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R-matrix theory of nuclear reactions.

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R-matrix theory of nuclear reactions

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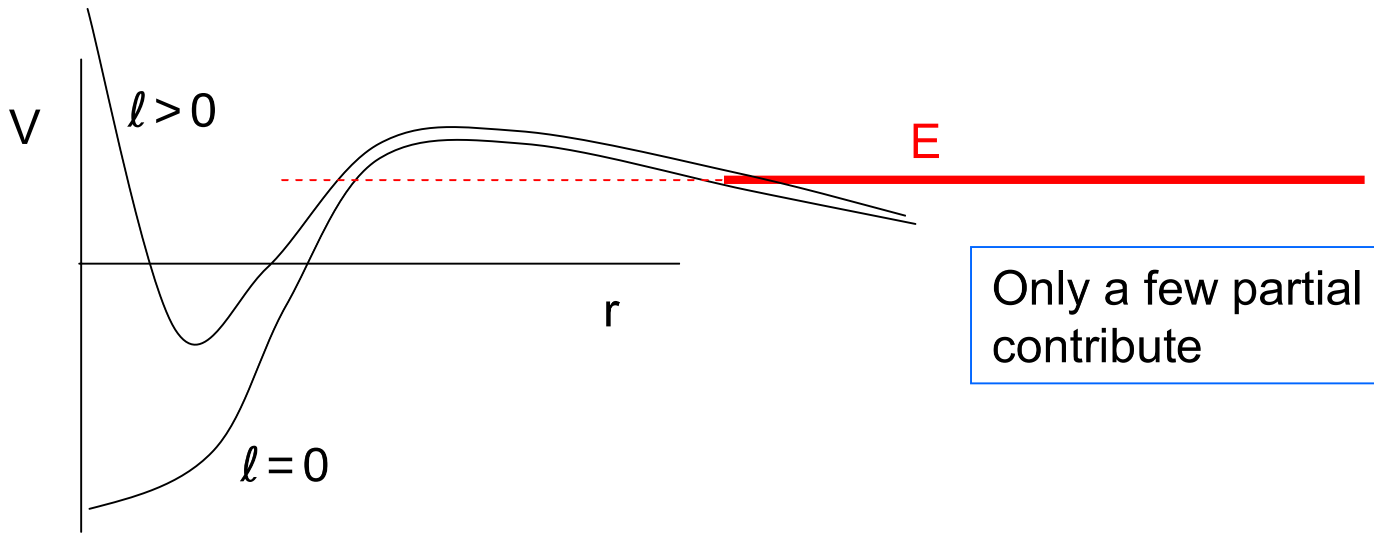
1. Introduction
2. General collision theory: elastic scattering
3. Phase-shift method
4. R-matrix theory (→ theory)
5. Phenomenological R matrix (→ experiment)
6. Conclusions

1.Introduction

General context: two-body systems

- Low energies (E $\lesssim$ Coulomb barrier), few open channels (one)
- Low masses (A $\lesssim$ 15-20)
- Low level densities ($\lesssim$ a few levels/MeV)
- Reactions with neutrons AND charged particles



Only a few partial waves contribute

Low-energy reactions

- Transmission ("tunnelling") decreases with ℓ $\rightarrow$ few partial waves
  $\rightarrow$ semi-classic theories not valid
  - astrophysics
  - thermal neutrons

- Wavelength $\lambda = \hbar c/E$
  $\lambda(\mathrm{fm}) \sim 197/E(\mathrm{MeV})$

  example: 1 MeV: $\lambda \sim 200$ fm
  radius ~ 2-4 fm
  → quantum effects important

Some references:

- C. Joachain: Quantum Collision Theory, North Holland 1987
- L. Rodberg, R. Thaler: Introduction to the quantum theory of scattering, Academic Press 1967
- J. Taylor : Scattering Theory: the quantum theory on nonrelativistic collisions, John Wiley 1972

Different types of reactions

1. Elastic collision : entrance channel=exit channel

   A+B → A+B: Q=0

covered here

2. Inelastic collision (Q≠0)

   A+B → A*+B (A*=excited state)
   A+B → A+B*
   A+B → A*+B*
   etc..

3. Transfer reactions

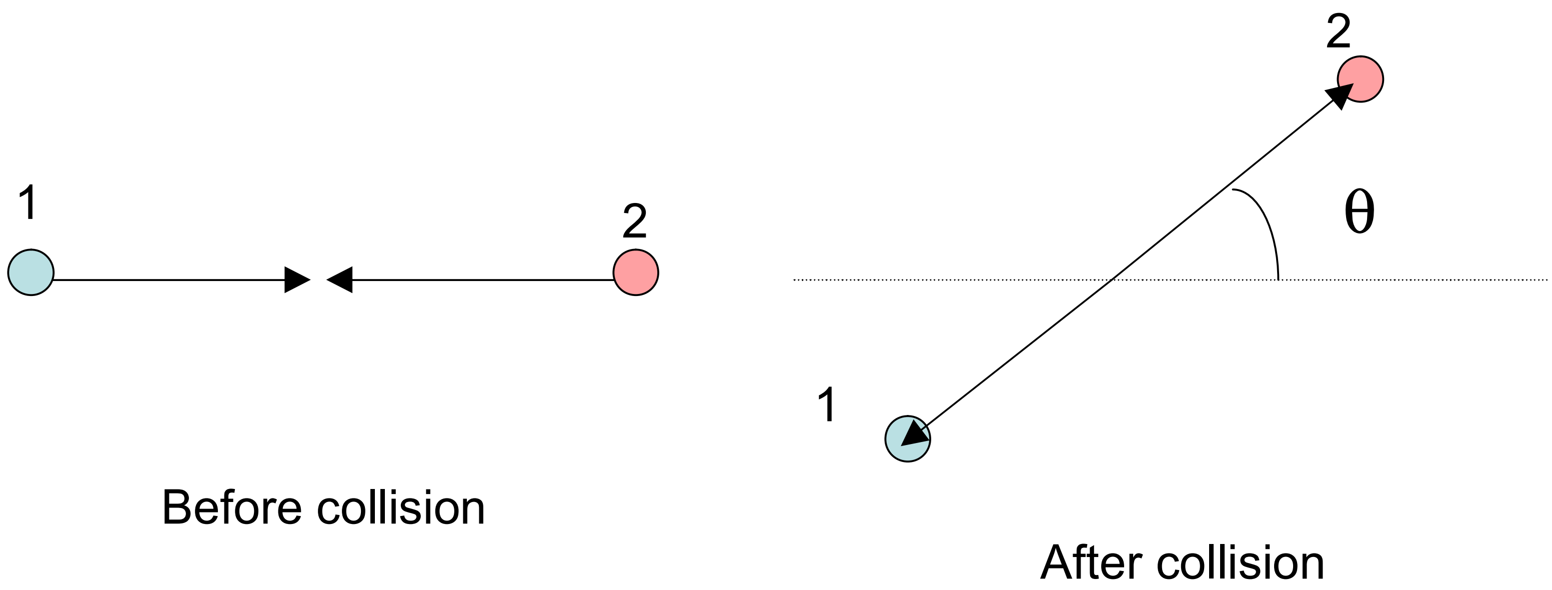
   A+B → C+D
   A+B → C+D+E
   A+B → C*+D
   etc…

4. radiative capture reactions
   A+B→ C + γ

NOT covered here

2. Collision theory: elastic scattering

Assumptions: elastic scattering
no internal structure
no Coulomb
spins zero



Center-of-mass system

Scattering wave functions

Schrödinger equation: V(r)=interaction potential

$$H\psi(r) = [-\frac{\hbar^2}{2\mu}\Delta + V(r)]\psi(r) = E\psi(r) \quad \text{with } E > 0$$

Assumption: the potential is real and decreases faster than 1/r

A large distances : V(r)→ 0

2 independent solutions : $\psi(r) \rightarrow A\left(\exp(i\mathbf{k}\cdot\mathbf{r}) + f(\theta)\frac{\exp(ikr)}{r}\right)$

Incoming plane wave

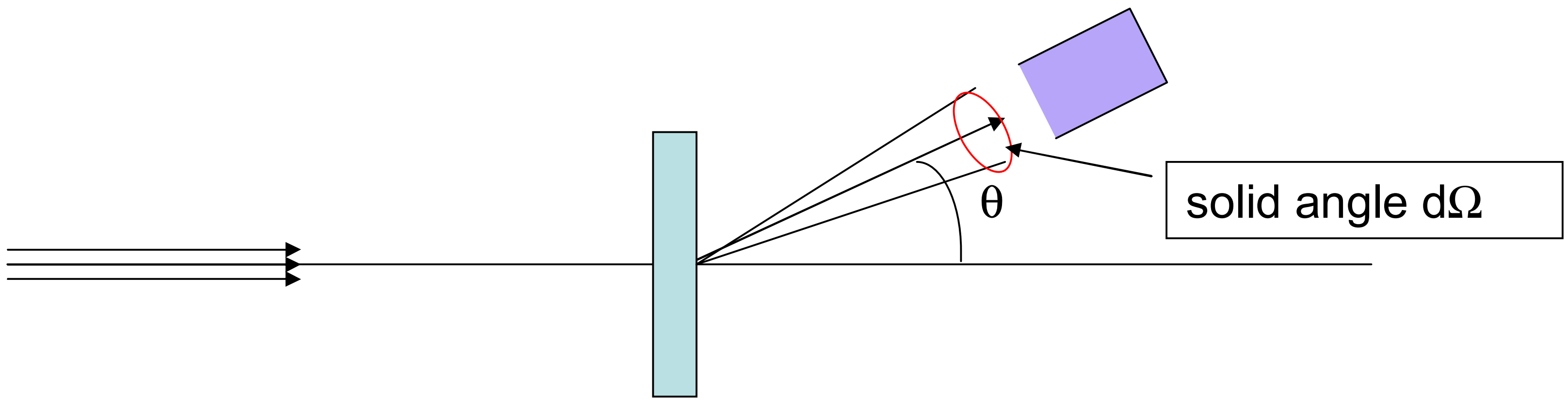
Outgoing spherical wave

where: k=wave number: $k^2=2\mu E/\hbar^2$

A=amplitude (scattering wave function is not normalized)

f(θ) =scattering amplitude (length)

Cross sections



Cross section ⟹

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2, \quad \sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

- Cross section obtained from the asymptotic part of the wave function
- "Direct" problem: determine σ from the potential
- "Inverse" problem : determine the potential V from σ
- Angular distribution: E fixed, θ variable
- Excitation function: θ variable, E fixed,

How to solve the Schrödinger equation for E>0?

$$H\psi(r) = (-\frac{\hbar^2}{2\mu}\Delta + V(r))\psi(r) = E\psi(r)$$

With $\psi(\mathbf{r}) \rightarrow \exp(ikz) + f(\theta)\frac{\exp(ikr)}{r}$ (*z* along the beam axis)

Several methods:

- Formal theory: Lippman-Schwinger equation ➔ approximations
    - Eikonal approximation
    - Born approximation
- Phase shift method: well adapted to low energies
- Etc…

Lippman-Schwinger equation:

$$\psi(\mathbf{r}) \rightarrow \exp(ikz) + f(\theta)\frac{\exp(ikr)}{r}$$

$$\psi(\mathbf{r}) = \exp(ikz) + \int G(\mathbf{r},\mathbf{r}')V(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}'$$ is solution of the Schrödinger equation

With $G(\mathbf{r},\mathbf{r}') = -\frac{2\mu}{\hbar^2}\frac{\exp(ik|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|}$ =Green function

r is supposed to be large (r >> r')

$$f(\theta) = -\frac{2\mu}{4\pi\hbar^2}\int \exp(-ikr'\cos\theta)V(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}'$$

- $\Psi(r)$ must be known
- $\Psi(r)$ only needed where V(r)≠0 → approximations are possible
- Born approximation : $\psi(\mathbf{r}) = \exp(ikz)$
- Eikonal approximation: $\psi(\mathbf{r}) = \exp(ikz) \times \tilde{\psi}(\mathbf{r})$

Valid at high energies:
V(r) ≪ E

3. Phase-shift method

a. Definitions, cross sections

   Simple conditions: neutral systems
   spins 0
   single-channel

b. Extension to charged systems

c. Extension to multichannel problems

d. Low energy properties

e. General calculation

f. Optical model

3.a Definition, cross section

The wave function is expanded as

$$\Psi(\mathrm{r}) = \sum_{l,m} \frac{u_l(r)}{r} Y_l^m(\Omega)$$

When inserted in the Schrödinger equation

$$-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right)u_l(r) + V(r)u_l(r) = Eu_l(r)$$

To be solved for any potential (real)

For r→∞ V(r) =0

$$u_l'' - \frac{l(l+1)}{r^2}u_l(r) + k^2 u_l(r) = 0, \quad \text{with} \quad k^2 = \frac{2\mu E}{\hbar^2}$$

=Bessel equation → $u_l(r)= j_l(kr), n_l(kr)$

For small x $j_l(x) \to \frac{x^l}{(2l+1)!!}$

$n_l(x) \to -\frac{(2l-1)!!}{x^{l+1}}$

For large x $j_l(x) \to \frac{1}{x}\sin(x - l\pi/2)$

$n_l(x) \to -\frac{1}{x}\cos(x - l\pi/2)$

Examples: $j_0(x) = \frac{\sin(x)}{x}, \quad n_0(x) = -\frac{\cos(x)}{x}$

At large distances: $u_l(r)$ is a linear combination of $j_l(kr)$ and $n_l(kr)$:

$$u_l(r) \to j_l(kr) - \tan\delta_l * n_l(kr)$$

With δ_l = phase shift (information about the potential)
If V=0 → δ=0

Cross section:

$$\frac{d\sigma}{d\Omega} = |f(\theta, E)|^2, \text{ with } f(\theta, E) = \frac{1}{2ik}\sum_l (2l+1)\left[\exp(2i\delta_l(E)) - 1\right]P_l(\cos\theta)$$

Provide the cross section

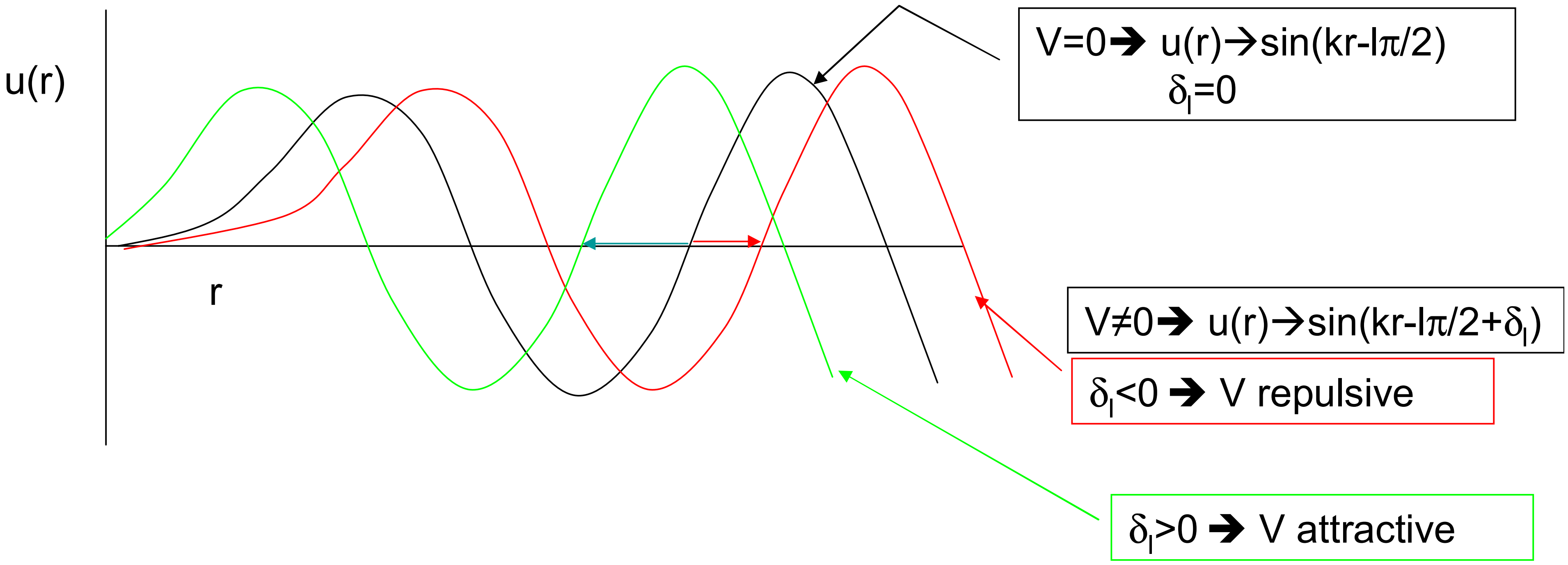
$$\frac{d\sigma}{d\Omega} = |f(\theta,E)|^2 \text{ , with } f(\theta,E) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\left[\exp(2i\delta_l(E))-1\right]P_l(\cos\theta)$$

➔ factorization of the dependences in E and θ

General properties of the phase shifts

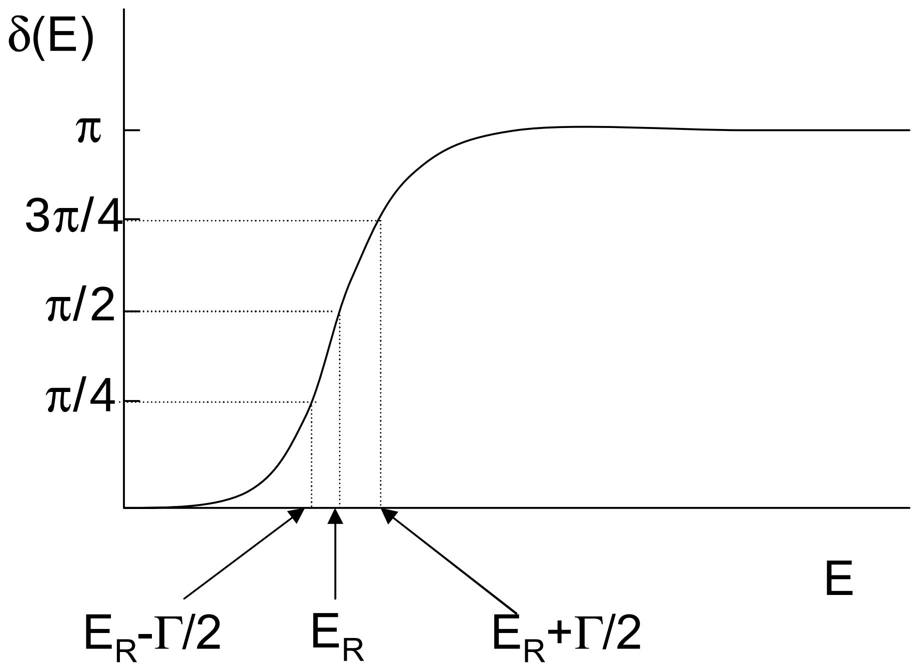
1. Expansion useful at low energies: small number of ℓ values
2. The phase shift (and all derivatives) are continuous functions of E
3. The phase shift is known within nπ: $exp(2i\delta)=exp(2i(\delta+n\pi))$
4. Levinson theorem
   - $\delta_l(E=0)$ is arbitrary
   - $\delta(E=0) - \delta(E=\infty) = N\pi$, where N is the number of bound states
   - Example: p+n, ℓ =0: $\delta(E=0) - \delta(E=\infty) = \pi$ (bound deuteron)

Interpretation of the phase shift from the wave function



Resonances: $\delta_R(E) \approx \arctan \frac{\Gamma}{2(E_R - E)}$ =Breit-Wigner approximation

E_R=resonance energy
Γ=resonance width



- Narrow resonance: Γ small
- Broad resonance: Γ large

Example: ^{12}C+p

With resonance: E_R=0.42 MeV, Γ=32 keV → lifetime: t=}/Γ~2x10^{-20} s

Without resonance: interaction range d~10 fm →interaction time t=d/v ~1.1x10^{-21} s

3.b Generalization to the Coulomb potential

The asymtotic behaviour $\Psi(r) \rightarrow \exp(ikz) + f(\theta)*\exp(ikr)/r$

Becomes: $\Psi(r) \rightarrow \exp(ikz+\eta \log(kr)) + f(\theta)*\exp(ikr-\eta \log(2kr))/r$

