Semiclassical Study of Continuum Effects in Transfer Reactions

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Introduction
Transfer reactions

In the study of transfer nuclear reactions, calculations are frequently made restricting the problem by taking two channels: Elastic and transfer channels.

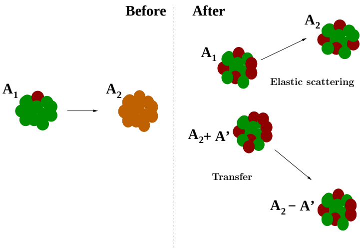

Figure: Both of the channels that are frequently taken into account in transfer reactions.

- But we can consider others bound states!
- It is necessary to include continuum states?

[2] R. A. Broglia and A. Winter *Heavy Ion Reactions*, Volume I. Addison Wesley Publishing company, 1999.

- What happens with nuclei next to the *Drip lines*?

Introduction

Transfer reactions

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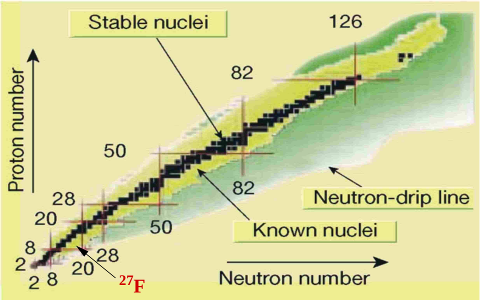

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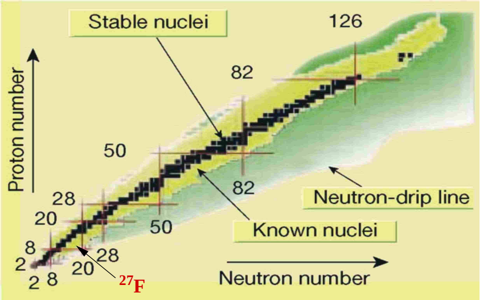

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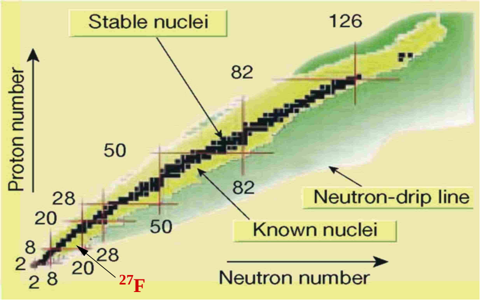


- The breakup energy can be of a few hundred of keV.
- The breakup channel might influence other channels. Therefore it is necessary to include continuum states.

Introduction

Transfer reactions

- A method that include continuum states is the CDCC (*Continuum Discretized Coupled Channel*).
- It is a non perturbative and complete quantum description. Therefore it might lead to difficult calculations!
- It is convenient to develop a simpler semiclassical approximation.

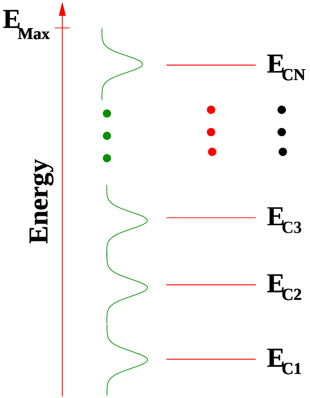


Figure: Illustration showing how the Continuum could be discretized in the CDCC method.

The Marta *et al.* model

In [3] the authors develop a semiclasssical model to study the continuum effects in a transfer reaction of the kind (Z,A+1)+(A,Z)→ (Z,A)+(Z,A+1).

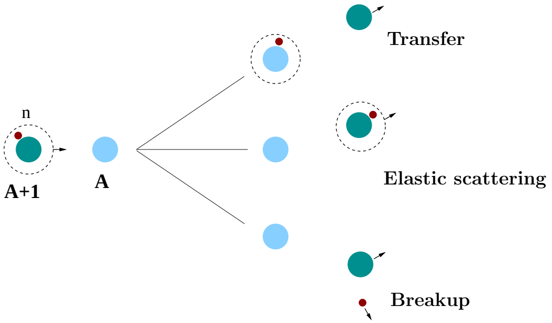
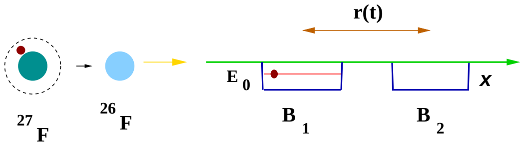


Figure: Processes that are taken into account by Marta et al.: Transfer reaction of a neutron, elastic scattering and breakup.

