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Non-unitary CP Violation of Majorana ν's

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Non-unitary CP Violation of Majorana ν's

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Motivation

In the SM, unitarity is the only constraint imposed on the CKM matrix. Current data leave little room for the unitarity violation (Branco's talk).

$$\begin{aligned} |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 &= 0.9992 \pm 0.0011 \quad \text{(1st row)} \\ |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 &= 1.003 \pm 0.027 \quad \text{(2nd row)} \\ |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 &= 1.001 \pm 0.005 \quad \text{(1st column)} \end{aligned}$$

PDG 2007

But the origin of neutrino masses must be beyond the SM. In this case, whether the MNS matrix is unitary or not relies on the model or theory.

The complete Lagrangian in a theory of neutrino mass generation must be unitary, and thus probability is conserved.

Unitarity Violation is a typical low-energy signal of new physics, when data are analyzed in the framework of the SM gauge group with 3 light species of charged fermions and neutrinos + neutrino mass terms.

The 3×3 MNS matrix will be non-unitary, if 3 known (light) neutrinos are mixed with other degrees of freedom, either light or heavy.

Motivation

Example A: **light sterile neutrinos --- no good TH / EX motivation today.**

Example B: **heavy Majorana neutrinos --- popular seesaw mechanisms.**

Example C: **whole tower of KK states --- models with extra dimensions.**

The scheme of Minimal Unitarity Violation (Antusch *et al* 07):

---- Only 3 light neutrino species are considered;

---- Sources of non-unitarity are allowed only in those terms of the SM Lagrangian which involve neutrinos.

Constraint on the 3×3 ν-mixing matrix *V* ---- data on ν-oscillations, *W* and *Z* decays, rare LFV modes and lepton universality tests, ... (Antusch *et al* 07):

$$|VV^\dagger| \approx \begin{pmatrix} 0.994 \pm 0.005 & < 7.0 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.0 \cdot 10^{-5} & 0.995 \pm 0.005 & < 1.0 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.0 \cdot 10^{-2} & 0.995 \pm 0.005 \end{pmatrix}$$

$$|V^\dagger V| \approx \begin{pmatrix} 1.00 \pm 0.032 & < 0.032 & < 0.032 \\ < 0.032 & 1.00 \pm 0.032 & < 0.032 \\ < 0.032 & < 0.032 & 1.00 \pm 0.032 \end{pmatrix}$$

Motivation

Lesson A: unitarity is good or bad (maybe violated) at the percent level.

Lesson B: Extra CPV phases due to unitarity violation of V are entirely unconstrained and may give rise to new CPV effect at the percent level.

My concern: non-unitarity of V in a class of seesaw models with one or more TeV-scale Majorana neutrinos:

---- heavy Majorana neutrinos might be produced at the LHC;

---- strength of non-unitarity of V might be as large as several percent.

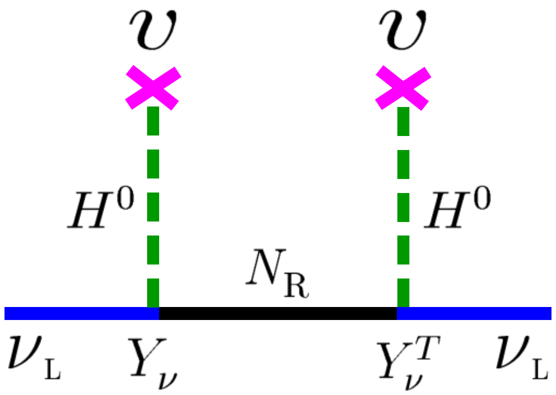
OUTLINE

- ν-N Mixing in TeV-scale Seesaws
- Non-unitary CPV in ν Oscillations
- Comments on CPV in N Decays
- Summary

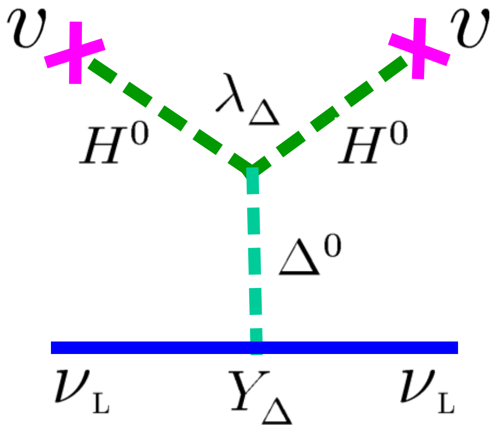
Why Seesaws?

A natural theoretical way to understand why 3 ν-masses are very small.

Type-one Seesaw (Minkowski 77, Yanagida 79, Glashow 79, Gell-Mann, Ramond, Slanski 79, Mohapatra, Senjanovic 80)



$$\boxed{M_\nu \approx -v^2 Y_\nu \frac{1}{M_R} Y_\nu^T}$$



$$\boxed{M_\nu \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}}$$

Triplet Seesaw (Magg, Wetterich 80, Schechter, Valle 80, Lazarides, Shafi, Wetterich 81, Mohapatra, Senjanovic 81, Gelmini, Roncadelli 81)

Type-II Seesaw (a few right-handed Majorana neutrinos and one Higgs triplet are both added into the SM)

Why TeV Seesaws?

