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Physics Prospects of Left-Right Intermediate Gauge Symmetry

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PLAN :

- **Status of the minimal “triplet” Model in SUSY $SO(10)$**
- **The new particle spectrum for gauge coupling unification**
- **Predictions from mass spectra analysis in $SO(10) \times S_4$**
- **Fermion masses and mixings in $SO(10) \times S_4$**
- **Summary and Outlook**

I. Introduction

- **We search for intermediate gauge sym.:** $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C (g_L = g_R) \equiv G_{2213}$

- If we break LR gauge sym to MSSM by RH triplet in $\overline{126} \subset SO(10)$, the R-parity is automatically conserved. This is important for proton stability and neutralino DM.

- With parity cons. at the intermediate scale the LH-triplet

  is also at the intermediate scale,

  leading to natural Type II seesaw, in addition to Type I seesaw, or an admixure of the two.

At first we consider the minimal spectrum:

$$SO(10) \xrightarrow[M_U]{210 \oplus 54} G_{2213}$$

$$\xrightarrow[M_R]{126 + \overline{126}} G_{213}$$

$$\xrightarrow[M_W]{10} U(1)_{em} \times SU(3)_C$$

To discuss gauge coupling unification we use the RGEs from M_Z to M_U,

$$\frac{1}{\alpha_Y(M_Z)} = \frac{1}{\alpha_G} + \frac{a_Y}{2\pi}\ln\frac{M_R}{M_Z} + \frac{1}{10\pi}\left(3a'_{2L} + 2a'_{BL}\right)\ln\frac{M_U}{M_R}$$

$$\frac{1}{\alpha_{2L}(M_Z)} = \frac{1}{\alpha_G} + \frac{a_{2L}}{2\pi}\ln\frac{M_R}{M_Z} + \frac{a'_{2L}}{2\pi}\ln\frac{M_U}{M_R}$$

$$\frac{1}{\alpha_{3C}(M_Z)} = \frac{1}{\alpha_G} + \frac{a_{3C}}{2\pi}\ln\frac{M_R}{M_Z} + \frac{a'_{3C}}{2\pi}\ln\frac{M_U}{M_R}$$

Here primes are for RG coeffs. in LRS theory and nonprimes for MSSM.

From these we obtain two equations:

$$L_\theta \equiv \frac{2\pi}{\alpha(M_Z)} \left(1 - \frac{8\, Sin^2\theta_W(M_Z)}{3}\right)$$

$$= A \ln \frac{M_U}{M_Z} + B \ln \frac{M_R}{M_Z},$$

$$L_S \equiv \frac{2\pi}{\alpha(M_Z)} \left(1 - \frac{8\alpha(M_Z)}{3\alpha_{3C}(M_Z)}\right)$$

$$= A' \ln \frac{M_U}{M_Z} + B' \ln \frac{M_R}{M_Z}.$$

$$
\begin{aligned}
A &= \frac{2}{3}(a'_{BL} - a'_{2L}), \\
B &= \frac{5}{3}(a_Y - a_{2L}) - A, \\
A' &= 2a'_{2L} + \frac{2}{3}a'_{BL} - \frac{8}{3}a'_{3C}, \\
B' &= \frac{5}{3}a_Y + a_{2L} - \frac{8}{3}a_{3C} - A'.
\end{aligned}
$$

Since B and B' are coeffs. of $\ln(M_R/M_Z)$ **we expect lower values of int. scale to be permissible only if these coeffs. are small.**

Using PDG value of $\alpha(M_Z) = 127.9$**,** $Sin^2\theta_W(M_Z) = 0.2312$ **, and** $\alpha_{3C}(M_Z) = 0.1187$**, gives**
$L_S = 662.736, \;\; L_\theta = 308.305$**.**

The minimal spectrum and the associated beta function coefficients are
$\mu = M_Z$ **-** M_R**:**
$\Phi_u(2,1,1) \oplus \Phi_d(2,-1,1) \subset G_{213}$,
$a_Y = 33/5, a_{2L} = 1, a_{3C} = -3$

$\mu = M_R - M_U$**:**
$\Phi(2,2,0,1), \Delta_L(3,1,-2,1) \oplus \Delta_R(1,3,-2,1) \oplus \overline{\Delta_L}(3,1,2,1) \oplus \overline{\Delta_R}(3,1,2,1)$,

$a'_{BL} = 24,\ a'_{2L} = a'_{2R} = 5,$
$a'_{3C} = a_{3C} = -3$
$A = 38/3,\ B = -10/3,\ A' = 34,\ B' = -14,\ AB' - A'B = -64$ **, i,e B and B' are not small. In fact** $|B'| \simeq A$**.**

The solutions are
$M_R \simeq 10^{16}\ GeV,\ M_U = 2 \times 10^{16}\ GeV$ **Note:** M_R **is at least two orders larger than the seesaw scale.**

II. INT. SCALE IN SUSY $SO(10) \times S_4$

We note the following:

- **nonSUSY SM alone does not unify gauge couplings**
- **Unification takes place in non SUSY GUTS when SM is extended in the gauge and/or in particle spectrum**
- **With extn. in gauge sector to G_{2213} non SUSY $SO(10)$ unifies the couplings with this intermediate gauge symmetry.**
- **When nonSUSY SM is promoted to MSSM with extended particle spectrum unification takes place rejecting any intermediate symmetry.**
- **Failure of intermediate sym. at one loop level in SUSY GUT might be hinting at further extn. of particle spectrum**

- **We find that this appropriate extn. is realised when S_4 family symmetry is included with** $G_{2213} \to G_{2213} \times S_4$ **and** $SO(10) \to SO(10) \times S_4$

- **By using $SO(10)$ Higgs multiplets which are singlets under S_4 to break the gauge symmetries at the GUT scale and the intermediate scale we have $MSSM \times S_4$ as the eW scale sym. or TeV scale sym. In prticular we realize the Spont. Sym. Breaking pattern**

  $SO(10) \times S_4 \to G_{2213} \times S_4 \to MSSM \times S_4$ **with perfect one-loop gauge coupling unification and $M_R = 10^{9.5}$ GeV to 10^{15} GeV.**

Hagedorn, Lindner and Mohapatra(2006) have examined the rich structure for fermion masses and mixings in non-SUSY $SM \times S_4$ **model. One should look for gauge coupling unification in this model . Our model gives** $MSSM \times S_4$ **as the ew-scale sym. or TeV - scale sym. with left-right intermediate gauge sym. and perfect of gauge coupling unification. over a wide range of values of the intermediate scale using only Renormalizable Interactions.**

Keeping the MSSM particle spectrum from M_Z to M_R unaltered, the enlarged particle spectrum at the intermediate scale consistent with $G_{2213} \times S_4$ symmetry is:

$\mu \simeq M_R$:

$$\Delta_L(3,1,-2,1) \oplus \Delta_R(1,3,-2,1) \oplus$$
$$\overline{\Delta_L}(3,1,2,1) \oplus \overline{\Delta_R}(3,1,2,1)$$
$$6(2,2,0,1), 3(1,1,0,8)$$

These modify the beta fn. coeffs. with:
$a'_{BL} = 24,\ a'_{2L} = a'_{2R} = 10, a'_{3C} = 6$
$A = 28/3, A' = 20$, **and**
$B = B' = 0$

The fact that the two coeffcs. exactly vanish with such $G_{2213} \times S_4$ multiplets of Higgs scalars may have some underlying importance for the combination of P, R, and S_4 symmetries.

The numerical solution of successful gauge coupling unification with $M_R = 10^{13}$ GeV is shown in the Fig.1. Any value of Int. scale is now allowed with $M_R \geq 10^{9.5}$ GeV.

