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Neutron cross sections and related topics.
Part 1

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Part I : Neutron cross sections and related topics

Exercise I.1

The velocity v_n of a neutron can be determined from the time-of-flight t_n over a given flight distance L by:

$$v_n = \frac{L}{t_n}$$

The relativistic relation between the kinetic energy E_n and the velocity v_n of the neutron is:

$$E_n = m_n c^2 \left(\frac{1}{\sqrt{1-(v_n/c)^2}} - 1 \right) = m_n c^2 (\gamma - 1) \qquad \text{with } \gamma = \frac{1}{\sqrt{1-(v_n/c)^2}}$$

where m_n is the rest mass of the neutron and c is the velocity of light. The first term of a series expansion gives the classical expression :

$$E_n = \frac{1}{2} m_n v_n^2 = \alpha^2 \frac{L^2}{t_n^2}$$

(a) Calculate the value of the constant α when the energy E_n is given in eV, the distance L in m and the time t_n in μs

(b) Calculate the kinetic energy exactly and in the classical approximation for neutrons with a time of flight of 2μs and 2000μs for a flight path length of 30 m and 200m.

t_n / μs	L / m	$E_{n,clas}$ / eV	$E_{n,rel}$ / eV	$E_{n,rel}$ / $E_{n,clas}$
2	30			
2000	30			
2	200			
2000	200			

Exercise I.2

At a measurement station with a nominal flight path length of 30m the following observations were made: the signal from the gamma-flash was observed at a time-of-flight $t_{\gamma,exp}$ = 298.4 ns and the time-of-flight of the 20.864 eV resonance of ^{238}U at a time-of-flight $t_{n,exp}$= 467129.2 ns. The observed time-of-flight t_{exp} is related to the real time-of-flight t by the relation : $t_{exp} = t_o + t$

(a) Determine the t_o value

(b) Determine a more precise value of the flight path length.

Parameter
Flight path length, L
Time offset, t_o

Exercise I.3

The vertical displacement Δs of a body subject to the gravitational force is given by:

$$\Delta s = \frac{1}{2} g_n t_n^2$$

with g_n the standard acceleration of free fall and t_n the time.

Calculate the vertical displacement of a 25 meV and 1 eV and 100 eV neutron for flight path lengths L = 30 m and 200 m.
(Assume that the start velocity has only a horizontal component and only the gravitational force has on impact on the kinematics)

Neutron energy /eV	L = 30 m		L = 200 m	
	t_n / μs	Δs / mm	t_n / μs	Δs / mm
0.025				
1				
100				

Exercise I.4

The Maxwell-Boltzmann distribution of the velocity v of a particle with mass m in equilibrium at a temperature T is:

$$P(v)dv = C\, v^2 \exp(-\frac{mv^2}{2kT})dv$$

where k is the Boltzmann constant and C is a normalization constant of the distribution :

$$C = \sqrt{\frac{2}{\pi}}(\frac{m}{kT})^{3/2}$$

(a) Show that the maximum of the distribution P(v)dv occurs at a velocity v_{max} corresponding to a kinetic energy $E_{max} = kT$

(b) What is the energy E_{max} if the temperature is 300 K?

(c) Thermal cross sections σ_{th} are often given at a standard energy, e.g. corresponding to a neutron velocity v = 2200 m/s. What is the corresponding neutron energy and temperature.

Exercise I.5

The Maxwell-Boltzmann distribution can be used to describe the neutron flux in a thermal reactor :

$$\varphi(v)dv = C^{'} v P(v)dv$$

where C' is a normalization constant.

In the thermal energy region most of the absorption cross sections are directly proportional to 1/v. For 1/v cross sections, the cross section $\sigma(E)$ at an energy E relates to a reference cross section σ_R at an energy E_R as:

$$\sigma(E) = \sigma_R \sqrt{\frac{E_R}{E}}$$

(a) Show that the energy distribution of the neutron flux becomes:

$$\varphi(E)dE = C'' \, E \exp(-\frac{E}{kT})dE$$

where k is the Boltzman constant and C'' is a normalization constant.