With $\eta=Z_1Z_2e^2/\hbar v$ =Sommerfeld parameter, v=velocity

Bessel equation: $$u_l'' - \frac{l(l+1)}{r^2}u_l(r) + k^2 u_l(r) = 0$$

Coulomb equation $$u_l'' - \frac{l(l+1)}{r^2}u_l(r) + (k^2 - 2k\eta)u_l(r) = 0$$

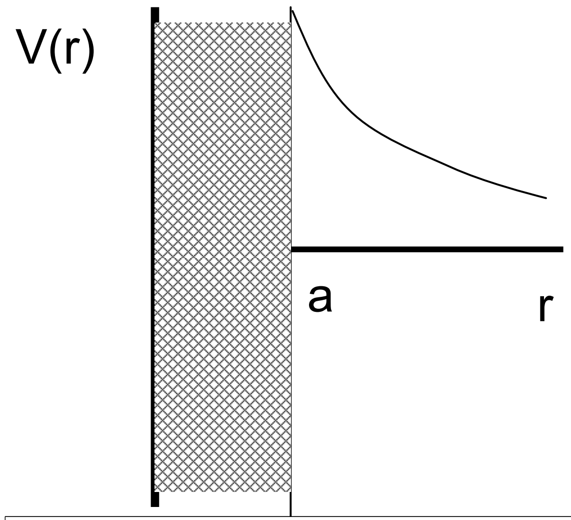
$$u_l(r) \rightarrow F_l(kr) + \tan\delta_l * G_l(kr)$$
$$\rightarrow I_l(kr) - U_l * O_l(kr), \text{with } U_l = \exp(2i\delta_l)$$
$$\rightarrow F_l(kr) * \cos\delta_l + G_l(kr) * \sin\delta_l$$

Solutions: $F_l(\eta,kr)$: regular, $G_l(\eta,kr)$: irregular

Ingoing, outgoing functions: $I_l = G_l - iF_l$, $O_l = G_l + iF_l$

$$I_l(x) \rightarrow \exp(-ix)$$
$$O_l(x) \rightarrow \exp(+ix)$$

Example: hard sphere

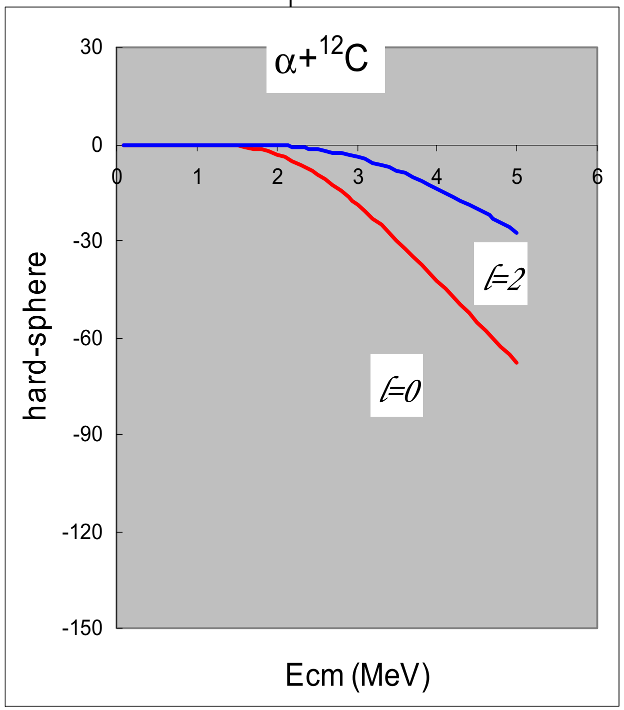


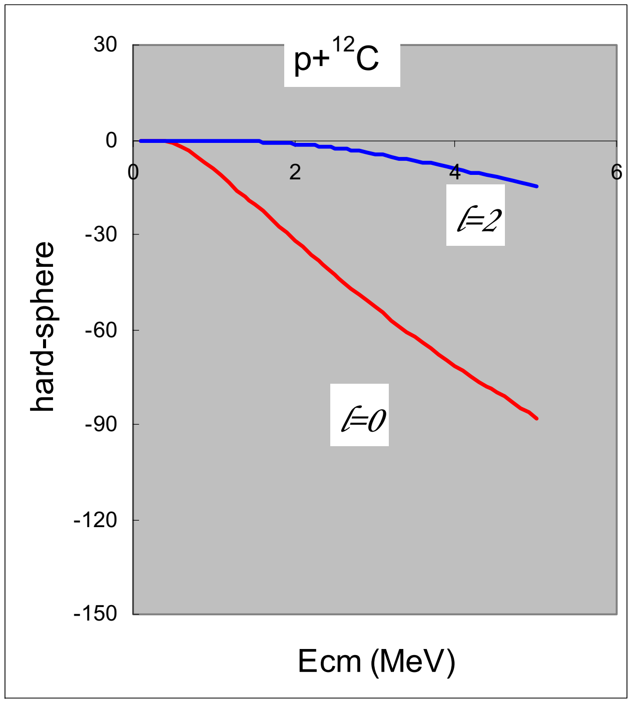
$$V(r) = \infty \quad \text{for } r<a$$
$$= Z_1 Z_2 e^2/r \quad \text{for } r>a$$

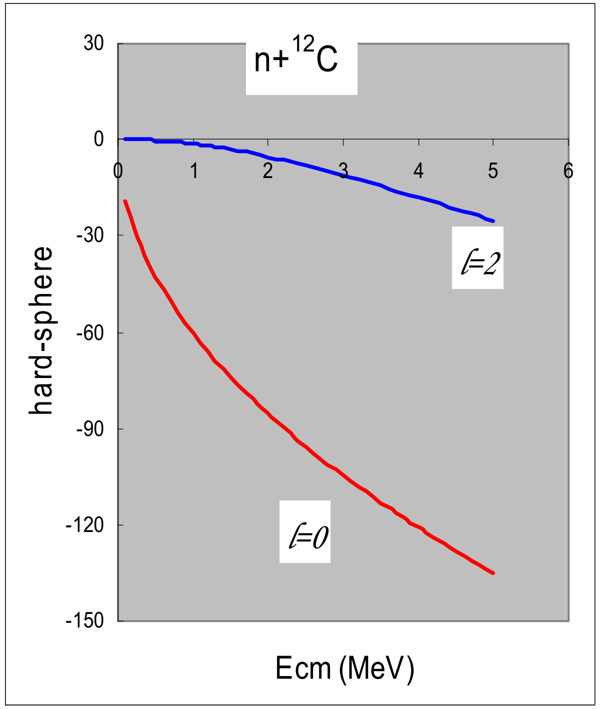
$$u_l(r) = F_l(kr) + \tan\delta_l G_l(kr)$$

$$u_l(a) = 0 \rightarrow \tan\delta_l = -\frac{F_l(ka)}{G_l(ka)}$$

= Hard-Sphere phase shift







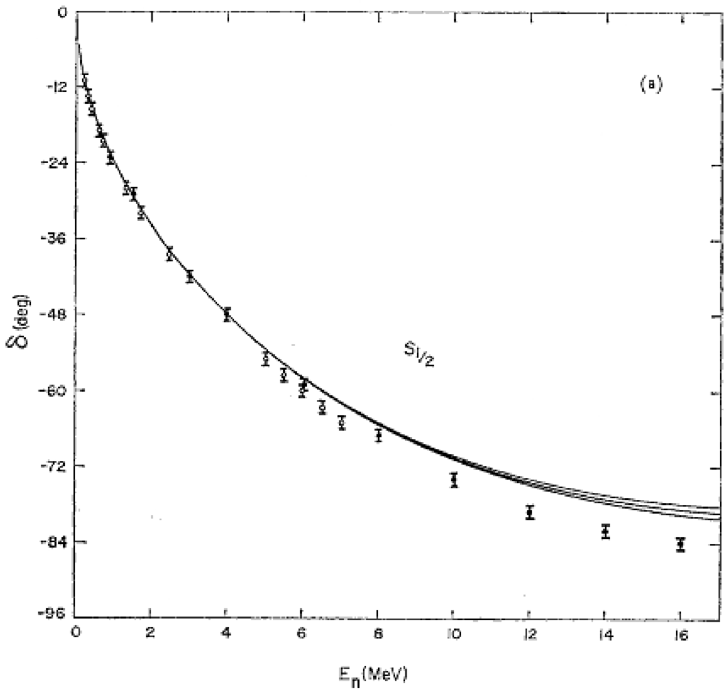
Special case: Neutrons, with ℓ=0:

$F_0(x)=\sin(x)$, $G_0(x)=\cos(x)$

→ $\delta=-ka$

example : α+n phase shift ℓ=0

R. A. ARNDT AND L. D. ROPER



➔ hard sphere is a good approximation

Elastic cross section with Coulomb:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2, \text{ with } f(\theta) = \frac{1}{2ik}\sum_l (2l+1)(\exp(2i\delta_l)-1)P_l(\cos\theta)$$

Still valid, but converges very slowly.

$$\delta_l = \delta_l^N + \delta_l^C$$

$$f(\theta) = \frac{1}{2ik}\sum_l (2l+1)[\exp(2i\delta_l)-1]P_l(\cos\theta)$$

$$= \frac{1}{2ik}\sum_l (2l+1)[\underbrace{\exp(2i\delta_l)-\exp(2i\delta_l^C)}_{\text{Nuclear}}+\underbrace{\exp(2i\delta_l^C)-1}_{\text{Coulomb: exact}}]P_l(\cos\theta)$$

Nuclear

Coulomb: exact

$$= f_N(\theta) + f_C(\theta)$$

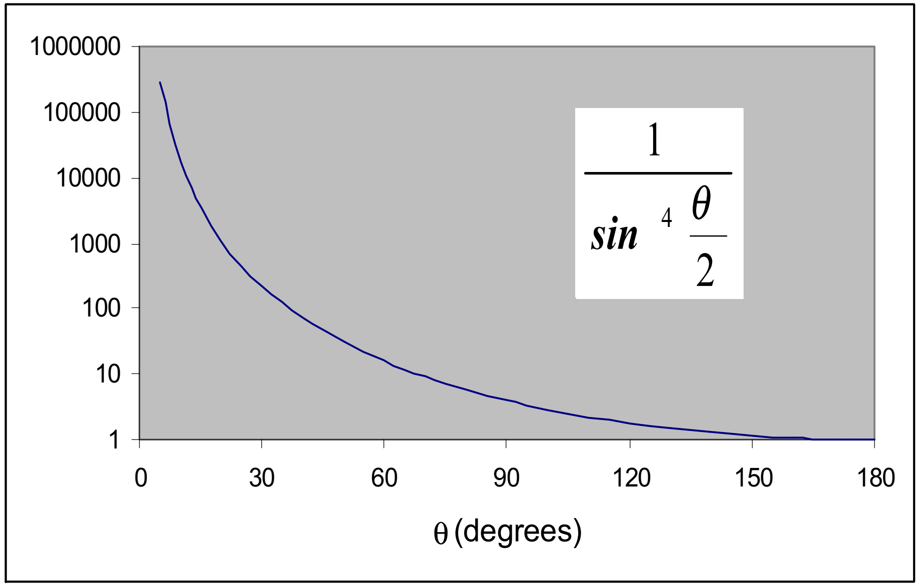
$$f_C(\theta) = \frac{-\eta}{2k\sin^2\theta/2}\exp(-i\eta\log(\sin^2\theta/2)) = \text{Coulomb amplitude}$$

$\frac{d\sigma_C}{d\Omega} = |f_C(\theta)|^2$ = Coulomb cross section: diverges for θ=0
increases at low energies

$$f(\theta) = f_C(\theta) + f_N(\theta)$$

- $f_C(\theta)$: Coulomb part: exact
- $f_N(\theta)$: nuclear part: converges rapidly

$$\frac{d\sigma_C}{d\Omega} = | f_C(\theta) |^2 \sim \frac{1}{E^2} \frac{1}{\sin^4 \frac{\theta}{2}}$$



- The total Coulomb cross section is not defined (diverges)
- Coulomb is dominant at small angles
  → used to normalize data
- Increases at low energies
- Minimum at θ=180° → nuclear effect maximum

3.c Extension to multichannel problems

- One channel: phase shift $\delta \rightarrow U=\exp(2i\delta)$
- Multichannel: *collision matrix* U_{ij}, (symmetric , unitary) with i,j=channels
- Good quantum numbers J=total spin π=total parity
- Channel *i* characterized by $I=I_1 \oplus I_2$ =channel spin
  $J=I \oplus \ell$ ℓ =angular momentum
- Selection rules: $|I_1 - I_2| \leq I \leq I_1 + I_2$
  $|\ell - I| \leq J \leq \ell + I$
  $\pi=\pi_1 * \pi_2 * (-1)^{\ell}$

Example of quantum numbers

$\alpha+{}^3He$ $\alpha=0^+$, ${}^3He=1/2^+$

J	I	ℓ	size
1/2+	1/2	0, ~~1~~	1
1/2-	1/2	~~0~~, 1	1
3/2+	1/2	~~1~~, 2	1
3/2-	1/2	1, ~~2~~	1

$p+{}^7Be$ ${}^7Be=3/2^-$, $p=1/2^+$

J	I	ℓ	size
0+	1 2	1 ~~2~~	1
0-	1 2	~~1~~ 2	1
1+	1 2	~~0~~, 1, ~~2~~ 1, ~~2~~, 3	3
1-	1 2	0, ~~1~~, 2 ~~1~~, 2, ~~3~~	3

Cross sections

One channel: $$\frac{d\sigma}{d\Omega} = | f(\theta) |^2, \text{ with } f(\theta) = \frac{1}{2ik} \sum_l (2l+1)(\exp(2i\delta_l) - 1) P_l(\cos\theta)$$

Multichannel $$\frac{d\sigma}{d\Omega} = \sum_{K_1, K_2, K_1', K_2'} | f_{K_1 K_2, K_1' K_2'}(\theta) |^2$$

With: K_1,K_2=spin orientations in the entrance channel

K'_1,K'_2=spin orientations in the exit channel

$$f_{K_1 K_2, K_1' K_2'}(\theta) = \sum_{J,\pi} \sum_{lI, l'I'} \ldots U^{J\pi}_{lI, l'I'} \; Y_{l'}(\theta, 0)$$

Collision matrix

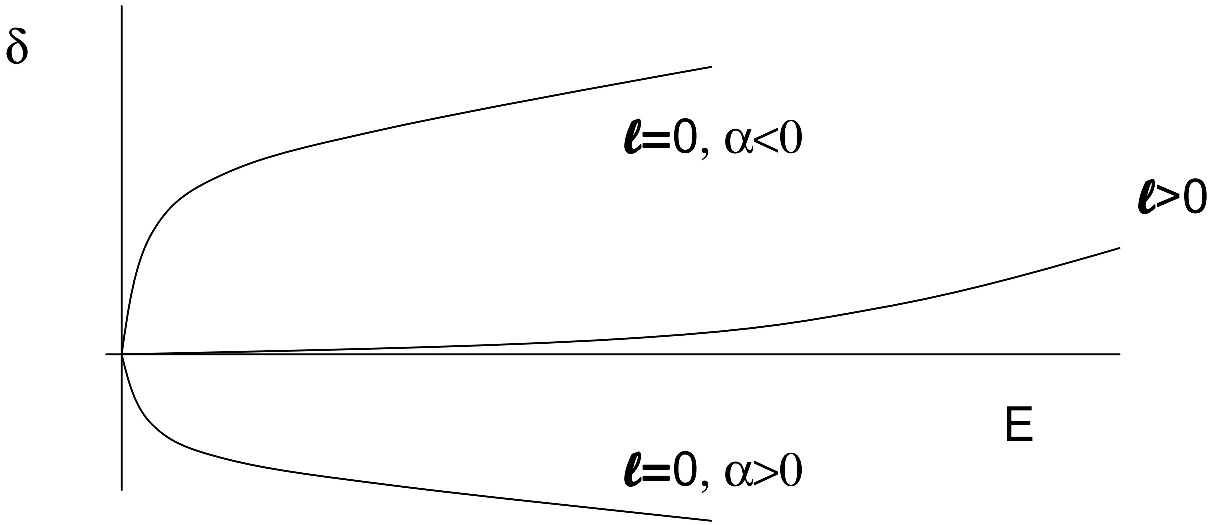
- generalization of δ: $U_{ij}=\eta_{ij}\exp(2i\delta_{ij})$
- determines the cross section

3.d Low-energy properties of the phase shifts

One defines $\gamma = \frac{1}{u(a)}\left(\frac{du}{dr}\right)_{r=a}$ =logarithmic derivative at *r*=*a* (*a* large)

Then, for very low E (neutral system): $\tan\delta_\ell \approx \frac{(ka)^{2\ell+1}}{(2\ell+1)!!(2\ell-1)!!}\left[\frac{\ell-\gamma a}{\gamma a+\ell+1}\right]$

For ℓ=0: $\alpha = -\lim_{k\to 0}\frac{\tan\delta_0(k)}{k} =$ scattering length



strong difference between ℓ=0 and ℓ>0

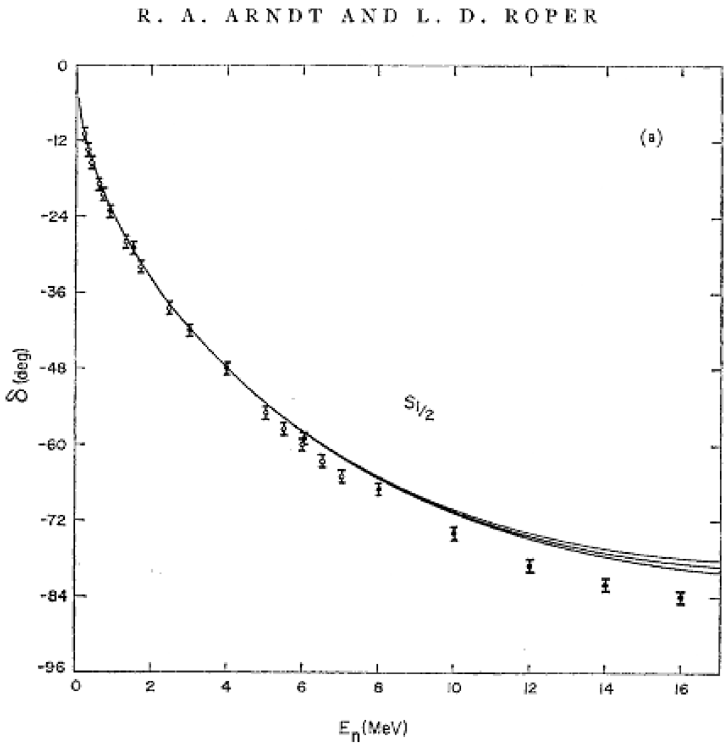
Generalization $k\cot\delta_0(k) = -\frac{1}{\alpha}+\frac{1}{2}r_e k^2+\ldots$ α=scattering length
r_e=effective range

Cross section at low E: $\sigma = \frac{4\pi}{k^2} \sum_\ell (2\ell+1) \sin^2 \delta_\ell \underset{k \to 0}{\longrightarrow} 4\pi\alpha^2$ (isotropic)

In general (**neutrons**): $\delta \sim k^{2\ell+1}$

$\sigma \sim k^{4\ell}$

Thermal neutrons (T=300K, E=25 meV): only ***ℓ***=0 contributes



example : α+n phase shift ***ℓ***=0

At low E: δ~-kα with α>0 ➔ replusive

3.e General calculation

For some potentials: analytic solution of the Schrödinger equation

In general: no analytic solution → numerical approach

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}u_\ell(r) + (V(r) - E)u_\ell(r) = 0$$

with: $$V(r) = V_N(r) + \frac{Z_1 Z_2 e^2}{r} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}$$

$$u_\ell(r) \rightarrow F_\ell(kr,\eta)\cos\delta_\ell + G_\ell(kr,\eta)\sin\delta_\ell$$

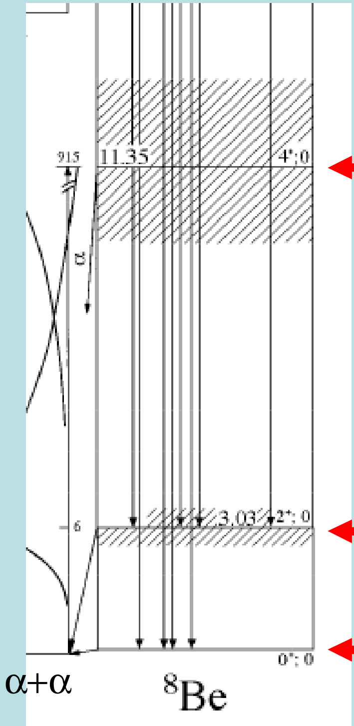
Numerical solution : discretization N points, with mesh size h

