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Generation and Dynamics of Ordered Sheared Zonal Flows from Drift Turbulence.

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Generation and Dynamics of Ordered Sheared Zonal Flows from Drift Turbulence

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This talk is made possible by the many discussions and contribution of materials and results from collaborators and colleagues:

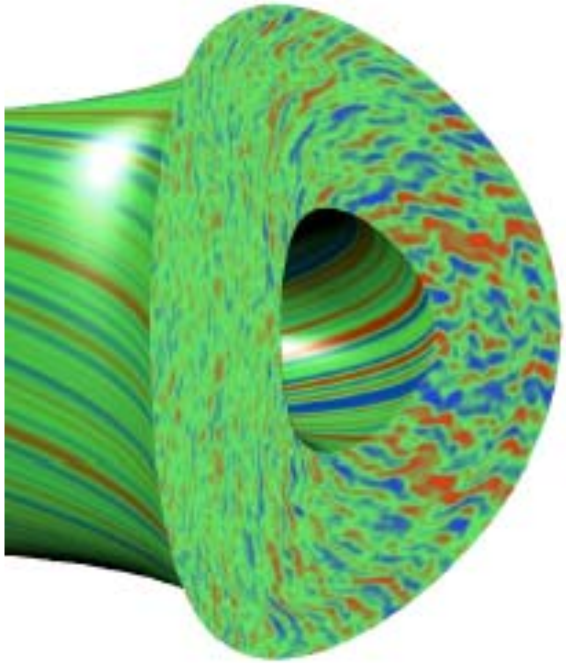
J. Boedo, M. Burin, P.H. Diamond, R. Fonck, A. Fujisawa, O. Gurcan, C. Holland, K. Itoh, S. Itoh, G. McKee, R. Moyer, J. Yu, S. Zweben

Background and Motivation

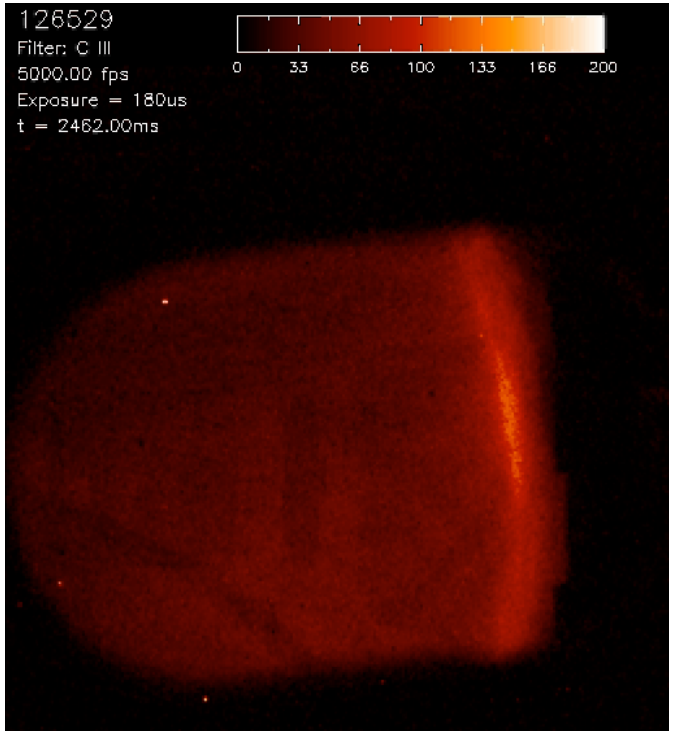
- **Turbulence in MFE Plasmas**
- **Why Is Turbulence of Interest in Fusion?**
- What are Zonal Flows?
- Why Care About the Nonlinear Interactions Between Turbulence and Zonal Flows?