(b) Show that for a 1/v cross section the average cross section $<\sigma(T)>$ in a flux with a Maxwellian distribution, which is characterized by a temperature T, can be expressed as a function of the cross section σ_R at a given temperature T_R by:

$$< \sigma(T) >= \frac{\int \sigma(E)\varphi(E)dE}{\int \varphi(E)dE} = \frac{\sqrt{\pi}}{2}\sigma_R \sqrt{\frac{T_R}{T}}$$

Note that : $\int x^{\alpha-1}e^{-x}dx = \Gamma(\alpha)$ with $\Gamma(2) = 1$ and $\Gamma(3/2) = \sqrt{\pi}/2$

(c) How much does the average cross section $<\sigma(T)>$ deviates from the cross section $\sigma(kT)$ at temperature kT.

Exercise I.6

A disc-shaped sample of natural gold has a diameter of (40.00 ± 0.10) mm and a mass of (5.0000 ± 0.0002)g. The atomic mass of ^{197}Au is (m_{Au}=196.966543 ± 0.000004) u.

(a) Calculate the mass per unit area or the thickness of the sample in g/cm^2

(b) Calculate the number of atoms per unit area in atoms per barn (at/b)

Isotope	m_X / u	isotopic abundance, a_i	thickness / (g/cm^2)	thickness / (at/b)	m / g
^{197}Au	196.966543	1			

Exercise I.7

A sample is made out of a pure element $_{Z}X$ containing the isotopes $^{A_i}_{Z}X$ $i = 1,...,n$. The isotopic abundance x_i of the isotope $^{A_i}_{Z}X$ in the element $_{Z}X$ is defined as the fraction of the number of nuclei $^{A_i}_{Z}X$ in element $_{Z}X$. The mass fraction w_i corresponds to the ratio of the mass ($m(X_i)$) of isotope $^{A_i}_{Z}X$ to the total mass $m(X)$ of element X.

(a) Determine the atomic mass (m_X) of element $_{Z}X$ in case the isotopic abundance x_i and atomic mass m_{Xi} of the isotopes are given

(b) Determine the relation between the isotopic abundance x_i and the weight fraction w_i of the isotopes $^{A_i}_{Z}X$ of element $_{Z}X$.

Exercise I.8

A disc-shaped sample of natural silver (^{nat}Ag) has a diameter of 80 mm and a mass of 50g. Natural silver consists of ^{107}Ag and ^{109}Ag. The natural isotopic abundance and atomic mass of ^{107}Ag and ^{109}Ag are given in the table.

Isotope	atomic mass / u	isotopic abundance	thickness / (at/b)	weight fraction	mass /g
^{107}Ag	106.905093	0.51839			
^{109}Ag	108.904756	0.48161			
Ag	107.868151	1		1	50

(a) Calculate the atomic mass of ^{nat}Ag
(b) Calculate the number of ^{nat}Ag atoms per unit area (in at/b)
(c) Calculate the number of atoms for each isotope per unit area (in at/b)
(d) Calculate the mass fraction and mass of each isotope

Exercise I.9

A disc-shaped sample of ZrO_2 has a diameter of 2.54 cm and a mass of 6.595 g. The isotopic abundance of Zr and O are given in the table together with the atomic mass of the Zr- and O-isotopes present in the sample..

Element	Isotope	atomic mass / u	isotopic abundance	fraction	thickness / (at/b)	weight fraction	mass (g)
Zr	^{90}Zr	89.90470	0.0229				
	^{91}Zr	90.90564	0.1861				
	^{92}Zr	91.90504	0.1895				
	^{93}Zr	92.9	0.1998				
	^{94}Zr	93.90632	0.2050				
	^{96}Zr	95.90828	0.1967				
O	^{16}O	15.99491463	0.99762				
	^{17}O	16.9991312	0.00038				
	^{18}O	17.9991603	0.00200				
ZrO_2				1		1	6.595

(a) Calculate the atomic mass of Zr and O for the isotopic abundance given in the table.
(b) Calculate the molar mass of ZrO_2
(c) Calculate the number of ZrO_2 molecules in the target
(d) Calculate the number of Zr- and O-isotopes per unit area (in at/b)
(e) Calculate the weight fractions and mass of the Zr- and O-isotopes

Exercise I.10

For a neutron induced reaction on a target nucleus with spin I and parity π_o, the angular momentum J of a resonance is given by the vector sum of the angular momenta:

$$\vec{J} = \vec{I} + \vec{i} + \vec{\ell}$$

with i the spin of the neutron and ℓ the angular momentum of the incident neutron. A neutron with angular momentum quantum number ℓ = 0, 1 and 2 is denoted as a s-, p-and d-wave neutron, respectively.

