

Combinatorial properties of the simplest Markov partitions for a 2-torus

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We consider a classical example of hyperbolic system, an Anosov linear diffeomorphism of the 2-torus:

$$T: x \mapsto Ax \pmod{\mathbb{Z}^2},$$

where $A \in GL_2(\mathbb{Z})$ is hyperbolic.

This system (with Lebesgue measure) is conjugated to a Markov chain. Proof is based on a construction of a *Markov partition* $\mathbb{T}^2 = M_1 \sqcup M_2 \sqcup \dots \sqcup M_k$ where M_j are parallelograms and $TM_i \cap M_j$ consists of not more than one connectivity component.

This partition is obtained from the pre-Markov partition $\mathbb{T}^2 = P_1 \sqcup P_2$, that is a pair of parallelograms with the following conditions: segment of its boundary parallel to unstable eigenvector maps into itself by T and segment parallel to stable one maps into itself by T^{-1} . In this case connectivity components of $TP_i \cap P_j$ form Markov partition.

It is natural to say that two such pre-Markov partitions are equivalent if one is mapped into another by $B \in GL_2(\mathbb{Z})$ with B commuting with A .

In the talk we shall describe structure of the set of equivalence classes for pre-Markov partitions and show its relation to a continuous fraction representation for slope of eigenvectors for A .