



*The Abdus Salam*  
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**1951-13**

**Workshop on the original of P, CP and T Violation**

***2 - 5 July 2008***

**Minimal SO(10) Splits Supersymmetry.**

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# **MINIMAL $SO(10)$ SPLITS SUPERSYMMETRY\***

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**CPT@ICTP**

July 3, 2008

**\*Work in progress in collaboration with Borut Bajc and Miha Nemevšek.**

# **OUTLINE**

- THE MODEL: MAIN FEATURES**
- LOW SCALE SUSY AND  $V$  MASS**
- SPLIT SUSY AND  $V$  MASS  
(EXPERIMENTAL SIGNATURES)**
- CONCLUSIONS**

# MINIMAL SUSY $SO(10)$ – MSGUT\*

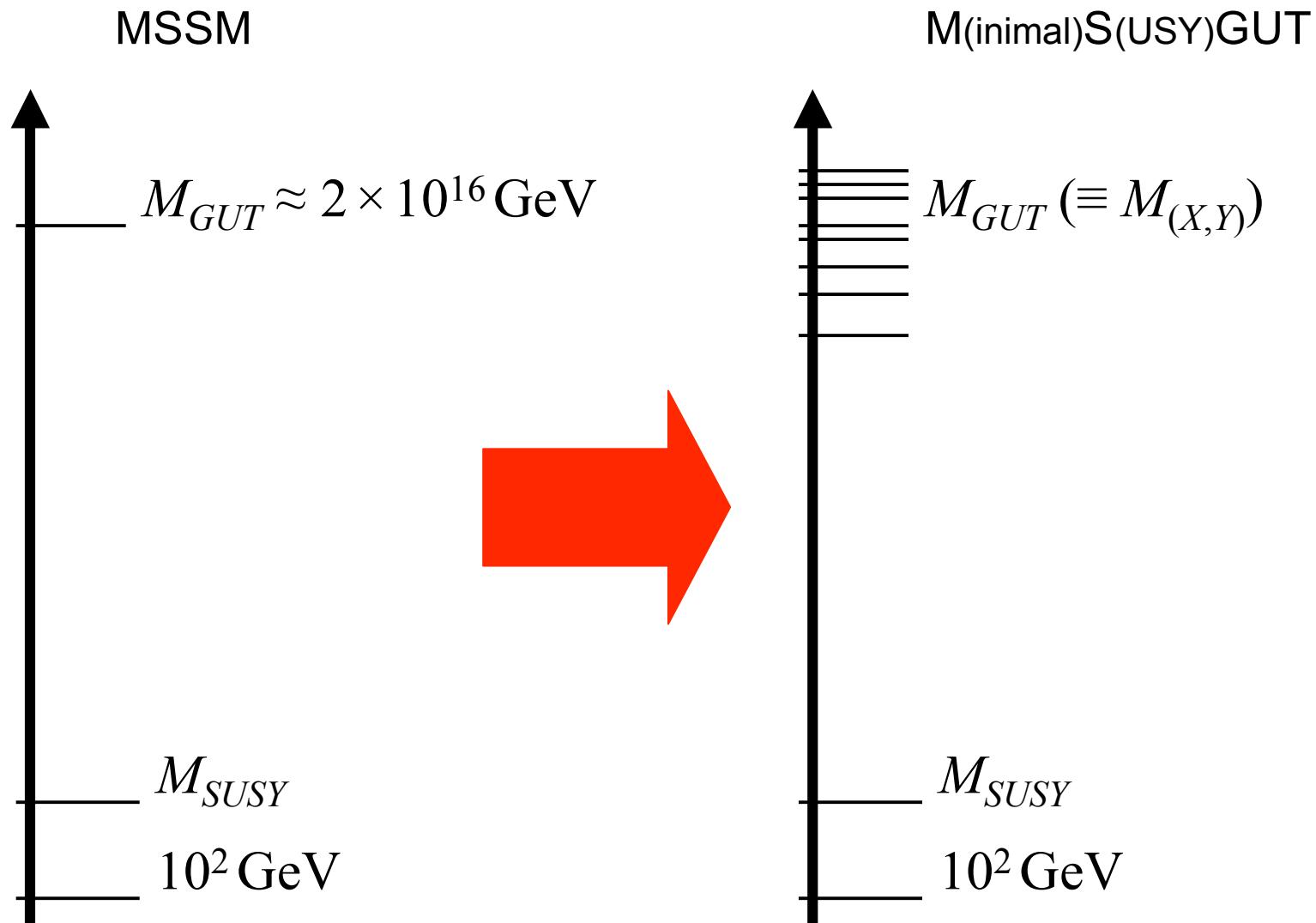
**REPRESENTATIONS:**  $16_{F,i}$ , ( $i = 1, 2, 3$ )     $210_H$      $126_H$      $\overline{126}_H$      $10_H$

$$W_H = \frac{m}{4!} 210_H^2 + \frac{\lambda}{4!} 210_H^3 + \frac{M}{5!} 126_H \overline{126}_H + \frac{\eta}{5!} 126_H 210_H \overline{126}_H + m_H^2 {10_H}^2 + \frac{1}{4!} 210_H 10_H (\alpha 126_H + \overline{\alpha} \overline{126}_H)$$

(unification)

\*T. E. Clark, T. K. Kuo and N. Nakagawa, Phys. Lett. B 115 (1982) 26; K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 70 (1993) 2845; C. S. Aulakh, B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, Phys. Lett. B 588 (2004) 196; ...