Defining with s the channel spin, the momenta satisfy the relations:

$$\vec{J} = \vec{\ell} + \vec{s} \qquad |\ell - s| \leq J \leq |\ell + s|$$

$$\vec{s} = \vec{I} + \vec{i} \qquad |I - i| \leq J \leq |I + i|$$

Since the neutron has a positive parity, the resonance parity π is defined by:

$$\pi = \pi_o \ (-1)^{\ell}$$

(a) Calculate the possible spin and parity combinations J^{π} of resonances induced by a s-, p-and d-wave neutron on a target nucleus with spin and parity $I^{\pi o} = 0^+$, $1/2^+$ and 1^+

(b) Determine also the spin factor g_J, which is defined by:

$$g_J = \frac{2J+1}{2(2I+1)}$$

Target nucleus I^{π}	Incident neutron i	ℓ	Channel spin s	Resonance J^{π}	g_J	$\Sigma\, g_J$
0^+	1/2	0				
		1				
		2				
$1/2^+$	1/2	0				
		1				
		2				
1^+	1/2	0				
		1				
		2				

Exercise I.11

Neutron induced cross sections in the resonance region are determined by resonance parameters corresponding to the properties of excited nuclear levels. The cross section for a reaction (n,r) of an isolated resonance with spin J for a non-fissile nucleus can in first approximation be described by the Single Level Breit-Wigner (SLBW) form:

$$\sigma_r(E_n) = \frac{\pi}{k^2} g_J \frac{\Gamma_n \Gamma_r}{(E_n - E_R)^2 + \frac{(\Gamma_n + \Gamma_r)^2}{4}}$$

where E_R is the resonance energy, Γ_n is the neutron width, Γ_r is the reaction width, g_J is the statistical factor and k is the angular wave number of the neutron.

The reaction cross section at thermal ($E_n = E_{th} = 0.025$ eV) is composed out of a contribution from unbound and bound ("negative resonances") states. Based on the SLBW expression (assuming $|E_R| > E_n$ and $\Gamma_n + \Gamma_r << E_R$) the cross section (in units of a barn) at thermal is a sum over all contributions:

$$\sigma_r(E_{th}) \approx 4.099\text{x}10^6 \left(\frac{A+1}{A}\right)^2 \sum_{i=1}^{N} \frac{g_{J,i}\, \Gamma_{n,i}\, \Gamma_{r,i}}{|E_{R,i}|^{5/2}}$$

with A nucleus-neutron mass ratio In this expression the resonance energy, reduced neutron and radiation width are expressed in eV.

(a) Calculate the contribution of the positive s-wave resonances for $^{197}Au(n,\gamma)$ with $I^\pi = 3/2^+$, which are given in the table.

(b) Adjust the neutron width of a negative resonance (or bound state) to match the capture cross section at thermal ($E_{th} = 0.025$ eV), which is $\sigma(E_{th},\gamma) = 98.66$ b. (Assume that the direct capture component can be neglected)
- for a negative resonance at -60 eV with spin J= 2 and radiation width $\Gamma_\gamma = 0.125$ eV
- for a negative resonance at -120 eV with spin J=2 and radiation width $\Gamma_\gamma = 0.125$ eV

E_R / eV	J	Γ_n / eV	Γ_γ / eV	Contribution to the thermal cross section $\sigma_\gamma(E_{th})$ / b	Relative
4.890	2	0.01520	0.124		
57.921	1	0.00435	0.112		
60.099	2	0.06640	0.110		
78.271	1	0.01667	0.120		
107.000	2	0.00760	0.110		

Exercise I.12

The total cross section for an s-wave in the SLBW-formalism is given by:

$$\sigma_{tot}(E_n) = g_J \frac{\pi}{k_n^2} \frac{\Gamma_n \Gamma}{(E_n - E_R)^2 + (\Gamma/2)^2} + g_J \frac{4\pi}{k_n} \frac{\Gamma_n (E_n - E_R) R}{(E_n - E_R)^2 + (\Gamma/2)^2} + g_J 4\pi R^2$$

The last term in this equation is the contribution due to the potential scattering (σ_{pot}).

