The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1951-2

Workshop on the original of P, CP and T Violation

2 - 5 July 2008

CP Violation in Kaon Decays

A. PICH

Universidad de Valencia, Facultad de Fisica
46100 Valencia, SPAIN

CP Violation in Kaon Decays

A. Pich
IFIC , Valencia

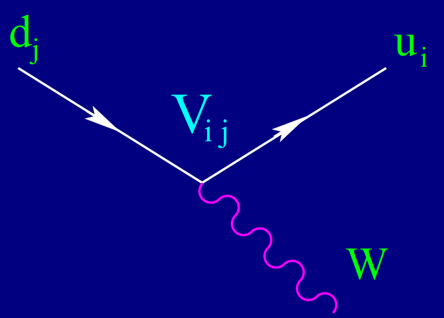
Workshop on the origins of P, CP and T violation (cpt@ictp)

ICTP, Trieste, Italy, July 2 to 5 2008

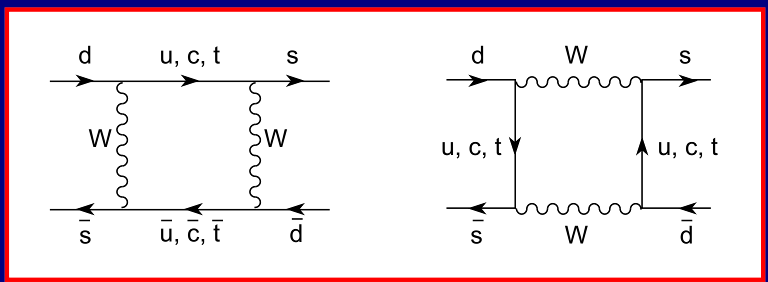
- **Superweak** $\not{CP}$: Wolfenstein 1964

K^0-$\bar{K}^0$ Mixing is the only source of $\not{CP}$

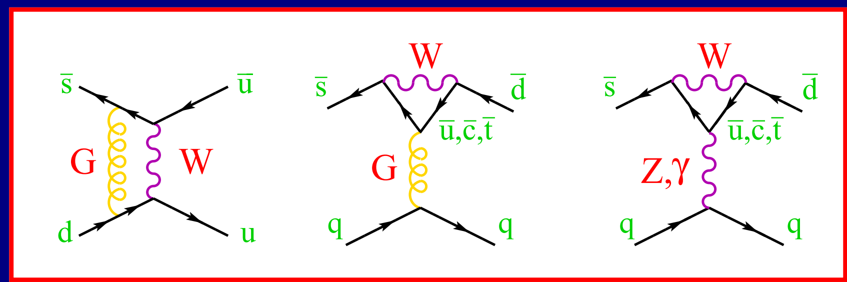
- **Standard Model** $\not{CP}$: Kobayashi–Maskawa 1973



$\not{CP}$ ⟷ **Quark Mixing**



$\Delta S = 2$



$\Delta S = 1$

DIRECT CP VIOLATION

$$\eta_{+-} \equiv \frac{T(K_L \to \pi^+\pi^-)}{T(K_S \to \pi^+\pi^-)} \approx \varepsilon_K + \varepsilon'_K \quad ; \quad \eta_{00} \equiv \frac{T(K_L \to \pi^0\pi^0)}{T(K_S \to \pi^0\pi^0)} \approx \varepsilon_K - 2\,\varepsilon'_K$$

$$\varepsilon_K = (2.229 \pm 0.010) \times 10^{-3}\;\; \mathrm{e}^{i\,\phi_{\varepsilon_K}} \quad ; \quad \phi_{\varepsilon_K} = (43.51 \pm 0.05)^\circ$$

$$\mathrm{Re}\left(\frac{\varepsilon'_K}{\varepsilon_K}\right) \approx \frac{1}{6}\left\{1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right\} = (16.5 \pm 2.6) \times 10^{-4}$$

$10^4\;\mathrm{Re}\,(\varepsilon'_K/\varepsilon_K)$

23.0 ± 6.5	NA31	1993
7.4 ± 5.9	E731	1993
14.7 ± 2.2	NA48	2002
20.7 ± 2.8	KTeV	2003

Long History of Theoretical Predictions

- Earlier estimates: $|\varepsilon'_K/\varepsilon_K| \lesssim 0.002$ Ellis-Gaillard-Nanopoulos '76
- First LO calculations: $|\varepsilon'_K/\varepsilon_K| \sim 0.01$ Gilman-Wise '79, Guberina-Peccei '80
- Electroweak penguins: Bijnens-Wise '84, Donoghue et al '86, Buras-Gerard '87
- First estimates of isospin breaking: Donoghue et al '86, Buras-Gerard '87, Lusignoli '89
- First matrix elements from $1/N_C$: Buras-Bardeen-Gerard '87
- Heavy top mass: Big cancellation Flynn-Randall '89, Buchalla-Buras-Harlander '90 Paschos-Wu '91, Lusignoli et al '92
- NLO short-distance calculation: $\mathrm{Re}(\varepsilon'_K/\varepsilon_K) \sim 7 \cdot 10^{-4}$ Buras et al '93, Ciuchini et al '93
- Modelling matrix elements: Dortmund, Lund, München, Roma, Trieste, ...
- χPT_{FSI} & $1/N_C$: $\mathrm{Re}(\varepsilon'_K/\varepsilon_K) \sim 1.7 \cdot 10^{-3}$ Pallante-Pich '00, Pallante-Pich-Scimemi '01
- Isospin breaking $(m_u - m_d,\, \alpha)$ Ecker et al '00, Cirigliano et al '03
- Matrix elements at NLO in $1/N_C$: Ongoing analytical / lattice effort Amherst, Barcelona, Caltech, CP-PACS, Granada, Lund, Marseille, Montpellier, RBC, Roma, ...

$K \to 2\pi$ ISOSPIN AMPLITUDES

$$A[K^0 \to \pi^+\pi^-] \equiv A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2}$$

$$A[K^0 \to \pi^0\pi^0] \equiv A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2}$$

$$A[K^+ \to \pi^+\pi^0] \equiv \frac{3}{2} A_2^+\, e^{i\chi_2^+}$$

$$A_0\, e^{i\chi_0} = \mathcal{A}_{1/2}$$

$$A_2\, e^{i\chi_2} = \mathcal{A}_{3/2} + \mathcal{A}_{5/2}$$

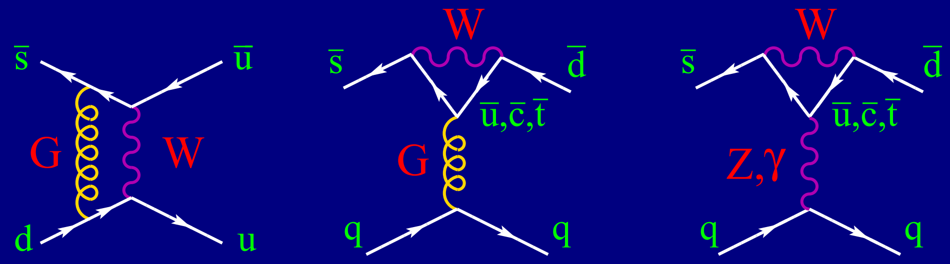
$$A_2^+\, e^{i\chi_2^+} = \mathcal{A}_{3/2} - \frac{2}{3}\mathcal{A}_{5/2}$$

