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Introductory School on Gauge Theory/Gravity Correspondence

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Non-AdS/non-CFT correspondence
Lecture 3

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... Gauge/gravity duality checks (continued)

• Confinement & "QCD"-string tension

SYM as YM is believed to undergo (strict) confinement, $T_s \sim \Lambda^2 \rightarrow V(r) = T_s r$.

One expects an area-law for the Wilson loop in this case, which we can compute in the dual supergravity background using Maldacena prescription (see G. Ferretti lectures)

$\hookrightarrow W(\square) \sim e^{-6TL} = e^{-S_{NG}}$

The NG action reads:

$$S_{NG} = \frac{1}{2\pi\alpha'} \int dx\, dt \sqrt{f(\rho)^2 + g(\rho)^2 \left(\frac{d\rho}{dx}\right)^2} = \int dt\, E$$

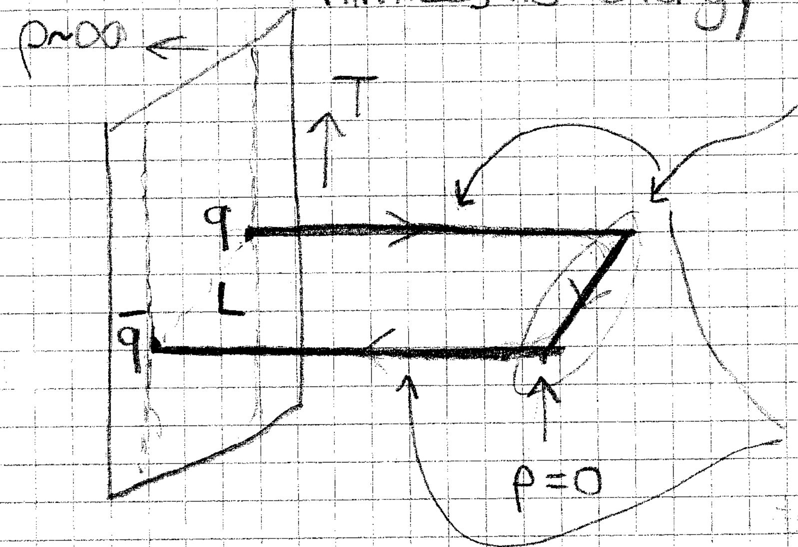
$$f(\rho)^2 \equiv G_{tt} G_{xx} \overset{MN}{=} \frac{\sinh 2\rho}{2e^{h(\rho)}}$$

$$g(\rho)^2 \equiv G_{tt} G_{\rho\rho} \overset{MN}{=} \alpha' g_s N f(\rho)^2$$

$$S = \frac{1}{2\pi\alpha'} \int dx\, dt\, f(\rho) \sqrt{1 + \alpha' g_s N \left(\frac{d\rho}{dx}\right)^2} = \int dt\, E$$

$$f(\rho) = f(0) + a_2 \rho^2 + O(\rho^4) \; ; \; g(\rho) = \sqrt{\alpha' g_s N}\, f(0) + O(\rho^2)$$

minimum: the string streches down to $\rho = 0$ where it minimizes its energy



- relevant contribution to the potential Energy comes from a string localized at $\rho = 0$
- infinite energy corresponding to q & $\bar{q}$ bare masses, $m_q + m_{\bar{q}} = 2\int_0^\infty g(\rho)\, d\rho$

$$E = \frac{1}{2\pi\alpha'} f(0) L - 2\kappa + O(e^{-\alpha L}) \longrightarrow T_s = \frac{1}{2\pi\alpha'} f(0) \underset{MN}{=} \frac{1}{2\pi\alpha'}$$

(general rule: $T_s \sim h^{-1/2}(0)$)

AdS $h^{-1/2}(0) = 0$!

- <u>The k-string tensions</u>

In $SU(N)$ SYM there are different types of confining strings (depending on the representatio[n] external massive fermions transform):

$$T_k \,, \quad k = 1, \dots, N-1 \quad ; \quad T_k = T_{N-k}$$

n-ality

it connects k massive q with k massive $\bar{q}$ ($k=1$ is the previous case)

A natural question for the stability of such confining flux-tubes is whether:

$$T_{k+k'} < T_k + T_{k'}$$

which would imply that a k-string would not decay into strings with smaller k.

