The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA
International Atomic Energy Agency

1951-7

**Workshop on the original of P, CP and T Violation**

*2 - 5 July 2008*

# Possible CPT Violation in the Neutrino Sector under Gravitational Field and its Implication

B. MUKHOPADHYAY

*Indian Institute of Science, Department of Physics*
*Bangalore 560 012, INDIA*

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

# Possible CPT violation in the Neutrino Sector under gravitational field and its implication

***Banibrata Mukhopadhyay***

Department of Physics

**Indian Institute of Science**

# Motivations

- Several astrophysical and cosmological problems are involved with neutrino physics.
- Examples: leptogenesis and then baryogenesis, neutrino cooled accretion disks around black holes, r-process nucleosynthesis in supernova explosions etc.
- For successful description of any such scenario one must investigate neutrinos in curved spacetime.
- One should start with Dirac equation in curved spacetime.

# Dirac Lagrangian in curved spacetime

$$\mathcal{L} = \sqrt{-g}\left(\frac{i}{2}\Psi\gamma^a \overleftrightarrow{D}_a \Psi - \Psi m \Psi\right),$$

where the covariant derivative is

$$D_a = \left(\partial_a - \frac{i}{4}\omega_{bca}\sigma^{bc}\right)$$

and the spin-connections are

$$\omega_{bca} = e_{b\lambda}\left(\partial_a e^{\lambda}{}_c + \Gamma^{\lambda}_{\gamma\mu} e^{\gamma}_c e^{\mu}_a\right).$$

Here

$$\sigma^{bc} = \frac{i}{2}\left[\gamma^b, \gamma^c\right].$$

Thus the Lagrangian can be rewritten as

$$\mathcal{L} = det(e)\,\bar{\Psi}\left(\frac{i}{2}\gamma^a \overleftrightarrow{\partial}_a - m + \gamma^a\gamma^5 B_a\right)\Psi,$$

with

CPT violating

$$B^d = \epsilon^{abcd}\omega_{bca}.$$

In em field

$$D_a = (\partial_a - ieA_a)$$

$$\bar{\Psi}\left(\frac{i}{2}\gamma^a\partial_a - m + e\gamma^a A_a\right)\Psi$$

Mohanty, BM, Prasanna 2002
BM 2005
Debnath, BM, Dadhich 2006

# Majorana neutrino

Standard model: neutrino is solely left-handed

antineutrino is solely right-handed

Particle is self-conjugate: its own antiparticle

$$\psi = \psi_L + \psi_R = \psi_L + \psi_L{}^c = \psi^c$$

In Weyl representation for a left handed neutrino $\Psi = \begin{pmatrix} \psi_L^c \\ \psi_L \end{pmatrix}$

Mass term: $m\left(\bar{\Psi}_L \Psi_{L^c} + \bar{\Psi}_{L^c} \Psi_L\right)$

violates lepton number

# Majorana neutrino

$$\mathcal{L} = \sqrt{-g}\left[\left(i\overline{\psi}_L\gamma^a\partial_a\psi_L + i\overline{\psi^c_L}\gamma^a\partial_a\psi^c_L\right) - m\left(\overline{\psi^c_L}\psi_L + \overline{\psi}_L\psi^c_L\right) + \left(\overline{\psi^c_L}\gamma^a\psi^c_L - \overline{\psi}_L\gamma^a\psi_L\right)B_a\right]$$

CPT violating

$$(-g)^{-1/2}\mathcal{L} = \begin{pmatrix}\psi^{c\dagger} & \psi^\dagger\end{pmatrix}\frac{i}{2}\gamma^0\gamma^\mu\overleftrightarrow{\mathcal{D}}_\mu\begin{pmatrix}\psi^c\\ \psi\end{pmatrix} + \begin{pmatrix}\psi^{c\dagger} & \psi^\dagger\end{pmatrix}\gamma^5 B_0\begin{pmatrix}\psi^c\\ \psi\end{pmatrix} - \begin{pmatrix}\psi^{c\dagger} & \psi^\dagger\end{pmatrix}\gamma^0 m\begin{pmatrix}\psi^c\\ \psi\end{pmatrix}$$