# UNIFICATION IN MINIMAL $SO(10)^*$

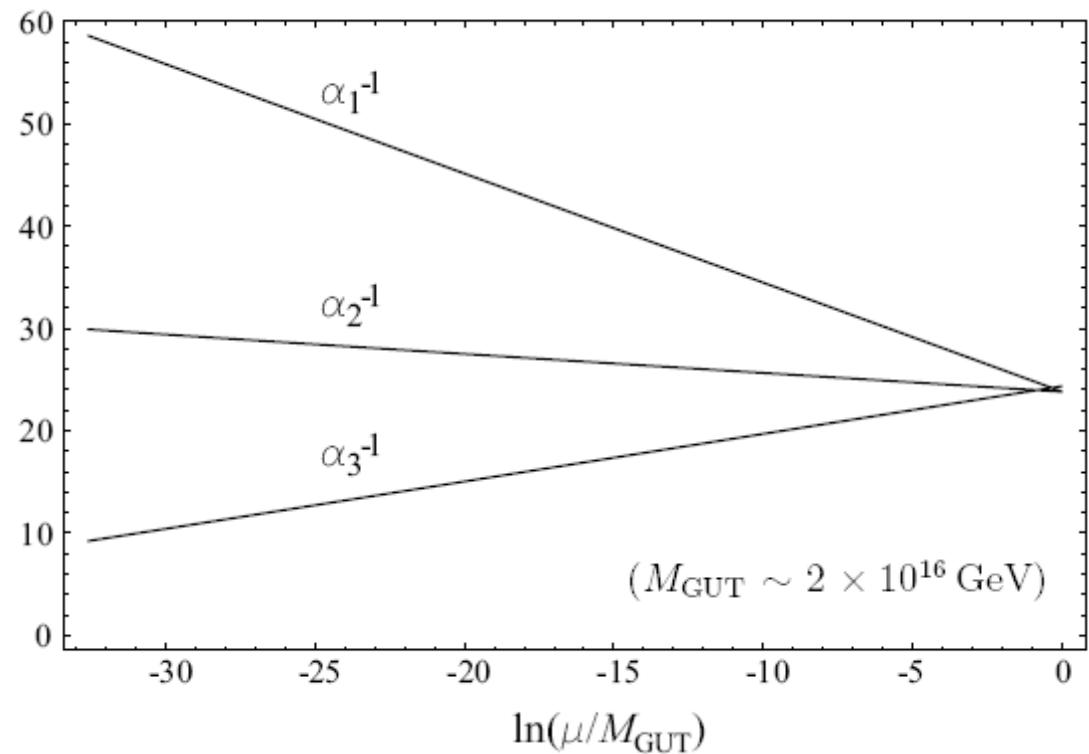


# MSSM: GAUGE COUPLING UNIFICATION

$$\alpha_1(M_Z) = 0.016949 \pm 0.000005$$

$$\alpha_2(M_Z) = 0.033816 \pm 0.000027$$

$$\alpha_3(M_Z) = 0.1176 \pm 0.0020$$



# **MSGUT: GAUGE COUPLING UNIFICATION\***

?

\*B. Bajc et al, hep-ph/0402122; C. S. Aulakh and S. K. Garg, hep-ph/0512224; S. Bertolini, T. Schwetz and M. Malinsky, hep-ph/0605006; ...

# MINIMAL SUSY $SO(10)$

**CONDITIONS\*:**

$$M = m \frac{\eta}{\lambda} \frac{3 - 14x + 15x^2 - 8x^3}{(x - 1)^2} \quad \dots$$



**RELEVANT PARAMETERS\*:**

$m, \alpha, \bar{\alpha}, |\lambda|, |\eta|, \phi = \arg(\lambda) \xrightarrow{0} -\arg(\eta), x = \text{Re}(x) + i\text{Im}(x), g_{\text{GUT}}, M_{\text{SUSY}}$

**RGEs: gauge coupling constants**

$\tan \beta$

**RGEs: Yukawa couplings**

\*C. S. Aulakh, B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, Phys. Lett. B 588 (2004) 196.

# FERMION MASSES IN MINIMAL $SO(10)^*$

$$M_u = \frac{N_u}{N_d} \tan \beta [M_d + \xi(M_d - M_e)]$$

$$M_n = \frac{v}{m} \frac{\sin^2 \beta}{\cos \beta} \alpha \sqrt{\frac{|\lambda|}{|\eta|}} \frac{N_u^2}{N_d} [m_I f_I + m_{II} f_{II}]$$

$$m_I = M_e(M_d - M_e)^{-1}M_e - 6\xi M_e + 9\xi^2(M_d - M_e)$$

$$m_{II} = M_d - M_e$$

**OUR CONVENTION:**  $\hat{M}_j = U_j^T M_j U_j$ ,  $j = u, d, e, n$

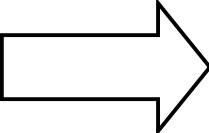
\*B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, hep-ph/0511352.