ΔI $= 1/2$ Rule: $\qquad \omega \equiv \frac{\mathrm{Re}\,(A_2)}{\mathrm{Re}\,(A_0)} \approx \frac{1}{22}$

Strong Final State Interactions: $\qquad \chi_0 - \chi_2 \approx \delta_0 - \delta_2 \approx 45^\circ$

$$\varepsilon'_K = \frac{-i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\, \omega \left\{ \frac{\mathrm{Im}\,(A_0)}{\mathrm{Re}\,(A_0)} - \frac{\mathrm{Im}\,(A_2)}{\mathrm{Re}\,(A_2)} \right\}$$

$\Delta S = 1$ TRANSITIONS



$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, V_{us}^* \sum_i C_i(\mu)\; Q_i(\mu)$$

$$Q_1 = (\bar s_\alpha u_\beta)_{\rm V-A}\,(\bar u_\beta d_\alpha)_{\rm V-A} \qquad Q_2 = (\bar s u)_{\rm V-A}\,(\bar u d)_{\rm V-A}$$

$$Q_{3,5} = (\bar s d)_{\rm V-A} \sum_q (\bar q q)_{\rm V\mp A} \qquad Q_4 = (\bar s_\alpha d_\beta)_{\rm V-A} \sum_q (\bar q_\beta q_\alpha)_{\rm V-A}$$

$$Q_{7,9} = \tfrac{3}{2}\,(\bar s d)_{\rm V-A} \sum_q e_q\,(\bar q q)_{\rm V\pm A} \qquad Q_{10} = \tfrac{3}{2}\,(\bar s_\alpha d_\beta)_{\rm V-A} \sum_q e_q\,(\bar q_\beta q_\alpha)_{\rm V-A}$$

$$Q_6 = -8 \sum_q (\bar s_L q_R)\,(\bar q_R d_L) \qquad Q_8 = -12 \sum_q e_q\,(\bar s_L q_R)\,(\bar q_R d_L)$$

- $q > \mu$: $\quad C_i(\mu) = z_i(\mu) - y_i(\mu)\,(V_{td}\,V_{ts}^*/V_{ud}\,V_{us}^*)$

  $O(\alpha_s^n t^n)$, $O(\alpha_s^{n+1} t^n)$ $\qquad [t \equiv \log(M/m)]$ $\qquad$ (Munich / Rome)

- $q < \mu$: $\quad \langle\pi\pi|Q_i(\mu)|K\rangle$ $\quad$? $\qquad$ Physics does not depend on μ

$$\varepsilon'_K/\varepsilon_K \;\sim\; \mathrm{Im}\,(V_{ts}^* V_{td})\; \left[P^{(1/2)} - P^{(3/2)}\right]$$ (Buras et al)

$$P^{(1/2)} = r \sum_i y_i(\mu)\, \langle Q_i(\mu)\rangle_0\, (1-\Omega_{IB}) \quad ; \quad P^{(3/2)} = \frac{r}{\omega} \sum_i y_i(\mu)\, \langle Q_i(\mu)\rangle_2$$

$$r = \frac{G_F\,\omega}{2\,|\varepsilon_K|\,\mathrm{Re}\,(A_0)} \quad ; \quad \omega = \frac{\mathrm{Re}\,(A_2)}{\mathrm{Re}\,(A_0)} \approx \frac{1}{22} \quad ; \quad \mathrm{Re}\,(A_0) = 3.37\times 10^{-7}\,\mathrm{GeV}$$

$\langle Q_i(\mu)\rangle \equiv \langle Q_i\rangle_{vs}\, B_i(\mu)$

→ **$\mathbf{m_t}$ large**

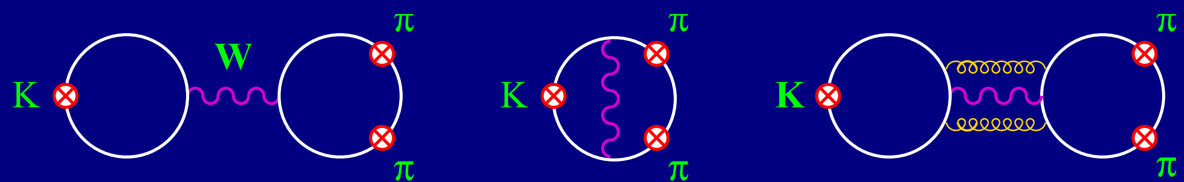
$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{110\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left\{B_6^{(1/2)}\,(1-\Omega_{IB}) - 0.4\, B_8^{(3/2)}\right\}$$

Delicate Cancellation. Strong Sensitivity to:

- m_s (quark condensate)
- Isospin Breaking ($m_u \neq m_d$, e.m. effects)
- Penguin Matrix Elements (χPT corrections)

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	$\Downarrow$ OPE	
$\lesssim m_c$	$\gamma, g \,;\, \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}, \; \mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	$\Downarrow$ $N_C \to \infty$	
M_K	$\gamma \,;\, \mu, e, \nu_i$ π, K, η	χPT

Weak Currents Factorize at Large N_C



$O(N_C^2)$ $\qquad$ $O(N_C)$ $\qquad$ $O(1)$

$$A[K^0 \to \pi^0\pi^0] = 0 \quad \Longrightarrow \quad A_0 = \sqrt{2}\, A_2$$

No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_C$

- **Multiscale problem:** **OPE** $\qquad \frac{1}{N_C}\,\log\left(\frac{M_W}{\mu}\right) \sim \frac{1}{3} \times 4$

Short-distance logarithms must be summed

- **Large χPT logarithms:** **FSI** $\qquad \frac{1}{N_C}\,\log\left(\frac{\mu}{M_\pi}\right) \sim \frac{1}{3} \times 2$

Infrared logarithms must also be included $\;[\delta_I \sim O(1/N_C)\,,\; \delta_0 - \delta_2 \approx 45^\circ]$

CHIRAL PERTURBATION THEORY (χPT)

- Expansion in powers of p^2/Λ_χ^2 : $\mathcal{A} = \sum_n \mathcal{A}^{(n)}$ $(\Lambda_\chi \sim 4\pi F_\pi \sim 1.2\ \mathrm{GeV})$
- Amplitude structure fixed by chiral symmetry
- Short-distance dynamics encoded in low-energy couplings (LECs)
- **O(p^2) χPT:** $\delta_0 = \delta_2 = 0$

$$\mathcal{A}_{1/2} = \sqrt{2}\, F_\pi \left(G_8 + \frac{1}{9}\, G_{27} \right) (M_K^2 - M_\pi^2)$$

$$\mathcal{A}_{3/2} = \frac{10}{9}\, F_\pi\, G_{27}\, (M_K^2 - M_\pi^2) \qquad ; \qquad \mathcal{A}_{5/2} = 0$$

- Loop corrections (χPT logarithms) unambiguously predicted
- LECs can be determined at $\mathbf{N_C} \to \infty$

$\mathbf{O(p^2, e^2p^0)}$ χPT

$\mathcal{Q} = \mathrm{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$

$$\mathcal{L}_2^{\Delta S=1} = G_8 F^4 \langle \lambda L_\mu L^\mu \rangle + G_{27} F^4 \left(L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) + e^2 F^6 G_8 g_{ew} \langle \lambda U^\dagger \mathcal{Q} U \rangle$$