Images of Turbulence in Tokamaks

GYRO

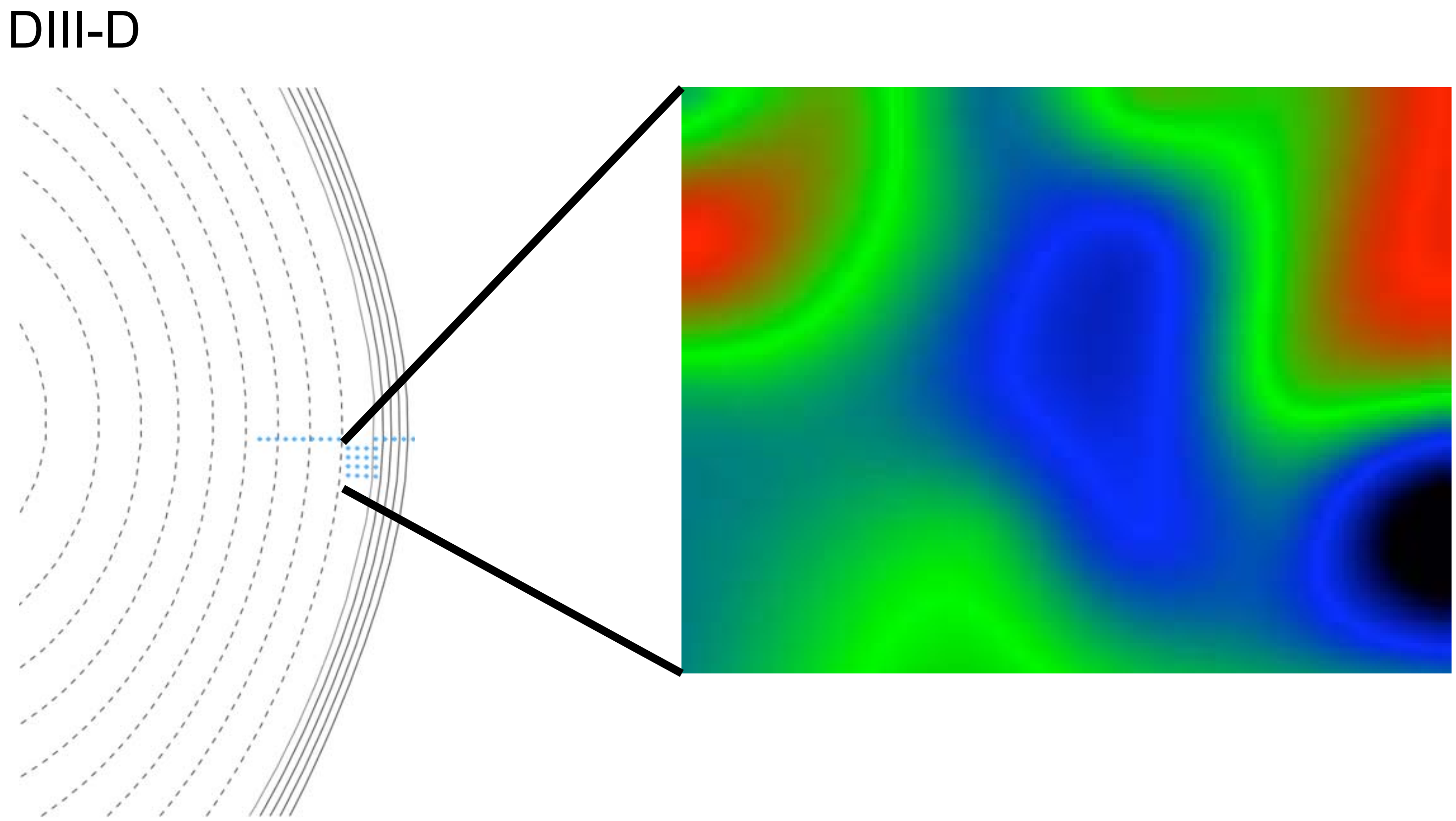


DIII-D



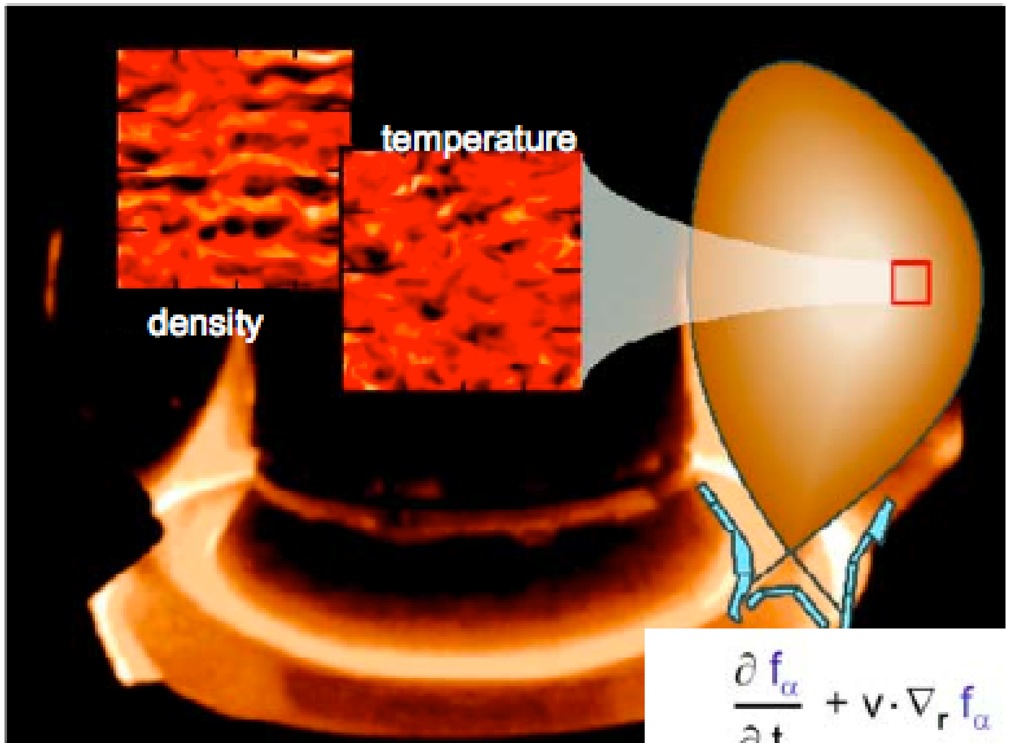
J. Yu C-III Emission (UCSD)

