The Abdus Salam
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United Nations
Educational, Scientific and
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IAEA
International Atomic Energy Agency

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Joint ICTP-IAEA Workshop on Nuclear Reaction Data for Advanced Reactor Technologies

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Neutron induced resonance reactions.

F. Gunsing
CEA Saclay
Gif sur Yvette
France

IRFU

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Frank Gunsing

CEA/Saclay

DSM/IRFU/SPhN

F-91911 Gif-sur-Yvette, France

gunsing@cea.fr

IRFU

Neutron-nucleus reactions

Reaction:

- $X + a \rightarrow Y + b$
- X(a,b)Y
- X(a,b)

Examples of equivalent notations:

$^{10}B + {}^{1}n \rightarrow {}^{7}Li + {}^{4}He$
$^{10}B + n \rightarrow {}^{7}Li + \alpha$
$^{10}B(n,\alpha)$

$^{238}U + n \rightarrow {}^{239}U^{*}$
$^{238}U + n \rightarrow {}^{239}U + \gamma$
$^{238}U(n,\gamma)$

Reaction cross section σ, expressed in barns, $1\ b = 10^{-28}\ m^2$

Neutron induced nuclear reactions:

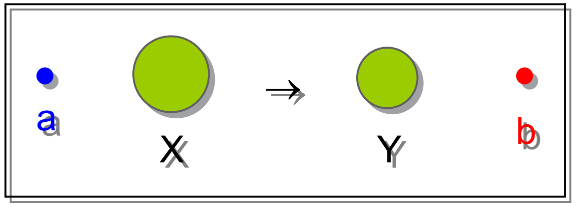
- elastic scattering (n,n)
- inelastic scattering (n,n')
- capture (n,γ)
- fission (n,f)
- particle emission (n,α), (n,p), (n,xn)

Total cross section σ_{tot}: sum of all reactions

Neutron-nucleus reactions

Reaction:

- X + a → Y + b
- X(a,b)Y



Cross section:
function of the kinetic energy of the particle a

$$\sigma(E_a) = \int\int \frac{d^2\sigma(E_a, E_b, \Omega)}{dE_b d\Omega} dE_b d\Omega$$

Differential cross section:
function of the kinetic energy of the particle a
and function of the kinetic energy **or** the angle
of the particle b

$$\frac{d\sigma(E_a, E_b)}{dE_b} \qquad \frac{d\sigma(E_a, \Omega)}{d\Omega}$$

Double differential cross section:
function of the kinetic energy of the particle a
and function of the kinetic energy **and** the angle
of the particle b

$$\frac{d^2\sigma(E_a, E_b, \Omega)}{dE_b d\Omega}$$

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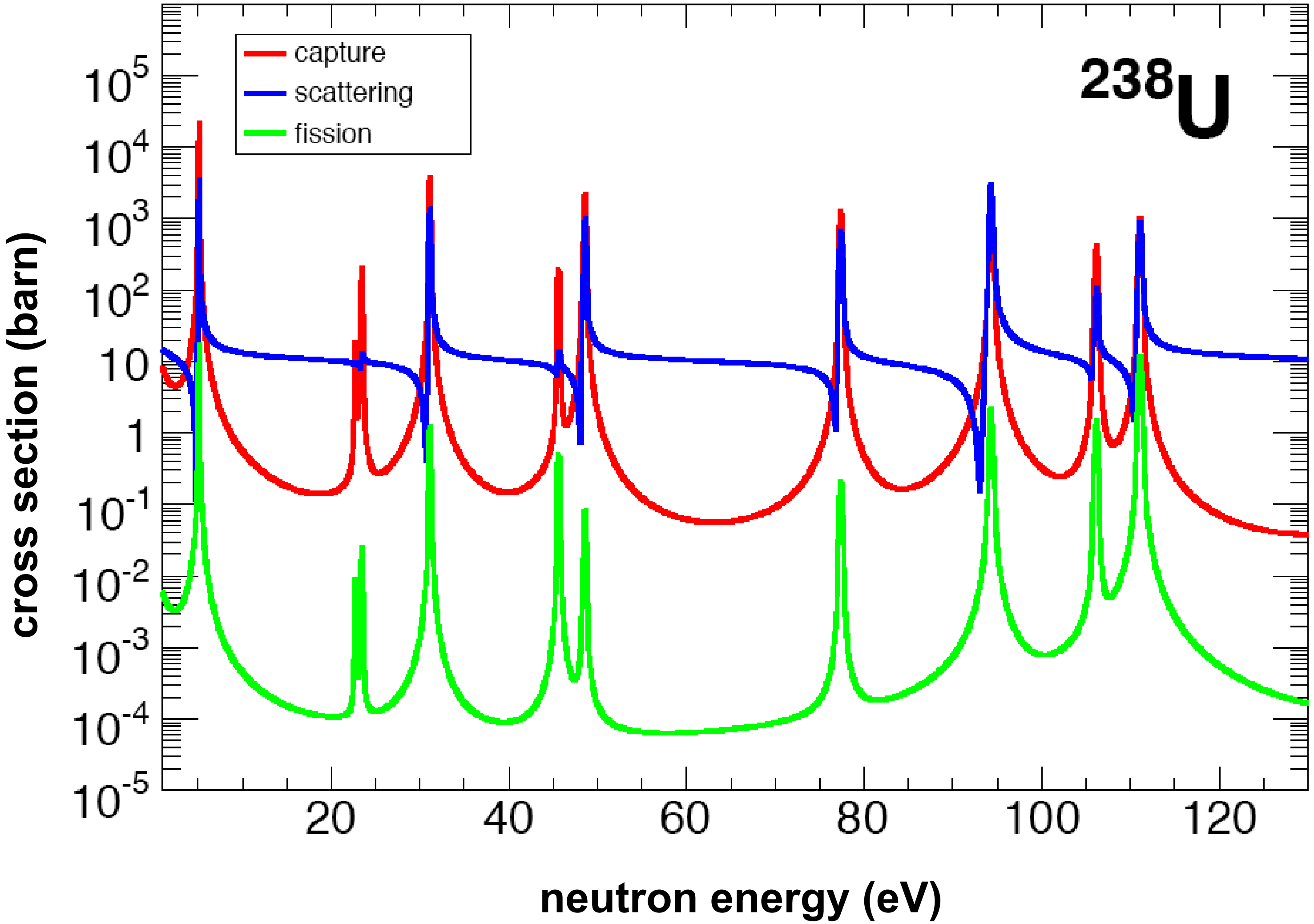
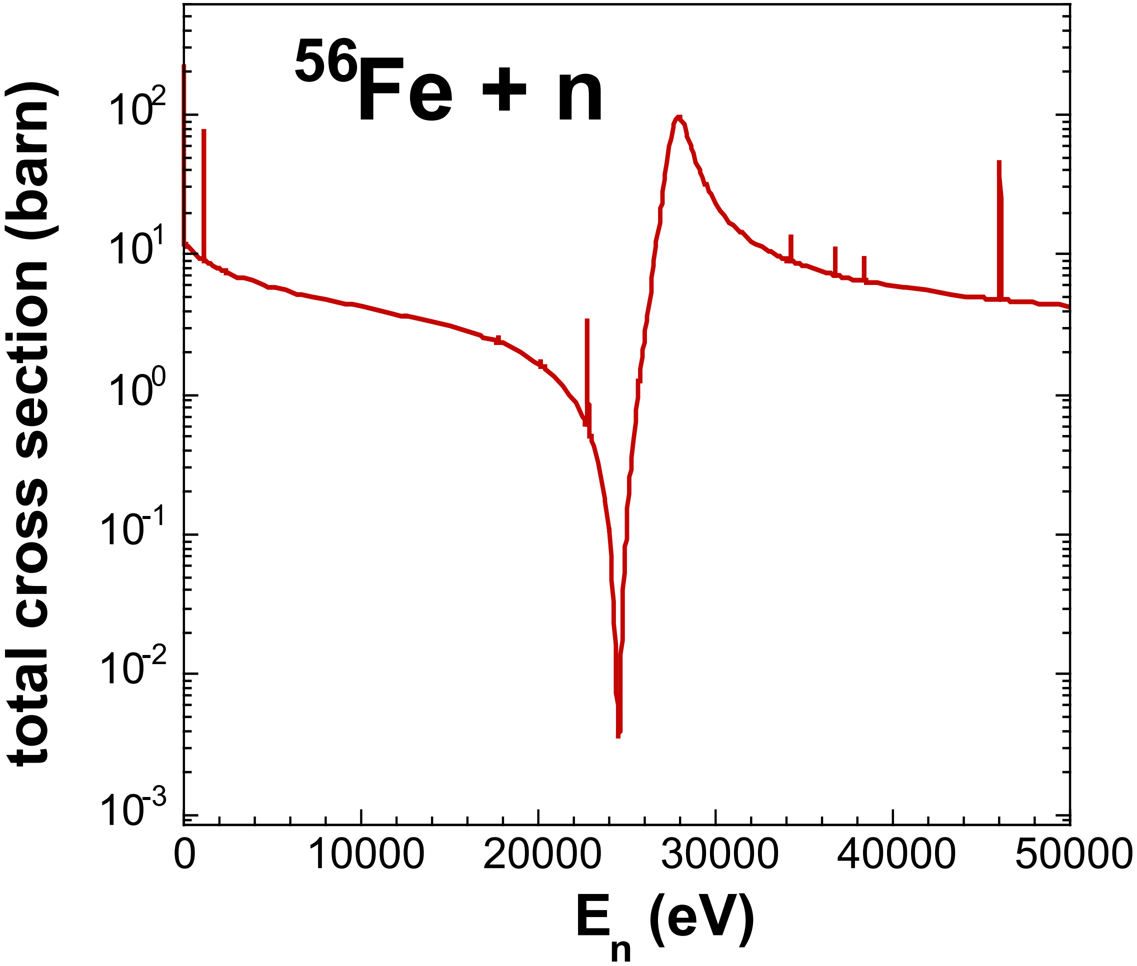
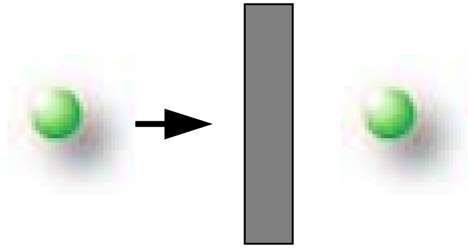
Cross sections σ_γ, σ_n et σ_f



Interference of $\sigma_{potential}$ and σ_n



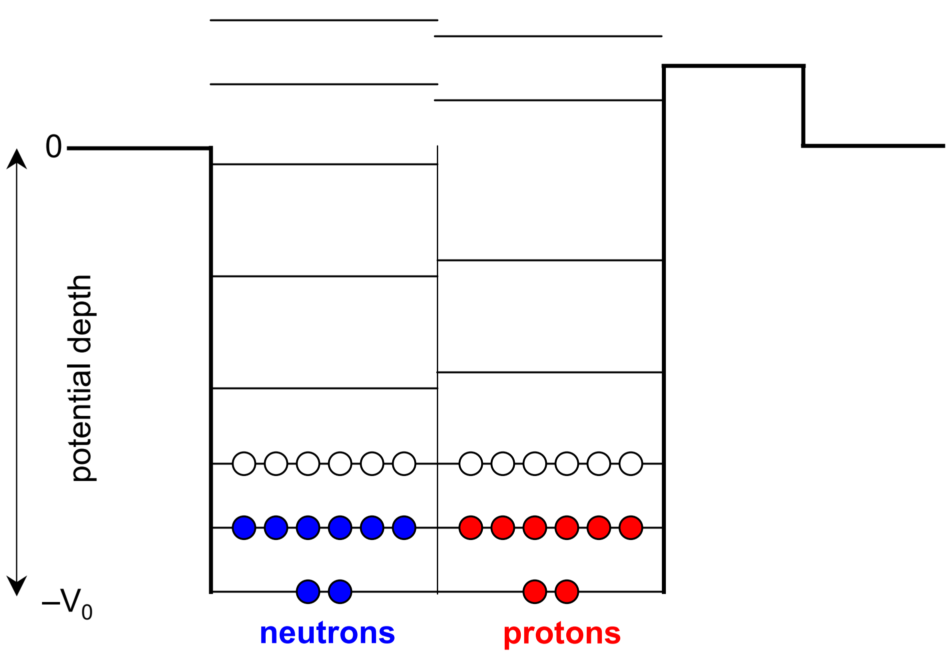


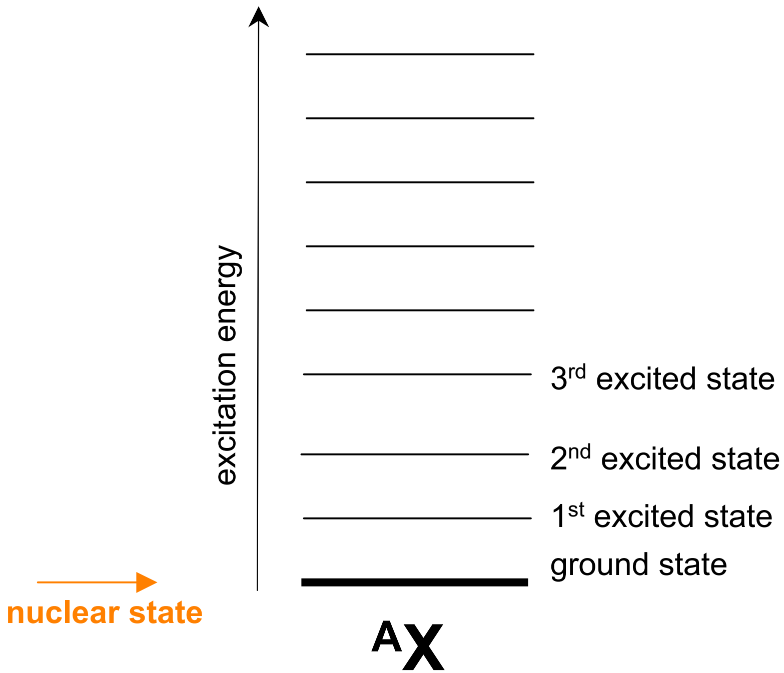
transmission
$T = \exp(-n \cdot \sigma_T)$
$0<T<1$

The nucleus as a quantum system

shell model representation:
configuration of nucleons in their potential

level scheme representation:
excited states of a nucleus
(shell model and other states)





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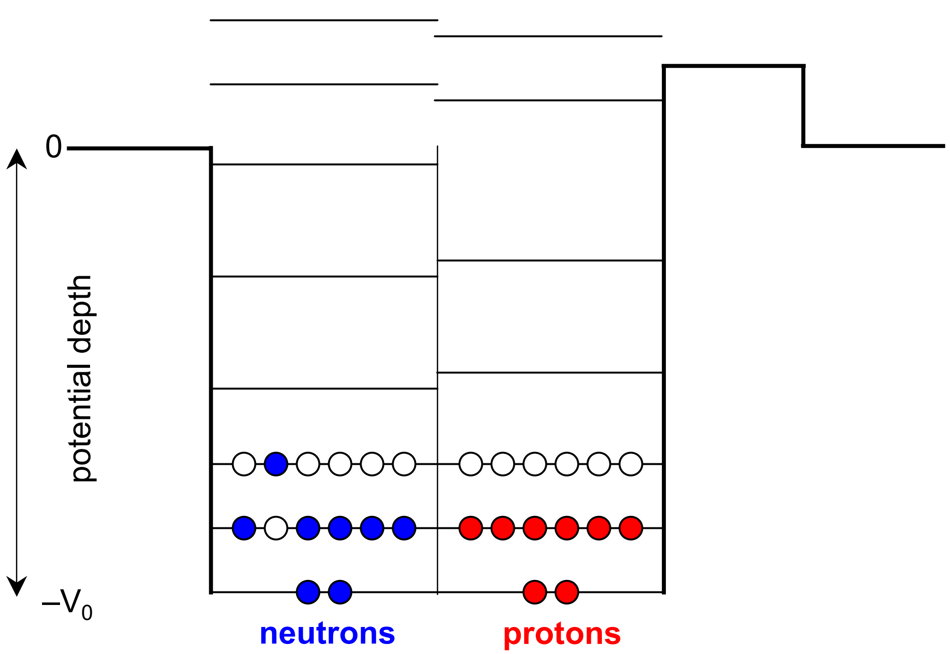

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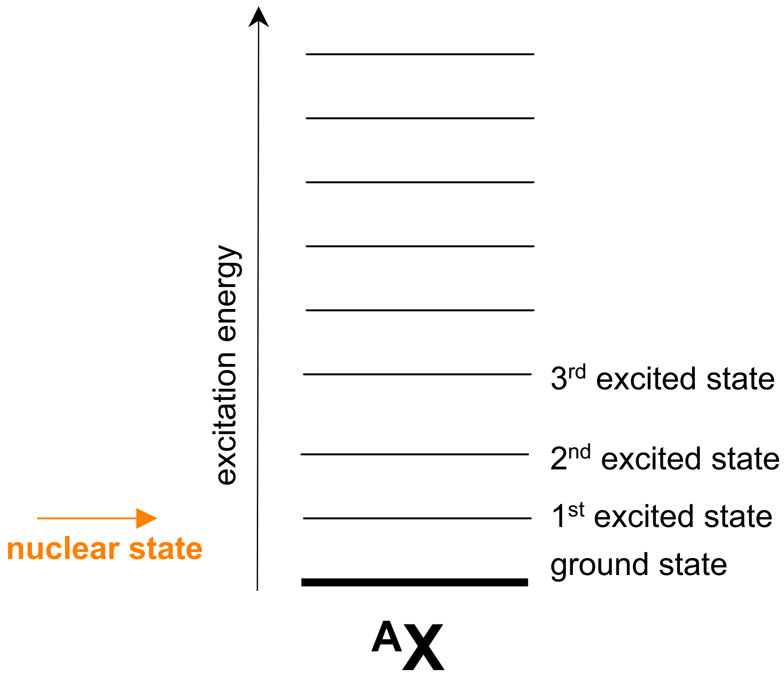

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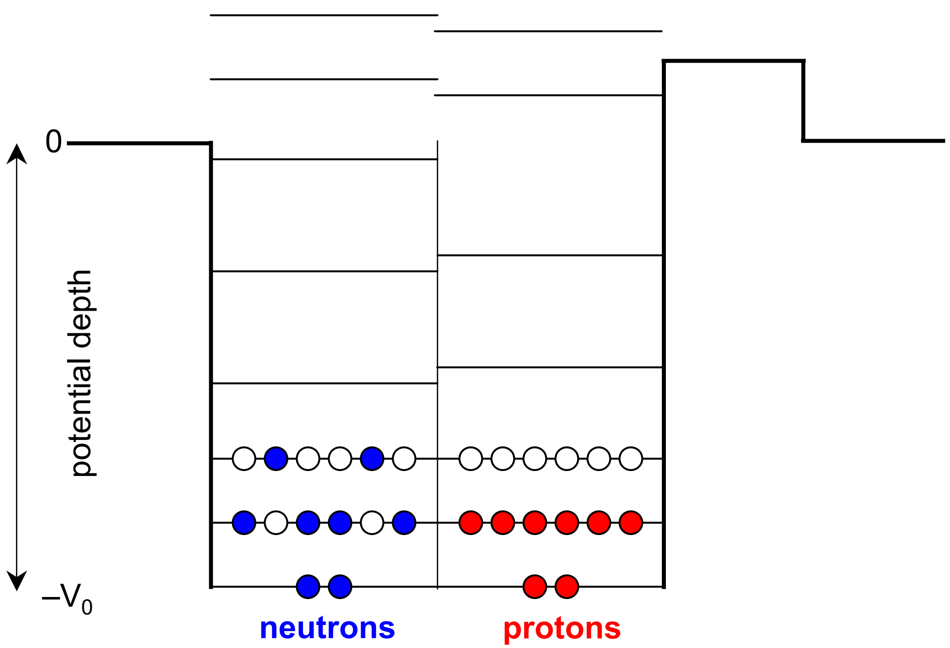

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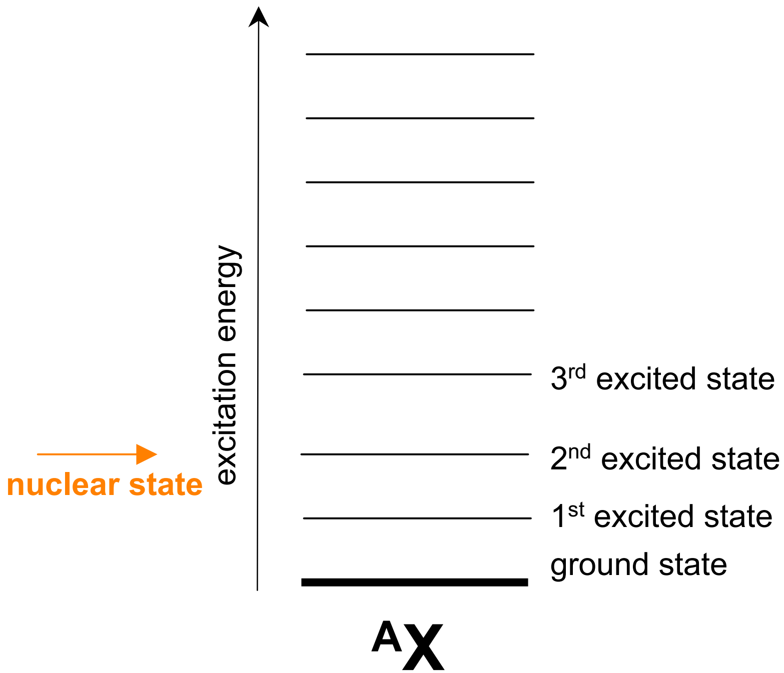