In several string inspired models the above seems to hold; more precisely it has been conjectured that:

$$\frac{T_k}{T_{k'}} = \frac{\sin(\pi k/N)}{\sin(\pi k'/N)}$$

which has been supported for YM by lattice computations.

- In the gravity dual the confining k-string is described by k coincident fundamental strings placed at $\rho = 0$ $\rightarrow$ due to $F_3 \neq 0$ at the tip through the S^3, the strings undergo

Myers effect and blow up into a D3 wrapping an S^2 inside the S^3 at a K-dependent azimutal angle ψ.

One can compute the tension of such object and find that it is minimal for $\psi = \frac{\pi K}{N}$ and equal to:

$$T_K \sim \sin \frac{\pi K}{N}$$

$$\left(T_K(\psi) \sim \left[\sin^4 \psi + \left(\psi - \frac{\sin 2\psi}{2} - \frac{\pi K}{N} \right)^2 \right]^{1/2} \right)$$

- Domain walls

There exist domain wall configurations interpolating between the N vacua of $N=1$ SYM; they are BPS and their tension is related to the different vev's of $\langle\lambda\lambda\rangle$ at the two sides of the wall $|W(i) - W(j)|$

$$T_{DW} \sim N \left| (\lambda\lambda)_i - (\lambda\lambda)_j \right| \sim N^2 \Lambda^3 \sin \frac{(i-j)\pi}{N} \underset{N\to\infty}{\sim} N\Lambda^3$$

In the gravity dual they are described by D5 wrapped on the blown-up S^3 and extending along x^0, x^1, x^2. One can show:

- The D5's are (magnetically) charged under C_2: crossing the wall there is a shift which precisely matches the expected ψ shift:

$$\Delta \psi = 2\pi \frac{k}{N} \qquad k = 1, \dots, N-1$$

- The D5's minimize their tension at $\rho = 0$, where the volume is minimum, in agreement of domain walls being and IR effect.

- The fact that k-strings can end on a domain wall is also reproduced, as F-strings can end on D5-branes.

$$T_{DW} \sim \frac{1}{\alpha'^3 g_s} \int_{S^3} e^{-\phi} \sqrt{G} \underset{\rho \to 0}{\sim} \frac{1}{\alpha'^{3/2}} (g_s N)^{3/2} \cdot N$$

- Glueball spectrum

Masses of bound states can be extracted from the correlation functions of gauge invariant operators, and looking for particle poles (G inv. O's are free at large N)

$$\langle \mathcal{O}(x)\,\mathcal{O}(y)\rangle \underset{|x-y|\gg 1}{\simeq} \sum_i a_i\, e^{-M_i|x-y|}$$

glueball masses

E.g.: consider the lowest mass glueball state, $J^{PC} = 0^{++}$. The corresponding glueball $\mathcal{O}$ (i.e. a local gauge inv. $\mathcal{O}$ with same quantum numbers) with lowest dimension is $\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}$.

AdS/CFT : $\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} \longleftrightarrow \phi$ (dilaton)

The computation of the 2-point function amou[nts] to solve the field EoM in the bulk.

$$\partial_\mu\left(\sqrt{G}\, G^{\mu\nu}\partial_\nu \phi\right) = 0$$

where $\phi(x,y) = \sum_I \tilde{\phi}_I(x)\, Y^I(y)$

(x^μ, ρ) angular coords.

Lowest mass modes correspond to s-waves, i.

y-indendent solutions. Expanding in plane wave

$$\tilde{\phi}_0(x) = \tilde{\phi}(\rho)\, e^{ikx} , \qquad m^2 \sim -k^2$$

4d momentum

The eigenvalues of the bulk equation are the corresponding glueball masses.