Imaging of Turbulent Density Fluctuations in the Core Region of DIII-D Tokamak



Ref: G. McKee, private communication

Result: Turbulent Transport in Confined Plasmas



“small”-scale turbulence:

- •strongly influenced by geometry
- •single particle effects important which not captured by simple fluid picture

$$\frac{\partial f_\alpha}{\partial t} + v\cdot\nabla_r f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v}\times\mathbf{B})\,\nabla_{\mathbf{v}} f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_{coll.}$$

distribution function   Electric field Lorentz force   Influence of collisio

together with Maxwell’s equations

Ref: Lackner, DEISY Talk 2005

Turbulence Leads to Cross-field Transport

$$\frac{\partial \bar{n}}{\partial t} + \nabla \cdot \tilde{\Gamma} = 0$$

$$\boxed{\tilde{\Gamma} = -\frac{\langle \tilde{n} \nabla \tilde{\phi} \rangle \times \bar{\mathbf{B}}}{B^2}}$$

$$m\bar{n}\frac{\partial \bar{\mathbf{v}}}{\partial t} + \nabla \cdot \vec{\mathbf{r}}_R + = \frac{q}{m}\left(\bar{\mathbf{E}} + \bar{\mathbf{v}} \times \bar{\mathbf{B}}\right) - \nabla\left(\bar{n}\bar{T}\right)$$

$$\boxed{\vec{\mathbf{r}}_R \equiv \left\langle \left(-\frac{\nabla \tilde{\phi} \times \bar{\mathbf{B}}}{B^2} \right) \left(-\frac{\nabla \tilde{\phi} \times \bar{\mathbf{B}}}{B^2} \right) \right\rangle}$$

$$\frac{3}{2}\frac{\partial \bar{n}\bar{T}}{\partial t} + \frac{3}{2}\frac{\partial \langle \tilde{n}\tilde{T} \rangle}{\partial t} + \nabla \cdot \left[\tilde{\mathbf{Q}}_{cond} + \tilde{\mathbf{Q}}_{conv} \right] = -\nabla \cdot \bar{\mathbf{q}}_{class} + P_{rad}$$

$$\boxed{\tilde{\mathbf{Q}}_{cond} \equiv \frac{3}{2}\bar{n}\left(-\frac{\langle \tilde{T} \nabla \tilde{\phi} \rangle \times \bar{\mathbf{B}}}{B^2} \right)} \qquad \boxed{\tilde{\mathbf{Q}}_{conv} \equiv \frac{3}{2}\bar{T}\left(-\frac{\langle \tilde{n} \nabla \tilde{\phi} \rangle \times \bar{\mathbf{B}}}{B^2} \right)}$$

Neglecting DC Convection, Magnetic Fluctuations, Parallel flow fluctuations, Viscosity, …