## TYPE II CONTRIBUTION TO $\nu$ MASS

$$F_{II} = \frac{v}{m} \frac{\sin^2 \beta}{\cos \beta} \alpha \sqrt{\frac{|\lambda|}{|\eta|}} \frac{N_u^2}{N_d} |f_{II}|$$

$$F_{II} \sim 0.2 \times 10^{-9}$$

**$b-\tau$  UNIFICATION**

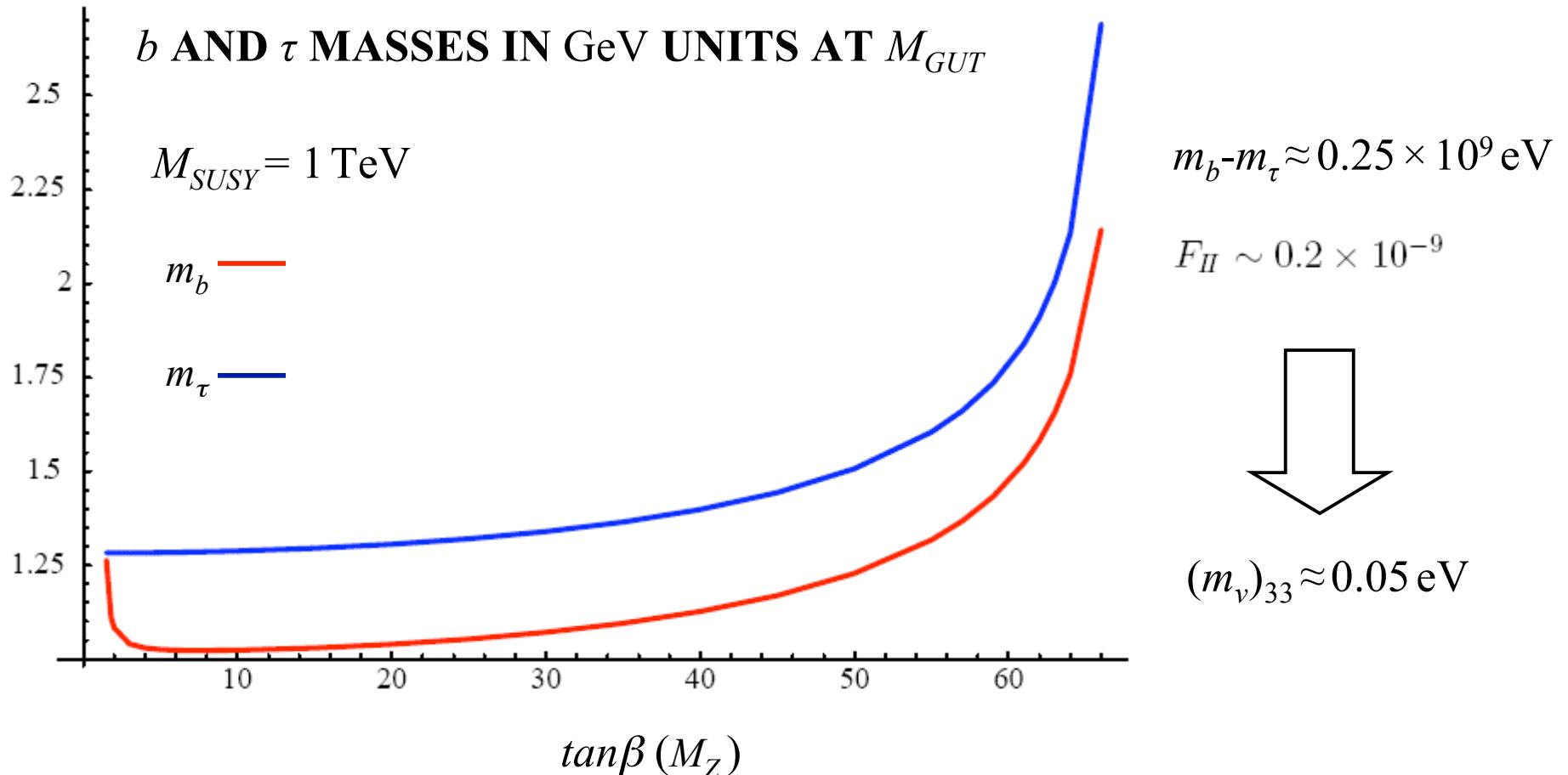
$$m^{-1} < \frac{f(x)}{|\lambda|} \frac{\sqrt{\pi}}{5 \times 10^{15} \text{ GeV} \sqrt{A_S}}$$


UPPER BOUND ON  $\nu$  MASS  
FROM PROTON STABILITY!

$$M_{(X,Y)} = m \frac{g_{GUT}}{|\lambda|} \sqrt{4 \left| \frac{1-x-2x^2}{x-1} \right|^2 + 2 \left| \frac{2x(2x^2+x-1)}{(x-1)^2} \right|^2} = m g_{GUT} \frac{f(x)}{|\lambda|}$$

**PROTON DECAY ( $d=6$ )**

# $b$ - $\tau$ UNIFICATION



THE  $M_Z$  DATA ARE FROM hep-ph/0607208 by I. D, P. P. Pérez and G. Rodrigo.