Is the seesaw scale very close to a fundamental physics scale?

How heavy are the heavy Majorana neutrinos or the Higgs triplet?

Planck

GUT — **to unify strong, weak & electromagnetic forces?**

Conventional (Type-one) Seesaw Picture: close to the GUT scale

TeV Seesaw Idea: driven by testability at LHC

TeV — **to solve the unnatural gauge hierarchy problem?**

Fermi

Naturalness? ⟷ **Testability?**

Type-I Seesaw

Type-I Seesaw: add 3 right-handed Majorana neutrinos into the SM.

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

or

$$-\mathcal{L}_{\rm mass} = \overline{e_{\rm L}} M_l E_{\rm R} + \frac{1}{2}\overline{(\nu_{\rm L} \quad N^{\rm c}_{\rm R})} \begin{pmatrix} 0 & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} \nu^{\rm c}_{\rm L} \\ N_{\rm R} \end{pmatrix} + {\rm h.c.}$$

Diagonalization (flavor basis ⇒ mass basis):

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} 0 & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \overline{M}_\nu & 0 \\ 0 & \overline{M}_{\rm R} \end{pmatrix}$$

$$V^\dagger V + S^\dagger S = V V^\dagger + R R^\dagger = 1$$

Hence *V* is not unitary

Seesaw:

$$M_\nu \equiv V \overline{M}_\nu V^T \approx -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$$

$$R \sim S \sim M_{\rm D} / M_{\rm R}$$

Strength of Unitarity Violation

$$V \approx \left(1 - \frac{1}{2} R R^\dagger\right) V_{\rm unitary}$$

Natural or Unnatural?

Natural case: no large cancellation in the leading seesaw term.

$$M_\nu \approx M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$$

0.01 eV

10^{15} GeV

100 GeV

$$R \sim S \sim M_{\rm D}/M_{\rm R} \sim 10^{-13}$$

$$\text{Unitarity Violation} \sim 10^{-26}$$

Unnatural case: large cancellation in the leading seesaw term.

$$M_\nu \approx M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$$

0.01 eV

1 TeV

100 GeV

$$R \sim S \sim M_{\rm D}/M_{\rm R} \sim 10^{-1}$$

$$\text{Unitarity Violation} \sim 10^{-2}$$

TeV-scale (right-handed) Majorana neutrinos: small masses of light Majorana neutrinos come from sub-leading perturbations.

Structural Cancellation

Given diagonal *M*_R with 3 eigenvalues *M*_1, *M*_2 and *M*_3, the leading (i.e., type-I seesaw) term of the light neutrino mass matrix vanishes, if and only if *M*_D has rank 1,

$$\boldsymbol{M}_{\mathrm{D}} = m \begin{pmatrix} y_1 & y_2 & y_3 \\ \alpha y_1 & \alpha y_2 & \alpha y_3 \\ \beta y_1 & \beta y_2 & \beta y_3 \end{pmatrix}$$

and if

$$\frac{y_1^2}{M_1} + \frac{y_2^2}{M_2} + \frac{y_3^2}{M_3} = 0$$

$$M_\nu \approx M_{\mathrm{D}} M_{\mathrm{R}}^{-1} M_{\mathrm{D}}^T = 0$$

(Buchmueller, Greub 91; Ingelman, Rathsman 93; Heusch, Minkowski 94; ……; Kersten, Smirnov 07).

Tiny ν-masses can be generated from tiny corrections to this complete "structural cancellation", by deforming *M*_D or *M*_R .

Simple example:

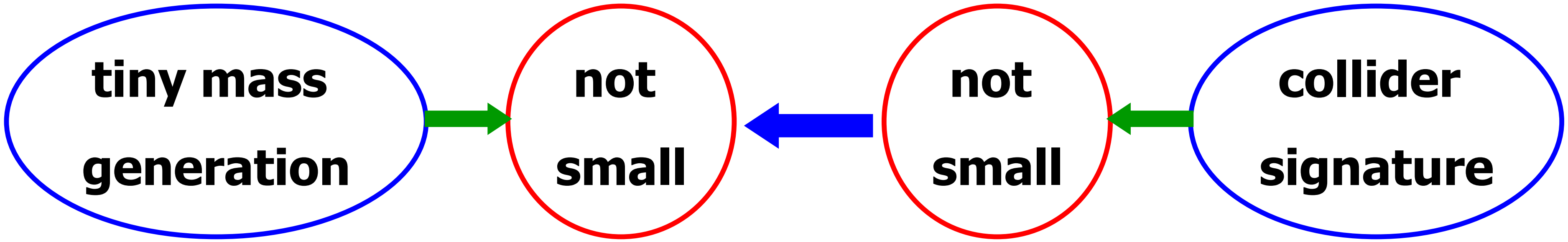
$$M'_{\mathrm{D}} = M_{\mathrm{D}} + \epsilon X_{\mathrm{D}}$$

$$\begin{aligned} M'_\nu &= M'_{\mathrm{D}} M_{\mathrm{R}}^{-1} {M'}_{\mathrm{D}}^{T} \\ &\approx \epsilon \left(M_{\mathrm{D}} M_{\mathrm{R}}^{-1} X_{\mathrm{D}}^T + X_{\mathrm{D}} M_{\mathrm{R}}^{-1} M_{\mathrm{D}}^T \right) + \mathcal{O}(\epsilon^2) \end{aligned}$$