In the SLBW-formalism the peak cross section σ_o, which reflects the maximum of the resonance part of the total cross section, is:

$$\sigma_o = \frac{4\pi}{k_n^2} \frac{g_J \Gamma_n}{\Gamma} \approx \frac{2.608\text{x}10^6}{E_R} \left(\frac{A+1}{A}\right)^2 \frac{g_J \Gamma_n}{\Gamma}$$

where in the last expression the peak cross section is given in barn and the resonance energy in eV. This peak cross section can be used to estimate the maximum total ($\sigma_{max,tot}$), capture ($\sigma_{max,\gamma}$) and elastic ($\sigma_{max,n}$) cross section becomes:

$$\sigma_{max,tot} = \sigma_o + \sigma_{pot}$$

$$\sigma_{max,\gamma} = \sigma_o \frac{\Gamma_\gamma}{\Gamma}$$

$$\sigma_{max,n} = \sigma_o \frac{\Gamma_n}{\Gamma}$$

(a) Calculate the peak cross sections σ_o, $\sigma_{o\gamma}$ and σ_{on} for the resonances of ^{238}U given in the table.
(b) Compare the peak cross section σ_o with the maximum of the cross sections given in the figure.
(c) Can the parity of the resonances at 66.02 eV and 80.73 eV be determined from the shape of the total cross sections?
(d) What about the resonances at 83.68 eV and 89.24 eV?
(e) How much does the potential scattering contribute to the total cross section for the resonance at 66.02 eV ? (the effective scattering radius for ^{238}U: R= 9.6 fm).

^{238}U: $I^\pi = 0^+$ R = 9.6 fm

E_R (eV)	J	g_J	Γ_n meV	Γ_γ meV	σ_o barn	$\sigma_{max,\gamma}$ barn	$\sigma_{max,n}$ barn
66.02	1/2	1	24.6	24.0			
80.73	1/2	1	1.8	25.0			
83.68	1/2	1	0.01	25.0			
89.24	1/2	1	0.09	25.0			

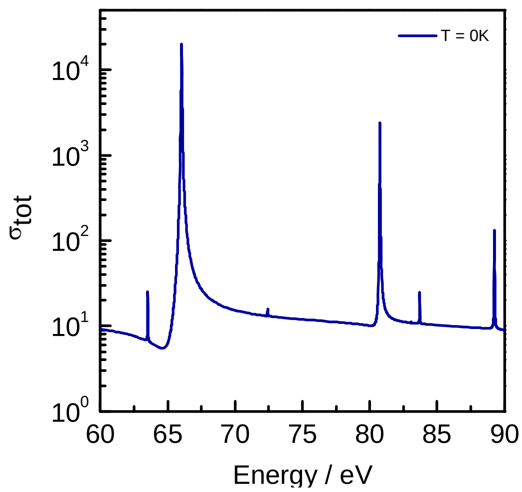


Figure : The neutron induced total cross section for 238U in the energy region between 60 eV and 90 eV.

Exercise I.13

To calculate reaction probabilities the thermal motion of the target nucleus has to be taken into account. Therefore, for practical applications resonance cross sections are mostly needed in Doppler broadened form. In the most simple approximation, the classical ideal gas model, it is assumed that the target nuclei have the same velocity distribution as an ideal gas at an effective temperature T_{eff}. The thermal motion of the target nuclei gives rise to a broadening Δ_D:

$$\Delta_D = \sqrt{\frac{4kT_{eff}E_R}{A}}$$

with A the nucleus-neutron mass ratio en k the Boltzman constant.

(a) Calculate the Doppler broadening for the resonances in the table

(b) Compare the Doppler broadening with the total natural line width of the resonance

^{238}U: $I^{\pi} = 0^{+}$ R = 9.6 fm

E_R (eV)	J	g_J	Γ_n meV	Γ_γ meV	$\Gamma_\gamma + \Gamma_n$ meV	Δ_D meV
66.02	1/2	1	24.6	24.0		
80.73	1/2	1	1.8	25.0		
83.68	1/2	1	0.01	25.0		
89.24	1/2	1	0.09	25.0		

Exercise I.14

The self-shielding factor for a parallel neutron beam on a target with target thickness n (in at/b) is defined by:

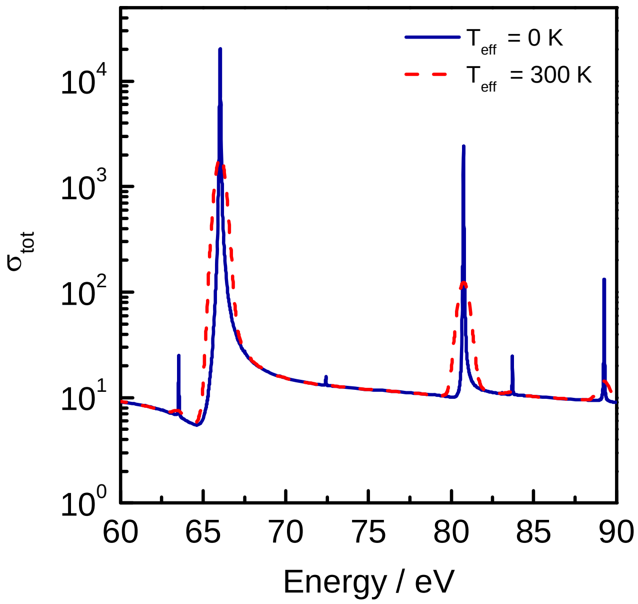
$$f = (1 - e^{-n\sigma_{tot}})$$

For practical application the calculation of the self-shielding factor requires the Doppler broadened cross sections. In figure 14.1 the total nuclear cross for ^{238}U+n is compared with the Doppler broadened cross section. In the figure the peak cross sections are indicated.

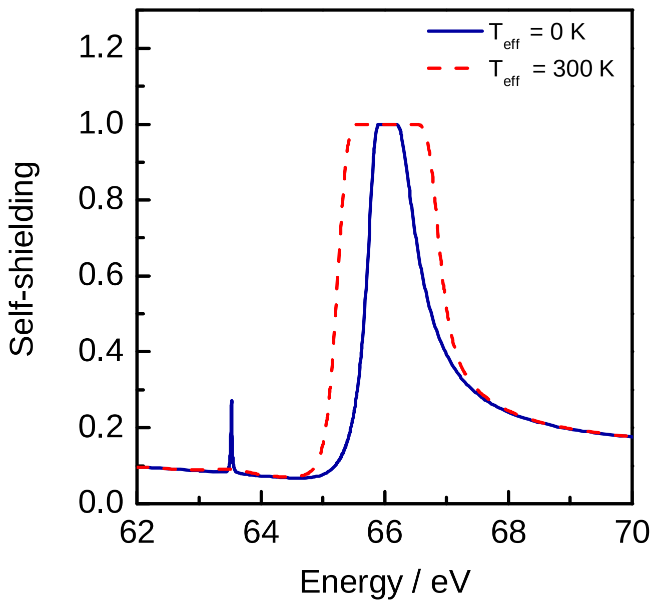
(a) Calculate the self-shielding factor for a parallel neutron beam on a 0.5 cm thick UO_2 sample for the resonances at 66.02, 80.73 and 89.24 eV in ^{238}U. Perform the calculations for the nuclear and Doppler broadened total cross sections. The sample is made of natural uranium and has a density of 10 g/cm^3.

(b) Discuss the impact of an increase in temperature on the self-shielding factor around the resonance at 66 eV (see figure)

E_R (eV)	J	σ_o (0 K) barn	σ_o (300 K) barn	f (0 K)	f (300 K)
66.02	1/2	20245	1900		
80.73	1/2	2420	125		
89.24	1/2	135	14		



The nuclear total cross section (T = 0K) compared with the Doppler broadened cross section for T = 300 K.