× Rough estimate : focus on the small ρ region (which corresponds to the IR), the metric is

$$ds^2 = dx^2_{1,3} + \alpha' g_s N\, d\rho^2$$

and we get

$$\Box_4 \tilde{\phi}_0(x) + G^{\rho\rho}\, \partial^2_\rho\, \tilde{\phi}_0(x) \propto \tilde{\phi}(\rho)'' - \alpha' g_s N\, \underset{\beta^2}{\underset{|||}{k^2}}\, \tilde{\phi}(\rho) = 0$$

$$\hookrightarrow \quad m^2_{gb} \sim \frac{1}{\alpha' g_s N} \qquad (\tilde{\phi} \sim e^{i\beta\rho})$$

Note : the $k=0$ ($\beta=0$) case is excluded by the boun-

dary conditions $\tilde{\phi}$ should satisfy ($k^2 < k_0^2 < 0$).

however the spectrum turns out to be discrete

The decoupling problem and the KK mixing

Computing the curvature of the MN model one finds at small ρ

$$R \sim \frac{1}{\alpha' g_s N}$$

$\rightarrow$ to make supergravity approx. valid we need small curvatures in α' units, hence

$$\boxed{g_s N \gg 1}$$

- The KK states coming from the S^3 have a mass spectrum which is roughly

$$M^2_{KK} \sim \frac{1}{R^2_{S^3}} = \frac{1}{\alpha' g_s N}$$

These states are KK glueball-like states which are charged under (the geometric dual of) $U(1)_T \times U(1)_R$ ($\subset SU(2)_R$) and so they are bound states of FT fields involving also FT KK-states

- at $\rho < 1$ (IR) we are probing distances of order $\sqrt{\alpha' g_s N}$ (recall $\rho \equiv r/\sqrt{\alpha' g_s N}$) so they cannot be disregarded
- Recalling the result we found for the string tension we have

$$M_{KK}^2 \sim T_s / g_s N$$

so in order to decouple these KK-modes we should take $g_s N \ll 1$, which is beyond the supergravity limit: this is the decoupling problem.

The above problem can be rephrased in the following way. For ordinary SYM we expect:

$$M_{gb} \sim \Lambda \quad , \quad T_s \sim \Lambda^2 \quad , \quad T_{DW} \sim N \Lambda^3$$

By taking our results and setting $M_{gb} \sim \Lambda$, i.e. $M_{gb} \sim 1/\sqrt{\alpha' g_s N} \equiv \Lambda$, we get:

$$M_{gb} \sim \Lambda \quad , \quad T_s \sim \Lambda^2 (g_s N) \quad , \quad T_{DW} \sim N \Lambda^3 (g_s N)^2$$

→ The theories we get display confinement, mass gap, chiral symmetry breaking, etc... but are not pure $N=1$ SYM : we have a family of string backgrounds dual to gauge theories that are a ONE-PARAMETER $(g_s N)$ generalization of ordinary SYM → in this family pure SYM corresponds to

$$g_s N \sim 1$$

so to a STRONGLY coupled string background.

• The gauge theory the MN model describes is roughly:

$$L = \mathrm{Tr}\Big[\overbrace{\frac{1}{2} F_{\mu\nu}^2 + 2i \lambda D \lambda}^{\text{pure SYM}} \overbrace{- D_\mu \Phi^2 + \Psi (i D - M_{KK}) \Psi + M_{KK}^2 \Phi^2 + V[\Phi, \Psi]}^{\text{extra stuff}} \Big]$$

Note : the sugra dual is expected to be dual to a 4d theory only for small ρ. At high energy the theory becomes 6d and then, since the ϕ diverges for $\rho \to \infty$, one has to S-dualize D5 $\to$ NS5. Therefore, the UV completion of the theory dual to the MN background is LST, not even a field theory!

- Given the above, a particularly surprising result is the β-function computation (independently on the use of the $\lambda \leftrightarrow a(\rho)$ radius/energy relation or the more "UV" one, $\lg \mu \leftrightarrow \rho$).

- Proposal to disentangle KK-contribution (GN): implement a β-deformation on the background (SL(2,R) transformation) on $U(1)_T \times U(1)_R$ under which ONLY KK-states are charged and see which quantities get affected

The β-function computation is not!

This is also understood from dual geometric p.o.v. : new solutions have been found with a different large ρ asymptotic wrt the original MN background.

They behave 6d at large enough ρ, while the original solution keeps on behaving as a "4d", from the dilaton p.o.v.