Type-II Seesaw

Incomplete cancellation between two leading terms of the light neutrino mass matrix in type-II seesaw scenarios. The residue of this incomplete cancellation generates the neutrino masses:

$$\boxed{M_\nu \approx M_{\rm L} - M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T}$$



(Chao, Luo, Z.Z.X., Zhou 08)

Discrete flavor symmetries may be used to arrange the textures of two mass terms, but fine-tuning seems unavoidable in the (Big – Big) case.

Unitarity violation:

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_{\rm N} \end{pmatrix}$$

$$V \approx \left(1 - \frac{1}{2} R R^\dagger\right) V_{\rm unitary}$$

Charged Current Interactions

The *standard* charged current interactions in the lepton flavor basis

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}} \gamma^\mu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the presence of heavy right-handed Majorana neutrinos, the overall 6×6 neutrino mass matrix can be diagonalized by a unitary matrix:

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_{\rm N} \end{pmatrix}$$

either Type-I or Type-II seesaw.

Neutrino flavor states in terms of light/heavy neutrino mass states:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L}$$

Correlated CC-interactions:

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \left[\overline{(e, \mu, \tau)_{\rm L}} \, V \gamma^\mu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + \overline{(e, \mu, \tau)_{\rm L}} \, R \gamma^\mu \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} W^-_\mu \right]$$

Correlation between *V* and *R*

R **: production & detection of heavy Majorana neutrinos at LHC;**
V **: oscillations & other phenomena of light Majorana neutrinos.**

They are two 3×3 sub-matrices of the 6×6 unitary matrix, hence they must be correlated with each other. This correlation characterizes the relationship between neutrino physics and collider physics.

Strategy: parametrizing the 6×6 unitary matrix in terms of 15 rotation angles and 15 phase angles. The common parameters shared by *R* and *V* measure their correlation --- a general and useful approach.

2-dimensional rotation matrices in 6-dimensional complex space

$$c_{ij} \equiv \cos\theta_{ij} \qquad \hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin\theta_{ij}$$

$$O_{12} = \begin{pmatrix} c_{12} & \hat{s}^*_{12} & 0 & 0 & 0 & 0 \\ -\hat{s}_{12} & c_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{13} = \begin{pmatrix} c_{13} & 0 & \hat{s}^*_{13} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\hat{s}_{13} & 0 & c_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{23} & \hat{s}^*_{23} & 0 & 0 & 0 \\ 0 & -\hat{s}_{23} & c_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Standard Parametrization

Parametrization:

$$\begin{pmatrix} V & R \\ \\ S & U \end{pmatrix} \equiv \begin{pmatrix} A & R \\ \\ B & U \end{pmatrix} \begin{pmatrix} V_0 & \mathbf{0} \\ \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

$$\begin{pmatrix} A & R \\ B & U \end{pmatrix} = O_{56} O_{46} O_{36} O_{26} O_{16} O_{45} O_{35} O_{25} O_{15} O_{34} O_{24} O_{14} \,,$$

$$\begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} = O_{23} O_{13} O_{12} \,.$$

V_0 is the standard form of the 3×3 unitary neutrino mixing matrix:

$$V_0 = \begin{pmatrix} c_{12} c_{13} & \hat{s}^*_{12} c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12} c_{23} - c_{12} \hat{s}_{13} \hat{s}^*_{23} & c_{12} c_{23} - \hat{s}^*_{12} \hat{s}_{13} \hat{s}^*_{23} & c_{13} \hat{s}^*_{23} \\ \hat{s}_{12} \hat{s}_{23} - c_{12} \hat{s}_{13} c_{23} & -c_{12} \hat{s}_{23} - \hat{s}^*_{12} \hat{s}_{13} c_{23} & c_{13} c_{23} \end{pmatrix}$$

V = A V_0

Unitarity Violation

$V \to V_0$ in the limit of $A \to 1$

$$V V^\dagger = A A^\dagger = 1 - R R^\dagger$$

Exact Results of *A* and *R*

They share 9 rotation angles & 9 phase angles: *V*—*R* correlation.

