The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

1944-5

Joint ICTP-IAEA Workshop on Nuclear Reaction Data for Advanced Reactor Technologies

19 - 30 May 2008

Neutron Induced Cross Section Measurements.

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Neutron Induced

Cross Section Measurements

Workshop on Nuclear Reaction Data for Advanced Reactor Technologies

Trieste, Italy, 19 – 30 May 2008

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Joint Research Centre (JRC)
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Geel - Belgium
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- **Neutron cross section measurements (Part 1)**
- **Data reduction and uncertainties (Part 2)**
  - Experimental observables
  - Definitions and terminology
  - Fitting a mathematical model to experimental data
  - Basic operations
  - AGS
  - Example

ISO Guide, “Guide to the expression of uncertainty in measurement”, Geneva, Switzerland, ISO,1995

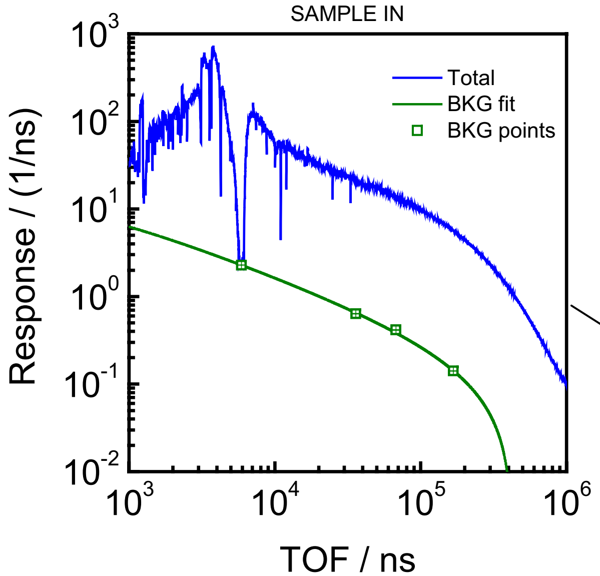
Mannhart, “A Small Guide to Generating Covariances of Experimental Data”, Juni 1981, PTB-FMRB-84

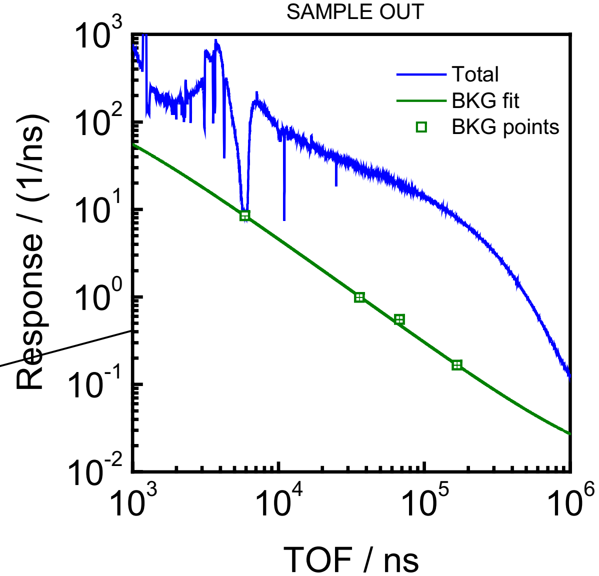
F.H. Fröhner, Nucl. Sci. Eng. 126 (1997) 1 -18

Experimental observables

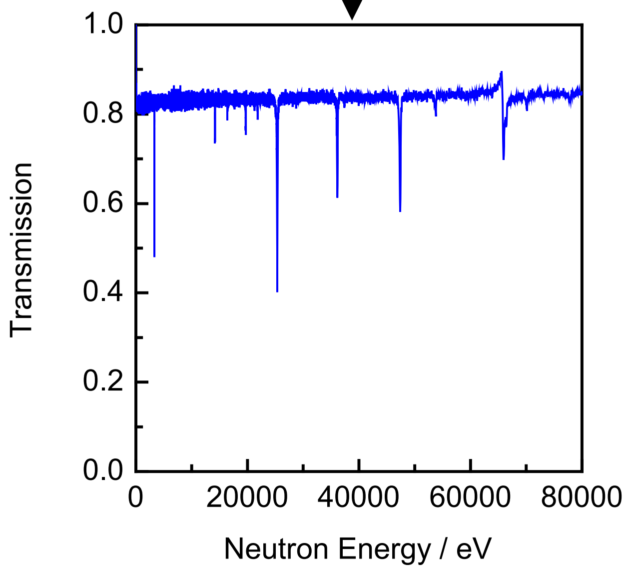
- **Experimental observables**
  - Transmission
  - Reaction yields
- **Definitions and terminology**
- **Fitting a mathematical model to data**
- **Basic operations**
- **AGS**
- **Examples**

Transmission





$$\boxed{T_{exp} = N_T \frac{C'_{in} - B'_{in}}{C'_{out} - B'_{out}}}$$



Transmission : data reduction

Sample in

(1) Dead time correction

(2) Background subtraction

Sample out

(1) Dead time correction

(2) Background subtraction

$$T_{exp} = N_T \frac{C'_{in} - B'_{in}}{C'_{out} - B'_{out}}$$

RRR

Resonance shape analysis

$$T_{exp}(T_n) = \int R_T(T_n, E_n)\, e^{-\sigma_{tot}(E_n)}\, dE_n$$

URR

Correction for resonance structure

$$< e^{-n\sigma_{tot}} > \approx e^{-<n\sigma_{tot}>}\left(1 + \frac{n^2}{2} \operatorname{var} \sigma_{tot} - ...\right)$$

$$< T_{exp} > = < e^{-n\sigma_{tot}} >$$

Reaction yield : data reduction

Reaction measurement

(1) Dead time correction

(2) Background subtraction

Flux measurement

(1) Dead time correction

(2) Background subtraction

$$Y_{exp} = N_r \frac{\sigma_\varphi}{\varepsilon_r} \frac{C_r' - B_r'}{C_\varphi' - B_\varphi'}$$

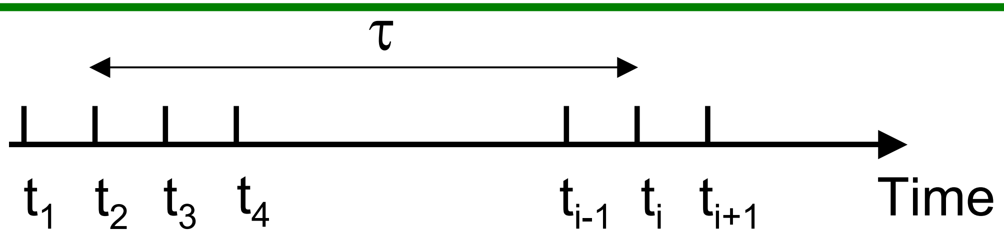
RRR
Resonance shape analysis

$$\underline{Y_{exp}(T_n)} = N_r \int R_T(T_n, E_n)\, \varepsilon_r(E_n)\, Y_r(E_n)\, dE_n$$

URR
Correction for self-shielding and multiple scattering (F_c)

$$< \sigma(n,r) > = F_c \, \underline{< Y_{exp} >}$$

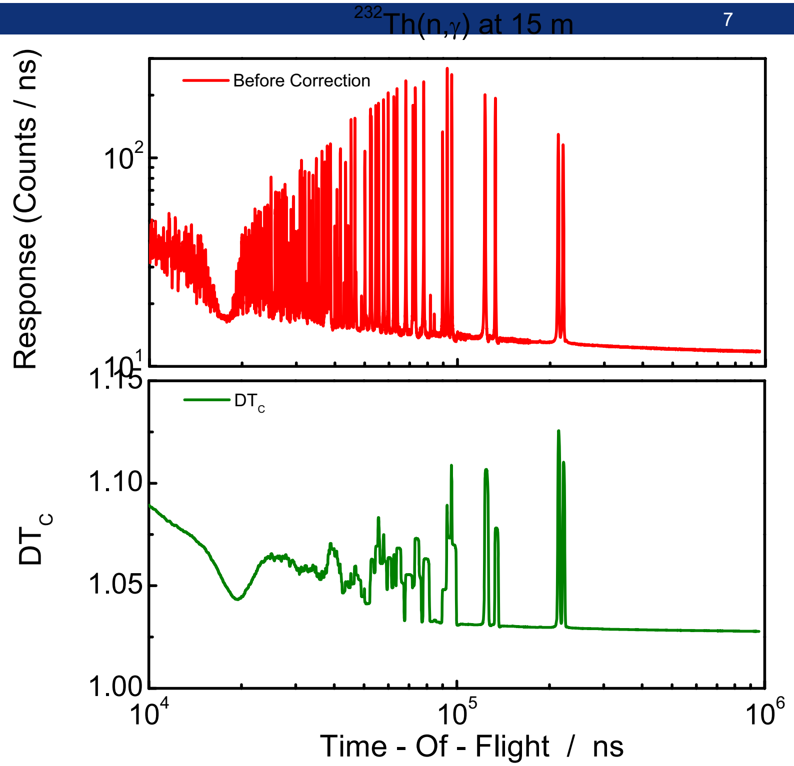
Dead time correction (1/2)