[3] H. D. Marta *et al*, Phys ReV. C., 73, (2006).

The Marta *et al.* model

In the description of the head on colission ^{27}F+^{26}F they considered a one-dimensional model



The dynamics of the neutron is described by quantum mechanics

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[T + V_1(x, r(t)) + V_2(x, r(t))\right]\Psi(x,t).$$

Where $T \rightarrow$ Kinetic Energy, $V_1 + V_2 \rightarrow$ Two square well potential.

The relative movement of the cores, $r(t)$, is treated classically.

$$r = r_{ca} + \frac{1}{2}a_{ca}t^2, \quad a_{ca} = \frac{2E_{c.m.}(r_{ca} - a)}{\mu r_{ca}^2}, \quad a = \frac{r_{ca}}{2}, \quad r_{ca} = \frac{z_1 z_2 e^2}{E_{c.m.}}.$$

The Marta *et al.* model

Calculation in coordinate space (with continuum effects)

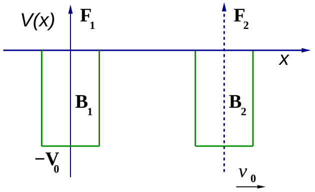
The Schrödinger equation is solved numerically (Cranck Nicholson Alg. [4])

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = H\Psi(x,t),$$

Probabilities

$$P_{el} = \left|\langle\Phi(x)|\Psi(x,t=\infty)\rangle\right|^2, \quad P_{tr} = \left|\langle\Psi_2(x,t=\infty)|\Psi(x,t=\infty)\rangle\right|^2,$$

$$\Psi_2(x,t) = \Phi(x-r(t))\ \exp\left\{\frac{i}{\hbar}\left[mv_0x - \left(\epsilon + \frac{1}{2}mv_0^2\right)t\right]\right\},\ v_0 = \sqrt{\frac{2E_{c.m.}}{\mu}}.$$



$\Phi(x) \rightarrow$ Bound state function of the neutron in the core B_1.

$\Phi(x-r(t)) \rightarrow$ Bound state function of the neutron in the core B_2.

$\Psi_2(x,t) \rightarrow$ Bound state function of the neutron in the core B_2 described in F_1.

[4] A. Askar and S. Cakmak. *J. Chem. Phys.*, 68, 2794, (1978).

Suggested model

- The solution of the Schrödinger equation in coordinate space has the advantage to include all continuum effects. However, this method does not allow a detailed study of those effects.

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- Which are the continuum states that affect the cross section the most?
- Our model is suggested with this purpose!
- We generalize the two level Marta et al. model by including the continuum in the simplest form: Taking a single state by symmetry.
- In order to keep the matrix elements and the inner products of the calculations finite, the continuum states are taken as wave packets with a specific symmetry.

Suggested model

Differences between Suggested model and the Marta et al model.

Potential

In order to simplify the calculations and to make them analytical we took the two square wells in the limit $V_0 \rightarrow \infty$ and $d \rightarrow 0$. Here V_0 is the depth and d the width of any potential well.

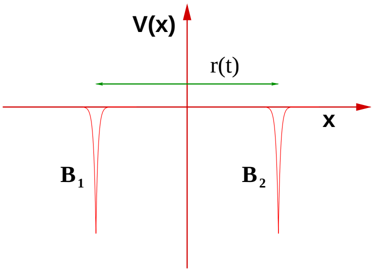

Figure: Effective potential under goes the neutron in our transfer reaction model in the head on collision $^{27}F + ^{26}F \rightarrow ^{27}F + ^{26}F$.

This limit is taken in such a way that it is possible to keep the same separation energy the neutron had in the square well.

