massless scalar field $\phi = \phi(x,y)$

$$\phi(x, y+2\pi R) = \phi(x,y) \rightarrow \phi(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} e^{i\frac{n}{R}y} \phi_n(x)$$

$$\Box_5 \phi(x,y) = 0 \longrightarrow \Box_4 \phi_n(x) - \frac{n^2}{R^2}\phi_n(x) = 0$$

m_n^2

$\phi_0(x)$ <u>massless</u> + tower of KK states $m_{KK} \sim \frac{1}{R}$

$\rightarrow$ $E \ll 1/R$ the KK states decouple

Considering a D5 wrapped on S^2 we get a 4d SYM theory + a tower of KK states which get decoupled in the limit where $Vol(S^2)$ is very SMALL $\rightarrow$ The SUGRA requirement of small curvatures typically leads to opposite limit in the dual, where radii of non-contractible spheres are large.

OUTLINE :

1. Wrapped D5's . A case study : the MN model
   1.1 Derivation of FT theory
   1.2 Derivation of SUGRA dual
   1.3 Gauge/gravity duality checks

2. D-branes at CY singularities
   2.1 General properties
   2.2 The KS model

(3. Introduction of flavors (both in 1. & 2.))

1. Wrapped D-branes scenarios

General philosophy: consider a $D(p+p')$-brane wrapped on a topologically non-trivial p'-cycle of a smooth CY. The low energy effective theory living on its WV is a $(p+p'+1)$-dim SYM theory with ≤ 16 supercharges.

Large N SYM with ≤ 16 SUSY's in $\mathbb{R}^{1,p} \times \Sigma_{p'}$ $\Longleftrightarrow$ IIA/IIB in the near-horizon limit of the background generated by $N \gg 1$ (partially) wrapped $(p+p')$-branes

To suppress loops corrections

Note: in the limit $V(\Sigma_{p'}) \to 0$ the theory reduces to a gauge theory in flat $\mathbb{R}^{1,p}$ space

Our case: $p+p' = 5$, $p' = 2$ $\longrightarrow$ the FT we want to find the dual of is 4d ($\mathcal{N}=1$) SYM.

1.1 Field Theory on a D5 wrapped on $S^2 \subset CY_3$

Question: how to wrap a D-brane on a topolo= gically non-trivial cycle of a CY space and preserve some supersymmetry?

Have to understand how to deal with a supersym= metric field theory in a curved space.

SUSY $\longleftrightarrow$ $\exists$ covariantly constant spinors

$$D_M \epsilon = (\partial_M + \omega_M)\epsilon = 0 \qquad (9.1)$$

$\omega_M = \omega_M^{NP} \gamma_{NP}$: spin connection on the mani= fold in spinorial rep. (only contribution from S^2, in our case)

↳ NO solutions in general!

x IDEA: change the representation in which fields transform by redefing Lorentz group as a mixture of the usual one with the R-symmetry group in such a way that under new Lorentz group all curvature terms appear hitting fields that are now scalars

Note : this procedure changes Lorentz assignment hence the spin of fields. In our case, however, the twist acts only on the S^2 → only 6d fields get affected, 4d reps do not.

Procedure : the spin connection of S^2 transforms in reps of $SO(2) \simeq U(1)$. We couple our theory to an external $U(1)$-field which gauges a $U(1)$ subgroup of the R-symmetry group $SO(4)_R$ and choose the gauge connection to be OPPOSITE to the spin connection

$$A_M^{ij}\Gamma_{ij} \quad SO(4)_R \text{ indeces}$$

$$D_M \epsilon = (\partial_M + \omega_M + A_M)\epsilon = \partial_M \epsilon = 0$$

→ the SUSY parameter has now become a scalar!

This procedure is called TWISTING.

Let's proceed in steps ⟶

D5 : $SO(1,9) \to SO(1,5) \times SO(4) \to SO(1,3) \times U(1)_J \times SO(4)$

R-symmetry group of SUSY gauge theory living on flat D5's

tangent bundle of the S^2

	$SO(1,5) \times SO(4)$	$SO(1,3) \times U(1)_J \times SU(2)_L \times SU(2)_R$
V_M	$(6,1)$	$(4_0,1,1) \oplus (1_\pm,1,1)$
ϕ^A	$(1,4)$	$(1_0,2,2)$
ψ^+	$(4,2)$	$(2_+,2,1) \oplus (\bar{2}_-,2,1)$
ψ^-	$(4',2')$	$(\bar{2}_+,1,2) \oplus (2_-,1,2)$

complex Weyl

Now we should implement the twist by identify $U(1)_J \simeq U(1) \subset SU(2)_L \times SU(2)_R$ and consider the 0-mode in the KK reduction.