$G_R \equiv -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* g_R$; $L_\mu = -i U^\dagger D_\mu U$; $\lambda \equiv \frac{1}{2} \lambda_{6-i7}$; $U \equiv \exp\left\{ i \sqrt{2}\, \Phi / F \right\}$

$$\mathcal{A}_{1/2} = \sqrt{2} F_\pi \left\{ G_8 \left[(M_K^2 - M_\pi^2) \left(1 - \frac{2}{3\sqrt{3}} \varepsilon^{(2)} \right) - \frac{2}{3} F_\pi^2 e^2 \left(g_{ew} + 2 Z \right) \right] + \frac{1}{9} G_{27} (M_K^2 - M_\pi^2) \right\}$$

$$\mathcal{A}_{3/2} = \frac{2}{3} F_\pi \left\{ \left(\frac{5}{3} G_{27} + \frac{2}{\sqrt{3}} \varepsilon^{(2)} G_8 \right) (M_K^2 - M_\pi^2) - F_\pi^2 e^2 G_8 \left(g_{ew} + 2 Z \right) \right\}$$

$$\mathcal{A}_{5/2} = 0 \qquad ; \qquad \delta_0 = \delta_2 = 0$$

$\varepsilon^{(2)} = (\sqrt{3}/4)\,(m_d - m_u)/(m_s - \hat{m}) \approx 0.011$; $Z \approx (M_{\pi^\pm}^2 - M_{\pi^0}^2)/(2 e^2 F_\pi^2) \approx 0.8$

$\mathbf{O(p^2, e^2p^0)}$ χPT ; $\mathbf{N_C \to \infty}$

$$g_8 = \left(\frac{3}{5}\, C_2 - \frac{2}{5}\, C_1 + C_4\right) - 16\; L_5\; \left(\frac{\langle \bar{q}\, q\rangle(\mu)}{F_\pi^3}\right)^2 C_6(\mu)$$

$$g_{27} = \frac{3}{5}\,(C_2 + C_1)$$

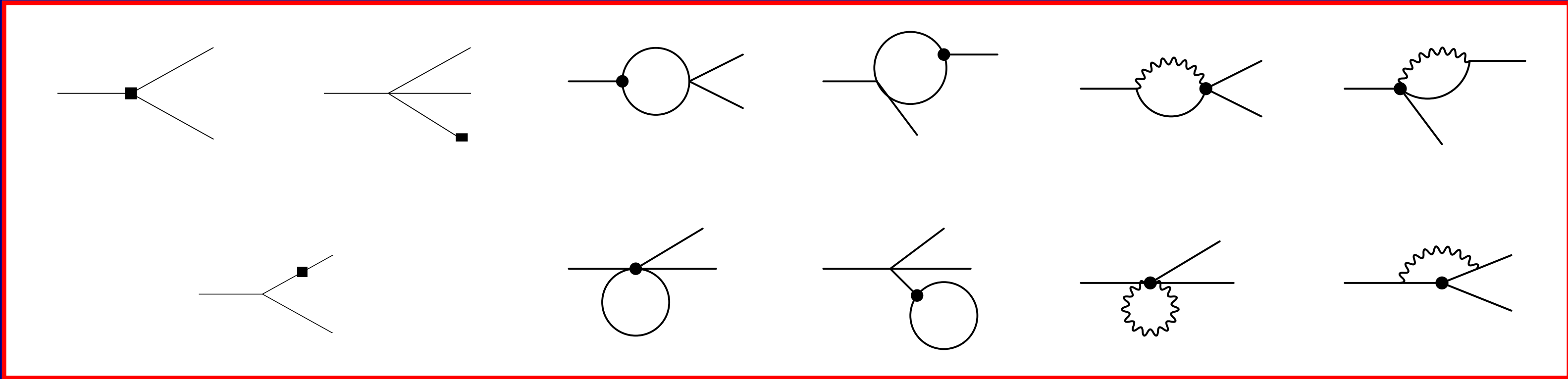
$$e^2\, g_8\, g_{ew} = -3\, \left(\frac{\langle \bar{q}\, q\rangle(\mu)}{F_\pi^3}\right)^2 \left[C_8(\mu) + \frac{16}{9}\, C_6(\mu)\, e^2\, (K_9 - 2\, K_{10})\right]$$

$$G_R = -\frac{G_F}{\sqrt{2}}\, V_{ud} V_{us}^*\; g_R \qquad (R = 8, 27)$$

$$\frac{\langle \bar{q}\, q\rangle(\mu)}{F_\pi^3} = \frac{M_{K^0}^2}{(m_s + m_d)(\mu)\, F_\pi} \left\{1 - \frac{8 M_{K^0}^2}{F_\pi^2}\,(2L_8 - L_5) + \frac{4 M_{\pi^0}^2}{F_\pi^2}\, L_5\right\}$$

- **Equivalent to standard calculations of $\mathbf{B_i}$**
- μ dependence only captured for $Q_{6,8}$

$\mathbf{O}\left[\mathbf{p}^4, (\mathbf{m_u} - \mathbf{m_d})\,\mathbf{p}^2, \mathbf{e}^2\mathbf{p}^2\right] \quad \chi$PT



- **Nonleptonic weak Lagrangian:** $O(G_F\, p^4)$

$$\mathcal{L}^{(4)}_{\rm weak} = \sum_i G_8\, N_i\, F^2\, O_i^8 + \sum_i G_{27}\, D_i\, F^2\, O_i^{27} + \text{h.c.}$$

- **Electroweak Lagrangian:** $O(G_F\, e^2 p^2)$

$$\mathcal{L}_{\rm EW} = e^2 \sum_i G_8\, Z_i\, F^4\, O_i^{EW} + \text{h.c.}$$

- $O(e^2 p^2)$ **Electromagnetic** $+$ $O(p^4)$ **Strong:** $K_i\, ,\; L_i$

- $K \to \pi\pi, \pi\pi\gamma$ Inclusive , DAPHNE

$$\mathcal{A}_n^{(X)} = a_n^{(X)} \left[1 + \Delta_L \mathcal{A}_n^{(X)} + \Delta_C \mathcal{A}_n^{(X)}\right]$$

① $O(p^4)$ χPT Loops: **Large correction** NLO in $1/N_C$

$\Delta_L \mathcal{A}_{1/2}^{(8)} = 0.27 \pm 0.05 + 0.47\, i$;	
$\Delta_L \mathcal{A}_{1/2}^{(27)} = 1.02 \pm 0.60 + 0.47\, i$;	$\Delta_L \mathcal{A}_{3/2}^{(27)} = -0.04 \pm 0.05 - 0.21\, i$
$\Delta_L \mathcal{A}_{1/2}^{(g)} = 0.27 \pm 0.05 + 0.47\, i$;	$\Delta_L \mathcal{A}_{3/2}^{(g)} = -0.50 \pm 0.20 - 0.21\, i$

Pallante-Pich-Scimemi

② All local $O(p^4)$ couplings fixed at $N_C \to \infty$ ➡ $\Delta_C \mathcal{A}_n^{(X)}$

Small correction to $O(p^2)$ results

③ Isospin Breaking: $O\left[(m_u - m_d)\, p^2, e^2 p^2\right]$ **Sizeable corrections**

Cirigliano-Ecker-Neufeld-Pich

④ $\mathrm{Re}(g_8)$, $\mathrm{Re}(g_{27})$, $\chi_0 - \chi_2$ fitted to data

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left\{ B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}}) - 0.4\ B_8^{(3/2)} \right\}$$

Delicate Cancellation. Strong Sensitivity to:

- m_s (quark condensate) $m_s(2\ \mathrm{GeV}) = 105 \pm 20$ MeV
- Isospin Breaking $(m_u \neq m_d\ ,\ \alpha)$ $\Omega_{\mathrm{eff}} = 0.06 \pm 0.08$

  Cirigliano-Ecker-Neufeld-Pich
- Penguin Matrix Elements

χPT Loops (FSI): $B_{6,\infty}^{(1/2)} \times (1.35 \pm 0.05)$; $B_{8,\infty}^{(3/2)} \times (0.54 \pm 0.20)$

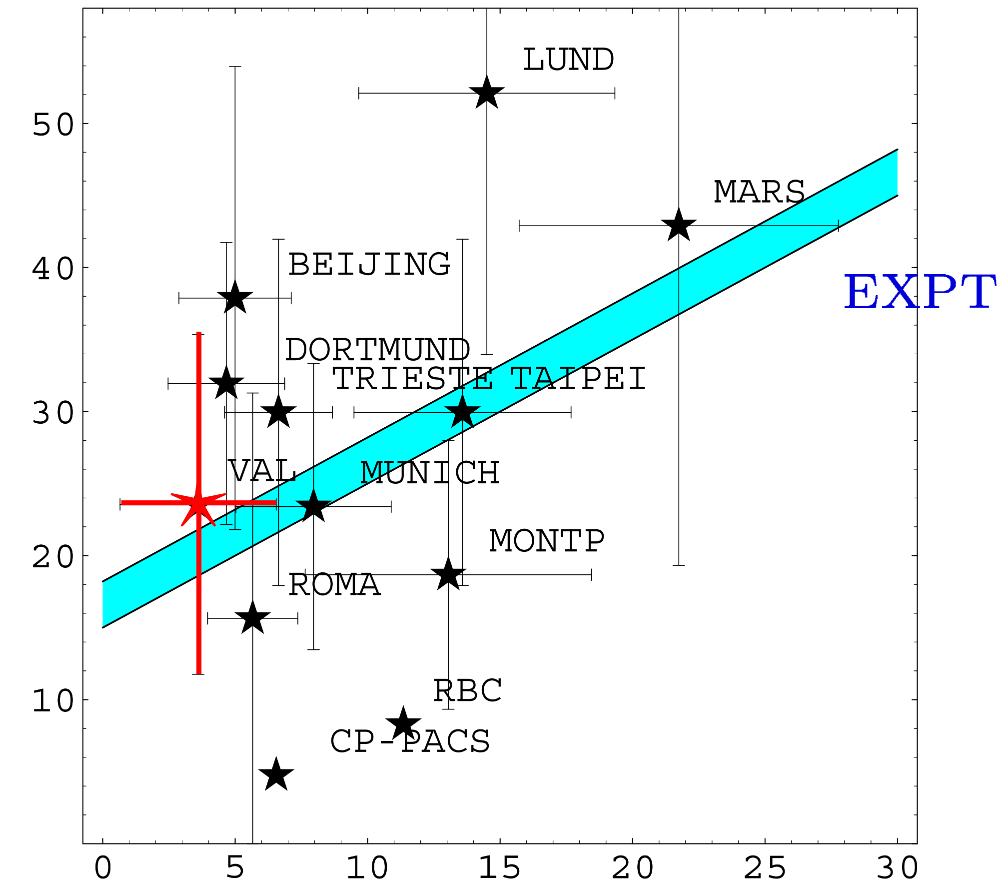
$$\Longrightarrow \quad \mathrm{Re}\,(\varepsilon'/\varepsilon) = \left(19 \pm 2_{\,\mu}\ {}^{+9}_{-6}{}_{\,m_s} \pm 6_{\,1/N_C}\right) \times 10^{-4}$$

Pallante-Pich-Scimemi ’01 (updated)

Exp. world average: $\mathrm{Re}\,(\varepsilon'/\varepsilon) = (16.5 \pm 2.6) \times 10^{-4}$

Challenge: Control of subleading $1/N_C$ corrections to χPT couplings

$$|\varepsilon'/\varepsilon'| \;=\; \mathrm{Im}\,(V_{ts}^* V_{td}) \;\times\; \left[P^{(1/2)} - P^{(3/2)} \right]$$



V. Cirigliano

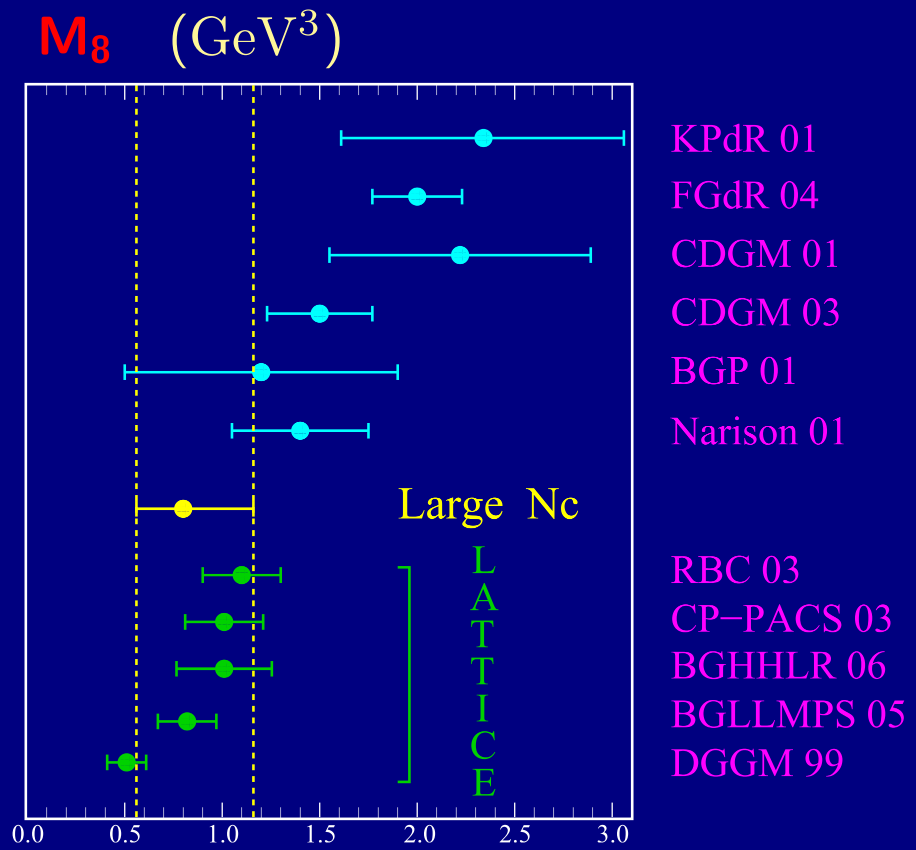
$\langle Q_i \rangle_I$ from original calculations