Assuming electrostatic ExB dynamics for velocity

Background and Motivation

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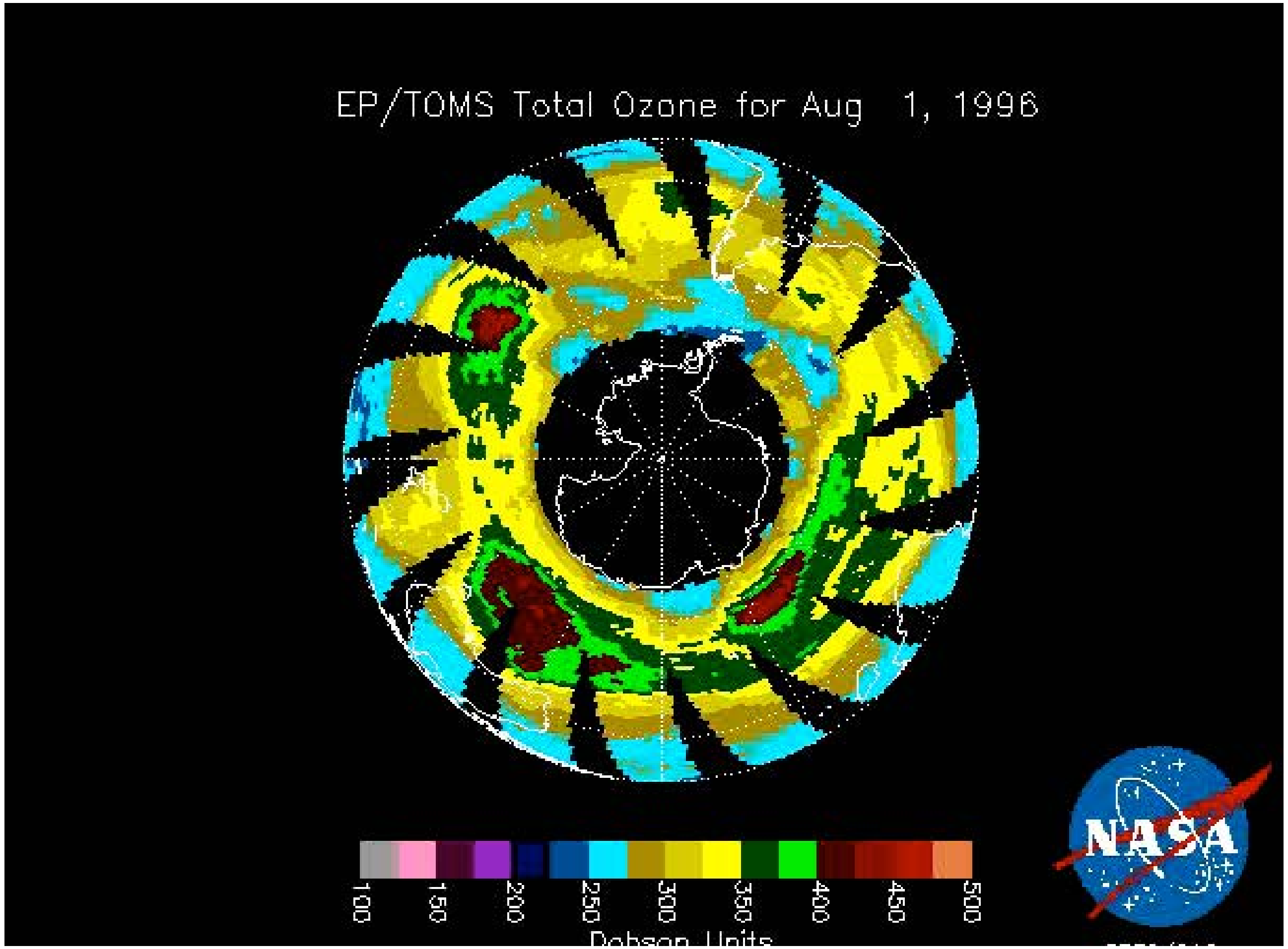
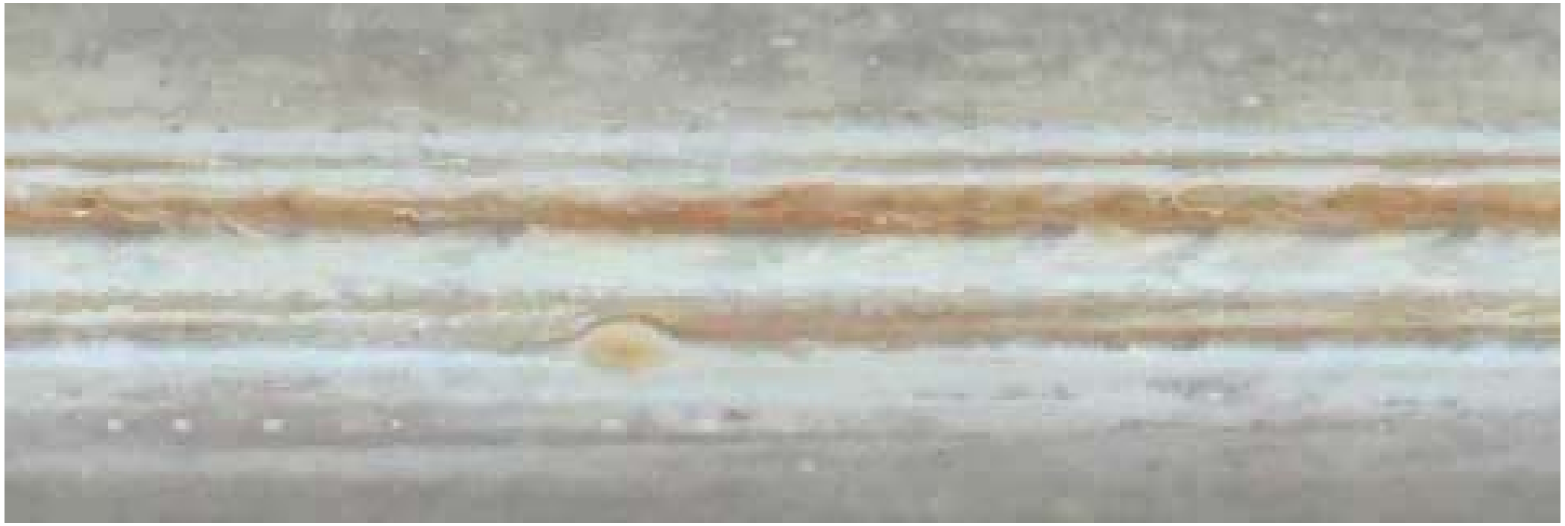
Zonal Flows are Common & Effect Transport in Many Systems