Massless fields in 4d will be the 0-modes UNCHARGED under the $U(1)_T$ (a charge under $U(1)$ acts a a mass term from the 4d p.o.v.)

$U(1) \equiv U(1)_L \subset SU(2)_L$

	$SO(1,3) \times U(1)_T$
V_M	$4_0 \oplus 1_+ \oplus 1_-$
ϕ^A	$2 \times 1_+ \oplus 2 \times 1_-$
ψ^+	$2_{++} \oplus 2_0 \oplus \bar{2}_0 + \bar{2}_{--}$
ψ^-	$2 \times \bar{2}_+ \oplus 2 \times 2_-$

The (massless) field content is that of $\mathcal{N}=1$ gauge multiplet : (A_μ, λ), the R-symmetry group being $U(1)_L$.

The above procedure could seem ad hoc. In fact it is not: this is precisely what happens when trying to wrap a D5-brane on a topologically non-trivial 2-cycle inside a CY while preserving SUSY. There is a precise geometric counterpart of the twisting procedure.

What the connection to wrapped D-branes on a CY?

Rough idea:

1. Given a manifold X, calibrated cycles are those minimizing their volume within their homology class ($\Sigma_p \simeq \Sigma'_p$ if $\Sigma_p - \Sigma'_p = \partial \Xi_{p+1}$)

2. A supersymmetric cycle is a cycle which a D-brane can wrap while preserving supersymmetry. SUSY requires that the energy of a state is minimum for its given charges, so a necessary condition (one can prove is also sufficient!) for susy preservation is that the cycle is calibrated

In a CY there exist non-contractible calibrated 2, 3 and 4 cycles]

→ putting everything together this implies that the twisting mechanism is naturally realized by wrapping a D-brane on a calibrated cycle.

We considered a D5 wrapped on a S^2 : the way the S^2 is embedded into the CY is specified by the way the normal bundle N_{S^2} is fibered on the tangent bundle T_{S^2}.
The mixing between the spin connection on S_2 and the gauge connection on the normal bundle (whose U(1) gauge field A_M we introduced is part of) is just what takes into account the fibration and tells how the WV theory couples to the external A_M field:

Each twist $\longleftrightarrow$ Each possible susy cycle

$U(1) \equiv U(1)_L \subset SO(2)_L$ $\quad S^2 \subset CY_3 \quad \mathcal{N}=1$ SYM

$U(1) \equiv U(1)_D \subset D[SU(2)_L \times SU(2)_R]$ $\quad S^2 \subset CY_2 \times \mathbb{R}^2 \quad \mathcal{N}=2$ SYM

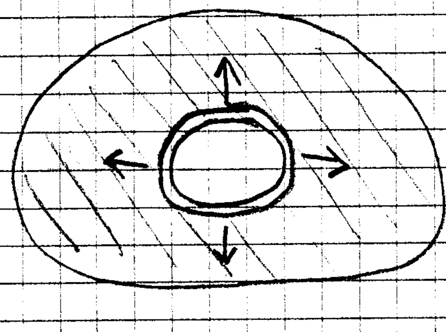
rigid cycle $\qquad$ non-rigid cycle

<u>Recall</u>: Transverse directions to a D-brane correspond to scalars of the effective WV theory (think of $\mathcal{N}=4$ SYM and the 6 scalars of the $\mathcal{N}=4$ supermultiplet) -

sections of the normal bundle N_{S^2}

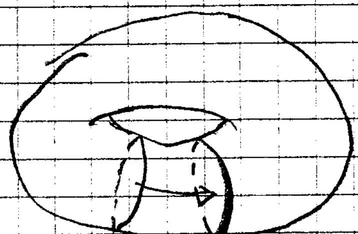
for wrapped branes not all scalars remain massless, but only those associated to directions along which one can move the cycles by keeping it calibrated (i.e SUSY)

- RIGID cycle $\rightarrow$ no other susy cycles in the same homology class



$S^2 \subset CY_3$ is rigid: $\mathcal{N}=1$ SYM does NOT have scalars!

- NON-RIGID cycle $\rightarrow$ $\exists$ other susy cycles in the same homology class



$S^2 \subset CY_2 \times \mathbb{R}^2$ is not rigid. $\mathcal{N}=2$ SYM has a complex scalar!