Ω_{eff} Updated

ELECTROWEAK PENGUINS contribute at $O(p^0)$ $(m_q, p \to 0)$

$$e^2 g_8 g_{ew} \, F^6 = 6\, C_7(\mu)\, \langle \mathcal{O}_1(\mu) \rangle - 12\, C_8(\mu)\, \langle \mathcal{O}_2(\mu) \rangle \xrightarrow{N_C \to \infty} -\frac{1}{3}\, C_8(\mu)\, \langle \bar{q} q(\mu) \rangle^2$$

$$\langle \mathcal{O}_1(\mu) \rangle \equiv \langle 0 | (s_L \gamma^\mu d_L)(\bar{d}_R \gamma_\mu s_R) | 0 \rangle \quad ; \quad \langle \mathcal{O}_2(\mu) \rangle \equiv \langle 0 | (s_L s_R)(\bar{d}_R d_L) | 0 \rangle$$



$$\mathbf{M_8} \equiv \langle (2\pi)_{I=2} | Q_8(\mu_0) | K^0 \rangle \Big|_{m_q = p = 0}$$

$$= \frac{8}{F^3} \langle \mathcal{O}_2(\mu_0) \rangle$$

$$\mu_0 = 2 \text{ GeV}$$

$$\mathbf{M_8} \xrightarrow{N_C \to \infty} \frac{2}{F^3} \langle \bar{q} q(\mu_0) \rangle^2$$

$$\approx \frac{2 M_K^4 F^3}{(m_s + m_q)^2(\mu_0)\, F_\pi^2}$$

$\not{CP}$ Charge Asymmetry in $K^{\pm} \to (3\pi)^{\pm}$ Decays

$$|T(u,v)|^2 \propto 1 + g\, u + h\, u^2 + k\, v^2 + \cdots$$

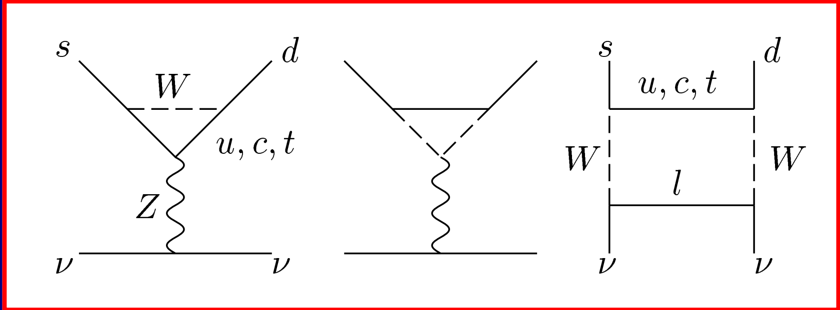
$u \equiv (s_3 - s_0)/m_\pi^2 \quad ; \quad v \equiv (s_1 - s_2)/m_\pi^2 \quad ; \quad s_0 \equiv (s_1 + s_2 + s_3)/3 \quad ; \quad s_i \equiv (p_K - p_{\pi_i})^2 \quad ; \quad 3 = \text{odd } \pi$

NA48/2: $$A_g \equiv \frac{g^+ - g^-}{g^+ + g^-} = \begin{cases} (-1.5 \pm 2.1) \cdot 10^{-4} & \pi^\pm \pi^+ \pi^- \\ (1.8 \pm 1.9) \cdot 10^{-4} & \pi^\pm \pi^0 \pi^0 \end{cases}$$

Theory: $A_g^{\pi^\pm \pi^+ \pi^-} = (-0.24 \pm 0.12) \cdot 10^{-4} \quad ; \quad A_g^{\pi^\pm \pi^0 \pi^0} = (0.11 \pm 0.07) \cdot 10^{-4}$

(Gámiz-Prades-Scimemi)

$K \to \pi\nu\bar{\nu}$



$$T \sim F(V_{is}^* V_{id}, \frac{m_i^2}{M_W^2})\, (\bar{\nu}_L \gamma_\mu \nu_L)\, \langle \pi | \bar{s}_L \gamma^\mu d_L | K \rangle$$

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) \;=\; (8.0 \pm 1.1) \times 10^{-11} \;\sim\; A^4 \left[\eta^2 + (1.4 - \rho)^2\right]$$

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) \;=\; (2.8 \pm 0.4) \times 10^{-11} \;\sim\; A^4\, \eta^2$$

Buras et al

Long-distance contributions are negligible

$\mathcal{A}(K_L \to \pi^0 \nu\bar{\nu}) \neq 0$ ⟹ **Direct** $\not{C}\not{P}$

BNL-E787: few events! ⟹ $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) \;=\; \left(1.47\,^{+1.30}_{-0.89}\right) \times 10^{-10}$

KEK-E391a: $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) \;<\; 2.1 \times 10^{-7}$ (90% CL)

New Experiments Needed

SUMMARY

- Qualitative understanding of ε'/ε within the Standard Model
- Quantitative prediction using the $1/N_C$ expansion and χPT
- Large chiral corrections generated by infrared logarithms
- Detailed analysis of isospin breaking corrections
- Good agreement with experiment (but large uncertainties)

$$\mathrm{Re}\left(\varepsilon'/\varepsilon\right) \; = \; \left(19 \pm 2_{\,\mu} \; {}^{+9}_{-6}{}_{\,m_s} \; \pm 6_{\,1/N_C}\right) \times 10^{-4}$$

Challenge: Control of subleading $1/\mathrm{N_C}$ corrections to χPT couplings

On-going theoretical efforts using both analytical & lattice tools

BACKUP SLIDES

Isospin Breaking in ε'/ε

$$\epsilon'_K \sim \omega_+ \left\{ \frac{\mathrm{Im}\, A_0^{(0)}}{\mathrm{Re}\, A_0^{(0)}} \left(1 + \Delta_0 + f_{5/2}\right) - \frac{\mathrm{Im}\, A_2}{\mathrm{Re}\, A_2^{(0)}} \right\}$$

$$\sim \omega_+ \left\{ \frac{\mathrm{Im}\, A_0^{(0)}}{\mathrm{Re}\, A_0^{(0)}} \left(1 - \Omega_{\mathrm{eff}}\right) - \frac{\mathrm{Im}\, A_2^{\mathrm{emp}}}{\mathrm{Re}\, A_2^{(0)}} \right\}$$

$$\omega \equiv \frac{\mathrm{Re}\, A_2}{\mathrm{Re}\, A_0} = \omega_+ \left(1 + f_{5/2}\right) \quad ; \quad \omega_+ \equiv \frac{\mathrm{Re}\, A_2^+}{\mathrm{Re}\, A_0} \quad ; \quad \Omega_{IB} = \frac{\mathrm{Re}\, A_0^{(0)}}{\mathrm{Re}\, A_2^{(0)}} \frac{\mathrm{Im}\, A_2^{\mathrm{non-emp}}}{\mathrm{Im}\, A_0^{(0)}}$$