$$t_k = t_i - \tau \qquad s_i = \sum_{j=t_k}^{t_i} N_j$$

$$DT_C(t_i) = \frac{1}{1 - \frac{s_i}{T_o}}$$

T_o is the number of bursts



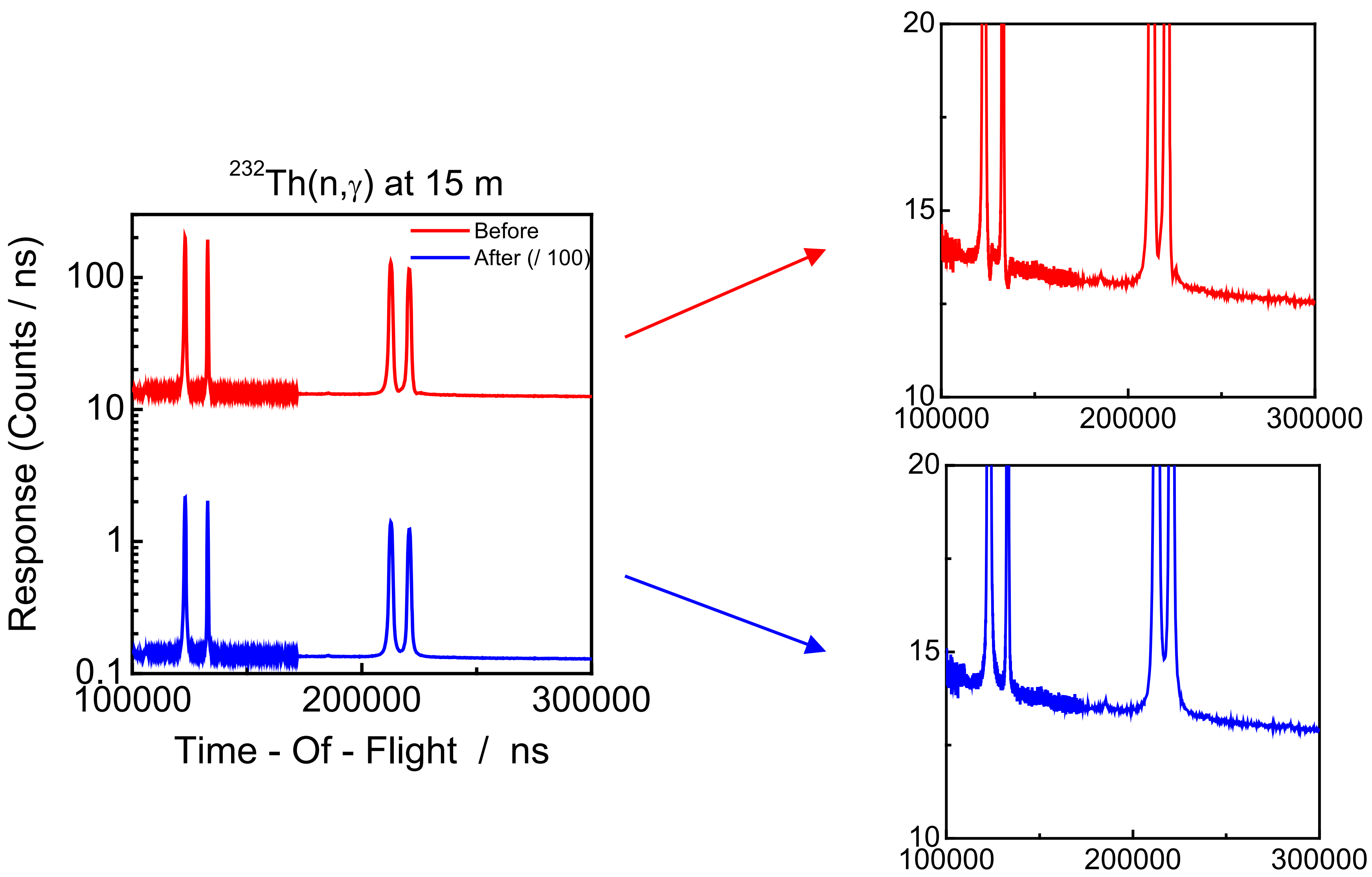
System dead time for on-line data processing (time-of-flight + amplitude):

- Acqiris DC282 (digitizer) = 350 ns
- CAEN N1728B (digitizer) = 560 ns

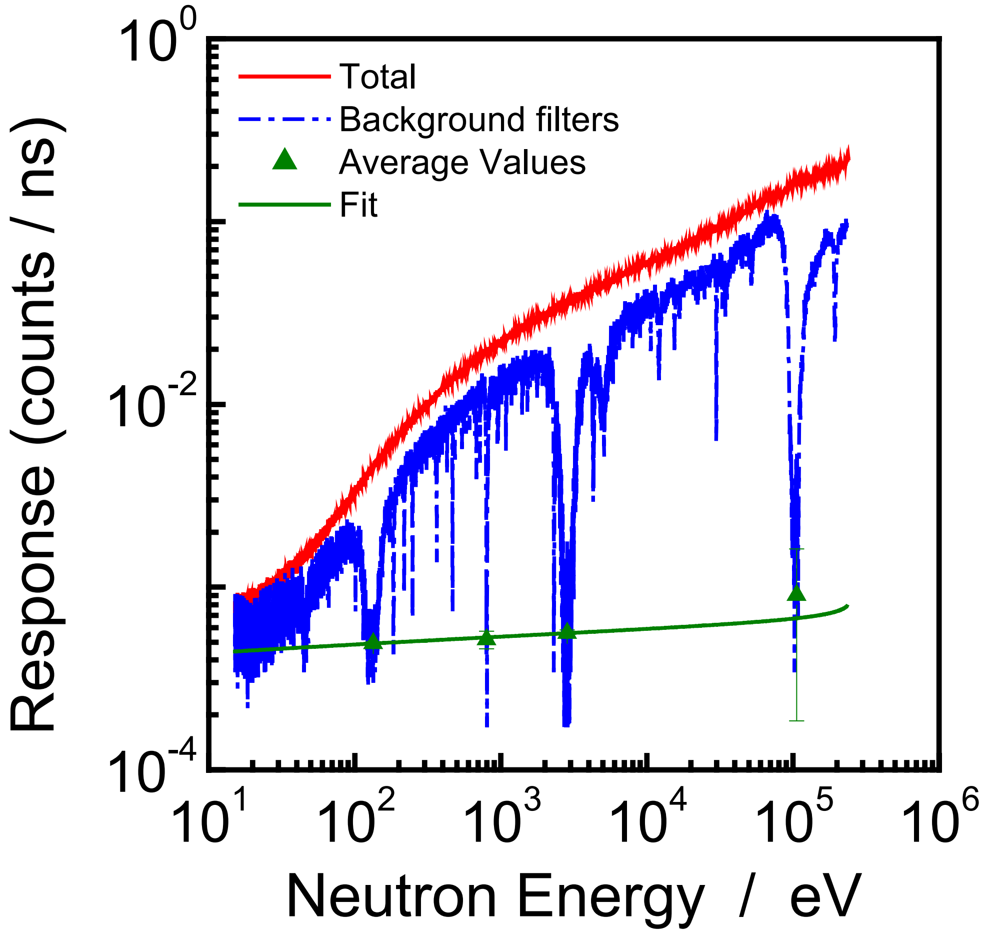
- Conventional system DAQ2000 = 2800 ns (without amplitude similar to digitizers)

Moore, Nucl. Instr. Meth. 169 (1980) 245
Mihailescu et al., submitted to NIM A

Dead time correction (2/2)



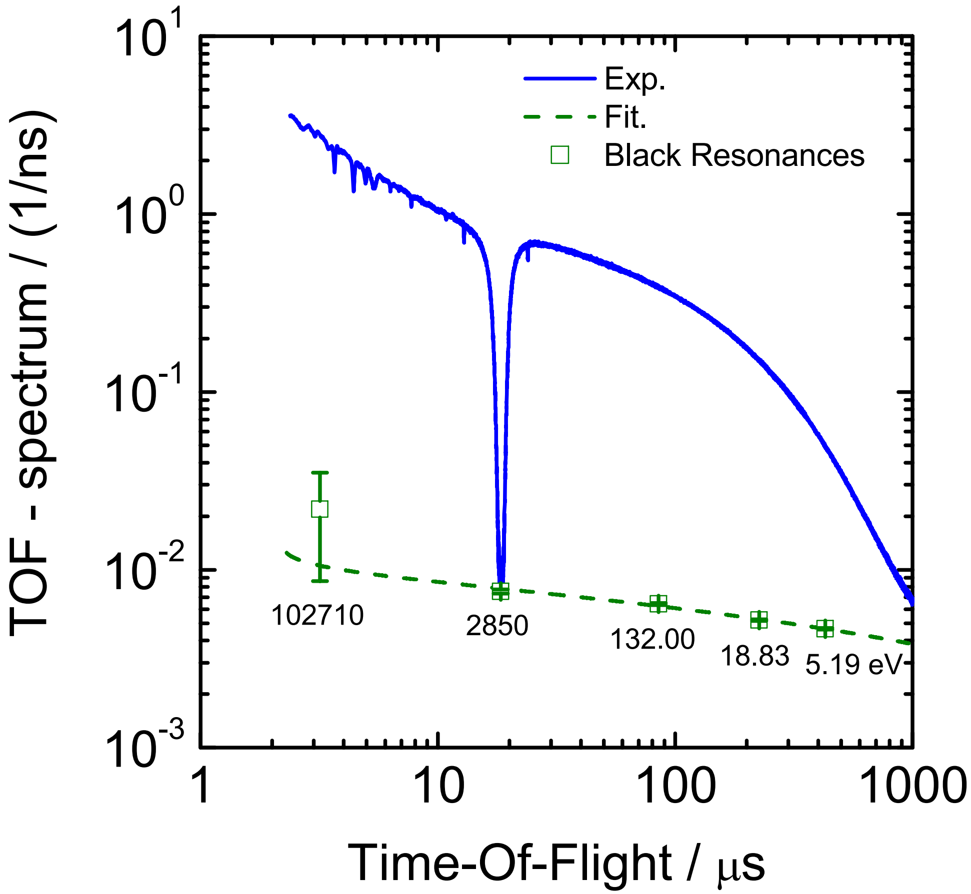
Background correction (1/3)



Black Resonance Filters

Element	Energy / eV
Ag	5.19
W	18.83
Co	132.00
Na	2850.00
S	102710.00

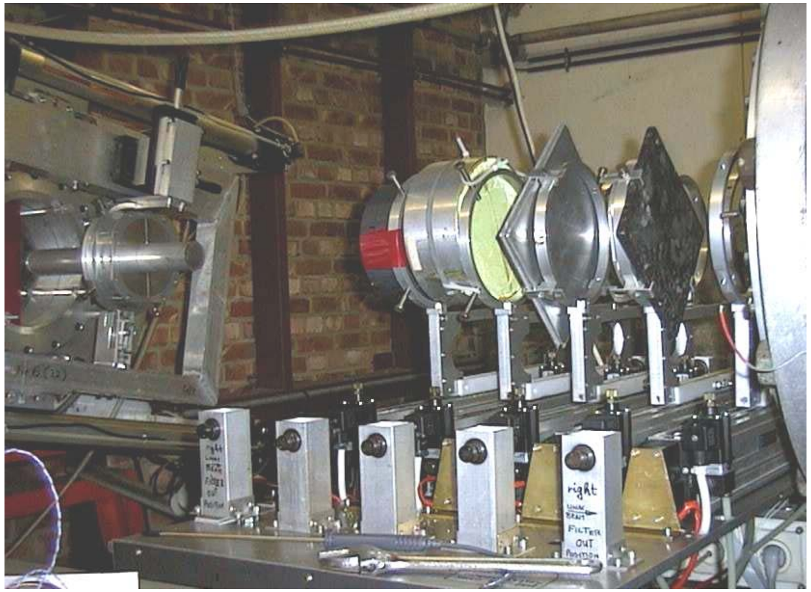
Background correction (2/3)