where $\mathcal{D}_\mu \equiv (\partial_0, \partial_i + \gamma^5 B_i).$

**Dispersion relation $(p^a \pm B^a)^2 = m^2$**

$$E_\nu = \sqrt{(\vec{p}-\vec{B})^2 + m^2} + B_0,$$

$$E_{\nu^c} = \sqrt{(\vec{p}+\vec{B})^2 + m^2} - B_0$$

Debnath, BM, Dadhich 2006
BM 2007

# Neutrino equation in curved spacetime

The Euler-Lagrange equation in two-component form for a Majorana neutrino

$$(-g)^{-1/2}\mathcal{L} = \begin{pmatrix}\psi^{c\dagger} & \psi^\dagger\end{pmatrix}\frac{i}{2}\gamma^0\gamma^\mu\overleftrightarrow{\mathcal{D}}_\mu\begin{pmatrix}\psi^c\\ \psi\end{pmatrix} + \begin{pmatrix}\psi^{c\dagger} & \psi^\dagger\end{pmatrix}\gamma^5 B_0\begin{pmatrix}\psi^c\\ \psi\end{pmatrix} - \begin{pmatrix}\psi^{c\dagger} & \psi^\dagger\end{pmatrix}\gamma^0 m\begin{pmatrix}\psi^c\\ \psi\end{pmatrix}$$

where $\mathcal{D}_\mu \equiv (\partial_0, \partial_i + \gamma^5 B_i).$

Mass matrix: $\mathcal{M} = \begin{pmatrix}-B_0 & -m\\ -m & B_0\end{pmatrix}$

$$m_{(e,\mu)1} = -\sqrt{B_0^2 + m_{e,\mu}^2},$$
$$m_{(e,\mu)2} = \sqrt{B_0^2 + m_{e,\mu}^2}$$

In presence of a Majorana mass, they appear as mixed states: their mass eigenstates are linear combination of neutrino and antineutrino states.

Similar to neutral kaon.

$$|\nu_1\rangle = \frac{1}{N}\{(B_0 + \sqrt{B_0^2 + m^2})|\psi^c\rangle + m|\psi\rangle\},$$
$$|\nu_2\rangle = \frac{1}{N}\{-m|\psi^c\rangle + (B_0 + \sqrt{B_0^2 + m^2})|\psi\rangle\}$$

Neutrino-antineutrino states couple together with modified energy

$$E_\nu = \sqrt{(\vec{p} - \vec{B})^2 + m^2} + B_0,$$
$$E_{\nu^c} = \sqrt{(\vec{p} + \vec{B})^2 + m^2} - B_0$$

Due to difference in energy (effective mass), their time evolution is different: Results in possible asymmetry and oscillation.

BM 2007; Sinha, BM 2008

# Neutrino Asymmetry

1) Possible under gravity if spacetime deviates from spherical symmetry
2) Lagrangian must be CPT violating: B is a constant or an even function of space-time
3) Lepton number violating interactions must be present: If processes take place during GUT or inflation when primordial fluctuations are present:

$$\Delta n = \frac{g}{(2\pi)^3} \int d^3\mathbf{p} \left[ \frac{1}{1+\exp(E_\nu/T)} - \frac{1}{1+\exp(E_{\bar{\nu}}/T)} \right]$$

$$\Delta n \sim g T^3 \left( \frac{\bar{B}_0}{T} \right)$$

Asymmetry per entropy is proportional to gravitational scalar potential

Mohanty, BM, Prasanna 2002
Singh, BM 2003
BM 2005

# Gravity Wave perturbation in Early Universe

$$g_{\mu\nu} = a(\tau)^2 \begin{pmatrix} 1+2\phi & -\omega_1 & -\omega_2 & -\omega_3 \\ -\omega_1 & -(1+2\psi)+h_+ & h_\times & 0 \\ -\omega_2 & h_\times & -(1+2\psi)-h_+ & 0 \\ -\omega_3 & 0 & 0 & -(1+2\psi) \end{pmatrix}$$

- The space-time metric:

  $ds^2 = -(1 + 2\,\phi)\,dt^2 - \omega_i\,dx^i\,dt + a(t)^2\,[(1 + 2\,\Psi - h_+)\,dx^2$

  $+ (1 + 2\,\Psi + h_+)\,dy^2 - 2\,h_x\,dx\,dy + (1 + 2\,\Psi)\,dz^2]$

- Components of gravitational coupling:

  $B^0 = \partial_z\,h_x,\ B^i = (\partial \times \omega)^i + \partial_t\,h_x\,\delta^{iz}$

$$\langle B_0 \rangle \equiv B_0 \simeq A_\times k \simeq A_\times \left(1.66 g_*^{1/2} \frac{T^2}{M_{Pl}}\right)$$

At GUT: $T \sim 10^{15}$ GeV

$M_{pl} \sim 10^{19}$ GeV

$B_0 \sim A_x\,10^{12}$ GeV

For $A_x \sim 10^{-7}$, $\Delta n/s \sim 10^{-10}$

(COBE)

Mohanty, BM, Prasanna 2002

Sinha, BM 2008

# Anisotropic phase of Early Universe

- Bianchi II model: Axially symmetric Universe

  $ds^2 = -dt^2 + S(t)^2 dx^2 + R(t)^2 [dy^2 + f(y)^2 dz^2]$

  $– S(t)^2 h(y) [2dx – h(y) dz] dz$

  where $f(y) = y$ and $h(y) = -y^2/2$

- At a particular situation: $R \sim (t/t_0)^{1/2}$, S is a constant

$$B_0 \sim \frac{S^2}{y}\left(\frac{t_0}{t}\right)$$

- "Mass varying neutrinos?"

Sinha, BM 2008

# Accretion disk around a rotating compact object

- Kerr geometry:
  $ds^2 = \eta_{ij}\, dx^i\, dx^j - [2\alpha/\rho\, s_i\, v_j + \alpha^2\, v_i\, v_j]\, dx^i\, dx^j$

$$\alpha = \frac{\sqrt{2Mr}}{\rho},\ \rho^2 = r^2 + \frac{a^2 z^2}{r^2},$$

$$v_i = \left(1,\ \frac{ay}{a^2+r^2},\ \frac{-ax}{a^2+r^2},\ 0\right),\ s_i = \left(0,\ \frac{rx}{\sqrt{r^2+a^2}},\ \frac{ry}{\sqrt{r^2+a^2}},\ \frac{z\sqrt{r^2+a^2}}{r}\right)$$

- Gravitational scalar potential coupling:

$$B^0 = e_{1\lambda}\left(\partial_3 e_2^\lambda - \partial_2 e_3^\lambda\right) + e_{2\lambda}\left(\partial_1 e_3^\lambda - \partial_3 e_1^\lambda\right) + e_{3\lambda}\left(\partial_2 e_1^\lambda - \partial_1 e_2^\lambda\right) = -\frac{4a\sqrt{M}z}{\bar{\rho}^2\sqrt{2r^3}}$$

where $\sigma^2 = 2\,r^2 + a^2 - x^2 - y^2 - z^2$

BM 2005
BM 2007

# Bounds on $B_a$

- In nonrelativistic limit, gravitational interaction becomes: s.B
  ≡ interaction between fermion spin and external field
- Eot-Wash II experiment (Adelberger et al. 1999): macroscopic number of fermions can be polarized in same direction → $\Delta E=|B|$: difference in energy between fermion spins polarized parallel and antiparallel to B
- This measures B upto $10^{-28}$GeV (Bluhm, Kostelecky 2000)

❑ Measuring net magnetization in a paramagnetic materials using a squid (Ni et al. 1999): External field B appears as an effective magnetic field $B_{eff} = B/\mu_B$

❑ $B_{eff}$ can be probed in this experiment ~ $10^{-12}$ G → B ~ $10^{-29}$ GeV

➢ In an inner accretion disk around a $10M_{Sun}$ black hole: B ~ $10^{-23}$ GeV

➢ In a satellite orbiting Earth with $v_\Phi$ ~ 1 km/sec: B ~ $10^{-37}$ GeV

# Oscillations in $\nu_1$ and $\nu_2$

**This is very similar to neutral kaon anti-kaon oscillation**

$$|\nu_1\rangle = \frac{1}{N}\{(B_0 + \sqrt{B_0^2 + m^2})|\psi^c\rangle + m|\psi\rangle\},$$
$$|\nu_2\rangle = \frac{1}{N}\{-m|\psi^c\rangle + (B_0 + \sqrt{B_0^2 + m^2})|\psi\rangle\}$$

$$E_\nu = \sqrt{(\vec{p} - \vec{B})^2 + m^2} + B_0,$$
$$E_{\nu^c} = \sqrt{(\vec{p} + \vec{B})^2 + m^2} - B_0$$