Cirigliano-Ecker-Neufeld-Pich

$\times\ 10^{-2}$	$\alpha = 0$ LO	$\alpha = 0$ NLO	$\alpha \neq 0$ LO	$\alpha \neq 0$ NLO
Ω_{IB}	11.7	15.9 ± 4.5	18.0 ± 6.5	22.7 ± 7.6
Δ_0	-0.004	-0.41 ± 0.05	8.7 ± 3.0	8.4 ± 3.6
$f_{5/2}$	0	0	0	8.3 ± 2.4
Ω_{eff}	11.7	16.3 ± 4.5	9.3 ± 5.8	6.0 ± 7.7

$$\Omega_{\mathrm{eff}} = 0.06 \pm 0.08$$
$$\equiv \Omega_{IB} - \Delta_0 - f_{5/2}$$

$$\Omega_{\mathrm{IB}}^{\pi^0\eta} = 0.16 \pm 0.03$$

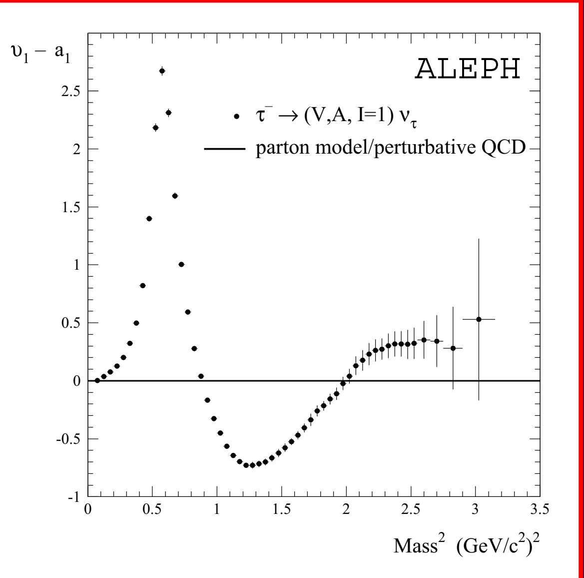
ELECTROWEAK PENGUINS contribute at $\mathbf{O(p^0)}$ $(m_q, p \to 0)$

$$e^2 g_8 g_{ew} \, F^6 = 6 \, C_7(\mu) \, \langle \mathcal{O}_1(\mu) \rangle - 12 \, C_8(\mu) \, \langle \mathcal{O}_2(\mu) \rangle \;\xrightarrow{N_C \to \infty}\; -\frac{1}{3} \, C_8(\mu) \, \langle \bar{q} q(\mu) \rangle^2$$

$$\langle \mathcal{O}_1(\mu) \rangle \equiv \langle 0 | (s_L \gamma^\mu d_L)(\bar{d}_R \gamma_\mu s_R) | 0 \rangle \quad ; \quad \langle \mathcal{O}_2(\mu) \rangle \equiv \langle 0 | (s_L s_R)(\bar{d}_R d_L) | 0 \rangle$$

These D=6 vacuum condensates appear in the left-right correlator:

$$\Pi^{\mu\nu}_{LR}(q) \equiv 2i \int d^4x \, e^{iqx} \, \langle 0 | T(L^\mu(x), R^\nu(0)^\dagger) | 0 \rangle \equiv \left(-g^{\mu\nu} q^2 + q^\mu q^\nu \right) \Pi_{LR}(-q^2)$$



$$\Pi_{LR}(Q^2) = \underbrace{\frac{1}{\pi} \int_0^\infty dt \, \frac{\mathrm{Im}\Pi_{LR}(t)}{t + Q^2}}_{\text{Data}} = \underbrace{\frac{1}{2} \sum_{n=1}^{\infty} \frac{\langle \widetilde{\mathcal{O}}_{2n+4} \rangle}{(Q^2)^{n+2}}}_{\text{QCD OPE}}$$

$$\lim_{Q^2 \to \infty} -Q^6 \, \Pi_{LR}(Q^2) = 4\pi\alpha_s \left[4 \, \langle \mathcal{O}_2 \rangle + \frac{2}{N_C} \, \langle \mathcal{O}_1 \rangle \right]$$

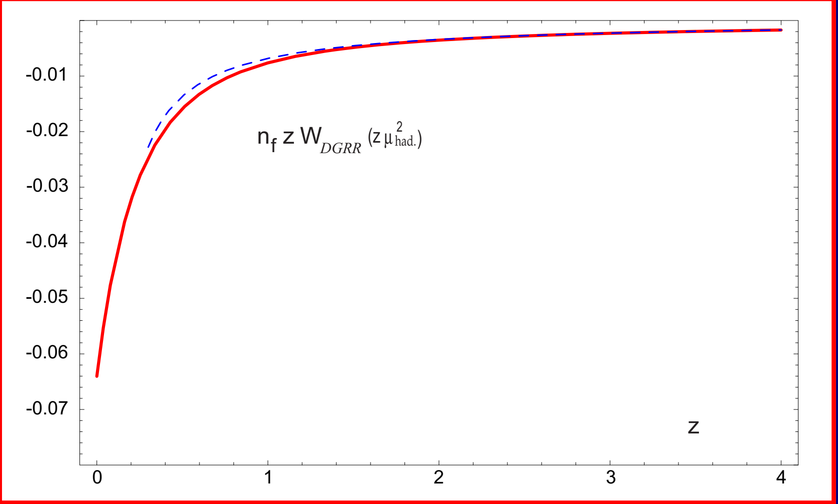
$$\langle \mathcal{O}_1 \rangle \sim \int_0^\infty dQ^2 \, Q^D \, \Pi_{LR}(Q^2)$$

n_f/N_C Correction to QCD PENGUIN $(m_q \to 0)$

(Hambye-Peris-de Rafael 03)

$$\mathrm{Im}(g_8) \doteq \mathrm{Im}[C_6(\mu)] \left\{ -16 L_5 \left(\frac{\langle\bar q q\rangle}{F^3}\right)^2 + \frac{8 n_f}{16\pi^2 F^4} \int_0^\infty dQ^2\, Q^{D-2}\, \mathcal{W}_{DGRR}(Q^2) \right\}$$

$$\left(\frac{q^\alpha q^\beta}{q^2} - g^{\alpha\beta}\right) \mathcal{W}_{DGRR}(-q^2) = \int d\Omega_q\, d^4x\, d^4y\, d^4z\, e^{iqx} \langle\, T\left[(\bar s_L q_R)(x)\,(\bar q_R d_L)(0)\,(\bar d_R \gamma_\alpha u_R)(y)\,(\bar u_R \gamma^\alpha s_R)(z)\right]\rangle_{\mathrm{con}}$$



Available theoretical information: (very poor)

$$\lim_{Q^2\to\infty} Q^2\, \mathcal{W}_{DGRR}(Q^2) = -\frac{F^4 \pi \alpha_s}{6 Q^2} \left[1 - 16 L_5 \left(\frac{\langle\bar q q\rangle}{F^3}\right)^2\right]$$

$$\lim_{Q^2\to 0} Q^2\, \mathcal{W}_{DGRR}(Q^2) = \left(\frac{\langle\bar q q\rangle}{F^2}\right)^2 \left\{ \frac{F^2}{8Q^2} - \left(L_5 - \frac{5}{2} L_3\right) \right\}$$

Big enhancement (~ 3) claimed

Infrared unstability from pion pole: $\int_0^\infty dQ^2\, \frac{Q^{-\epsilon}}{Q^2 + m_\pi^2} \sim \frac{2}{\epsilon\, m_\pi^\epsilon}$

Large non-factorizable contribution claimed before (Bardeen et al, Bijnens-Prades)

Phenomenological $K \to \pi\pi$ Fit

Cirigliano-Ecker-Neufeld-Pich

PDG + KLOE 02 $\quad [\Gamma(K_S \to \pi^+\pi^-(\gamma)/\Gamma(K_S \to \pi^0\pi^0)]$