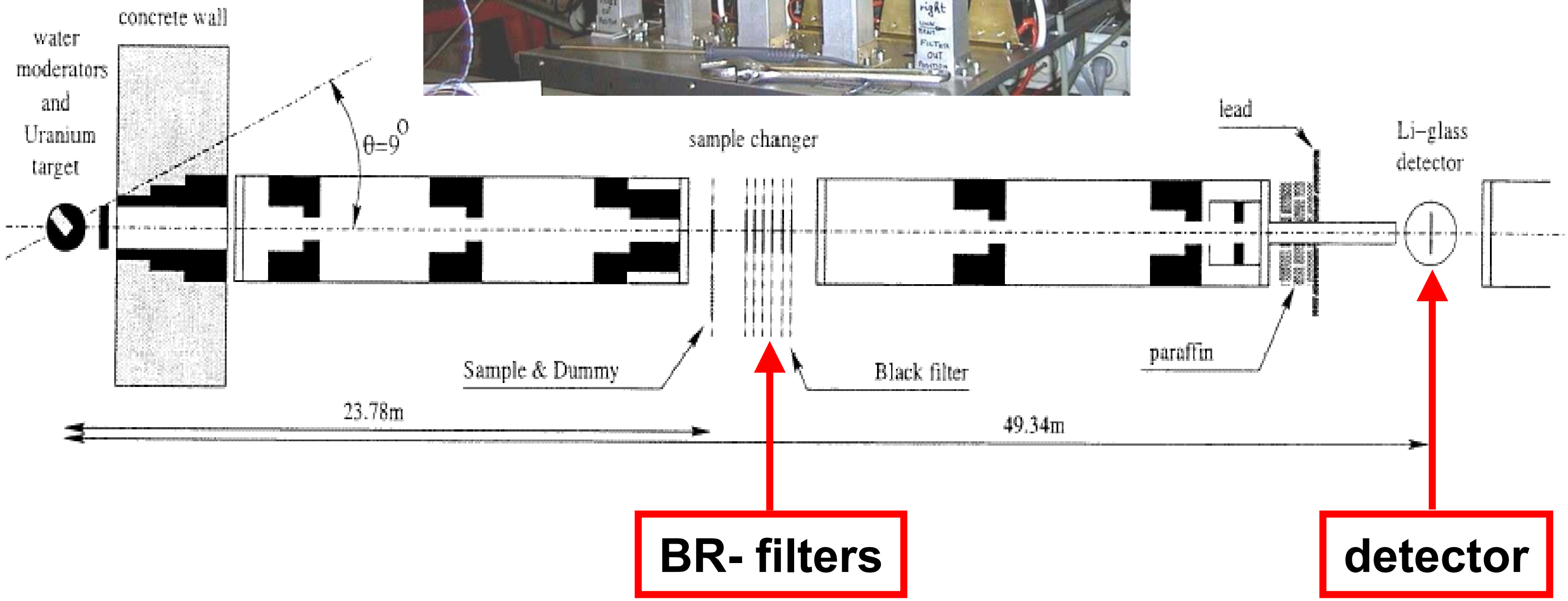


Black Resonance Filters

Element	Energy / eV
Ag	5.19
W	18.83
Co	132.00
Na	2850.00
S	102710.00

Background measurement (3/3)





- **Experimental observables**
- **Definitions and terminology**
  - Probability and statistical quantities
  - Error and Uncertainty
  - Propagation of uncertainties
- **Fitting a mathematical model to data**
- **Basic operations**
- **AGS**
- **Examples**

Probability and statistical quantities

P(x) theoretical probability density function (PDF) of x

P(x,y) theoretical probability density function of (x,y)

- **Mean** $\mu_x = \langle x \rangle = \int x\, P(x)\, dx$

- **Variance** $\sigma_x^2 = \left\langle (x-\mu_x)^2 \right\rangle = \int (x-\mu_x)^2\, P(x)\, dx$

- **Covariance** $\sigma_{xy}^2 = \left\langle (x-\mu_x)(y-\mu_y) \right\rangle = \int (x-\mu_x)(y-\mu_y) P(x,y)\, dxdy$

- **Correlation** $\rho(x,y) = \dfrac{\sigma_{xy}^2}{\sigma_x\, \sigma_y}$

Observed statistical quantities

n observations x_1, …, x_n of variable x

m observations (x_1,x_1) …, (x_n,y_n) of variable (x,y)

- **Sample mean** $$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$
- **Sample variance** $$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$
- **Sample covariance** $$s_{xy}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)$$
- **Sample correlation** $$r(x,y) = \frac{s_{xy}}{s_x\, s_y}$$

Function of variables : linear function

n observations x_1, …, x_n of variable x → z = f(x_1,…,x_n)

- **Linear function** $$z = f(x_i) = \sum_{i=1}^{k} a_i\ x_i$$

- **Mean** $$\mu_z = \langle f(x_i) \rangle = \sum_{i=1}^{k} a_i\ \mu_i$$

- **Variance** $$\sigma_z^2 = \left\langle \left(f(x_i) - \mu_f \right)^2 \right\rangle = \sum_i a_i^2\ \sigma_{x_i}^2 + \sum_{i\neq}\sum_j a_i\ a_j\ \sigma_{x_i x_j}$$

- **Taylor expansion (1st order !)**

$$z = f(x_i) \cong f(\mu_i) + \sum_i \left.\frac{\partial f}{\partial x_i}\right|_{\mu} (x_i - \mu_i)$$

$$z = f(x_i) \cong f(\mu_i) + \sum_i g_i (x_i - \mu_i) \qquad g_i = \left.\frac{\partial f}{\partial x_i}\right|_{\mu}$$

- **Mean**

$$\mu_z = \langle f(x_i) \rangle \cong f(\mu_i)$$

- **Variance**

$$\sigma_z^2 = \left\langle \left(f(x_i) - \mu_f\right)^2 \right\rangle \cong \sum_i g_i^2 \, \sigma_{x_i}^2 + \sum_{i \neq} \sum_j g_i \, g_j \, \sigma_{x_i x_j}$$

Matrix notation

- **Linear function** $\vec{z} = f(\vec{x}) = \underline{A}\vec{x}$ $\qquad a_{ik} = \frac{\partial f_i}{\partial x_k}$
  - Mean $\vec{\mu}_z = \underline{A}\,\vec{\mu_x}$
  - Variance $\underline{V}_z = \underline{A}\,\underline{V}_x\,\underline{A}^T$
- **Non-linear function** $\vec{z} = f(\vec{x}) \cong f(\vec{\mu}_x) + \underline{G}_x(\vec{x} - \vec{\mu}_x)$ $\qquad g_{x,ik} = \frac{\partial f_i}{\partial x_k}$

  (1st order Taylor !)

  sensitivity matrix
  design matrix
  - Mean $\vec{\mu}_z \cong f(\vec{\mu}_x)$
  - Variance $\underline{V}_z \cong \underline{G}_x\,\underline{V}_x\,\underline{G}_x^T$

Error and Uncertainty (ISO Guide)

- **Uncertainty (dispersion (or width), always > 0)**

"Uncertainty is a parameter associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand"

- **Error (difference between two quantities, can be + or -)**

"Error is the result of a measurement minus a true value of the measurand"

Error

$$\boxed{T_{exp} = N_T \, \frac{C'_{in} - B'_{in}}{C'_{out} - B'_{out}}}$$

- **Systematic error**

  Mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions minus a true value of the measurand.

  The expectation value of the error arising from a systematic effect = 0

- **Random error**

  Result of a measurement minus the mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions.

  The expectation value of a random error = 0

Systematic error (ISO Guide)

- **If a systematic error arises from a recognized effect, the effect can be quantified and a correction or correction factor should be applied.**
- **The uncertainty on the correction factor <u>should not</u> be quoted as a systematic error.**
- **One <u>should not</u> enlarge the uncertainty to compensate for a systematic effect that is not recognized!**

Evaluation of uncertainty components

- **Type A evaluation**
  - Method of evaluation of uncertainty by the statistical analysis of series of observations
  - Is obtained from an **observed** frequency distribution
- **Type B evaluation**
  - Method of evaluation of uncertainty by means other than the statistical analysis of series of observations.
  - Is obtained from an **assumed** probability density function (subjective probability) .
  - The evaluation is based on scientific judgement
    - Theoretical distribution
    - Experience from previous measurement data
    - Data provided in calibration or other certificates
- **Propagation of uncertainties ⇒ combined uncertainty**

$$\underline{V}_f \cong \underline{G}_x \, \underline{V}_x \, \underline{G}_x^T \qquad g_{x,ik} = \frac{\partial f_i}{\partial x_k}$$

non-linear problem : approximation!

Example : Counting experiment

- **Type A evaluation**
  - Perform repeated measurements and record the number of events in time t
  - Calculate the uncertainty from the standard deviation of the observed frequency distribution

$$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

- **Type B evaluation : Poison statistics**
  - The observed quantity is distributed as a Poison distribution
  - The uncertainty is defined by the standard deviation of a Poison distribution

$$\sigma_x^2 = \mu_x$$

How to quote uncertainties

- **Evaluation of uncertainty components**
  - Type A : statistical analysis of repeated measurements
  - Type B : scientific judgement
- **All uncertainties are statistical !**

  (uncertainty due to counting statistics, uncertainty on the normalization, …)

  (correlated or not-correlated uncertainties)
- **Standard (u_x) or expanded uncertainty**
  - Standard uncertainty : $(\bar{x} \pm u_x)$ with $u_x = \frac{s_x}{\sqrt{n}}$ $\lambda_p = 1 \Rightarrow$ p = 0.68
  - Expanded uncertainty : $\lambda_p > 1$

    e.g. $\lambda_p = 1.96 \Rightarrow$ p = 0.95

$$P\left(\left|\bar{x} - \mu_x\right| \le \lambda_p \frac{\sigma}{\sqrt{n}}\right) = p$$

Fitting a mathematical model to data

- **Definitions and terminology**
- **Experimental observables**
- **Fitting a mathematical model to data**
  - Least squares – Maximum likelihood
  - Generalized least squares method
- **Basic operations**
- **AGS**
- **Examples**

Central limit theorem

Central limit theorem:

The sum of a large number of independent and identically-distributed random variables will be approximately normally distributed

If $x_1, \ldots, x_n$ are **independent** and **identically** distributed with mean μ_x and variance σ^2 then :

for large n :

the distribution of $\bar{x} = \frac{\sum_i x_i}{n}$ is the normal distribution with parameters $(\mu_x, \frac{\sigma}{\sqrt{n}})$

$$P(\bar{x} \mid \mu_x, \sigma_x)\, d\bar{x} = \frac{1}{\sqrt{2\pi\sigma_x}} \exp\left(-\frac{1}{2}\left(\frac{\bar{x}-\mu_x}{\sigma_x}\right)^2\right) d\bar{x} \quad \text{with } \sigma_x = \frac{\sigma}{\sqrt{n}}$$

Central limit theorem

General: based on maximum entropy principle

Fröhner, Nucl. Sci. Eng. 126 (1997) 1

For observables $\vec{x} = (x_1, \ldots, x_n)$ with covariance matrix $\underline{V}_x$

which are **identically** distributed with mean $\vec{\mu}_x$

$$P\left(\vec{x} \middle| \vec{\mu}_x, \underline{V}_x\right) d\vec{x} = \frac{1}{\sqrt{\det(2\pi \underline{V}_x)}} \exp\left(-\frac{1}{2} (\vec{x} - \vec{\mu}_x)^T \underline{V}_x^{-1} (\vec{x} - \vec{\mu}_x) \right) d\vec{x}$$

Fitting a mathematical model to data

Problem: n data points (x_1,z_1), …, (x_n, z_n) and a model function $\vec{z} = f(\vec{x},\vec{a})$ that in addition to the variable x also depends on k parameters $a_1,\ldots,a_k$ with n>k.

