The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1943-8

Introductory School on Gauge Theory/Gravity Correspondence

19 - 30 May 2008

Non-AdS/non-CFT correspondence

Lecture 5

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2.2 The KS model :

<u>set-up</u> : a set of N D3-branes and M fractional D3-branes at a conifold singularity

$$\begin{tikzcd} \underset{1}{SU(N+M)} \arrow[r, bend left, "A_i"] & \underset{2}{SU(N)} \arrow[l, bend left, "B_i"] \end{tikzcd}$$

$$W = \lambda\, \varepsilon^{ij} \varepsilon^{kl}\, \mathrm{Tr}\, A_i B_k A_j B_l$$

$$(M \neq 0 : \beta \neq 0 ,\ U(1)_R \to \mathbb{Z}_{2M})$$

Let us first discuss / analyze the SUGRA solution

Expected : $G_{\mu\nu}$, F_5 & ISD $G_3 = F_3 + i H_3$

$$\begin{cases} ds^2 = h^{-1/2}(r)\, dx_\mu dx^\mu + h^{1/2}(r)\, (dr^2 + r^2 ds^2(T^{1,1})) \\[2ex] h(r) = \dfrac{27\pi\alpha'^2}{4} \cdot \dfrac{\left[g_s N + \frac{3}{2\pi} (g_s M)^2 \ln r/r_0 + \frac{3}{2\pi} (g_s M)^2/4 \right]}{r^4} \\[2ex] F_5 = 27\pi\alpha'^2 \cdot \left(N + \dfrac{3}{2\pi}\, g_s M^2 \ln r/r_0 \right) \mathrm{vol}(T^{1,1}) \\[2ex] F_3 = \dfrac{M\alpha'}{2}\, \omega_3 \ , \quad B_2 = \dfrac{3 g_s M \alpha'}{2}\, \omega_2 \ln r/r_0 \\[2ex] \displaystyle\int_{S^2} \omega_2 = 4\pi \qquad \int_{S^3} \omega_3 = 8\pi^2 \end{cases}$$

gauge couplings run , as expected

Comments :

- As anticipated, the are RR running (as well as not running) fluxes

$$\frac{1}{4\pi^2\alpha'}\int_{S^3} F_3 = M \quad \leftarrow \sim \# \text{ of } D3_f : \text{constant}$$

$$\frac{1}{(4\pi^2\alpha')^2}\int_{T^{1,1}} F_5 = N_{eff}(r) = N + \frac{3}{2\pi} g_s M^2 \ln r/r_0$$

$$\left(Vol(T^{1,1}) = \frac{16}{27}\pi^3 \right)$$

$\sim \#$ of D3 : decreases towards $r \to 0$

r_0 is a reference scale where # D3's = N

- There are other interesting scales :

1. $r = \tilde{r}$ where $N_{eff}(\tilde{r}) = 0$

"effective # of D3's vanishes

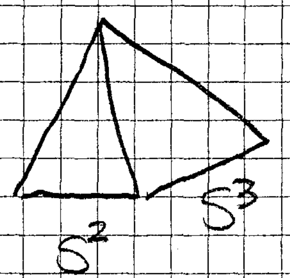
2. $r = r_s$ where $h(r_s) = 0$ where the solution has a naked singularity !

$$r_s < \tilde{r} \ll r_0 \quad \text{for } N/M \gg 1$$

This solution (KT) is the analoge of the singular MN (CV) solution (i.e. $a(\rho)=0$). KS found a smooth singularity-free solution following a similar strategy:

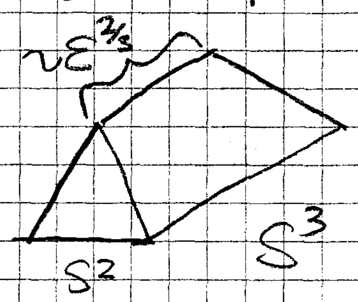
Singular conifold $\longrightarrow$ Deformed conifold

$z_1 z_2 - z_3 z_4 = 0$ $\qquad\qquad$ $z_1 z_2 - z_3 z_4 = \varepsilon^2$



$[z_i] = L^{3/2}$

$[\varepsilon] = L^{3/2}$



$$\varepsilon \longleftrightarrow a(0)$$

One finds a new solution (KS) with the following properties:

1. it is regular (i.e. free of singularities)
2. the metric at $r\to 0$ is $\mathbb{R}^{1,7}\times S^3$ where $R^2(S^3) \sim \frac{\alpha'}{\varepsilon^{4/3}} \cdot \alpha' g_s M$
3. at large distances $(r\to\infty)$ it approaches the singular solution

- The KS solution should be used to understand the deep IR. It suggests (more later) that in such region one is left with fractional branes only, hence pure SU(M) SYM.