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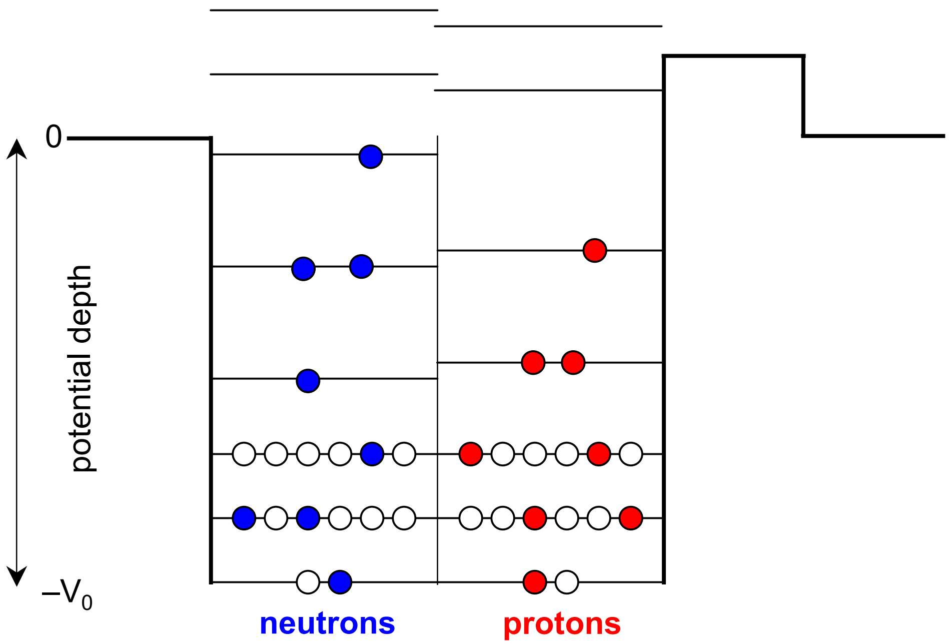


Decay of a nuclear state

state with a life time τ:

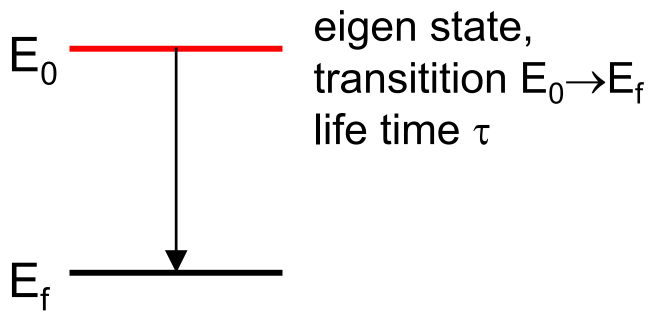
$$\Psi(t) = \Psi_0 e^{-iE_0 t/\hbar} e^{-t/2\tau}$$

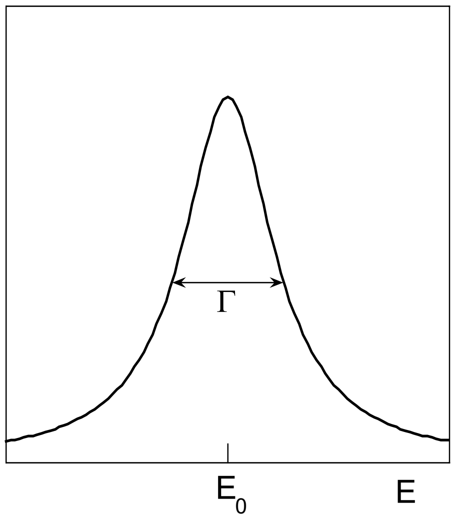
definition (Heisenberg):

$$\Gamma = \frac{\hbar}{\tau}$$

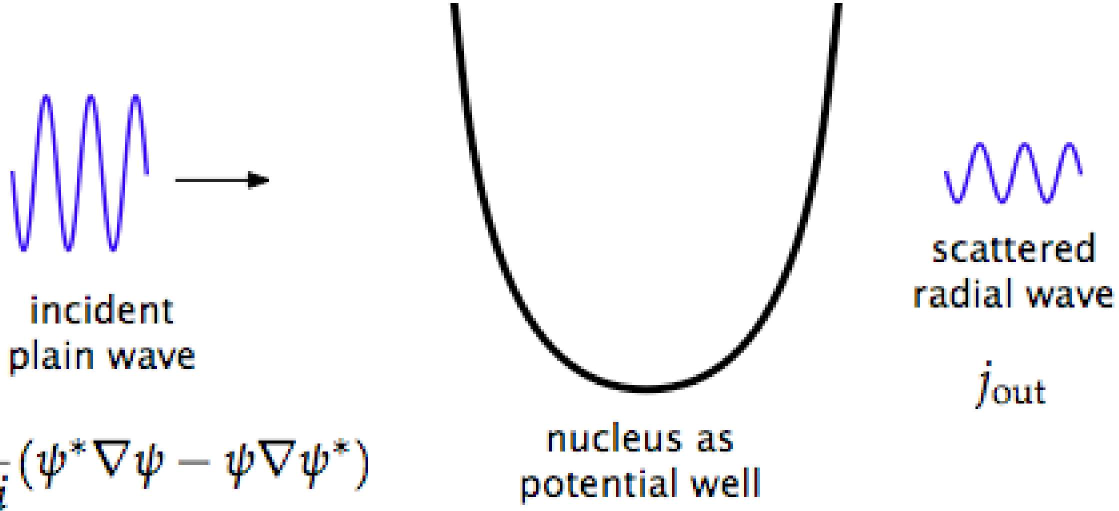
Fourier transform gives energy profile:

$$I(E) = \frac{\Gamma / 2\pi}{(E - E_0)^2 + \Gamma^2 / 4}$$





Neutron-nucleus reactions



$$j_{\text{inc}} = \frac{\hbar}{2mi}(\psi^* \nabla \psi - \psi \nabla \psi^*)$$

Conservation of probability density: $$\sigma(\Omega) = \frac{r^2\, j_{\text{out}}(r,\Omega)}{j_{\text{inc}}}$$

Solve Schrödinger equation of system to get cross sections.
Shape of wave functions of in- and outgoing particles are known,
potential is unknown. Two approaches:

- calculate potential (optical model calculations, smooth cross section)
- use eigenstates (R-matrix, resonances)

R-matrix formalism

partial incoming wave functions: $\mathcal{I}_c$

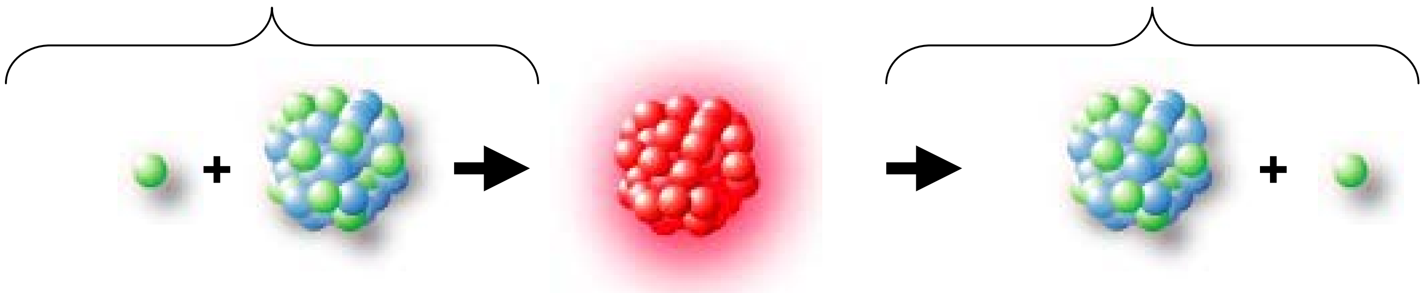
partial outgoing wave functions: $\mathcal{O}_{c'}$

related by collision matrix: $U_{cc'}$

cross section:

$$\sigma_{cc'} = \pi \lambda_c^2 |\delta_{c'c} - U_{c'c}|^2$$

External region (r>a_c, well separated particles):
• no interaction, Schrödinger equation solvable.



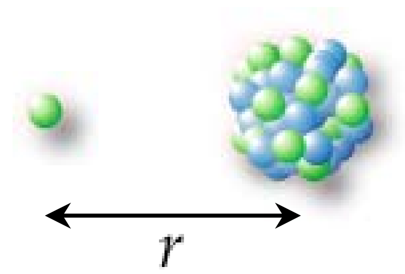
entrance channel
$c = \{\alpha, \ell, j, J, m_J\}$

compound nucleus

exit channel
$c' = \{\alpha', \ell', j', J', m_J'\}$

Internal region (r<a_c compound nucleus):
• wave function is expansion of eigenstates λ.

Find the wave functions



separation distance

$r > a_c$ external region

$r < a_c$ internal region

$r = a_c$ match value and derivate of Ψ

$$\left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2m_c}{\hbar^2}(V(r) - E)\right] R(r) = 0$$

External region: **easy**, solve Schrödinger equation

central force, separate radial and angular parts.

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$$

solution: solve Schrödinger equation of relative motion:

- Coulomb functions
- special case of neutron particles (neutrons): fonctions de Bessel

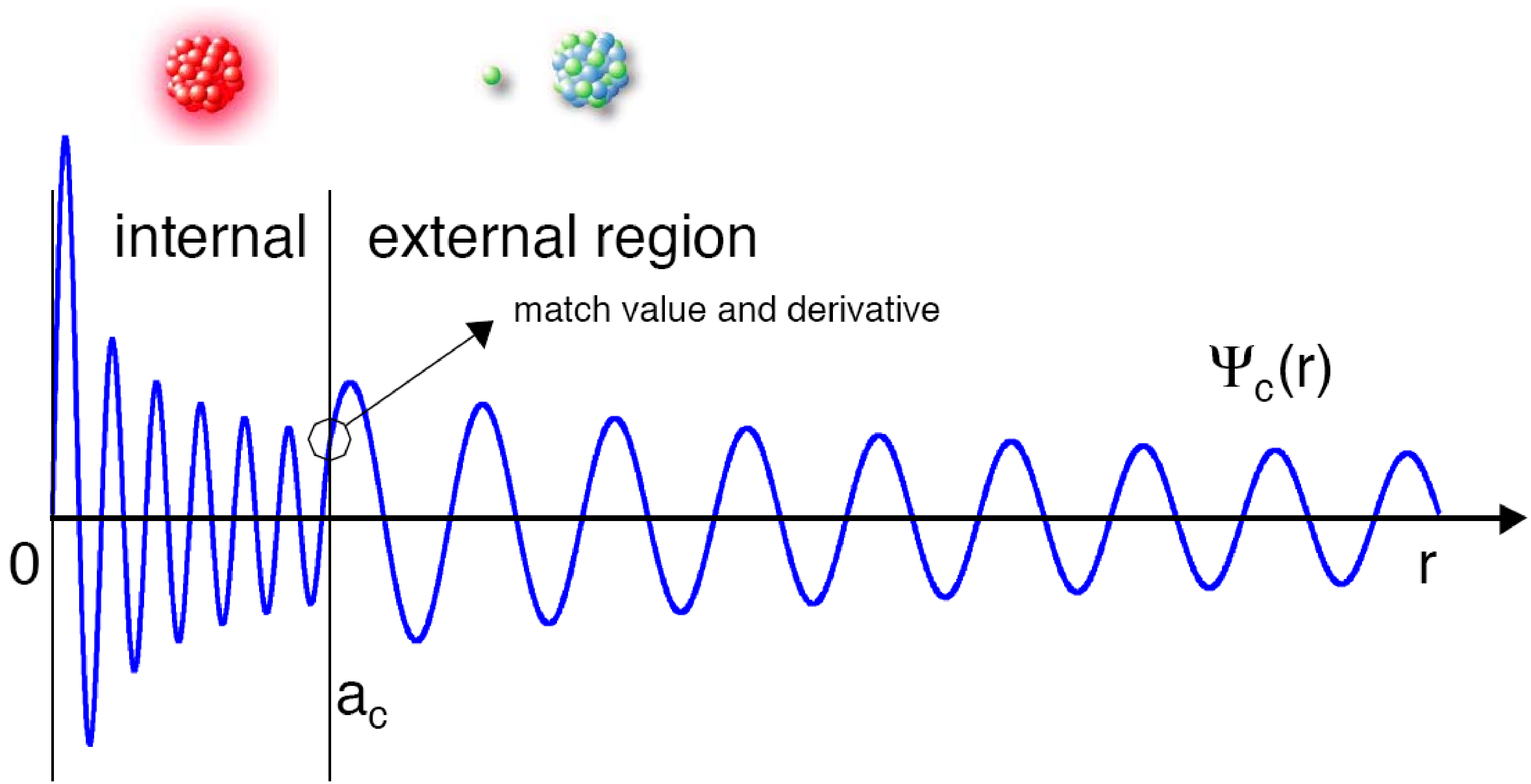
Internal region: **very difficult**, Schrödinger equation cannot be solved directly

solution: expand the wave function as a linear combination of its eigenstates.

using the R-matrix:

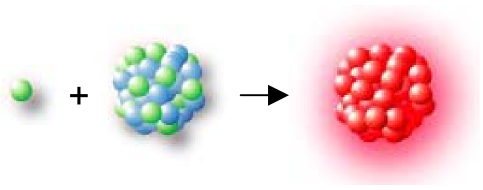
$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

The R-matrix formalism

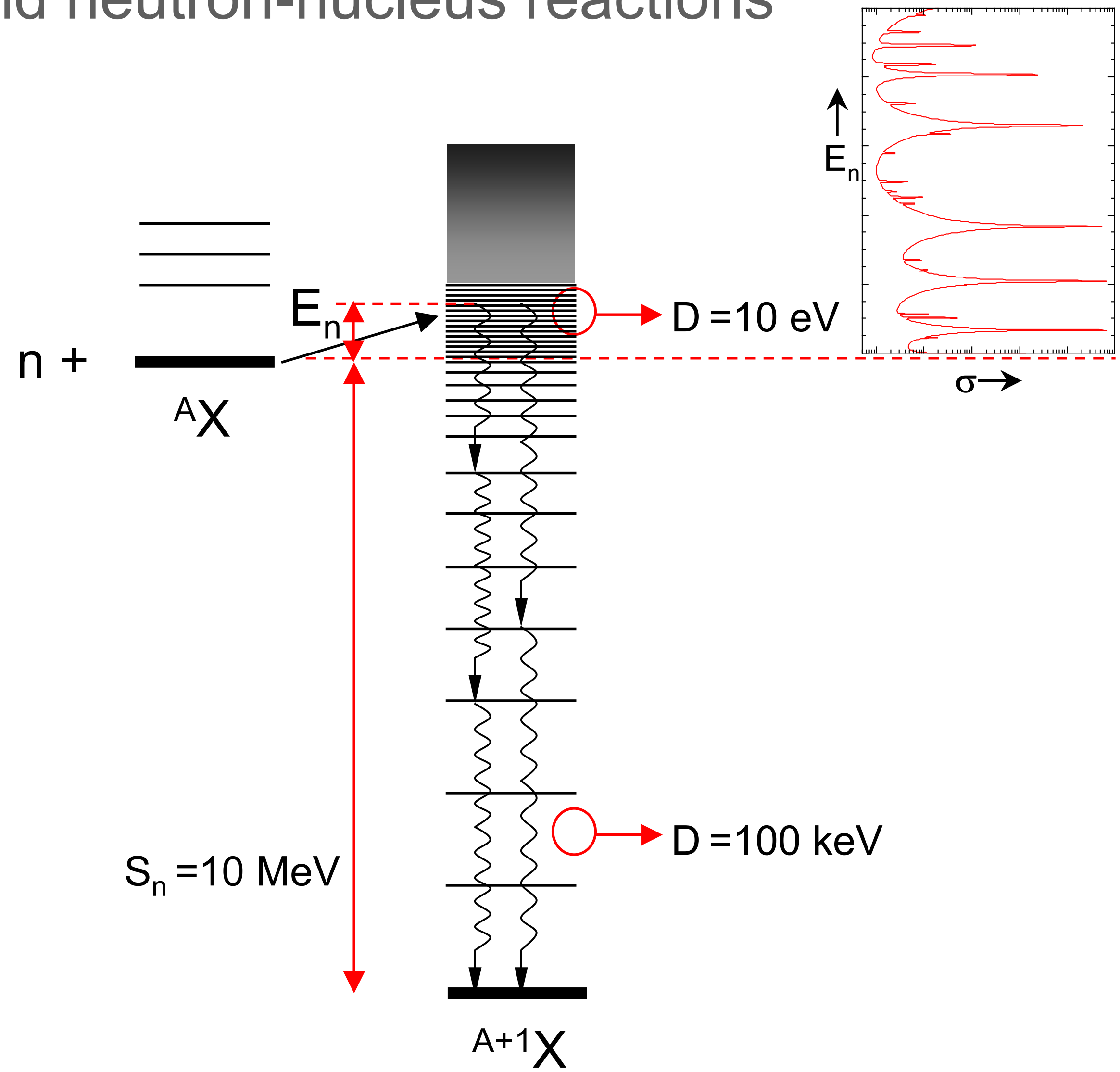


Compound neutron-nucleus reactions

compound
nucleus reaction



$\tau \sim 10^{-16}$
$E_n < 10$ MeV



The R-matrix formalism

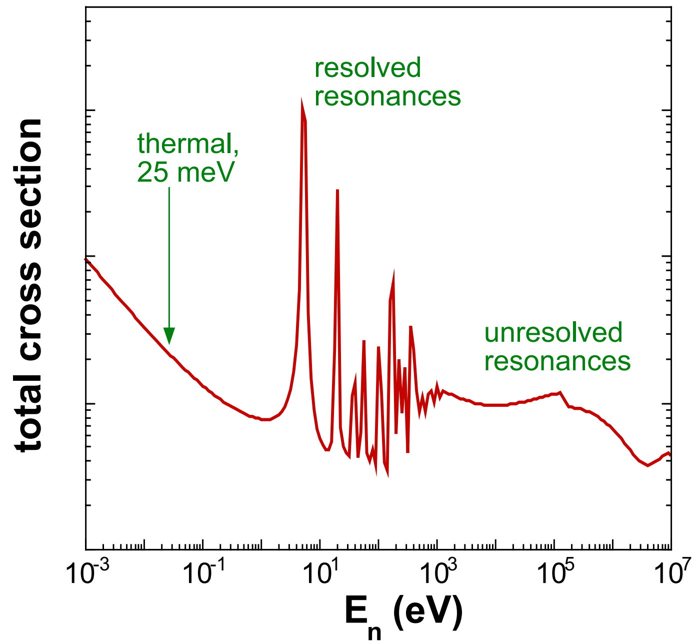
- The R-matrix formalism is adapted to describe compound nucleus reactions.
- Typically used for neutron-induced reactions at low energy (E_n<10 MeV, resonance region).
- The resonance parameters are properties of the excited nuclear levels:

- in the resolved resonance region (RRR), to each level (resonance) corresponds a set of parameters:

$$E, J^{\pi}, \Gamma_n, \Gamma_\gamma$$

- in the unresolved resonance region (URR) average parameters are used:

$$<D_\ell>, <\Gamma_n^\ell>, <\Gamma_\gamma^\ell>$$



Resonance parameters

- A same set of resonance parameters is used to produce all resonant reactions,
- at low energies mainly elastic scattering and capture (and fission).

$$\sigma_{r,t} = \sigma_{r,t}(\text{resonance parameters})$$

- In a measurement, one does not measure a cross section, but a reaction yield or
- transmission factor.

$$Y_r = (1 - e^{-n\sigma_t})\frac{\sigma_r}{\sigma_t} \qquad 0 < Y_r < 1$$

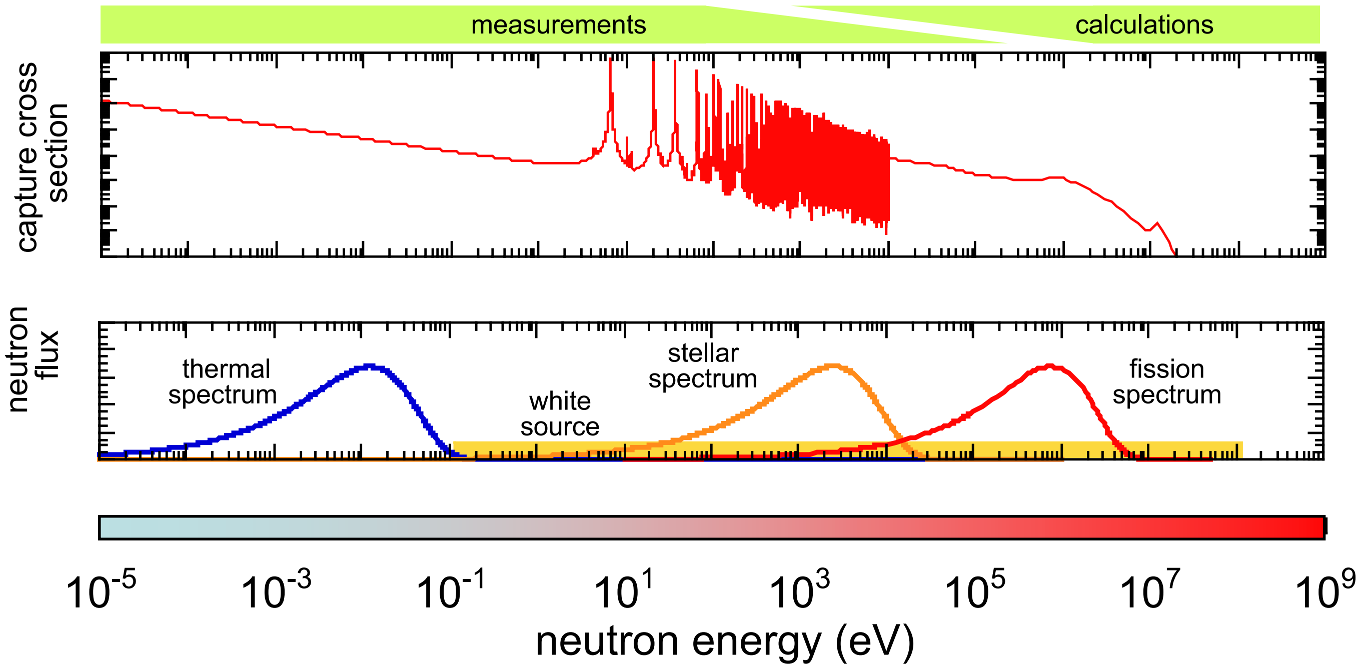
$$T = e^{-n\sigma_t} \qquad 0 < T < 1$$

- The measured reaction yield is not equally sensitive to all parameters, additional
- constraints can be necessary to extract RP from measurement.