Solution:

(1) Least squares : find the vector $\vec{\mathbf{a}}$ such that the curve fits best the given data in the least squares sense, that is, the weigthed sum of squares is minimized:

$$\chi^2(\vec{a}) = (\vec{z}_{exp} - f(\vec{x},\vec{a}))^T \, \underline{V}_Z^{-1} \, (\vec{z}_{exp} - f(\vec{x},\vec{a}))$$

(2) Maximum likelihood: find the vector $\vec{\mathbf{a}}$ that maximizes the likelihood function.

When the PDF is the normal distribution :

$$P\left(\vec{a} \middle| \vec{z}, \underline{V}_z\right) d\vec{a} \propto \exp\left(-\frac{1}{2}(\vec{z}_{exp} - f(\vec{x},\vec{a}))^T \underline{V}_z^{-1} (\vec{z}_{exp} - f(\vec{x},\vec{a}))\right) d\vec{a}$$

Least squares method is equivalent to Maximum likelihood

Generalized least squares

Input from experiment : z_{exp} and V_z, x_{exp} and V_x

- **Linear model**

$$z_m = f(\vec{x},\vec{a}) = \underline{G}_a \vec{a} \qquad g_{a,ij} = \frac{\partial f_i}{\partial a_j}$$

$$\underline{V} = (\underline{V}_z + \underline{G}_x \underline{V}_x \underline{G}_x^T) \qquad g_{x,ij} = \frac{\partial f_i}{\partial x_j}$$

$$\vec{a} = (\underline{G}_a^T \underline{V}^{-1} \underline{G}_a)^{-1} (\underline{G}_a^T \underline{V}^{-1} \vec{z}_{exp})$$

$$\underline{V}_a = (\underline{G}_a^T \underline{V}^{-1} \underline{G}_a)^{-1}$$

Van der Zijp (proceedings ESARDA)

- **Non-linear model (1st order Taylor)**

$$z_m = f(\vec{x},\vec{a}) \cong f(\vec{x},\vec{a}_o) + \underline{G}_a (\vec{a} - \vec{a}_o)$$

$$\underline{V} = (\underline{V}_z + \underline{G}_x \underline{V}_x \underline{G}_x^T)$$

$$(\vec{a} - \vec{a}_o) = (\underline{G}_a^T \underline{V}^{-1} \underline{G}_a)^{-1} \underline{G}_a^T \underline{V}^{-1} (\vec{z}_{exp} - f(\vec{x},\vec{a}_o))$$

$$\underline{V}_{a-a_o} = (\underline{G}_a^T \underline{V}^{-1} \underline{G}_a)^{-1}$$

solved by iteration

Basic operations

- **Definitions and terminology**
- **Fitting a mathematical model to data**

- **Basic operations (+ , - , x , /)**
  - Background
  - Normalization
  - Fit correlated data

- **AGS**
- **Examples**

Background

Example: y vector dimension 2 $\rightarrow$ z = f(y_1, y_2, b)

$$\vec{z} = \vec{y} - b$$

$\mathbf{V}_y$ uncorrelated uncertainties from counting statistics : $V_y = \begin{bmatrix} \sigma^2_{y_1} & 0 \\ 0 & \sigma^2_{y_2} \end{bmatrix}$

$\mathbf{V}_b$ uncertainty on background (not correlated with V_y) : $V_b = \sigma^2_b$

$$\mathbf{V}_z = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sigma^2_{y_1} & 0 & 0 \\ 0 & \sigma^2_{y_2} & 0 \\ 0 & 0 & \sigma^2_b \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} \sigma^2_{y_1} + \sigma^2_b & \sigma^2_b \\ \sigma^2_b & \sigma^2_{y_2} + \sigma^2_b \end{bmatrix}$$

$$\underline{V}_z = \underline{A}\ \underline{V}_x\ \underline{A}^T \qquad a_{ik} = \frac{\partial f_i}{\partial x_k}$$

Background

$(y_1\text{-}b,\ y_2\text{-}b) = (z_1,z_2) \rightarrow (z_1+z_2,\ z_1\text{-}z_2)$

$$V_{(z_1,z_2)} = \begin{bmatrix} \sigma_{y_1}^2 + \sigma_b^2 & \sigma_b^2 \\ \sigma_b^2 & \sigma_{y_2}^2 + \sigma_b^2 \end{bmatrix}$$

$$V_{(z_1+z_2,z_1-z_2)} = \begin{bmatrix} \sigma_{y_1}^2 + \sigma_{y_2}^2 + 4\sigma_b^2 & \sigma_{y_1}^2 - \sigma_{y_2}^2 \\ \sigma_{y_1}^2 - \sigma_{y_2}^2 & \sigma_{y_1}^2 + \sigma_{y_2}^2 \end{bmatrix}$$

$$\sigma_{z_1+z_2}^2 = \sigma_{y_1}^2 + \sigma_{y_2}^2 + 4\sigma_b^2$$

$$\sigma_{z_1-z_2}^2 = \sigma_{y_1}^2 + \sigma_{y_2}^2$$

Background

$(y_1\text{-}b,\ y_2\text{-}b) = (z_1,z_2) \rightarrow (z_1+z_2,\ z_1-z_2)$

	Covariance	**Only diagonal terms**
$\sigma^2_{z_1+z_2} =$	$\sigma^2_{y_1} + \sigma^2_{y_2} + 4\sigma^2_b$	$\sigma^2_{y_1} + \sigma^2_{y_2} + 2\sigma^2_b$
$\sigma^2_{z_1-z_2} =$	$\sigma^2_{y_1} + \sigma^2_{y_2}$	$\sigma^2_{y_1} + \sigma^2_{y_2} + 2\sigma^2_b$

$$(y_1\text{-}b,\ y_2\text{-}b) = (z_1,z_2) \rightarrow (z_1+z_2,\ z_1\text{-}z_2) = (v_1,v_2) \rightarrow (v_1+v_2) = 2z_1$$

Covariance

$$\sigma^2_{2z_1} = V_{v_1+v_2} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \sigma^2_{y_1} + \sigma^2_{y_2} + 4\sigma^2_b & \sigma^2_{y_1} - \sigma^2_{y_2} \\ \sigma^2_{y_1} - \sigma^2_{y_2} & \sigma^2_{y_1} + \sigma^2_{y_2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 4(\sigma^2_{y_1} + \sigma^2_b) = 4\sigma^2_{z_1}$$

Only diagonal terms

$$\sigma^2_{2z_1} = 2\sigma^2_{y_1} + 2\sigma^2_{y_2} + 4\sigma^2_b = 2\sigma^2_{z_1} + 2\sigma^2_{z_2}$$

Example: y vector dimension 2 $\rightarrow$ z = f(y_1, y_2, N)

$$\vec{z} = N\,\vec{y}$$

V_y uncorrelated uncertainties from counting statistics : $V_y = \begin{bmatrix} \sigma^2_{y_1} & 0 \\ 0 & \sigma^2_{y_2} \end{bmatrix}$

V_N uncertainty on normalization (not correlated with V_y) : $V_N = \sigma^2_N$

$$V_z \approx \begin{bmatrix} N & 0 & y_1 \\ 0 & N & y_2 \end{bmatrix} \begin{bmatrix} \sigma^2_{y_1} & 0 & 0 \\ 0 & \sigma^2_{y_2} & 0 \\ 0 & 0 & \sigma^2_N \end{bmatrix} \begin{bmatrix} N & 0 \\ 0 & N \\ y_1 & y_2 \end{bmatrix} = \begin{bmatrix} N^2\,\sigma^2_{y_1} + y_1^2\,\sigma^2_N & y_1\,y_2\,\sigma^2_N \\ y_1\,y_2\,\sigma^2_N & N^2\,\sigma^2_{y_2} + y_2^2\,\sigma^2_N \end{bmatrix}$$

not -linear

Normalization

$$(Ny_1, Ny_2) = (z_1,z_2) = \rightarrow (z_1z_2, z_1/z_2)$$

$$V_{(z_1,z_2)} = \begin{bmatrix} N^2\,\sigma^2_{y_1} + y_1^2\,\sigma_N^2 & y_1\,y_2\,\sigma_N^2 \\ y_1\,y_2\,\sigma_N^2 & N^2\,\sigma^2_{y_2} + y_2^2\,\sigma_N^2 \end{bmatrix}$$

$$V_{(z_1z_2,z_1/z_2)} = \begin{bmatrix} N^4y_2^2\sigma^2_{y_1} + N^4y_1^2\sigma^2_{y_2} + 4N^2y_1^2y_2^2\sigma_N^2 & N^2\sigma^2_{y_1} - \frac{y_1^2}{y_2^2}N^2\sigma^2_{y_2} \\ N^2\sigma^2_{y_1} - \frac{y_1^2}{y_2^2}N^2\sigma^2_{y_2} & \frac{\sigma^2_{y_1}}{y_2^2} + \frac{y_1^2}{y_2^4}\sigma^2_{y_2} \end{bmatrix}$$

$$\sigma^2_{z_1z_2} = N^4y_2^2\sigma^2_{y_1} + N^4y_1^2\sigma^2_{y_2} + 4N^2y_1^2y_2^2\sigma_N^2$$

$$\sigma^2_{z_1/z_2} = \frac{\sigma^2_{y_1}}{y_2^2} + \frac{y_1^2}{y_2^4}\sigma^2_{y_2}$$

Normalization

(Ny1, Ny2) = (z_1,z_2) = → (z_1z_2, z_1/z_2)

Covariance

$$\sigma^2_{z_1 z_2} = N^4 y_2^2 \sigma^2_{y_1} + N^4 y_1^2 \sigma^2_{y_2} + 4N^2 y_1^2 y_2^2 \sigma^2_N$$

$$\sigma^2_{z_1/z_2} = \frac{\sigma^2_{y_1}}{y_2^2} + \frac{y_1^2}{y_2^4}\sigma^2_{y_2}$$

Only diagonal terms

$$N^4 y_2^2 \sigma^2_{y_1} + N^4 y_1^2 \sigma^2_{y_2} + 2N^2 y_1^2 y_2^2 \sigma^2_N$$

$$\frac{\sigma^2_{y_1}}{y_2^2} + \frac{y_1^2}{y_2^4}\sigma^2_{y_2} + 2\frac{\sigma^2_N}{N^2}\frac{y_1^2}{y_2^2}$$

Fit correlated data: common "offset " uncertainty

z_1 and z_2 are observables of the same quantity K which are deduced from the experimental data (y_1,y_2) affected by a common "offset error" with uncertainty σ_b and mean =0.