This is (almost) correct and one can perform all checks we did in the MN model finding qualitative similar results: confinement, chiral symmetry breaking and gaugino condensation ($\langle\lambda\lambda\rangle \sim \varepsilon^2/\alpha'^3$), confining string tensions, ...

- Distinctive different w/rt MN: there is NO mass gap, i.e. there exists a massless gauge invariant chiral superfield!

However, this does not come from the gauge sector. In order to understand its origin we should understand the RG-flow for which the UV singular solution is sufficient.

The RG-flow has a novel behavior, which supergravity <u>exactly</u> reproduces, which is known as the "Seiberg duality CASCADE".

- Let us first check the (two) β-functions.

For $N \gg M$ we have that the χ_{sf} anomalous dimension:

$$\gamma = -\frac{1}{2} + O\left(\frac{M}{N}\right)^2$$

For $N \gg M$ the quiver is invariant under $M \to -M$, $N \to N$, so γ cannot depend on odd powers in M/N-expansion.

$$\begin{cases} \beta_{8\pi^2/g_1^2} = 3(N+M) - 2N\left(1+\frac{1}{2}\right) = 3M > 0 \\ \beta_{8\pi^2/g_2^2} = 3N - 2(N+M)\left(1+\frac{1}{2}\right) = -3M < 0 \end{cases}$$

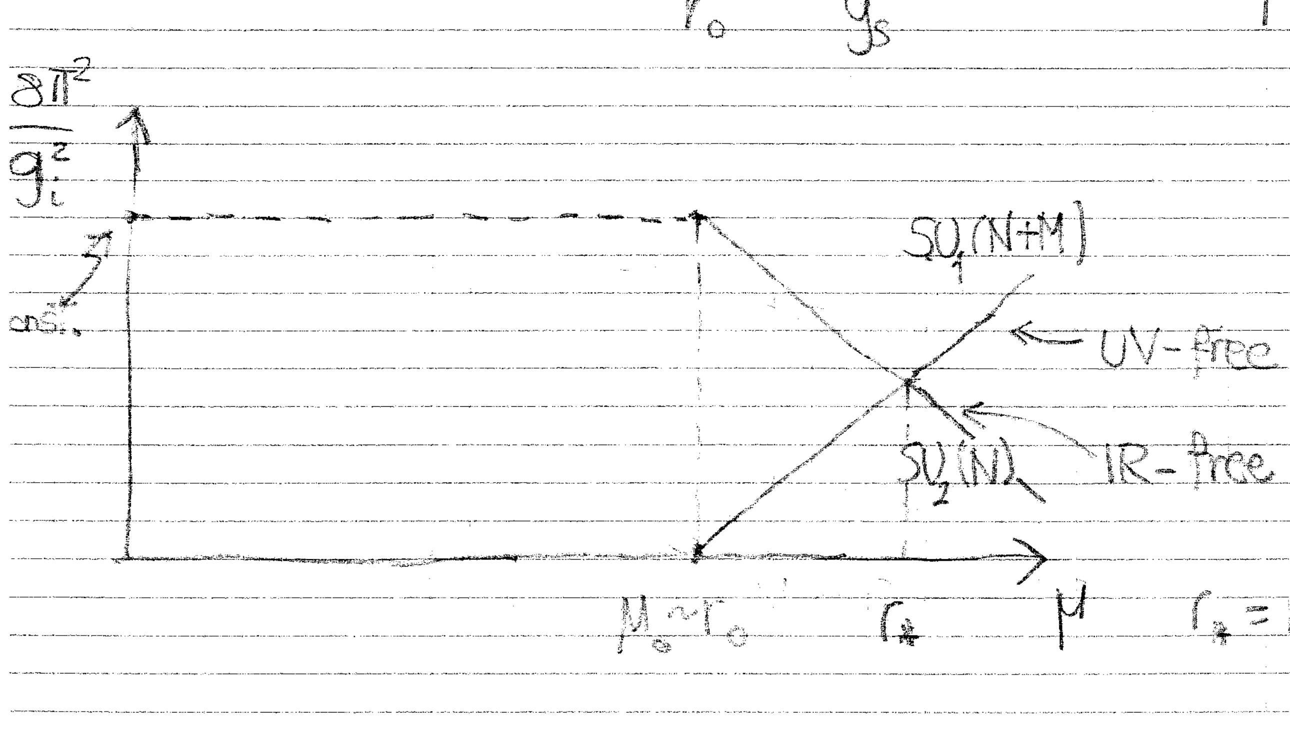
From the above we get:

$$\begin{cases} \dfrac{8\pi^2}{g_1^2} + \dfrac{8\pi^2}{g_2^2} = \text{const.} \\[2ex] \dfrac{8\pi^2}{g_1^2} - \dfrac{8\pi^2}{g_2^2} = 6M \ln \dfrac{\mu}{\mu_0} - \text{const.} \end{cases}$$

This is <u>exactly</u> matched by the SUGRA solution:

$$\begin{cases} \dfrac{8\pi^2}{g_1^2} + \dfrac{8\pi^2}{g_2^2} = \dfrac{2\pi}{g_s} \\[2ex] \dfrac{8\pi^2}{g_1^2} - \dfrac{8\pi^2}{g_2^2} = \dfrac{4\pi}{g_s}\dfrac{1}{4\pi^2\alpha'}\displaystyle\int B_2 - \dfrac{2\pi}{g_s} = \end{cases} \quad = \quad \begin{cases} \dfrac{8\pi^2}{g_1^2} = 3M \ln \dfrac{r}{r_0} \\[2ex] \dfrac{8\pi^2}{g_2^2} = -3M \ln \dfrac{r}{r_0} + \dfrac{2\pi}{g_s} \end{cases}$$

$$= 6M \ln \frac{r}{r_0} - \frac{2\pi}{g_s} \qquad (r \sim \mu)$$



$$r_* = r_0\, e^{\frac{\pi}{3Mg_s}}$$

To understand what happens at the scale μ_0 one could invoke Seiberg duality: one can think of $SU_2(N)$ so weakly coupled that it may act as a flavor symmetry at leading order:

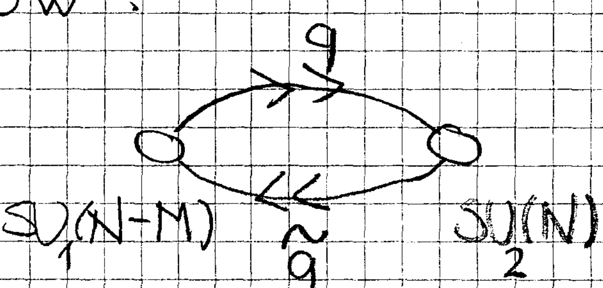
$$SU(N_c)\ ,\ N_F \xrightarrow{SD} SU(N_F - N_c)\ ,\ N_F\ \&$$