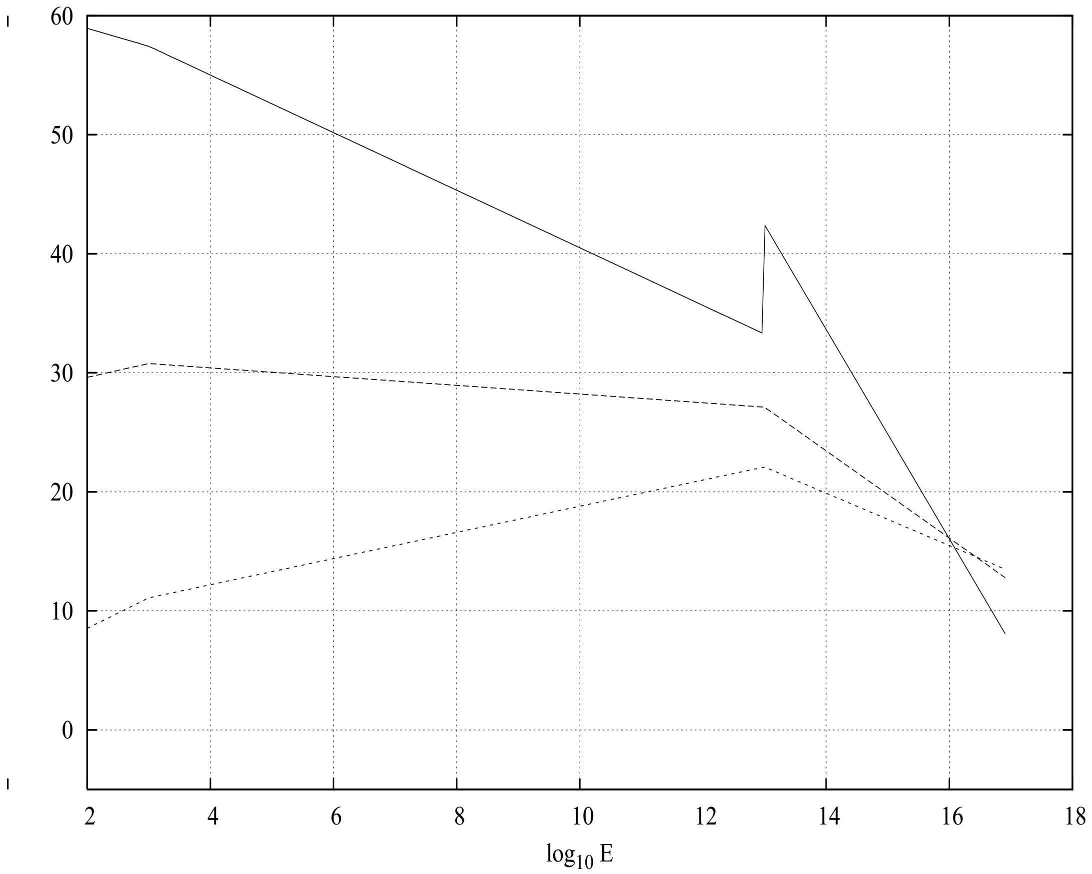

Figure 1: The evolution of the gauge couplings with $G_{2213} \times S_4$ intermediate gauge symmetry in SUSY $SO(10) \times S_4$ model

We need six bidoublets which may originate from six tens of $SO(10) \times S_4$ where $6 = 3 + 2 + 1$. That means we have one triplet, one doublet, and one singlet of S_4 Similarly the three octets under $SU(3)_C$ may originate from a triplet of 45 of $SO(10) \times S_4$.

We have carried out mass spectra analysis to show that such a spectrum is permitted by the theory below the GUT scale by tuning the parameters of the superpotential.

Under $SO(10) \times S_4$ **they are:**

$\Phi(210,1), S(54,1), \Sigma_0(126,1), \overline{\Sigma}_0(\overline{126},1)$
$A_i(45,3), H_0(10,3), H_D(10,2), H_T(10,3)$

Higgs Superpotential:

$W_0 = \frac{1}{2}m_\Phi \Phi^2 + \frac{1}{2}m_S S^2 + \frac{1}{2}m_A \Sigma_i A_i^2$

$+m_\Sigma \Sigma\overline{\Sigma} + \frac{1}{2}m_{H_0} H_0^2 + + \frac{1}{2}m_{H_D} {H_D}^2$

$+\frac{1}{2}m_{H_T} {H_T}^2 + \lambda_0 \Phi^3 + \lambda_1 \Phi\Sigma\overline{\Sigma}$

$+(\lambda_2 \Sigma + \lambda_3 \overline{\Sigma}) H_0 \Phi + \lambda_4 \Sigma_i A_i^2 \Phi + S(\lambda_5 S^2 + \lambda_6 \Sigma_i A_i^2 + \lambda_7 \Phi^2$

$+\lambda_8 \Sigma^2 + \lambda_9 \overline{\Sigma}^2 + \lambda_{10} H_0^2 + \lambda_{11} H_D^2 + \lambda_{12} H_T^2) + \ldots.$

Using vacuum expectation values $< S >= v_S, < \Phi >= v_\Phi < \Delta_R >= \sigma, < \overline{\Delta_R} >= \overline{\sigma}$**, the vanishing F-terms yield ,**

$$m_\Phi v_\Phi + \frac{{}_0 v_\Phi^2}{3\sqrt{2}} + \frac{{}_1 \sigma\overline{\sigma}}{10\sqrt{2}} - \frac{2{}_7 v_\Phi v_S}{\sqrt{15}} = 0,$$

$$m_S v_S + \frac{\sqrt{3}\lambda_5 v_S^2}{2\sqrt{5}} - \frac{\lambda_7 v_\Phi^2}{\sqrt{15}} = 0,$$

$$\left[m_\Sigma + \frac{\lambda_1 v_\Phi}{10\sqrt{2}}\right]\sigma = 0$$

Vanishing D-term gives, $\sigma = \overline{\sigma}$**. Also for the desired hierarchy** $\sigma = \overline{\sigma} <<< S >\simeq \quad < \Phi >$ **leadind to**

$$m_\Phi + \frac{\lambda_0 v_\Phi}{3\sqrt{2}} - \frac{2\lambda_7 v_S}{\sqrt{15}} = 0$$

v_S **satisfies a quadratic equation,**

$$p v_S^2 + q v_S - r = 0$$

$$p = \frac{\sqrt{3}\lambda_5}{2\sqrt{5}} - \frac{24\lambda_7^3}{5\sqrt{15}\lambda_0^2},$$

$$q = m_S + \frac{24\lambda_7^2}{5\lambda_0^2} m_\Phi$$

$$r = \frac{18\lambda_7}{\sqrt{15}\lambda_0^2} m_\Phi^2$$

Goldstone Bosons are 30 in number

A.1. $(1, 1, 3, 2/3) + (c.c) \subset (1, 1, 15) \subset 210$ **as 6 Goldstone Bosons.They have masses** $m_{G1} = m_\Phi + \frac{\lambda_0 v_\Phi}{3\sqrt{2}} - \frac{2\lambda_7 v_S}{\sqrt{15}} = 0$

A.2.$(2, 2, 3, 1/3)+(c.c)$ **as remaining** 24 **Goldstone Bosons from a linear combination of states in** $(2, 2, 6) \subset 54$ **and** $(2, 2, \overline{10}) \subset 210$**. To achieve this finetuning of** λ_5 **is needed.**