The covariance matrix for (z_1,z_2)= (y_1-0,y_2-0) is :

$$V_{(z_1,z_2)} = \begin{bmatrix} \sigma_{y_1}^2 + \sigma_b^2 & \sigma_b^2 \\ \sigma_b^2 & \sigma_{y_2}^2 + \sigma_b^2 \end{bmatrix}$$

where σ_{y1} and σ_{y2} are the uncorrelated uncertainty components of y_1 and y_2 (e.g. due to counting statistics).

The best value K is obtained by minimizing the expression:

$$\chi^2(k) = (\vec{z}_{exp} - K)^T \; \underline{V}^{-1}_{(z_1,z_2)} \; (\vec{z}_{exp} - K)$$

Fit correlated data: common “offset” uncertainty

Solution : $$\chi^2(k) = (\vec{z}_{exp} - K)^T \, \underline{V}^{-1}_{(z_1,z_2)} \, (\vec{z}_{exp} - K)$$

The best value k: $$K = \frac{z_1 \sigma^2_{y_2} + z_2 \sigma^2_{y_1}}{\sigma^2_{y_1} + \sigma^2_{y_2}}$$ (weighted average)

with uncertainty : $$\sigma^2_K = \frac{\sigma^2_{y_1} \, \sigma^2_{y_2}}{\sigma^2_{y_1} + \sigma^2_{y_2}} + \sigma^2_b$$

Fit correlated data: common "normalization" uncertainty

z_1 and z_2 are observables of the same quantity K which are deduced from the experimental data (y_1,y_2) affected by a common "normalization" with uncertainty σ_N and mean N = 1.

The covariance matrix for (z_1,z_2) = (N y_1, N y_2) with N = 1 is :

$$V_{(z_1,z_2)} = \begin{bmatrix} \sigma^2_{y_1} + y_1^2\sigma_N^2 & y_1 y_2 \sigma_N^2 \\ y_1 y_2 \sigma_N^2 & \sigma^2_{y_2} + y_2^2\sigma_N^2 \end{bmatrix}$$

where σ_{y1} and σ_{y2} are the uncorrelated uncertainty components (e.g. due to counting statistics).

The best value k is obtained by minimizing the expression:

$$\chi^2(k) = (\vec{z}_{exp} - K)^T \, \underline{V}^{-1}_{(z_1,z_2)} \, (\vec{z}_{exp} - K)$$

Fit correlated data: common "normalization" uncertainty

Solution : $\chi^2(k) = (\vec{z}_{exp} - Z)^T \, \underline{V}^{-1}_{(z_1,z_2)} \, (\vec{z}_{exp} - Z)$

The best value K: $K = \dfrac{z_1\sigma^2_{y_2} + z_2\sigma^2_{y_1}}{\sigma^2_{y_1} + \sigma^2_{y_2} + \underline{(y_1 - y_2)^2 \; \sigma^2_N}}$ ($\neq$ weighted average)

with uncertainty : $\sigma^2_Z = \dfrac{\sigma^2_{y_1}\,\sigma^2_{y_2} + (y_1^2\sigma^2_{y_2} + y_2^2\sigma^2_{y_1})\,\sigma^2_N}{\sigma^2_{y_1} + \sigma^2_{y_2} + (y_1 - y_2)^2 \; \sigma^2_N}$

Only in case $(y_1 - y_2)^2\sigma_N^2$ is small K approaches the weighted average.

$\Rightarrow$ Known as : "Peelle's Pertinent Puzzle", see Fröhner NSE 126 (1997) 1

$\Rightarrow$ Due to "non-linearities"

Fit correlated data: common "normalizaton" uncertainty

Solution : treat the normalization as a separate observable

Observables: (N'_{exp}, y_1, y_2) with covariance matrix $V_{(N', y_1, y_2)}$

The function is : $(N'_{exp}, y_1, y_2) = (N', KN', KN') = f(N', K)$ (note $N' = 1/N$)

$$V_{(N',y_1,y_2)} = \begin{bmatrix} \sigma^2_{N'} & 0 & 0 \\ 0 & \sigma^2_{y_1} & 0 \\ 0 & 0 & \sigma^2_{y_2} \end{bmatrix}$$

(N', K) are determined by minimizing:

$$\chi^2(N',K) = ((N'_{exp}, y_1, y_2) - (N', N'K, N'K))^T \, \underline{V}^{-1}_{(N',y_1,y_2)} \, ((N'_{exp}, y_1, y_2) - (N', N'K, N'K))$$

Example

Two measurements (z_1, z_2) of variable Z each with 10% uncorrelated uncertainty and 20% uncertainty on the calibration factor N:

$z_1 = 1.5$

$z_2 = 1.0$

$$V_{z_1,z_2} = \begin{bmatrix} 0.15^2 + (1.15\text{x}0.20)^2 & 1.0\text{x}1.5\text{x}0.20^2 \\ 1.0\text{x}1.5\text{x}0.20^2 & 0.10^2 + (1.0\text{x}0.20)^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1125 & 0.06 \\ 0.06 & 0.05 \end{bmatrix}$$

Best value for Z ?

Weighted average (only uncorrelated terms) : 1.154 $\pm$ 0.083

Least square without covariance : 1.154 $\pm$ 0.083

Least squares with covariance : 0.882 $\pm$ 0.218

Least squares with calibration factor as parameter : 1.154 $\pm$ 0.254

Least squares without covariance + adding u_N afterwards : 1.154 $\pm$ 0.245

Summary : Z = N (C – B) (1/2)

- **Uncertainties**
  - V_C : diagonal, uncorrelated uncertainties due do counting statistics Poison (Type B), or preferably by repeated measurements (Type A)
  - B : background subtraction → introduces correlated uncertainties
  - N : normalization → introduces correlated uncertainties

- **N and B can be considered as due to systematic effects. The value for B and N are the systematic corrections. These corrections have their uncertainties (or better covariance matrix)**

- **Not correcting for N and/or B implies that the value Z can not be quoted !**

Summary : Z = N (C – B) (2/2)

- **Be careful with large uncertainties on the normalization.**
- **Quote all the components involved in the data reduction process**
- **Separate as much as possible the components which create correlated uncertainties**
- **Include the normalization in the model**

General : $Z_i = f(x_1,\ldots,x_n, c_1,\ldots,c_m)$

- x_i : $\underline{V}_X$ only uncorrelated uncertainties (counting statistics)
- c_i : $\underline{V}_c$ correlated uncertainties (normalization, background, …)

$$g_{ik} = \frac{\partial f_i}{\partial Y_k}$$

$$\vec{Y} = \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \\ c_1 \\ . \\ . \\ . \\ c_m \end{bmatrix} \qquad \underline{V}_Y = \left[\begin{array}{ccccc:c} \sigma^2_{x_1} & 0 & \ldots & 0 & & 0 \\ 0 & \sigma^2_{x_2} & \ldots & 0 & & 0 \\ \ldots & \ldots & \ldots & \ldots & & \ldots \\ 0 & 0 & \ldots & \sigma^2_{x_n} & & 0 \\ \hdashline 0 & 0 & \ldots & 0 & & \underline{V}_c \end{array}\right] \Rightarrow \underline{V}_Z \cong \underline{G}_Y^T \underline{V}_Y \underline{G}_Y$$

Experimental observables

$T = f(N_T, \tau_d, C_{in}, B_{in}, C_{out}, B_{out})$ **$Y_r = f(N_r, \varepsilon_r, \tau_d, C_r, B_r, C_\varphi, B_\varphi)$**

- **C** **counting statistics (not correlated)**
- **τ_d** **$u_{\tau d}$ from least square fit of time interval distribution**
- **B** **least square fit (correlated uncertainties)**
- **N_T** **$u_{N_T} \approx 0.5$ % (alternating sequences of "in-out" measurements)**

  Borella et al., Phys. Rev. C 76 (2007) 014605
- **ε_r** **depends on method**
- **N_r** **depends on method (related to ε_r)**

  (n,γ) for total energy principle with PHWT $u_{N_r} \approx 2\%$

  Borella et al. Nucl. Instr. Meth. A577, (2007) 626

AGS: Analysis of Generic $_{TOF}$-Spectra

Reaction yield

$$\boxed{Y_{exp} = N \frac{\sigma_\varphi}{\varepsilon_r} \frac{C_r' - B_r'}{C_\varphi' - B_\varphi'}}$$

Transmission

$$\boxed{T_{exp} = N \frac{C_{in}' - B_{in}'}{C_{out}' - B_{out}'}}$$

C' dead time corrected counts

B' background contribution

N normalization factor

Histogram operations + Covariance information (AGS)

$$\boxed{\begin{array}{l} Y_{exp} + \text{covariance} \\ T_{exp} + \text{covariance} \end{array}}$$

input

Reaction models : Resonance parameters + covariances

Cross sections + covariances

Uncertainty propagation in AGS

–

Z : function of vectors $\mathbf{Z_1},\mathbf{Z_2}$, and parameter vector **b**:

$$\mathbf{Z} = F(\mathbf{b}, \mathbf{Z_1}, \mathbf{Z_2})$$

Uncertainty propagation results in:

$$V_Z = \left(\frac{\partial F}{\partial b}\right) V_b \left(\frac{\partial F}{\partial b}\right)^T + \left(\frac{\partial F}{\partial Z_1}\right) V_{Z_1} \left(\frac{\partial F}{\partial Z_1}\right)^T + \left(\frac{\partial F}{\partial Z_2}\right) V_{Z_2} \left(\frac{\partial F}{\partial Z_2}\right)^T + \ldots$$

AGS $\Rightarrow$

$$\boxed{V_Z = S_Z\, S_Z^{\;T} + D_Z}$$

correlated part
dimension: n x k

uncorrelated part
diagonal : n values

n : length of vector **Z**
k : number of common sources of uncertainties

Uncertainty in AGS : $Z_1 = F(a_1, Y_1)$

$$\mathbf{Z_1} = F(\,\mathbf{a_1}\,,\,\mathbf{Y_1}\,)$$

Z_1, Y_1 : dimension n

a_1 : dimension k_1

$$V_{Z_1} = \left(\frac{\partial F}{\partial a_1}\right) V_{a_1} \left(\frac{\partial F}{\partial a_1}\right)^T + \left(\frac{\partial F}{\partial Y_1}\right) V_{Y_1} \left(\frac{\partial F}{\partial Y_1}\right)^T$$