$$
A = \begin{pmatrix}
c_{14}c_{15}c_{16} & 0 & 0 \\
\begin{array}{l} -c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ -\hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & c_{24}c_{25}c_{26} & 0 \\
\begin{array}{l} -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + \hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} \\ -\hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & c_{34}c_{35}c_{36}
\end{pmatrix}
$$

$$
R = \begin{pmatrix}
\hat{s}^*_{14}c_{15}c_{16} & \hat{s}^*_{15}c_{16} & \hat{s}^*_{16} \\
\begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ +c_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & -\hat{s}^*_{15}\hat{s}_{16}\hat{s}^*_{26} + c_{15}\hat{s}^*_{25}c_{26} & c_{16}\hat{s}^*_{26} \\
\begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} - c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} + c_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -\hat{s}^*_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +c_{15}c_{25}\hat{s}^*_{35}c_{36} \end{array} & c_{16}c_{26}\hat{s}^*_{36}
\end{pmatrix}
$$

Approximations of *A* and *R*

All 9 rotation angles are expected to be small, at most of O(0.1), 9 CPV phases may be large to generate new CP-violating effects.

$$A = 1 - \begin{pmatrix} \frac{1}{2}\left(s_{14}^2 + s_{15}^2 + s_{16}^2\right) & 0 & 0 \\ \hat{s}_{14}\hat{s}_{24}^* + \hat{s}_{15}\hat{s}_{25}^* + \hat{s}_{16}\hat{s}_{26}^* & \frac{1}{2}\left(s_{24}^2 + s_{25}^2 + s_{26}^2\right) & 0 \\ \hat{s}_{14}\hat{s}_{34}^* + \hat{s}_{15}\hat{s}_{35}^* + \hat{s}_{16}\hat{s}_{36}^* & \hat{s}_{24}\hat{s}_{34}^* + \hat{s}_{25}\hat{s}_{35}^* + \hat{s}_{26}\hat{s}_{36}^* & \frac{1}{2}\left(s_{34}^2 + s_{35}^2 + s_{36}^2\right) \end{pmatrix} + \mathcal{O}(s_{ij}^4)$$

$$R = 0 + \begin{pmatrix} \hat{s}_{14}^* & \hat{s}_{15}^* & \hat{s}_{16}^* \\ \hat{s}_{24}^* & \hat{s}_{25}^* & \hat{s}_{26}^* \\ \hat{s}_{34}^* & \hat{s}_{35}^* & \hat{s}_{36}^* \end{pmatrix} + \mathcal{O}(s_{ij}^3) \; ,$$

Observations:

If the unitarity violation of *V* is close to the percent level, then elements of *R* can reach order of 0.1, leading to appreciable collider signatures for TeV-scale Majorana neutrinos.

New CP-violating effects, induced by the non-unitarity of *V*, may show up in (short-baseline) neutrino oscillations.

Such a parametrization turns out to be very useful in ν–phenomenology.

UV-induced CP Violation

Example: ***V*_0** **takes the tri-bimaximal mixing pattern which has**

$$\tan\theta_{12} = 1/\sqrt{2},\ \theta_{13} = 0 \text{ and } \theta_{23} = \pi/4$$

$$\delta_{12} = \delta_{13} = \delta_{23} = 0$$

Non-unitary *V* takes the simple form

$$V \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}}\ (1+2X) & \sqrt{\frac{1}{3}}\ (1-X) & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}}\ (1-2Y+Z) & -\sqrt{\frac{1}{3}}\ (1+Y+Z) & \sqrt{\frac{1}{2}}\ (1-Z) \end{pmatrix}$$

CP violation (9 Jarlskog invariants):

$$X = \hat{s}_{14}\hat{s}^*_{24} + \hat{s}_{15}\hat{s}^*_{25} + \hat{s}_{16}\hat{s}^*_{26}$$
$$Y = \hat{s}_{14}\hat{s}^*_{34} + \hat{s}_{15}\hat{s}^*_{35} + \hat{s}_{16}\hat{s}^*_{36}$$
$$Z = \hat{s}_{24}\hat{s}^*_{34} + \hat{s}_{25}\hat{s}^*_{35} + \hat{s}_{26}\hat{s}^*_{36}$$

$$J^{ij}_{\alpha\beta} \equiv \mathrm{Im}(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i})$$