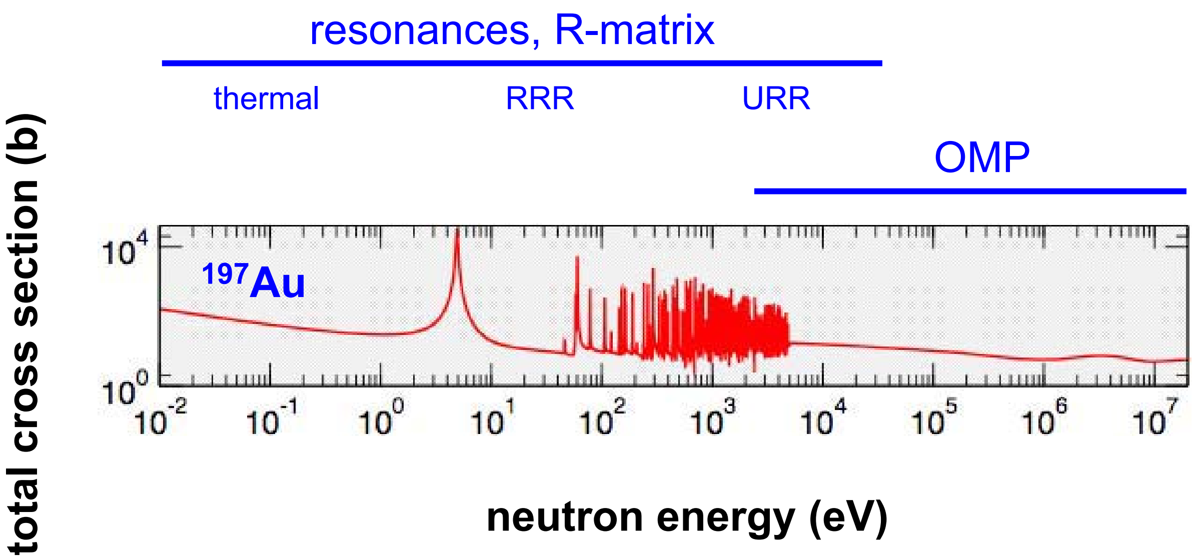
Neutron-nucleus reactions

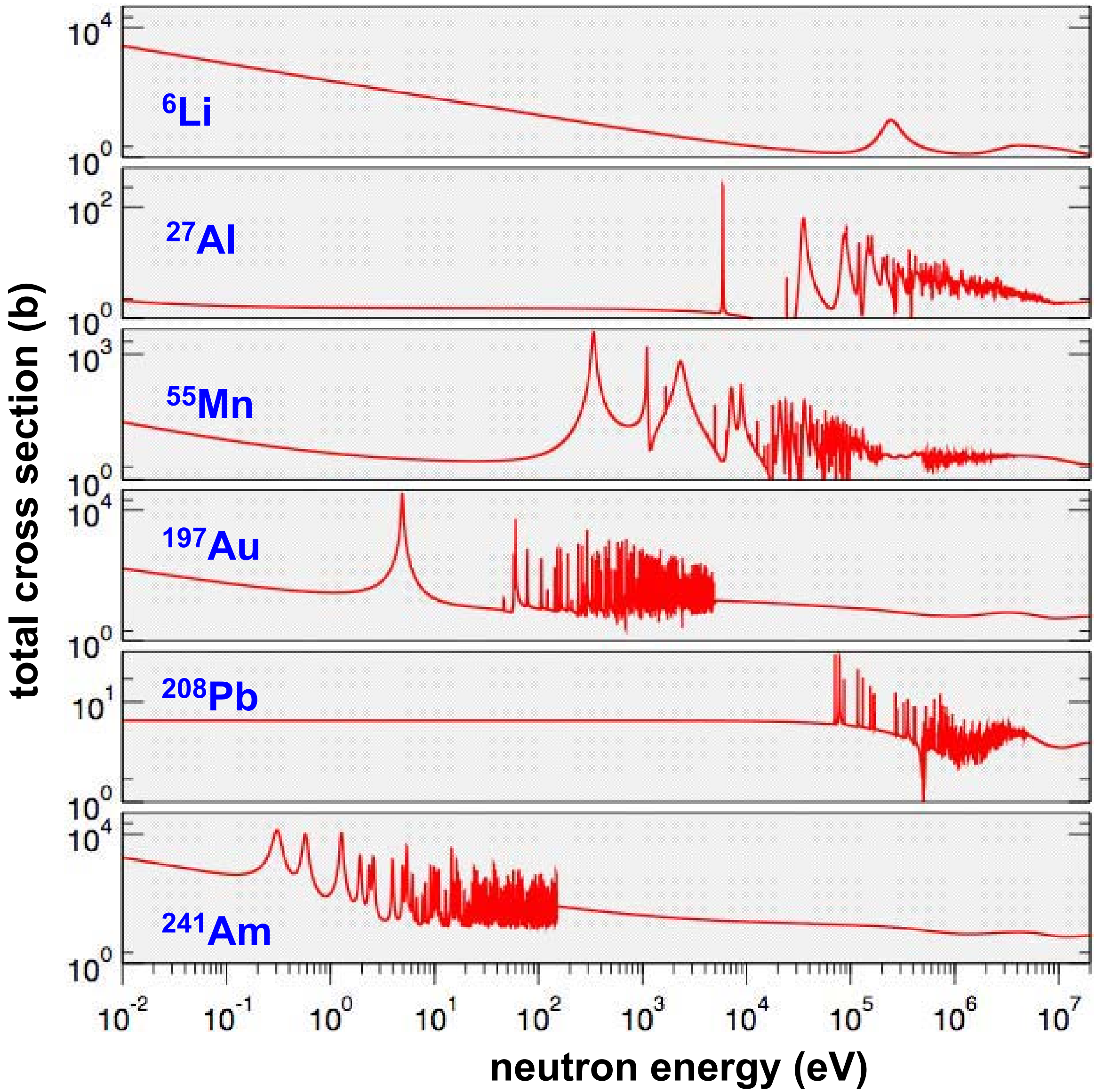
R-matrix resonance description
- fine structure (resonances)
- highly fluctuating cross sections
- resonance parametrization
- low energy, few channels

Optical model calculations
- gross structure
- average cross sections
- optical model potential
- high energy, many channels



Compound nucleus reactions





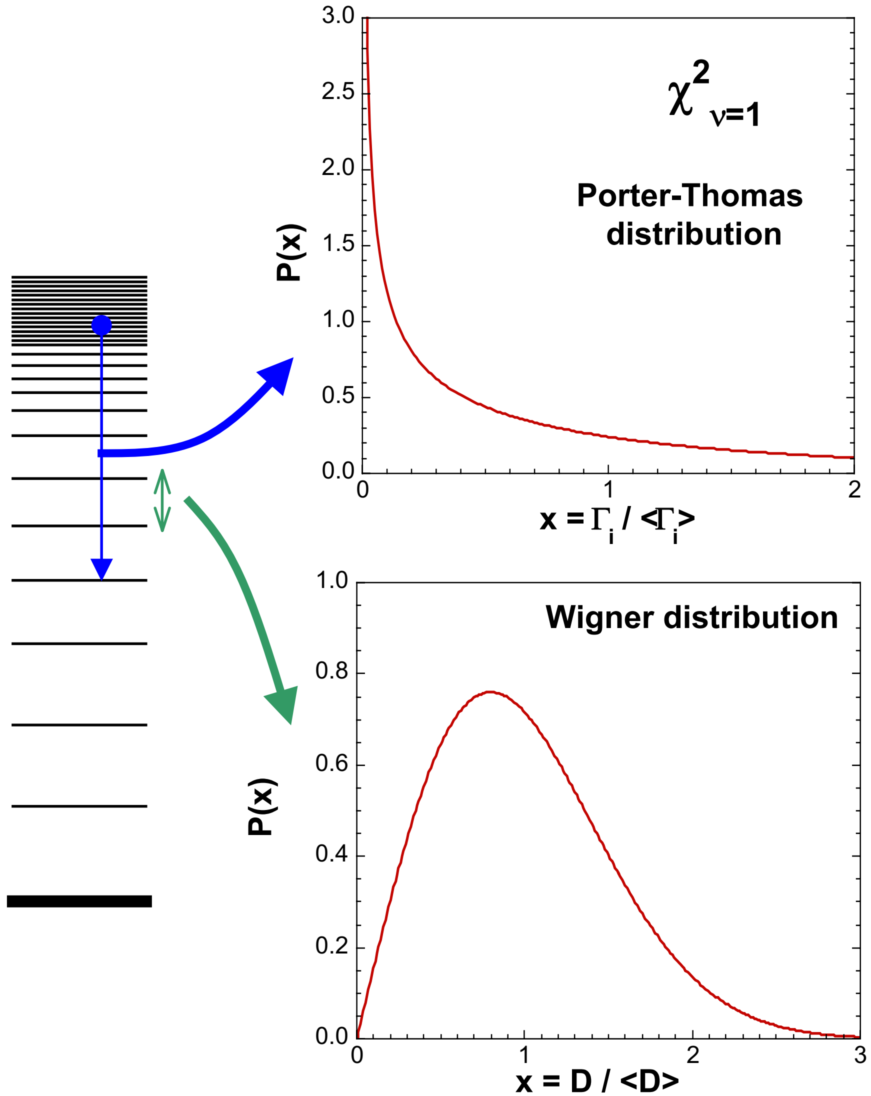
total cross section (b)
neutron energy (eV)
^{6}Li
^{27}Al
^{55}Mn
^{197}Au
^{208}Pb
^{241}Am
10^{4}
10^{0}
10^{2}
10^{3}
10^{1}
10^{-2}
10^{-1}
10^{5}
10^{6}
10^{7}

Statistical model

The nucleus in the vicinity of S_n is described by the Gaussian Orthogonal Ensemble (GOE)

The matrix elements are random variables with a Gaussian distribution.

- Consequences:
  - The partial width have a Porter-Thomas distribution.
  - The spacing of levels with the same J^{π} have approximately a Wigner distribution.



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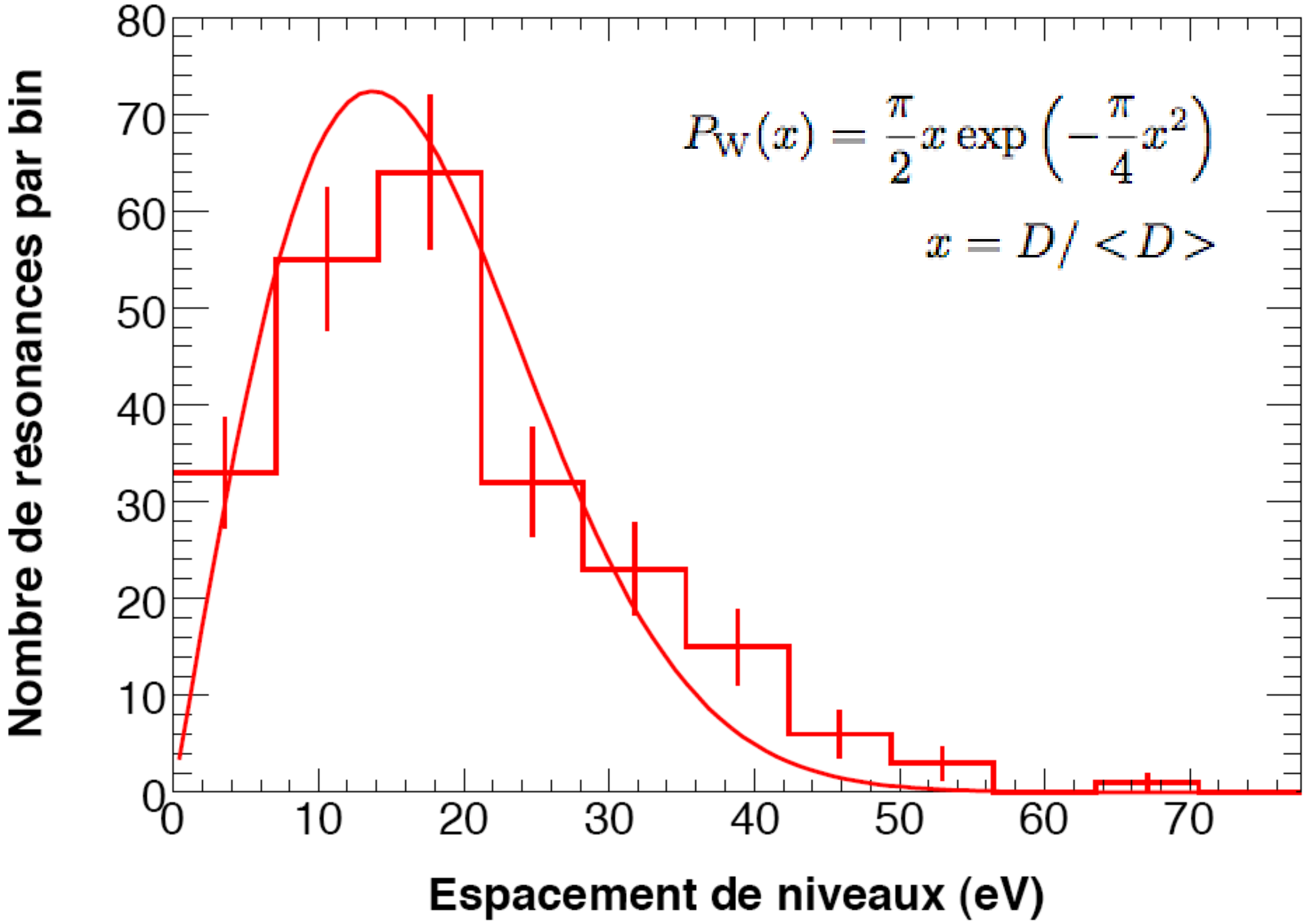
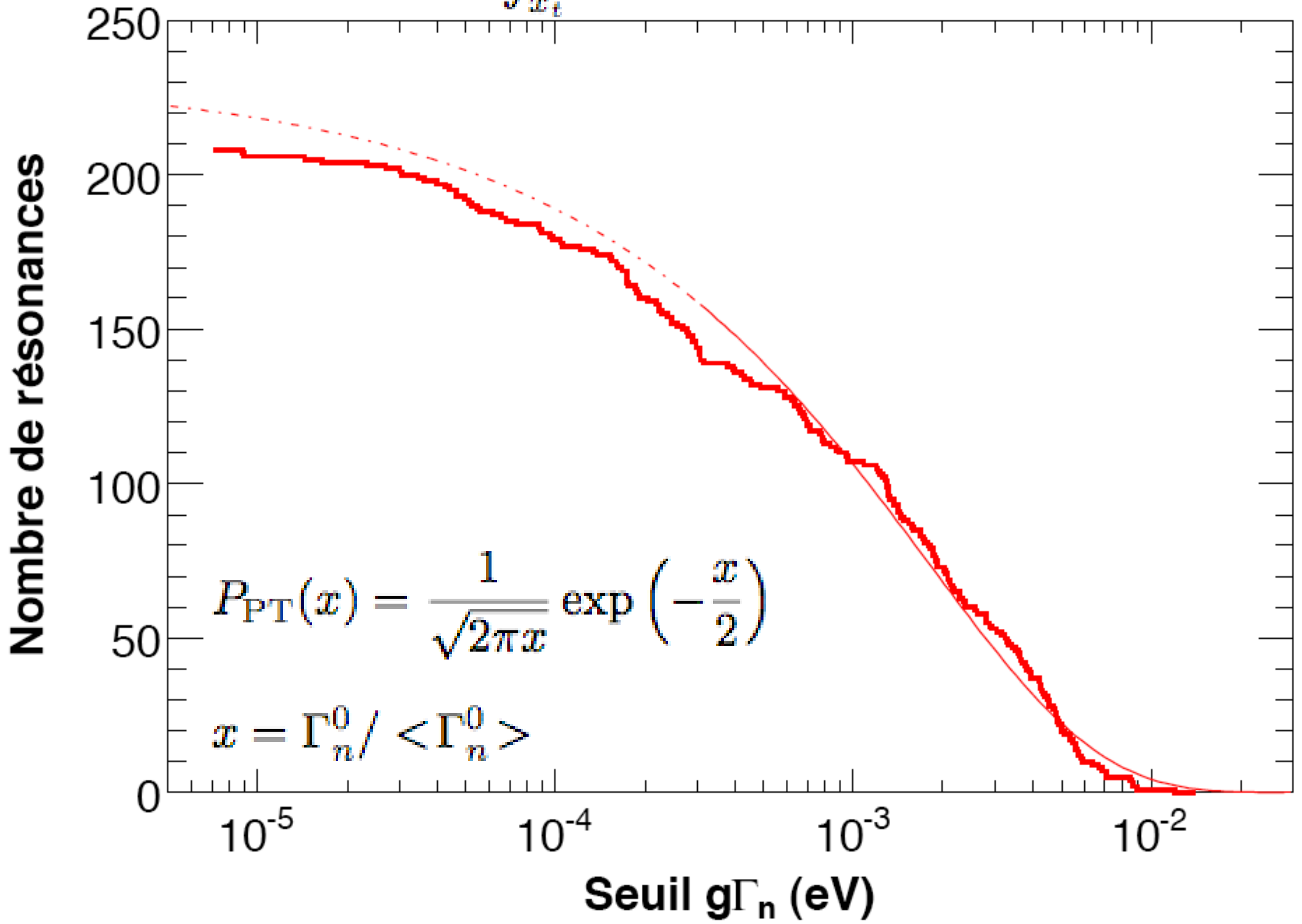
Distribution of the spacing of two consecutive levels



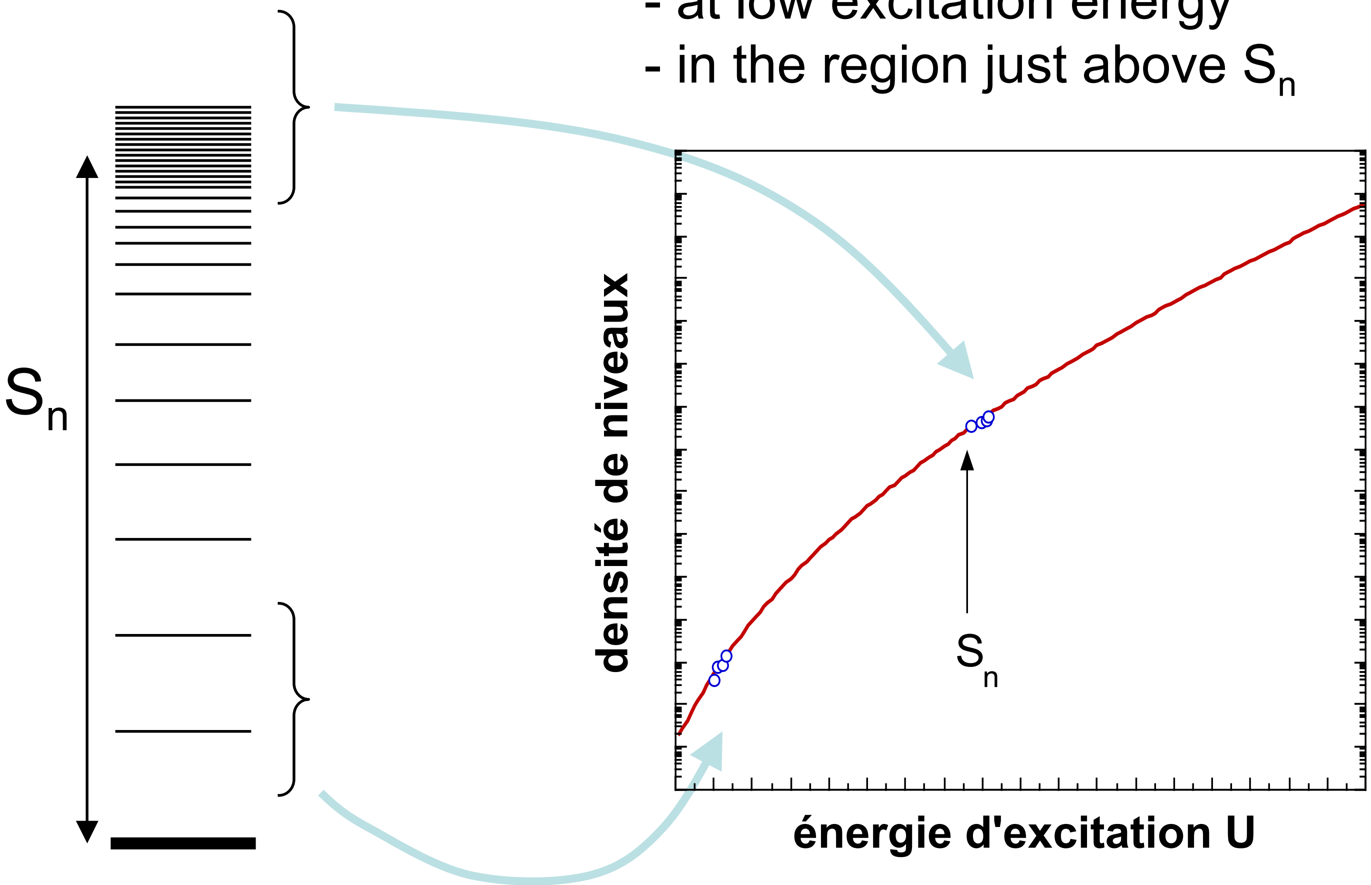
Distribution of neutron widths

$$N(x_t) = N_0 \int_{x_t}^{\infty} P_{\mathrm{PT}}(x)dx = N_0(1 - \mathrm{erf}\sqrt{x_t/2})$$