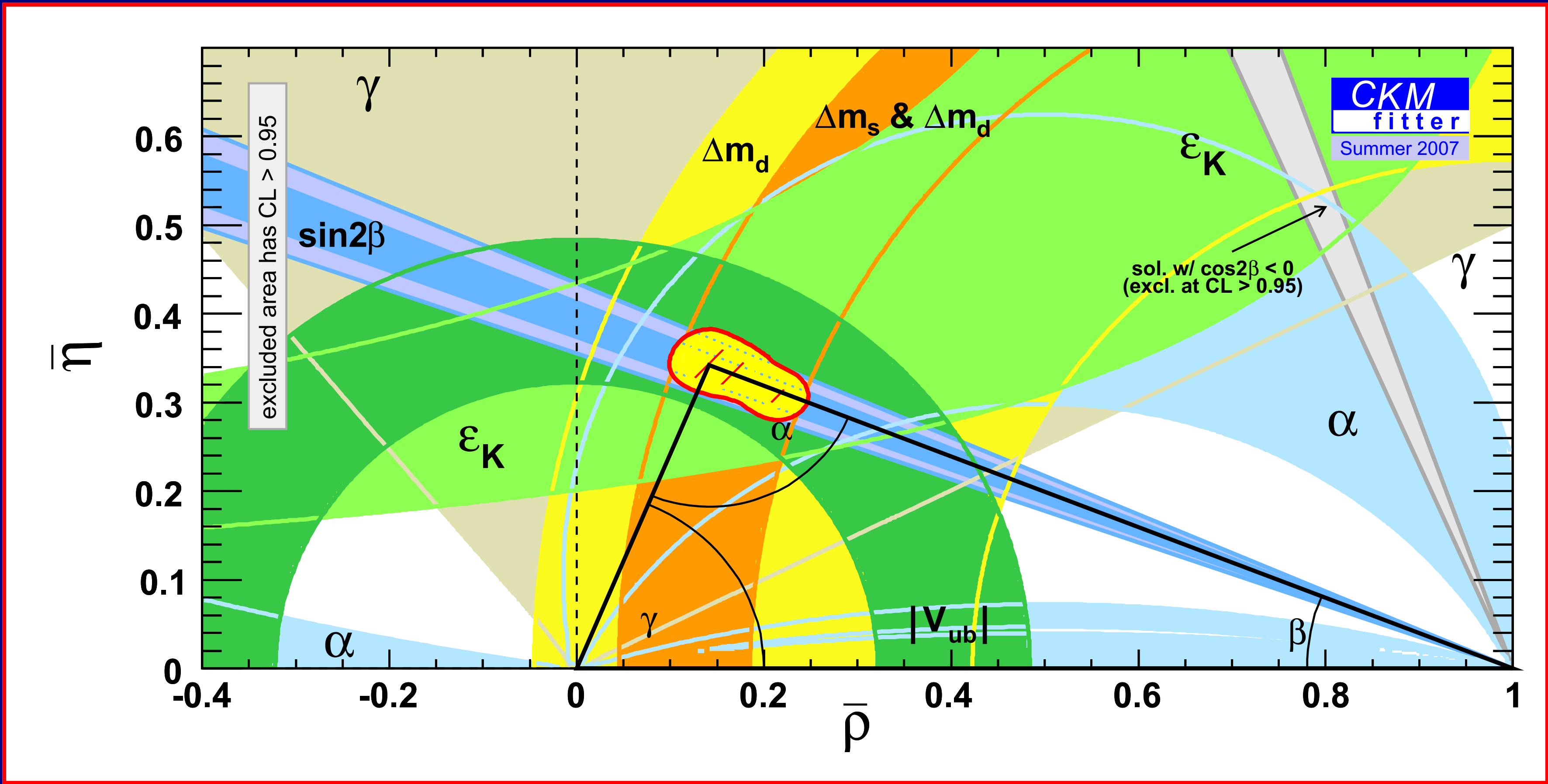
	LO-IC	LO-IB	NLO-IC	NLO-IB
Re g_8	5.09 ± 0.01	5.11 ± 0.01	3.67 ± 0.14	3.65 ± 0.14
Re g_{27}	0.294 ± 0.001	0.270 ± 0.001	0.297 ± 0.014	0.303 ± 0.014
$\chi_0 - \chi_2$	$(48.6 \pm 2.6)^\circ$	$(48.5 \pm 2.6)^\circ$	$(48.6 \pm 2.6)^\circ$	$(54.6 \pm 2.4)^\circ$

IC $\equiv$ $[m_u - m_d = \alpha = 0]$; IB $\equiv$ $[m_u - m_d \neq 0 \, , \; \alpha \neq 0]$

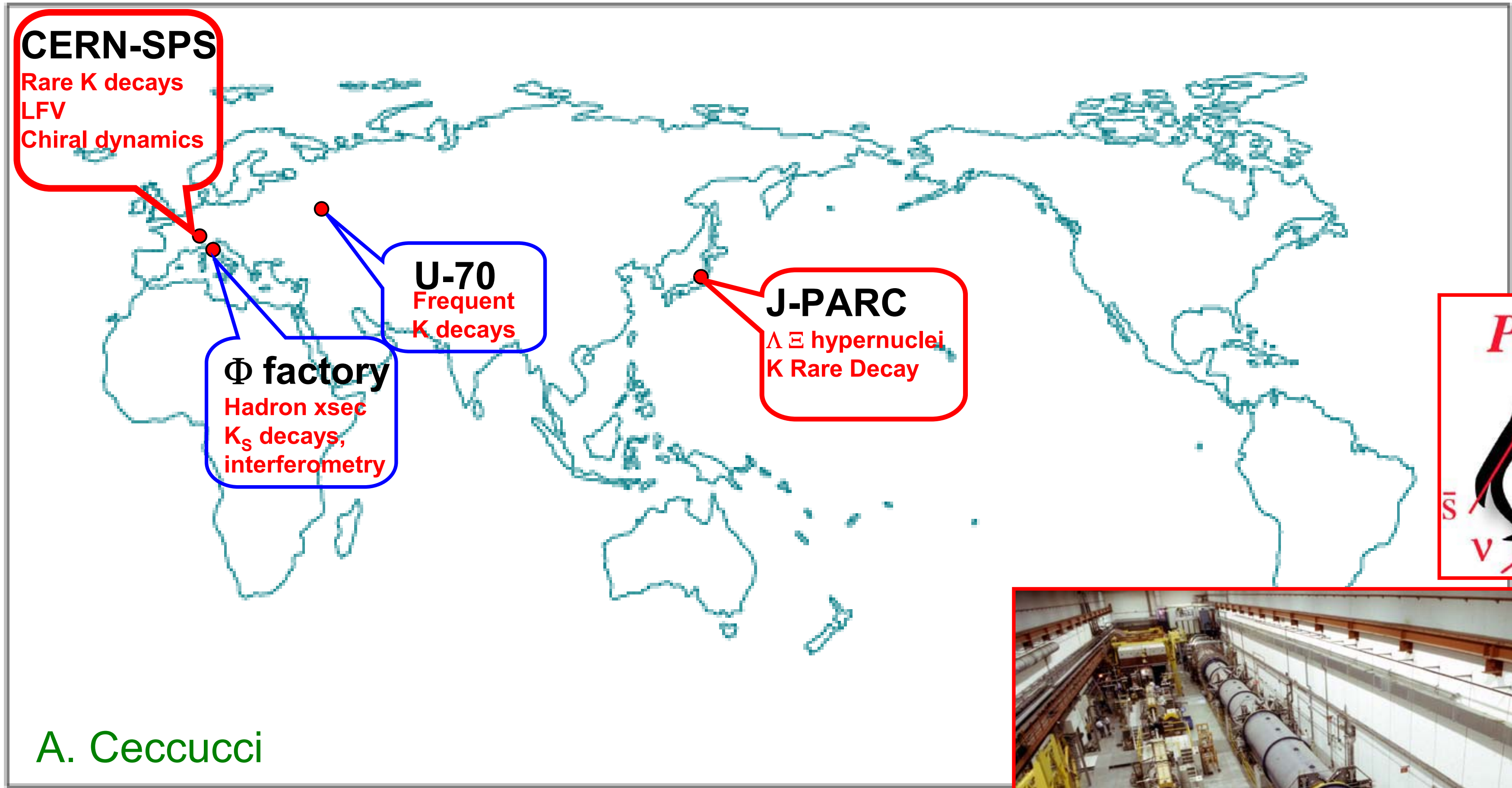
$\pi\pi \to \pi\pi$: $\quad \delta_0 - \delta_2 = (47.7 \pm 1.5)^\circ$ $\quad$ Colangelo–Gasser–Leutwyler '01