# PROTON DECAY ( $d = 6$ )

THEORY:

$$\begin{aligned}\Gamma^{p \rightarrow \pi^0 e^+} &= \frac{m_p}{16\pi f_\pi^2} A_L^2 |\alpha|^2 (1 + D + F)^2 \\ &\quad \left[ A_{SR}^2 \left| k_1^2 V^{11} + k_2^2 (V_q)^{*11} (V_q V)^{11} \right|^2 + A_{SL}^2 k_1^4 \left| V^{11} + (V_q)^{*11} (V_q V)^{11} \right|^2 \right]\end{aligned}$$

EXPERIMENT:

$$\tau(p \rightarrow \pi^0 e^+) > 1.6 \times 10^{33} \text{ years}$$

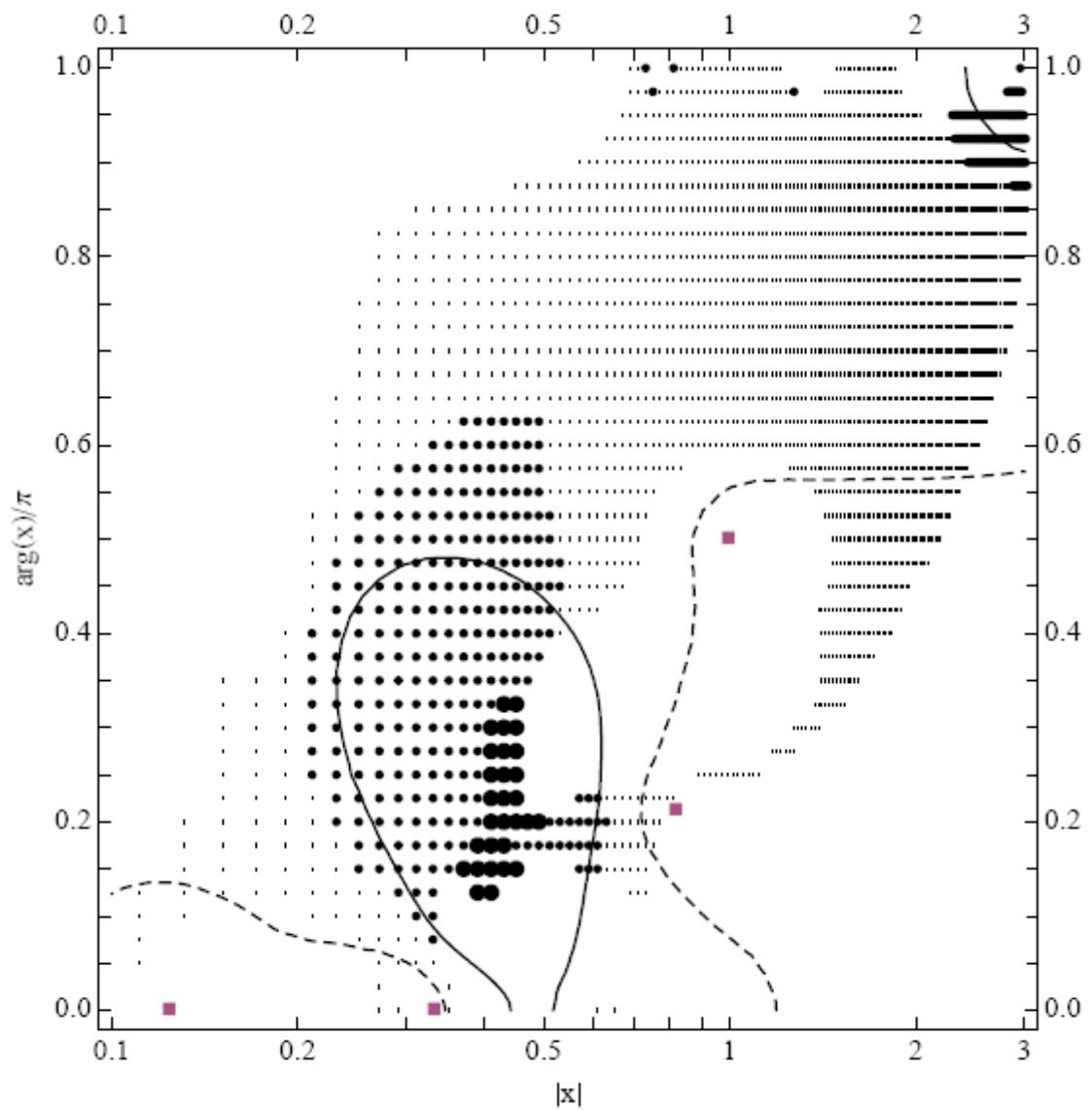
$$M_{(X,Y)} = m \frac{g_{GUT}}{|\lambda|} \sqrt{4 \left| \frac{1 - x - 2x^2}{x - 1} \right|^2 + 2 \left| \frac{2x(2x^2 + x - 1)}{(x - 1)^2} \right|^2} = m g_{GUT} \frac{f(x)}{|\lambda|}$$