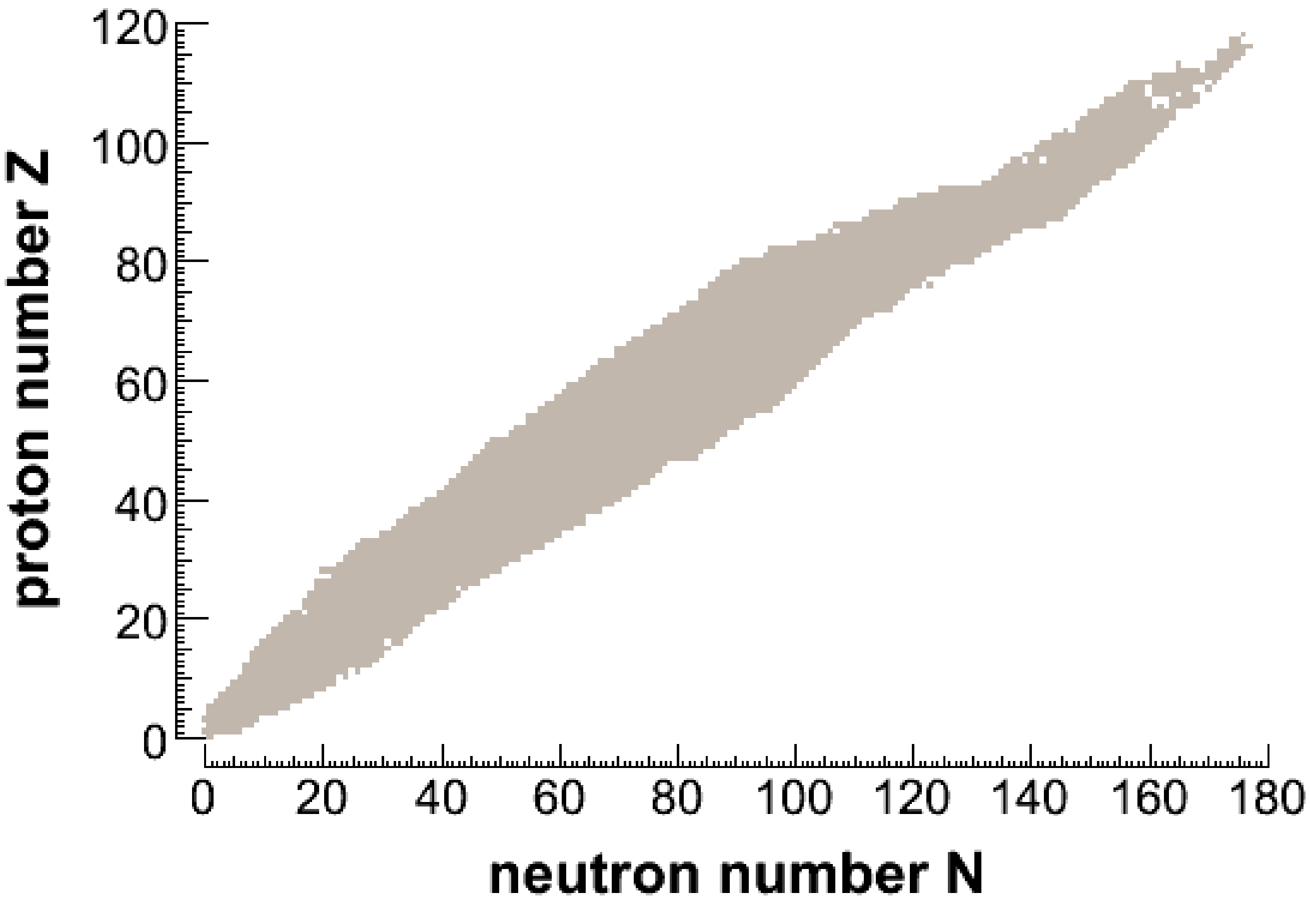


Level density

Separated states are available
- at low excitation energy
- in the region just above S_n



Known nuclei



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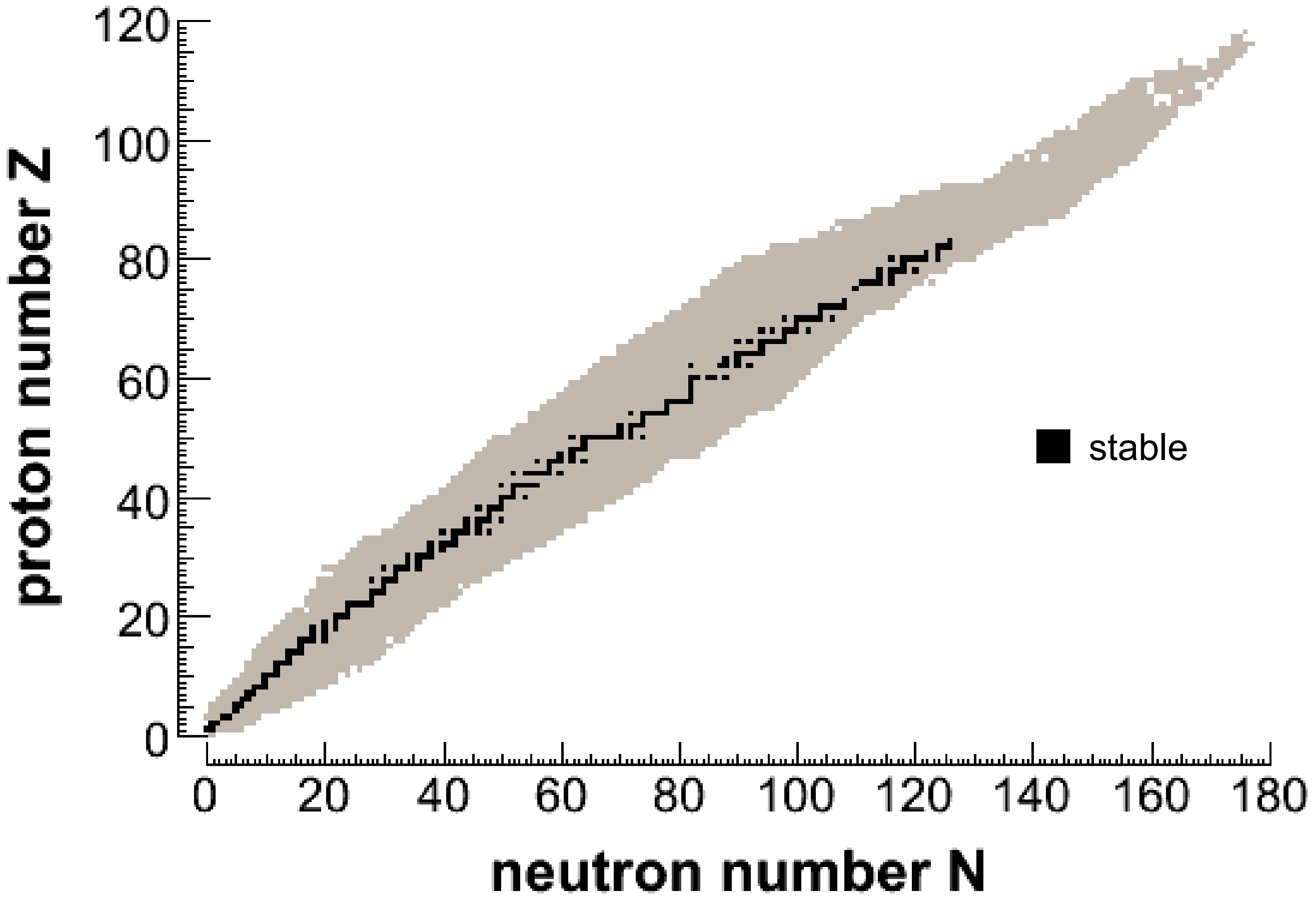
Known nuclei



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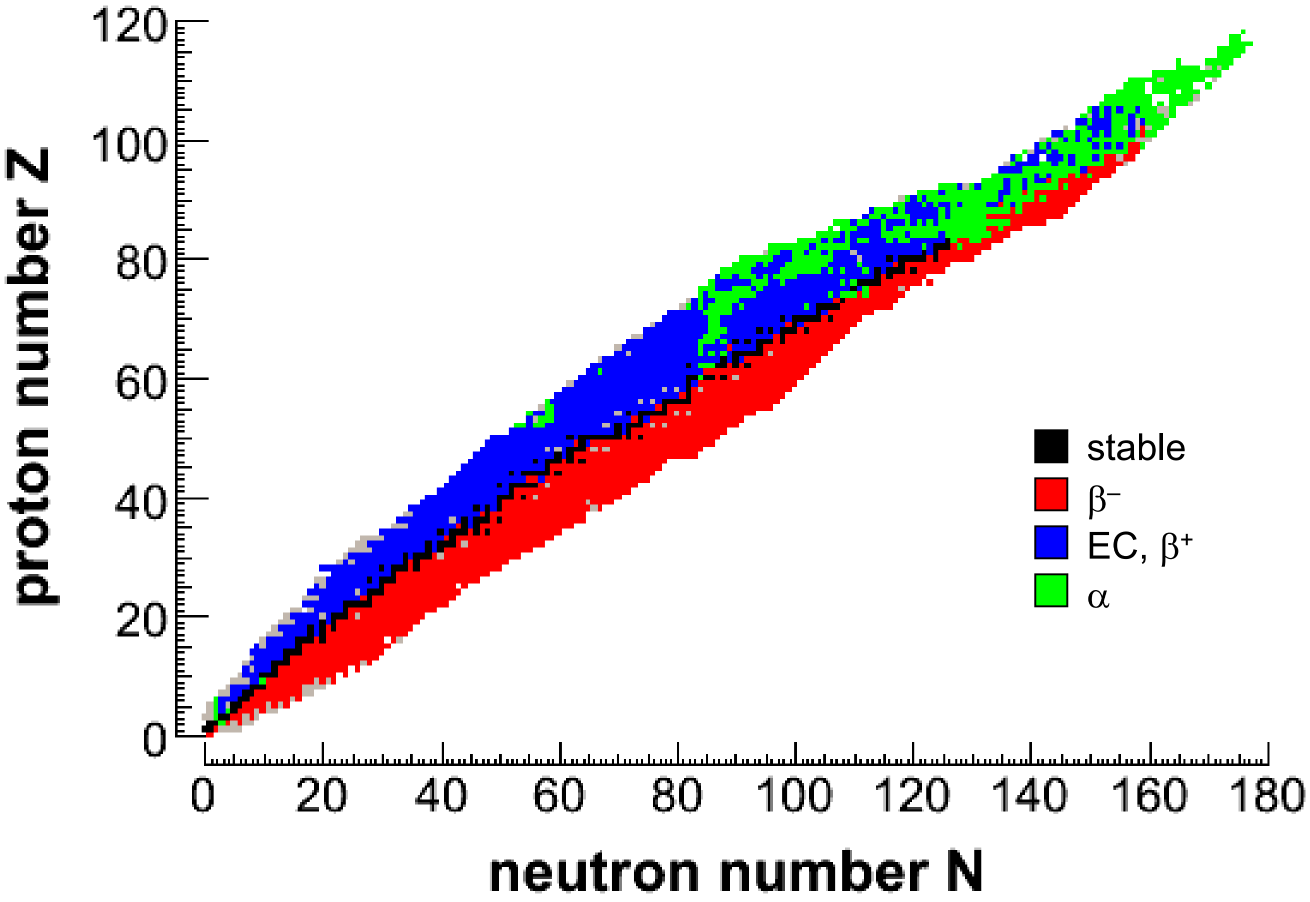
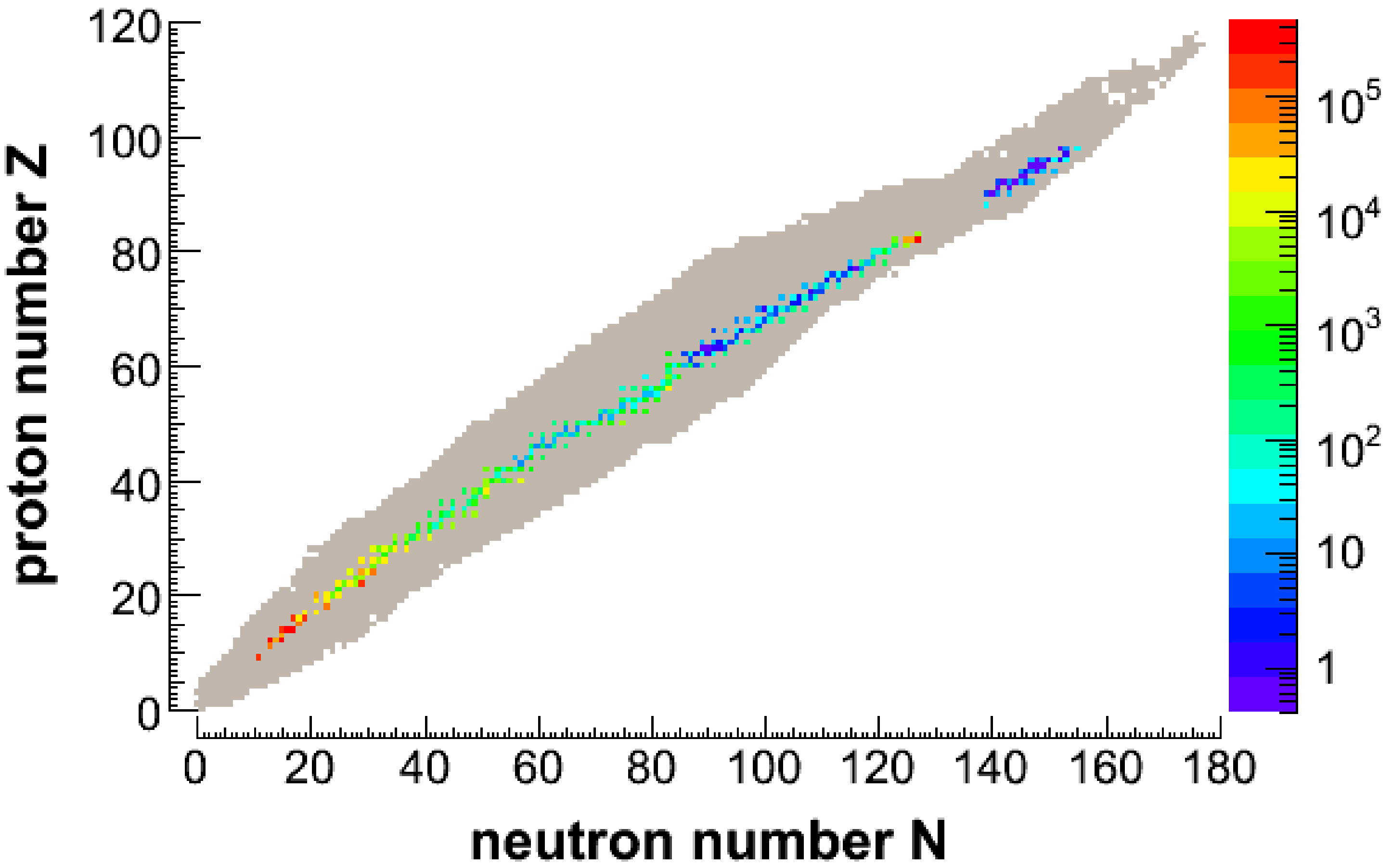


Level spacing D_0



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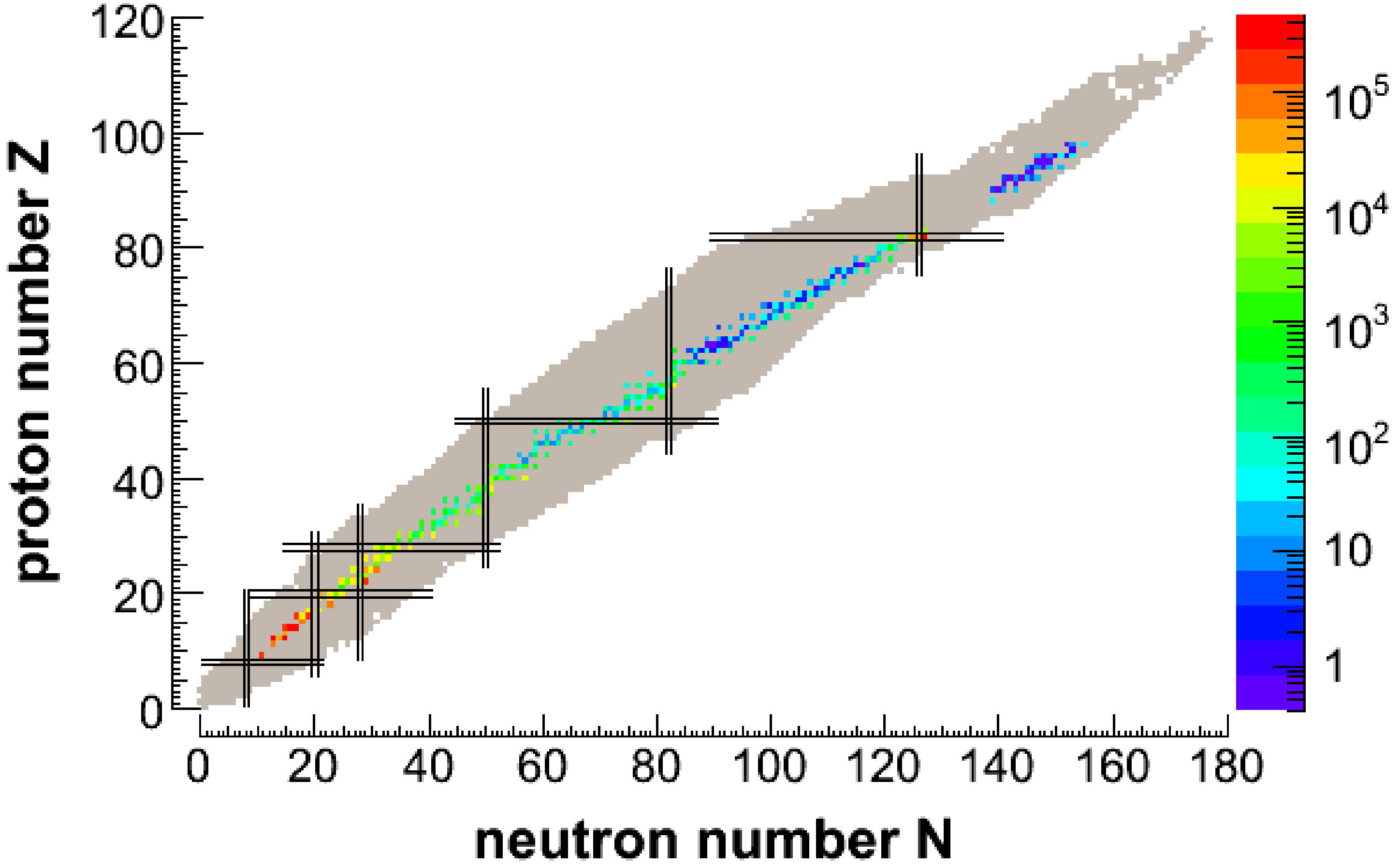

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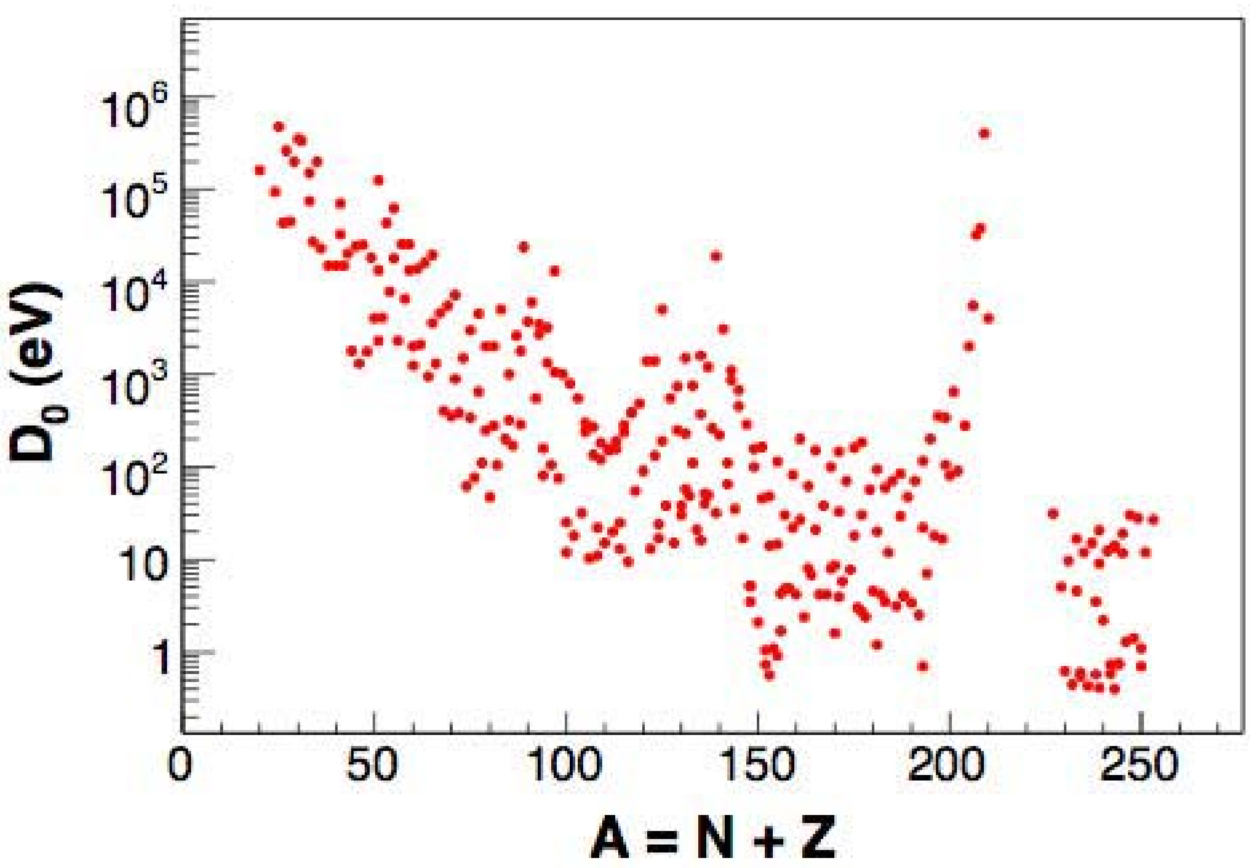
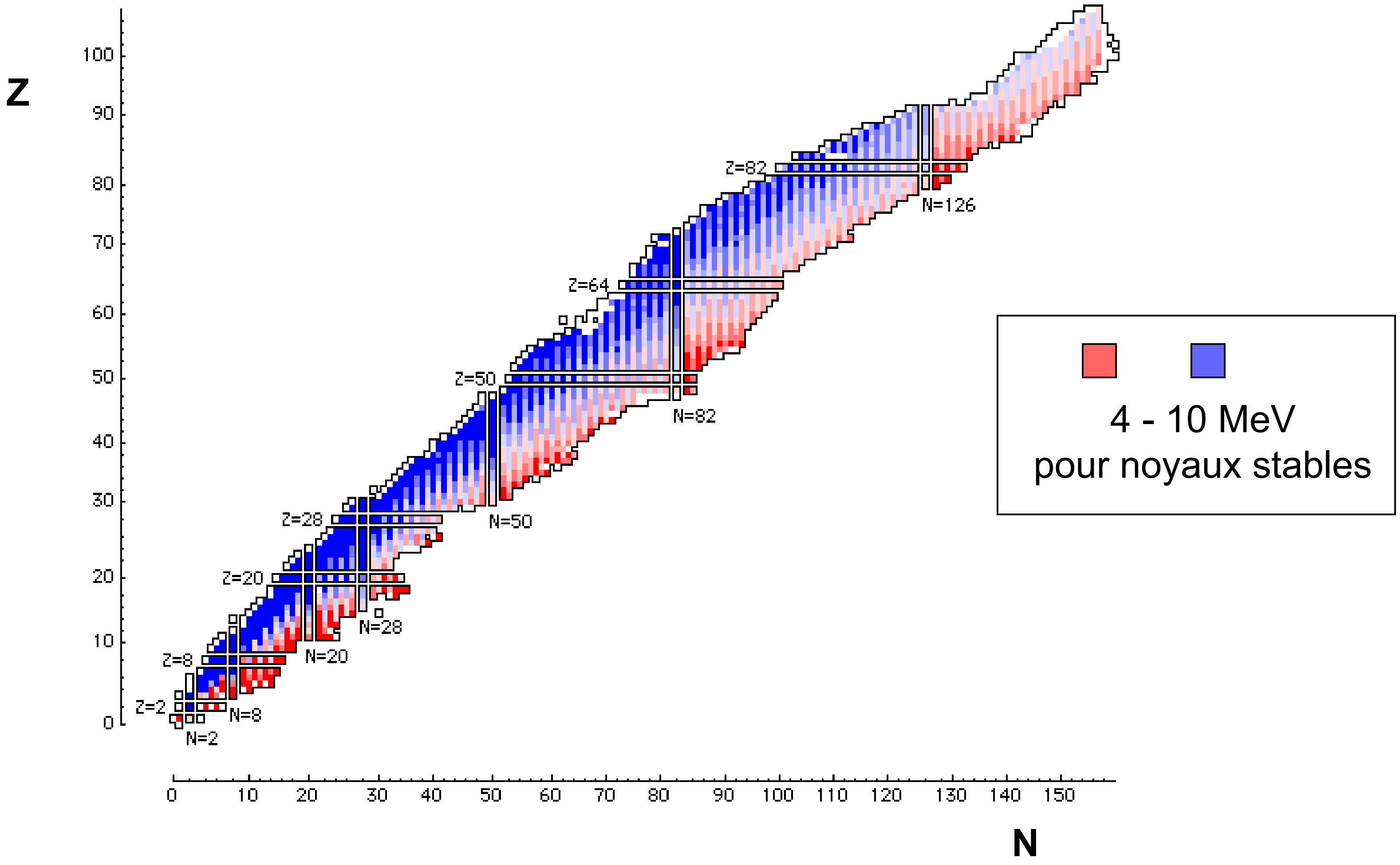