$$J^{23}_{e\mu} = J^{23}_{\tau e} = J^{31}_{e\mu} = J^{31}_{\tau e} = 0$$

$$J^{12}_{e\mu} \approx \mathrm{Im}X/3$$

$$J^{12}_{\tau e} \approx \mathrm{Im}Y/3$$

New CPV

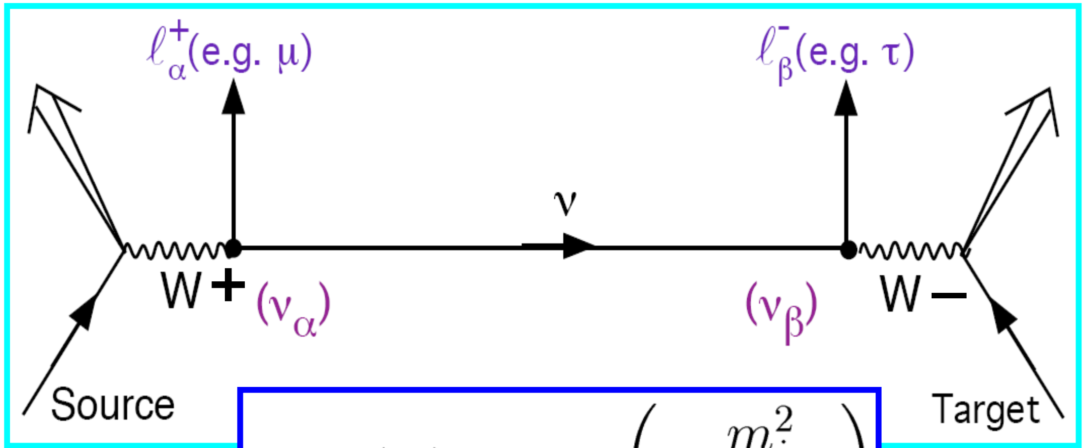
O(≤1%)

$$J^{12}_{\mu\tau} \approx \left(\mathrm{Im}X + \mathrm{Im}Y\right)/6\ ,$$
$$J^{23}_{\mu\tau} \approx \left(\mathrm{Im}X + \mathrm{Im}Y + 2\mathrm{Im}Z\right)/6\ ,$$
$$J^{31}_{\mu\tau} \approx \left(\mathrm{Im}X + \mathrm{Im}Y - \mathrm{Im}Z\right)/6\ .$$

Neutrino Oscillations

Production and detection of a neutrino beam via CC weak interactions:

$$A(W^+ \to l_\alpha^+ \nu_i) = \frac{V_{\alpha i}^*}{\sqrt{(VV^\dagger)_{\alpha\alpha}}}$$



$$A(\nu_i W^- \to l_\beta^-) = \frac{V_{\beta i}}{\sqrt{(VV^\dagger)_{\beta\beta}}}$$

$$\sum_i |A(W^+ \to l_\alpha^+ \nu_i)|^2 = 1$$

$$\text{Prop}(\nu_i) = \exp\left(-i\frac{m_i^2}{2E}L\right)$$

$$\sum_i |A(\nu_i W^- \to l_\beta^-)|^2 = 1$$

$$\begin{aligned} A(\nu_\alpha \to \nu_\beta) &= \sum_i \left[A(W^+ \to l_\alpha^+ \nu_i) \cdot \text{Prop}(\nu_i) \cdot A(\nu_i W^- \to l_\beta^-)\right] \\ &= \frac{1}{\sqrt{(VV^\dagger)_{\alpha\alpha}(VV^\dagger)_{\beta\beta}}} \sum_i \left[V_{\alpha i}^* \exp\left(-i\frac{m_i^2}{2E}L\right) V_{\beta i}\right] \end{aligned}$$

Like *non-standard* interactions in initial & final states (Lindner's talk).

Neutrino Oscillations

Oscillation probability in vacuum (e.g., Antusch *et al* 06, Z.Z.X. 08):

$$P(\nu_\alpha \to \nu_\beta) = \frac{\sum_i |V_{\alpha i}|^2 |V_{\beta i}|^2 + 2\sum_{i<j} \mathrm{Re}\left(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}\right) \cos\Delta_{ij} - 2\sum_{i<j} J^{ij}_{\alpha\beta} \sin\Delta_{ij}}{\left(VV^\dagger\right)_{\alpha\alpha} \left(VV^\dagger\right)_{\beta\beta}}$$

$$\Delta_{ij} \equiv \Delta m^2_{ij} L/(2E) \text{ with } \Delta m^2_{ij} \equiv m^2_i - m^2_j$$

$$|\Delta m^2_{13}| \approx |\Delta m^2_{23}| \gg |\Delta m^2_{12}|$$

Jarlskog invariants of CP violation:

$$J^{ij}_{\alpha\beta} \equiv \mathrm{Im}(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i})$$

Unitary: universal Jarlskog invariant = 2 area of each unitarity triangle.

Non-unitary: 9 different Jarlskog invariants, and triangles → polygons.

"Zero-distance" (near-detector) effect at *L* = 0 :

$$P(\nu_\alpha \to \nu_\beta)\,|_{L=0} = \frac{|(VV^\dagger)_{\alpha\beta}|^2}{(VV^\dagger)_{\alpha\alpha}(VV^\dagger)_{\beta\beta}}$$

Example

A non-unitary deviation from the tri-bimaximal mixing pattern:

$$V \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}}\ (1+2X) & \sqrt{\frac{1}{3}}\ (1-X) & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}}\ (1-2Y+Z) & -\sqrt{\frac{1}{3}}\ (1+Y+Z) & \sqrt{\frac{1}{2}}\ (1-Z) \end{pmatrix}$$

$$J^{12}_{\mu\tau} \approx \left(\mathrm{Im}X + \mathrm{Im}Y\right)/6 \,,$$
$$J^{23}_{\mu\tau} \approx \left(\mathrm{Im}X + \mathrm{Im}Y + 2\mathrm{Im}Z\right)/6 \,,$$
$$J^{31}_{\mu\tau} \approx \left(\mathrm{Im}X + \mathrm{Im}Y - \mathrm{Im}Z\right)/6 \,.$$

