A SCATTERING MAP FOR THE INTERACTION OF TWO CHARGES IN A UNIFORM MAGNETIC FIELD

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The interaction of two charges moving in $\mathbb{R}^3$ in a magnetic field $\boldsymbol{B}$ can be formulated as a Hamiltonian system with six degrees of freedom. Assuming that the magnetic field is uniform and the interaction potential has rotation symmetry we reduce this system to one with three degrees of freedom. For special values of the conserved quantities, choices of parameters or restriction to the coplanar case, we obtain systems with two degrees of freedom. Specialising to the case of Coulomb interaction, these reductions enable us to obtain many qualitative features of the dynamics. For charges of the same sign, the gyrohelices either "bounce–back", "pass–through", or exceptionally converge to coplanar solutions. For charges of opposite signs, we decompose the state space into "free" and "trapped" parts with transitions only when the particles are coplanar. A scattering map is defined for those trajectories which come from and go to infinite separation along the field direction. It determines the asymptotic parallel velocities, guiding centre field lines, magnetic moments and gyrophases for large positive time from those for large negative time. In regimes where gyrophase averaging is appropriate, the scattering map has a simple form, conserving the magnetic moments and parallel kinetic energies (in a frame moving along the field with the centre of mass) and rotating or translating the guiding centre field lines. When the gyrofrequencies are in low order resonance, however, gyrophase averaging is not justified and transfer of perpendicular kinetic energy is shown to occur. In the extreme case of equal gyrofrequencies an additional integral helps us to analyse further and prove that there is typically also transfer between perpendicular and parallel kinetic energy.

References

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