Neutron separation energy



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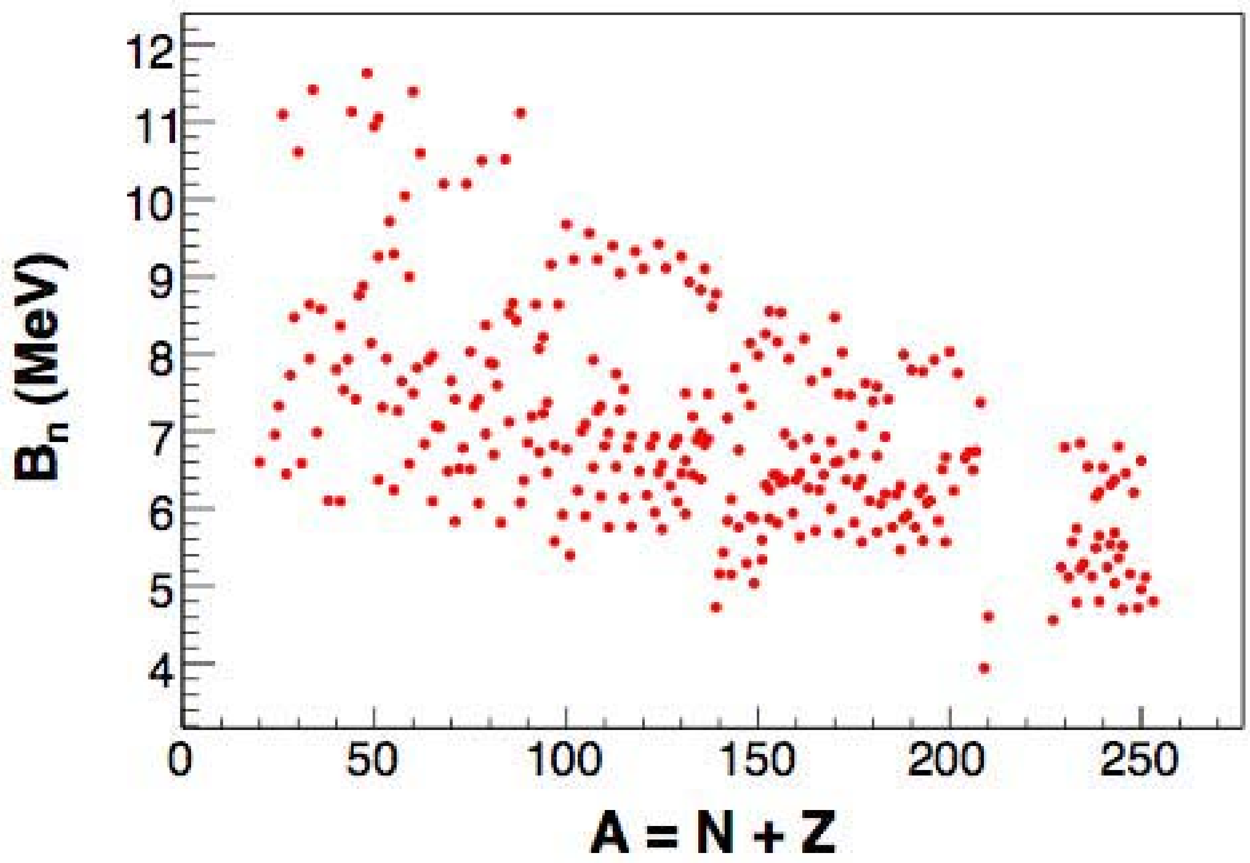
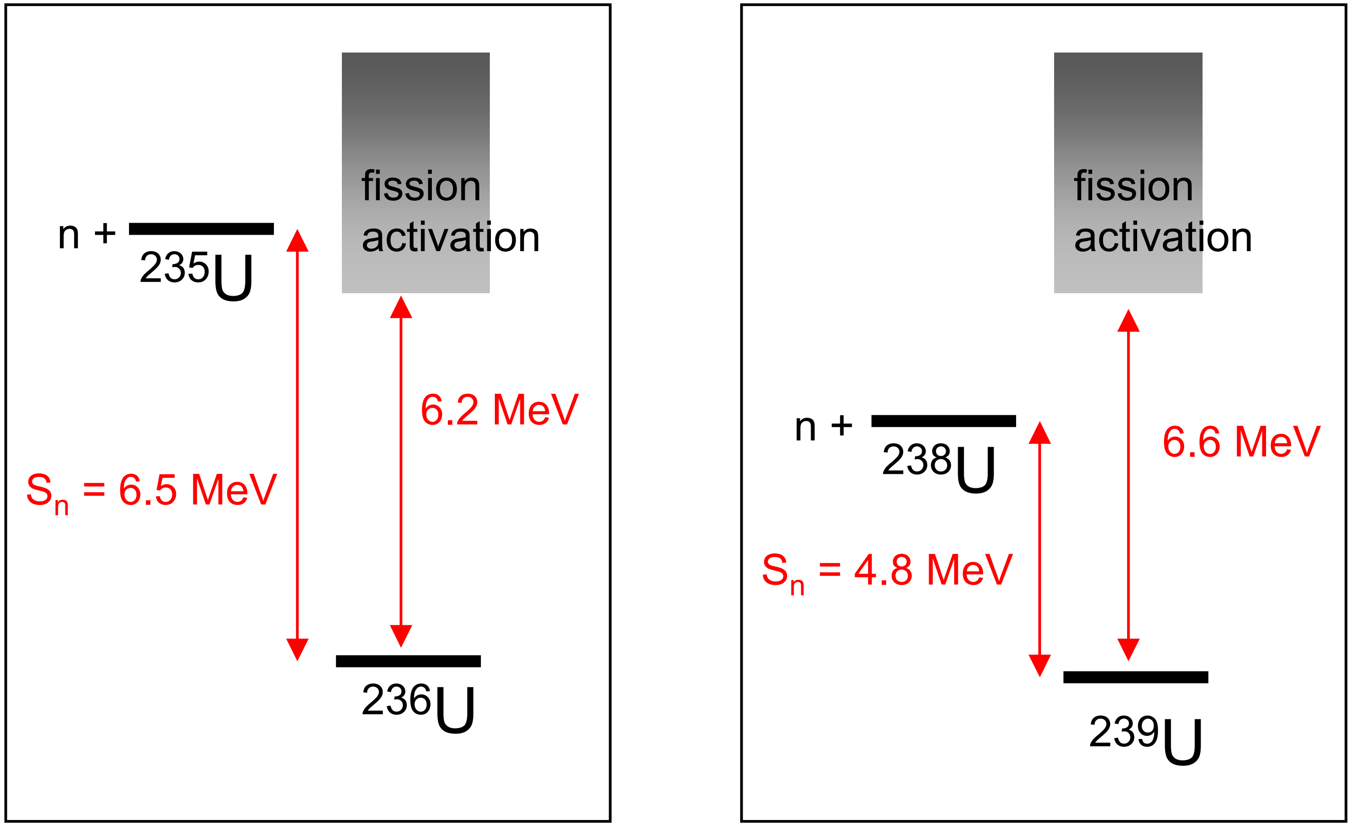
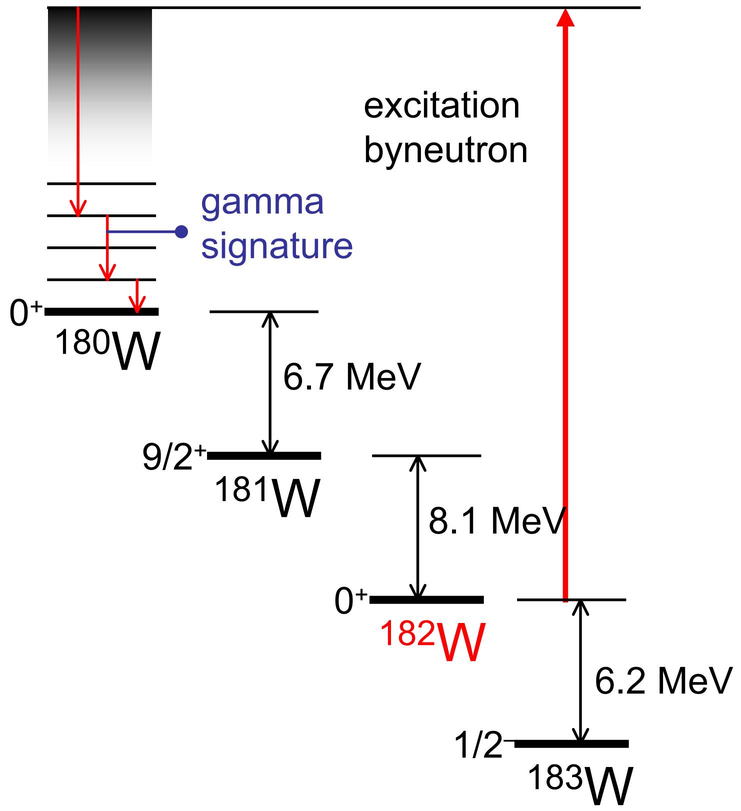


Fission of ^{235}U+n and ^{238}U+n

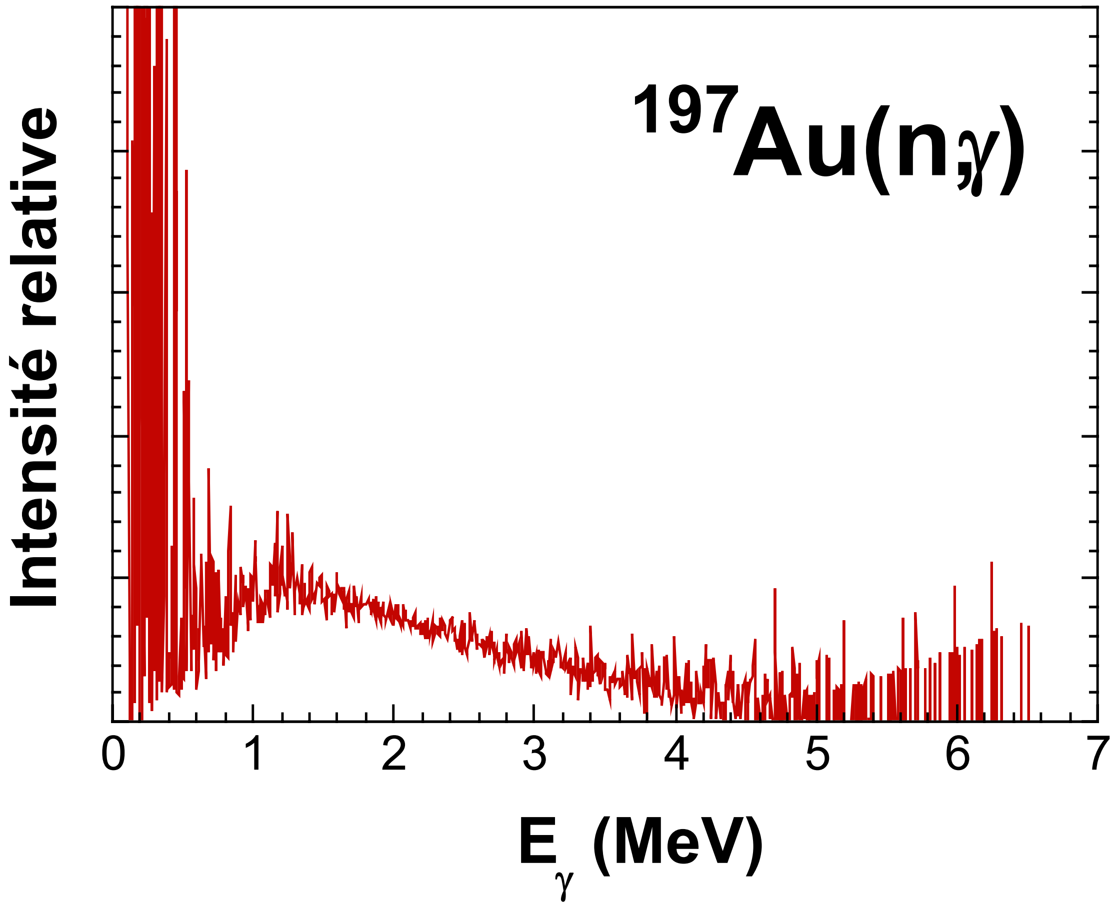


Measurement of (n.xn) by gamma-ray spectroscopy



example: $^{182}W(n,3n)^{180}W$

Simulated gamma-ray spectrum



Moderation of neutrons

Maxwell-Boltzmann velocity distribution:

$$\frac{dn}{dv} \sim v^2 \exp(-\frac{mv^2}{2k_B T})$$

$$v_{\max} = \sqrt{2k_B T/m}$$

$$E_{\max} = \frac{1}{2}mv^2 = k_B T$$

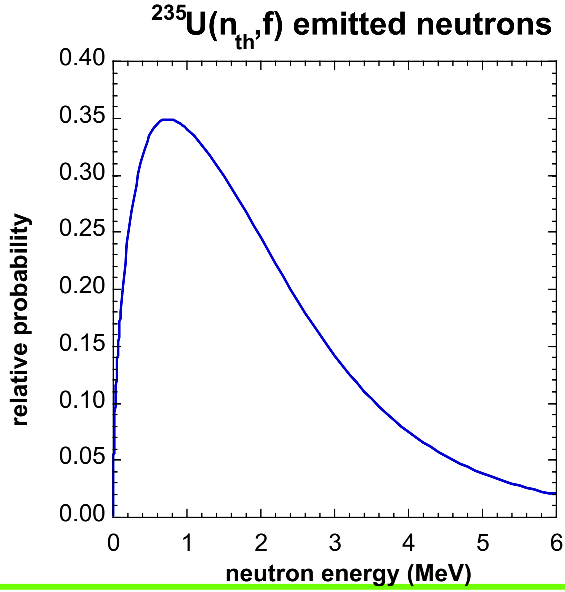
thermal neutrons:

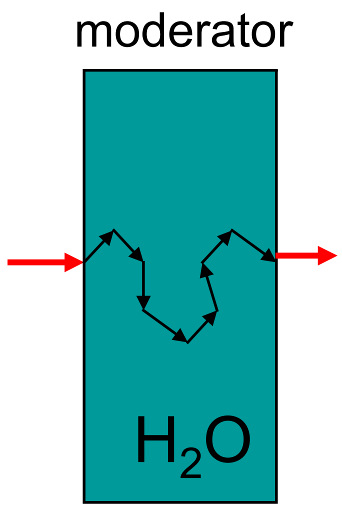
$$v_{\max} = 2200 \text{ m/s (def.)}$$

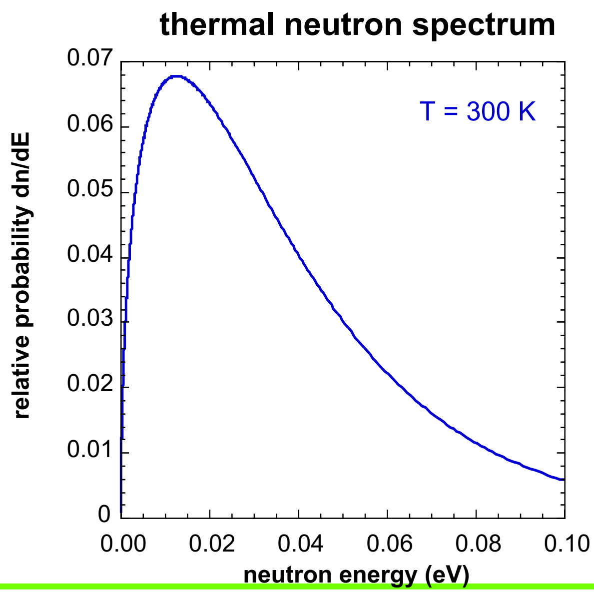
$$E_{\max} = 25.3 \text{ meV}$$

$$T = \frac{1}{2}mv^2/k_B = 293.6 \text{ K}$$

$$\frac{dn}{dE_n} \rightarrow E_{\max} = \frac{1}{2}k_B T$$







Evaluated nuclear data libraries

Libraries

- JEFF - Europe
- JENDL - Japon
- ENDF/B - US
- BROND - Russia
- CENDL - China

Common format:

ENDF-6

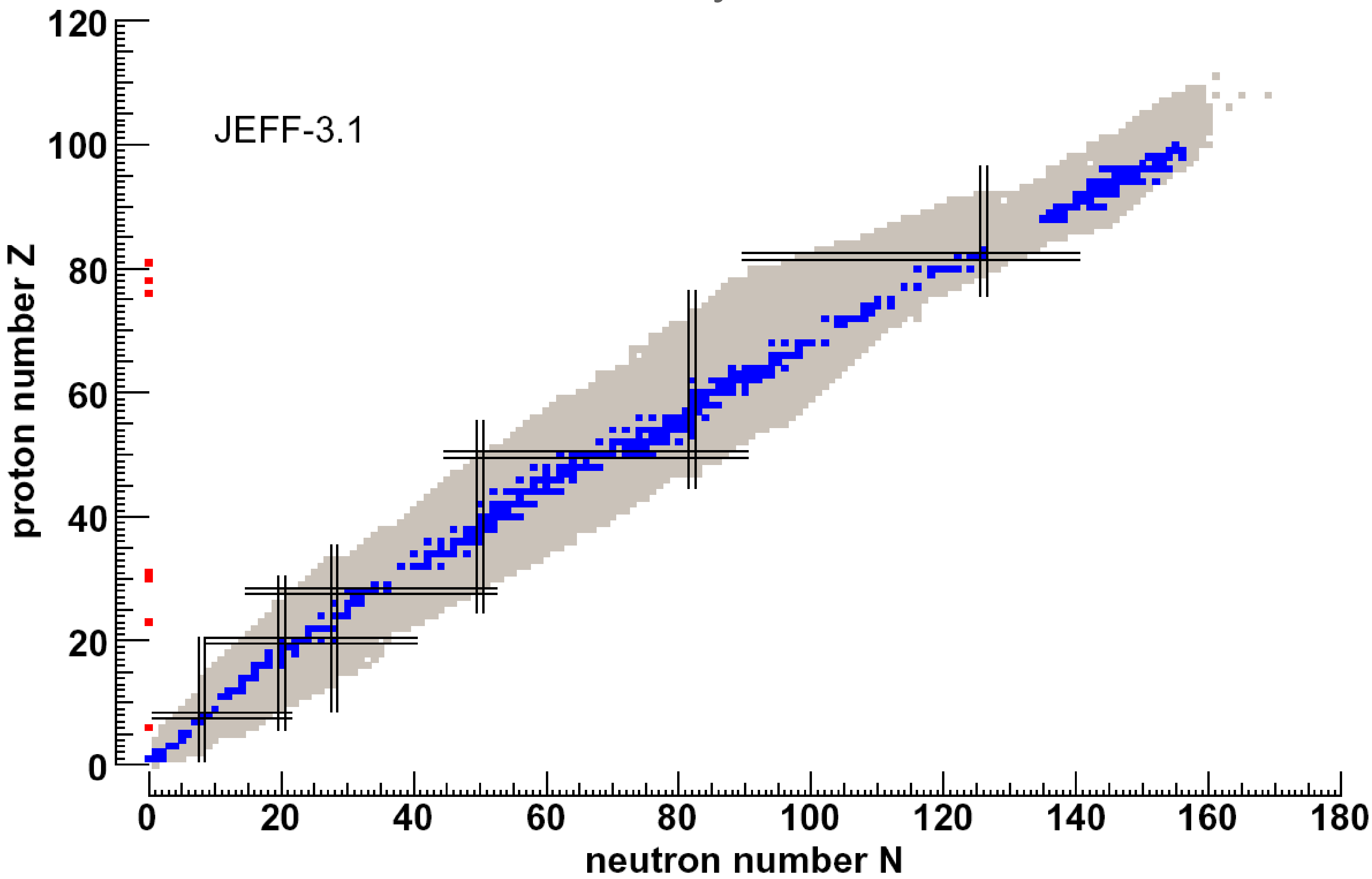
Contents:

Data for particle-induced reactions (neutrons, protons, gamma, other)
but also radioactive decay data

Data are indentified by “materials”
(isotopes, isomeric states, (compounds))

ex.	^{16}O:	mat = 825
	^{nat}V:	mat = 2300
	^{242m}Am:	mat = 9547

The library JEFF-3.1

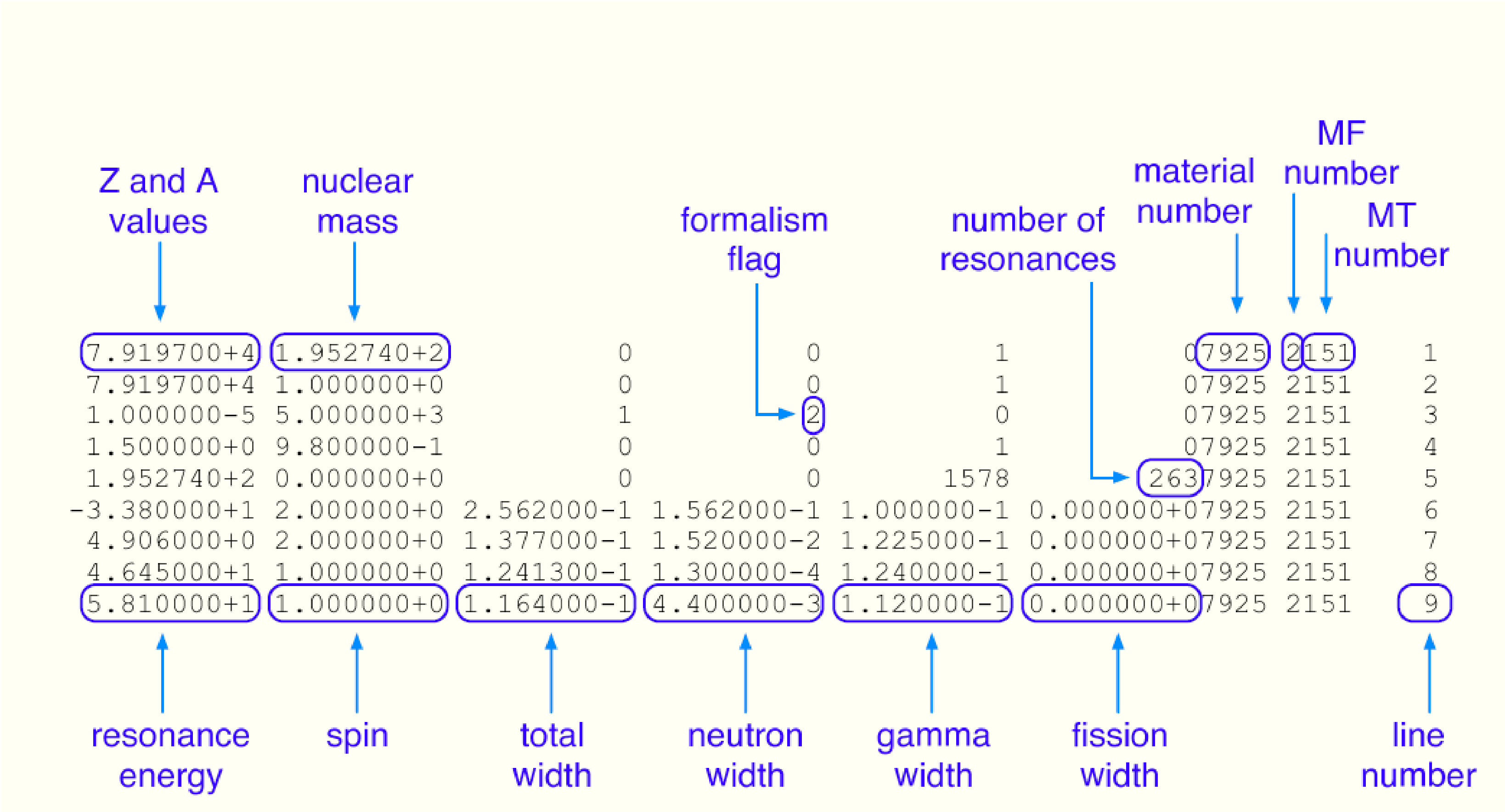


Files for a material

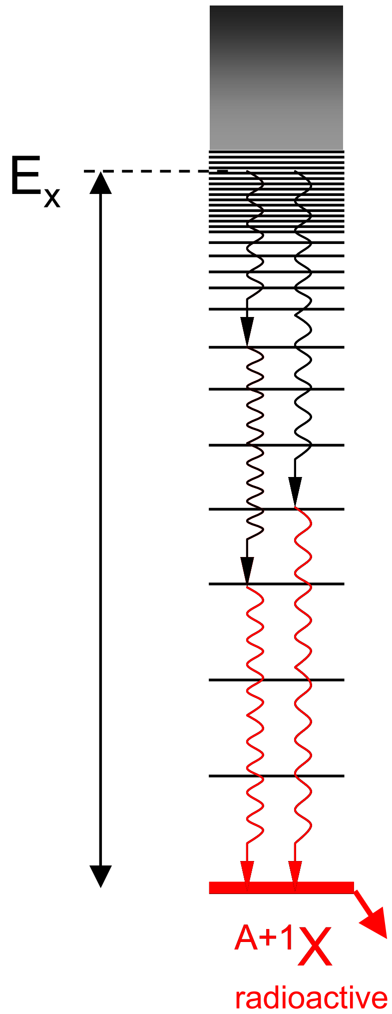
from report ENDF-102

1 General information
2 Resonance parameter data
3 Reaction cross sections
4 Angular distributions for emitted particles
5 Energy distributions for emitted particles
6 Energy-angle distributions for emitted particles
7 Thermal neutron scattering law data
8 Radioactivity and fission-product yield data
9 Multiplicities for radioactive nuclide production
10 Cross sections for photon production
12 Multiplicities for photon production
13 Cross sections for photon production
14 Angular distributions for photon production
15 Energy distributions for photon production
23 Photo-atomic interaction cross sections
27 Atomic form factors or scattering functions for photo-atomic interactions
30 Data Covariances obtained from parameter covariances and sensitivities
31 Data covariances for nubar
32 Data covariances for resonance parameters
33 Data covariances for reaction cross sections
34 Data covariances for angular distributions
35 Data covariances for energy distributions
39 Data covariances for radionuclide production yields
40 Data covariances for radionuclide production cross sections

Example: part of an evaluated data file

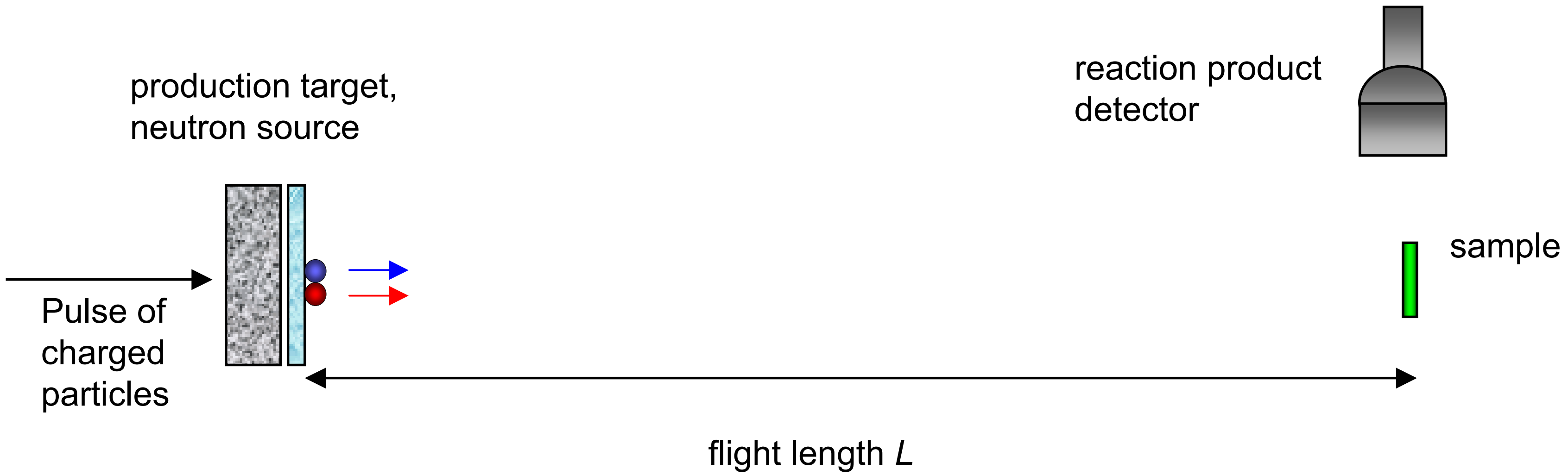


Neutron Capture Gamma-Ray Detection



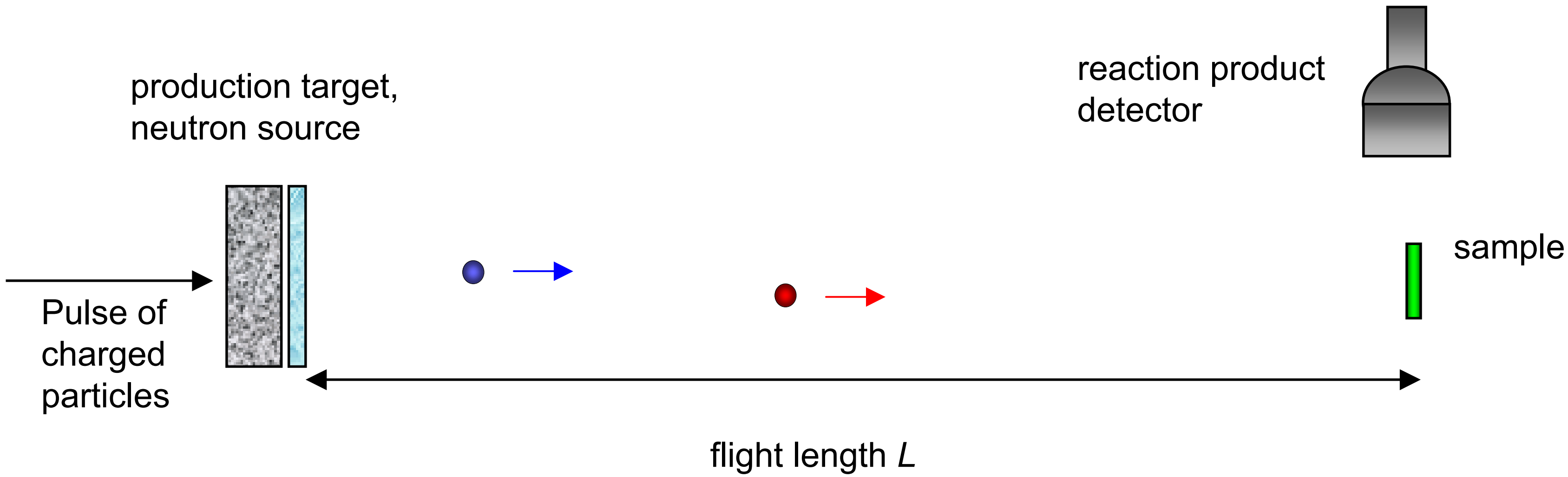
- **Activation**
  - cross sections integrated over known neutron spectrum
  - applicable to some nuclei only
  - no time of flight
- **Level population spectroscopy**
  - applicable to some nuclei only
  - feasible with HPGe detectors,
- **Total energy detection**
  - $\varepsilon_c \sim E_x$, requires weighting function
  - neutron insensitive detector example: C_6D_6 liquid scintillator
- **Total absorption detection**
  - requires $\Omega = 4\pi$, efficiency 100%
  - capture/fission discrimination in possible, example BaF_2 total absorption calorimeter

Measuring a reaction yield using the time-of-flight technique

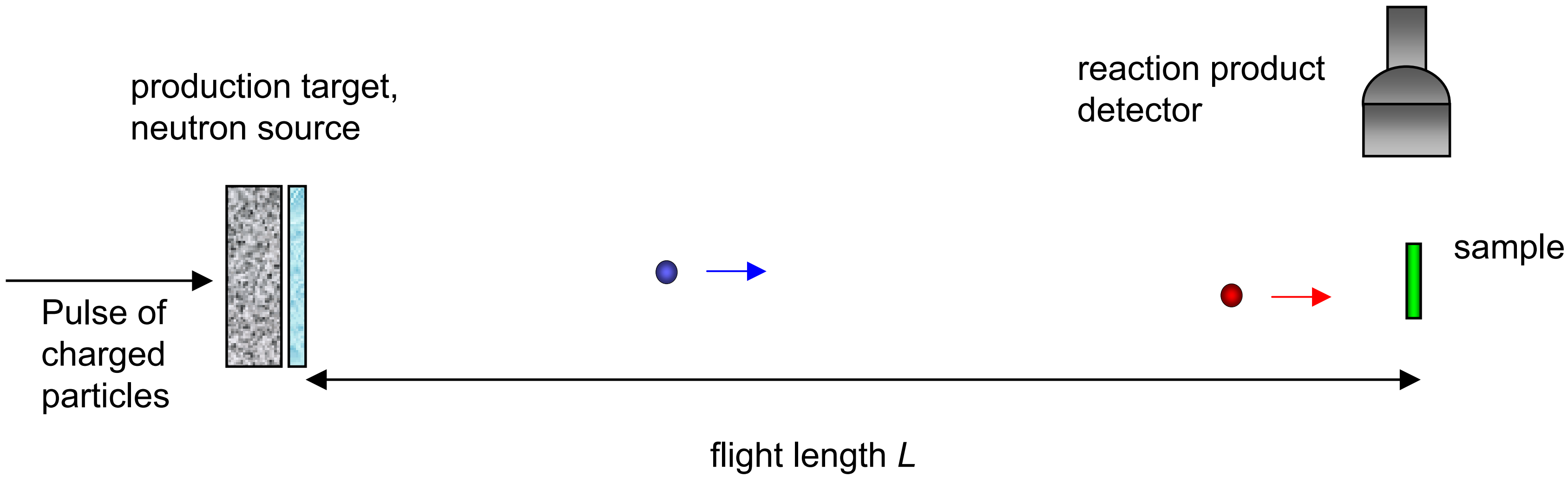


time zero

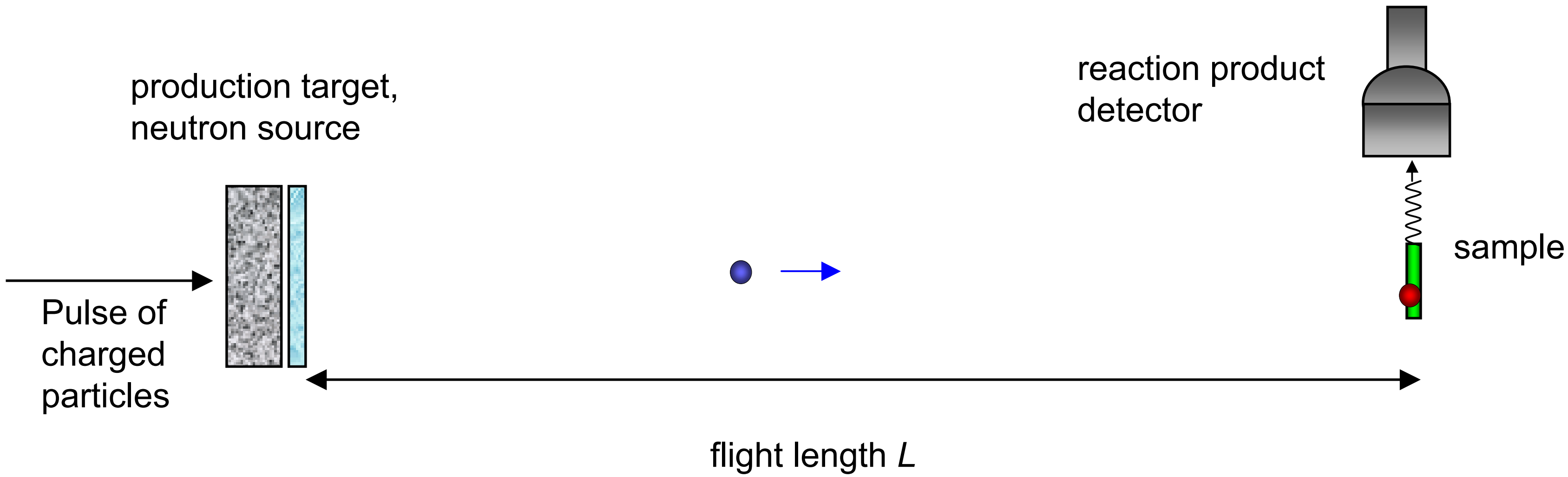
Measuring a reaction yield using the time-of-flight technique



Measuring a reaction yield using the time-of-flight technique

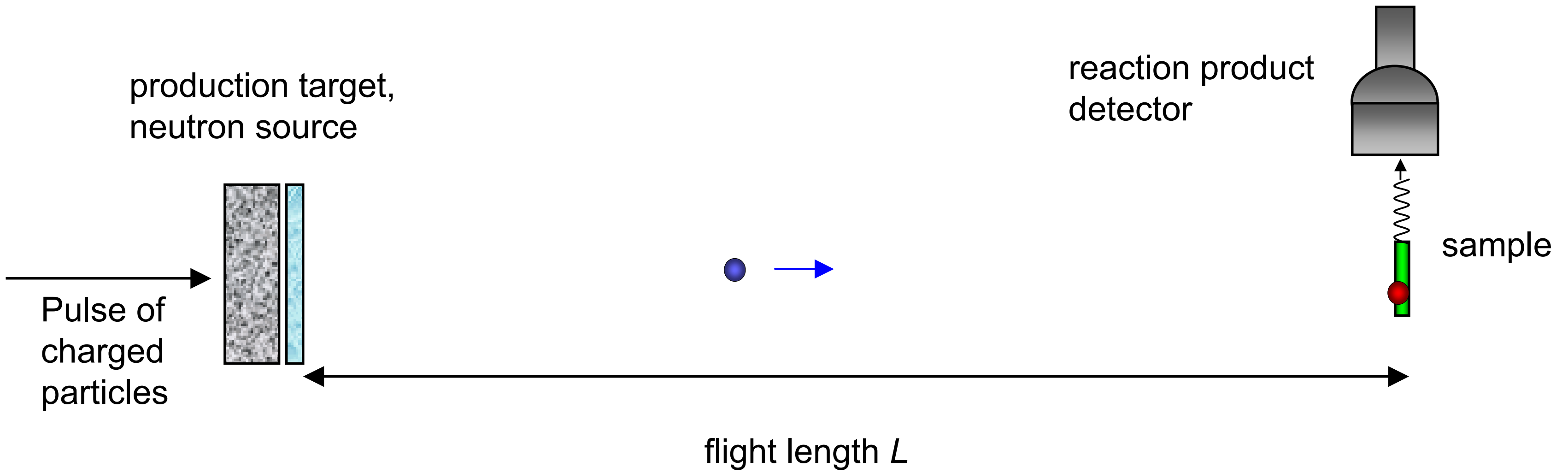


Measuring a reaction yield using the time-of-flight technique



time of flight *t*

Measuring a reaction yield using the time-of-flight technique



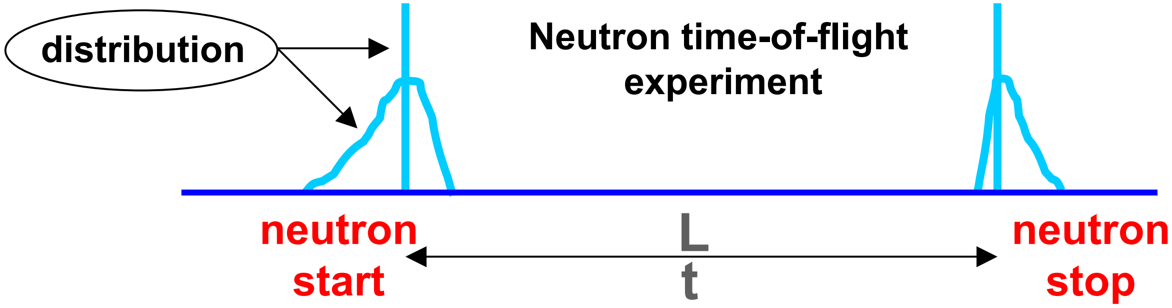
time of flight *t*

Kinetic energy of the neutron by time-of-flight

$$E_n = E_{tot} - mc^2 = c^2p^2 + m^2c^4 - mc^2 = mc^2(\gamma - 1) \qquad \gamma = (1 - v^2/c^2)^{-1/2}$$

$$E_n = \frac{1}{2}mv^2 = \alpha^2 \cdot \frac{L^2}{t^2}$$

Resolution



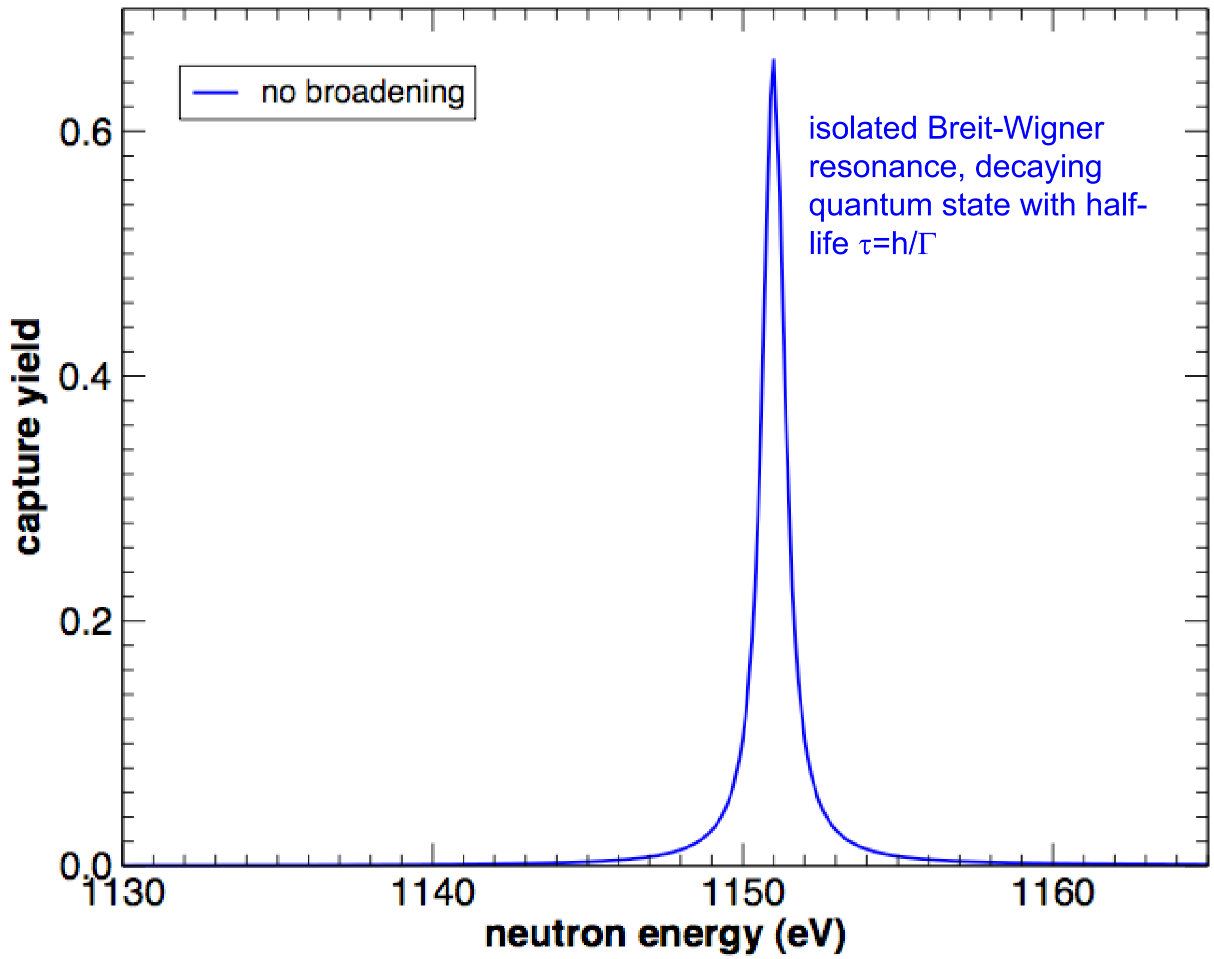
time-energy relation $\sqrt{E} = \alpha \frac{L}{t}$