- $u_\ell(0)=0$
- $u_\ell(h)=1$ (or any constant)
- $u_\ell(2h)$ is determined numerically from $u_\ell(0)$ and $u_\ell(h)$ (Numerov algorithm)
- $u_\ell(3h)$,… $u_\ell(Nh)$
- for large r: matching to the asymptotic behaviour → phase shift

Bound states: same idea

Example: α+α

Experimental spectrum of ^{8}Be

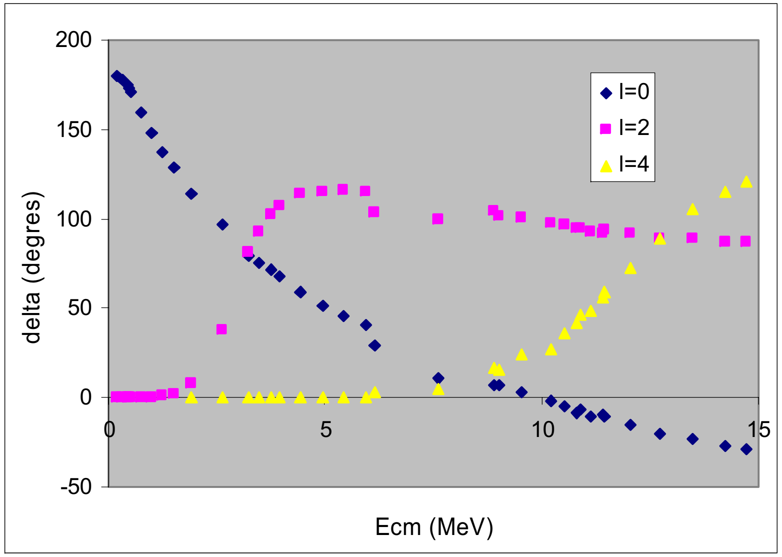


4^+
E~11 MeV
Γ~3.5 MeV

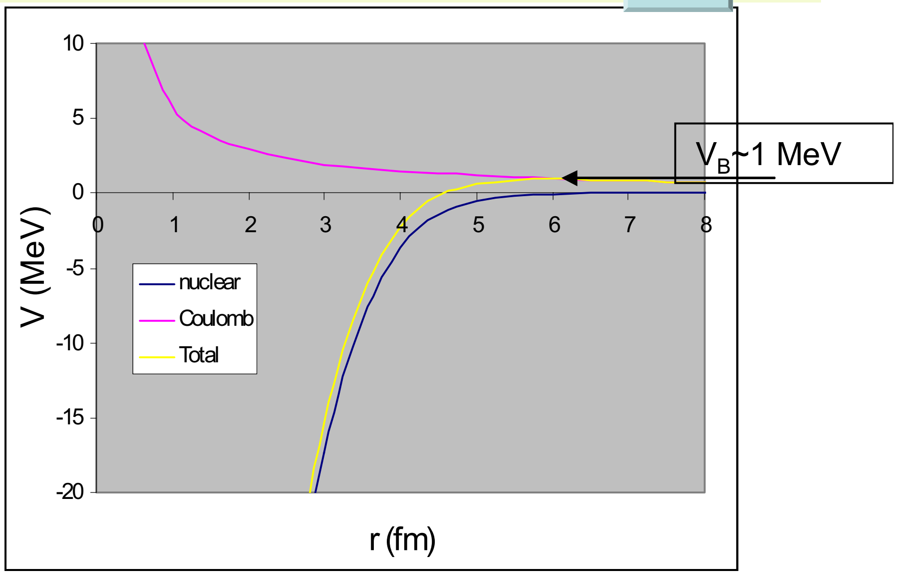
2^+
E~3 MeV
Γ~1.5 MeV

0^+
E=0.09 MeV
Γ=6 eV

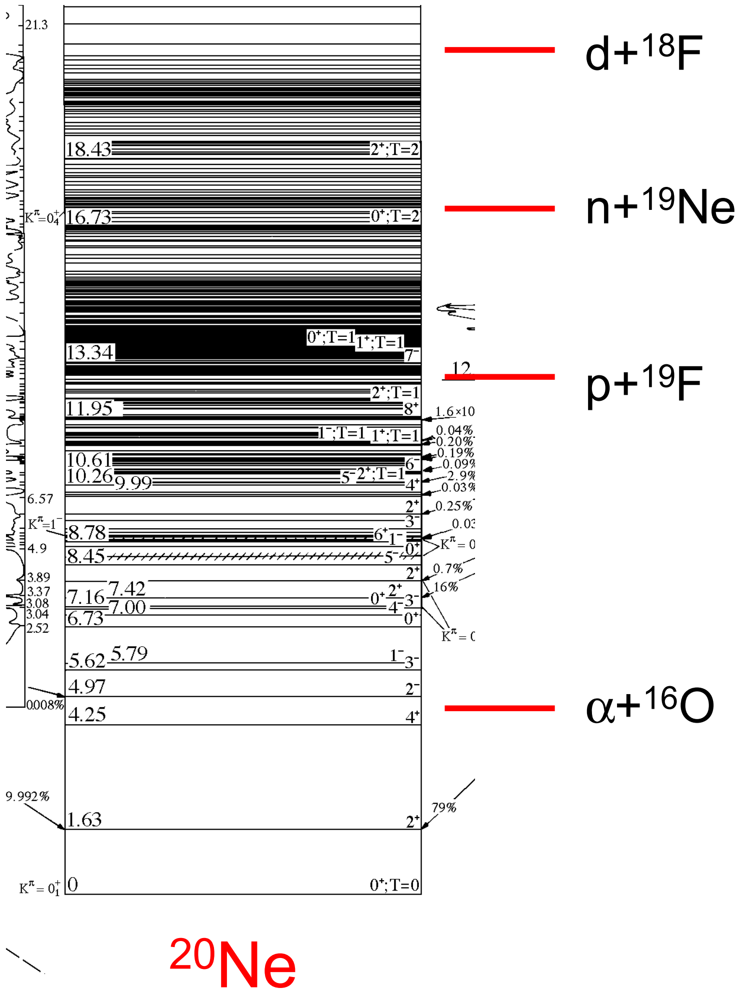
Experimental phase shifts



Potential: $V_N(r)=-122.3*\exp(-(r/2.13)^2)$



Goal: to simulate absorption channels



High energies:

- many open channels
- strong absorption
- potential model extended to complex potentials (« optical »)

Phase shift is complex: $\delta=\delta_R+i\delta_I$

collision matrix: $U=\exp(2i\delta)=\eta\exp(2i\delta_R)$
where $\eta=\exp(-2\delta_I)<1$

Elastic cross section

$$\frac{d\sigma}{d\Omega}=\frac{1}{4k^2}\left|\sum_\ell(2\ell+1)(\eta_\ell\exp(2i\delta_\ell)-1)P_\ell(\cos\theta)\right|^2$$

Reaction cross section:

$$\sigma=\frac{\pi}{k^2}\sum_\ell(2\ell+1)(1-\eta_\ell^2)$$

4. The R-matrix Method

Goals:

1. To solve the Schrödinger equation (E>0 or E<0)
    1. Potential model
    2. 3-body scattering
    3. Microscopic models
    4. Many applications in nuclear and atomic physics

2. To fit cross sections
    1. Elastic, inelastic → spectroscopic information on resonances
    2. Capture, transfer → astrophysics

References:

- A.M. Lane and R.G. Thomas, Rev. Mod. Phys. 30 (1958) 257
- F.C. Barker, many papers

Principles of the R-matrix theory

Standard variational calculations

- Hamiltonian $H\Psi = E\Psi$
- Set of *N* basis functions $u_i(r)$ with $\Psi(r) = \sum_i c_i u_i(r)$

➔Calculation of $H_{ij}=<u_i|H|u_j>$ over the full space
$N_{ij}=<u_i|u_j>$

(example: gaussians: $u_i(r)=\exp(-(r/a_i)^2)$)

$$N_{ij} = \int_0^\infty u_i(r) u_j(r) dr$$

$$H_{ij} = \int_0^\infty u_i(T+V) u_j dr$$

- Eigenvalue problem : $\sum_i (H_{ij} - EN_{ij})c_i = 0$ $\rightarrow$ upper bound on the energy
- But: Functions $u_i(r)$ tend to zero $\rightarrow$ not directly adapted to scattering states

Principles of the R-matrix theory

Extension to the R-matrix: includes boundary conditions

a. Hamiltonian

$$H\Psi = E\Psi, \text{ with } H \to T + \frac{Z_1 Z_2 e^2}{r}$$

b. Wave functions

Set of *N* basis functions $u_i(r)$ $\Psi(r) = \sum_i c_i u_i(r)$ valid in a limited range

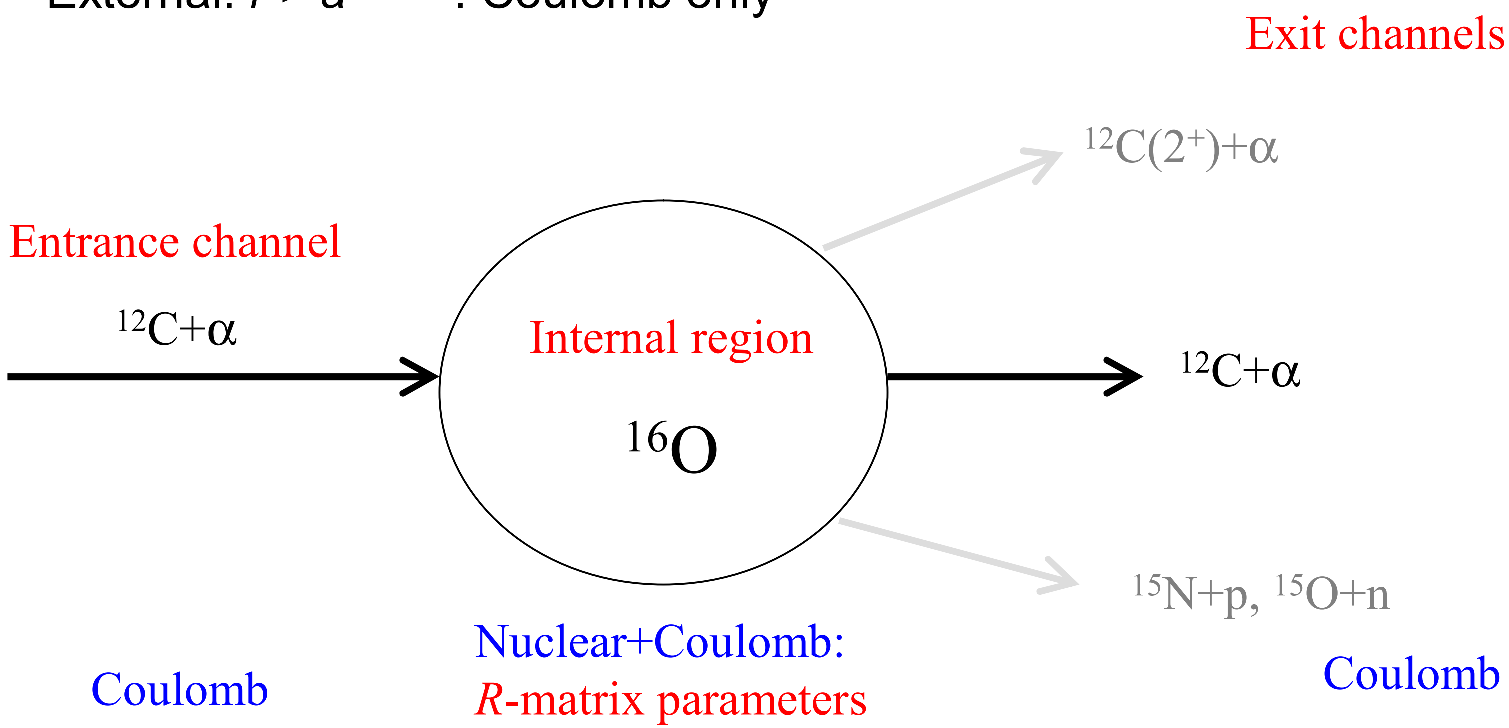
$$\text{with } \Psi \to \left(I_l(kr) - U_l * O_l(kr)\right) Y_l^m(\Omega)$$

➔ correct asymptotic behaviour (E>0 and E<0)

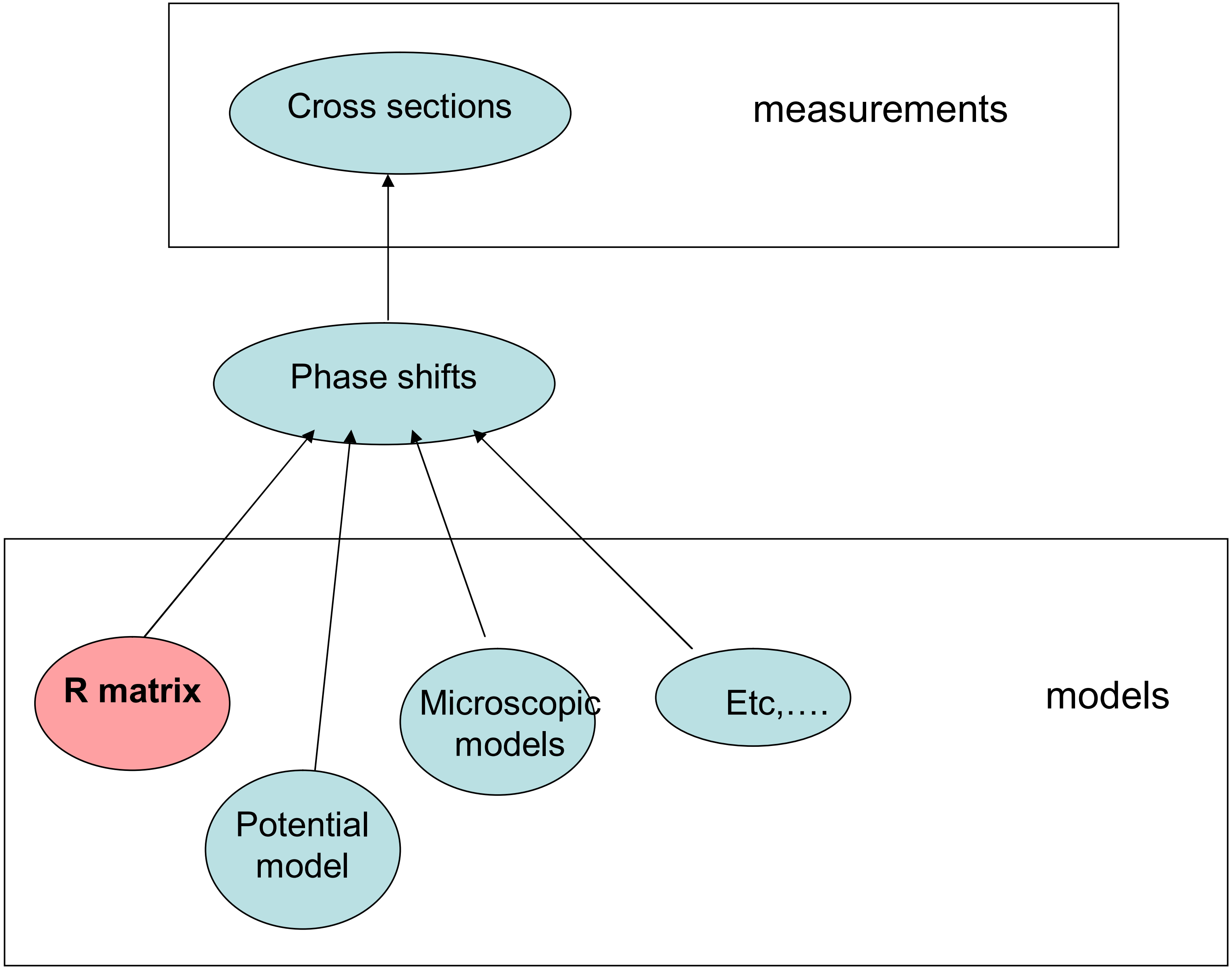
- Spectroscopy: short distances only
  Collisions : short *and* large distances

- R matrix: deals with collisions

- Main idea: to divide the space into 2 regions (radius *a*)
  - Internal: $r \leq a$: Nuclear + coulomb interactions
  - External: $r > a$: Coulomb only



Here: elastic cross sections



Plan of the lecture:

- The *R*-matrix method: two applications
  - ❑ Solving the Schrödinger equation (essentially E>0)
  - ❑ Phenomenological *R*-matrix (fit of data)

- Applications

- Other reactions: capture, transfer, etc.

c. R matrix

- N basis functions $u_i(r)$ are valid in a *limited range* ($r \leq a$)
  $\Rightarrow$ in the internal region:

$$\Psi_{int}(r) = \sum_{i=1}^{N} c_i u_i(r), \text{ for } r \leq a \qquad (c_i\text{=coefficients})$$

- At large distances the wave function is Coulomb
  $\Rightarrow$ in the external region:

$$\Psi_{ext}(r) = A\left(I_l(kr) - U_l * O_l(kr)\right), \text{ for } r > a$$

- Idea: to solve $\sum_i (H_{ij} - EN_{ij})c_i = 0$

with

$$H_{ij} = \langle u_i | H | u_j \rangle_{\text{int}}$$

$$N_{ij} = \langle u_i | u_j \rangle_{\text{int}}$$

defined in the internal region

- But T is not hermitian over a finite domain,

$$\langle u_i | T | u_j \rangle_{\text{int}} \neq \langle u_j | T | u_i \rangle_{\text{int}}$$

$$\int_0^a u_i \frac{d^2}{dr^2} u_j dr \neq \int_0^a u_j \frac{d^2}{dr^2} u_i dr, \text{ if } a \neq \infty$$

- Bloch-Schrödinger equation:

$$(H - E + \mathcal{L}(L_0))\,\Psi^{\ell m} = \mathcal{L}(L_0)\Psi^{\ell m},$$

- With $\mathcal{L}(L_0)$=Bloch operator (surface operator) – constant L_0 arbitrary

$$\mathcal{L}(L_0) = \frac{\hbar^2}{2\mu m_N a}\delta(\rho - a)\left(\frac{d}{d\rho} - \frac{L_0}{\rho}\right)\rho,$$

- Now we have T+$\mathcal{L}(L_0)$ hermitian:

$$< u_i \,|\, T + \mathcal{L}(L_0) \,|\, u_j >_{int} = < u_j \,|\, T + \mathcal{L}(L_0) \,|\, u_i >_{int}$$

- Since the Bloch operator acts at *r*=*a* only:

$$(H - E + \mathcal{L}(L_0))\,\Psi^{\ell m}_{int} = \mathcal{L}(L_0)\Psi^{\ell m}_{ext},$$

- Here L_0=0 (boundary-condition parameter)

- Summary: $(H - E + \mathcal{L}(0))\,\Psi^{\ell m}_{int} = \mathcal{L}(0)\Psi^{\ell m}_{ext}, \quad (1)$

$$\Psi^{\ell m}_{int} = \sum_i c^\ell_i\, u^\ell_i(r) \quad (2)$$

$$\Psi^{\ell m}_{ext} = (I_\ell(kr) - U^\ell(E) O_\ell(kr)) \quad (3)$$

- Using (2) in (1) and the continuity equation:

$$\sum_i c^\ell_i \underbrace{< u^\ell_j | H + \mathcal{L}(0) - E | u^\ell_i >_{int}}_{D_{ij}(E)} = < u^\ell_j | \mathcal{L}(0) | \Psi^\ell_{ext} >,$$