Zonal Flows are Common & Effect Transport in Many Systems

CASSINI Imaging Team, NASA



2D Dynamics in Magnetized Plasmas & Rotating Fluids

Navier-Stokes Eqn for *Rotating Fluid:*

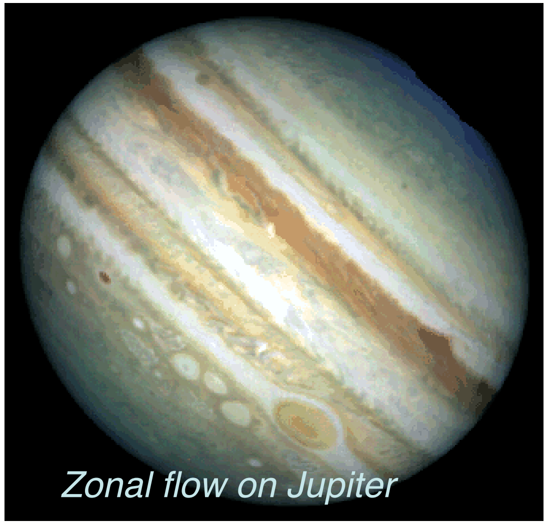
$$\left(\frac{\partial}{\partial t}+\mathbf{v}\bullet\nabla\right)\mathbf{v}+2\mathbf{\Omega}\times\mathbf{v}=-\frac{1}{\rho}\nabla p-\nabla\Phi+\nu\,\nabla^2\mathbf{v}$$

Momentum Eqn for *Magnetized Plasma:*

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\bullet\nabla\right)\mathbf{v}+\mathbf{B}\times\mathbf{v}=-\frac{1}{\rho}\nabla p-\nabla\Phi+\nu\,\nabla^2\mathbf{v}$$

I.e. *momentum conservation same form for rotating fluid And magnetized plasma ➔> DYNAMICS ARE SAME !*

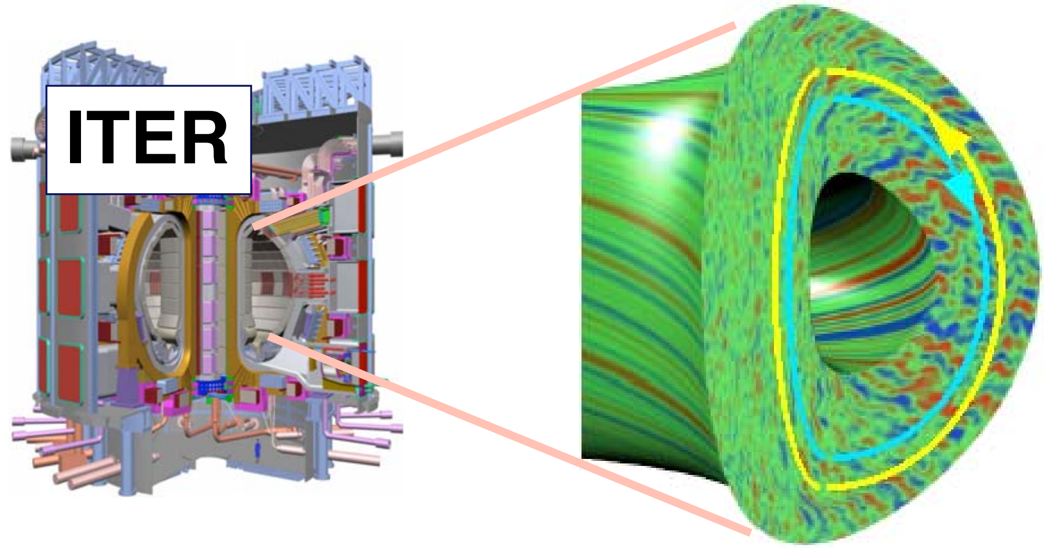
Result: Strong Similarity Between Planetary Flows & Magnetized Plasma Flows



Zonal flow on Jupiter

ExB flows

$m=n=0,\ k_r = \text{finite}$



Ref: K. Itoh, APS 2005 Invited Talk, PoP May 2006

ZFs are "mode", but:

1. Turbulence driven
2. No linear instability
3. No direct radial transport

Why ZFs are Important for Fusion ?

Ref Itoh APS 2005

DW Turb. Coexists with Zonal Flows.

Issues in fusion

1. Self-regulation

$$\chi = \mathcal{R}\,\chi_{gB}$$

$\tau_E = H\ \tau_E{}^L$

"Intrinsic" H-factor (w/o barrier)