- neutron time-of-flight: $t + \delta t$
- flight length: $L + \delta L$
- neutron kinetic energy: $E + \delta E$

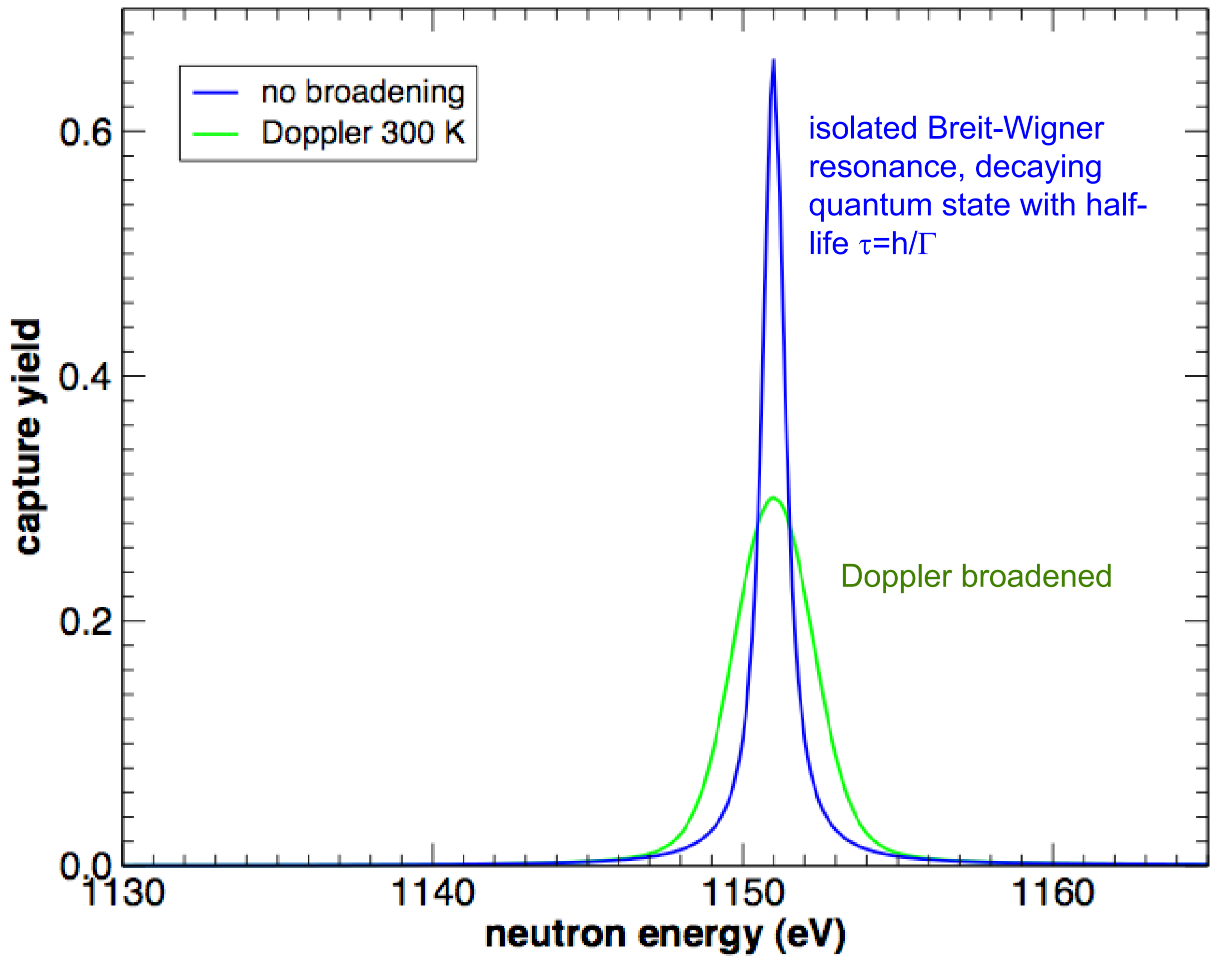
The resolution can be expressed equivalenty in time, distance and energy:

$$R_t(\delta t)d\delta t = R_L(\delta L)d\delta L = R_E(\delta E)d\delta E$$

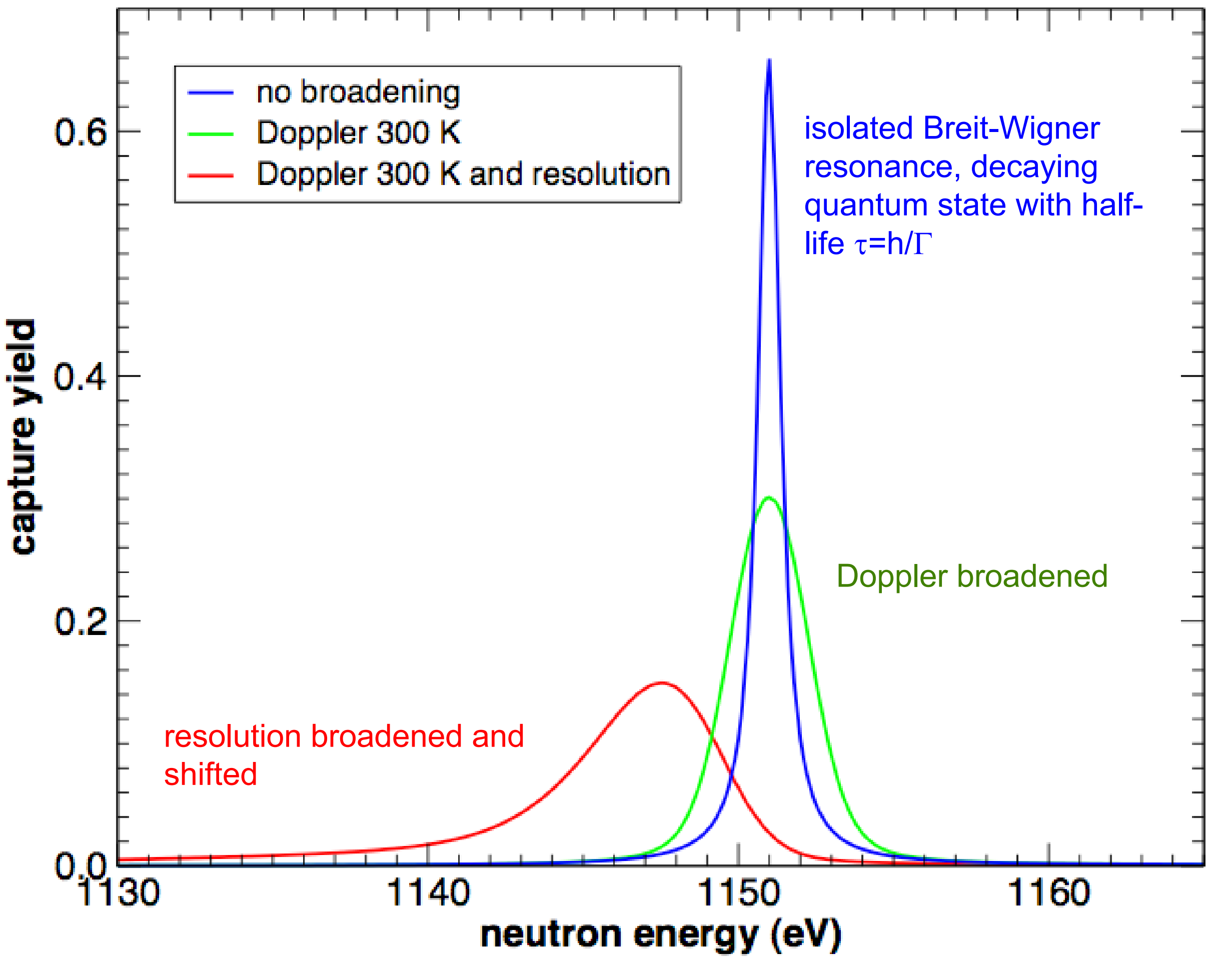
Measured reaction yield



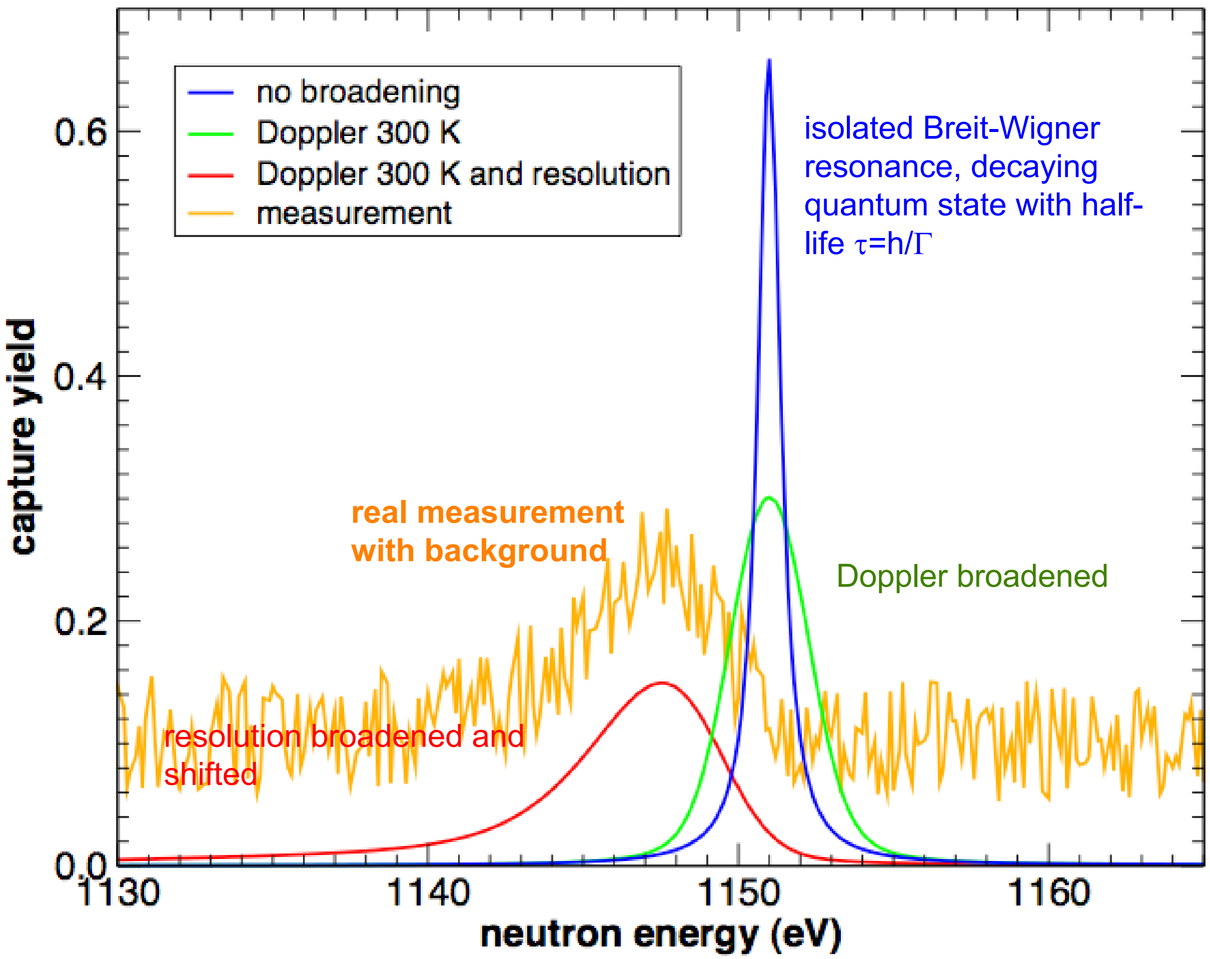
Measured reaction yield



Measured reaction yield



Measured reaction yield



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Further Reading

Books/Papers

- K. S. Krane, *Introductory Nuclear Physics*, Wiley & Sons, (1988).
- G. F. Knoll, *Radiation Detection and Measurement*, Wiley & Sons, (2000).
- P. Reus, *Précis de neutronique*, EDP Sciences, (2003).
- J. E. Lynn, *The Theory of Neutron Resonance Reactions*, Clarendon Press, Oxford, (1968).
- F. Fröhner, *Evaluation and analysis of nuclear resonance data*, JEFF Report 18, OECD/NEA (2000).
- C. Wagemans, *The Nuclear Fission Process*, CRC, (1991).
- A. M. Lane, R. G. Thomas, “R-matrix theory of nuclear reactions”, *Rev. Mod. Phys.* **30** (1958) 257.
- G. Wallerstein, et al., “Synthesis of the elements in stars: forty years of progress”, *Rev. Mod. Phys.* **69** (1997) 995.

Web sites

www.nea.fr
www.nndc.bnl.gov
wwwiaea.org
www.cern.ch/ntof
www.irmm.jrc.be

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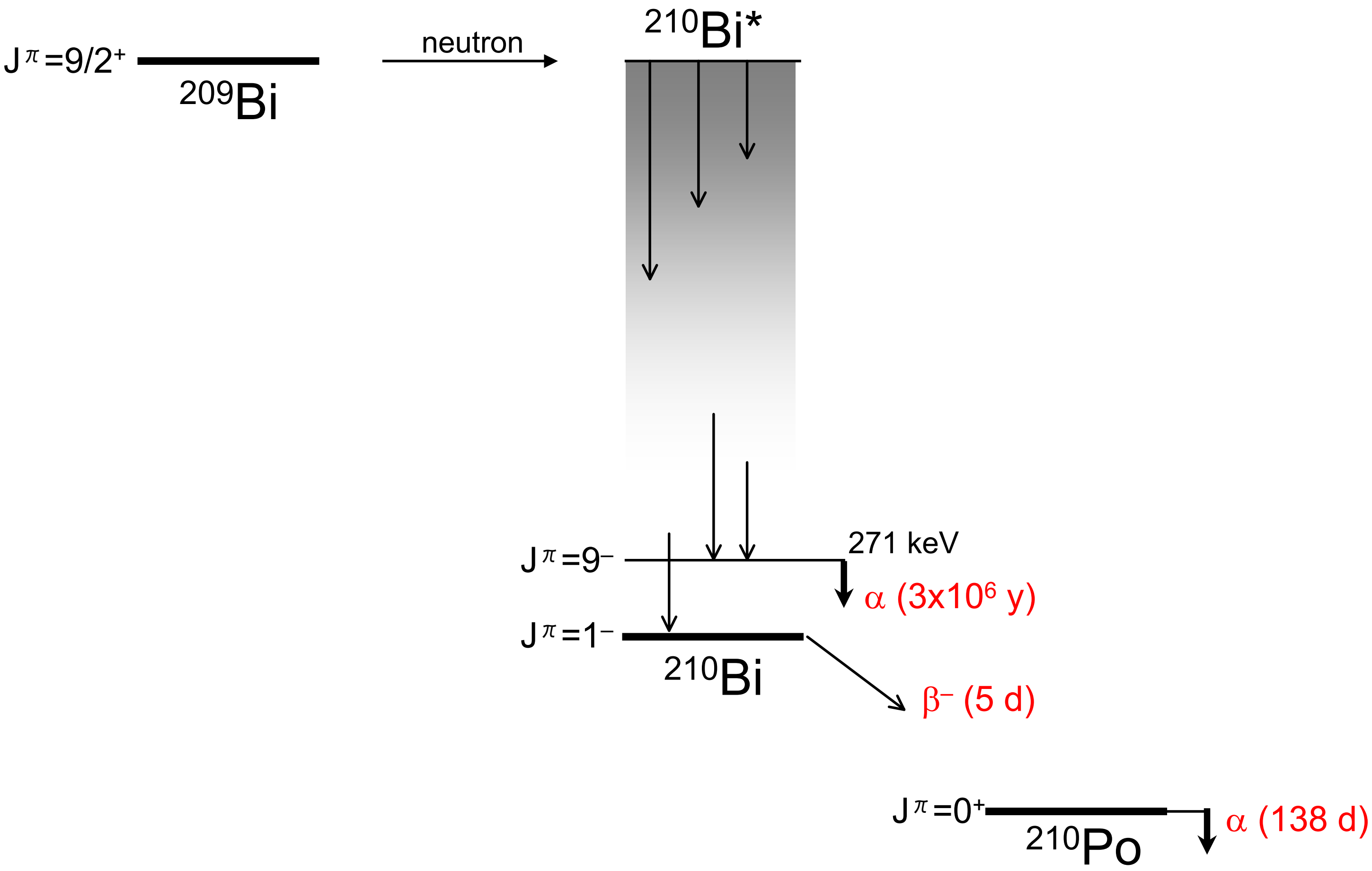
Conclusion

- Neutron induced reactions are important nuclear data necessary for a wide range of fields ranging from nuclear structure and astrophysics to advanced nuclear technology applications.
- The R-matrix formalism is adapted to describe compound nucleus reactions at low energy (E_n<10 MeV, resonance region).
- Resolved resonances need to be measured accurately, they cannot be predicted by nuclear models.