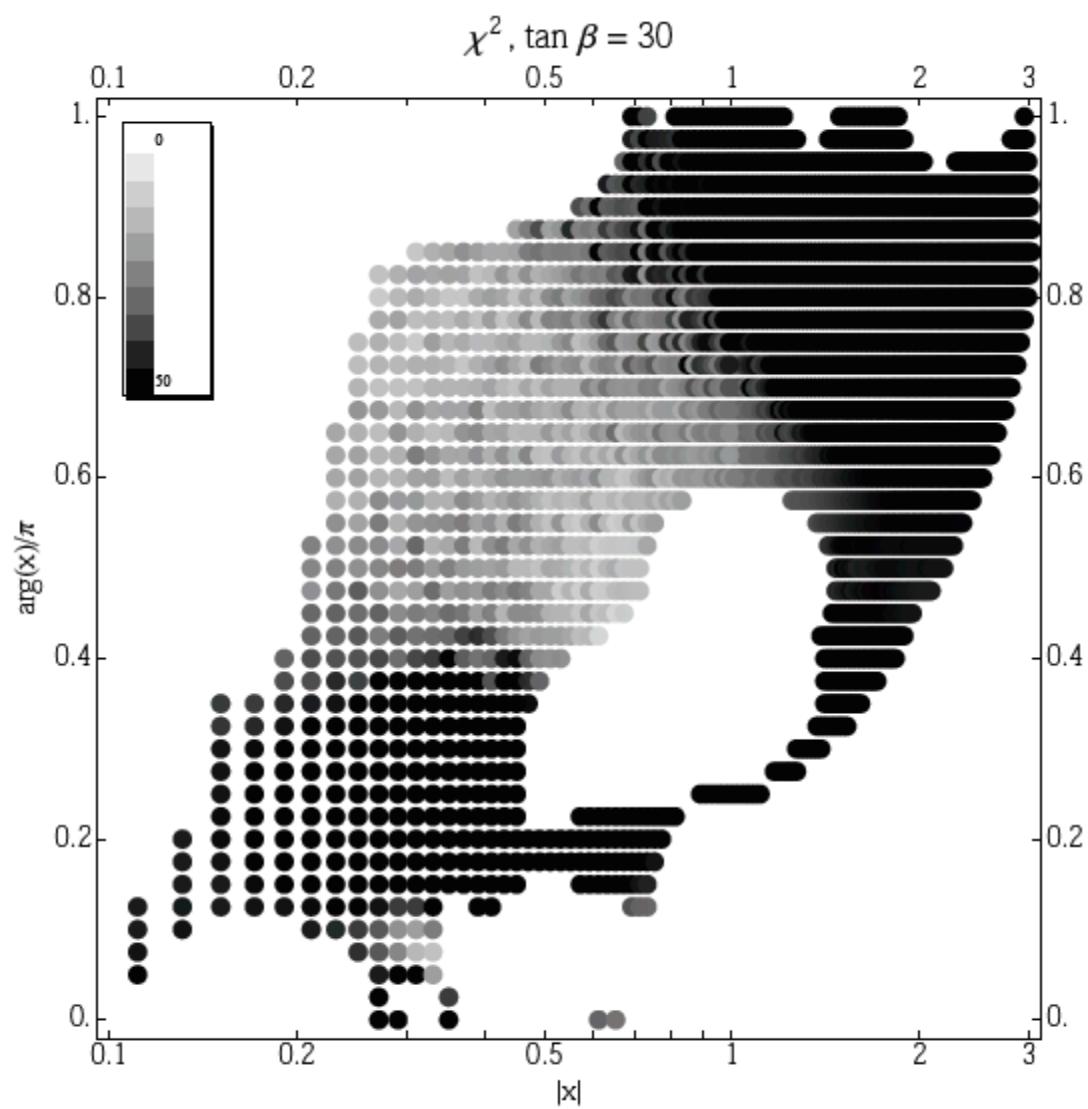
$$V = U_d^\dagger U_e \quad V_q = U_u^\dagger U_d \quad k_{1(2)}^2 = \frac{2\pi\alpha_{GUT}}{