$\mathbf{V_{a_1}}$ covariance matrix (symmetric & positive definite)

$\mathbf{V_{a_1}} = \mathbf{L_{a_1}}\,\mathbf{L_{a_1}}^{\mathbf{T}}$ (Cholesky decomposition)

L_a : lower triangular matrix

$$S_{a1} = \left(\frac{\partial F}{\partial a_1}\right) L_{a_1}$$

$\mathbf{V_{Y_1}}$ only diagonal terms : $\mathbf{D_{Y_1}} = \mathbf{V_{Y_1}}$

$\left(\frac{\partial F}{\partial Y_1}\right)$ only diagonal terms

$$D_{Z1} = \left(\frac{\partial F}{\partial Y_1}\right) \underbrace{D_{Y_1}}_{\to\ \sigma^2_{Y1u}} \left(\frac{\partial F}{\partial Y_1}\right)^T$$

$$\boxed{V_{Z_1} = \underbrace{S_{a_1}}_{\text{dimension: } n \times k_1}\, S_{a_1}{}^T + \underbrace{D_{Z_1}}_{\text{n values (diagonal)}}}$$

Example : $Z = a\,Y$

$$\mathbf{Z} = a\,Y$$

Z, Y : dimension n
a : dimension 1

$$V_Z = \left(\frac{\partial F}{\partial a}\right) V_a \left(\frac{\partial F}{\partial a}\right)^T + \left(\frac{\partial F}{\partial Y}\right) V_Y \left(\frac{\partial F}{\partial Y}\right)^T$$

$$V_a = \sigma_a\,\sigma_a$$

$$\frac{\partial F}{\partial a} = Y$$

$$S_a = Y\,\sigma_a$$

$\mathbf{V_Y}$ only diagonal terms : $\mathbf{D_Y} = \mathbf{V_Y}$

$\left(\frac{\partial Z}{\partial Y}\right)$ only diagonal terms

$$D_Z = a^2\,D_Y \rightarrow \mathbf{a^2\sigma^2{}_{Y\,u}}$$

$$\boxed{V_Z = S_a\,S_a{}^T + D_Z}$$

dimension: n x 1

n values (diagonal)

Uncertainty in AGS : $Z = F(b, Z_1, Z_2)$

$$\mathbf{Z} = \mathrm{F}(\,\mathbf{b}\,,\mathbf{Z_1}\,,\mathbf{Z_2})$$

Z, Z_1, Z_2 : dimension n
b : dimension k_b

$$\mathbf{V_b = L_b\, L_b^T}$$

$$\mathbf{S_b} = \left(\frac{\partial \mathbf{Z}}{\partial \mathbf{b}}\right)\mathbf{L_b}$$

$$\mathbf{V_{Z_1} = S_{a_1}\, S_{a_1}^{\;T} + D_{Z_1}}$$

dim $S_{a_1} = n \times k_1$

$$\mathbf{V_{Z_2} = S_{a_2}\, S_{a_2}^{\;T} + D_{Z_2}}$$

dim $S_{a_2} = n \times k_2$

$$V_Z = \left(\frac{\partial F}{\partial b}\right)V_b\left(\frac{\partial F}{\partial b}\right)^T + \left(\frac{\partial F}{\partial Z_1}\right)V_{Z_1}\left(\frac{\partial F}{\partial Z_1}\right)^T + \left(\frac{\partial F}{\partial Z_2}\right)V_{Z_2}\left(\frac{\partial F}{\partial Z_2}\right)^T$$

$$V_Z = S_b S_b^{\;T} + \left(\frac{\partial F}{\partial Z_1}\right)S_{a_1}S_{a_1}^{\;T}\left(\frac{\partial F}{\partial Z_1}\right)^T + \left(\frac{\partial F}{\partial Z_2}\right)S_{a_2}S_{a_2}^{\;T}\left(\frac{\partial F}{\partial Z_2}\right)^T + \left(\frac{\partial F}{\partial Z_1}\right)D_{Z_1}\left(\frac{\partial F}{\partial Z_1}\right)^T + \left(\frac{\partial F}{\partial Z_2}\right)D_{Z_2}\left(\frac{\partial F}{\partial Z_2}\right)^T$$

$$S_Z = \begin{pmatrix} S_b & \left(\frac{\partial F}{\partial Z_1}\right)S_{a_1} & \left(\frac{\partial F}{\partial Z_2}\right)S_{a_2}\end{pmatrix}$$

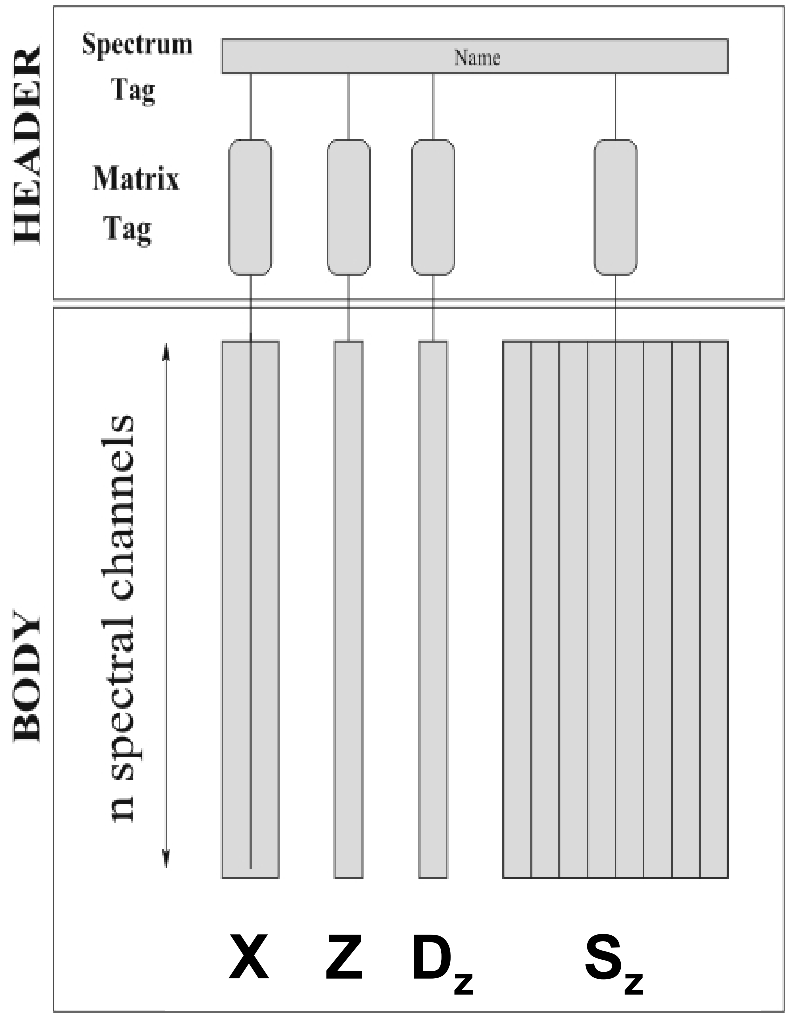
$$D_Z = \left(\frac{\partial F}{\partial Z_1}\right)D_{Z_1}\left(\frac{\partial F}{\partial Z_1}\right)^T + \left(\frac{\partial F}{\partial Z_2}\right)D_{Z_2}\left(\frac{\partial F}{\partial Z_2}\right)^T$$

$$\boxed{V_Z = S_Z\, S_Z^{\;T} + D_Z}$$

dimension: n x k
$k = k_b + k_1 + k_2$

n values (diagonal)

AGS, File format



Storage and calculus

Covariance matrix

n^2 elements (e.g. 32k x 8 bytes → 8 Gb !)

$2n^2$ mult. & sum. /step (20 steps → 8×10^{10} flops!)

AGS representation

n (k+1) elements (32k, 20 corr. → 5 Mb)

n (k+1) mult. & sum./step (20 steps → 7×10^3 flops)

AGS commands

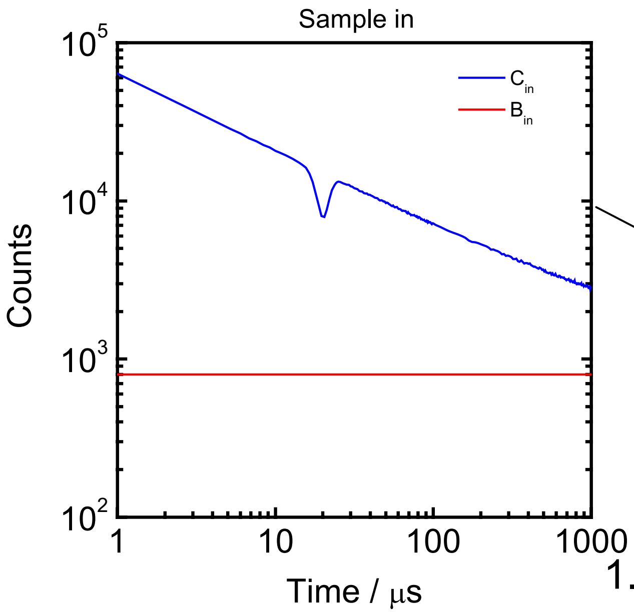
	Write only commands
`ags_mpty`	Create an empty AGS file
`ags_getA`	Import spectra from another AGS file
`ags_getE`	Import/interpolate evaluated data from an ENDF file
`ags_getXY`	Import histogram data from an ASCII file
	Read/Write commands : Operations on spectra
`ags_addval`	Add a constant value to all Y-values of a spectrum
`ags_avgr`	Average Y values per channel
`ags_func`	Calculates the Y values for a special function
`ags_idtc`	Determine the dead time correction of a TOF-spectrum
`ags_divi`	Divide a spectrum by another
`ags_mult`	Multiply a spectrum with another
`ags_lico`	Linear combination of n spectra with n constants
`ags_ener`	Build energy from TOF X-vector
`ags_fit`	Non-linear fit of spectra
`ags_fxyp`	User-programmed function
	Read Only commands
`ags_edit`	Edit constants and scalers attached to a spectrum
`ags_list`	List Y values of spectra with common X values
`ags_putX`	Export final result to an ASCII file
`ags_scan`	Scan the contents of an AGS file