Light Scalrs from Mass Spectra Analysis:

(A). $\Delta_L(3,1,-2,1), \Delta_R(1,3,-2,1) \subset 126$ **and their conjugates acquire degenerate masses,**

$$M_R = m_\Sigma + \frac{\lambda_1 v_\Phi}{10\sqrt{2}}$$

$M_R << M_U$ **is guaranted by tuning** λ_1**.**

(B). The five bi-doublts from five 10**-plets are teated to have**
masses near M_R **in the usual fashion by some doublet triplet splitting**
mechanism or by tuning the parameters (e.g. λ_{11}**, and** λ_{12}**)**
. The sixth bidoublet in the sixth 10**-plet which is a siglet under** S_4 **istreated to have mass near** 100 **GeV (e,g by finetuning of** λ_{10}**). Equivalently by suaitable unitary transforamation and fine tunings one can keep five linear combinations of bi-doublets at the** M_R **scale while the sixth linear combination is kept at the ew-scale. .**

(C). Choosing the basis $(A_i)^{1,1,0,8}_{1,1,15}, S^{1,1,0,8}_{1,1,20'}, \Phi^{1,1,0,8}_{1,1,15}$, we find that their are 3 number of unmixed states in A_i with masses,

$m_A - \frac{\sqrt{2}\lambda_4 v_\Phi}{3} - \frac{2\lambda_6 v_S}{\sqrt{15}}$ **Clearly the advatage of A_i being the members of $\mathbf{3} \subset \mathrm{S}_\mathbf{4}$ is that the tuning of the single parameter λ_4 makes all the three octets light at the M_R scale**

.

It is found that the other two states mix through the mass matrix

,

$$M_2 = \begin{bmatrix} m_S - \frac{2\sqrt{3}\lambda_5 v_S}{\sqrt{5}} & -\frac{\lambda_7 v_\Phi}{\sqrt{6}} \\ -\frac{\lambda_7 v_\Phi}{\sqrt{6}} & m_\Phi - \frac{\lambda_0 v_\Phi}{3\sqrt{2}} - \frac{2_7 v_S}{\sqrt{15}} \end{bmatrix}.$$

(1)

The eigen values emerging from this eq. are in general at the
GUT scale and we do not adopt any further fine-tuning.

Thus by tuning the parameters of the super-potential it is possible to obtain five bidoublets, three color octets and left- and right handed triplets at the M_R scale necessary for successful gauge coupling unification with G_{2213} intermediate symmetry with $M_R = 5 \times 10^5 - 10^{16}$ GeV.

One bidoublet is kept at the ew scale.

III. FERMION MASSES AND MIXINGS

Details can be found in arXiv:0804.4571.

For our analysis of fermion masses we adopt the following strategy:

We add a doublet of $(126 \oplus 1\bar{2}6)_{1,2}$ **having GUT-scale masses for all their components.**

(i) All the ew bi-doublets in $(126 \oplus 1\bar{2}6)_{0,1,2}$ **are kept heavy at the GUT scale.**

(ii) In this way the five bi-doublets at the intermediate scale acquire VEVs $\simeq 100$ **GeV but the three weak bidoublets in** $(1\bar{2}6)_{0,1,2}$ **aquire a somewhat suppressed induced VEV's** $\simeq 10 - 100$ **MeV.**

All these contribute to fermion masses and mixing and are adequate for our purpose.

The three fermion generations are treated as a triplet under S_4.

Under $SO(10) \times S_4, \Psi_i = (16, 3)$.The super-potential for fermion-Higgs Yukawa interaction is written as,

$$
\begin{aligned}
W_{Yuk} = {} & (\Psi_1\Psi_1 + \Psi_2\Psi_2 + \Psi_3\Psi_3)(y_0 H_0 + f_0\overline{\Sigma}_0) \\
& + \tfrac{1}{\sqrt{2}}(\Psi_2\Psi_2 - \Psi_3\Psi_3)(y_1 H_1 + f_1\overline{\Sigma}_1) \\
& + \tfrac{1}{\sqrt{6}}(-2\Psi_1\Psi_1 + \Psi_2\Psi_2 + \Psi_3\Psi_3)(y_1 H_2 + f_1\overline{\Sigma}_2) \\
& + y_3[(\Psi_2\Psi_3 + \Psi_3\Psi_2)H_3 + (\Psi_1\Psi_3 + \Psi_3\Psi_1)H_4 \\
& + (\Psi_1\Psi_2 + \Psi_2\Psi_1)H_5].
\end{aligned}
$$

