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Workshop on the original of P, CP and T Violation

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CP in neutrino oscillations and neutrino oscillograms of the Earth

Eugene AKHMEDOV

Max-Planck-Institut fuer Kernphysik
Postfach 103980
Saupfercheckweg 1, 69117
Heidelberg
Germany

$\not{C}\not{P}$ in neutrino oscillations and neutrino oscillograms of the Earth

Evgeny Akhmedov

MPI-K, Heidelberg & Kurchatov Inst., Moscow

High-E neutrino oscillations in the Earth

In collaboration with Michele Maltoni and Alexei Smirnov

Based on

JHEP 0705 (2007) 077 `[hep-ph/0612285]`

JHEP 06 (2008) 072 `[arXiv:0804.1466]`

& work in progress

Neutrino oscillograms of the Earth

Contours of equal osc. probabilities in (Θ_ν, E_ν) plane

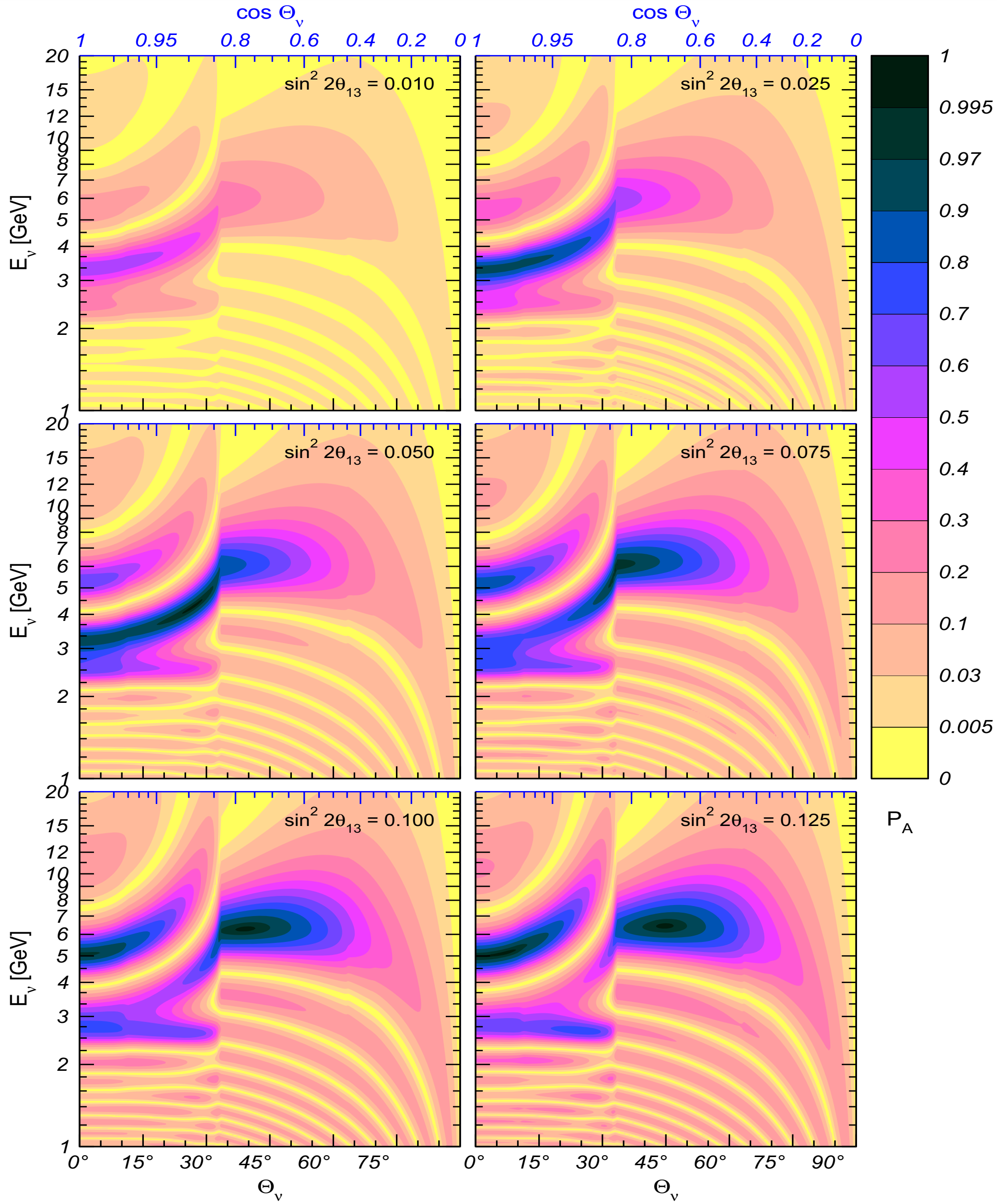
Θ_{13} - dependence of P_2 $\Rightarrow$

P_2 – effective 2f transition probability $(\Delta m^2_{sol} \to 0)$

$$P_{e\mu} = s^2_{23} P_2$$

$$P_{e\tau} = c^2_{23} P_2$$

(E.A., Maltoni & Smirnov, 2006)

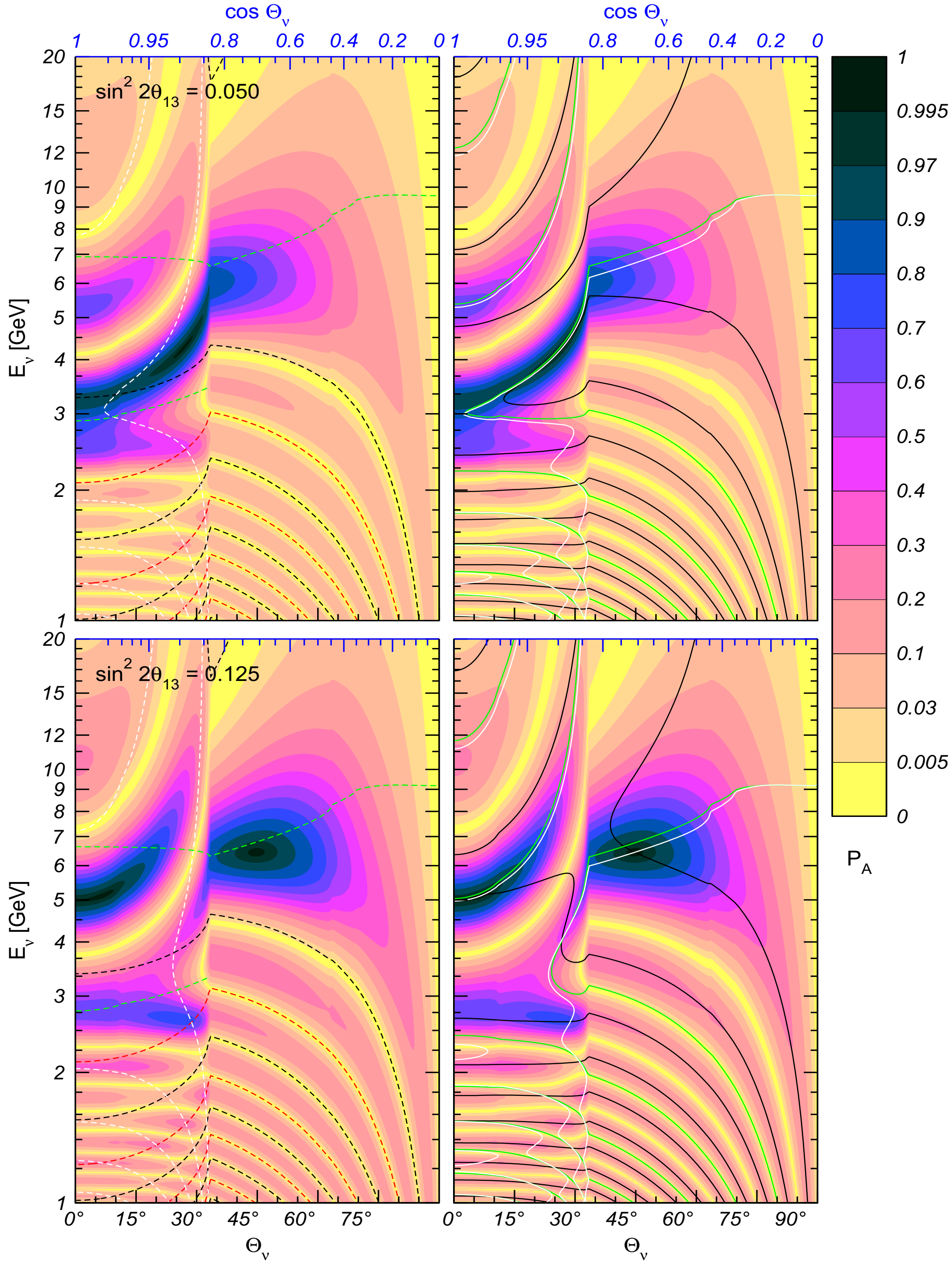


In the right panels:

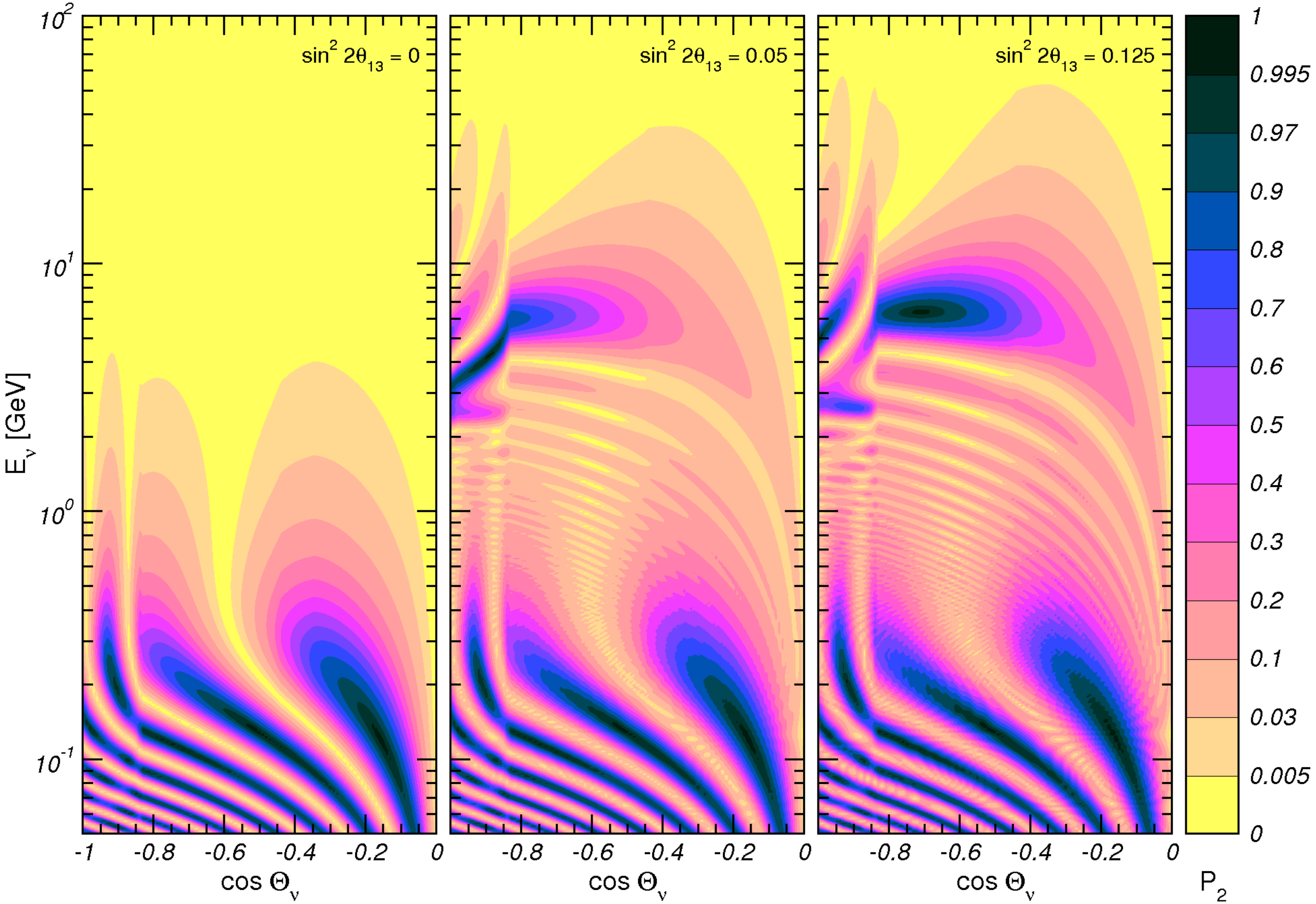
– alignment (collin.) cond.