Oscillation probability $\mathcal{P}(t) = \frac{m^2}{B_0^2 + m^2}\sin^2\{(B_0 - |\vec{B}|)t\}$

Oscillation length $\lambda = \frac{\pi}{B_0 - |\vec{B}|}$

- **Considering neutrinos coming out off inner accretion disks around a black hole of mass $10M_{Sun}$: Length ~ 10km ~ Schwarzschild radius**
- **For a supermassive black hole of mass $10^8M_{Sun}$: Length ~ $10^9$km**

BM 2007
Sinha, BM 2008

# Flavor Mixing

- Neutrino Lagrangian under gravity:

$$(-g)^{-1/2}\mathcal{L} = (\psi^{c\dagger}\psi^{\dagger})\frac{i}{2}\gamma^0\gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} + (\psi^{c\dagger}\psi^{\dagger})\gamma^5 B_0 \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} - (\psi^{c\dagger}\psi^{\dagger})\gamma^0 m \begin{pmatrix} \psi^c \\ \psi \end{pmatrix}$$

- The modified masses:

$$\mathcal{M} = \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix}$$

$$m_{(e,\mu)1} = -\sqrt{B_0^2 + m_{e,\mu}^2},$$
$$m_{(e,\mu)2} = \sqrt{B_0^2 + m_{e,\mu}^2}$$

$$|\nu_1\rangle = \frac{1}{N}\{(B_0 + \sqrt{B_0^2 + m^2})|\psi^c\rangle + m|\psi\rangle\},$$
$$|\nu_2\rangle = \frac{1}{N}\{-m|\psi^c\rangle + (B_0 + \sqrt{B_0^2 + m^2})|\psi\rangle\}$$

In presence of a Majorana mass, neutrinos appear as mixed states: their mass eigenstates are linear combination of neutrino and antineutrino.

$$(-g)^{-1/2}\mathcal{L}_m = -\frac{1}{2}\Big(\nu_{e1}^\dagger m_{e1}\nu_{e1} + \nu_{e2}^\dagger m_{e2}\nu_{e2} + \nu_{\mu1}^\dagger m_{\mu1}\nu_{\mu1} + \nu_{\mu2}^\dagger m_{\mu2}\nu_{\mu2}$$
$$+ \nu_{\mu1}^\dagger m_{\mu e}\nu_{e1} + \nu_{\mu2}^\dagger m_{\mu e}\nu_{e2} + \nu_{e1}^\dagger m_{\mu e}\nu_{\mu1} + \nu_{e2}^\dagger m_{\mu e}\nu_{\mu2}\Big).$$

- Mass Lagrangian of $\nu_\mu$, $\nu_e$ mixed by a Majorana mass $m_{e\mu}$

Sinha, BM 2008

# Effect of modified mass

- Modified masses and mass eigenstates

Thus we obtain all together four mass eigenstates $\chi_1$, $\chi_2$, $\chi_3$ and $\chi_4$ described as

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \mathcal{F}_1^\dagger \begin{pmatrix} \nu_{e1} \\ \nu_{\mu 1} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \chi_3 \\ \chi_4 \end{pmatrix} = \mathcal{F}_2^\dagger \begin{pmatrix} \nu_{e2} \\ \nu_{\mu 2} \end{pmatrix}$$

$$\mathcal{F}_{1,2} = \begin{pmatrix} \cos\phi_{1,2} & -\sin\phi_{1,2} \\ \sin\phi_{1,2} & \cos\phi_{1,2} \end{pmatrix}.$$

$$\tan\phi_{1,2} = \frac{m_{\mu e}}{m_{i(1,2)} + \sqrt{m_{i(1,2)}^2 + m_{\mu e}^2}}$$

$$2m_{i(1,2)} = m_{\mu(1,2)} - m_{e(1,2)}$$

Sinha, BM 2008

# Flavor Oscillation

- Flavor states are described by

$$\begin{aligned} |\nu_{e1}\rangle &= \cos\phi_1|\chi_1\rangle - \sin\phi_1|\chi_2\rangle \\ |\nu_{\mu 1}\rangle &= \sin\phi_1|\chi_1\rangle + \cos\phi_1|\chi_2\rangle \end{aligned}$$

and

$$\begin{aligned} |\nu_{e2}\rangle &= \cos\phi_2|\chi_3\rangle - \sin\phi_2|\chi_4\rangle \\ |\nu_{\mu 2}\rangle &= \sin\phi_2|\chi_3\rangle + \cos\phi_2|\chi_4\rangle . \end{aligned}$$