Just below the GUT-scale for $\mu \sim M_U$,

$$
\begin{aligned}
W_{Yuk} \;=\; & \Sigma_{k=1}^{3}[Q_k^T\tau_2(y_0H_0^\phi + f_0\overline{\Delta}_0^\phi)Q_k^C \\
& +L_k^T\tau_2(y_0H_0^\phi - 3f_0\overline{\Delta}_0^\phi)L_k^C] \\
& +\frac{1}{\sqrt{2}}[Q_2^T\tau_2(y_1H_1^\phi + f_1\overline{\Delta}_1^\phi)Q_2^C \\
& -Q_3^T\tau_2(y_1H_1^\phi + f_1\overline{\Delta}_2^\phi)Q_3^C \\
& +L_2^T\tau_2(y_1H_1^\phi - 3f_1\overline{\Delta}_1^\phi)L_2^C \\
& -L_3^T\tau_2(y_1H_1^\phi - 3f_1\overline{\Delta}_2^\phi)L_3^C] \\
& +\frac{1}{\sqrt{6}}[-2Q_1^T(y_1H_2^\phi + f_1\overline{\Delta}_2^\phi)Q_1^C \\
& -2L_1^T\tau_2(y_1H_2^\phi - 3f_1\overline{\Delta}_2^\phi)L_1^C \\
& +Q_2^T\tau_2(y_1H_2^\phi + f_1\overline{\Delta}_2^\phi)Q_2^C \\
& L_2^T\tau_2(y_1H_2^\phi - 3f_1\overline{\Delta}_2^\phi)L_2^C \\
& +Q_3^T\tau_2(y_1H_2^\phi + f_1\overline{\Delta}_2^\phi)Q_3^C \\
& +L_3^T\tau_2(y_1H_2^\phi - 3f_1\overline{\Delta}_2^\phi)L_3^C]
\end{aligned}
$$

$$+y_3[Q_2^T\tau_2 H_3^\phi Q_3^C + Q_3^T\tau_2 H_3^\phi Q_2^C$$
$$+Q_1^T\tau_2 H_4^\phi Q_3^C$$
$$+Q_3^T\tau_2 H_4^\phi Q_1^C + Q_1^T\tau_2 H_5^\phi Q_2^C$$
$$+Q_2^T\tau_2 H_5^\phi Q_1^C + (Q \to L)].$$

where all $\overline{\mathbf{\Delta}}_{\mathbf{i}}^{\phi}(\mathbf{i}=\mathbf{0},\mathbf{1},\mathbf{2})$ are heavy weak bi-doublets in $(\mathbf{2},\mathbf{2},\mathbf{15})_{\mathbf{i}} \subset \overline{\mathbf{126}}_{\mathbf{i}}(\mathbf{i}=\mathbf{0},\mathbf{1},\mathbf{2})$. All other six bi-doublets are lighter than M_U; five of them being at M_R and the sixth one near M_Z.

Adding their contributions, the mass matrices of quarks and leptons have the well known forms,

$M_u = M_u^{(10)} + M_u^{(126)}$, $M_d = M_d^{(10)} + M_d^{(126)}$,

$M_l = M_d^{(10)} - 3M_d^{(126)}$, $M_\nu^D = M_u^{(10)} - 3M_u^{(126)}$,

The Type-I seesaw contribution to light neutrino mass matrx is,

$M_\nu = -{M^D_\nu}^T M_\nu^D / M_N$,

$$M_u^{(10)} = \left[\begin{array}{ccc} \alpha_0 - 2\alpha_2 & \alpha_5 & \alpha_4 \\ \alpha_5 & \alpha_0 + \alpha_1 + \alpha_2 & \alpha_3 \\ \alpha_4 & \alpha_3 & \alpha_0 - \alpha_1 + \alpha_2 \end{array} \right],$$

$$M_d^{(10)} = \left[\begin{array}{ccc} \beta_0 - 2\beta_2 & \beta_5 & \beta_4 \\ \beta_5 & \beta_0 + \beta_1 + \beta_2 & \beta_3 \\ \beta_4 & \beta_3 & \beta_0 - \beta_1 + \beta_2 \end{array} \right]$$

$$M_u^{(126)} = \left[\begin{array}{ccc} \gamma_0 - 2\gamma_2 & 0 & 0 \\ 0 & \gamma_0 + \gamma_1 + \gamma_2 & 0 \\ 0 & 0 & \gamma_0 - \gamma_1 + \gamma_2 \end{array} \right]$$

$$M_d^{(126)} = \left[\begin{array}{ccc} \delta_0 - 2\delta_2 & 0 & 0 \\ 0 & \delta_0 + \delta_1 + \delta_2 & 0 \\ 0 & 0 & \delta_0 - \delta_1 + \delta_2 \end{array} \right]$$

In these equations
$\alpha_i \equiv y_i < H_i^u >, \beta_i \equiv y_i < H_i^d >$,
$\gamma_i \equiv f_i < \Delta_i^u >, \delta \equiv f_i < \Delta_i^d >$.
(i not summed).
The choice of diagonal basis in the down quark sector which automatically also leads to the diagonal basis in the charged lepton sector, enables to choose the six parameters, $\beta_i, \delta_i (i = 0, 1, 2)$ **to be real and** $\beta_3 = \beta_4 = \beta_5 = 0$**. All other parameters are, in general, complex. Analytically the six real parameters in terms of down-quark and charged lepton mass eigen-values at the see-saw scale (**$\mu = M_R$**) are,**