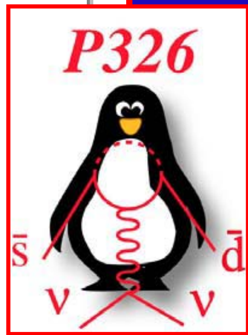
Before KLOE 02 $\quad (\chi_0 - \chi_2)_{\mathrm{LO-IC}} = 57^\circ$

UNITARITY TRIANGLE CONSTRAINTS



Future Kaon Initiatives





NA48/2 → 2 ×10^{11} kaon decays

P-326 (NA48/3) → > 10^{13} kaon decays

$\Delta[Br(K^+ \to \pi^+ \nu\nu)] \sim 0.10$

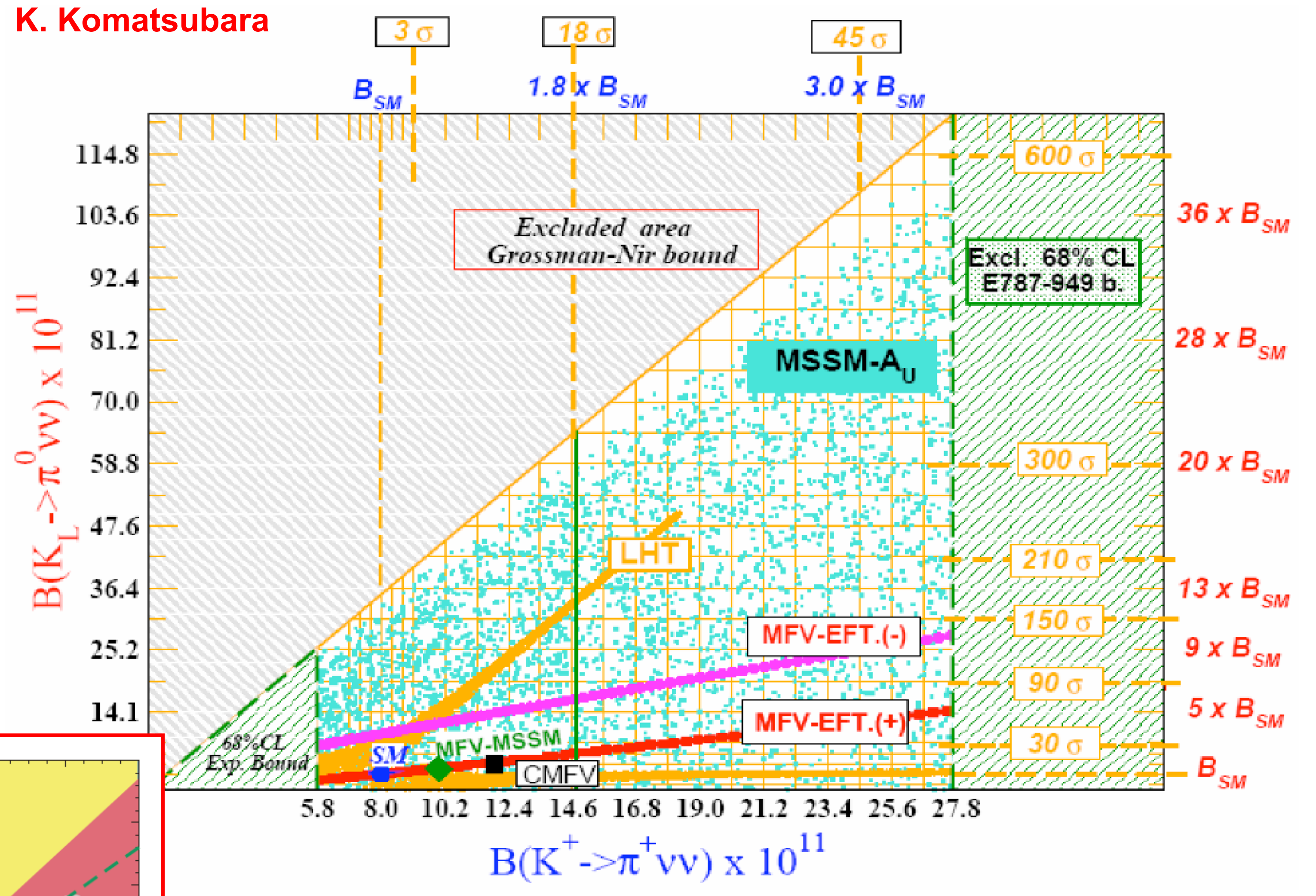
Plans for $K^+ \to \pi^+ \bar{\nu}\nu$

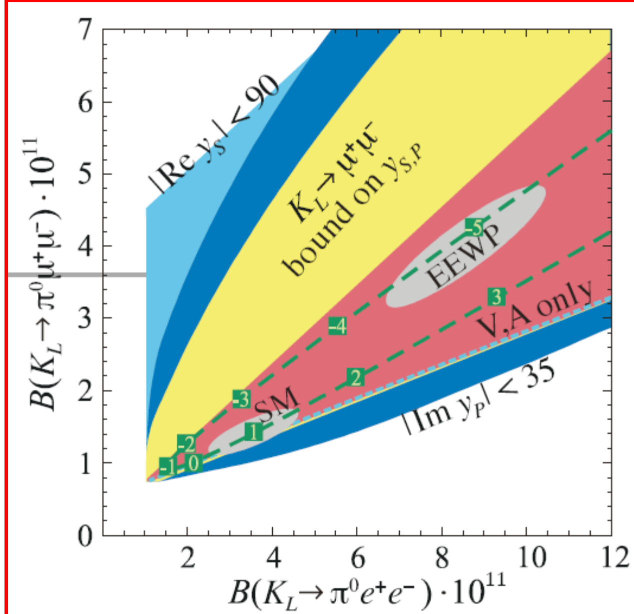
- J-PARC: LoI ; plans to use the BNL-E949 detector
- CERN: P-326 ; about 80 SM events in two years

Plans for $K_L \to \pi^0 \bar{\nu}\nu$

- KEK: E391a ; data taking completed (three runs)
  - Present limit $< 2.1\ 10^{-7}$ 90% CL (10% of Run-1 data)
  - Aims to reach the Grossman-Nir bound ($\sim 10^{-9}$)
- J-PARC: proposal (>2010)
  - Step I: E391a detector at J-PARC ~ SM sensitivity
  - Step II: New detector & dedicated beam-line ~ 100 SM events
- CERN: would need an upgraded proton complex

$K_L \to \pi^0 l^+ l^-$





Plenty of Room for New Physics