→ Provides c_i (inversion of D)

$$\sum_i c^\ell_i\, u^\ell_i(a) = (I_\ell(ka) - U^\ell(E) O_\ell(ka))$$

Continuity at *r=a*

→

$$U^\ell = \frac{I_\ell(ka) - kaI'_\ell(ka)R^\ell}{O_\ell(ka) - kaO'_\ell(ka)R^\ell}$$

$$U^\ell = \frac{I_\ell(ka) - kaI'_\ell(ka)R^\ell}{O_\ell(ka) - kaO'_\ell(ka)R^\ell}$$

$$= \frac{I_\ell(ka)}{O_\ell(ka)} \frac{1 - L^\star R^\ell}{1 - LR^\ell} = \exp(2i\delta^\ell)$$

$$L = ka\frac{O'_\ell(ka)}{O_\ell(ka)}$$

With the R-matrix defined as $$R^\ell(E) = \frac{\hbar^2 a}{2\mu} \sum_{ij} u_i^\ell(a)(D^\ell(E))^{-1}_{ij} u_j^\ell(a)$$

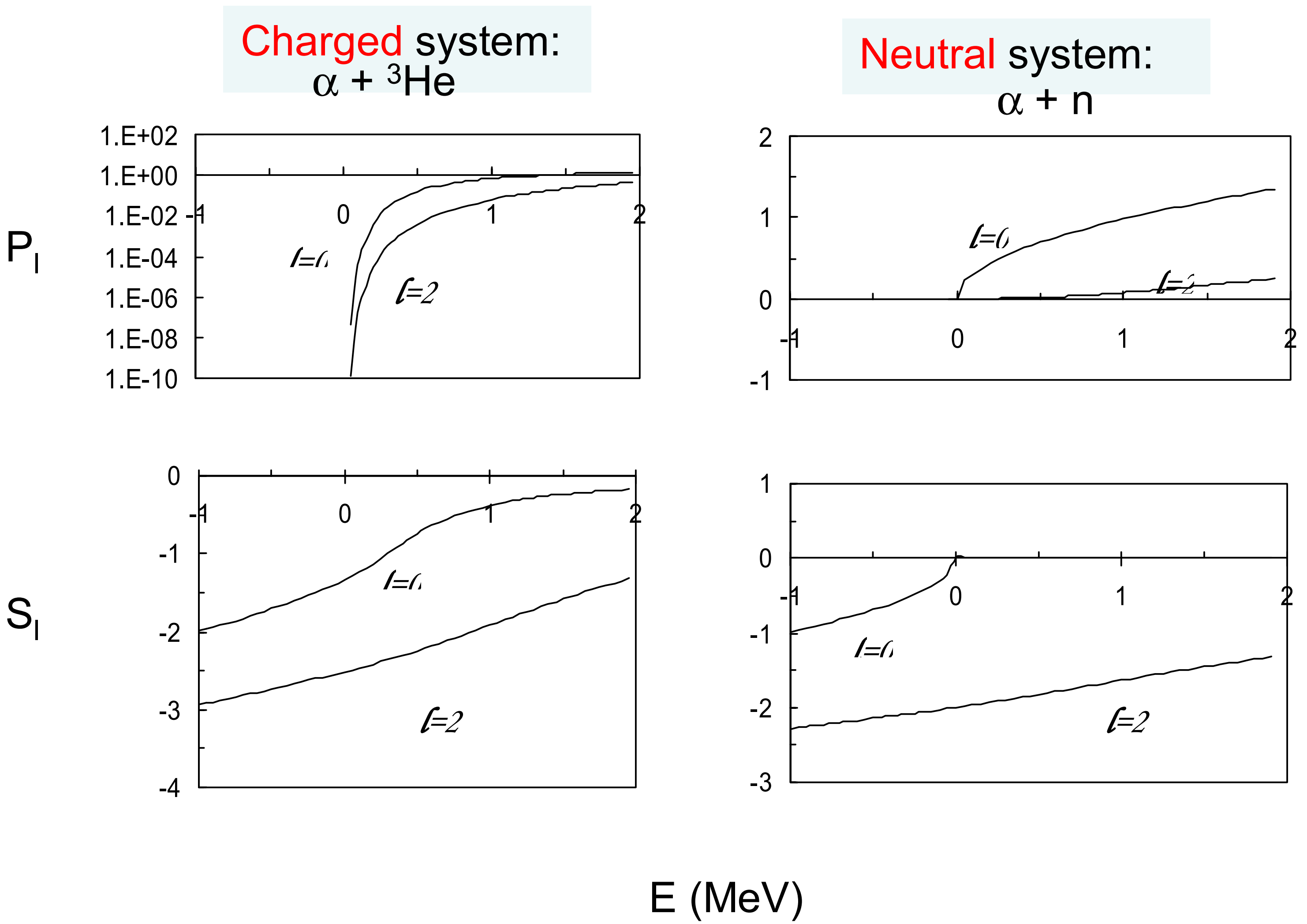
If the potential is real: *R* real,
|*U*|=1, δ real

Procedure (for a given ℓ):

1. Compute matrix elements $D^\ell_{ij}(E) =< u_i^\ell|H + \mathcal{L}(0) - E|u_j^\ell >_{int}$
2. Invert matrix D
3. Compute R matrix
4. Compute the collision matrix U

$$L = ka\frac{O'_\ell(ka)}{O_\ell(ka)} = S(E) + iP(E)$$

S(E)=shift factor
P(E)=penetration factor



Interpretation of the R matrix

$$\Psi^{\ell}_{ext}(r) = A(I_\ell(kr) - U^{\ell}(E)O_\ell(kr)) \qquad (1)$$

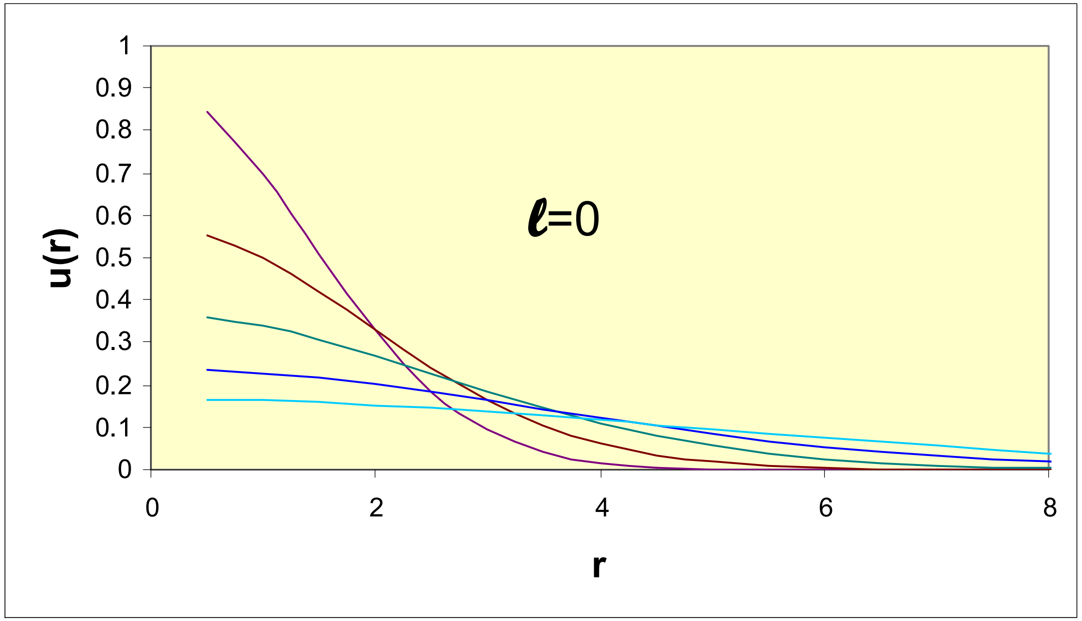
$$U^{\ell} = \frac{I_\ell(ka) - kaI'_\ell(ka)R^{\ell}}{O_\ell(ka) - kaO'_\ell(ka)R^{\ell}} \rightarrow R^{\ell} = \frac{I_\ell(ka) - U^{\ell}O_\ell(ka)}{ka(I'_\ell(ka) - U^{\ell}O'_\ell(ka))}$$

From (1):
$$\frac{\Psi'^{\ell}(a)}{\Psi^{\ell}(a)} = k\frac{I'_\ell(ka) - U^{\ell}O'_\ell(ka)}{I_\ell(ka) - U^{\ell}O_\ell(ka)} = \frac{1}{aR^{\ell}}$$

Then: $R^{\ell} = \dfrac{1}{a(\log \Psi^{\ell}(a))'}$ =inverse of the logarithmic derivative at *a*

Example $u_i^\ell(r) = r^\ell \exp(-(r/a_i)^2)$

Gaussians with different widths



$$
\begin{aligned}
N_{ij} &= < u_i^\ell | u_j^\ell >_{int} \\
&= \int_0^a r^{2\ell} \exp(-(r/a_i)^2 - (r/a_j)^2)\, dr
\end{aligned}
$$

Can be done exactly
(incomplete γ function)

Matrix elements of H can be calculated analytically for gaussian potentials

Other potentials: numerical integration

Input data

- Potential
- Set of *n* basis functions (here gaussians with different widths)
- Channel radius *a*

Requirements

- *a* large enough : $V_N(a) \sim 0$
- *n* large enough (to reproduce the internal wave functions)
- *n* as small as possible (computer time)

  → compromise

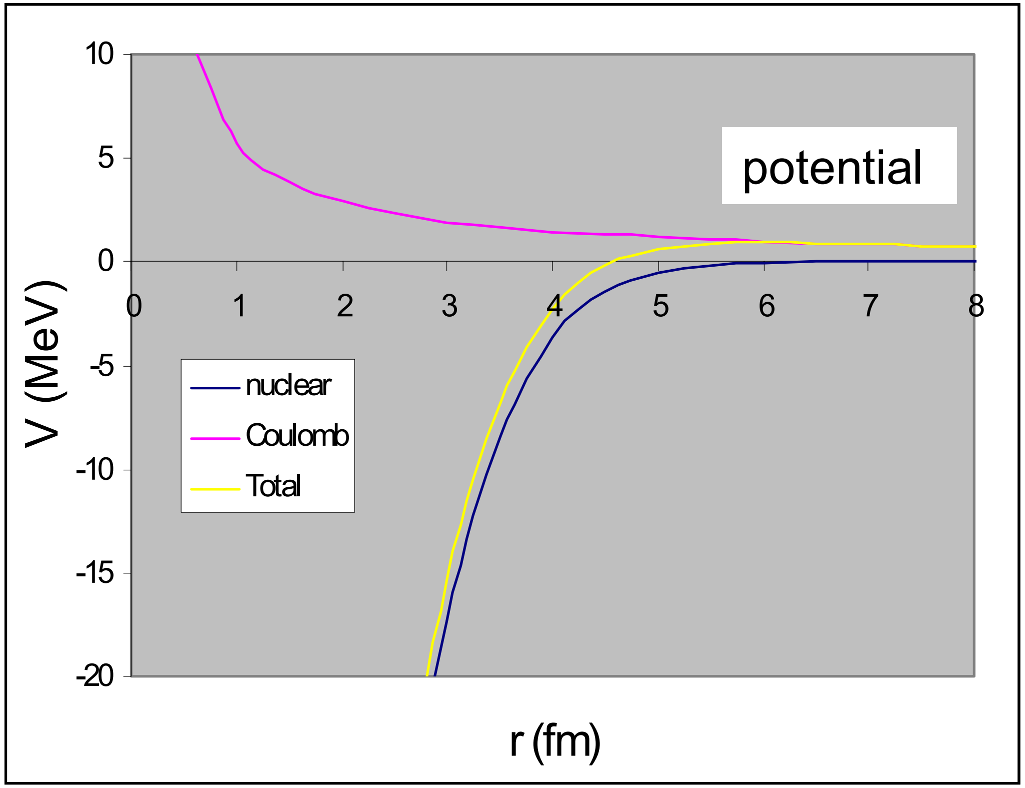
Tests

- Stability of the phase shift with the channel radius *a*
- Continuity of the derivative of the wave function

Results for $\alpha+\alpha$

- potential : $V(r)=-126*\exp(-(r/2.13)^2)$ (Buck potential)

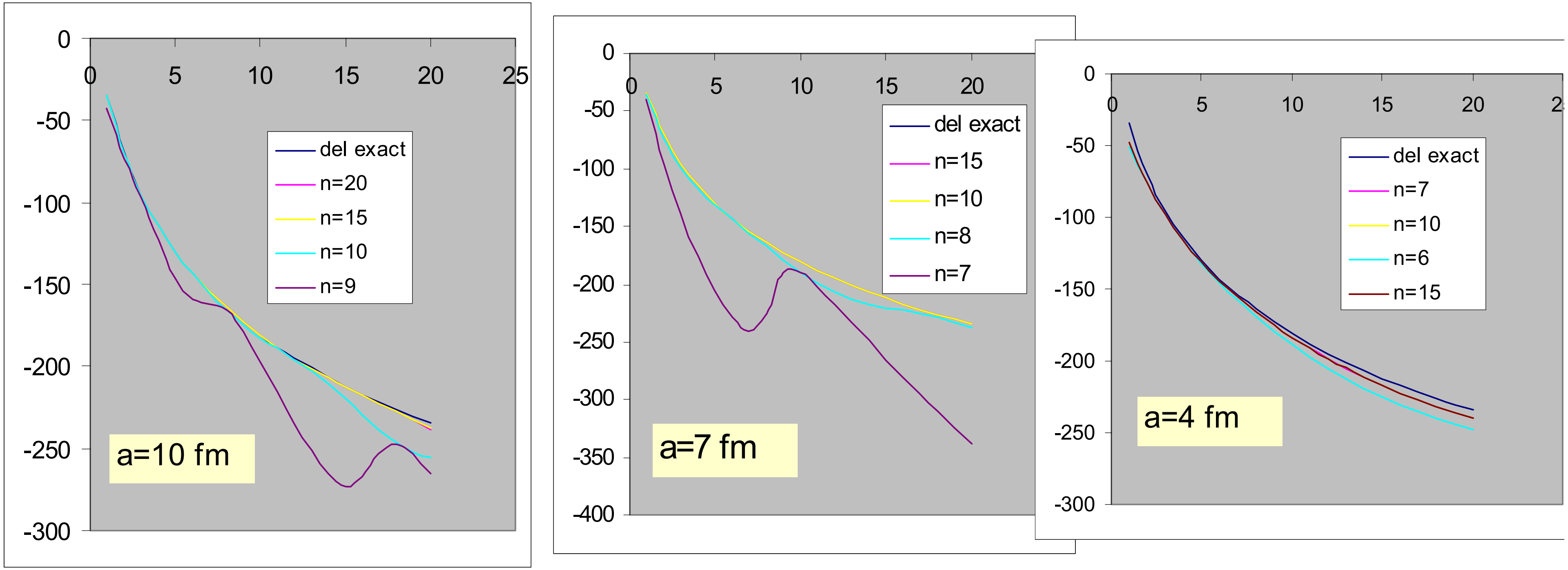
- Basis functions: $u_i(r)=r^{\ell}*\exp(-(r/a_i)^2)$
  with $a_i=x_0*a_0^{(i-1)}$ (geometric progression)
  typically $x_0=0.6$ fm, $a_0=1.4$



$V_N(a)=0$

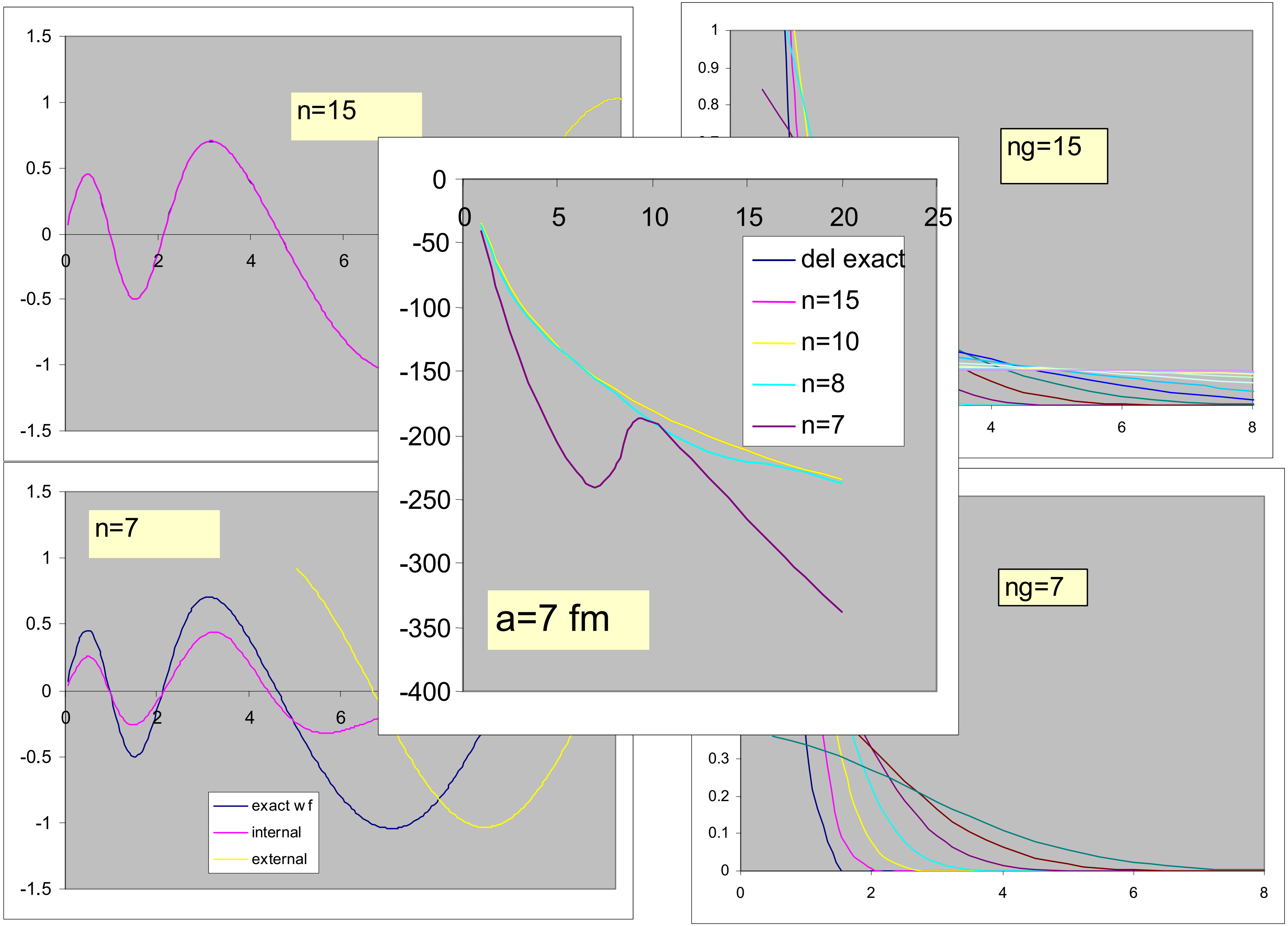
→ $a > 6$ fm

Elastic phase shifts



- a=10 fm too large (needs too many basis functions)
- a=4 fm too small (nuclear not negligible)
- a=7 fm is a good compromise