$$
\begin{aligned}
\beta_0 &= \left[3(m_b^0 + m_s^0 + m_d^0) + m_\tau^0 + m_\mu^0 + m_e^0\right]/12, \\
\beta_1 &= \left[-3m_b^0 + 3m_s^0 - m_\tau^0 + m_\mu^0\right]/8, \\
\beta_2 &= \left[3m_b^0 + 3m_s^0 - 6m_d^0 + m_\tau^0 + m_\mu^0 - 2m_e^0\right]/24, \\
\delta_0 &= \left[m_b^0 + m_s^0 + m_d^0 - (m_\tau^0 + m_\mu^0 + m_e^0)\right]/12, \\
\delta_1 &= \left[-m_b^0 + m_s^0 + m_\tau^0 - m_\mu^0\right]/8, \\
\delta_2 &= \left[m_b^0 + m_s^0 - 2m_d^0 - m_\tau^0 - m_\mu^0 + 2m_e^0\right]/24.
\end{aligned}
$$

Table 1: Renormalisation Group extrapolated running masses of quarks and charged leptons of three generations at the intermediate scale $M_{\rm R} = 10^{13}$ GeV as estimated.

$\tan\beta$	10	55
$m_{\rm u}$ (MeV)	$0.8882\pm^{0.1694}_{0.1794}$	$0.8882\pm^{0.1675}_{0.1795}$
$m_{\rm c}$ (MeV)	$258.0945\pm^{23.8287}_{25.8339}$	$258.2929\pm^{23.3295}_{25.8144}$
$m_{\rm t}$ (GeV)	$94.3698\pm^{22.5572}_{25.8339}$	$104.2363\pm^{32.7015}_{18.2028}$
$m_{\rm d}$ (MeV)	$1.8290\pm^{0.5111}_{0.2779}$	$1.8219\pm^{0.5054}_{0.2755}$
$m_{\rm s}$ (MeV)	$36.4261\pm^{5.1588}_{5.4807}$	$36.2891\pm^{5.0777}_{5.4340}$
$m_{\rm b}$ (GeV)	$1.2637\pm^{0.1189}_{0.0893}$	$1.5768\pm^{0.2640}_{0.1685}$
$m_{\rm e}$ (MeV)	$0.3911\pm^{0.0002}_{0.0002}$	$0.3893\pm^{0.0005}_{0.0002}$
m_{μ} (MeV)	$82.5539\pm^{0.0346}_{0.0330}$	$82.2064\pm^{0.0468}_{0.1024}$
m_{τ} (GeV)	$1.4085\pm^{0.0009}_{0.0008}$	$1.6574\pm^{0.0188}_{0.0148}$

We utilise the RG-extrapolated values of the running charged fermion masses at the intermediate scale $\mu = M_R \approx v_R \approx 10^{13}$ GeV as shown in Table.1 for $\tan\beta = 10, 55$ [?]. In the present model the definition $\tan\beta = v_u/v_d$ is valid in the presence of MSSM below the intermediate scale.

Using the down quark and charged lepton masses from Table 1 and eqs. for β_i and δ_i, we obtain,

$$\beta_0 = 449.773\,\mathrm{MeV},\ \beta_1 = -625.971\,\mathrm{MeV},$$

$$\beta_2 = 224.155\,\mathrm{MeV},\ \delta_0 = -15.791\,\mathrm{MeV},$$

$$\delta_1 = 12.334\,\mathrm{MeV},\ \delta_2 = -8.074\,\mathrm{MeV}.$$

Using low-energy values of CKM matrix elements with its phase $\delta = 60^\circ$ and using the renormalisation factor $r_N = exp[-(y_{top}^2 ln(v_R/m_{top})/16\pi$ 0.86 leads to the CKM matrix at $\mu = M_R = 10^{13}$ GeV. We obtain elements of M_u in terms of the running up-qurk mass eigen-values and CKM elements via,

$$M_u = V_{CKM}^T \mathrm{diag}(m_u^0, m_c^0, m_t^0) V_{CKM}$$

For $\tan\beta = 10$, using eqs for $M_U^{(10)}, M_U^{(126)}$, and $M_u = M_U^{(10)} + M_U^{(126)}$ and the elements of M_u obtained from V_{CKM} and eigen values of running up-quark masses at intermediate scale, determines the three parameters $\alpha_i (i = 3, 4, 5)$ while three equations are obtained among the

other six complex parameters, $\alpha_i (i = 0, 1, 2)$ **and** $\gamma_i (i = 0, 1, 2)$**,**

$$
\begin{aligned}
\alpha_0 + \alpha_1 + \alpha_2 + \gamma_0 + \gamma_1 + \gamma_2 &= 369.41477 \pm^{53.55023}_{42.84781} \\
-i(4.583805 \pm^{1.099412}_{0.702248}), \\
\alpha_0 - \alpha_1 + \alpha_2 + \gamma_0 - \gamma_1 + \gamma_2 &= 94204.06225 \pm^{22517.33}_{14450.55} \\
-i8.79720 \times 10^{-6}, \\
\alpha_0 - 2\alpha_2 + \gamma_0 - 2\gamma_2 &= 17.687675 \pm^{3.08543}_{1.83276} \\
-i(3.619279 \pm^{0.867322}_{0.556762}),
\end{aligned}
$$