– generalized res. cond. $(\mathrm{Im}\,\alpha^{(2)} = 0)$

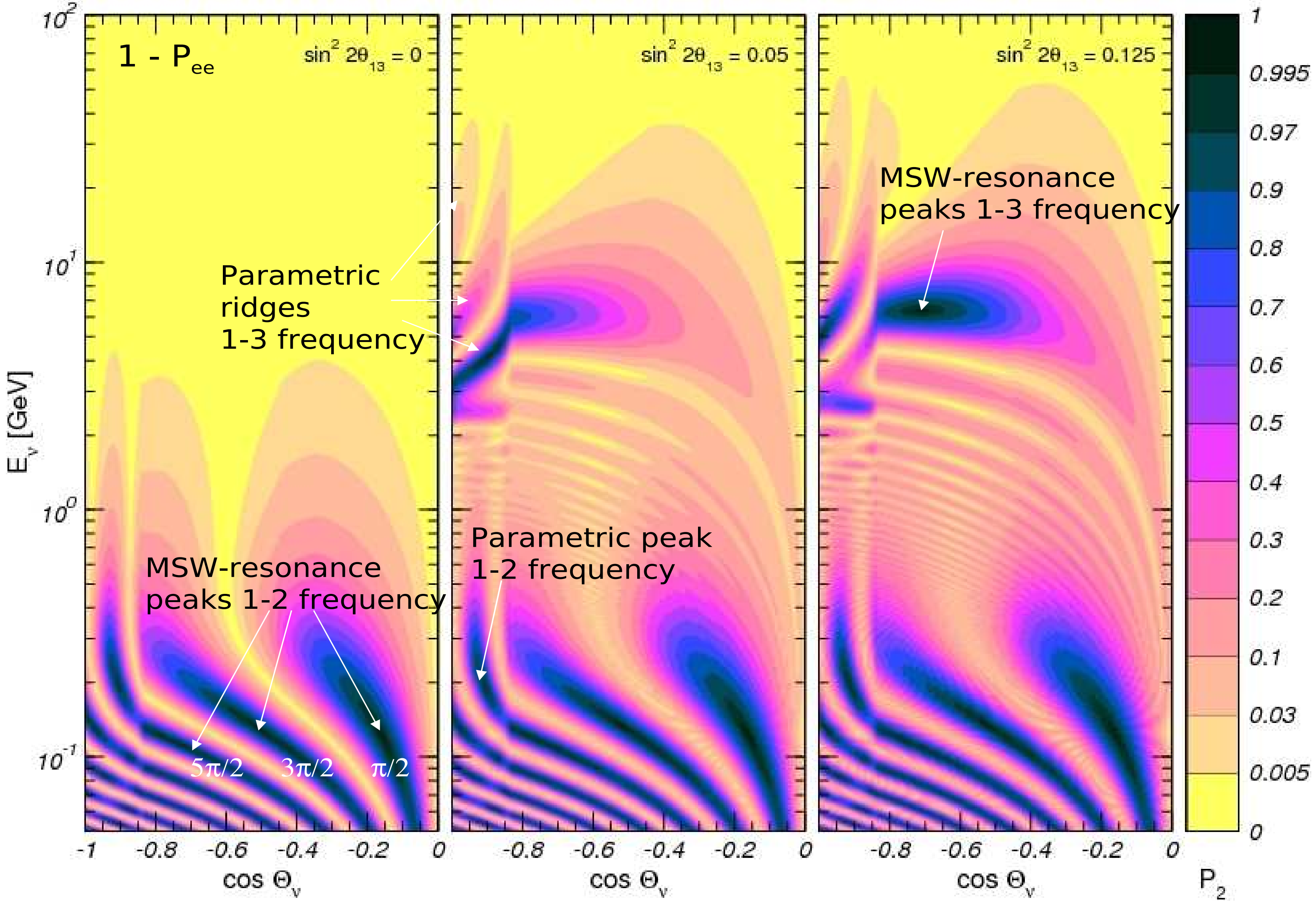
– generalized phase cond.



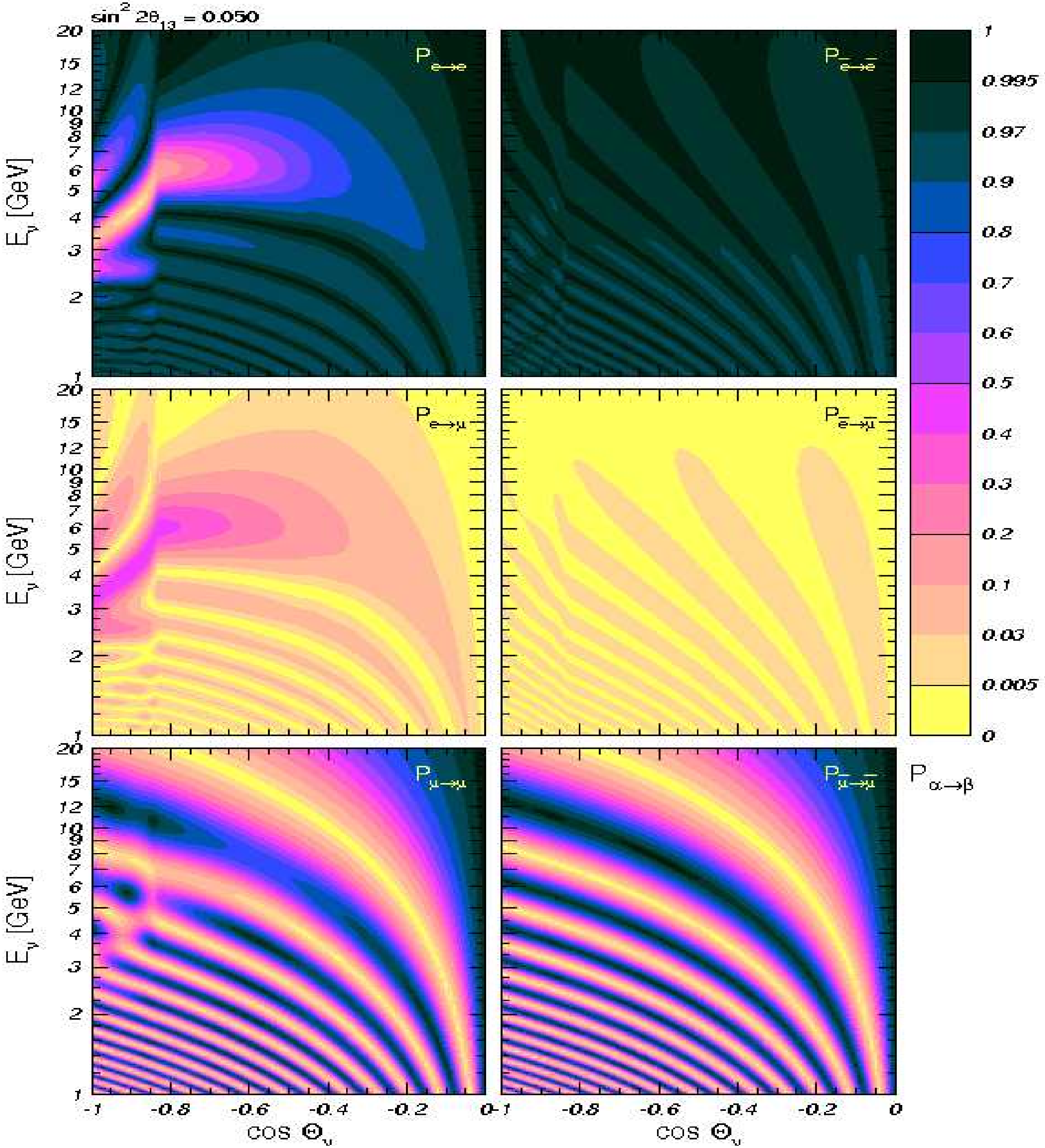
Including the effects of $\Delta m^2_{\rm sol}$: $(1 - P_{ee})$



Including the effects of Δm^2_{sol}: $(1 - P_{ee})$



Other osc. channels



Including the effects of $\Delta m^2_{\rm sol}$

Fundamental $\not{C}\not{P}$ and $\not{T}$; dependence of P_{ab} on $\delta_{\rm CP}$ (also in CP - and T - even terms) $\Rightarrow$ parameter correlations and degeneracies (e.g. θ_{13} and $\delta_{\rm CP}$)

"Magic" baseline: $L \simeq 7300$ km – dependence on $\delta_{\rm CP}$, $\Delta m^2_{\rm sol}$ disappears (Barger *et al.*, 2001; Huber & Winter, 2003; Huber *et al.*, 2006)

Physical interpretation (Smirnov, 2006):

$$\diamondsuit \qquad P_{\mu e} = |c_{23}\, A_{e2}\, e^{i\delta_{\rm CP}} + s_{23}\, A_{e3}|^2$$

To leading order in s_{13} and $\Delta m^2_{21}/\Delta m^2_{31}$: $(E^R_{12} \lesssim E \lesssim E^R_{13})$

$$A_{e2} \simeq A_S(\theta_{12}, \Delta m^2_{21})\,, \qquad A_{e3} \simeq A_A(\theta_{13}, \Delta m^2_{31})$$

Magic baseline: $A_S = 0$.

Including the effects of $\Delta m^2_{\rm sol}$

For $N_e \simeq const$: $|A_S| \simeq \sin 2\theta_{12}^m \sin\phi \quad \Rightarrow \quad L_{\rm magic}: \quad \phi = \pi n$

At high energies: $\boxed{VL = 2\pi n} \quad \Leftrightarrow \quad L = 2\pi n/V$

Atmospheric “magic” baselines (“magic curves”):

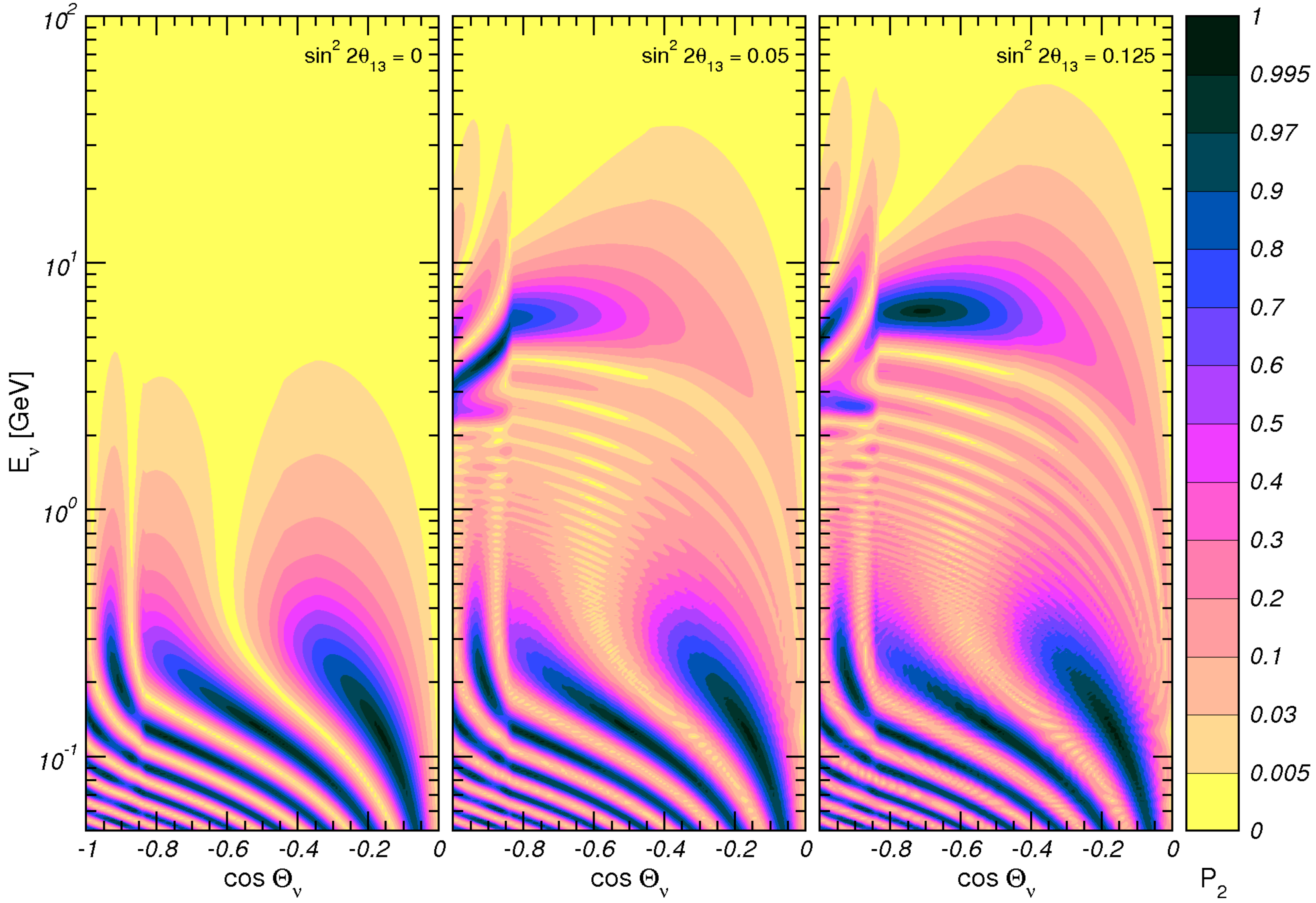
$$A_A = 0$$

– dependence on $\delta_{\rm CP}$, $\Delta m^2_{\rm atm}$, θ_{13} disappears. In a matter of $N_e \simeq const.$: $|A_A| \simeq \sin 2\theta_{13}^m |\sin(\omega_{13}L)|$,

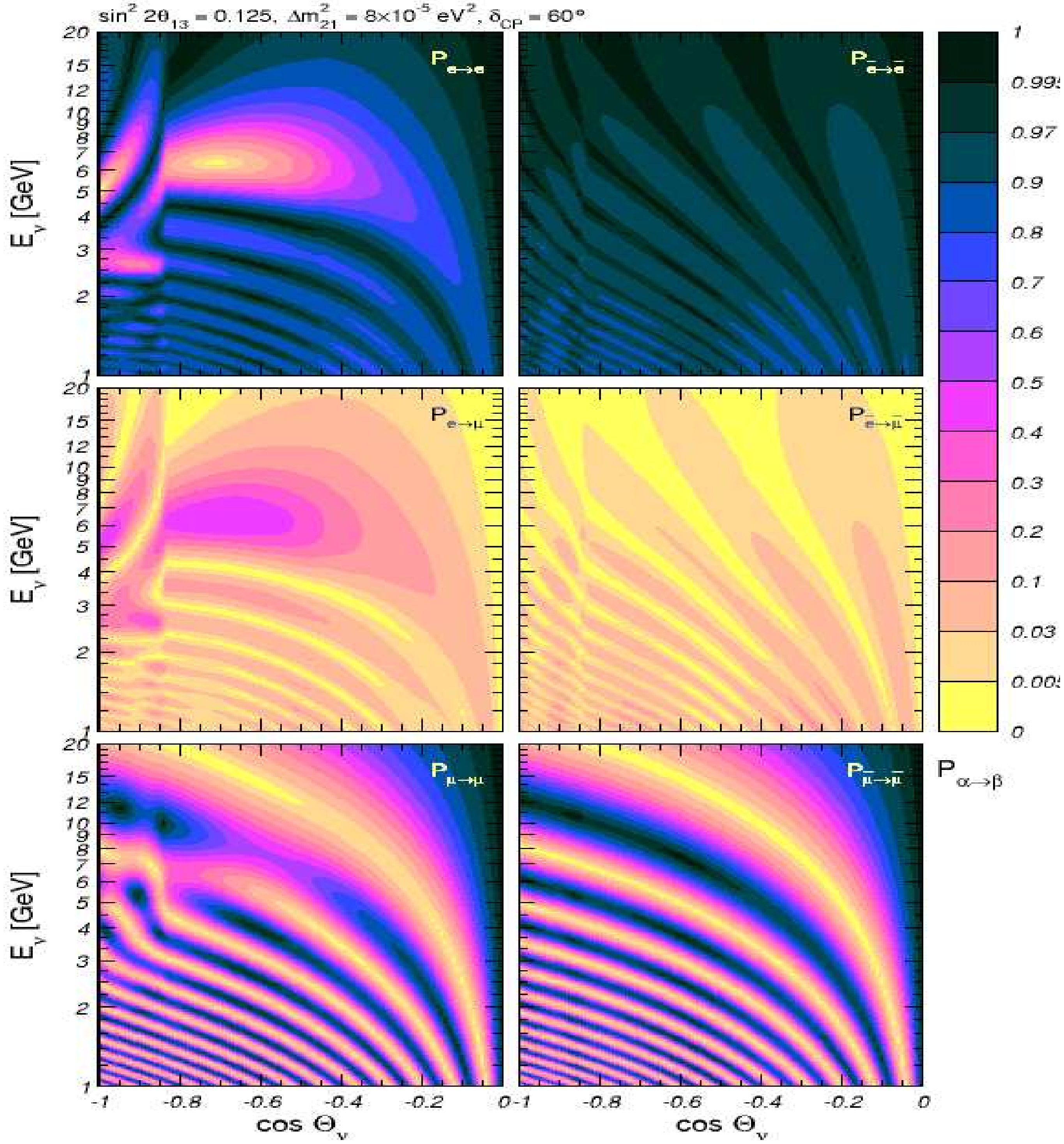
$$\omega_{13} = \sqrt{(\cos 2\theta_{13}\Delta - V/2)^2 + \sin^2 2\theta_{13}\Delta^2}\,, \qquad \Delta \equiv \Delta m^2_{31}/(4E)$$

Atmospheric magic curves: $\omega_{13}L \simeq \pi n$.