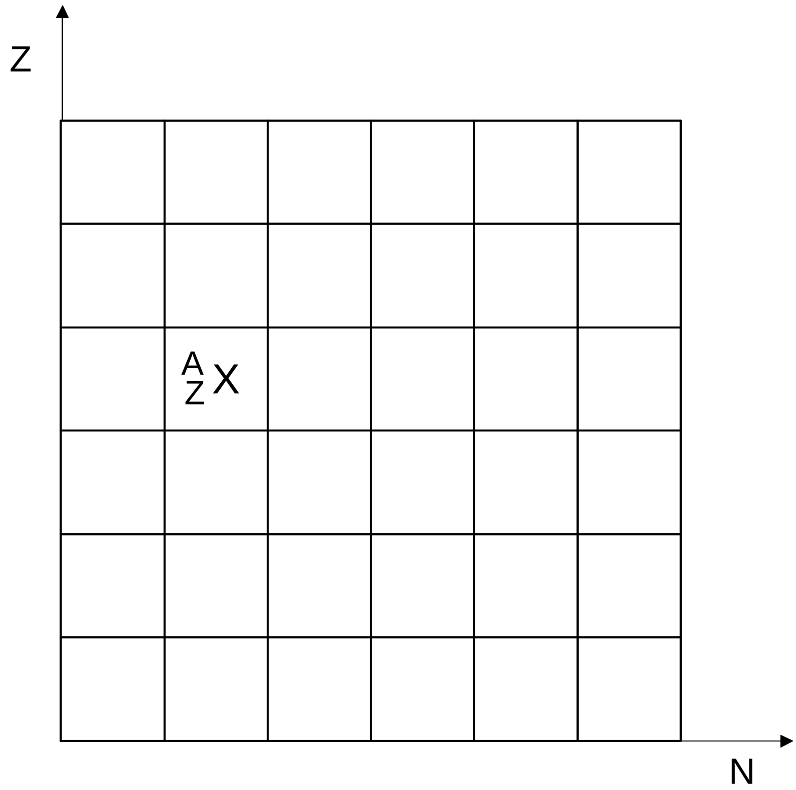


The self-shielding factor for a parallel neutron beam on a 0.5 cm thick UO_2 sample around the 66.02 eV resonance for T= 0 and 300 K.

Exercise I.15

Consider a neutron beam with a neutron flux 10^{14} cm^{-2} s^{-1} which hits a target consisting of nuclei ${}^{A}_{Z}X$. The nucleus ${}^{A}_{Z}X$ undergoes a (n,γ) reaction; the nucleus formed in this way immediately decays via β^- decay. The nucleus obtained after this decay undergoes a (n,γ) reaction, leading to an unstable nucleus. This nucleus in turn decays via electron capture (EC) with $T_{1/2}$ = 8 year but it also undergoes a (n,γ) reaction with an averaged cross section <σ(n,γ)> = 10 mb. Follow here the most probable path. The next nucleus again undergoes a (n,γ) reaction, leading to a short-living nucleus immediately decaying via α-emission. The daughter nucleus undergoes a (n,γ) reaction. On the nucleus formed in this way two neutron induced reactions are possible:
a (n, γ) reaction with an average cross section <σ(n,γ)> = 2.0 b and a (n,p) reaction with an average cross section <σ(n,γ)> = 1.8 mb. Follow the most dominant process.

(a) Draw the most probable path followed in the (N,Z)-diagram
(b) What is the final nucleus on this path?



(n,γ)	${}^{A}_{Z}X + n \rightarrow {}^{A+1}_{Z}X + \gamma$
(n,p)	${}^{A}_{Z}X + n \rightarrow {}^{A}_{Z-1}X + p$
β^-	${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}X + e^-$
β^+	${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}X + e^+$
EC	${}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}X$
α	${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X + \alpha$

Reaction rate (N = number of nuclei per volume) : $N\varphi\sigma$
Decay probability (decay constant $\lambda = \ln 2 / T_{1/2}$) : λN

Mills et al., "Quantities, Units and Symbols in Physical Chemistry"

Useful constants (SI Units)

Quantity	Symbol	Value
Speed of light in vacuum	c_o	299 792 458 m s^{-1} (defined)
Planck constant	h	6.626 075 5 (40) x 10^{-34} Js
Elementary charge	e	1.602 177 33 (45) x 10^{-19} C
Electron rest mass	m_e	9.109 389 7 (54) x 10^{-31} kg
Proton rest mass	m_p	1.672 623 1 (10) x 10^{-27} kg
Neutron rest mass	m_n	1.674 928 6 (10) x 10^{-27} kg
Atomic mass constant (unified atomic mass unit)	m_u = 1u	1.660 540 2 (10) x 10^{-27} kg
Avogadro constant	N_A	6.022 136 7 (36) x 10^{23} mol^{-1}
Boltzmann constant	k	1.380 658 (12) x 10^{-23} J K^{-1}
Standard acceleration of free fall	g_n	9.806 65 m s^{-2} (defined)

SI Prefixes

Submultiple	Prefix	Symbol
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-15}	femto	f

Multiple	Prefix	Symbol
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G

Conversion tables for units

Name	Symbol	Relation to SI
ångström	Å	= 10^{-10} m
barn	b	= 10^{-28} m^2
gram	g	= 10^{-3} kg
year	a	≈ 31 556 952 s
electronvolt	eV	= e x V ≈ 1.602 18 x 10^{-19} J
watt	W	= kg m^2 s^{-1}