Suggested model

Calculation without continuum states

In the Marta *et al.* two levels model

$$\Psi(x,t) = C_1\psi_1(x,t) + C_2\psi_2(x,t),$$

$$\phi_b^{(+)}(x,t) = \frac{1}{\sqrt{2}}\Big(\psi_2(x,t) + \psi_1(x,t)\Big),$$

$$\phi_b^{(-)}(x,t) = \frac{1}{\sqrt{2}}\Big(\psi_2(x,t) - \psi_1(x,t)\Big),$$

The state suggested for the neutron

$$\Psi(x,t) = b_+(t)e^{-iE_0t/\hbar}\varphi_b^{(+)}(x,t) + b_-(t)e^{-iE_0t/\hbar}\varphi_b^{(-)}(x,t),$$

where

$$\varphi_b^{(+)}(x,t) = \frac{1}{\sqrt{2}}\Big(\chi_2(x,t) + \chi_1(x,t)\Big),$$

$$\varphi_b^{(-)}(x,t) = \frac{1}{\sqrt{2}}\Big(\chi_2(x,t) - \chi_1(x,t)\Big),$$

$$\chi_1(x,t) = \sqrt{\kappa}\ e^{-\kappa|x+r(t)/2|},$$

$$\chi_2(x,t) = \sqrt{\kappa}\ e^{-\kappa|x-r(t)/2|}.$$

$$E_0 = -B \rightarrow \text{Neutron separation energy}, \qquad \kappa = \frac{\sqrt{2mB}}{\hbar}.$$

Suggested model

Calculation without continuum states

In order to find the elastic and transfer probabilities we introduce the suggested state

$$\Psi(x,t) = b_+(t)e^{-iE_0t/\hbar}\varphi_b^{(+)}(x,t) + b_-(t)e^{-iE_0t/\hbar}\varphi_b^{(-)}(x,t),$$

in the Schrödinger equation. It leads to

Differential coupled equation without continuum states

$$i\mathcal{N}_{bb}^{(\pm)}\dot{b}_\pm = \left[(\cosh w + 1)\,\frac{\mathcal{H}_{bb}^{(\pm)} - E_0\mathcal{N}_{bb}^{(\pm)}}{\varepsilon_0} - i\sinh w\; D_{bb}^{(\pm)}\right] b_\pm .$$

Here we use $\varepsilon_0 = \frac{\hbar v_\infty}{a}$ and the notations

$$\dot{b}_\pm = \dot{b}_\pm(w) = \frac{db_\pm}{dw},$$

$$\mathcal{N}_{bb}^{(\pm)} = \mathcal{N}_{bb}^{(\pm)}(w) = \langle\varphi_b^{(\pm)}|\varphi_b^{(\pm)}\rangle,$$

$$\mathcal{H}_{bb}^{(\pm)} = \mathcal{H}_{bb}^{(\pm)}(w) = \langle\varphi_b^{(\pm)}|\ \hat{H}\ |\varphi_b^{(\pm)}\rangle,$$

$$D_{bb}^{(\pm)} = D_{bb}^{(\pm)}(w) = a\ \langle\varphi_b^{(\pm)}|\frac{\partial\varphi_b^{(\pm)}}{\partial r}\rangle.$$

Suggested model

Calculation without continuum states

As the neutron is initially bounded to the well corresponding to core B_1. The initial conditions are

$$b_+(t \to -\infty) = \frac{1}{\sqrt{2}}, \qquad b_-(t \to -\infty) = -\frac{1}{\sqrt{2}}.$$

Solving the differential equations is possible to calculate the asymptotic values of b_- e b_+.

Elastic and transfer probabilities

$$P_{el} = |\langle \chi_1(t \to \infty) | \Psi(t \to \infty) \rangle|^2 = \frac{1}{2} \; |b_+(w \to \infty) - b_-(w \to \infty)|^2 ,$$

$$P_{tr} = |\langle \chi_2(t \to \infty) | \Psi(t \to \infty) \rangle|^2 = \frac{1}{2} \; |b_+(w \to \infty) + b_-(w \to \infty)|^2 .$$