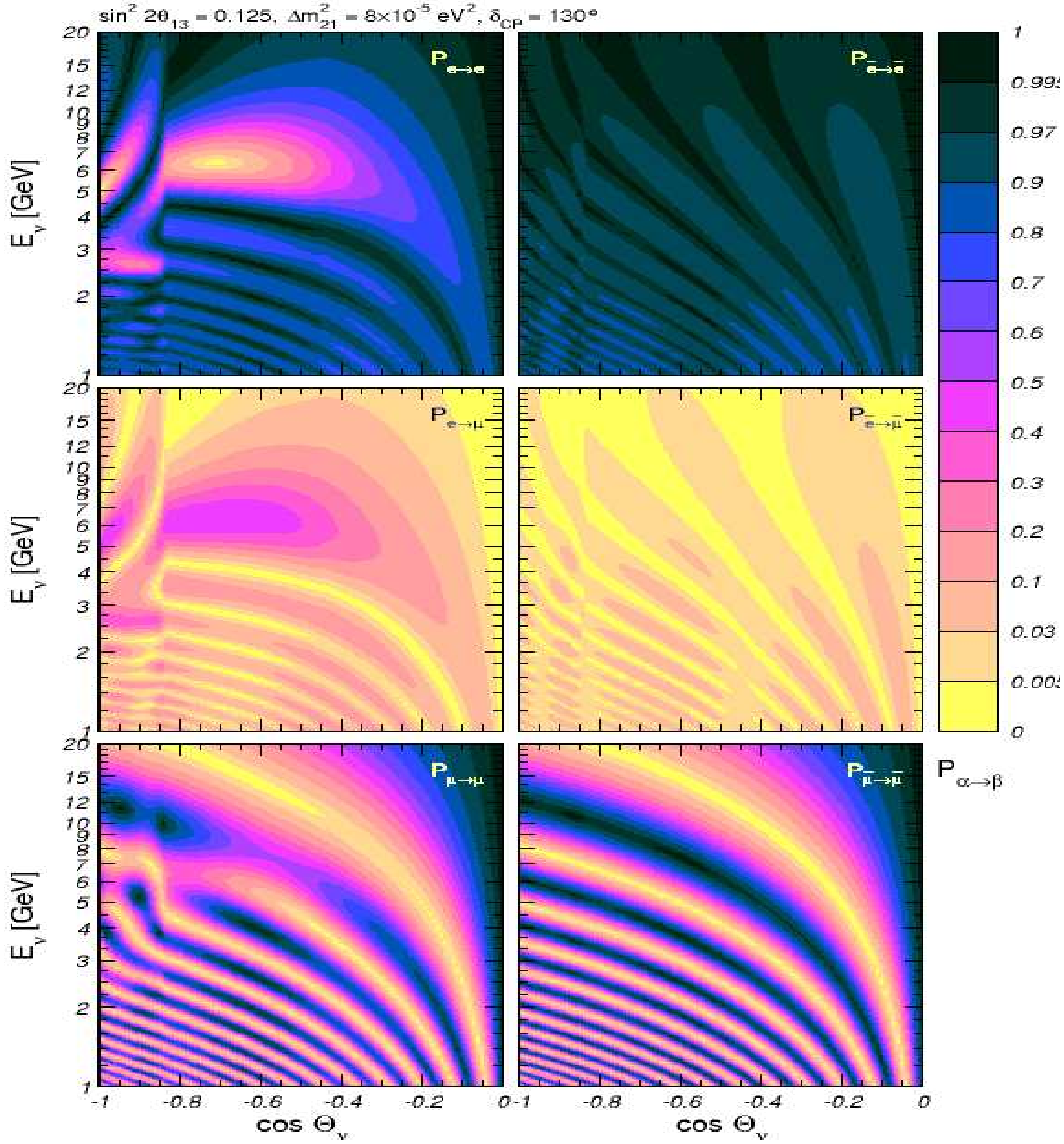
Including the effects of $\Delta m^2_{\rm sol}$: $(1 - P_{ee})$