$$H \propto \mathcal{R}^{-0.6}$$

Realizability of Ignition

$$L \propto \mathcal{R}^{0.8}$$

$$L \propto H^{-1.3}$$

2. Shift for onset of large transport → Operation of ITER near marginality

3. Identify transport control knob → Flow damping ? → P_{LH}- threshold

→ Intermittent flux, e.g., ELMs → Peak heat load

→ β-limit

4. Meso-scales and zonal field → Impacts on RWM and NTM → Steady state

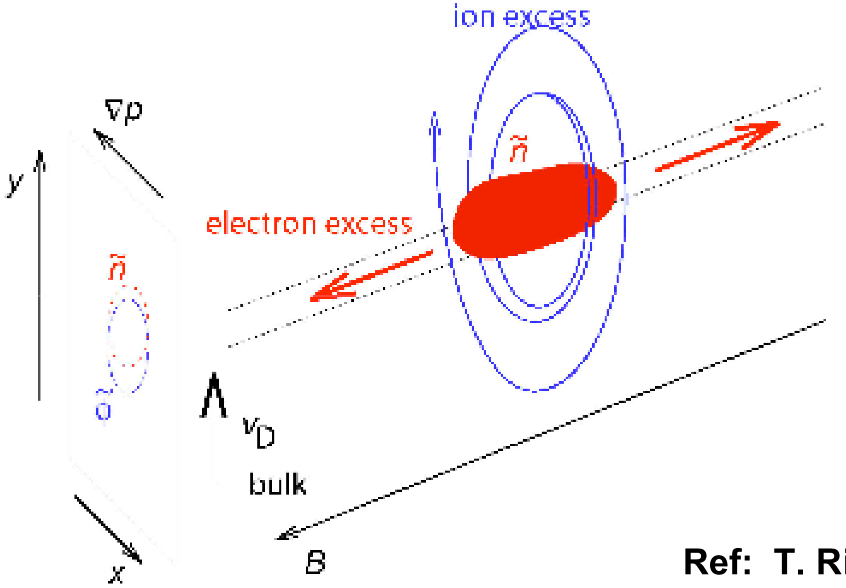
Outline

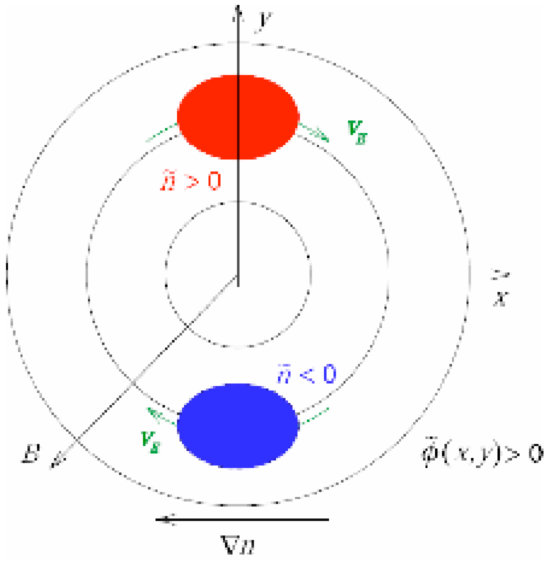
- **Basic Physics of Drift Instabilities & Zonal Flows**
- Intro to Statistical Tools for Turbulence/Zonal Flow Studies
- Laboratory Plasma Studies
- Confinement Experiment Studies of Turbulence and Zonal Flows
- Conclusions & Open Issues

The Basic Picture of Turbulent Transport

Drift waves:

- ExB drift & density profile: $\tilde{\phi}$ excites $\tilde{n}_e$
- || electron and ⊥ ion polarisation dynamics: $\tilde{\phi}$ tied to $\tilde{n}_e$
- Structure propagates in *y*-direction
- Resistivity: phase shift between $\tilde{n}_e$ and $\tilde{\phi}$
- Transport across field





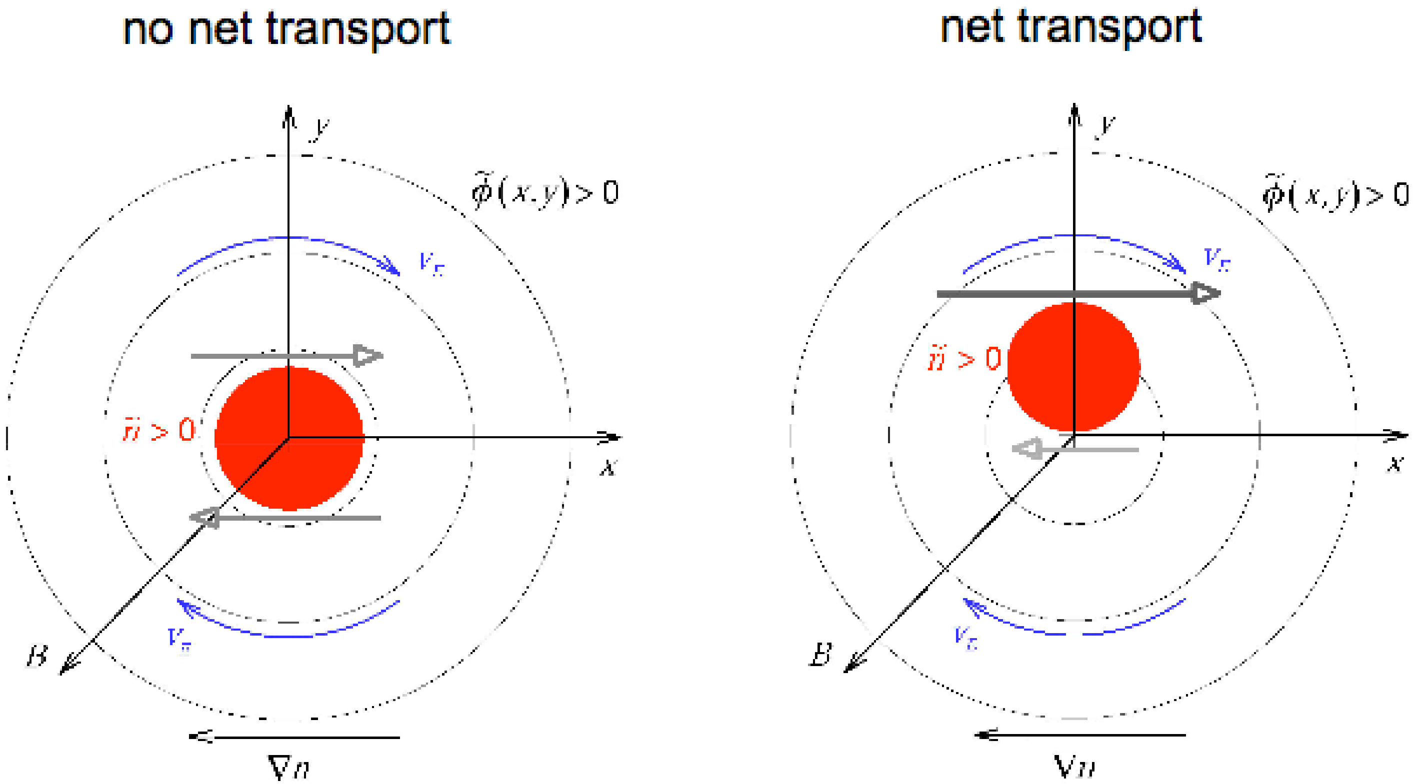
Interchange forcing:

- Compressibility ⊥ drifts due to inhomogeneous B:

  energy path between $\tilde{n}_e$ and $\tilde{\phi}$
- Bigger phase shift between $\tilde{n}_e$ and $\tilde{\phi}$
- Transport across field

Ref: T. Ribiero IPP-Garching 2005 Summer School

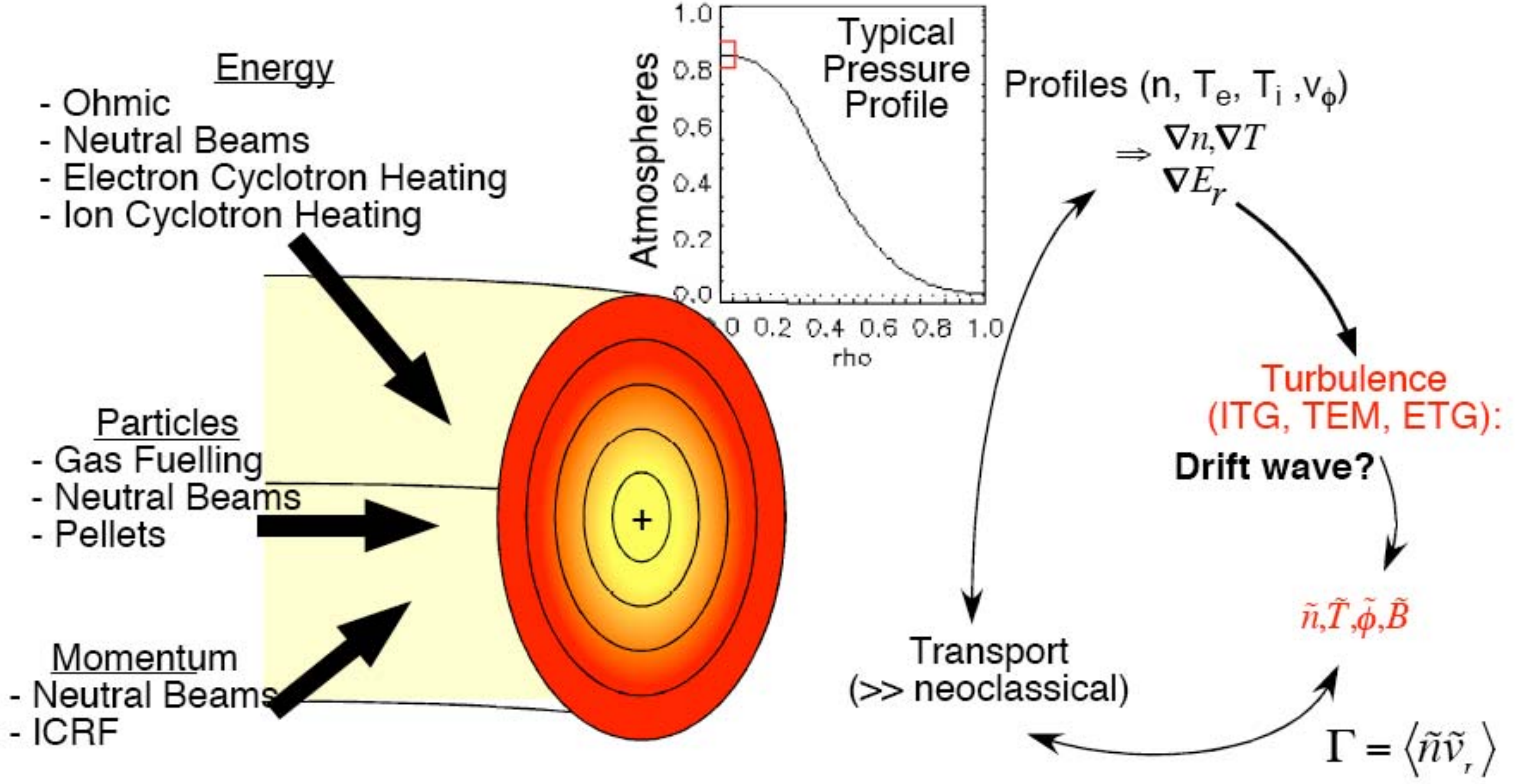
The Basic Picture of Turbulent Transport