Z.Z.X. 08, Luo 08 (matter effects), Z.Z.X., Zhou 08 (neutrino telescopes)

Short- or medium-baseline experiments in the neglect of matter effects (Fernandez-Martinez *et al* 07). In particular (Z.Z.X. 08),

$$P(\nu_\mu \to \nu_\tau) \approx \sin^2 2\theta_{23} \underbrace{\sin^2 \frac{\Delta_{23}}{2}}_{\approx 1} - 2\left(J^{23}_{\mu\tau} + J^{13}_{\mu\tau}\right) \sin\Delta_{23}$$

$$P(\overline{\nu}_\mu \to \overline{\nu}_\tau) \approx \sin^2 2\theta_{23} \sin^2 \frac{\Delta_{23}}{2} + 2\left(J^{23}_{\mu\tau} + J^{13}_{\mu\tau}\right) \sin\Delta_{23}$$

$$\begin{aligned} & J^{23}_{\mu\tau} + J^{13}_{\mu\tau} \\ = \; & J^{23}_{\mu\tau} - J^{31}_{\mu\tau} \\ \approx \; & \mathrm{Im}Z/2 \end{aligned}$$

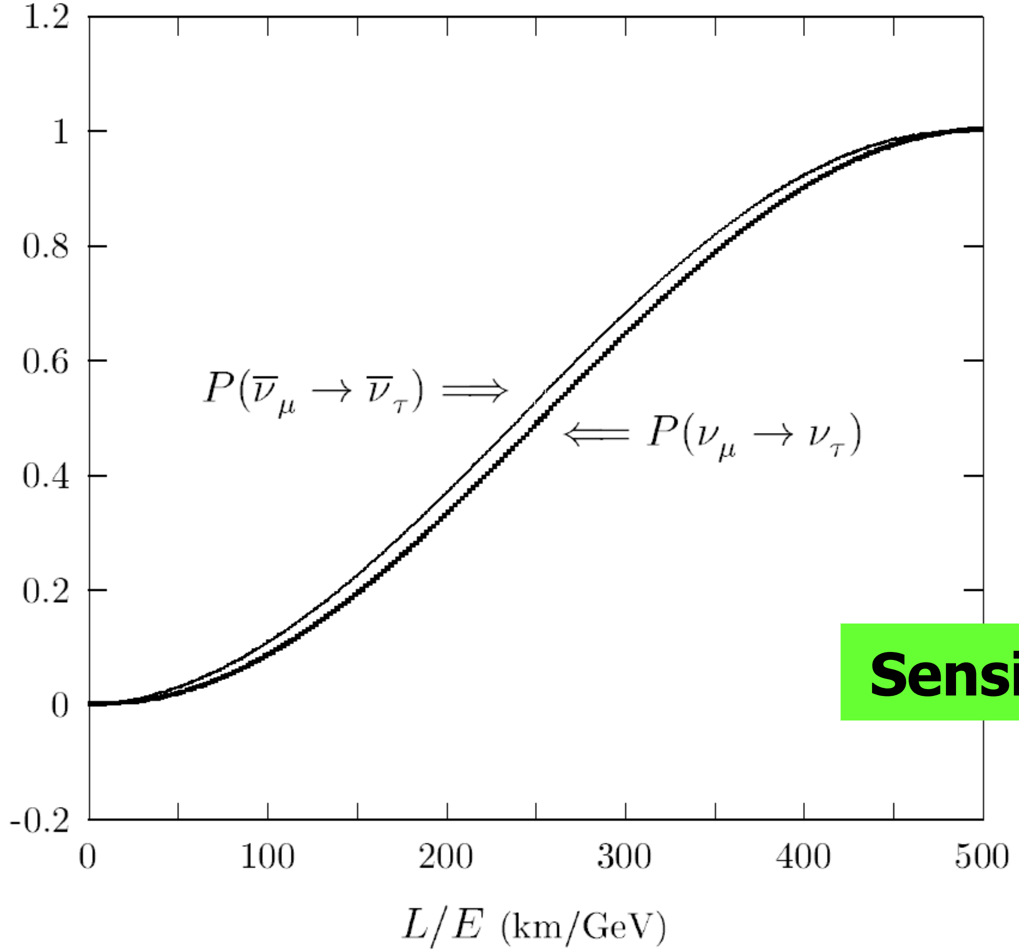
UV-induced CPV at 1% level?

$$2\left(J^{23}_{\mu\tau} + J^{13}_{\mu\tau}\right) \approx s_{24}s_{34}\sin\left(\delta_{24}-\delta_{34}\right) + s_{25}s_{35}\sin\left(\delta_{25}-\delta_{35}\right) + s_{26}s_{36}\sin\left(\delta_{26}-\delta_{36}\right)$$