Dependence on δ_{CP}

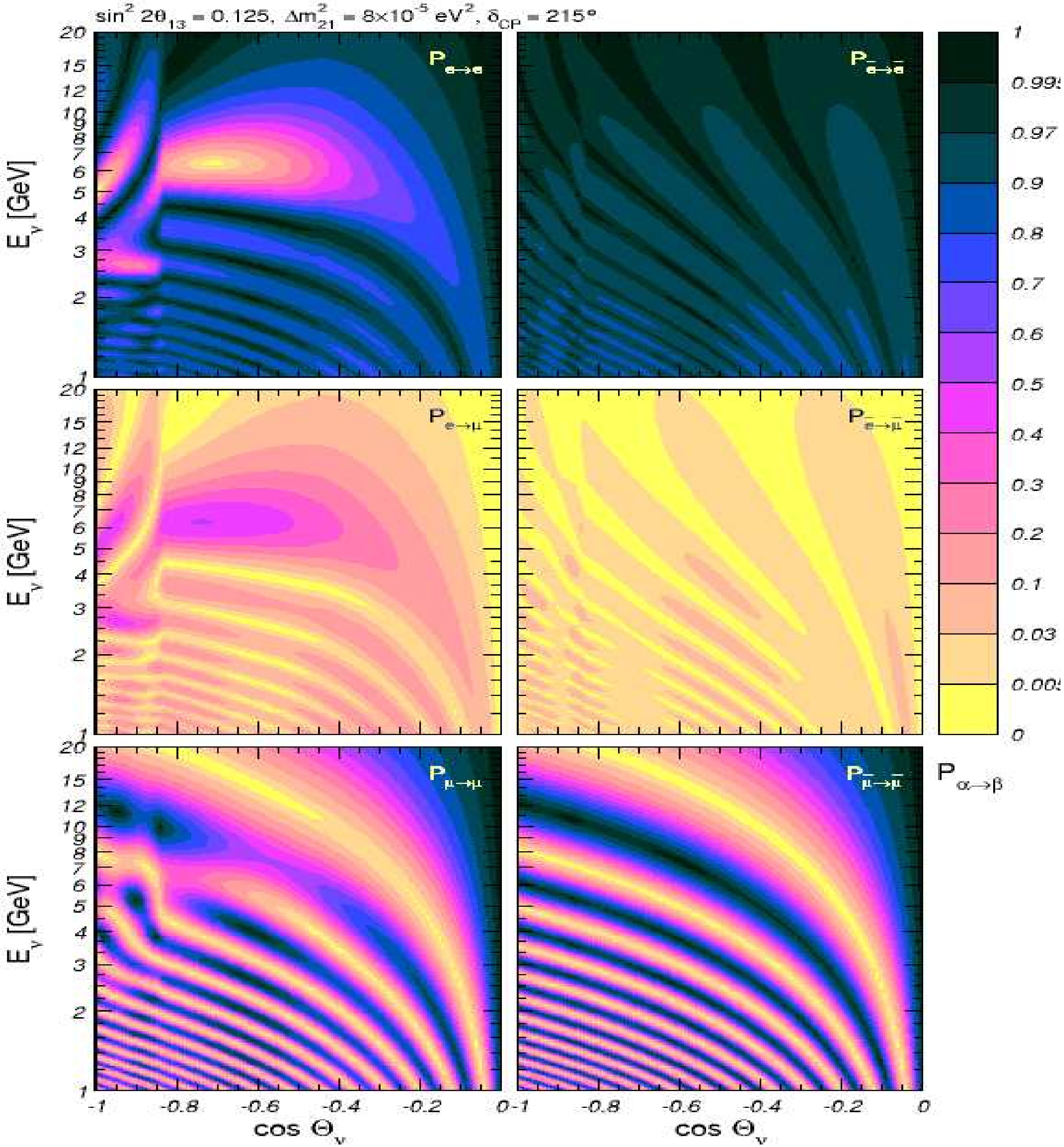


Dependence on δ_{CP}



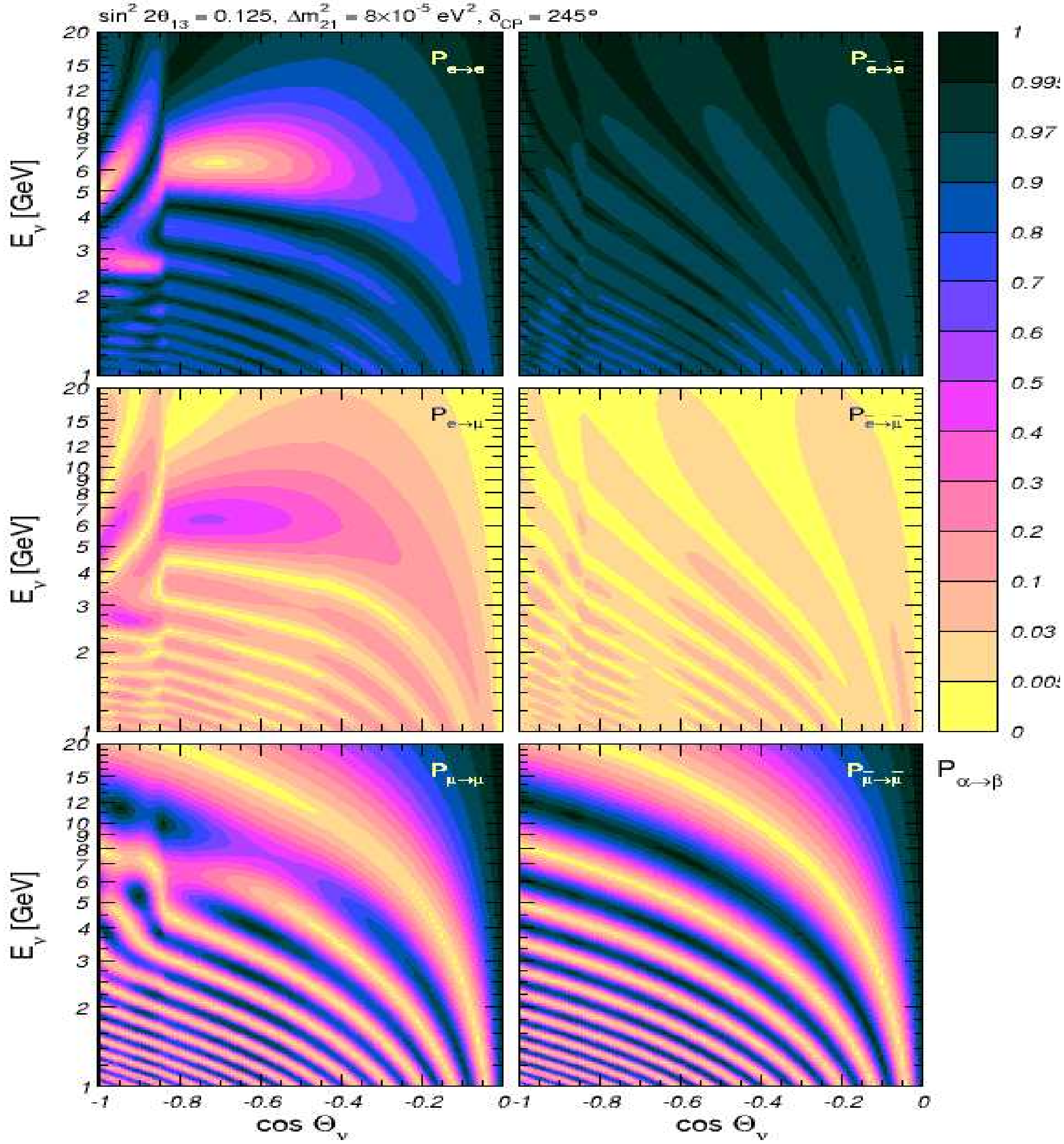
Dependence on

δ_{CP}



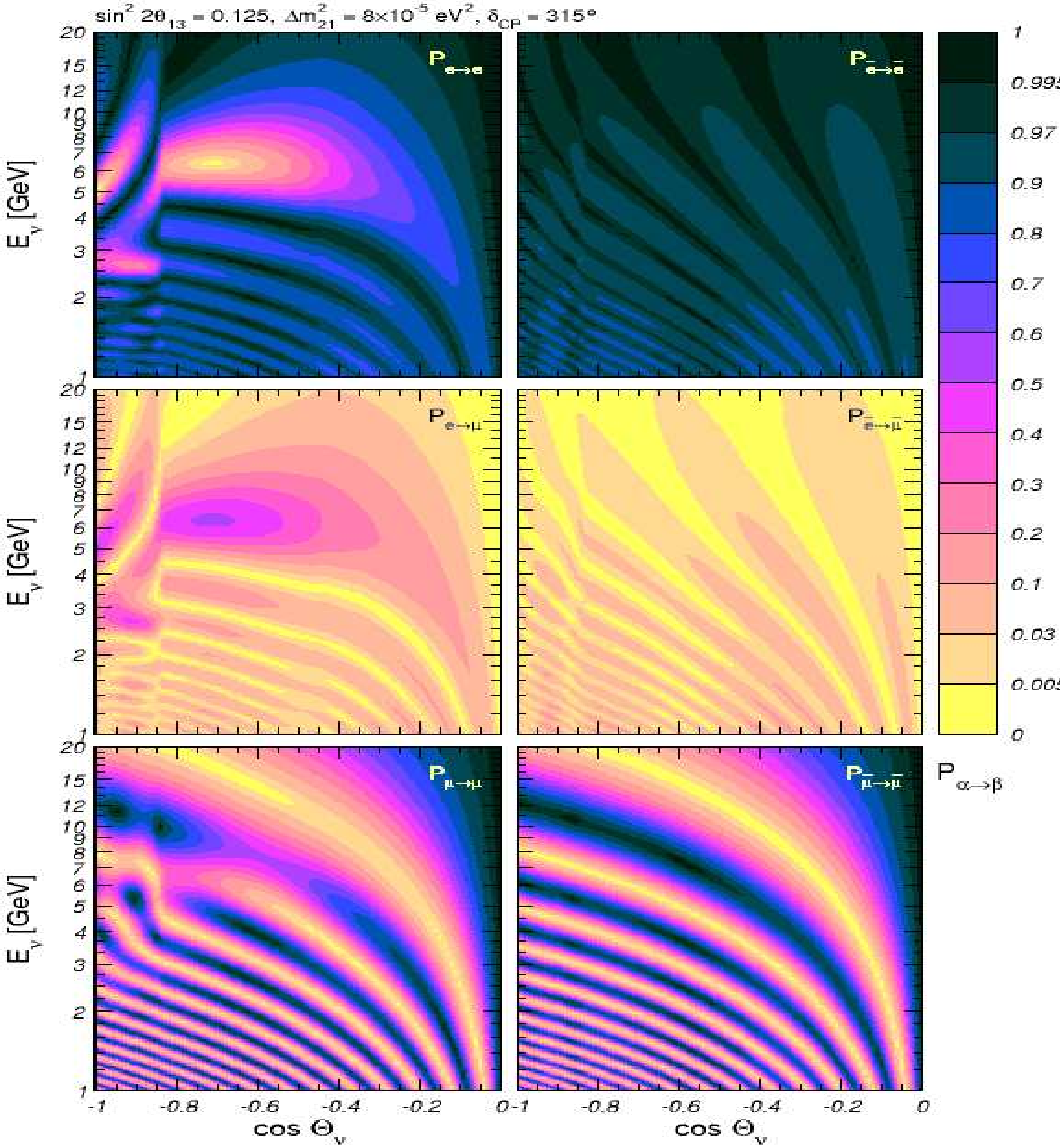
Dependence on

$\delta_{\rm CP}$



Dependence on

$\delta_{\rm CP}$



Oscilograms for event number differences

A convenient measure – χ^2 density $\xi^2 \equiv \rho(\chi^2)$ (independent of binning)

A set of true oscill. parameters $\{param\}_{true}$

A set of assumed oscill. parameters $\{param\}_{true}$

$\Rightarrow \quad P(\{param\}_{true}), \quad P(\{param\}_{th})$

Calculate (in the $(\Theta, \log E)$ plane) the "event numbers" – probabilities folded with fluxes, cross sections and Jacobian $(E \to \log E)$ $\Rightarrow$ "event number" densities $\rho_{ex}, \ \rho_{th}$.

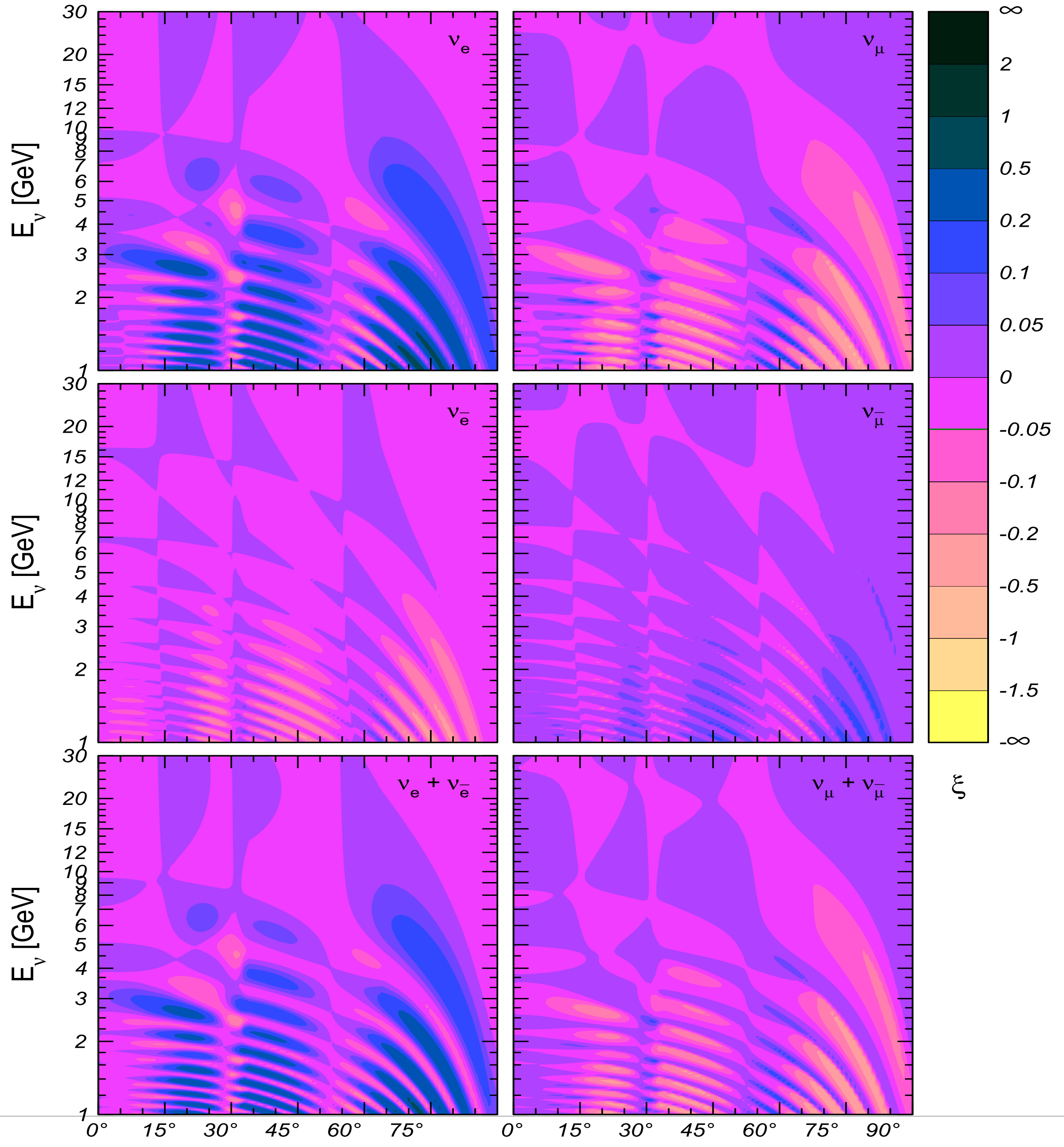
Event numbers: $E = \rho_{ex} dS, \quad T = \rho_{th} dS, \quad dS$ – bin area

$$\xi^2 = \frac{(E-T)^2}{E} = \frac{(\rho_{ex} - \rho_{th})^2}{\rho_{ex}} dS$$

Smearing due to finite energy/angle resolution can be readily included

Theoretical uncertainties and systematical errors (for each particular experiment) can be incorporated. Sign of $(E-T)$ can be included.

CP oscillogram for event # difference



Understanding CP oscillograms

3 grids of curves: $(A_{e2} \to A_S, \quad A_{e2} \to A_A)$

(1) Solar magic lines (vertical)

(2) Atmospheric magic curves (bent)

(3) Interference phase curves (similar to atmospheric curves, but steeper for ν's and less steep for $\bar{\nu}$'s).

$$P_{\mu e} = |c_{23} A_s e^{i\delta} + s_{23} A_a|^2$$

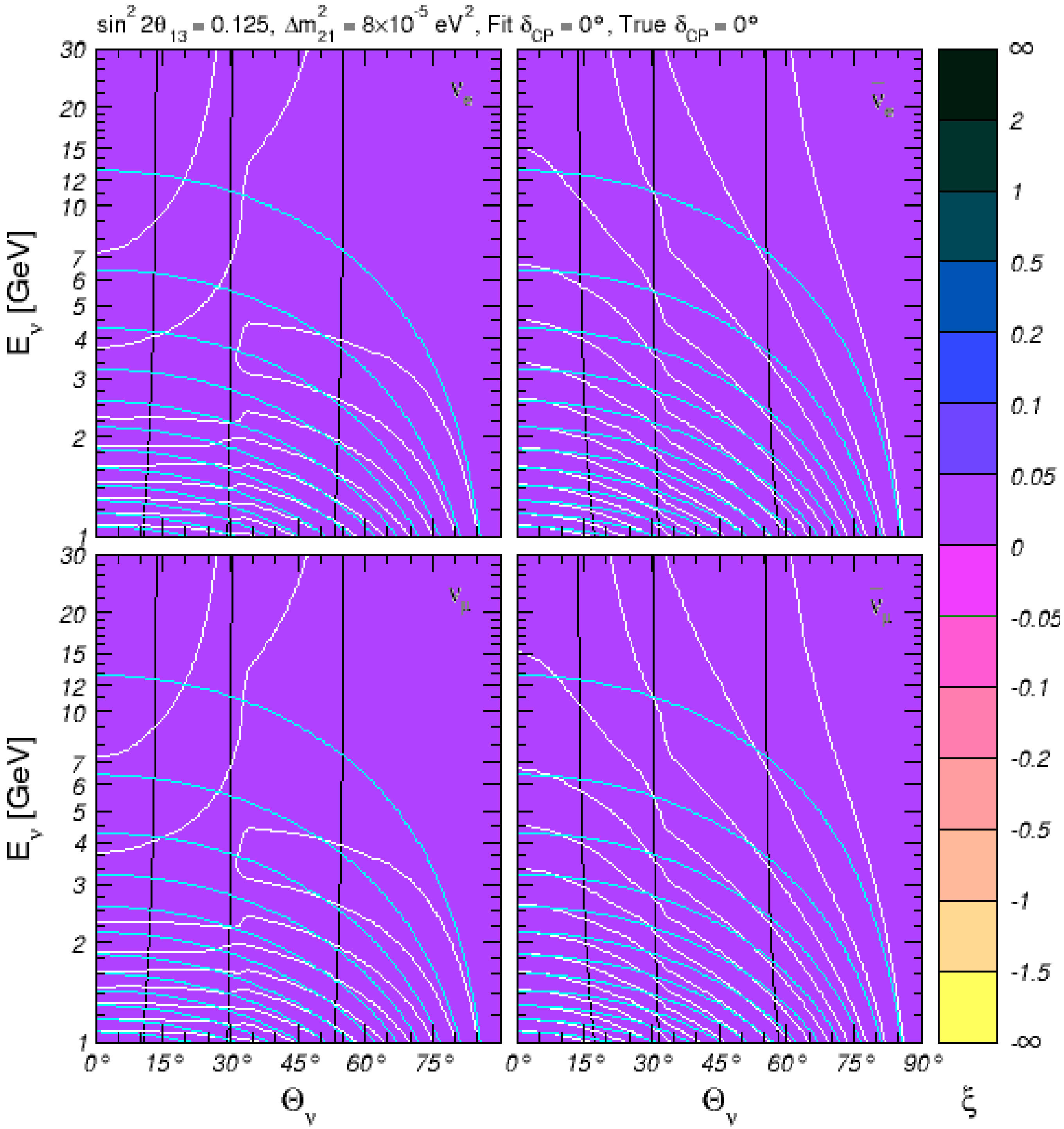
$$= c_{23}^2 |A_s|^2 + s_{23}^2 |A_a|^2 + 2 s_{23} c_{23} |A_s||A_a| \cos(\phi + \delta), \qquad \phi = arg(A_s A_a^*)$$

$$P(\delta_{true}) = P(\delta_{th}) \quad \Rightarrow \quad |A_s||A_a| \cos(\phi + \delta_{true}) = |A_s||A_a| \cos(\phi + \delta_{th})$$