a Meson singlet $M \sim Q\tilde{Q}$

with $W = q M \tilde{q}$

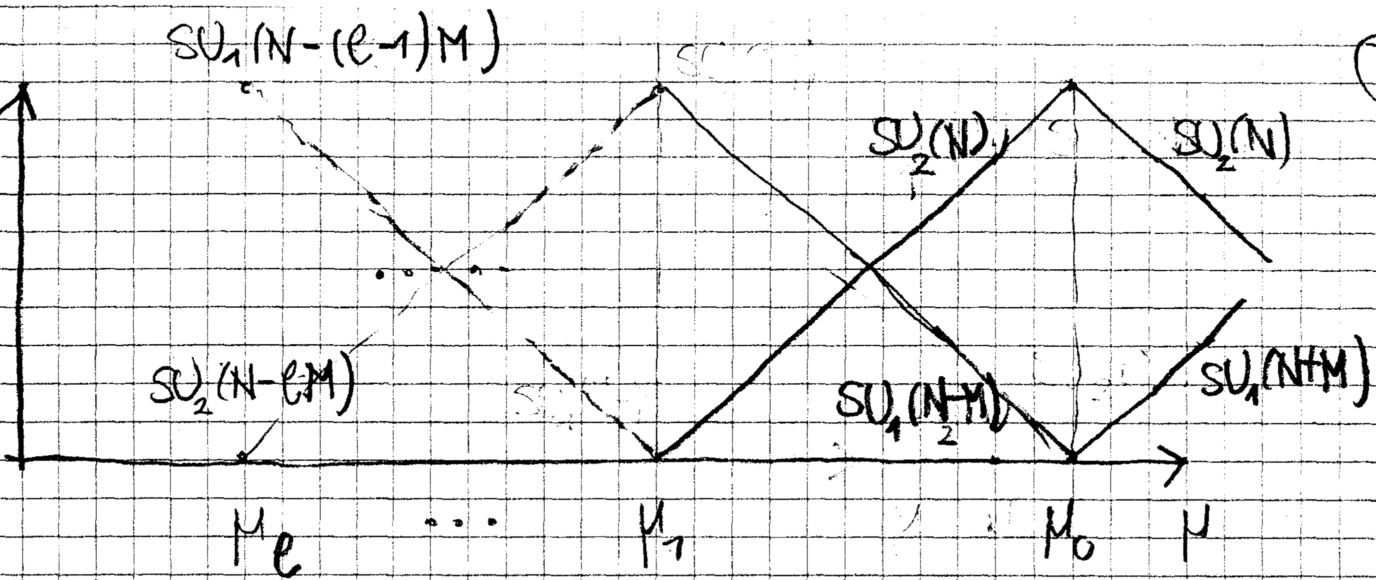
$$SU_1(N+M)\ ,\ 2N\ \&\ W \sim (Q\tilde{Q})^2_{A_i,\,B_i} \xrightarrow{SD} SU(N-M)\ ,\ 2N\ \&\ W = qM\tilde{q} + (M)^2$$

M it is massive and can be integrated out using its F-term eqs. The full theory is now:



$+\ W \sim (q\tilde{q})^2$

→ <u>IDENTICAL</u> as before but $N \to N' = N - M$

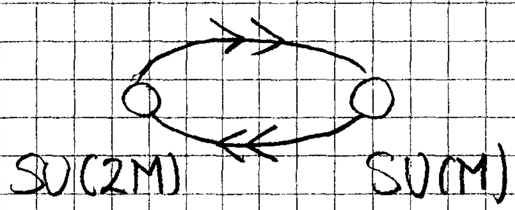


From μ_0 on the flow is inverted and is now the second gauge factor which has a QCD-like running. The theory is said to be SELF-SIMILAR passing through μ_0 and the flow QUASI-PERIODIC. This is known as the <u>Seiberg duality cascade</u>.

<u>Note 1</u> : M (# of FB) remains constant, N (# of RB) decreases. The above FT analysis makes precise the intuition we got from the SUGRA dual.

<u>Note 2</u> : the more N decreases, the less the $\gamma = -\frac{1}{2}$ approx. is valid and the cascade gets distorted.

Suppose now $\underline{N = KM}$. After $K-1$ cascade steps we get:



$\rightarrow \det \mathcal{M} - B\tilde{B} = \Lambda^{2N_c}$

SQCD with $N_f = N_c = 2M$: there is both a baryonic & a mesonic branch (which due to $W \sim (Q\tilde{Q})^2$ are separated). Along the baryonic branch:

$$B = [A]^{2M} \qquad \tilde{B} = [B]^{2M}$$

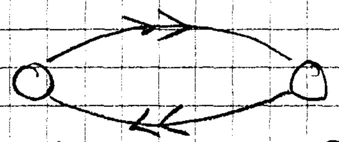
$$B\tilde{B} = \Lambda^{4M}_{SU(2M)} , \qquad \det M = 0$$

the $SU(2M)$ group confines and one is left with

$$\bigcirc_{SU(M)} \qquad (+\ \text{Goldston}\ \chi_{sf}\ !)$$

$\rightarrow$ as anticipated, the BOTTOM of the cascade it is as if RB had disappeared: the IR dynamics depends on fractional branes only which in this case, are deformation FB.