Numerical Illustration

Example: **an experiment with E ≥ a few GeV & L ~ a few 100 km.**

$$\theta_{23} \sim \pi/4,\ \Delta m^2_{23} \sim 2.5 \times 10^{-3}\ \mathrm{eV}^2\ \text{and}\ (J^{23}_{\mu\tau} + J^{13}_{\mu\tau}) \sim 0.01$$

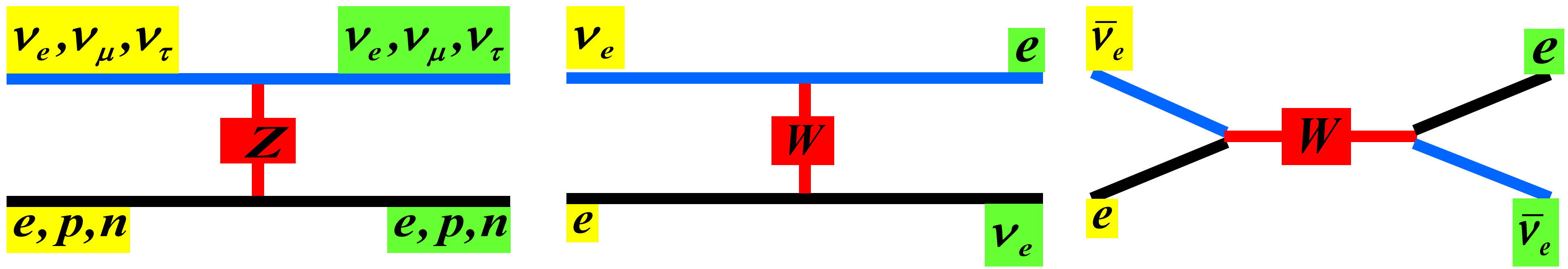


Neutrino Factory?

Sensitivity ≤ 1% ?

Matter Effects

Illustration: one heavy Majorana neutrino and constant matter density.



$$
\begin{aligned}
P(\nu_\nu \to \nu_\tau) &\approx \sin^2\frac{\Delta_{23}}{2} - s_{24}s_{34}\left[\sin\left(\delta_{24}-\delta_{34}\right) + A_{\rm NC}L\cos\left(\delta_{24}-\delta_{34}\right)\right]\sin\Delta_{23} \\
P(\overline{\nu}_\nu \to \overline{\nu}_\tau) &\approx \sin^2\frac{\Delta_{23}}{2} + s_{24}s_{34}\left[\sin\left(\delta_{24}-\delta_{34}\right) + A_{\rm NC}L\cos\left(\delta_{24}-\delta_{34}\right)\right]\sin\Delta_{23}
\end{aligned}
$$

(Goswami, Ota 08; Luo 08)

$$A_{\rm NC} = G_{\rm F}N_n/\sqrt{2}$$

Genuine CPV **Matter effect**

The same matter-effect term appears in ν_μ → ν_μ oscillations (Luo 08).

Interactions of *N*

The *standard* weak interactions of 3 ν's with *W*, *Z* and *H* in the flavor basis:

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^{\dagger} \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^{T} & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^{*} = \begin{pmatrix} \widehat{M}_{\nu} & \mathbf{0} \\ \mathbf{0} & \widehat{M}_{\rm N} \end{pmatrix}$$

$$\nu_{\rm L} = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L}$$

$$N_{\rm R} = S^{*} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm R} + U^{*} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm R}$$

$$\begin{aligned} -\mathcal{L}_{W} &= \frac{g}{\sqrt{2}} \overline{l_{\rm L}} \gamma^{\mu} \nu_{\rm L} W_{\mu}^{-} + {\rm h.c.} \\ -\mathcal{L}_{Z} &= \frac{g}{2\cos\theta_{\rm w}} \overline{\nu_{\rm L}} \gamma^{\mu} \nu_{\rm L} Z_{\mu} \\ -\mathcal{L}_{H} &= \frac{1}{\sqrt{2}} \overline{\nu_{\rm L}} Y_{\nu} N_{\rm R} H + {\rm h.c.} \end{aligned}$$

