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# Joint ICTP-IAEA Workshop on Nuclear Reaction Data for Advanced Reactor Technologies

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**exercises**

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# EXCEL PROGRAM Potential.xls

## 1. Introduction

The program allows to determine 3 quantities:

a. The phase shift (in degrees)
b. The wave function at a given energy (in cell I10)
c. The potential (in MeV)

1.1 Content of the excel file:

- Sheet “Potential”: main part of the program. The Schrödinger equation is solved with the Numerov’s algorithm.
- Sheet “Data”: contains experimental phase shifts.

1.2 Input data:

- Masses and charges (cells B1 to B4)
- Angular momentum ℓ (cell B5)
- The discretization values N (number of points, typically N=400, cell E1) and size h (typically h=0.1, cell E2)
- The potential given by a Gaussian form $V(r) = V_0 \exp(-(r/r_0)^2)$
  $V_0$ (in MeV) is given in cell H1, and r0 (in fm) in cell H2.

1.3 Note: a similar program is available online at
http://pntpm3.ulb.ac.be/Nacre/Programs/gamdep.htm

## 2. System $\alpha$+$\alpha$

a. Compute the phase shifts for $\ell$ =0,2,4 with the potential of Buck et al. [Nucl. Phys. A. 275 (1977) 246] with $V_0$=-122.62 MeV, $r_0$=2.132 fm.
Compare with the experimental phase shifts, and with the $^8$Be spectrum (below).

b. Evaluate the barrier height $V_B$.

Take E=$V_B$/10, E=$V_B$, E=10*$V_B$, and plot the wave functions for $\ell$ =0,2,4.
What do you conclude?

c. Use the theoretical phase shift to estimate the $\ell$ =2 resonance energy and width by 2 methods:

1. The Breit-Wigner approximation: $\tan\delta(E) \approx \dfrac{\Gamma/2}{E_R - E}$

2. Check that the following definitions are equivalent, and apply them to the $2^+$ resonance

$$E_R = \max(\frac{d\delta}{dE}), \quad \Gamma = 2 / \left.\frac{d\delta}{dE}\right)_{E=E_R}$$

Hint: tabulate the phase shift with a small energy step (~0.1 MeV) and compute the derivative numerically.

3. **Systems α+p and α+n**

a. Compute the α+p and α+n phase shifts for $\ell=0$ and $\ell=1$ ($J=1/2^-$ and $J=3/2^-$), and compare with experiment. The potentials are:

$\ell=0$: $V_0=-66.58$ MeV, $r_0=1.61$ fm

$\ell=1, J=3/2-$: $V_0=-89$ MeV, $r_0=1.82$ fm

$\ell=1, J=1/2-$: $V_0=-68$ MeV, $r_0=1.82$ fm

b. Determine the α+n scattering length $a$ from the low-energy expansion

$$k\cot(\delta) \simeq -\frac{1}{a} + \ldots$$

c. Evaluate the α+p barrier height.
Take $E=V_B/10$, $E=V_B$, $E=10*V_B$, and plot the wave functions for α+p and α+n ($\ell=0$). What do you conclude?

d. For α+n, $J=3/2^-$, evaluate the energy $E_R$ and width $\Gamma$ of the resonance.
Plot the wave function at $E=E_R$, $E=E_R-\Gamma/2$, and $E=E_R+\Gamma/2$. Check that the internal part of the wave function is maximal at the resonance energy.

## 4. Experimental spectra of $^8$Be, $^5$He and $^5$Li