Rapport de branchement ^{209}Bi + n



L'origine des déchets nucléaires

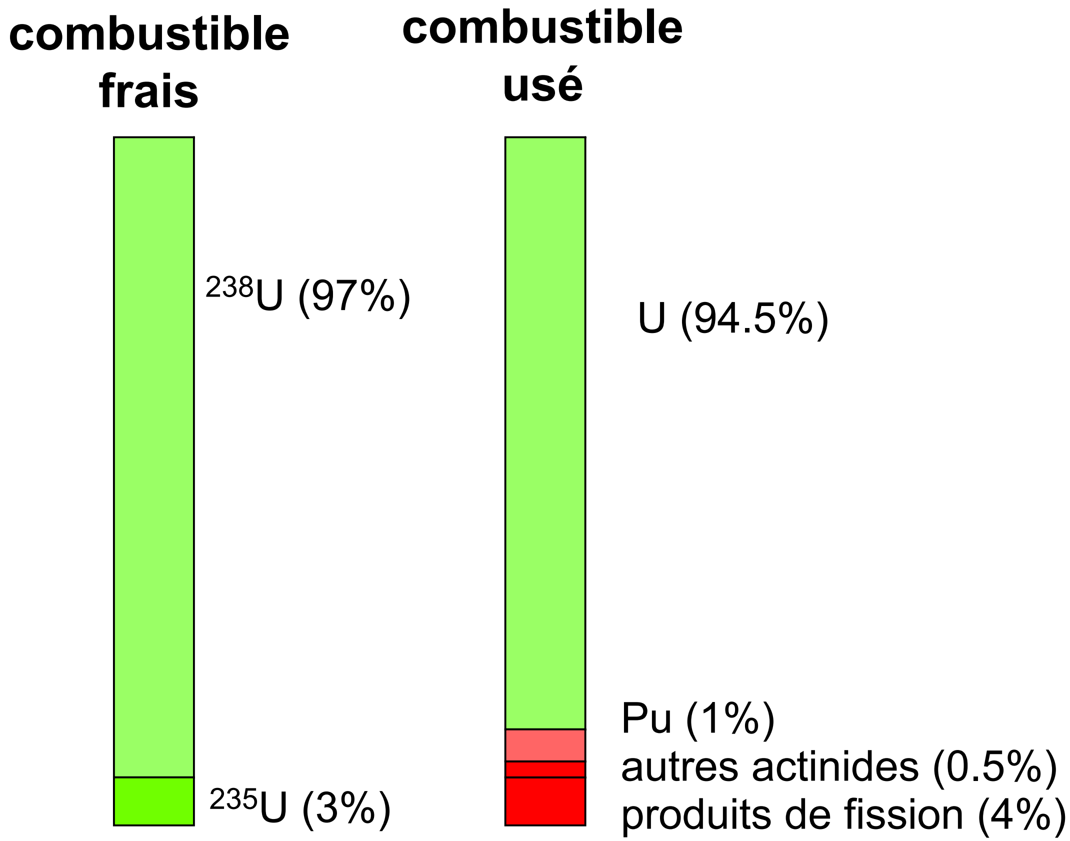
Quantité des déchets en France:

2500 kg/an par habitant
dont 1 kg radioactif
dont 20 g hautement radioactif

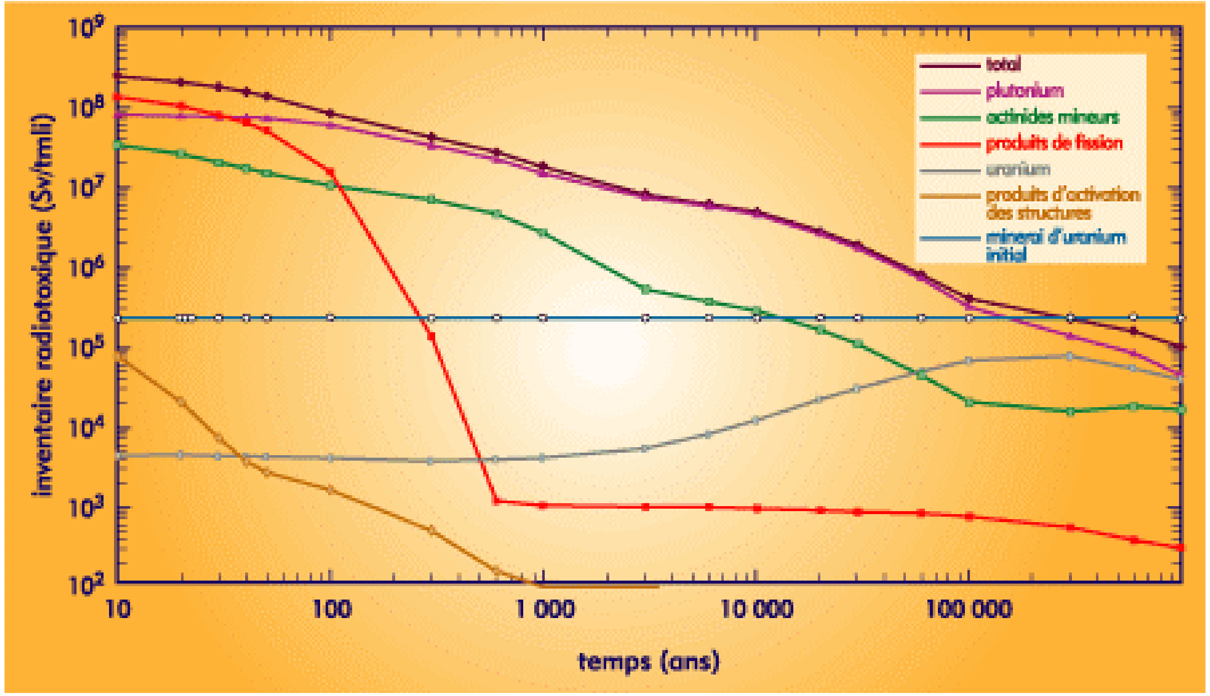
Loi Bataille:

recherche 1991 - 2006

- **séparation et transmutation**
- **stockage profonde**
- **conditionnement et entreposage**



Réduire la radiotoxicité: la transmutation en complément de stockage



Clefs CEA 46 (2002)

Composition des déchets

- **produits de fission**

transmutation par capture de neutrons dans flux de neutrons thermique

- **actinides mineurs**

transmutation par fission induite par neutrons dans flux de neutrons rapide

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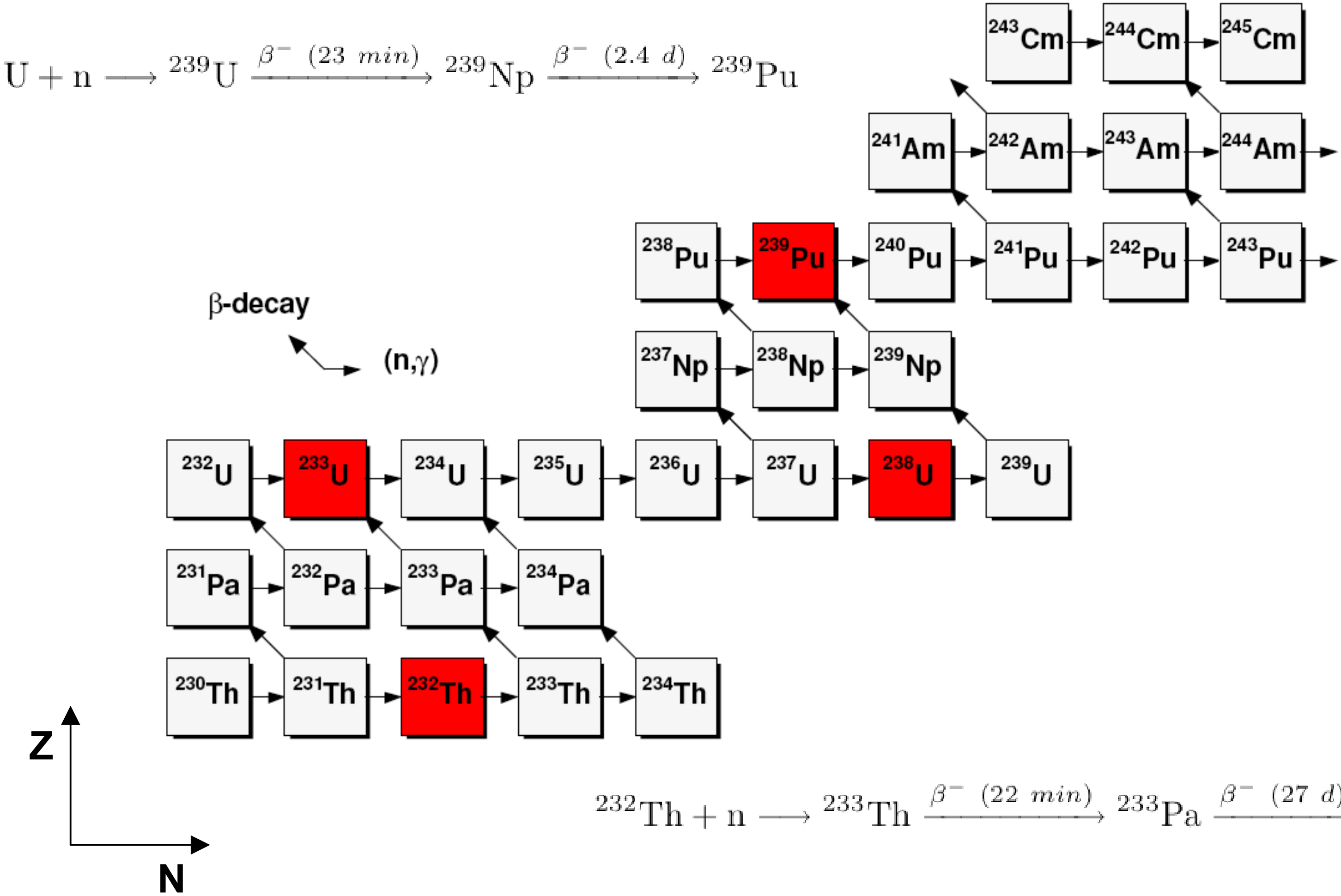
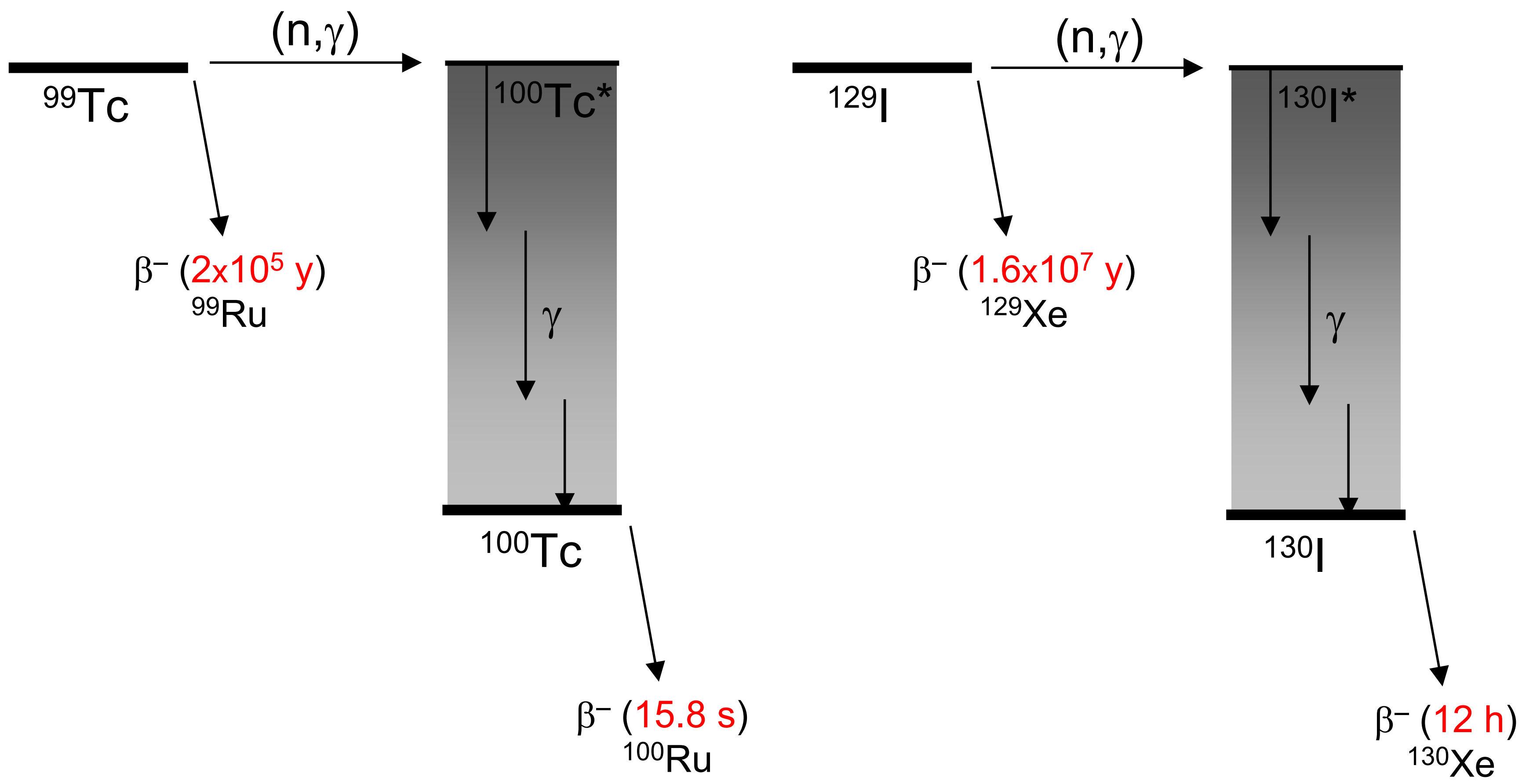
Réduire la radiotoxicité des déchets: le cylce du thorium

$$^{238}\mathrm{U} + \mathrm{n} \longrightarrow {}^{239}\mathrm{U} \xrightarrow{\beta^-\ (23\ min)} {}^{239}\mathrm{Np} \xrightarrow{\beta^-\ (2.4\ d)} {}^{239}\mathrm{Pu}$$



$$^{232}\mathrm{Th} + \mathrm{n} \longrightarrow {}^{233}\mathrm{Th} \xrightarrow{\beta^-\ (22\ min)} {}^{233}\mathrm{Pa} \xrightarrow{\beta^-\ (27\ d)} {}^{233}\mathrm{U}$$

Réduire la radiotoxicité des déchets: la transmutation



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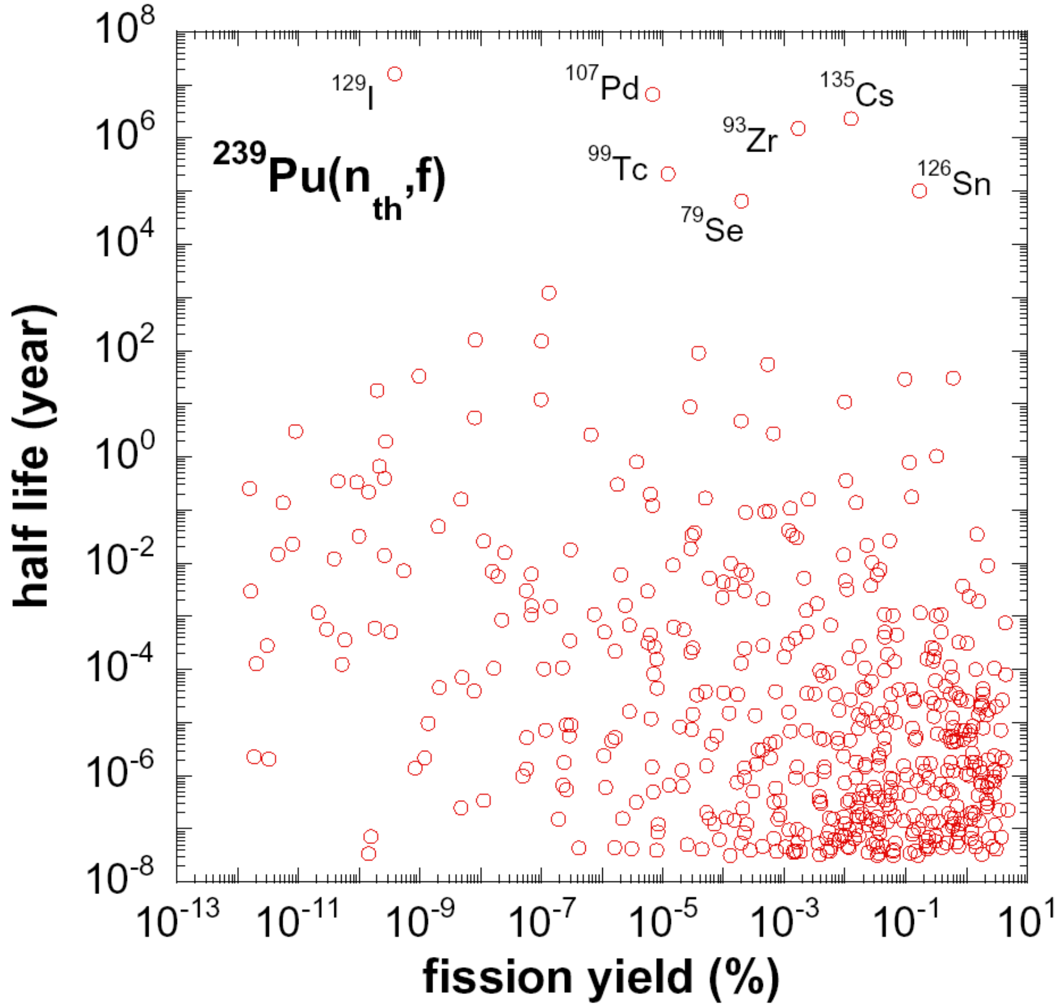
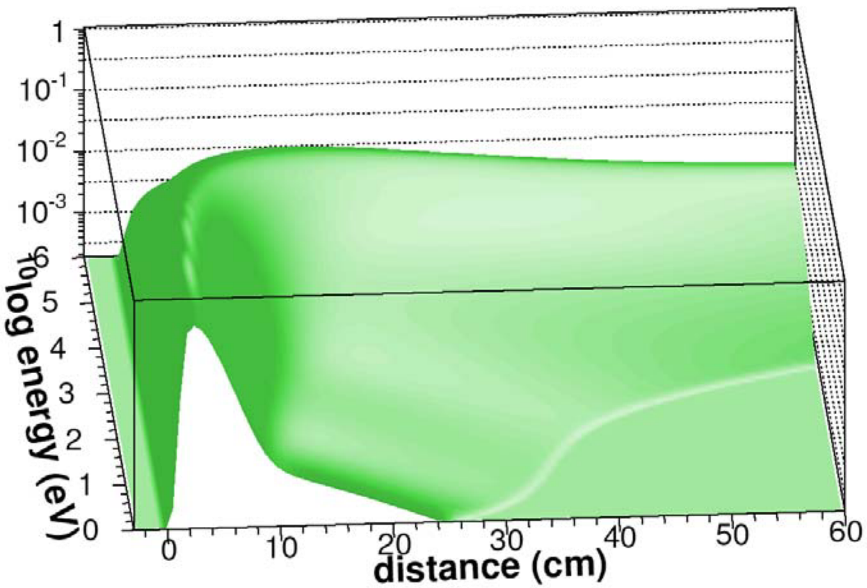
Produits de fission: temps de vie versus rendement



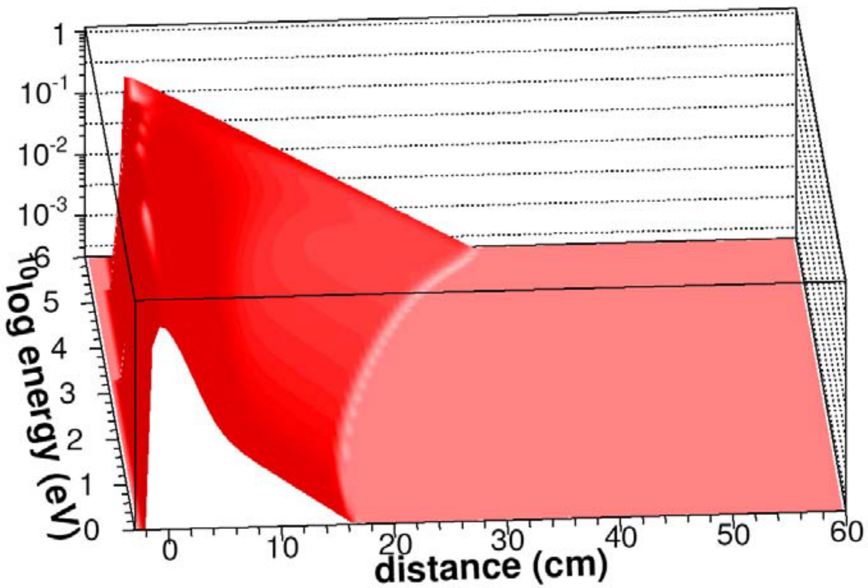
CEA IRFU

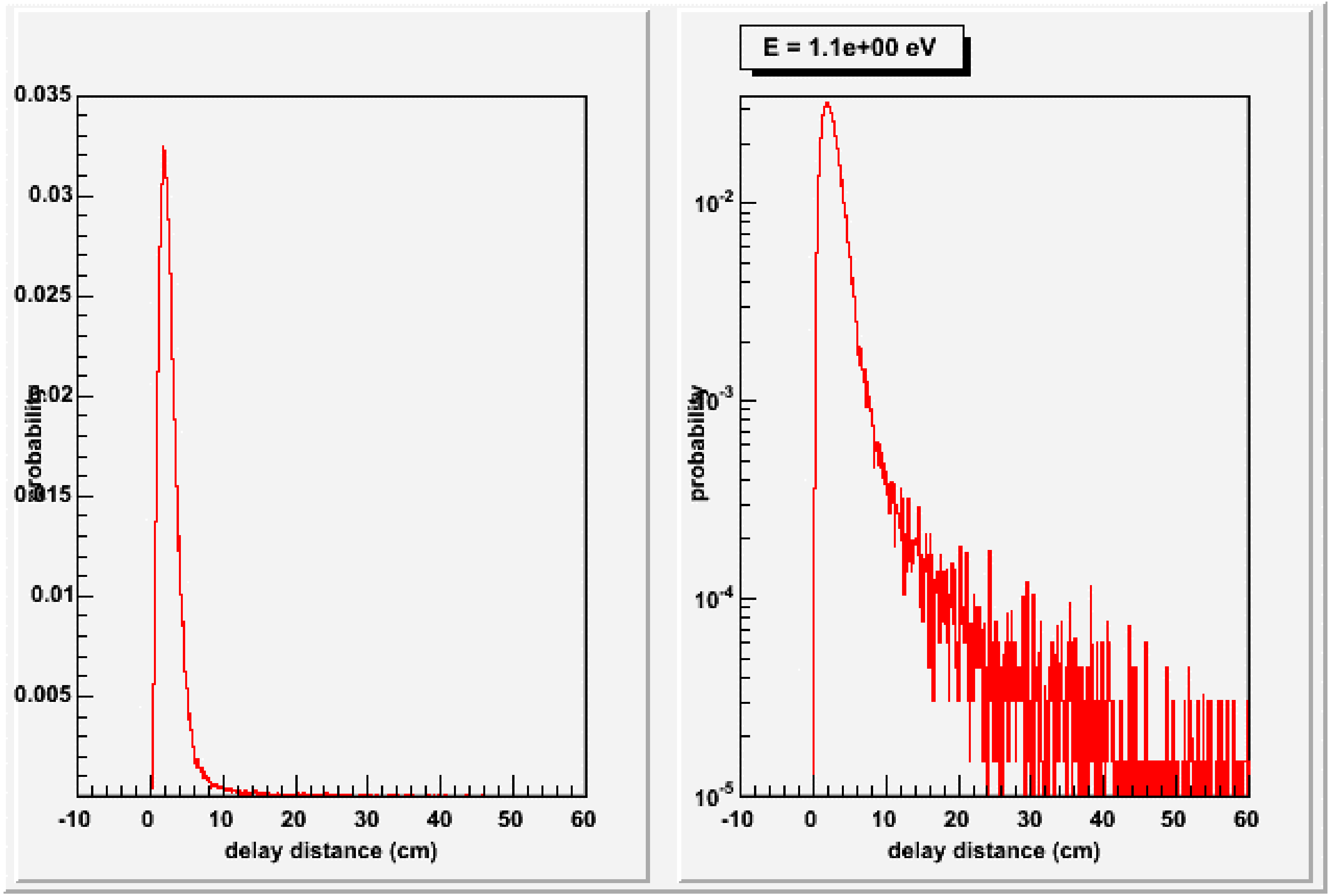
Resolution

n_TOF



GELINA (0°)




E = 1.1e+00 eV
0.035
0.03
0.025
0.02
0.015
0.01
0.005
probability
-10
0
10
20
30
40
50
60
delay distance (cm)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
probability
delay distance (cm)