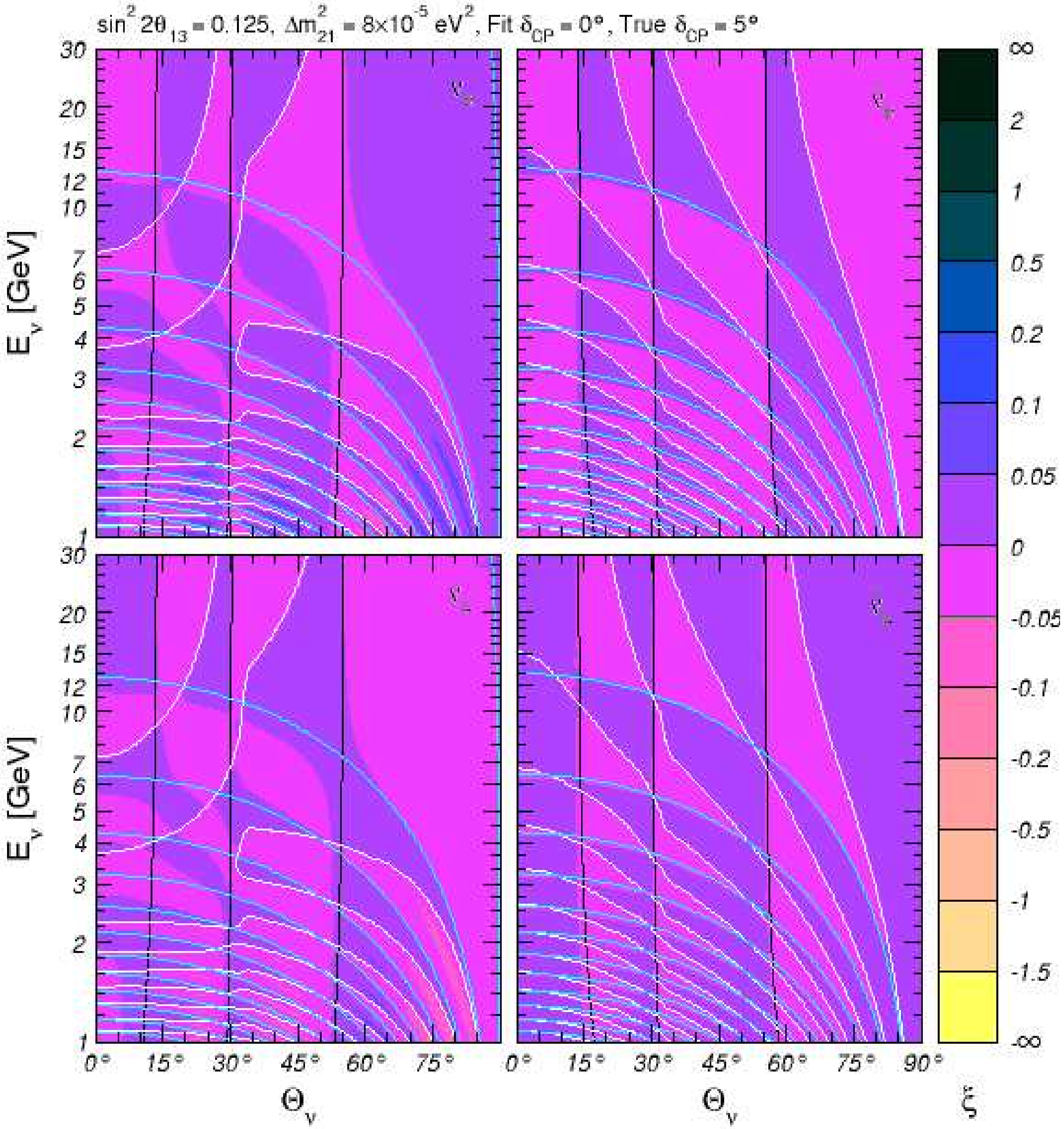
Realisations:

- $|A_s| = 0$ (solar "magic" lines)
- $|A_a| = 0$ (atm. "magic" curves)
- $\delta_{th} = \delta_{true} + 2\pi n$ (trivially, true and assumed values of δ coincide)
- $(\phi + \delta_{th}) = -(\phi + \delta_{true}) + 2\pi k$, or $\phi = -(\delta_{true} + \delta_{th})/2 + \pi k.$

