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Title:

Reversible systems verus Hamiltonian systems in R^4

Abstract:

Consider a class of vector fields having the form $x^{\cdot} = f(x)$, x 2 R4 where f(x) is a smooth function, f(0) = 0. The vector field (1) is called time-reversible if there is a linear involution R 2 L(R4), R2 = Id satisfying the relation f(R(x)) = −R(f(x)), x 2 R4. We assume that Fix(R) = {x 2 R4; Rx = x} is a 2-dimensional manifold. We show that normal forms of reversible vector fields in R4 at an elliptic equilibrium possess a formal Hamiltonian structure. In the non-resonant case we establish a formal conjugacy between reversible and Hamiltonian normal forms. Moreover, it is generically formally orbitally five-jet determined. In the more general case of p:q resonance we obtain formal orbital equivalence between reversible and Hamiltonian normal forms, which involve an additional time-reparametrization of orbits. In addition, we show that in the p:q resonance with p:q2 {1:1 or p+q>4} reversible normal forms are not formally conjugate to a Hamiltonian vector field.

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