$$
\begin{aligned}
\alpha_3 &= -(3423.16388\pm^{756.85493}_{525.63030}) - i(62.68593\pm^{14.98302}_{9.61636}) \\
\alpha_4 &= 634.28240\pm^{152.89360}_{96.74958} - i(268.69088\pm^{64.22568}_{41.21599}), \\
\alpha_5 &= -(80.01832\pm^{11.10225}_{9.05636}) + i(9.33456\pm^{2.18567}_{1.43414}),
\end{aligned}
$$

where all parameters are in MeV and the uncertainties in the RHS of these equations reflect the uncertainties in the low-energy data [?]. It is clear that the set of six eqs. leaves undetermined three($9-6$) complex (six real) parameters which provide a very rich structure to the model. Because of this, the model may be able to confront the present neutrino data and even the future precision data that may emerge from planned and ongoing oscillation experiments. On the other hand, it is also possible that the number of parameters may not ensure faithful representation of neutrino data because of highly non-linear nature of the problem emerging from see-saw mechanism.

In order to examine the efficiency of the model in representing the neutrino sector, we use the standard parametrization of the leptonic PMNS mixing matrix,

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, δ **is the Dirac phase and** φ_1, φ_2 **are Majorana phases of neutrinos. These phases have range from** 0 **to** 2π**.**

We use experimental data on neutino oscillations within the 3σ **limit [?]:**

$$
\begin{gathered}
0.29 \le \tan^2\theta_{12} \le 0.64, \\
0.49 \le \tan^2\theta_{23} \le 2.2, \\
\sin^2\theta_{13} \le 0.054, \\
5.2 \le \Delta m^2_{\odot}/10^{-5}eV^2 \le 9.8, \\
1.4 \le \Delta m^2_{atm}/10^{-3}eV^2 \le 3.4.
\end{gathered}
$$

For numerical analysis we exploit the well defined diagonalisation procedure for complex and symmetric mass matrices,

$$\begin{aligned} U^\dagger M_\nu U^* &= \text{diag}(m_1, m_2, m_3), \\ U^\dagger M_\nu M_\nu^\dagger U &= \text{diag}(m_1^2, m_2^2, m_3^2), \end{aligned}$$

where U is a unitary diagonalising matrix , the light neutrino mass matrix M_ν has been defined in by Type-I seesaw and $m_i (i = 1, 2, 3)$ are positive mass eigen values.

For the sake of simplicity we reduce the parameters of the model by treating the parameters $\gamma_i (i = 0, 1, 2)$ as real. Then the six eqs. for $\alpha_i - \gamma_i$ combinations, determines six real parameters out of a total nine, This choice of parameters implies that the CP-violation has its origin only in the quark sector as reflected in the CKM matrix [?].

Thus, in addition to the see-saw scale, we are left with three real parameters to fit the neutrino oscillation data on four quantities, $\Delta m^2_{\odot}, \Delta m^2_{atm}, \tan^2\theta_{12}$, and $\tan^2\theta_{23}$ and make predictions on $\sin\theta_{13}$, leptonic Dirac phase (δ) and Majorana phases(φ_1, φ_2), sum of the three light neutrino masses Σm_i, the effective matrix element for neutrinoless double beta decay, $< m_{ee} >$, and the kinematic neutrino mass m_β to be measured in beta decay where

$$< m_{ee} >= |\sum_{i=1}^{3}(U^{ei}_{PMNS})^2 m_i|,$$

$$m_\beta = (\sum_{i=1}^{3}|U^{ei}_{PMNS}|^2 m_i^2)^{1/2}.$$

Equivalently, the three real unknown parameters are defined as,

$$\xi = \gamma_0 - 2\gamma_2,$$
$$\eta = \gamma_0 + \gamma_1 + \gamma_2,$$
$$\zeta = \gamma_0 - \gamma_1 + \gamma_2.$$

We find that $\xi,\ \eta$ and ζ are quite efficient in describing the present neutrino oscillation data. Some examples of our fit to the data and model predictions are shown in Table 2.

We find that the see-saw scale is determined to be $M_N = 3.78 \times 10^{13}$ GeV for hierarchial neutrino masses. The first and the second columns show that for fixed values of η and ζ, the parameter ξ is very effective in controlling the value of the solar neutrino mixing angle (θ_{12}).