M_{(X,Y)((X',Y'))}^2}$$

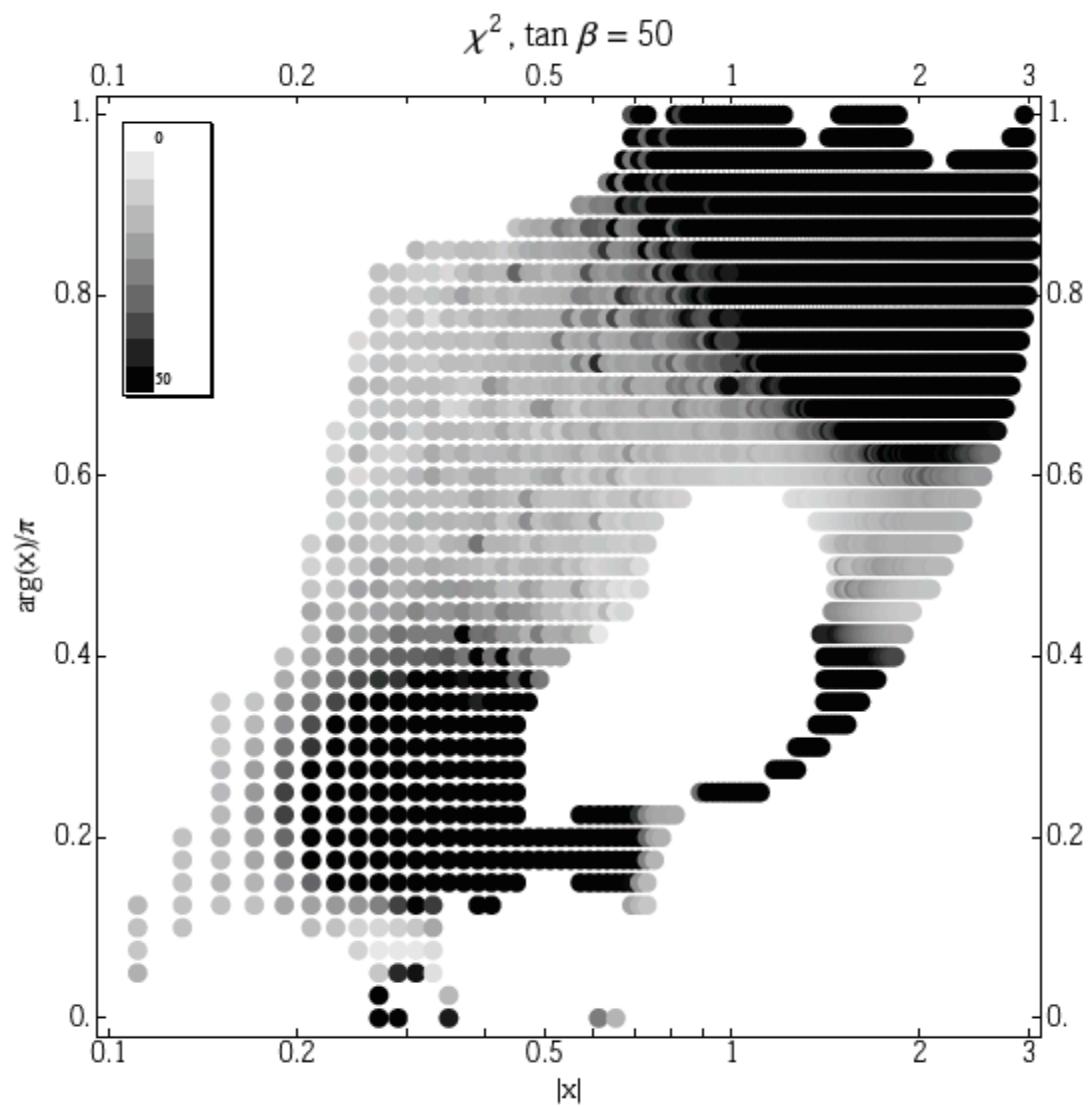
$$M_{(X,Y)} \approx M_{(X',Y')} \quad A_{SL} \approx A_{SR} = A_S$$