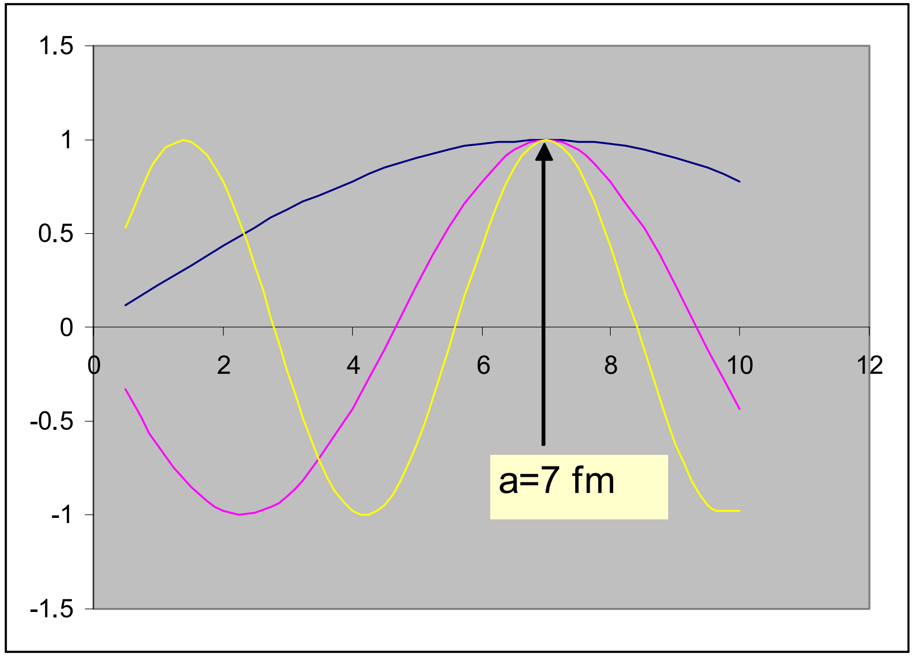
Wave functions at 5 MeV, a= 7 fm

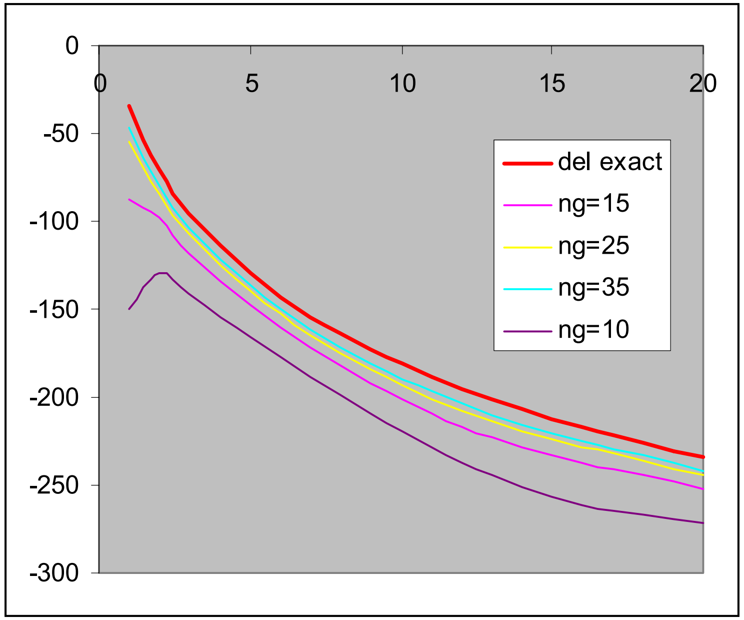


Other example : sine functions $u_i^\ell(r) = \sin \frac{\pi r}{a}(n - \frac{1}{2})$

• Matrix elements very simple

• Derivative $u_i'(a)=0$

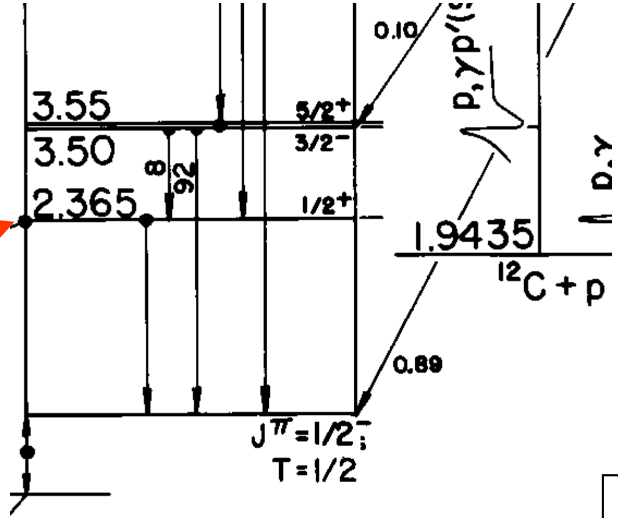




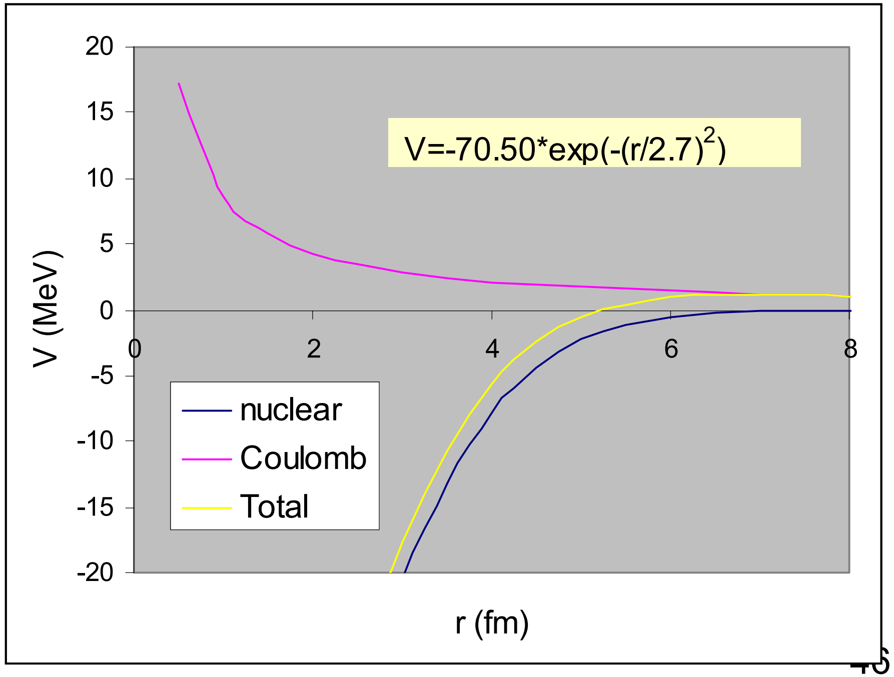
➔ Not a good basis (no flexibility)

Example of resonant reaction: $^{12}C+p$

Resonance
$\ell=0$
E=0.42 MeV
Γ=32 keV

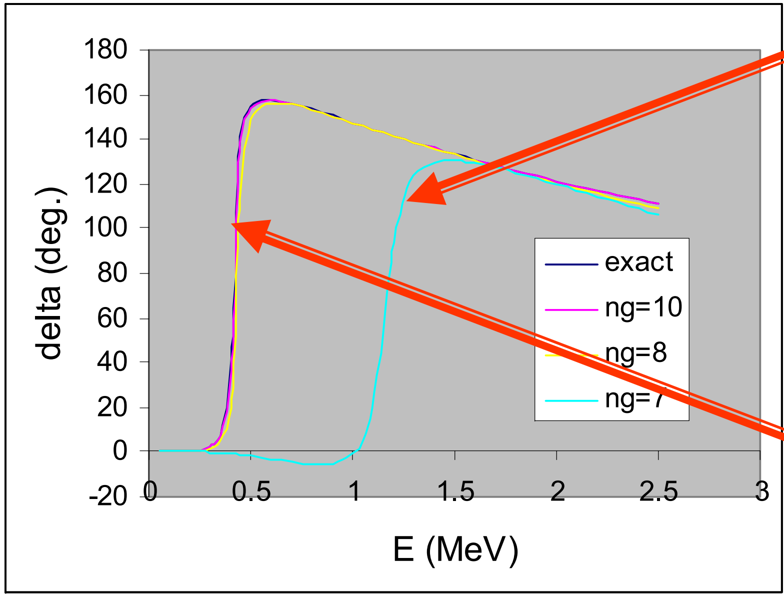


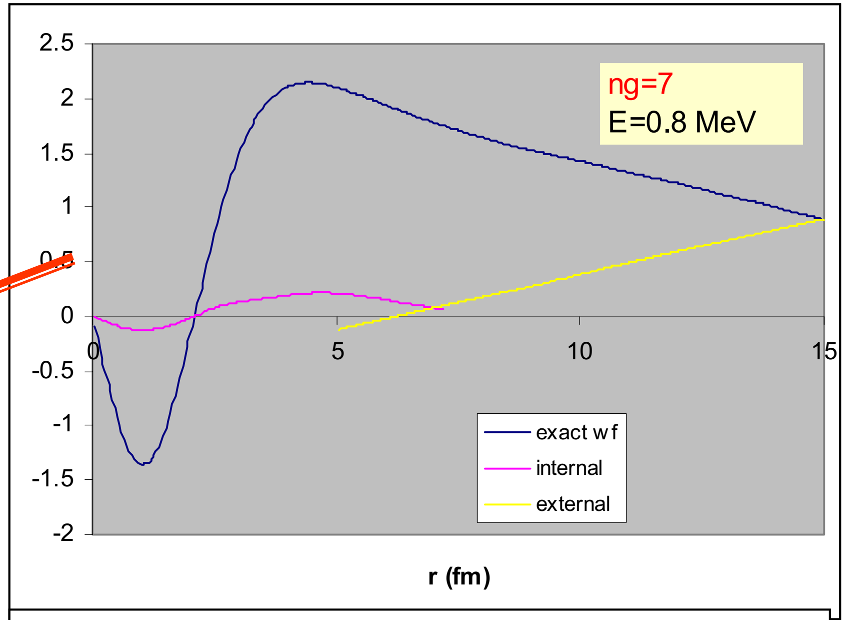
- potential : $V=-70.5*\exp(-(r/2.70)^2)$
- Basis functions: $u_i(r)=r^{\ell}*\exp(-(r/a_i)^2)$

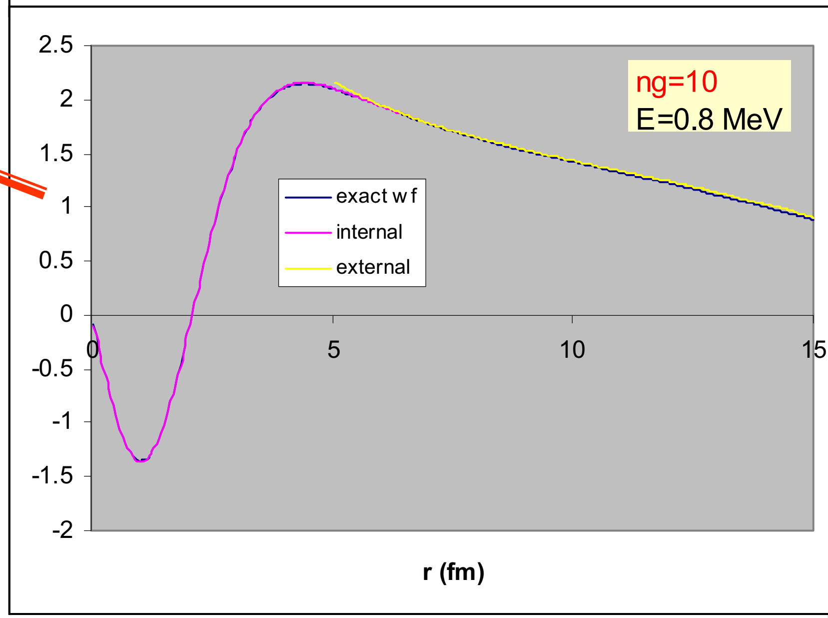


Wave functions

Phase shifts a=7 fm







Resonance energies

$$U^\ell = \frac{I_\ell(ka)}{O_\ell(ka)} \frac{1 - L^\star R^\ell}{1 - LR^\ell}, \quad \text{with } L(E) = S(E) + iP(E)$$
$$= \exp(2i\delta^\ell) = \exp(2i(\delta^\ell_{HS} + \delta^\ell_R))$$

with

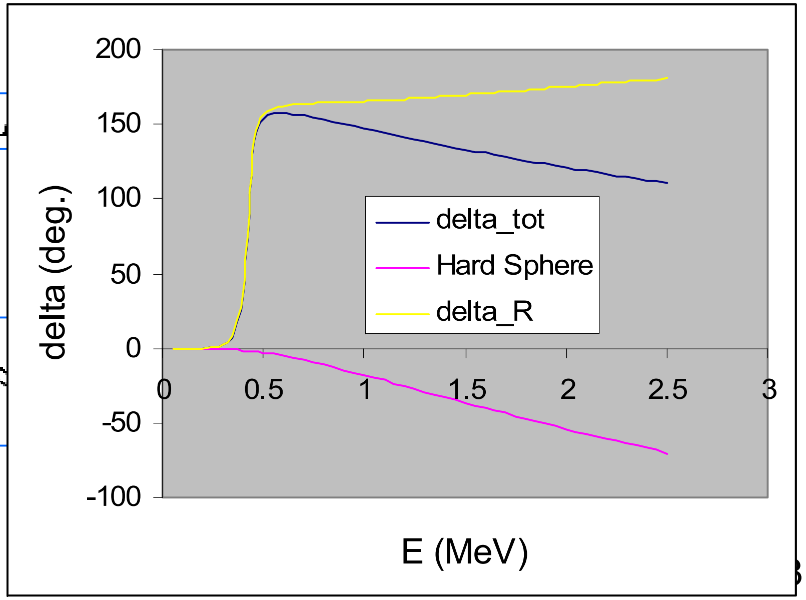
$$\exp(2i\delta^\ell_{HS}) = \frac{I_\ell(ka)}{O_\ell(ka)} \rightarrow \delta^\ell_{HS} = -\arctan\frac{F_\ell(ka)}{G_\ell(ka)}$$ Hard-sphere phase shift

$$\exp(2i\delta^\ell_R) = \frac{1 - L^\star R^\ell}{1 - LR^\ell} \rightarrow \delta^\ell_R = \arctan\frac{PR}{1 - SR}$$ R-matrix phase shift

- Resonance energy E_r defined by $1 - S(E_r)$

In general: must be solved numerically

- Resonance width Γ defined by $\tan\delta_R(E) \approx$



Application of the R-matrix to bound states

Positive energies: $u_l'' - \frac{l(l+1)}{r^2} u_l(r) + (k^2 - 2k\eta) u_l(r) = 0$ Coulomb functions F_ℓ, G_ℓ

Negative energies: $u_l'' - \frac{l(l+1)}{r^2} u_l(r) + (-k^2 - 2k\eta) u_l(r) = 0$ Whittaker functions $W_{-\eta,\ell+1/2}(2x)$

Asymptotic behaviour:

$$F_\ell(x) \to \sin(x - \ell\frac{\pi}{2} - \eta \log 2x)$$

$$G_\ell(x) \to \cos(x - \ell\frac{\pi}{2} - \eta \log 2x)$$

$$W_{-\eta,\ell+1/2}(2x) \to \frac{\exp(-x)}{x^\eta}$$

Starting point of the R matrix:

$$H\Psi = E\Psi$$

$$H \to T + \frac{Z_1 Z_2 e^2}{r}$$

$$\Psi \to W_{-\eta,l+1/2}(2kr) Y_l^m(\Omega)$$

- R matrix equations $$(H-E+\mathcal{L}(L))\,\Psi_{int}^{\ell m}=\mathcal{L}(L)\Psi_{ext}^{\ell m}, \qquad (1)$$

$$\Psi_{int}^{\ell m}=\sum_i c_i^\ell\, u_i^\ell(r) \qquad (2)$$

$$\Psi_{ext}^{\ell m}=CW_{-\eta,\ell+1/2}(2kr) \qquad (3)$$

With *C*=ANC (Asymptotic Normalization Constant): important in “external” processes

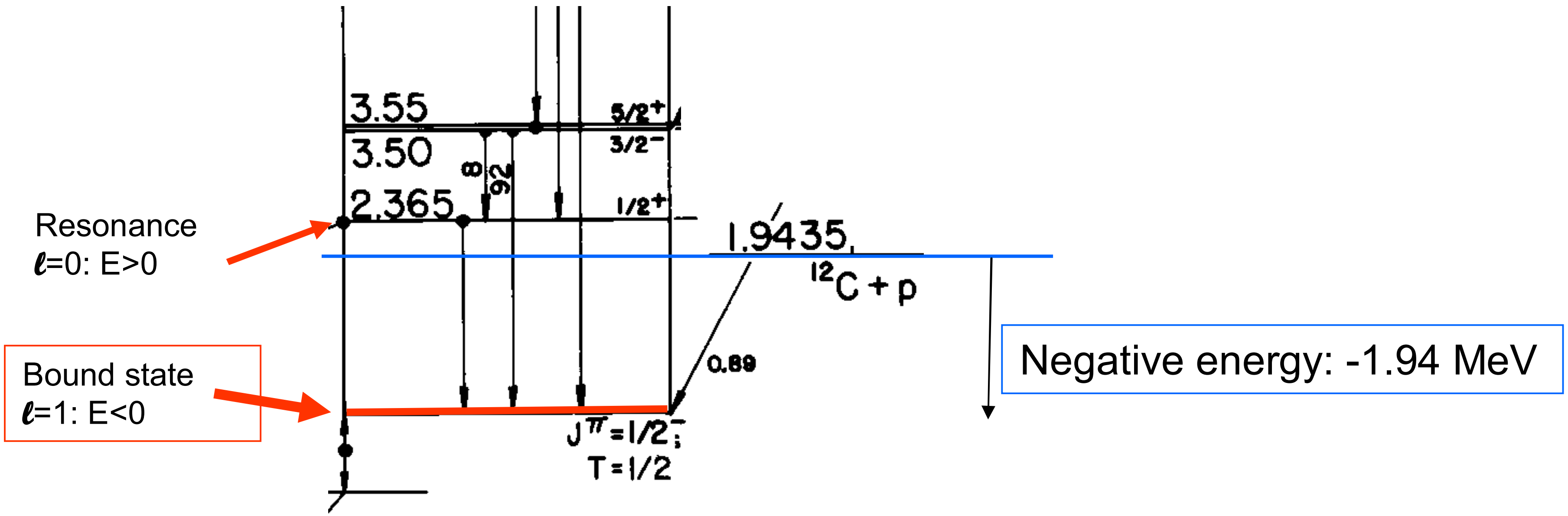
- Using (2) in (1) and the continuity equation:

$$\sum_i c_i^\ell \underbrace{< u_i^\ell|H+\mathcal{L}(L)-E|u_j^\ell >_{int}}_{D_{ij}(E)} = < u_j^\ell|\mathcal{L}(L)|\Psi_{ext}^\ell >= 0,$$

if $L=2ka\dfrac{W'(2ka)}{W(2ka)}$

- But: *L* depends on the energy, which is not known → iterative procedure

Application to the ground state of $^{13}N = {}^{12}C + p$



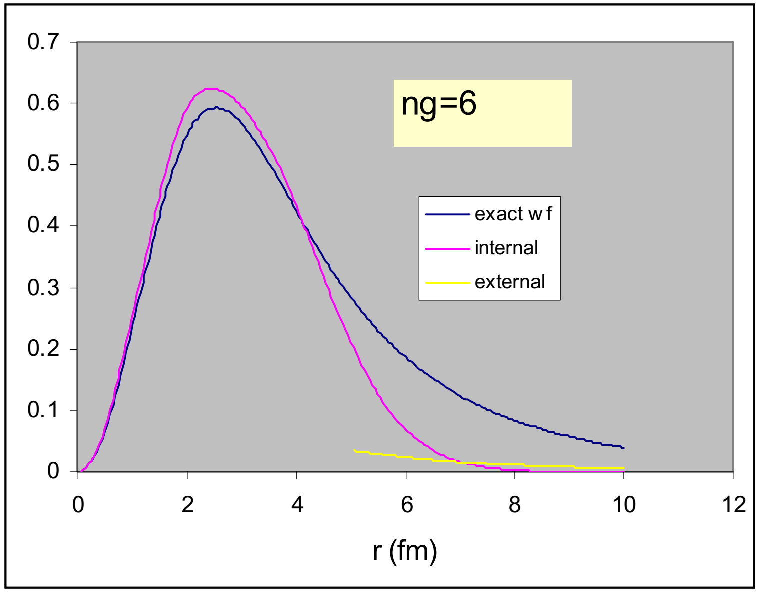
- Potential : $V=-55.3*\exp(-(r/2.70)^2)$
- Basis functions: $u_i(r)=r^{\ell}*\exp(-(r/a_i)^2)$ (as before)