where $M_{\rm D} = Y_{\nu} \langle H \rangle / \sqrt{2}$

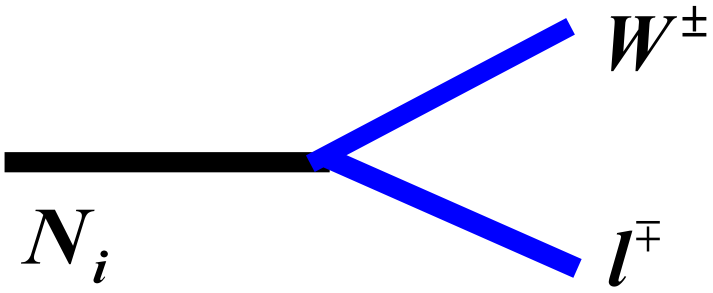
In the mass basis

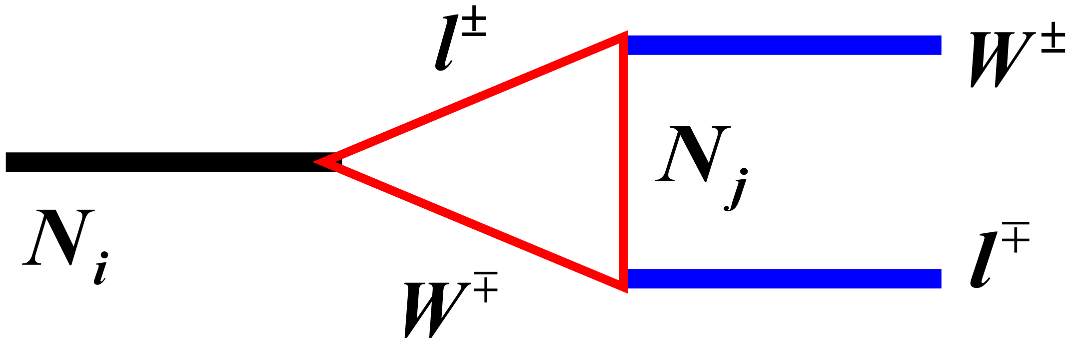
$$\begin{aligned} -\mathcal{L}_{W}^{l-N} &= \frac{g}{\sqrt{2}} \left[\overline{(e \;\; \mu \;\; \tau)_{\rm L}} \; \gamma^{\mu} R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} W_{\mu}^{-} + {\rm h.c.} \right] \\ -\mathcal{L}_{Z}^{\nu-N} &= \frac{g}{2\cos\theta_{\rm w}} \left[\overline{(\nu_1 \;\; \nu_2 \;\; \nu_3)_{\rm L}} \; \gamma^{\mu} (V^{\dagger} R) \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} + {\rm h.c.} \right] Z_{\mu} \\ -\mathcal{L}_{H}^{\nu-N} &\approx \frac{g}{2M_{W}} \left[\overline{(\nu_1 \;\; \nu_2 \;\; \nu_3)_{\rm L}} \; (V^{\dagger} R) \widehat{M}_{N} (U^{\dagger} U)^{*} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm R} + {\rm h.c.} \right] H \end{aligned}$$

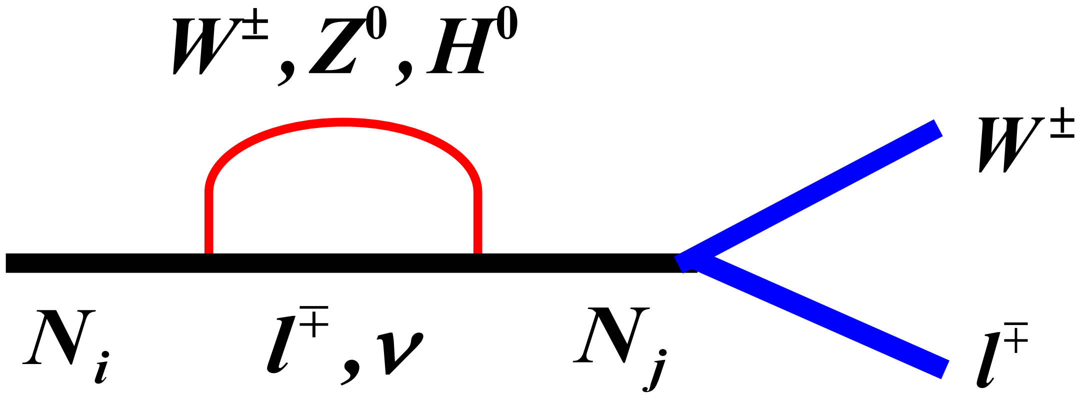
CP Violation

CP violation: interference between tree and one-loop amplitudes of *N*.

$$\boxed{RR^\dagger = \mathbf{1} - VV^\dagger}$$







Xing, Zhou (in progress)

$$\boxed{\begin{aligned} \mathrm{CPV} \quad &\equiv \quad \frac{\Gamma(N_i \to W^+ l^-) - \Gamma(N_i \to W^- l^+)}{\Gamma(N_i \to W^+ l^-) + \Gamma(N_i \to W^- l^+)} \\ &\propto \quad |R|^2 \;\sim\; 1\% \end{aligned}}$$

Resonant enhancement with 4 heavy Majorana neutrinos. (Bray, Lee, Pilaftsis 07)

Concluding Remarks

◆ Naturalness of the SM implies that there should exist a kind of new physics at the TeV scale. We wonder whether it is also responsible for the neutrino mass generation ---- TeV seesaws.

◆ It seems that people are struggling for a convincing reason to consider TeV seesaws ---- a balance between TH naturalness and EX testability as the guiding principle?

◆ Non-unitary CP Violation is a straightforward consequence of TeV seesaws ---- it might manifest itself in both the oscillations of light neutrinos and the decays of heavy neutrinos.

◆ An uneasy feeling ---- the generation of tiny neutrino masses seems always to be decoupled from appreciable collider signals of TeV Majorana neutrinos. Unnatural? Unnatural? Unnatural?