# LOW SCALE SUSY AND $\nu$ MASS

$$\eta = \lambda = \alpha = \bar{\alpha} = 1 \quad \tan \beta = 50$$





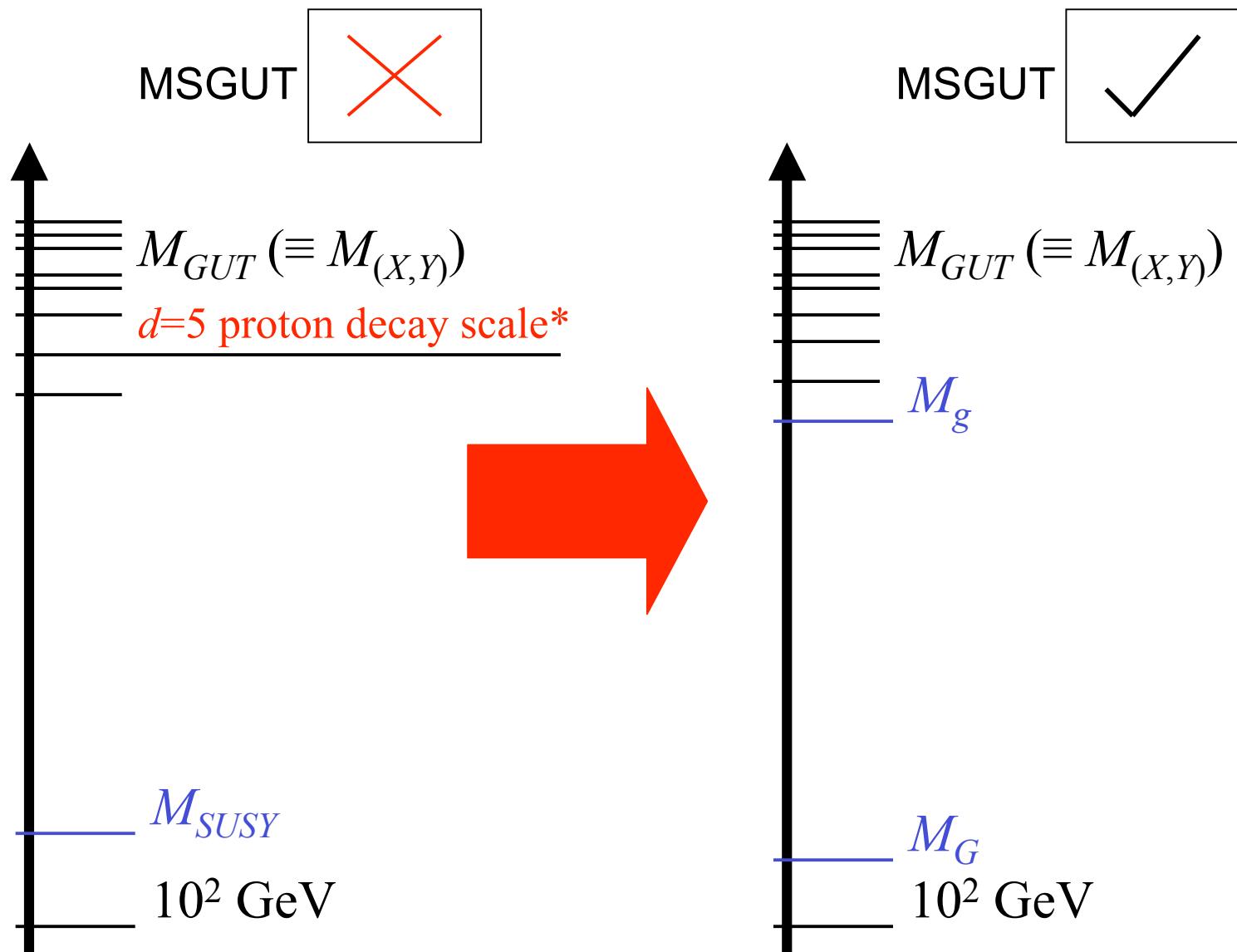


# SPLIT SUSY\* AND $\nu$ MASS

## RELEVANT STEPS:

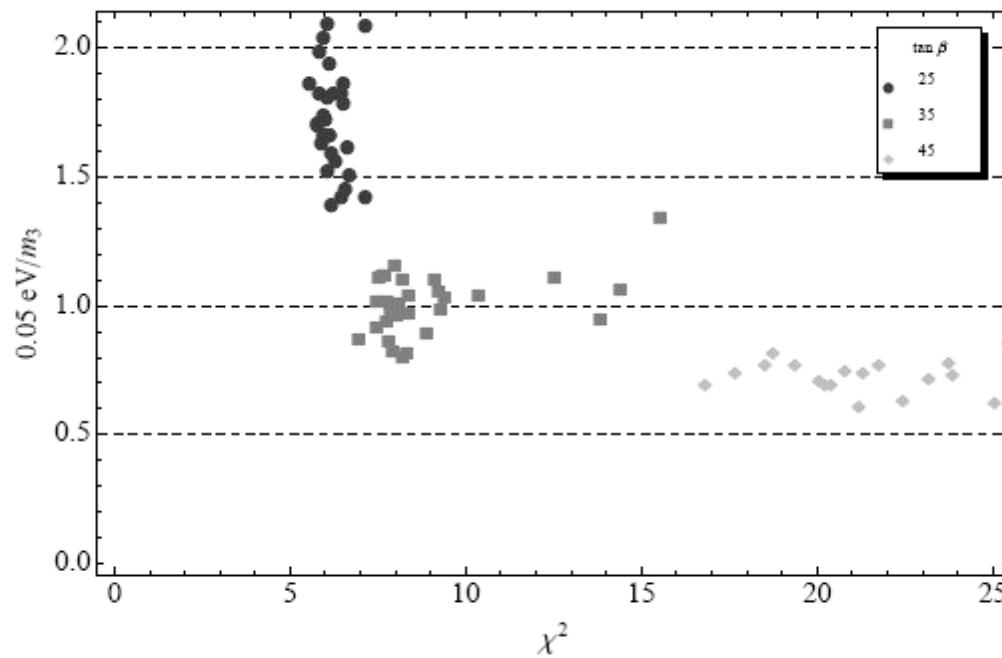
- i) We maximize  $F_{II}$  in order to have a viable neutrino scale.  
This fixes  $x$ ,  $m$  ( $M_{GUT}$ ),  $\alpha$ ,  $\bar{\alpha}$ ,  $\lambda$  and  $\eta$ .
- ii) We check whether unification takes place by solving three equations for the running of gauge couplings for three unknowns —  $g_{GUT}$ ,  $M_g$  and  $M_G$ .
- iii) We run masses of quarks and charged leptons and the CKM parameters to  $M_{GUT}$  for a given  $\tan\beta$  and perform numerical fit that also includes fitting of neutrino masses as well as solar and atmospheric angles.

\*N. Arkani-Hamed and S. Dimopoulos, hep-ph/0405159; G. F. Giudice and A. Romanino, hep-ph/0406088.



\*T. Fukuyama et al, hep-ph/0406068; B. Dutta et al, 0712.1206 [hep-ph].