• For $N = KM + p$ at the last step one gets



$SU(M+p)$ $\qquad$ $SU(p)$

→ one can show that the baryonic branch is LIFTED. The mesonic branch is described by p D3-branes probing the warped deformed conifold (KS).

• The mesonic branch exists also for $p=0$: M regular D3's probing the warped deformed conifold.

<u>Remark</u>: the KS solution describes the <u>baryonic branch</u> of the $N = KM$ duality cascade.

So, this is what the FT analysis says. What about the <u>gravity dual</u> description? Remarkably the KS background reproduces exactly all this non-trivial dynamics!

- The B_2-flux is not periodic in SUGRA. However as it goes throug a period $\left(\frac{1}{2\pi\alpha'}\int B_2 \to 2\pi + \frac{1}{2\pi\alpha'}\int B_2\right)$ string theory should be the same and the dual gauge theory, too. In fact:

$$2\pi \text{ period for } \frac{1}{2\pi\alpha'}\int B_2 = 3 g_s M \ln \frac{r}{r_0}$$

$$\frac{1}{2\pi\alpha'}\int B_2 \Big|_{r=r_0} = 0 \longrightarrow \frac{1}{4\pi^2\alpha'}\int B_2 \Big|_{r=r_1} = -2\pi$$

$$r_1 = r_0\, e^{-\frac{2\pi}{3 g_s M}}$$

What about $N_{eff}(r)$?

$$\frac{1}{(4\pi^2\alpha')^2}\int F_5 \Big|_{r=r_0} = N \to \frac{1}{(4\pi^2\alpha')^2}\int F_5 \Big|_{r=r_1} = N - M \;!!$$

which exactly matches the Seiberg duality cascade. A cascade step is

$$\frac{\Delta r}{r_0} = \left(1 - e^{-\frac{2\pi}{3 g_s M}}\right) \begin{cases} \text{large} & g_s M \ll 1 \\ \text{small} & g_s M \gg 1 \end{cases}$$

⊙ When $\int F_5 = 0$ the solution becomes non-physical and in fact there is a naked singularity
→ the IR region should be described by the regular KS solution.

- it describes all SYM IR dynamics
- it also describes the Goldston mode (a pseudoscalar Goldston boson → phase of $B\tilde{B}$, and a massless scalar glueball → magnitude of $B\tilde{B}$) in terms of a massless supergravity fluctuation, from δF_3 and a "$\delta G + \delta B$", respectively. The corresponding superpatner, a Weyl fermion, was also found.

→ the $SU(N+M) \times SU(N)$, $N = kM$ has a complex dimension one baryonic branch: there exists a complex dimension one set of KS-like sols (of which KS represents the symmetric point).

* The decoupling problem

The KS theory is described in the IR by pure SYM SU(M) + an infinite tower of massive states + a massless mode.

- Can we decouple what does not belong to pure SYM SU(M)?

- in the UV, the curvature depends on $g_s N_{eff}(r)$ and can be made small even if $g_s M \ll 1$: the cascade steps are well separated, the SUGRA solution (KS/KT) well describes the RG-flow

- in the IR, near the apex, $N_{eff}(r) \sim M$: the curvature depends on $g_s M$ and SUGRA is reliable if $g_s M \gg 1$: the cascade is dense, the cascade steps narrow, the massive states mix with the pure glue (like in the MN model).

Note 1: the massless χ_{sf} keeps being massless all the way. However, it is a composite of the SU(2M) dynamics and its interactions are suppressed by Λ_{2M}:

$g_s M \ll 1 \rightarrow$ the SYM SU(M) sector is exponentially weakly coupled to the χ_{sf} sector since

$$\Lambda_{2M} \sim e^{\frac{2\pi}{3 g_s M}} \Lambda_M \gg \Lambda_M$$

Note 2: the UV completion involves an infinite tower of massive states, as in MN, but their origin is different.
The dual gauge theory remains 4D all the way to the UV. Consistently with general argument (see Lecture I) the FT is NEVER weakly coupled. UV completion: "SCF fixed point with ∞ ranks, $SU(\infty) \times SU(\infty)$".