Where

$$\chi_1(x,t) = \frac{1}{\sqrt{2}} \left(\varphi_b^{(+)}(x,t) - \varphi_b^{(-)}(x,t) \right),$$

$$\chi_2(x,t) = \frac{1}{\sqrt{2}} \left(\varphi_b^{(+)}(x,t) + \varphi_b^{(-)}(x,t) \right).$$

Suggested model

Calculations with one continuum state by parity

We introduce the continuum in the simplest way. We add to the neutron state

$$\Psi(x,t) = b_+(t)e^{-iE_0t/\hbar}\varphi_b^{(+)}(x,t) + b_-(t)e^{-iE_0t/\hbar}\varphi_b^{(-)}(x,t)$$

one state in the continuum by parity

Neutron state

$$\begin{aligned}\Psi(x,t) =& b_+(t)\ e^{-iE_0t/\hbar}\varphi_b^{(+)}(x,t) + b_-(t)\ e^{-iE_0t/\hbar}\varphi_b^{(-)}(x,t)\\ &+ c_+(t)\ e^{-iE_ct/\hbar}\varphi_c^{(+)}(x) + c_-(t)\ e^{-iE_ct/\hbar}\varphi_c^{(-)}(x).\end{aligned}$$

$$E_0 = -B \rightarrow \text{Separation energy of the neutron},$$

$$E_c = \frac{\hbar^2 k_0^2}{2m} \rightarrow \text{Average energy of the wave packet.}$$

Suggested model
Calculations with one continuum state by parity

The continuum states are described by the wave packets

$$\varphi_c^{(+)}(x) = \int_{-\infty}^{\infty} dk\ \Gamma(k)\ \chi_k^{(+)}(x),$$

$$\varphi_c^{(-)}(x) = \int_{-\infty}^{\infty} dk\ \Gamma(k)\ \chi_k^{(-)}(x),$$

where

$$\chi_k^{(+)}(x) = \frac{1}{\sqrt{\pi}} \cos kx,$$

$$\chi_k^{(-)}(x) = \frac{1}{\sqrt{\pi}} \sin kx$$

and

$$\Gamma(k) = \begin{cases} \frac{1}{\sqrt{\Delta_k}} & k_0 - \frac{\Delta_k}{2} \leq k \leq k_0 + \frac{\Delta_k}{2}, \\ 0 & \text{otherwise.} \end{cases}$$

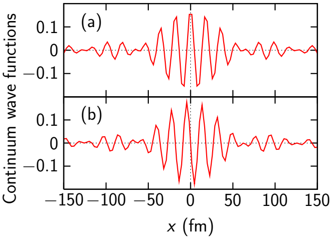

Figure: Continuum states with wave vector center, $k_0 = 0.35$ fm^{-1}, and width $\Delta_k = 0.1$ fm^{-1} of the wave packet .(a) Symmetric and (b) antisymmetric.

Suggested model

Calculations with one continuum state by parity

Introducing the neutron state in the Schrödinger equation

$$\Psi(x,t) = b_+(t)\ e^{-iE_0 t/\hbar}\varphi_b^{(+)}(x,t) + b_-(t)\ e^{-iE_0 t/\hbar}\varphi_b^{(-)}(x,t) + c_+(t)\ e^{-iE_c t/\hbar}\varphi_c^{(+)}(x) + c_-(t)\ e^{-iE_c t/\hbar}\varphi_c^{(-)}(x).$$

Differential equations including one continuum state by parity

$$A_{bb}^{(\pm)}\ \dot{b}_\pm(w) + A_{bc}^{(\pm)}\ \dot{c}_\pm(w) = B_{bb}^{(\pm)}\ b_\pm(w) + B_{bc}^{(\pm)}\ c_\pm(w),$$
$$A_{cb}^{(\pm)}\ \dot{b}_\pm(w) + A_{cc}^{(\pm)}\ \dot{c}_\pm(w) = B_{cb}^{(\pm)}\ b_\pm(w) + B_{cc}^{(\pm)}\ c_\pm(w).$$

The functions A and B involve inner products between the functions $\varphi_b^{(\pm)}$ and $\varphi_c^{(\pm)}$ and matrix elements of H.