# PRELIMINARY RESULTS: $\nu$ MASS VS. $\chi^2$



# PRELIMINARY RESULTS: $|U_{e3}|$ VS. $\chi^2$

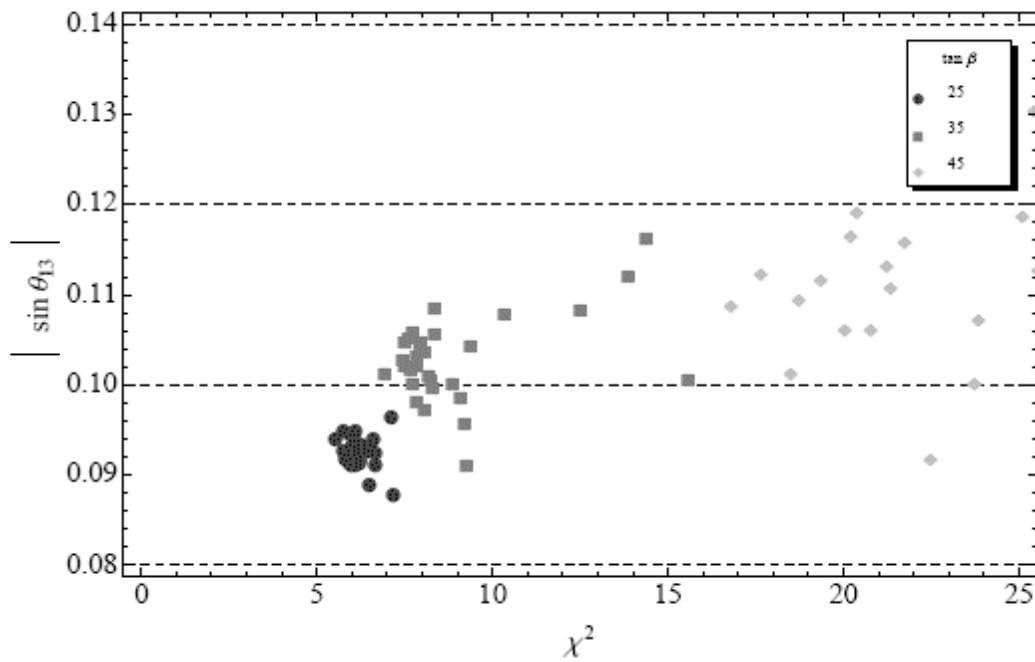


Table 1: Input and output parameters of the numerical fit for  $\tan \beta = 35$  with  $\chi^2 = 8.09319$ ,  $m_3 = 0.045\text{ eV}$ ,  $m_2 = 0.0088\text{ eV}$  and  $m_1 = 0.0015\text{ eV}$ . Unification takes place for  $|x| = 0.109648$ ,  $\arg(x)/\pi = 0.5$ ,  $\alpha = 2.25464$ ,  $\bar{\alpha} = 11.$ ,  $\eta = 10.7529$ ,  $\lambda = 0.03$ ,  $\ln M_{GUT}/M_g = 3.82438$ ,  $\ln M_{GUT}/M_G = 24.3486$ ,  $g_{GUT} = 1.22$  and  $M_{GUT} = 5.4 \times 10^{15}\text{ GeV}$ .

observable	FIT	RGE	pull
$m_e(\text{MeV})$	0.42491	0.42491	
$m_\mu(\text{GeV})$	0.08970	0.08970	
$m_\tau(\text{GeV})$	1.541	1.541	
$m_u(\text{MeV})$	0.55	0.55	0.019847
$m_c(\text{GeV})$	0.217	0.215	0.0739264
$m_t(\text{GeV})$	73.15	67.78	0.79109
$m_d(\text{MeV})$	0.26	1.2	-2.25182
$m_s(\text{MeV})$	26.4	22.0	0.58308
$m_b(\text{GeV})$	0.901	0.914	-0.136006
$s_{12}^{CKM}$	0.239956	0.227229	0.800175
$s_{23}^{CKM}$	0.0478599	0.0474469	0.217581
$s_{13}^{CKM}$	0.004492	0.004487	0.0120442
$\delta^{CKM}$	1.15172	0.994788	0.788766
$s_{12}^{PMNS}$	0.446992	0.55	-0.706773
$s_{23}^{PMNS}$	0.572297	0.69	-0.461712
$s_{13}^{PMNS}$	0.103476		
$m_2/m_3$	0.178027	0.18	-0.0986272

Table 2: Input and output parameters of the numerical fit for  $\tan \beta = 35$  with  $\chi^2 = 7.5$ ,  $m_3 = 0.049$  eV,  $m_2 = 0.0089$  eV and  $m_1 = 0.0015$  eV. Unification takes place for  $|x| = 0.1$ ,  $\arg(x)/\pi = 0.46$ ,  $\alpha = 2.15012$ ,  $\bar{\alpha} = 11.$ ,  $\eta = 10.9998$ ,  $\lambda = 0.03$ ,  $\ln M_{GUT}/M_g = 2.12491$ ,  $\ln M_{GUT}/M_G = 24.8055$ ,  $g_{GUT} = 1.22$  and  $M_{GUT} = 5.4 \times 10^{15}$  GeV.

observable	data at $M_Z$	FIT	RGE	pull
$m_e$ (MeV)	0.4866613	0.4433781	0.4433781	
$m_\mu$ (GeV)	0.10273	0.09359	0.09359	
$m_\tau$ (GeV)	1.746	1.601	1.601	
$m_u$ (MeV)	1.6	0.57	0.56	0.015
$m_c$ (GeV)	0.628	0.223	0.221	0.081
$m_t$ (GeV)	171.5	74.7	69.7	0.723
$m_d$ (MeV)	3.5	0.26	1.3	-2.28
$m_s$ (MeV)	62	27.2	22.6	0.576
$m_b$ (GeV)	2.89	0.927	0.932	-0.047
$s_{12}^{CKM}$	0.2272	0.2355	0.2272	0.519
$s_{23}^{CKM}$	0.0422	0.0481	0.0479	0.116
$s_{13}^{CKM}$	0.00399	0.00453	0.00453	-0.006
$\delta^{CKM}$	0.995	1.097	0.995	0.515
$s_{12}^{PMNS}$	0.55	0.46	0.55	-0.576
$s_{23}^{PMNS}$	0.69	0.56	0.69	-0.723
$s_{13}^{PMNS}$		0.103		
$m_2/m_3$		0.18	0.18	0.0

# PROTON DECAY SIGNATURES\*

$$\tau(p \rightarrow \pi^0 e^+)$$

$$\tau(p \rightarrow K^+ \bar{\nu})/\tau(p \rightarrow \pi^0 e^+) = 119 \pm 5$$

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ALL RELEVANT PHASES ARE DETERMINED FROM THE FIT.  
ALL RELEVANT MASSES ARE FOUND FROM UNIFICATION.

\*P. Filevies Perez, hep-ph/0403286.

# CONCLUSIONS

- Preliminary results indicate that the minimal  $SO(10)$  with low scale SUSY fails to accommodate relevant neutrino mass scale.
- Split SUSY case not only generates viable  $\nu$  scale but allows for consistent description of all known masses and mixing parameters in a particular region of parameter space.
- The model yields interesting signatures for the next generation of proton decay experiments and observable  $U_{e3}$ .