- Oscillation probability $\mathcal{P}_{fg} = \sin^2 2\phi \sin^2 \delta_{fg}(t)$

- In original Ψ $\mathcal{P}_{ig} = \sin^2 2\phi \left\{ (\sin\theta_e \sin\theta_\mu + \cos\theta_e \cos\theta_\mu)^2 \sin^2\left(\frac{\Delta M^2}{4E} t\right)\right\}$

$$\delta_{fg} = \frac{\Delta M^2}{4E} = \frac{|M_1{}^2 - M_2{}^2|t}{4E} = \frac{|M_3{}^2 - M_4{}^2|t}{4E}, L_{osc} = \frac{4\pi E}{\Delta M^2}$$

$$\Delta M^2 = \left(\sqrt{B_0^2 + m_\mu^2} + \sqrt{B_0^2 + m_e^2}\right) \sqrt{\left\{\left(\sqrt{B_0^2 + m_\mu^2} - \sqrt{B_0^2 + m_e^2}\right)^2 + 4m_{e\mu}^2\right\}}$$

- $B_0$ very large: $\Delta M^2 \longrightarrow 4B_0 m_{e\mu}$ : situation of GUT scale

Sinha, BM 2008

# Early Universe: GUT

- At GUT: $t \sim 10^{-35}$ sec, $B_0 \sim 10^{45}$ eV >> $m_e$, $m_\mu$, $m_{e\mu}$ ($\sim 10^{-2}$ eV)

$$P = 0.999 \sin^2\left(\frac{1.4\times 10^8 eV^2 \sec}{4 E \hbar}\right)$$

- Oscillation is completely controlled by gravity
- Oscillation takes place vigorously
- Enormous muon neutrinos produce: not understood if we don't consider gravity effect

Sinha, BM 2008

# Early Universe: Nucleosynthesis

- At BBN: t ~ 1 sec, $B^0 > 10^{-8}$ eV

  choose $B_0 \sim 5 \times 10^{-2}$ eV ~ $m_e$, $m_\mu$, $m_{e\mu}$

$$P = 0.999 \sin^2\left(\frac{7\times 10^{-4} eV^2 \sec}{4 E \hbar}\right)$$

- For TeV neutrinos gravity effect increases Probability two orders of magnitude, while for thermal neutrinos it is 1.5 times.

Sinha, BM 2008

# Around Primordial Black Holes

- Gravitational coupling: $$B^0 = -\frac{4a\sqrt{M}\,z}{\sigma^2\sqrt{2r}}$$

  when $\sigma^2 = 2r^2 + a^2 - x^2 - y^2 - z^2$

- Consider neutrinos at around 20 Schwarzschild radius around a black hole of $M \sim 10^{22}$ gm; $B_0 \sim 5\times10^{-2}$ eV

$$P = 0.999\sin^2\left(\frac{7\times10^{-4}eV^2 t}{4E\hbar}\right)$$

$$L = \frac{4\pi E\hbar c}{7\times10^{-4}eV^2}$$

L for thermal neutrinos decreases to 0.54cm from 4.6cm obtained without gravity

BM 2007
Sinha, BM 2008

# Summary

- Neutrino couples to spacetime curvature, violating CPT with a suitable gravitational background.
- This results in possible neutrino-antineutrino asymmetry and then leptogenesis/baryogenesis. This also leads to neutrino-antineutrino oscillation.
- Early curved Universe in presence of lepton number violating GUT processes is a very feasible situation to have this occurred.
- Conditions for asymmetry:
  1) Space-time must NOT be spherical symmetric.
  2) Background curvature coupling is either a constant or an even function of space-time.
  3) Lepton number violating interactions must be present.
- Gravitational field modifies mass (mass matrix) of neutrinos, mixing particle and antiparticle. This generates neutrino mass varying with time.
- Once gravity modifies mass of neutrinos, it severely affects flavor oscillation probability as well, and corresponding oscillation length.
- This possibly generates lots of high energetic muon neutrinos from electron neutrinos.