Elastic and transfer probabilities

$$P_{el} = |\langle \chi_1(t\to\infty)|\Psi(t\to\infty)\rangle|^2 = \frac{1}{2}\ |b_+(w\to\infty) - b_-(w\to\infty)|^2,$$
$$P_{tr} = |\langle \chi_2(t\to\infty)|\Psi(t\to\infty)\rangle|^2 = \frac{1}{2}\ |b_+(w\to\infty) + b_-(w\to\infty)|^2.$$

Results

Elastic and transfer probabilities

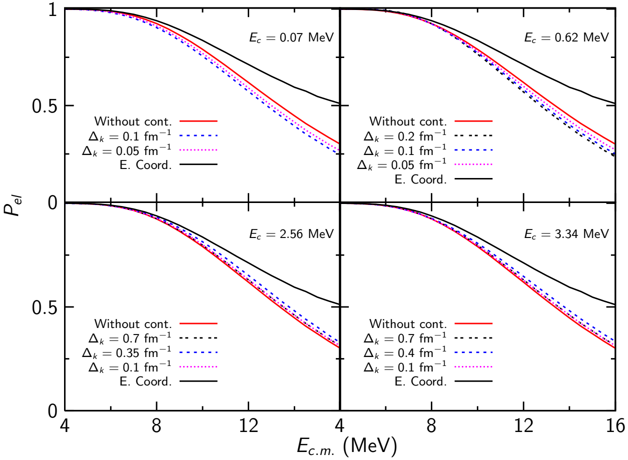

Figure: Elastic probabilities in head-on collisions for the $^{27}F + ^{26}F$ system as a function of the center-of-mass-collision energy $E_{c.m.}$.

Results

Elastic and transfer probabilities

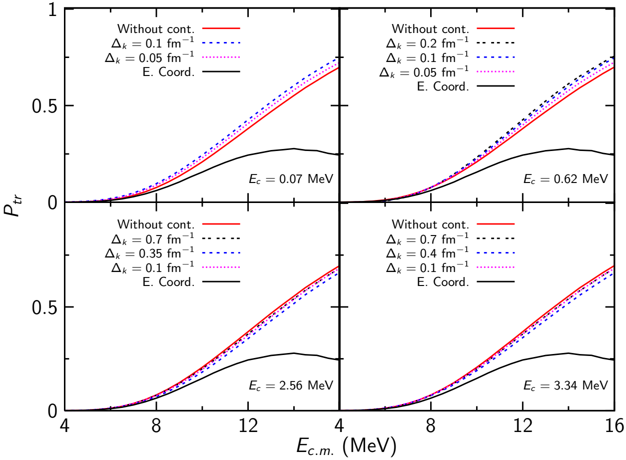

Figure: Transfer probabilities in head-on collisions for the ^{27}F+^{26}F system as a function of the center-of-mass-collision energy $E_{c.m.}$.

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Elastic and transfer probabilities

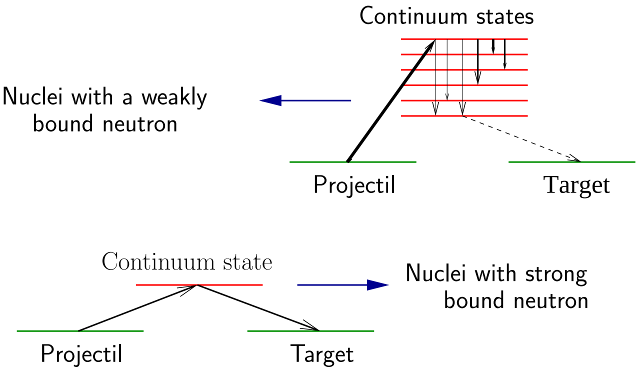

Figure: Ilustration explaining the effects caused by the inclusion of the continuum states in transfer reactions.

Conclusions and prospects

- We performed a large number of calculations varying the width Δ_k and the average energy E_c of the wave packet and in all cases the effects of including a single state in the continuum by parity are small. Therefore the inclusion of only one continuum state is not enough.

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- The next step is to understand the role of the different energies in the continuum and to include more continuum states. This can be done to optimize the coupled-channel codes.
- It is important to optimize our codes (given their complexity) including for instance, the energies that mostly influence the process and thus reducing the number of channels involved.

Thank you for your attention!.