AGS, Script for transmission data

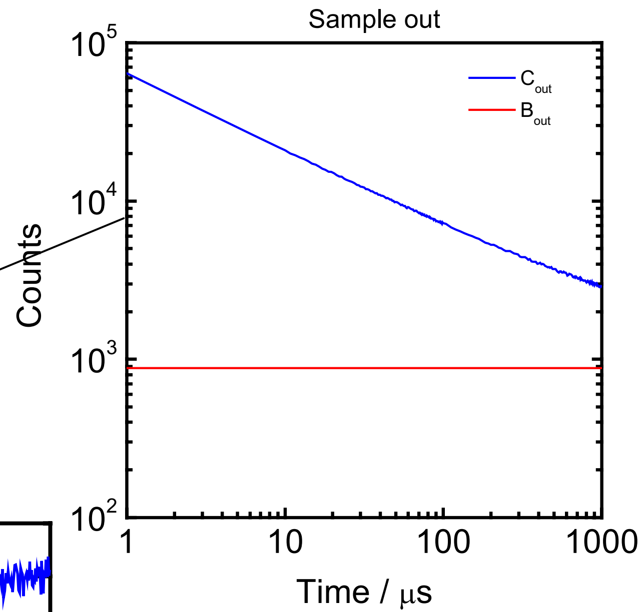

```
# create ags-file
ags_mpty TRFAK

# read sample out
scaler=TOout,CMout
ags_getXY TRFAK /SCALER=$scaler /FROM=spout.his /ALIAS=SOUT

# read sample in
scaler=TOin,CMin
ags_getXY TRFAK /SCALER=$scaler /FROM=spin.his /ALIAS=SIN /LIKE=A01SOUT

# dead time correction
dtcoef=DTCOEF
ags_idtc TRFAK,A01SOUT /DTIME=$dtcoef /LPSC=1
ags_idtc TRFAK,B01SIN /DTIME=$dtcoef /LPSC=1

# normalize to central monitor and divide by bin width
ags_avgr TRFAK,C01SOUT /CMSC=2
ags_avgr TRFAK,D01SIN /CMSC=2

#calculate background contribution
ags_func TRFAK /FUN=f01 /PARFILE=PAROUT /ALIAS=SBOUT /LIKE=A01SOUT
ags_func TRFAK /FUN=f01 /PARFILE=PARIN /ALIAS=SBIN /LIKE=A01SOUT

#subtract background
ags_lico TRFAK,E01SOUT,G01SBOUT /ALIAS=SOUTNET /PAR=1.0,-1.0
ags_lico TRFAK,F01SIN,H01SBIN /ALIAS=SINNET /PAR=1.0,-1.0

#create transmission factor
ags_divi TRFAK,I01SOUTNET,J01SINNET /ALIAS=TRFAK
```

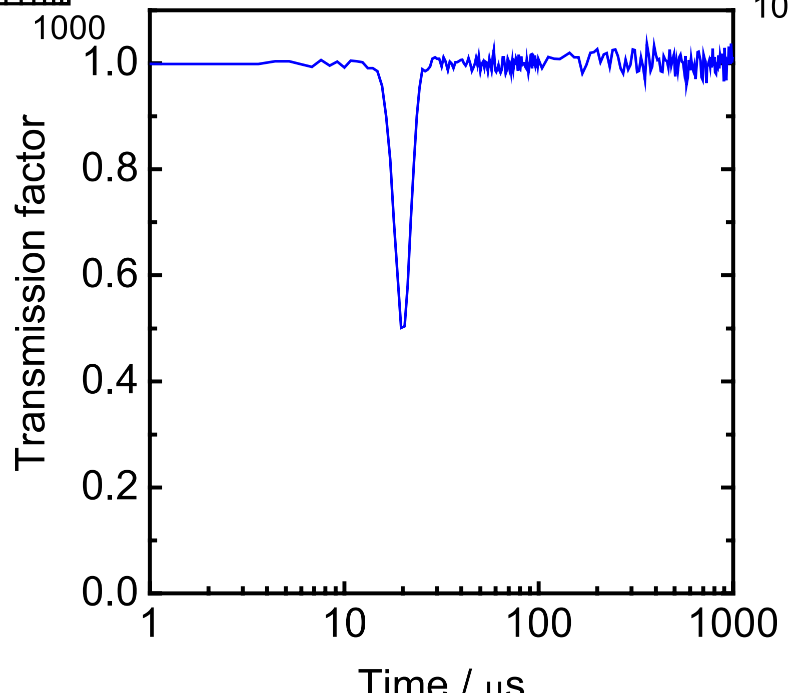
$$\boxed{T = \frac{C'_{in} - B'_{in}}{C'_{out} - B'_{out}}}$$

Calculation of transmission factor



$$T_{exp} = N \frac{C'_{in} - B'_{in}}{C'_{out} - B'_{out}}$$



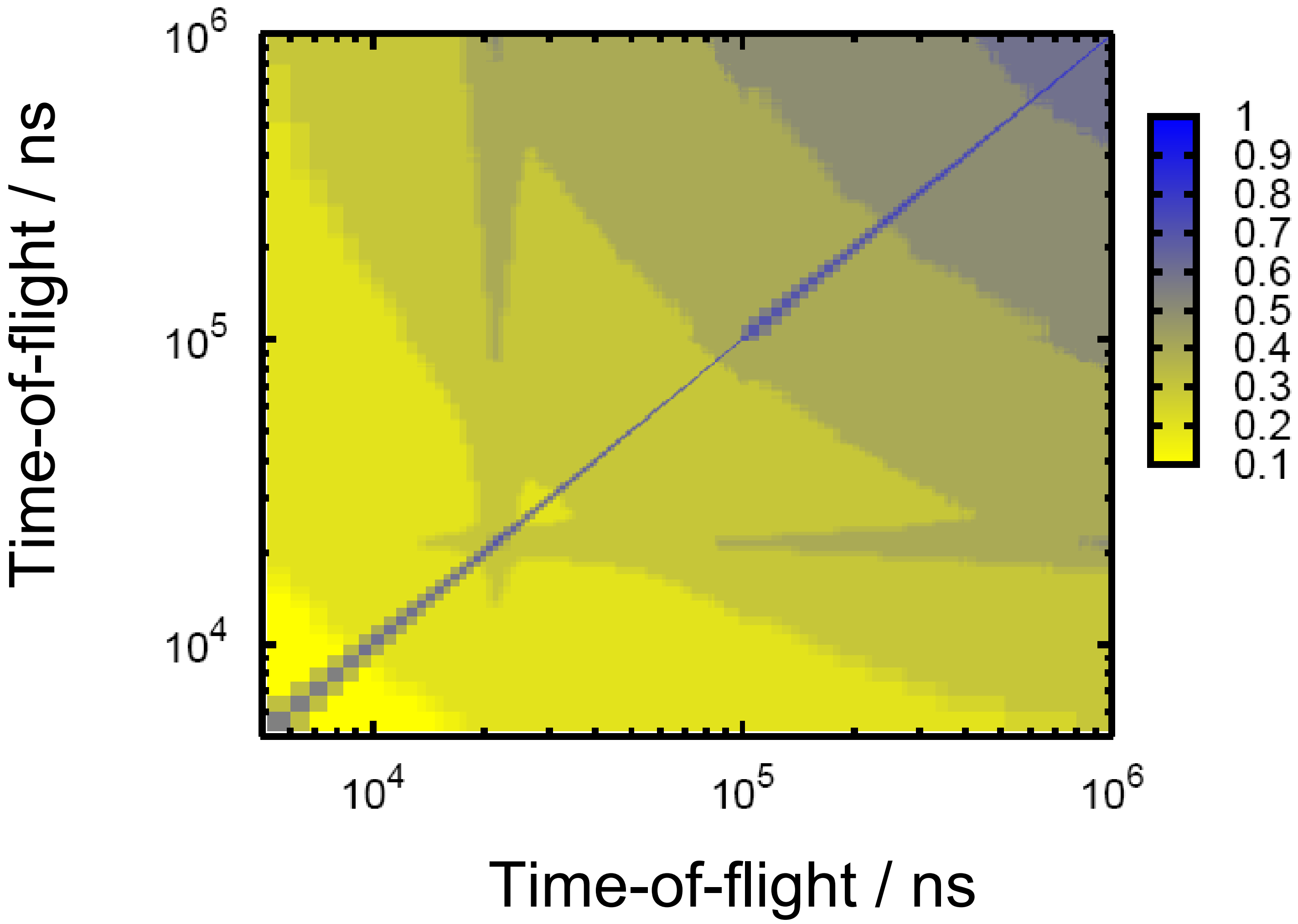


Output AGS_PUTX

$\delta B_{in} / B_{in}$: 10.0 %
$\delta B_{out} / B_{out}$: 5.0 %
$\delta N / N$: 0.5 %

X_L	X_H	Z	δZ	δZ_u	$V_Z = D_Z + S\,S^T$			
					D_Z	S		
					δZ_u^2	B_{in}	B_{out}	N
800	1600	0.999	0.79E-2	0.59E-2	0.35E-4	0.14E-2	-0.08E-2	0.50E-2
1600	2400	0.999	0.86E-2	0.67E-2	0.45E-4	0.18E-2	-0.10E-2	0.50E-2
2400	3200	0.999	0.92E-2	0.73E-2	0.54E-4	0.21E-2	-0.12E-2	0.50E-2
3200	4000	0.999	0.97E-2	0.78E-2	0.61E-4	0.24E-2	-0.13E-2	0.50E-2
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
16000	16800	0.899	1.30E-2	1.07E-2	1.15E-4	0.51E-2	-0.25E-2	0.45E-2
16800	17600	0.818	1.24E-2	1.02E-2	1.04E-4	0.53E-2	-0.24E-2	0.41E-2
17600	18400	0.701	1.15E-2	0.93E-2	0.86E-4	0.54E-2	-0.21E-2	0.35E-2
18400	19200	0.594	1.06E-2	0.84E-2	0.71E-4	0.55E-2	-0.18E-2	0.30E-2
19200	20000	0.501	0.98E-2	0.76E-2	0.57E-4	0.56E-2	-0.15E-2	0.25E-2
20000	20800	0.504	1.00E-2	0.77E-2	0.59E-4	0.57E-2	-0.16E-2	0.25E-2
20800	21600	0.581	1.09E-2	0.85E-2	0.73E-4	0.58E-2	-0.19E-2	0.29E-2
21600	22400	0.707	1.22E-2	0.98E-2	0.97E-4	0.60E-2	-0.23E-2	0.35E-2
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
964000	972000	0.999	5.91E-2	3.75E-2	14.06E-4	3.98E-2	-2.18E-2	0.50E-2
972000	980000	1.037	6.09E-2	3.89E-2	15.13E-4	4.04E-2	-2.31E-2	0.52E-2
980000	988000	1.001	6.01E-2	3.80E-2	14.46E-4	4.05E-2	-2.23E-2	0.50E-2
988000	996000	1.010	5.92E-2	3.77E-2	14.23E-4	3.96E-2	-2.20E-2	0.50E-2