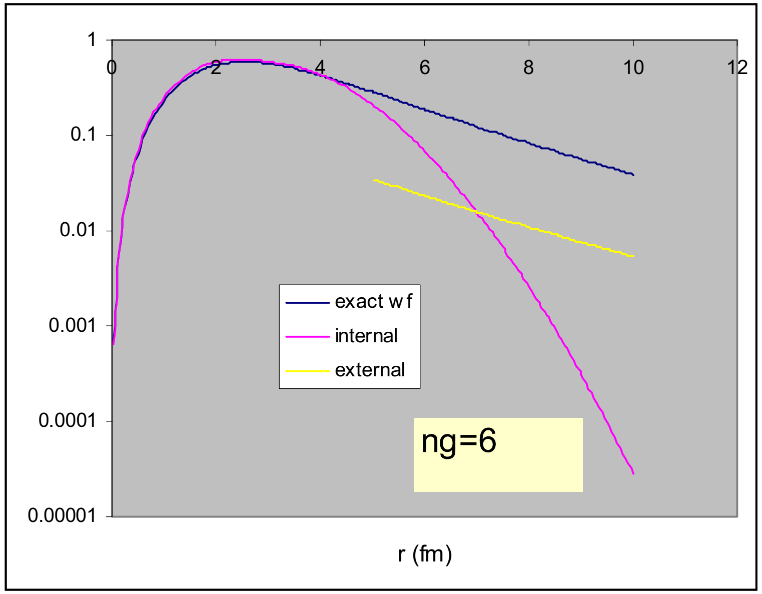
Calculation with a=7 fm

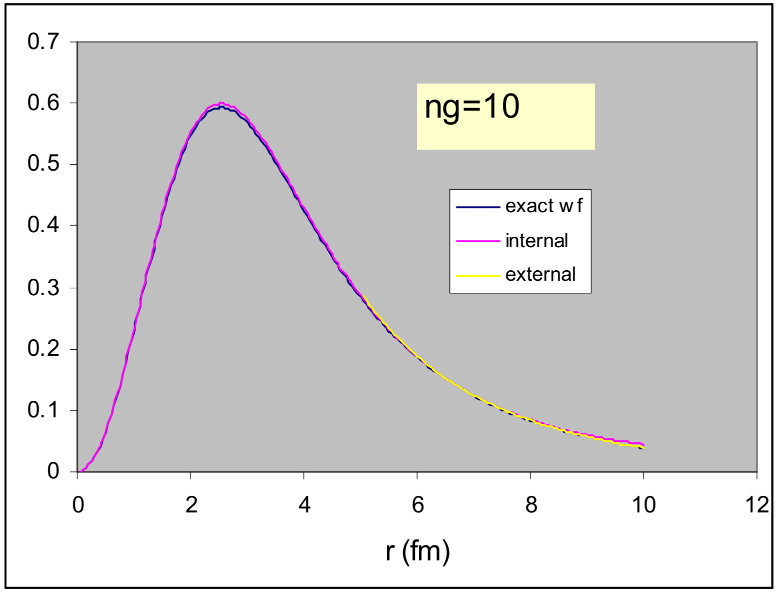
- ng=6 (poor results)
- ng=10 (good results)

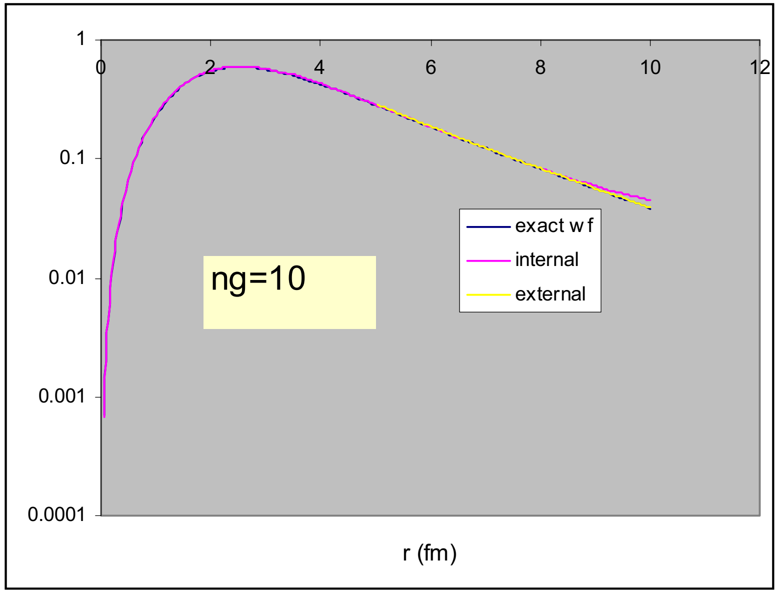
Iteration	ng=6	ng=10
1	-1.500	-2.190
2	-1.498	-1.937
3	-1.498	-1.942
		-1.942
Final	-1.498	-1.942
Exact	-1.942	
Left derivative	-1.644	-0.405
Right derivative	-0.379	-0.406

Wave functions (a=7 fm)









5. Phenomenological R matrix

- Goal: fit of experimental data
- Basis functions $\Psi^{\ell}_{int}(r) = \sum_i c^{\ell}_i \, u^{\ell}_i(r)$
- R matrix equations

$$R(E) = \frac{\hbar^2 a}{2\mu} \sum_{ij} u^{\ell}_i(a) (D^{\ell}(E))^{-1}_{ij} u^{\ell}_j(a)$$

with $$D^{\ell}_{ij}(E) = < u^{\ell}_i | H + \mathcal{L}(0) - E | u^{\ell}_j >_{int}$$

The choice of the basis functions is arbitrary

BUT must be consistent in *R(E)* and *D(E)*

Change of basis

- Eigenstates of $(H + \mathcal{L}(0))\Psi_\lambda = E_\lambda \Psi_\lambda$ *over the internal region*
- Ψ_λ expanded over the same basis: $\Psi_\lambda(r) = \sum_i d_i^\lambda u_i(r)$
- → standard diagonalization problem

$$\sum_i d_i^\lambda < u_i | H + \mathcal{L}(0) - E_\lambda | u_j >_{int} = 0$$

Instead of using $u_i(r)$, one uses $\Psi_l(r)$

$$R(E) = \frac{\hbar^2 a}{2\mu} \sum_{\lambda\lambda'} \Psi_\lambda(a) \left(D^\ell(E)\right)^{-1}_{\lambda\lambda'} \Psi_{\lambda'}(a)$$

$$\begin{aligned} D^\ell_{\lambda\lambda'}(E) &= < \Psi_\lambda | H + \mathcal{L}(0) - E | \Psi_{\lambda'} >_{int} \\ &= (E_\lambda - E)\delta_{\lambda\lambda'} \end{aligned}$$

$$R(E) = \frac{\hbar^2 a}{2\mu} \sum_\lambda \frac{|\Psi_\lambda(a)|^2}{E_\lambda - E} = \sum_\lambda \frac{\gamma_\lambda^2}{E_\lambda - E}$$

$$R(E) = \frac{\hbar^2 a}{2\mu} \sum_\lambda \frac{|\Psi_\lambda(a)|^2}{E_\lambda - E} = \sum_\lambda \frac{\gamma_\lambda^2}{E_\lambda - E}$$

Completely equivalent

With $\gamma_\lambda = \sqrt{\frac{\hbar^2 a}{2\mu}} \Psi_\lambda(a)$ = reduced width

→2 steps: computation of eigen-energies E_λ (=poles) and eigenfunctions
computation of reduced widths from eigenfunctions

Remarks:

• E_λ = pole energy: different from the resonance energy
depend on the channel radius

• γ_λ proportional to the wave function at *a*→ measurement of clustering (depend on *a*!)
large γ_λ → strong clustering

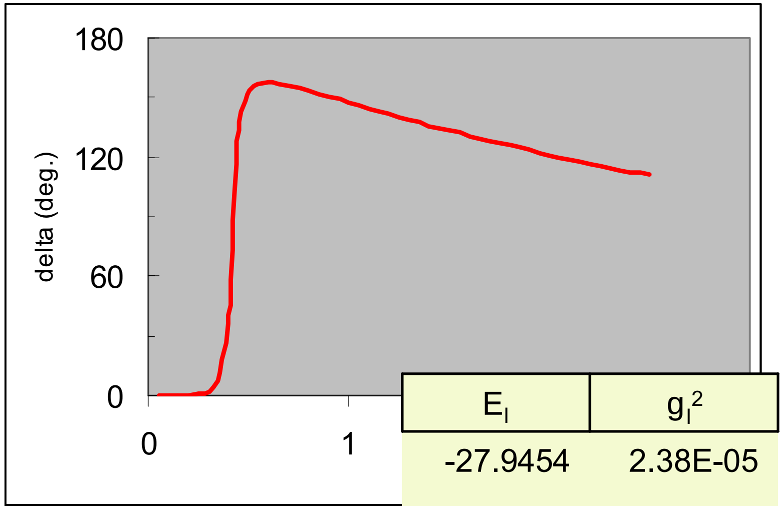
• dimensionless reduced width $\theta_\lambda^2 = \frac{\gamma_\lambda^2}{\gamma_W^2}$, with $\gamma_W^2 = \frac{3\hbar^2}{2\mu a^2}$ = Wigner limit

• http://pntpm.ulb.ac.be/Nacre/Programs/coulomb.htm: web page to compute reduced widths

Example : $^{12}C+p$

- potential : $V=-70.5*\exp(-(r/2.70)^2)$
- Basis functions: $u_i(r)=r^{\ell}*\exp(-(r/a_i)^2)$ with $a_i=x_0*a_0^{(i-1)}$

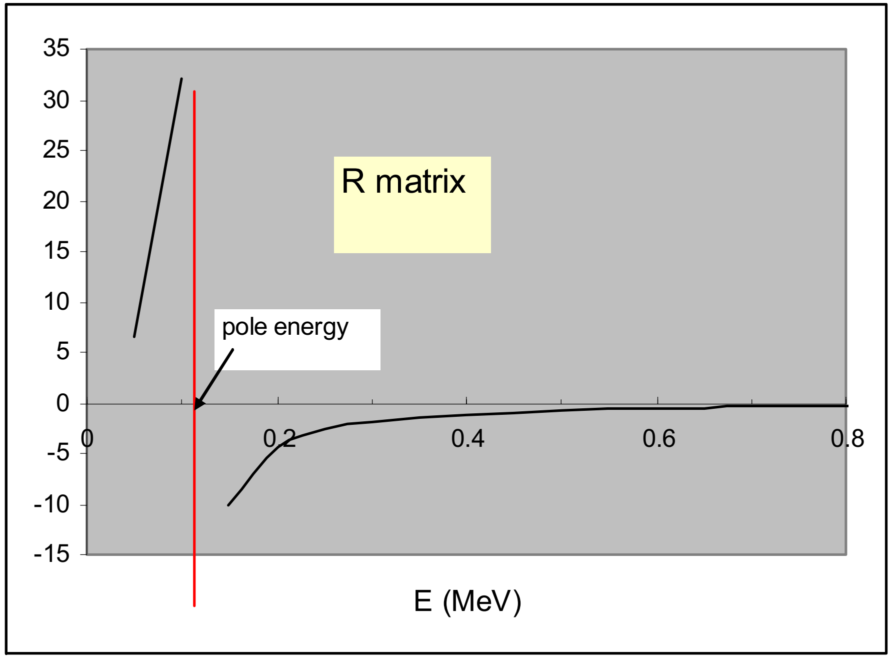
10 basis functions, a=8 fm



$$R(E) = \sum_{\lambda} \frac{\gamma_{\lambda}^2}{E_{\lambda} - E}$$

10 eigenvalues

E_l	g_l^2
-27.9454	2.38E-05
0.112339	0.392282
7.873401	1.094913
26.50339	0.730852
40.54732	0.010541
65.62125	1.121593
107.7871	4.157215
153.8294	1.143461
295.231	0.097637
629.6709	0.019883



Link between “standard” and “phenomenological” R-matrix:

- Standard R-matrix: parameters are **calculated from basis functions**
- Phenomenological R-matrix: parameters are **fitted to data**

Calculation: 10 poles

pole	E_l	γ_l^2
1	-27.95	2.38E-05
2	0.11	3.92E-01
3	7.87	1.09E+00
4	26.50	7.31E-01
5	40.55	1.05E-02
6	65.62	1.12E+00
7	107.79	4.16E+00
8	153.83	1.14E+00
9	295.23	9.76E-02
10	629.67	1.99E-02

Fit to data

Isolated pole (2 parameters)

$$\frac{\gamma_0^2}{E_0 - E}$$

Background (high energy): gathered in 1 term

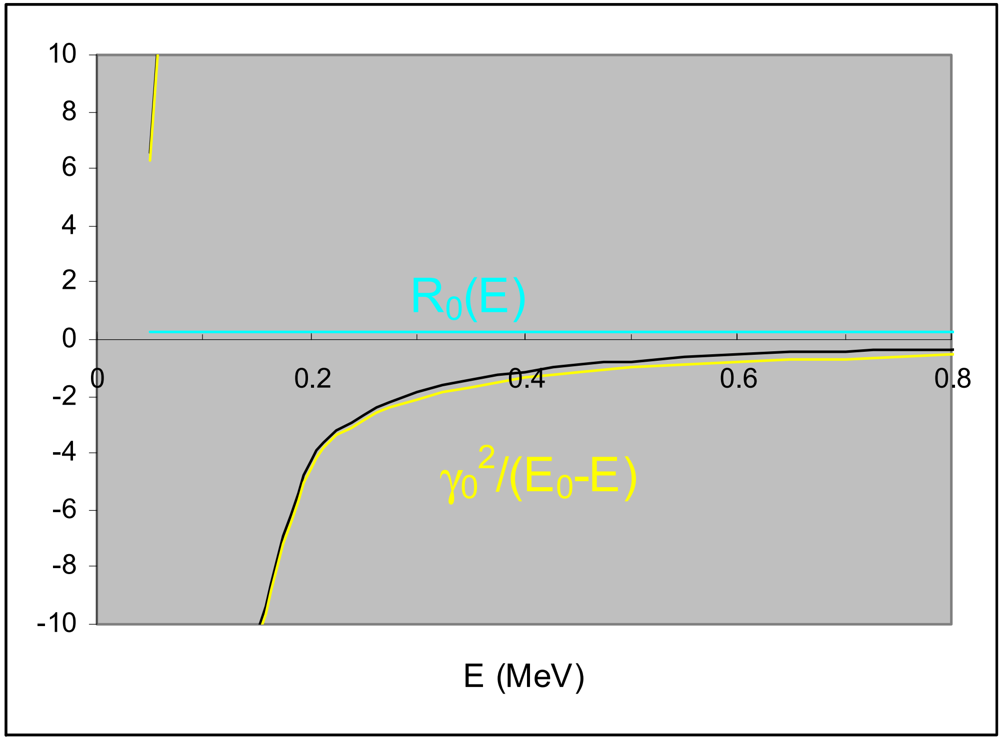
$$R_0(E) = \sum_{\lambda \neq 0} \frac{\gamma_\lambda^2}{E_\lambda - E}$$

$E << E_l \rightarrow R_0(E) \sim R_0$

➔ In phenomenological approaches (one resonance): $R(E) \approx \frac{\gamma_0^2}{E_0 - E} + R_0$

$^{12}C+p$

$$R(E) = \sum_\lambda \frac{\gamma_\lambda^2}{E_\lambda - E} = \frac{\gamma_0^2}{E_0 - E} + R_0(E)$$



Approximations: $R_0(E)=R_0$=constant (background)

$R_0(E)=0$: Breit-Wigner approximation: one term in the R matrix

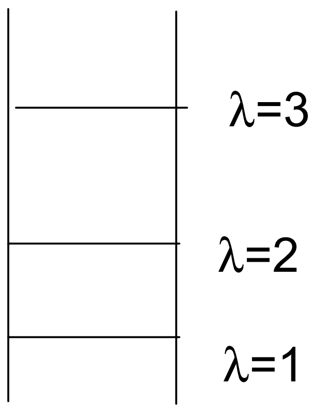
Remark: the R matrix method is NOT limited to resonances ($R=R_0$)

Summary

Solving the Schrödinger equation in a basis with *N* functions provides:

$$R(E) = \sum_{\lambda=1}^{N} \frac{\gamma_\lambda^2}{E_\lambda - E}$$

where E_λ, γ_λ are calculated from matrix elements between the basis functions



Each pole corresponds to a state (bound state or resonance)
Properties: energy, reduced width
!! *depend on a* !! (not physical)

Other approach: consider E_λ, γ_λ as free parameters (no basis)

→ Phenomenological R matrix

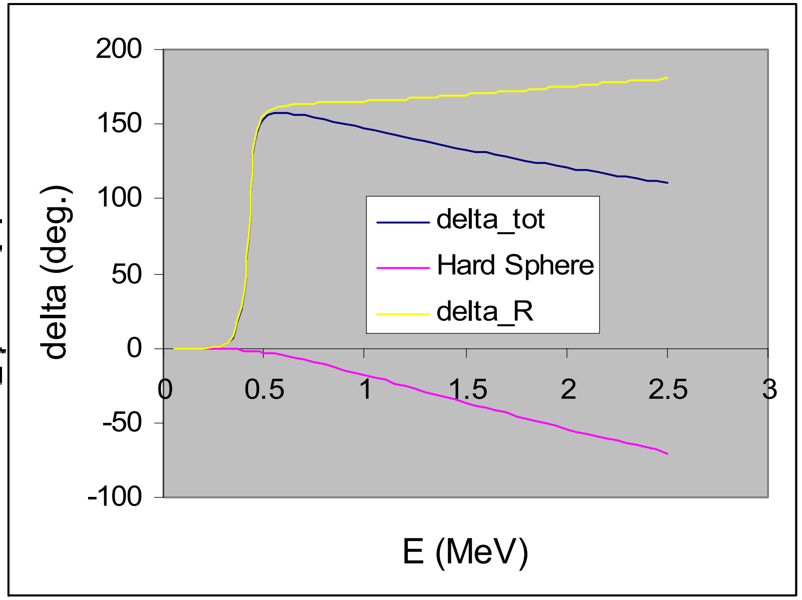
Question: how to relate R-matrix parameters with experimental information?