Dependence on δ_{true}

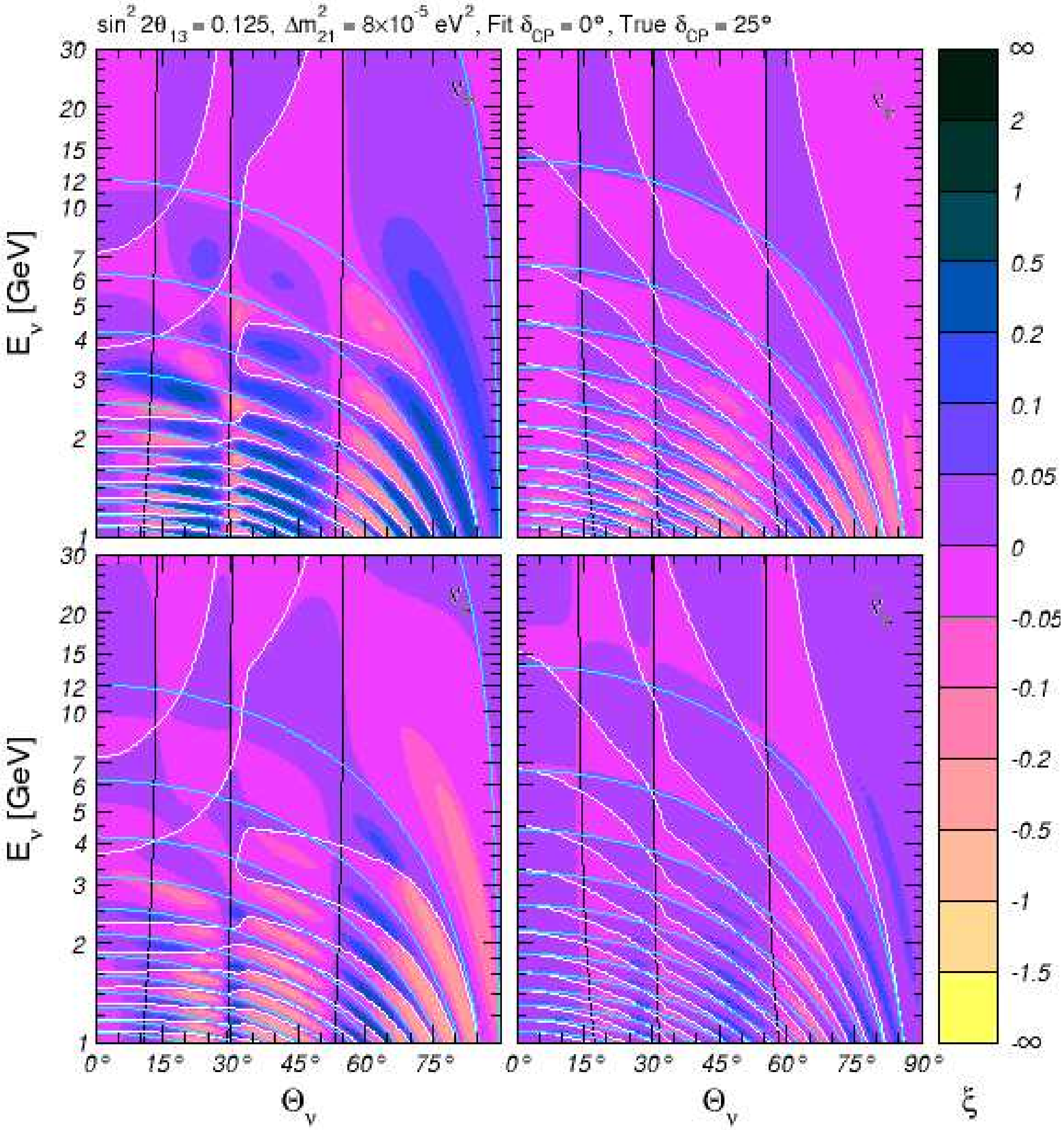


Dependence on δ_{true}



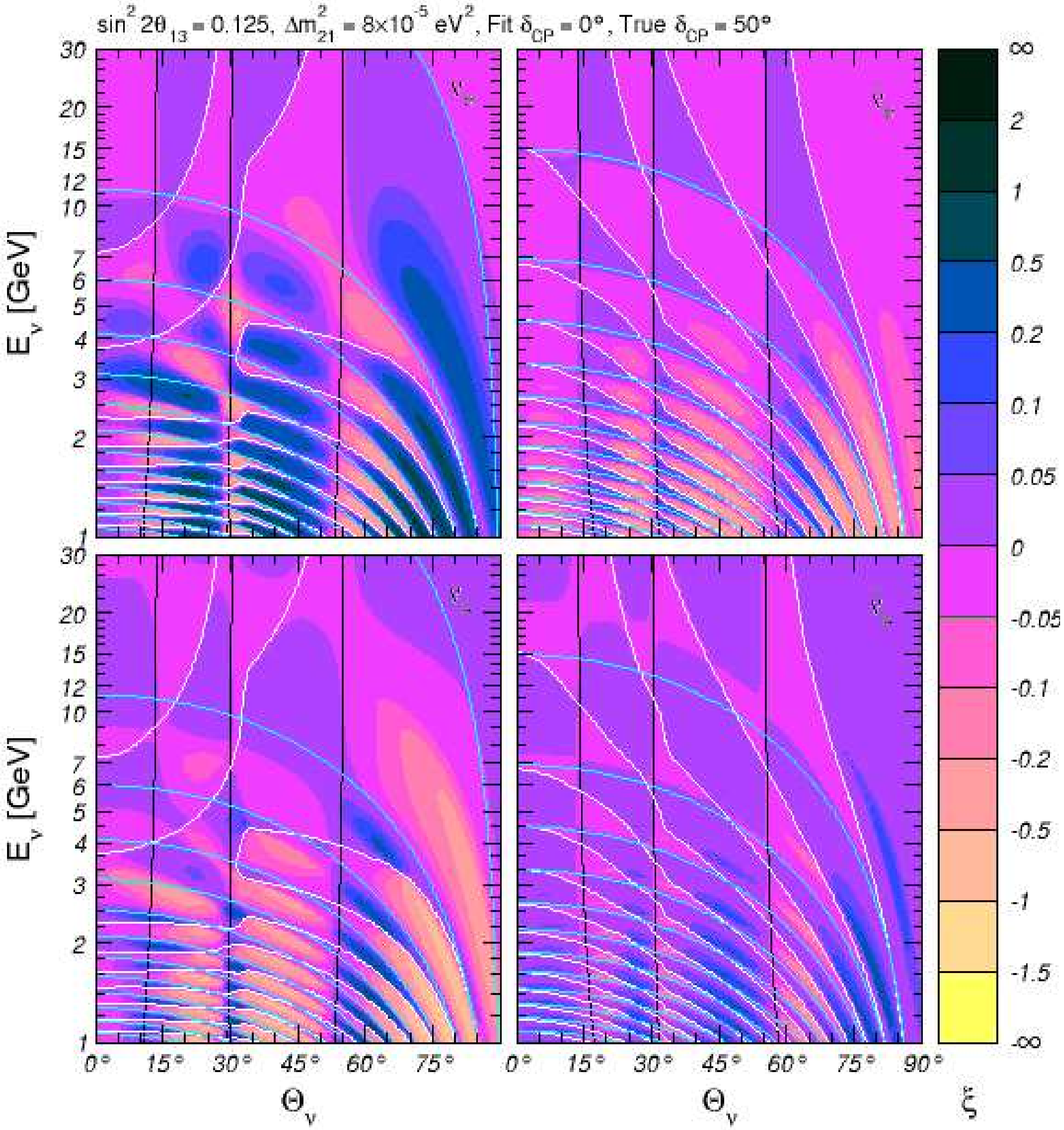
Dependence on

δ_{true}



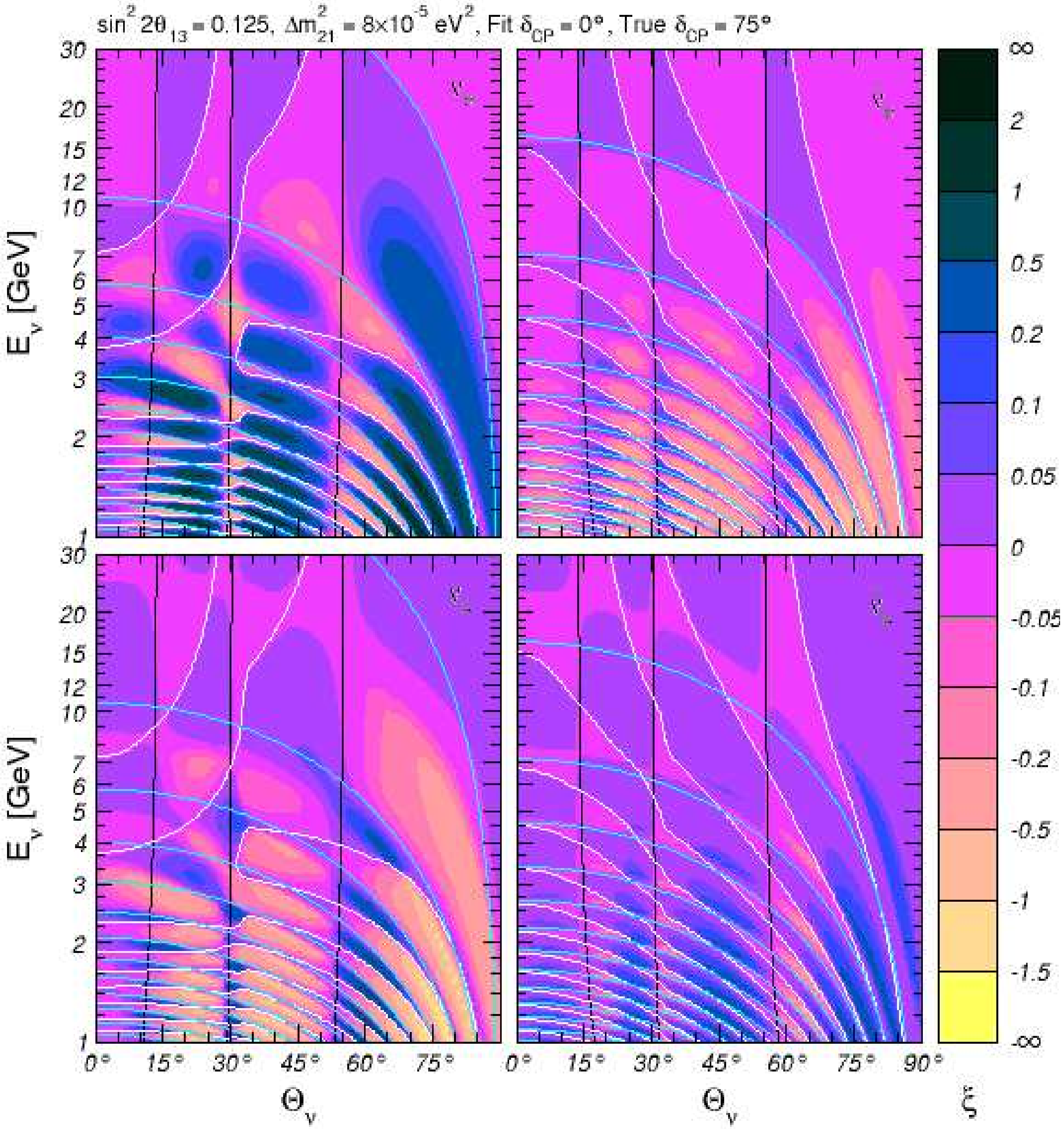
Dependence on

δ_{true}



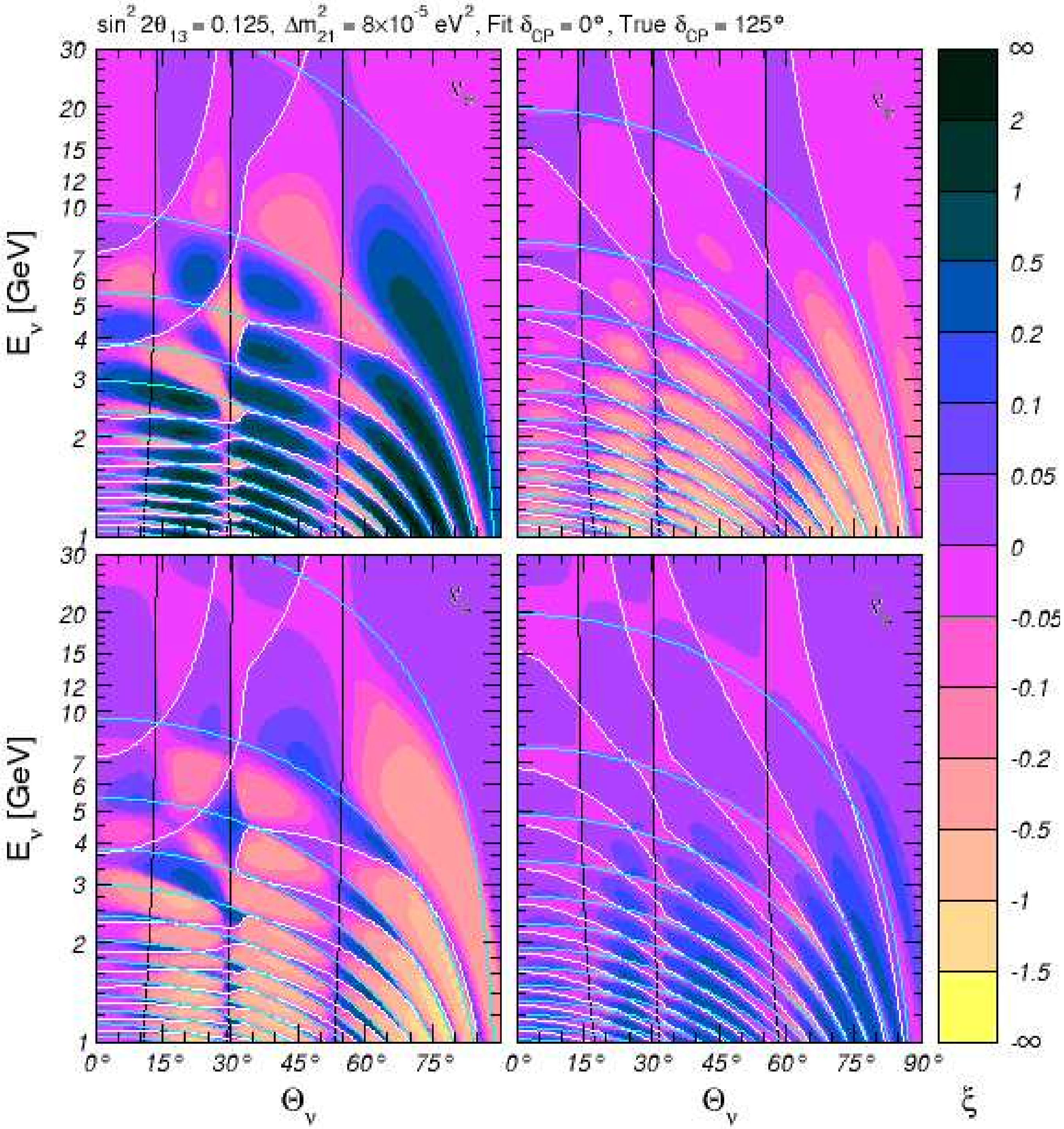
Dependence on

δ_{true}



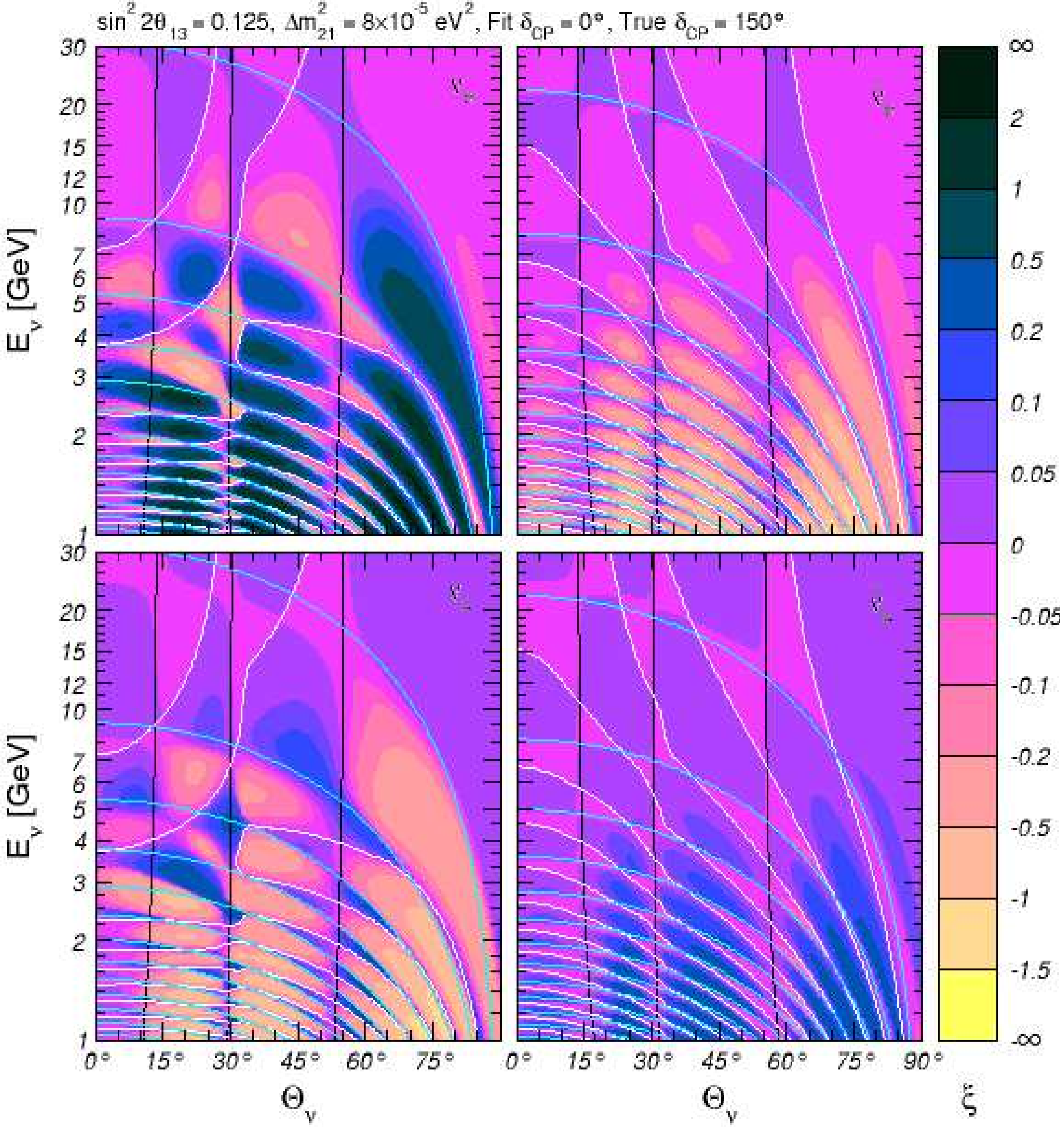
Dependence on

δ_{true}

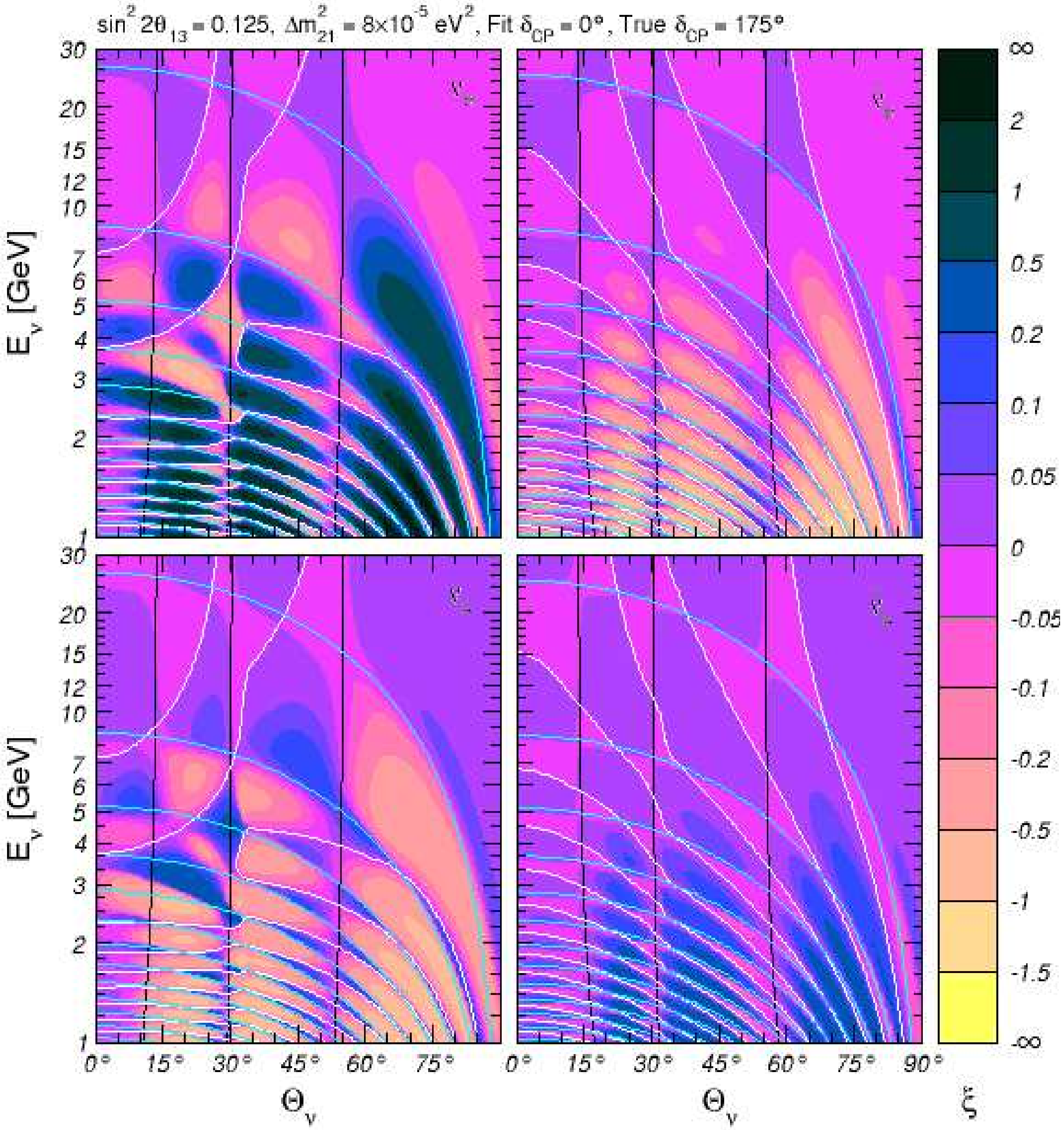


Dependence on

δ_{true}