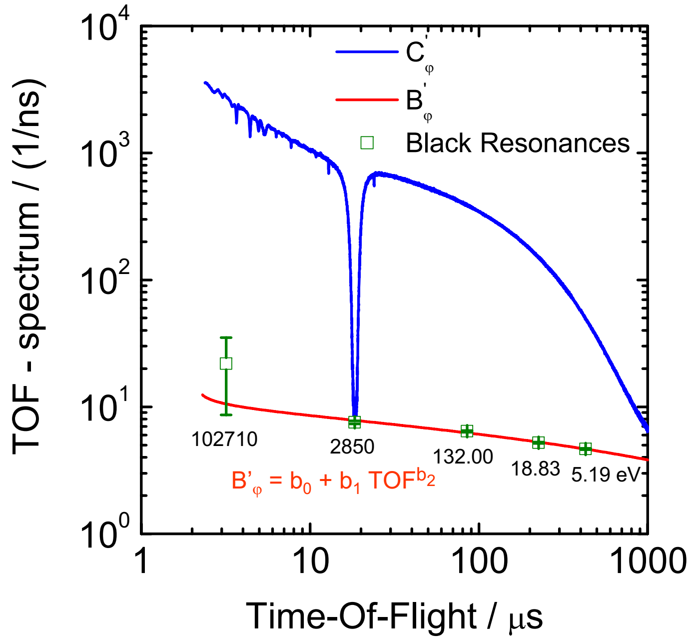
AGS Output -> Covariance matrix



EUROPEAN COMMISSION

Example : ^{232}Th(n,γ) in URR CRP IAEA "Th-U fuel cycle"

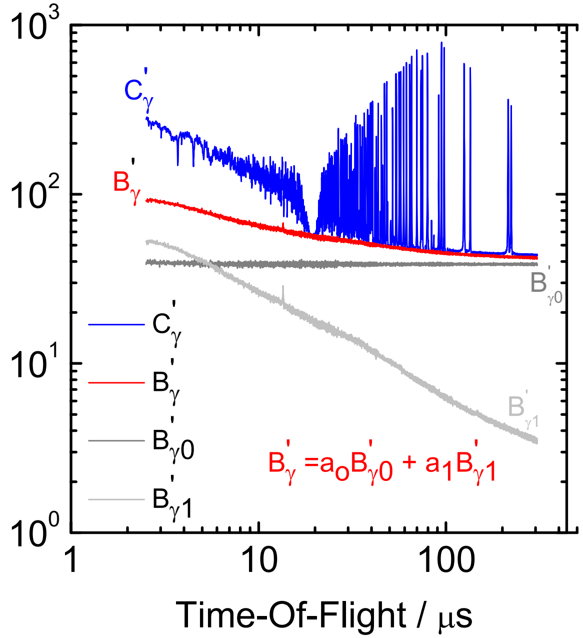
Institute for Reference Materials and Measurements



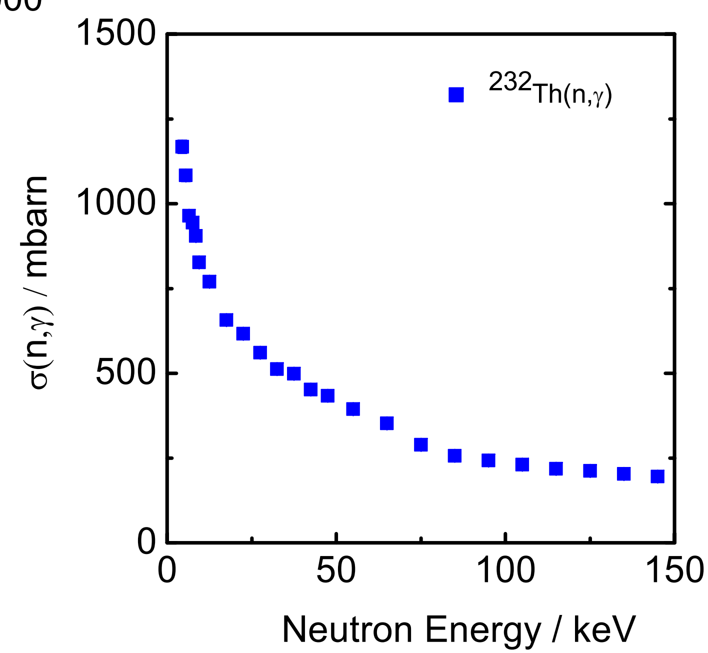
$$Y_{\gamma,exp} = N \frac{1}{\varepsilon_r} \frac{C'_\gamma - B_\gamma}{\varphi}$$

$$Y_{\gamma,exp} = N \frac{\sigma_\varphi}{\varepsilon_r} \frac{C'_\gamma - B_\gamma}{C'_\varphi - B_\varphi}$$

$$\sigma(n,\gamma) = F_c \; Y_{\gamma,exp}$$



A. Borella et al. NSE 152 (2006) 1-14



Example : ^{232}Th(n,γ) in URR

Correlated uncertainties due to background in capture data

E_{min} keV	E_{max} keV	σ_γ mb	$\delta\sigma_\gamma$ (%)	$\delta\sigma_{\gamma,u}$ (%)	ρ												
4	6	1107.9	0.49	0.17	1.00	0.60	0.57	0.58	0.55	0.53	0.62	0.58	0.56	0.61	0.58	0.54	0.51
6	8	934.2	0.44	0.19		1.00	0.55	0.56	0.53	0.51	0.60	0.57	0.55	0.60	0.56	0.53	0.49
8	10	845.1	0.43	0.21			1.00	0.54	0.51	0.49	0.58	0.55	0.52	0.57	0.54	0.51	0.47
10	15	749.1	0.38	0.15				1.00	0.52	0.50	0.59	0.56	0.54	0.59	0.55	0.52	0.49
15	20	638.7	0.39	0.18					1.00	0.48	0.57	0.54	0.52	0.56	0.53	0.50	0.47
20	30	571.3	0.36	0.14						1.00	0.55	0.52	0.50	0.54	0.51	0.48	0.45
30	40	490.3	0.32	0.18							1.00	0.61	0.59	0.64	0.61	0.57	0.54
40	50	429.6	0.31	0.19								1.00	0.56	0.61	0.58	0.54	0.51
50	60	382.9	0.33	0.22									1.00	0.59	0.56	0.52	0.49
60	80	311.4	0.30	0.18										1.00	0.61	0.57	0.54
80	100	242.5	0.33	0.22											1.00	0.54	0.51
100	120	217.8	0.33	0.23												1.00	0.48
120	140	201.6	0.33	0.24													1.00

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Example : ^{232}Th(n,γ) in URR

dead time, background + normalization (1.5%)

E_{min}	E_{max}	σ_γ	$\delta\sigma_\gamma$	$\delta\sigma_{\gamma,u}$	ρ												
keV	keV	mb	(%)	(%)													
4	6	1107.9	1.70	0.17	1.00	0.85	0.85	0.86	0.86	0.87	0.88	0.88	0.88	0.89	0.89	0.88	0.88
6	8	934.2	1.64	0.19		1.00	0.88	0.89	0.89	0.89	0.91	0.91	0.91	0.92	0.92	0.91	0.91
8	10	845.1	1.63	0.21			1.00	0.89	0.89	0.90	0.91	0.91	0.91	0.92	0.92	0.91	0.91
10	15	749.1	1.61	0.15				1.00	0.90	0.91	0.93	0.92	0.93	0.93	0.93	0.93	0.92
15	20	638.7	1.61	0.18					1.00	0.91	0.92	0.92	0.93	0.93	0.93	0.93	0.92
20	30	571.3	1.59	0.14						1.00	0.93	0.93	0.93	0.94	0.93	0.93	0.93
30	40	490.3	1.56	0.18							1.00	0.95	0.95	0.96	0.95	0.95	0.95
40	50	429.6	1.56	0.19								1.00	0.95	0.96	0.95	0.95	0.95
50	60	382.9	1.55	0.22									1.00	0.96	0.96	0.95	0.95
60	80	311.4	1.55	0.18										1.00	0.96	0.96	0.96
80	100	242.5	1.56	0.22											1.00	0.96	0.95
100	120	217.8	1.55	0.23												1.00	0.95
120	140	201.6	1.55	0.24													1.00

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Importance of covariance data

Application of reaction model to deduce model parameters:

REFIT, SAMMY	$\Rightarrow$	**resolved resonance parameters**
FITACS	$\Rightarrow$	**average resonance parameters**

Minimize χ^2 as a function of parameters a (resonance parameters)

$$\chi^2(\vec{a}) = (\vec{z}_{exp} - \vec{z}_M(\vec{a}))^T \underline{V}_{z,exp}^{-1} (\vec{z}_{exp} - \vec{z}_M(\vec{a}))$$

Covariance of parameters $\vec{a}$

$$\underline{V}_a = (\underline{G}_a^{\,T} \, \underline{V}_{z,exp}^{-1} \, \underline{G}_a)^{-1}$$

Covariance of calculated Z_M

$$V_{z,M} = \underline{G}_a \, \underline{V}_a \, \underline{G}_a^{\,T}$$

Average resonance parameters from data in URR

Z_{exp}	:	$<\sigma_\gamma>$	average capture cross section
Model	:	Generalized SLBW in URR	
$\vec{a}$	:	Γ_0 and Γ_1	average radiation width for $\ell = 0$, 1

	$\delta Z_{exp} = 1.6\%$ and $\rho = 0.0$ $\delta Z_u = 1.6$ % $\delta Z_c = 0.0$ %	$\delta Z_{exp} = 1.6\%$ and $\rho = 0.9$ $\delta Z_u = 0.5$ % $\delta Z_c = 1.5$ %
Γ_0 Γ_1	1.18 % 1.00 - 0.74 0.95 % 1.00	1.81 % 1.00 0.88 0.97 % 1.00
E_n / keV	$\delta\sigma_{\gamma,M}$ (%)	$\delta\sigma_{\gamma,M}$ (%)
5	0.48	0.88
50	0.33	0.80
100	0.35	0.97

Sirakov et al., Annals of Nuclear Energy, 2008doi:10.1016/j.anucene.2007.12.008,

Summary

- **Error ≠ uncertainty**
  - Random error : expectation value = 0
  - Systematic error (correction factor) : expectation value = 0
- **All uncertainties are statistical**
- **Evaluation of uncertainties**
  - Type A : statistical analysis of repeated measurements
  - Type B : scientific judgment
- **Importance of covariance information**
- **Quote all components involved in the data reduction process which create correlated uncertainties**
- **AGS, reduction of TOF-data with propagation of correlated and uncorrelated uncertainties including full reporting of the reduction process**