Resonance energies

!!! Different from pole energies!

$$U^\ell = \frac{I_\ell(ka)}{O_\ell(ka)}\,\frac{1 - L^\star R^\ell}{1 - L R^\ell},\ \text{wit}$$

$$= \exp(2i\delta^\ell) = \exp(2i(\delta$$



with

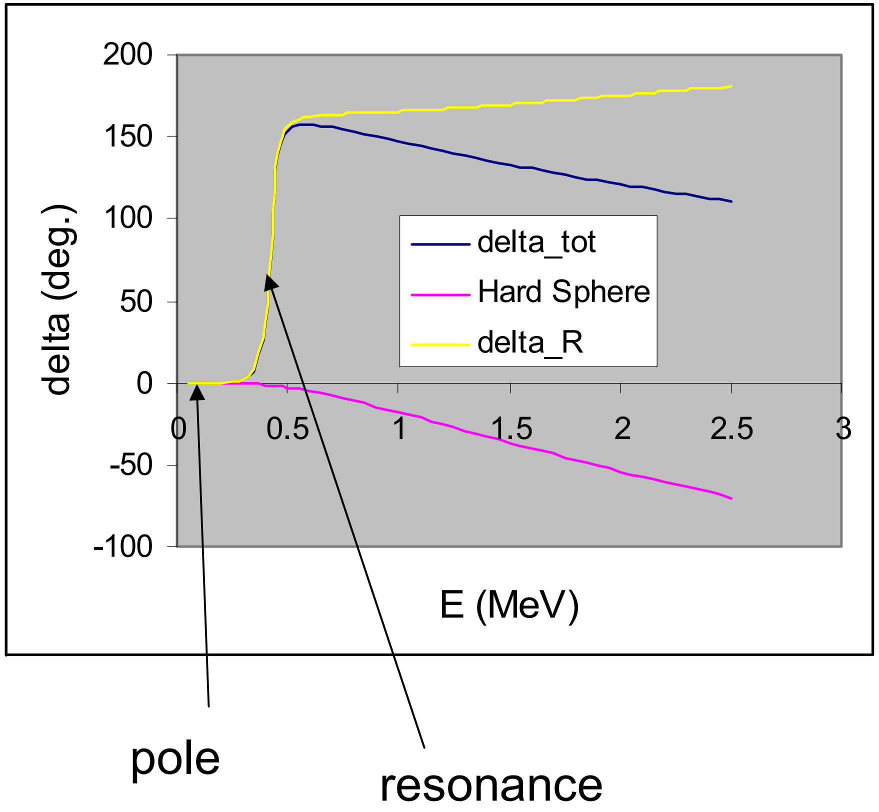
$$\exp(2i\delta^\ell_{HS}) = \frac{I_\ell(ka)}{O_\ell(ka)} \rightarrow \delta^\ell_{HS} = -\arctan\frac{F_\ell(ka)}{G_\ell(ka)}$$

$$\exp(2i\delta^\ell_{R}) = \frac{1 - L^\star R^\ell}{1 - L R^\ell} \rightarrow \delta^\ell_{R} = \arctan\frac{PR}{1 - SR}$$

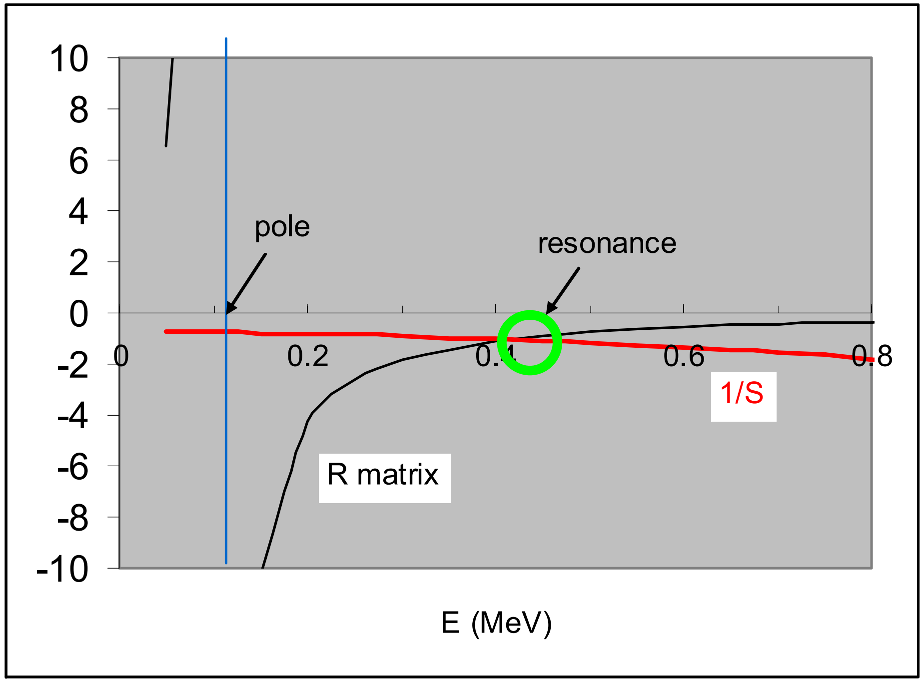
$$\delta_R^\ell(E) = \arctan \frac{P(E)R(E)}{1 - S(E)R(E)}$$

Resonance energy E_r defined by $1 - S(E_r)R(E_r) = 0$ →δ_R=90°

In general: must be solved numerically



Plot of R(E), 1/S(E)



The Breit-Wigner approximation

Single pole in the R matrix expansion $R(E) \approx \frac{\gamma_0^2}{E_0 - E}$

Phase shift: $\delta_R(E) = \arctan \frac{P(E)R(E)}{1 - S(E)R(E)} \approx \arctan \frac{\gamma_0^2 P(E)}{E_0 - E - \gamma_0^2 S(E)}$

Resonance energy E_r, defined by: $E_0 - E_r - \gamma_0^2 S(E_r) = 0$ Not solvable analytically

Thomas approximation: $S(E_r) \approx S(E_0) + S'(E_0)(E_r - E_0)$

Then: $E_r \approx E_0 - \frac{\gamma_0^2 S(E_0)}{1 + \gamma_0^2 S'(E_0)}$

Near the resonance energy E_r:

$$\tan \delta_R(E) \approx \frac{\gamma_0^2 P(E)}{E_0 - E - \gamma_0^2 S(E)} \approx \frac{\gamma_0^2 P(E)}{E_r - E + \gamma_0^2 (S(E_r) - S(E))}$$

$$\approx \frac{\gamma_0^2 P(E)}{E_r - E + \gamma_0^2 (E_r - E) S'(E_r)}$$

$$\tan \delta_R(E) \approx \frac{\gamma_0^2 P(E)}{E_r - E + \gamma_0^2 (E_r - E) S'(E_r)} \approx \frac{\Gamma(E)}{2(E_r - E)}$$

with $$\Gamma(E) = 2\frac{\gamma_0^2}{1 + \gamma_0^2 S'(E_r)} P(E) = 2\gamma_{obs}^2 P(E)$$

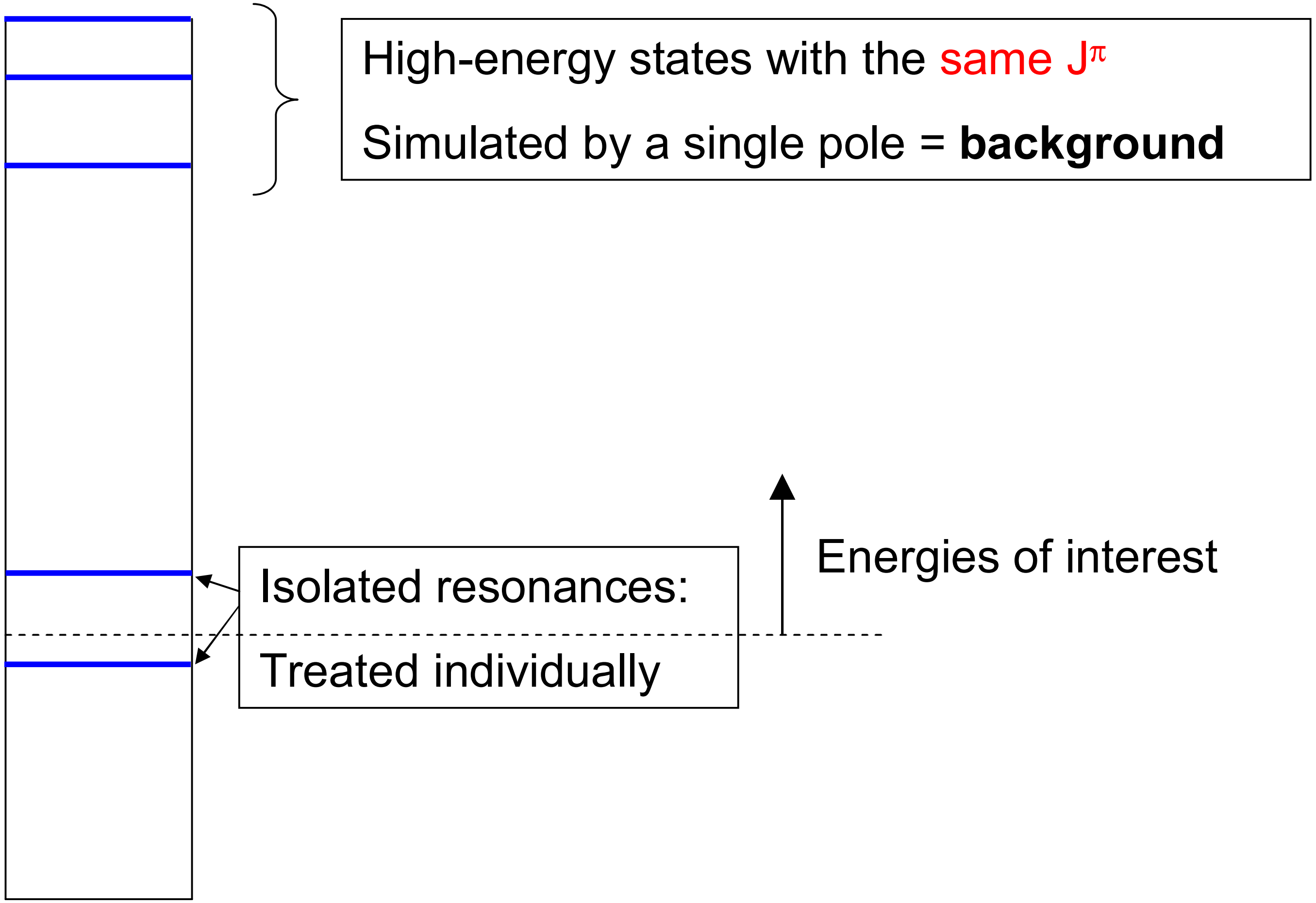
Remarks:

- γ_0, E_0 =" R-matrix", "calculated", "formal" parameters, needed in the R matrix
  - depend on the channel radius *a*
  - Defined for resonances *and* bound states *(Er<0)*
- γ_{obs}, E_r = "observed" parameters
  - model independent, to be compared with experiment
  - should not depend on *a*
- The total width Γ depends on energy through the penetration factor *P*

  → fast variation with *E*

  → low energy: narrow resonances (*but the reduced width can be large)*

Summary



Non-resonant calculations are possible: only a background pole

Link between "calculated" and "observed" parameters

One pole (N=1)

$$E_R = E_0 - \frac{S(E_0)\gamma_0^2}{1 + S'(E_0)\gamma_0^2}$$

$$\gamma_{obs}^2 = \frac{\gamma_0^2}{1 + S'(E_0)\gamma_0^2}$$

R-matrix parameters (calculated)

↕

Observed parameters (=data)

Several poles (N>1)

$$1 - S(E_r)R(E_r) = 0$$

Must be solved numerically

Generalization of the Breit-Wigner formalism:
link between observed and formal parameters when *N*>1

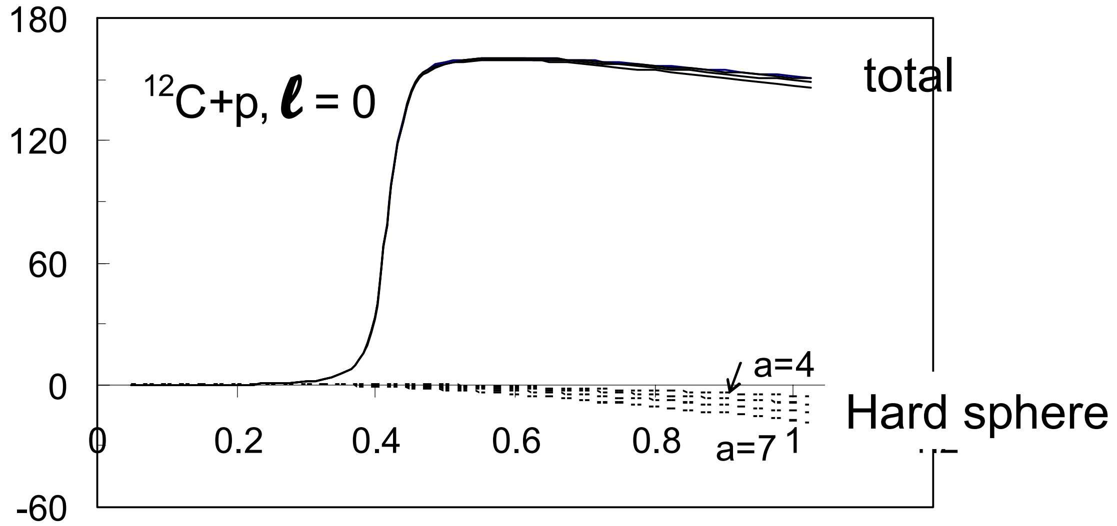
C. Angulo, P.D., Phys. Rev. C **61**, 064611 (2000) – single channel
C. Brune, Phys. Rev. C **66**, 044611 (2002) – multi channel

Examples: ^{12}C+p and ^{12}C+α

Narrow resonance: ^{12}C+p

^{12}C+p ($E^r = 0.42$ MeV, $\Gamma = 32$ keV, $J = 1/2^+, \ell = 0$)

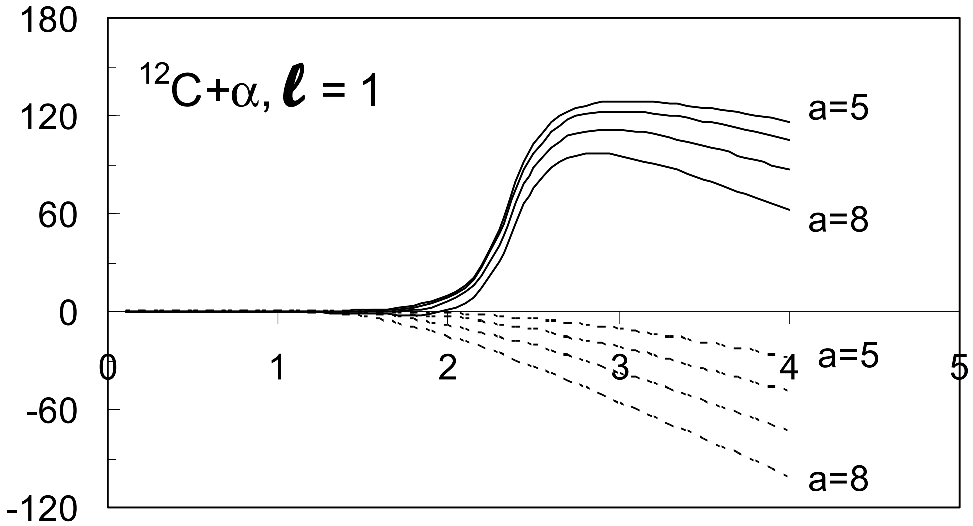
	$a = 4$ fm	$a = 5$ fm	$a = 6$ fm	$a = 7$ fm
γ^2_{obs} (MeV)	1.09	0.59	0.35	0.23
E_0 (MeV)	−2.15	−0.61	−0.11	0.11
γ^2_0 (MeV)	3.09	1.16	0.57	0.32



Broad resonance: $^{12}C+\alpha$

$^{12}C+\alpha$ ($E^r = 2.42$ MeV, $\Gamma = 0.42$ MeV, $J = 1^-, \ell = 1$)

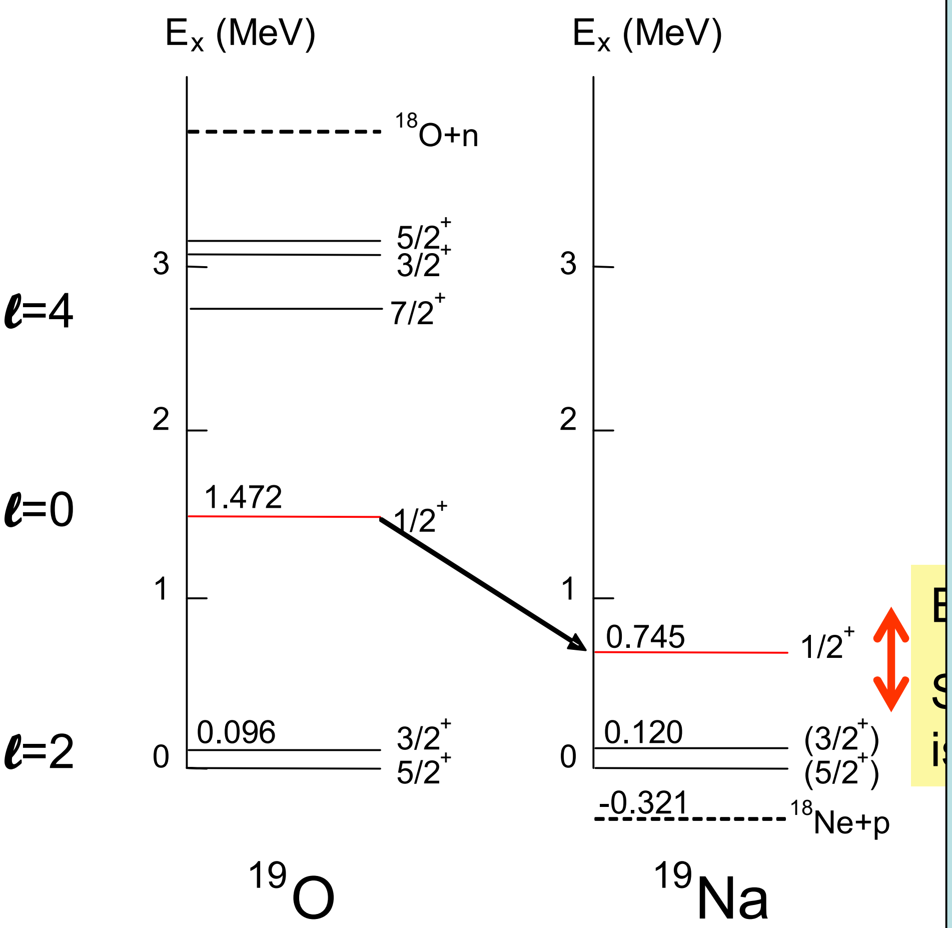
	$a = 5$ fm	$a = 6$ fm	$a = 7$ fm
γ^2_{obs} (MeV)	0.57	0.28	0.16
E_0 (MeV)	0.49	1.92	2.22
γ^2_0 (MeV)	1.17	0.37	0.19



^{18}Ne+p elastic scattering: *C. Angulo et al, Phys. Rev. C67 (2003) 014308*

- Experiment at Louvain-la-Neuve: ^{18}Ne+p elastic

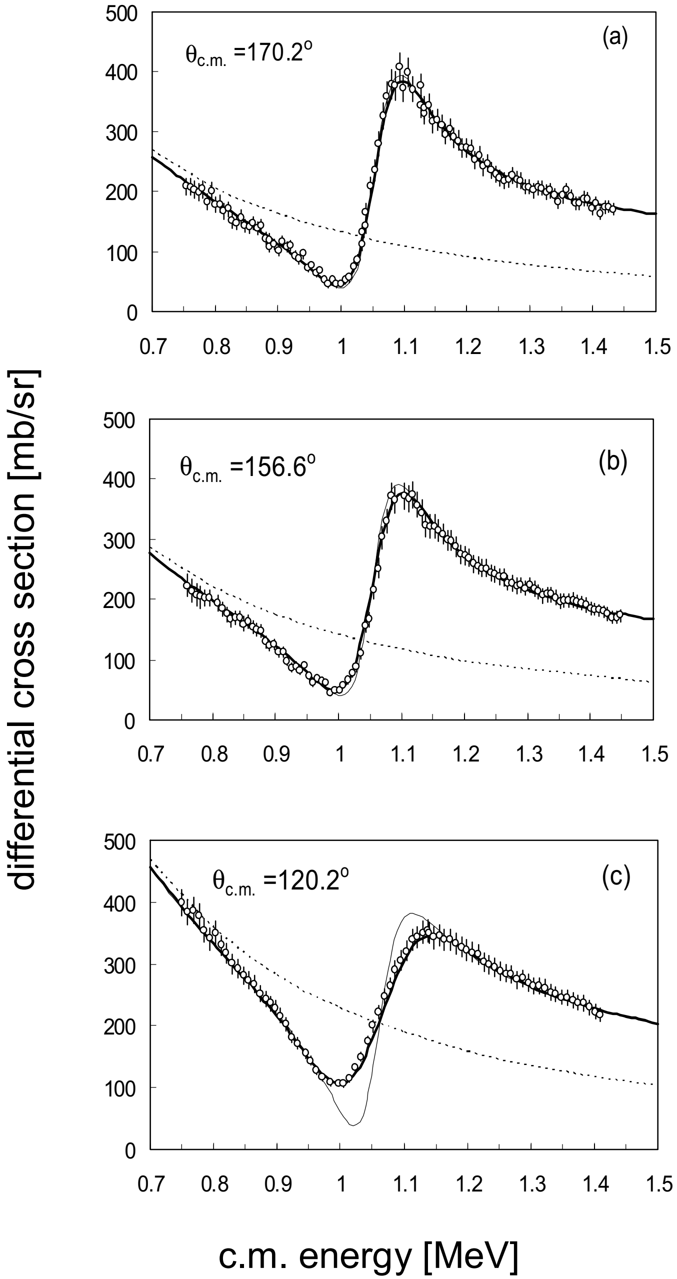
  → search for the mirror state of ^{19}O(1/2$^+$)



$$\frac{d\sigma}{d\Omega} = \frac{1}{4k^2} \left| \sum_l (2l+1)(\exp(2i\delta_l - 1) P_l(\cos\theta) \right|^2$$