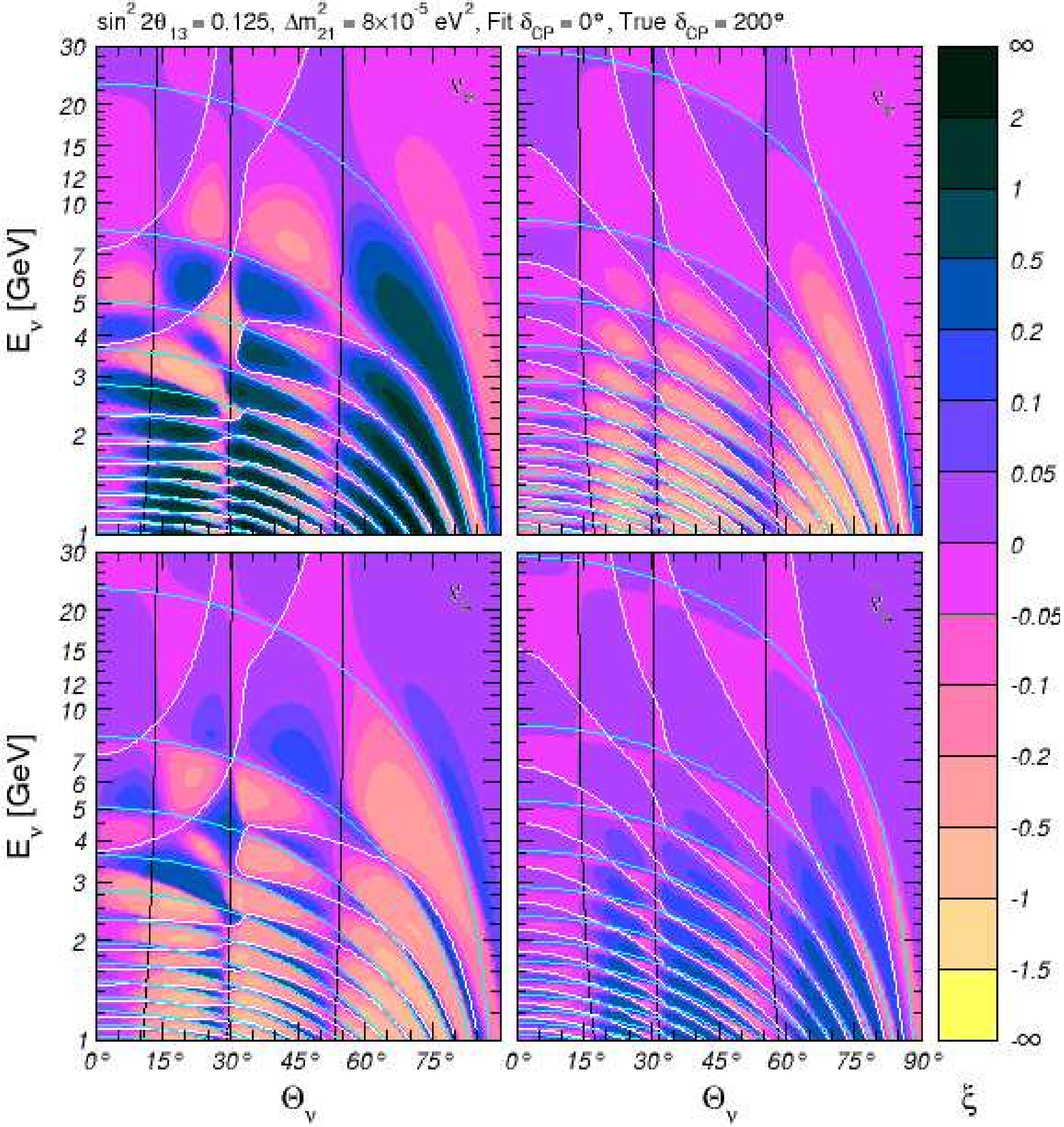


Dependence on δ_{true}



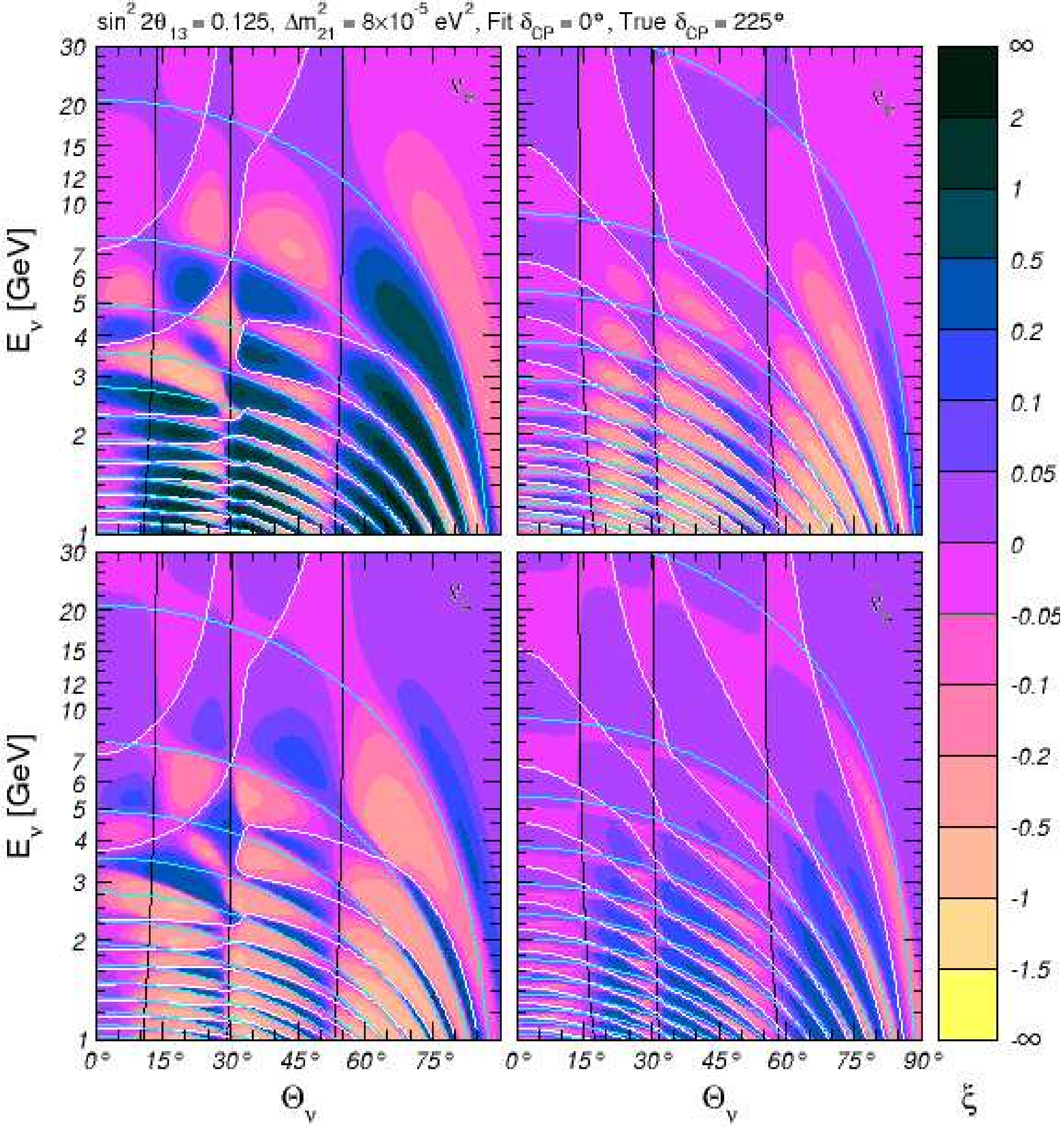
Dependence on

δ_{true}

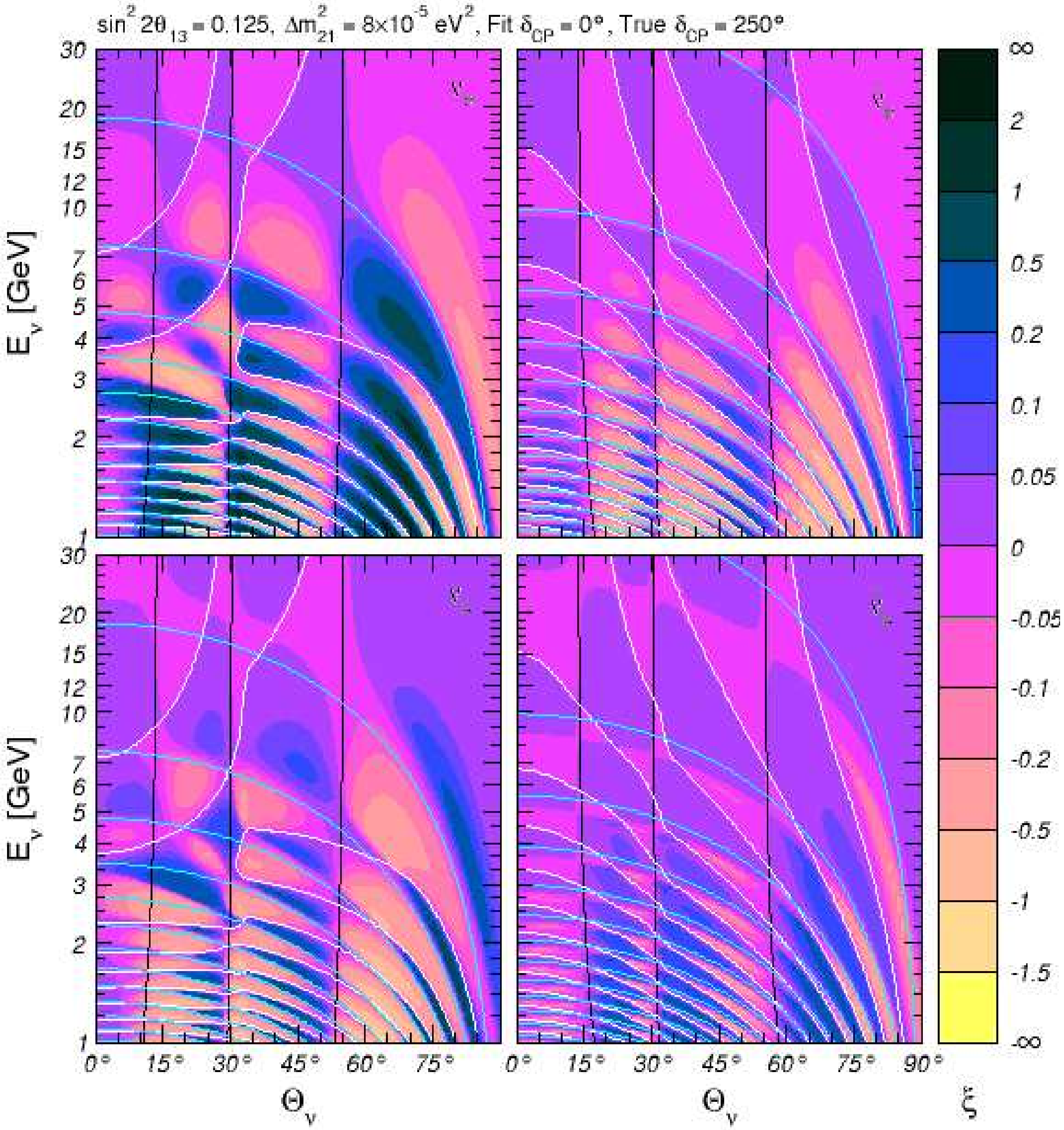


Dependence on

δ_{true}



Dependence on δ_{true}



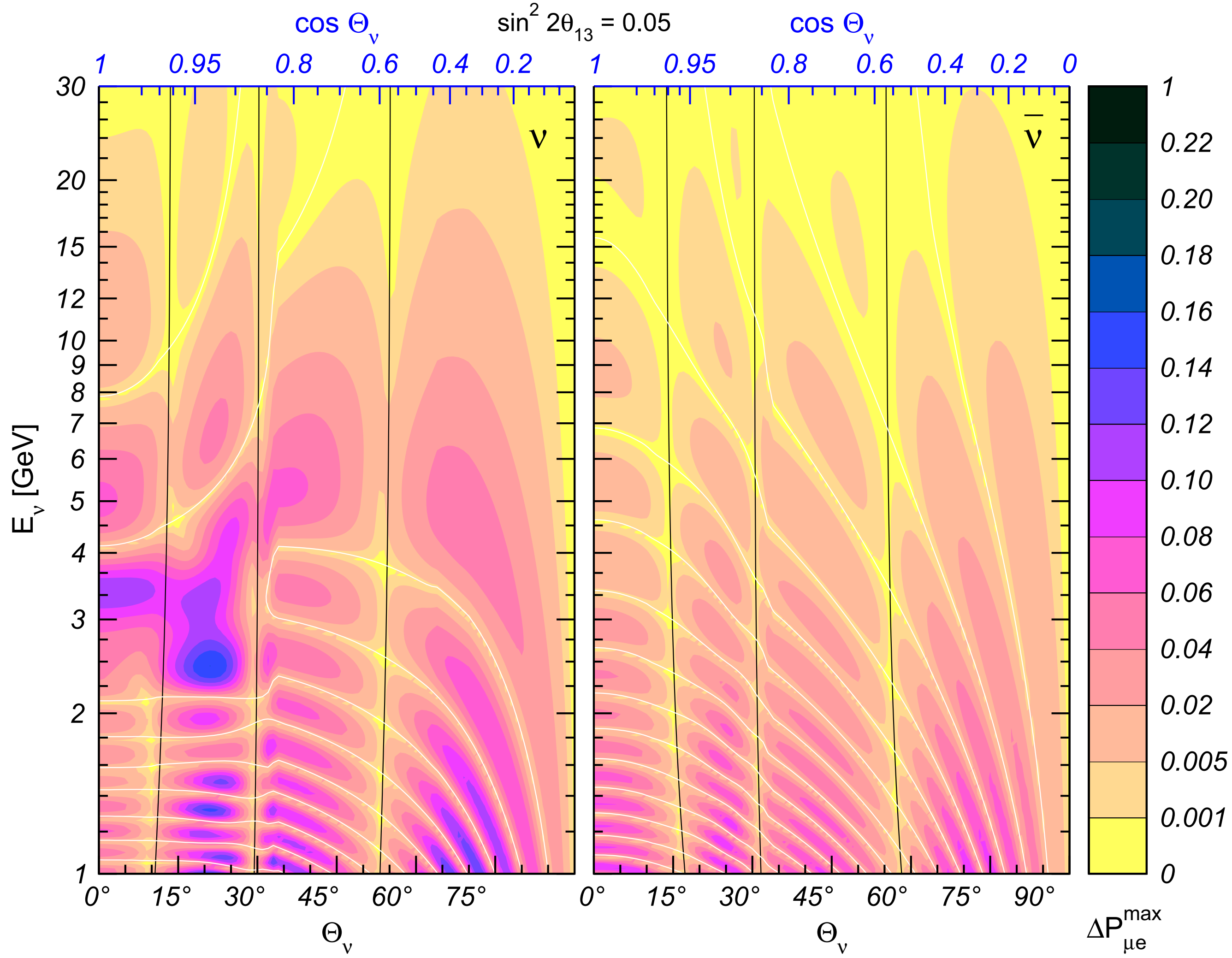
Sensitivity to $\not{C}\not{P}$ phase

Contour plots of

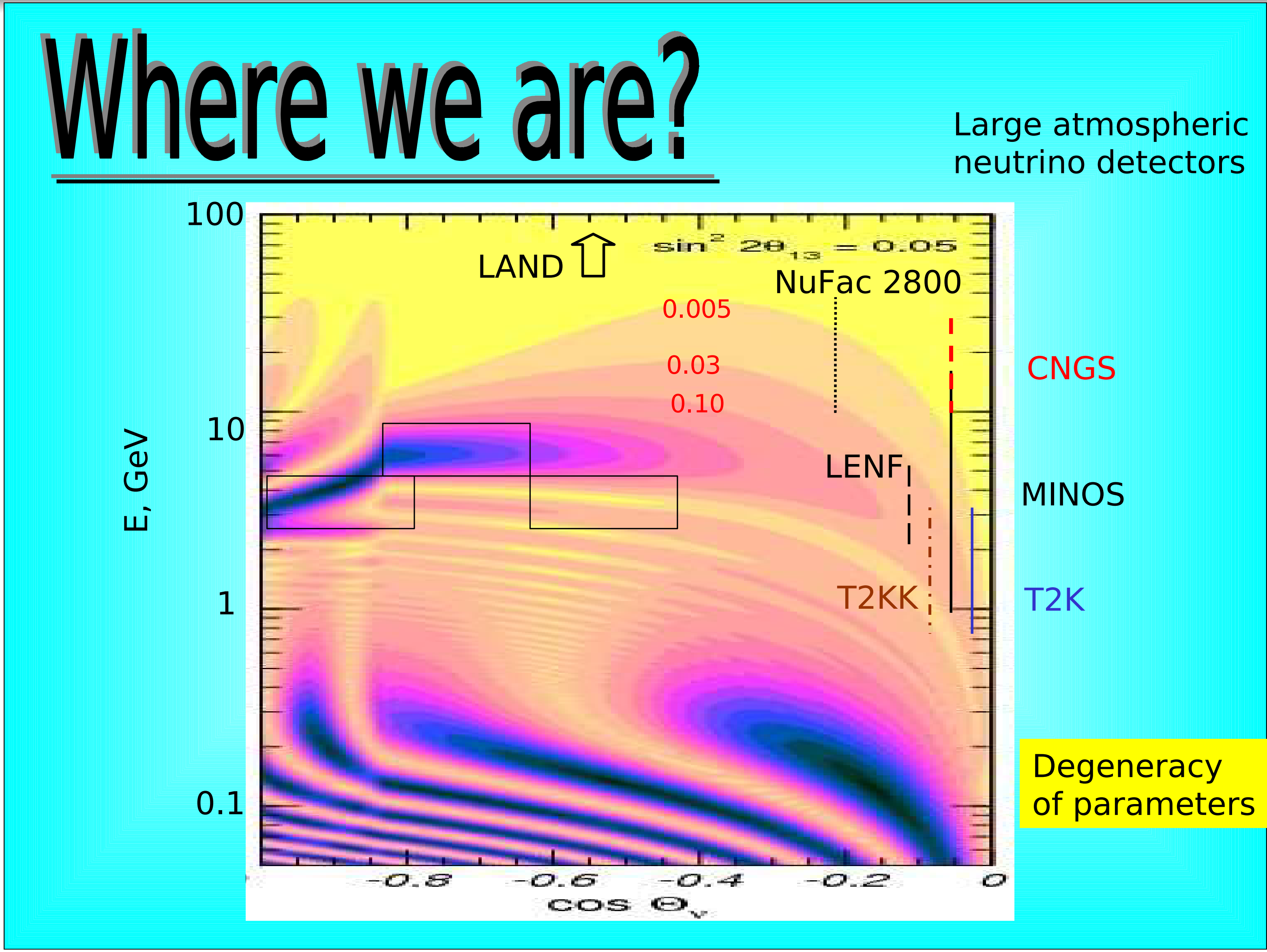
$\Delta P_{\mu e} = P_{\mu e}^{\max} - P_{\mu e}^{\min}$