Table 2: Fit to the available neutrino oscillation data and predictions of reactor mixing angle θ_{13}, Leptonic Dirac phase (δ), Majorana phases (φ_1, φ_2) and the CP violation parameter J_{CP} in the $SO(10) \times S_4$ model with see-saw scale at $M_R = 3.78 \times 10^{13}$ GeV and $\tan\beta = 10$

ξ (GeV)	1.025	1.100	1.235
η (GeV)	2.137	2.137	2.400
ζ (GeV)	25.529	25.529	25.700
m_1 (eV)	0.00536	0.00596	0.00801
m_2 (eV)	0.00920	0.00956	0.01268
m_3 (eV)	0.05000	0.05000	0.07860
$\sum_i m_{\rm i}$ (eV)	0.0645	0.0675	0.09929
$\Delta m^2_{\odot}$ (eV^2)	6×10^{-5}	6×10^{-5}	9.6×10^{-5}
Δm^2_{atm} (eV^2)	2.5×10^{-3}	2.5×10^{-3}	3.1×10^{-3}
$\sin\theta_{12}$	0.515	0.616	0.511
$\sin\theta_{23}$	0.718	0.718	0.736
$\sin\theta_{13}$	0.055	0.057	0.052
δ(radians)	3.096	3.048	3.100
ϕ_1(radians)	5.67	5.46	5.65
ϕ_2(radians)	5.59	5.39	5.65
J_{CP}	2.66×10^{-4}	6.49×10^{-4}	2.95×10^{-5}
$< m_{ee} >$ (eV)	0.00646	0.00742	0.00932
m_β (eV)	0.00462	0.00516	0.00600

The model is capable of accommodating both larger and smaller values of θ_{13}. In one example we find that the predicted reactor mixing angle occurs in the range $\theta_{13} \simeq 3^\circ - 5^\circ$ which is within the accessible limit of ongoing and planned experiments [?]. The sum of the three neutrino masses are found to be well within the cosmological bound [?]. The leptonic Dirac phase turns out to be closer to π with $\delta = 2.9 - 3.1$ radians and the two Majorana phases are within $5.3 - 5.7$ radians. The predicted values of matrix element for double beta decay and the kinematical mass for beta decay are found to be nearly two orders smaller than the current experimental bounds [?, ?, ?]. Similar conclusion has been also obtained for hierarchial neutrinos with S_4 flavor symmetry in the non-SUSY standard model [?]. The Jarlskog invariant [?] is found to vary between $J_{CP} \simeq 2.95 \times 10^{-5}$ and $J_{CP} \simeq 10^{-3}$ where the smaller (larger) value depends upon how much closer (farther) is the Dirac phase (δ) from π. We observe

that the predictions of this model in the neutrino sector made at the high see-saw scale is to remain stable under radiative corrections when extrapolated to low energies especially since the light neutrino mass eigen values are small [?].

1 IV. Summary and Outlook

- **In non-SUSY $SO(10)$ there is no problem in getting left-right intermediate gauge symmetry, but the theory has gauge hierachy problem.**

- **In minimal SUSY $SO(10)$ the RG constraint dictates the LR intermediate gauge sym. breaking scale to be of same oder as the GUT scale which is at least 2 orders larger than the seesaw scale for neutrino masses.**

- **If we achieve a LR-gauge sym. breaking scale $M_R = 10^{13}$ GeV to 10^{14}GeV , then the seesaw scale can be obtained without any adjustment of the Majorana coupling : i,e with $f_0 \simeq 1$.**

- **In this work we have addressed the question of possible existence of R-parity and Parity conserving left-right gauge theory as an intermediate symmetry in SUSY $SO(10)$ with manifest one-loop unification of the gauge couplings.**

- **We have realized this possibility but with the extensions:**
  $G_{2213} \rightarrow G_{2213} \times S_4$,
  $SO(10) \rightarrow SO(10) \times S_4$.
  and the pattern of sym. breaking is
  $SO(10) \times S_4 \rightarrow G_{2213} \times S_4$
  $\rightarrow MSSM \times S_4$

- **In addition to the minimal Higgs multiplets, we need contributions of five more bi-doublets and three $SU(3)_C$-octets at the intermediate scale. Thus, the total no of six bi-doublets and three octets are nicely fitted into the the representations ,$\mathbf{3+2+1=6}$, and $\mathbf{3}$, respectively, of the flavor group s_4.With almost perfect gauge coupling unification. The intermediate scale is found to vary over a wide range $10^{9.5} \rightarrow 10^{15}$ GeV**

- **SUSY $SO(10)$ with triplet of 45, six $10'$s , and a singlet of $(126 \oplus 1\bar{2}6)_0$ provide the right spectrum for gauge coupling unification.**

- **Three fermion generations are in the triplet of $16'$s. To get fermion masses we add a doublet of $(126 \oplus 1\bar{2}6)_{1,2}$ which have GUT scale masses. The weak bidoublets in $(126 \oplus 1\bar{2}6)_{0,1,2}$ get suppressed induced**

VEVs (See Babu and Mohapatra, 1993) and others get VEVs $\simeq 100$ GeV

- **The model fits all the fermion masses and mixings with good predictions of reactor mixing angle and CP-Violating parameters. It is very rich in structure and has the capability to confront more accurate data in near future.**

- **It would be interesting to investigate the experimentally testable light quasi-degenerate neutrino masses near their WMAP bounds in this model with possibility of high scale mixing unification or otherwise.**

- **With high degree of degeneracy acquired through S_4 symmetry it would be interesting to predict baryogenesis via RESONANT leptogenesis in this model.**

- **The idea of $b - \tau$ unification being applica-**

ble at or below the seesaw scale, it would be a good idea to see the success of $Type-II$ **seesaw dominance or by taking an admixure of Type-I and Type -II.**