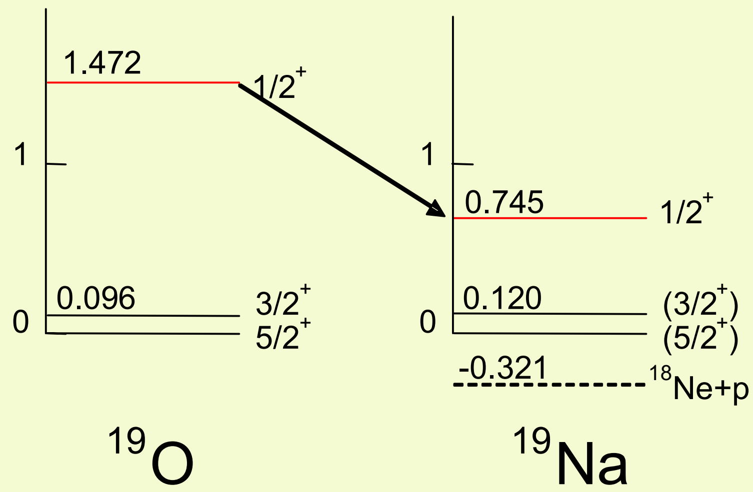
- Phase shifts d_l defined in the R-matrix (in principle from ℓ=0 to ∞)
- ℓ=0: one pole
  with 2 parameters: energy E_0
  reduced width γ_0
- ℓ=1, no resonance expected
  →hard-sphere phase shift
- ℓ=2 (J=3/2$^+$,5/2$^+$): very narrow resonances expected
  →weak influence
- ℓ>2: hard sphere

→ The cross section is fitted with 2 parameters



^{18}Ne+p elastic scattering

Final result
E_R = 1.066 ± 0.003 MeV
Γ_p = 101 ± 3 keV

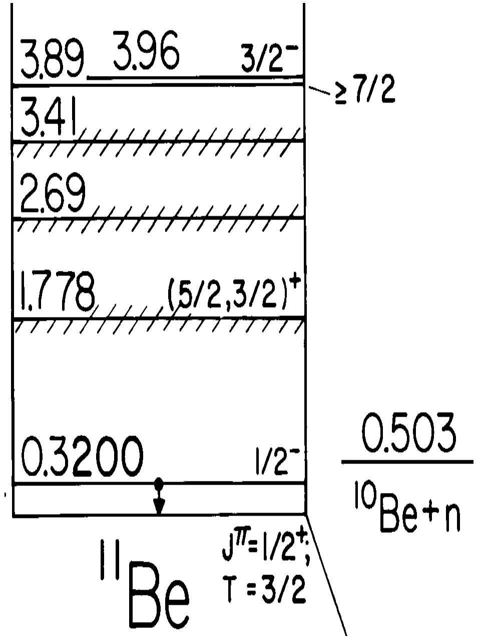


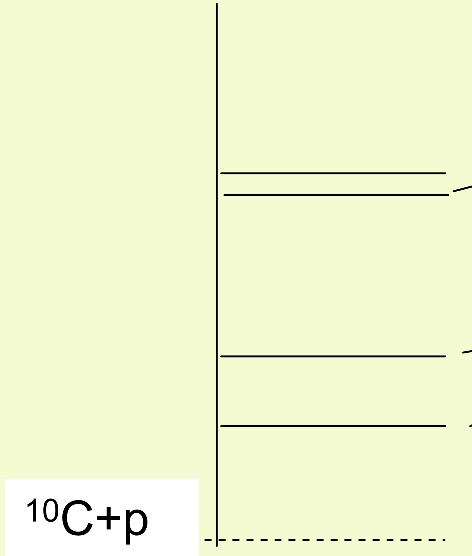
→ Very large Coulomb shift
From Γ=101 keV, γ^2=605 keV, θ^2=23%
Very large reduced width

Other case: $^{10}C+p \rightarrow ^{11}N$: analog of ^{11}Be

^{11}Be: neutron rich

^{11}N: proton rich

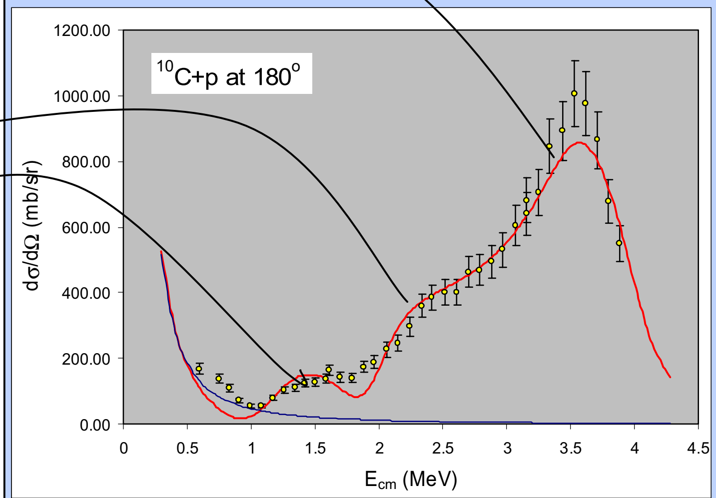




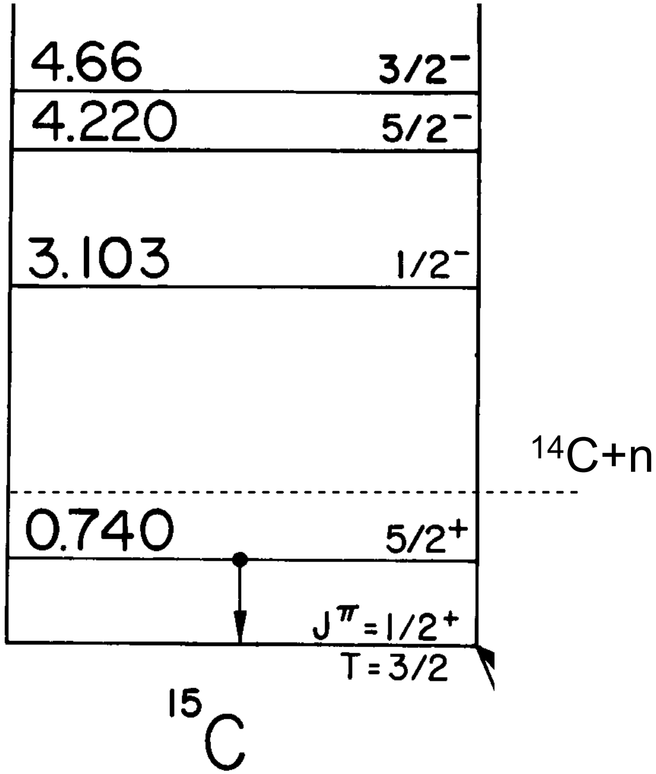
Expected ^{11}N (unbound)

Energies?
Widths?
Spins?

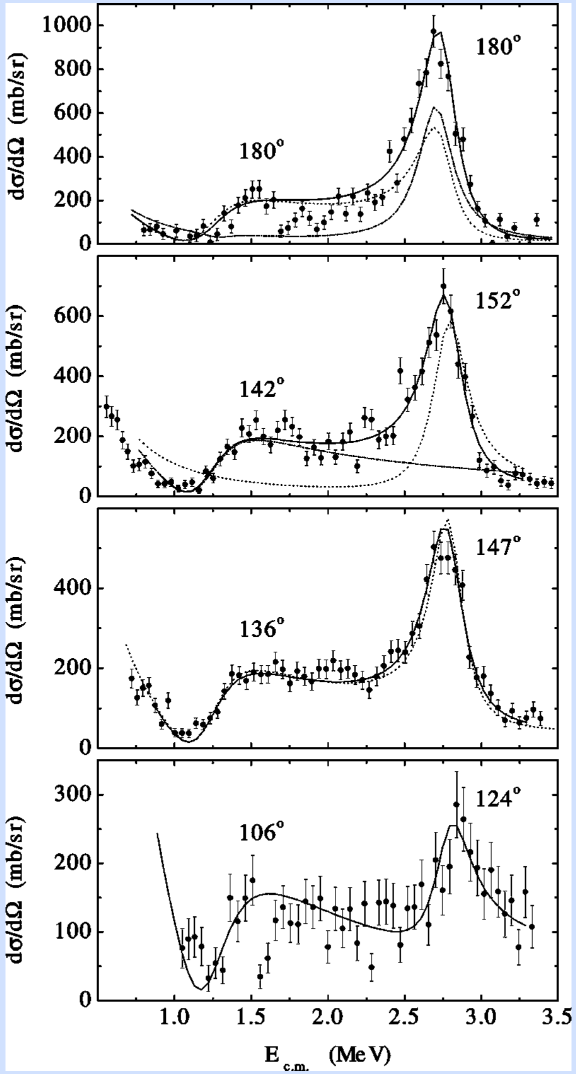
Markenroth et al (Ganil)
Phys.Rev. C62, 034308 (2000)



Other example: $^{14}O+p \rightarrow {}^{15}F$: analog of ^{15}C



^{14}O+p elastic: ground state of 15F unbound
Goldberg et al. Phys. Rev. C 69 (2004) 031302



Data analysis: general procedure for elastic scattering

$$\frac{d\sigma}{d\Omega} = \sum_{K_1,K_2,K_1',K_2'} |f_{K_1K_2,K_1'K_2'}(\theta)|^2$$

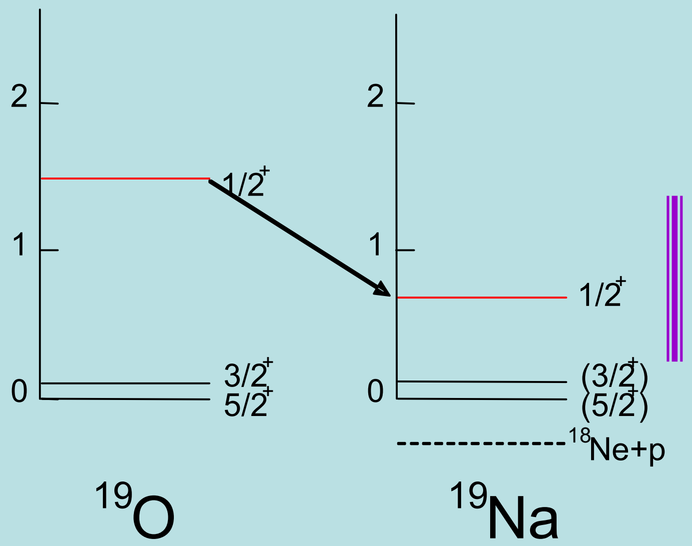
$$f_{K_1K_2,K_1'K_2'}(\theta) = \sum_{J,\pi} \sum_{lI,l'I'} \ldots U^{J,\pi}_{lI,l'I'} \; Y_{l'}(\theta,0)$$

- Only unknown quantity
- To be obtained from models

Problem: how to determine the collision matrix U?

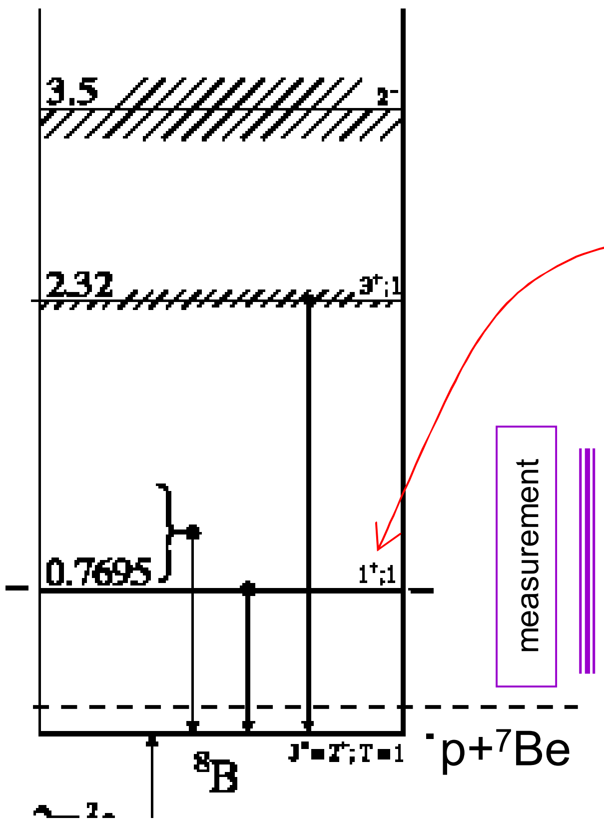
Consider each partial wave Jp

Simple case: ^{18}Ne+p: spins $I_1=0^+$ and $I_2=1/2^+$
collision matrix 1x1



J	ℓ	
1/2+	0	R matrix: one pole (2 parameters) *but could be 3 (background)*
1/2-	1	No resonance →Hard sphere
3/2+	2	Very narrow resonance → Hard sphere
3/2-	1	No resonance → Hard sphere
≥5/2	≥2	Neglected

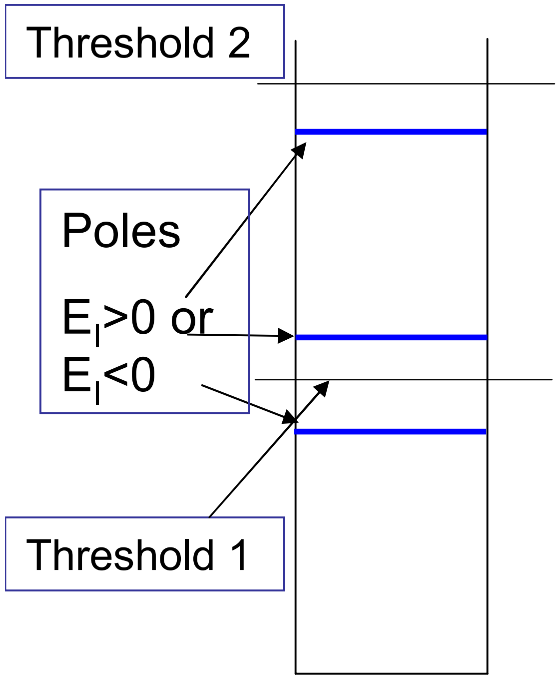
More complicated : ^7Be+p: spins $I_1=3/2^-$ and $I_2=1/2^+$
collision matrix: size larger that one (depends on J)



J	I, ℓ	
0^+	1,1	No resonance→hard sphere
0^-	2,2	No resonance → hard sphere
1^+	1,1 2,1 2,3	• Res. at 0.63 MeV → Rmatrix I=1 OR I=2 • Channel mixing neglected ($U_{ij}=0$ for $i\neq j$)
1^-	1,0 1,2 2,2	• Channel mixing neglected • ℓ=0: scattering length formalism • ℓ=2: hard sphere
2^+	1,1 1,3 2,1 2,3	• Channel mixing neglected • Hard sphere
2^-	1,2 2,0 2,2 2,4	• Channel mixing neglected • ℓ=0: scattering length formalism • ℓ=2,4: hard sphere

Finally, 4 parameters: E_R and Γ for the 1^+ resonance, 2 scattering lengths

Other processes: capture, transfer, inelastic scattering, etc.



Elastic scattering

$E_2, \gamma_2(1)$

$E_1, \gamma_1(1)$

$E_0, \gamma_0(1)$

$$R(E) = \sum_{\lambda=1}^{N} \frac{\gamma_\lambda^2(1)}{E_\lambda - E}$$

Inelastic scattering, transfer

$E_2, \gamma_2(1), \gamma_2(2)$

$E_1, \gamma_1(1), \gamma_1(2)$

$E_0, \gamma_0(1), \gamma_0(2)$

$$R_{ij}(E) = \sum_{\lambda=1}^{N} \frac{\gamma_\lambda(i)\gamma_\lambda(j)}{E_\lambda - E}$$

Pole properties: energy
reduced width in different channels (→ more parameters)
gamma width → capture reactions

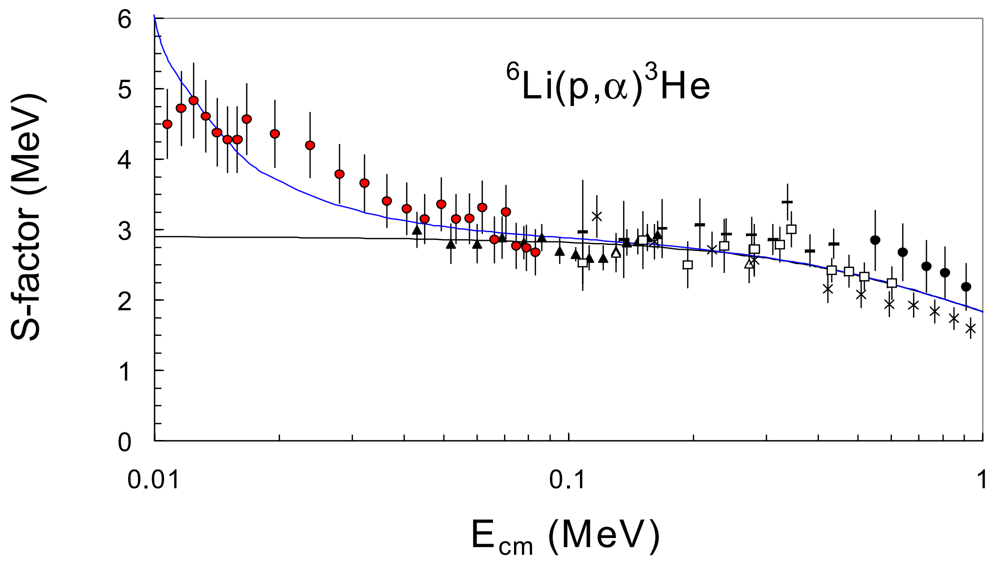
Example of transfer reaction: ^{6}Li(p,a)^{3}He (Nucl. Phys. A639 (1998) 733)

$$R = \begin{pmatrix} R_{pp} & R_{p\alpha} \\ R_{\alpha p} & R_{\alpha\alpha} \end{pmatrix}, \text{ with } R_{\alpha p} = R_{p\alpha}$$

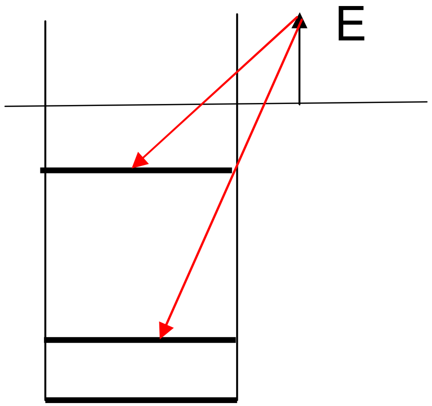
Non-resonant reaction: R matrix=constant

Collision matrix $U = \begin{pmatrix} U_{pp} & U_{p\alpha} \\ U_{\alpha p} & U_{\alpha\alpha} \end{pmatrix}$, deduced from the R matrix

Cross section: $\sigma \sim |U_{pa}|^2$



Radiative capture



Capture reaction=transition between an initial state at energy E to bound states

Cross section ~ $|\langle \Psi_f | H_\gamma | \Psi_i(E) \rangle|^2$

Additional pole parameter: gamma width

$\langle \Psi_f | H_\gamma | \Psi_\iota(E) \rangle = \langle \Psi_f | H_\gamma | \Psi_i(E) \rangle_{int} + \langle \Psi_f | H_\gamma | \Psi_i(E) \rangle_{ext}$

with $\langle \Psi_f | H_\gamma | \Psi_i(E) \rangle_{int}$ depends on the poles

$\langle \Psi_\phi | H_\gamma | \Psi_\iota(E) \rangle_{ext}$ =integral involving the external w.f.

More complicated than elastic scattering!

But: many applications in nuclear astrophysics

Microscopic models

- A-body treatment:

$$H=\sum_{i=1}^{A}T_i+\sum_{i<j=1}^{A}V_{ij}$$

with

T_i = kinetic energy of nucleon i

V_{ij} = nucleon - nucleon interaction

- wave function completely antisymmetric (bound and scattering states)
- not solvable when A>3
- Generator Coordinate Method (GCM)
  basis functions $\Phi_i^{\ell m}=\mathcal{A}\phi_1\phi_2\Gamma_\ell(\rho,R_i)Y_\ell^m(\Omega)$

  with ϕ_1, ϕ_2 = internal wave functions
  $\Gamma_\ell(\rho,R)$=gaussian function
  R=generator coordinate (variational parameter)
  ρ=nucleus-nucleus relative coordinate

- At $\rho=a$, antisymmetrization is negligible→ the same R-matrix method is applicable

6. Conclusions

1. One R-matrix for each partial wave (limited to low energies)
2. Consistent description of resonant and non-resonant contributions (not limited to resonances!)
3. The R-matrix method can be applied in two ways
   a) To solve the Schrödinger equation
   b) To fit experimental data (low energies, low level densities)

4. Applications a)
   - Useful to get phase shifts and wave functions of scattering states
   - Application in many fields of nuclear and atomic physics
   - 3-body systems
   - Stability with respect to the radius is an important test

5. Applications b)
   - Same idea, but the pole properties are used as free parameters
   - Many applications: elastic scattering, transfer, capture, beta decay, etc.