for δ varying

between 0 and $360°$



Measuring the oscillograms



A. Smirnov, Melbourne talk

Studying ν properties with atm. ν's

Huge atmospheric neutrino detectors may be necessary!

Would require :

- Very good energy and angle resolution
- Low threshold ($E_{\text{thr.}} \sim 2-3$ GeV)
- Charge discrimination (?)
- High statistics

Very ambitious, but the gain may be overwhelming $\Rightarrow$

It is worth studying the oscillograms with Huge Atmospheric Neutrino Detectors !

Conclusions

The potential of atmospheric neutrino experiments for studying neutrino properties is far from being exhausted!

Backup slides

Neutrino oscillations in the Earth

A coherent description in terms of different realizations of just 2 conditions – amplitude and phase conditions

Matter with $N_e = const$:

$$\diamondsuit \quad P_{\rm tr} \;=\; \sin^2 2\theta_m \,\sin^2\phi_m$$

- amplitude condition = MSW resonance condition $(\theta_m = 45^\circ)$
- phase condition: $\phi_m \;=\; \pi/2 + \pi n$

Neutrino oscillations in the Earth

"Castle wall" density profile:

$$\diamondsuit \quad P_{\rm tr}^{(n)} = \frac{X_1^2 + X_2^2}{X_1^2 + X_2^2 + X_3^2} \sin^2 n\Phi$$

Evolution matrix: $\nu(t) = U(t,\, t_0)\, \nu(0)$. For 2 layers:

$$U^{(2)}(t,\, t_0) = \begin{pmatrix} Y - iX_3 & -i(X_1 - iX_2) \\ -i(X_1 + iX_2) & Y + iX_3 \end{pmatrix}, \qquad Y^2 + \mathbf{X}^2 = 1$$

- amplitude condition = parametric resonance condition ($X_3 = 0$)
- phase condition: $\Phi \equiv \arccos Y = \pi/2 + \pi n$

The meaning of the amplitude condition

Alignment of the transitions amplitudes in different layers.

Evolution matrices for individual layers:

$$U_i(t,\,t_0) \;=\; \begin{pmatrix} \alpha_i & \beta_i \\ -\beta_i^* & \alpha_i^* \end{pmatrix}, \qquad |\alpha_i|^2 + |\beta_i|^2 \;=\; 1\,, \qquad i = 1,2,3$$

For 2 layers: $U^{(2)} = U_2 U_1$,

$$\beta^{(2)} \;=\; \alpha_2\,\beta_1 \;+\; \beta_2\,\alpha_1^*$$

Alignment (collinearity) condition:

$$\arg(\alpha_2\,\beta_1) \;=\; \arg(\beta_2\,\alpha_1^*) \qquad \mathrm{mod}\;(\pi)$$

– potentially leads to maximal trans. probability.

For 2 layers of const. densities: align. cond. $\Leftrightarrow$ $s_1 s_2 X_3 \;=\; 0$

How about 3 layers?

$U^{(3)} = U_3 U_2 U_1$.   For the Earth, $U^{(3)} = U_1^T U_2 U_1$.

Transition amplitude:

$$\beta^{(3)} = \alpha_1 \alpha_2 \beta_1 - \alpha_1^* \alpha_2^* \beta_1^* + |\alpha_1|^2 \beta_2 + |\beta_1|^2 \beta_2^*$$

$\Rightarrow$ If the 2-layer align. cond. is satisfied, so is the 3-layer one !

A consequence of

- The symmetry of the core density profile
- The symmetry of the overall density profile of the Earth (3rd layer's profile is the reverse of the 1st layer's one)

$\Rightarrow$ The generalized amplitude condition is the alignment condition in the case of non-constant density layers

Generalized phase condition

For constant density matter: $\phi = \pi/2 + \pi n \quad \Leftrightarrow \quad \mathrm{Im}\,\alpha^{(1)}\beta^{(1)*} = 0.$

$\Rightarrow$ Generalize to an arbitrary density profile:

$$\mathrm{Im}\,\alpha\beta^* = 0 \quad \Leftrightarrow \quad \frac{dP_{\mathrm{tr}}}{dL} = 0$$

The whole complex oscillation pattern:

- MSW resonances
- parametric resonances
- saddle points
- local maxima and minima
- absolute maxima and minima

can be understood in terms of the generalized amplitude and phase conditions ! (E.A., Maltoni & Smirnov, 2006)

Special points

- $\mathrm{Re}\,\alpha_1 \;=\; \mathrm{Re}\,\alpha_2 \;=\; 0$ (constant-density layers: $c_1 \;=\; c_2 \;=\; 0$)

$$\Rightarrow \qquad P_A \;=\; \sin^2(4\theta_m - 2\theta_c)$$

Maxima between the core and mantle MSW resonances for $\theta_c - \theta_m > \pi/4$ and above the MSW resonances, and saddle points below the MSW resonances and between the resonances for $\theta_c - \theta_m \leq \pi/4$.

- $\mathrm{Im}\,\beta_1 \;=\; \mathrm{Re}\,\alpha_2 \;=\; 0$ (constant-density layers: $s_1 \;=\; c_2 \;=\; 0$)

$$\Rightarrow \qquad P_A \;=\; \sin^2 2\theta_c$$

Lie below $2.5\ \mathrm{MeV}$. Local maxima. No (or almost no) oscillation effect in the mantle.

General properties of P_{ab}

3 flavours $\Rightarrow 3 \times 3 = 9$ probabilities

$$P_{ab} = P(\nu_a \to \nu_b),$$

plus 9 probabilities for antineutrinos $P_{\bar{a}\bar{b}}$.

Unitarity conditions (probability conservation):

$$\sum_b P_{ab} = \sum_a P_{ab} = 1 \qquad (a, b = e, \mu, \tau)$$

5 indep. conditions $\Rightarrow 9 - 5 = 4$ indep. probabilities left.

Additional symmetry: the matrix of matter-induced potentials $\mathrm{diag}(V(t), 0, 0)$ commutes with O_{23} $\Rightarrow$ additional relations between probabilities.

Dependence on θ_{23} and # of indep. P_{ab}

Define

$$\tilde{P}_{ab} = P_{ab}(s_{23}^2 \leftrightarrow c_{23}^2,\ \sin 2\theta_{23} \to -\sin 2\theta_{23})$$

(e.g., $\theta_{23} \to \theta_{23} + \pi/2$). Then

$$P_{e\tau} = \tilde{P}_{e\mu} \qquad P_{\tau\mu} = \tilde{P}_{\mu\tau} \qquad P_{\tau\tau} = \tilde{P}_{\mu\mu}$$

2 out of 3 conditions are independent $\Rightarrow$ $4 - 2 = 2$
indep. probabilities (e.g., $P_{e\mu}$ and $P_{\mu\tau}$) $\Rightarrow$

$\Diamond$ *All 9 neutrino ocillation probabilities can be expressed through just two!* (E.A., Johansson, Ohlsson, Lindner & Schwetz, 2004)

$$P_{\bar{a}\bar{b}} = P_{ab}(\delta_{\rm CP} \to -\delta_{\rm CP},\ V \to -V) \quad \Rightarrow$$

$\Diamond$ *All 18 ν and $\bar{\nu}$ probab. can be expressed through just two*

General dependence on $\delta_{\rm CP}$

Another use of essentially the same symmetry: rotate by

$$O'_{23} = O_{23} \times \mathrm{diag}(1, 1, e^{i\delta_{\rm CP}})$$

From commutativity of $\mathrm{diag}(V(t), 0, 0)$ with O'_{23} $\Rightarrow$
General dependence of probabilities on $\delta_{\rm CP}$:

$$P_{e\mu} = A_{e\mu} \cos\delta_{\rm CP} + B_{e\mu} \sin\delta_{\rm CP} + C_{e\mu}$$

$$P_{\mu\tau} = A_{\mu\tau} \cos\delta_{\rm CP} + B_{\mu\tau} \sin\delta_{\rm CP} + C_{\mu\tau}$$

$$+ D_{\mu\tau} \cos 2\delta_{\rm CP} + E_{\mu\tau} \sin 2\delta_{\rm CP}$$

(Yokomakura, Kimura & Takamura, 2002)

Evolution in the rotated basis

Evolution matrix $S(t,\, t_0)$: $\nu(t) \;=\; S(t,\, t_0)\, \nu(t_0)$. Satisfies

$$\diamondsuit \quad i\frac{d}{dt}S(t,\, t_0) \;=\; H\, S(t,\, t_0) \qquad \text{with} \quad S(t_0,\, t_0) \;=\; \mathbb{1}\,.$$

$$H \;=\; (O_{23}\,\Gamma_\delta\, O_{13}\,\Gamma_\delta^\dagger\, O_{12}\,)\,\mathrm{diag}(0,\,\delta,\,\Delta)\;(O_{12}^T\,\Gamma_\delta O_{13}^T\,\Gamma_\delta^\dagger\, O_{23}^T) \;+\; \mathrm{diag}(V(t),\,0,\,0)$$

$$=\; (O_{23}\,\Gamma_\delta\, O_{13}\;\, O_{12}\,)\,\mathrm{diag}(0,\,\delta,\,\Delta)\,(O_{12}^T\, O_{13}^T\,\Gamma_\delta^\dagger\, O_{23}^T) \;+\; \mathrm{diag}(V(t),\,0,\,0)$$

where

$$\delta \;\equiv\; \frac{\Delta m_{21}^2}{2E}\,, \qquad \Delta \;\equiv\; \frac{\Delta m_{31}^2}{2E}$$

Oscillation probabilities:

$$P_{ab} \;=\; |S_{ba}|^2$$

Define

$$O_{23}' = O_{23}\,\Gamma_\delta$$

The matrix $\mathrm{diag}(V(t), 0, 0)$ commutes with O_{23}' $\quad\Rightarrow\quad$ go to the rotated basis

Evolution in the rotated basis – contd.

$$\nu \;=\; O'_{23}\,\nu'\,, \qquad \text{or} \qquad S(t,\,t_0) \;=\; O'_{23}\,S'(t,\,t_0)\,O'_{23}{}^{\dagger}\,,$$

In the rotated basis $H' \;=\; O'_{23}\,H\,O'_{23}{}^{\dagger}$. Explicitly:

$$H'(t) = \begin{pmatrix} s_{12}^2 c_{13}^2\delta + s_{13}^2\Delta + V(t) & s_{12}c_{12}c_{13}\delta & s_{13}c_{13}\left(\Delta - s_{12}^2\delta\right) \\ s_{12}c_{12}c_{13}\delta & c_{12}^2\delta & -s_{12}c_{12}s_{13}\delta \\ s_{13}c_{13}\left(\Delta - s_{12}^2\delta\right) & -s_{12}c_{12}s_{13}\delta & c_{13}^2\Delta + s_{12}^2 s_{13}^2\delta \end{pmatrix}$$

Dependence on θ_{23} and $\delta_{\rm CP}$ can be obtained in the general case by rotating back to the original flavour basis. Also: easy to apply PT approximations

- If $\frac{\Delta m_{21}^2}{2E}L \ll 1$ – neglect $\delta \;=\; \frac{\Delta m_{21}^2}{2E}$
- If θ_{13} is very small – neglect s_{13}

or use expansion in these small parameters

Producing the oscillograms – contd.

Huge atmospheric neutrino detectors may be necessary!

Would require :

- Very good energy and angle resolution
- Low threshold ($E_{\rm thr.} \sim 3$ GeV)
- Charge discrimination (?)
- High statistics

Very ambitious, but the gain may be overwhelming $\Rightarrow$

It is worth studying the oscillograms with Huge Atmospheric Neutrino Detectors !

Conclusions

- Atmospheric neutrino experiments led to the first unambiguous evidence for neutrino oscillations
- About a half of atmospheric neutrinos traverse the Earth on their way to the detector
- Matter can strongly affect ν oscillations inside the Earth through the MSW and parametric resonance effects
- Neutrino oscillograms of the Earth carry a wealth of information both on neutrinos and the Earth

Conclusions

They :

- Depend strongly on the neutrino mass hierarchy and the value of θ_{13}
- Depend sensitively on the $\not{C}\not{P}$ phase δ_{CP} and on the Earth density profile
- Their specific structures (MSW resonances, parametric ridges, local and global extrema, saddle points) and their dependence on ν parameters can by fully described in terms of the amplitude and phase conditions
- This can be used for looking for best strategies for future ν oscillations experiments