Particle transport caused by gradient driven turbulence:
phase shift of pressure ahead of the electrostatic potential

Ref: T. Ribiero IPP-Garching 2005 Summer School

Turbulence Drives Cross-Field Transport of Particles, Energy and Momentum in a Magnetically-Confined Plasma



$$Q = n\langle \tilde{v}_r \tilde{T} \rangle + T\langle \tilde{n}\tilde{v}_r \rangle$$

Complex, Highly Nonlinear, Self-Regulating System

Requires fluctuation diagnostics to characterize, understand and ultimately help control turbulence

Origin of Zonal Flow Lies in 2D Dynamics

Navier-Stokes Eqn for *Rotating Fluid:*

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\bullet\nabla\right)\mathbf{v}+2\mathbf{\Omega}\times\mathbf{v}=-\frac{1}{\rho}\nabla p-\nabla\Phi+\nu\,\nabla^{2}\mathbf{v}$$

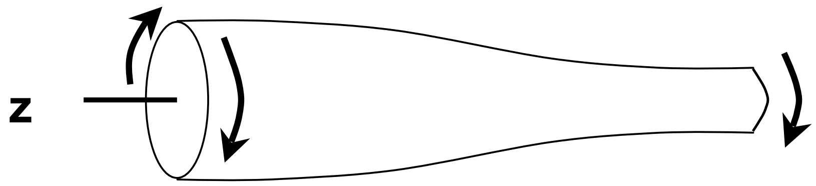
For $|\nabla\times\mathbf{v}|<<|\mathbf{\Omega}|\quad \nabla\cdot\mathbf{v}=0\quad \nu=0$

$$\boxed{\mathbf{\Omega}\cdot\nabla\mathbf{v}=0}$$

I.e. *no velocity gradient along direction of axis of rotation*

Flow Generation from Turbulence: the Vortex Merging Picture

3-D



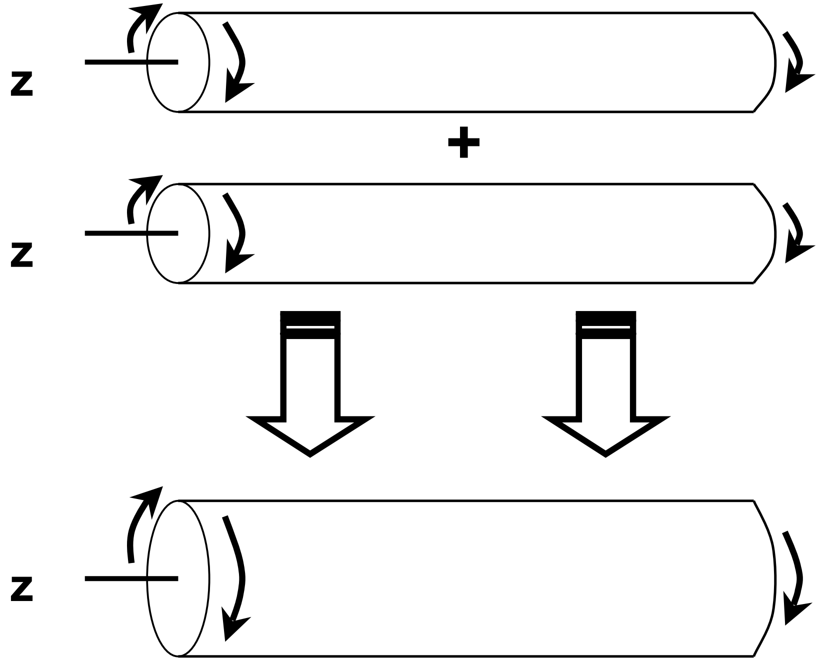
$$\frac{\partial v_z}{\partial z} \neq 0 \Rightarrow$$

==> Vortex Stretching

…Generate Flows on
Smaller Scales

2-D

$$\frac{\partial v_z}{\partial z} = 0 \Rightarrow$$ Vortex Merging



Flow Generation from Turbulence: Fourier Space

Usual Reynolds Stress Term in Simplified Momentum Eqn
(ala Diamond et al. PRL 1994 and others)

$$\frac{\partial \bar{u}_\theta}{\partial t} + \frac{\partial \langle \tilde{u}_r \tilde{u}_\theta \rangle}{\partial r} = -\nu_{damp} \bar{u}_\theta$$

"Radial Transport of Angular Momentum"

Consider a Zonal Flow to Have:

$$\mathbf{u} = u_\theta^Z \hat{\theta} \qquad \mathbf{k}_r^Z = k_r^Z \hat{\mathbf{r}}$$

$$|\mathbf{k}_r^Z| << |\mathbf{k}_1|, |\mathbf{k}_2|$$

$$\tau_Z \sim 1/k_r^Z u_\theta^Z >> t_{corr}$$

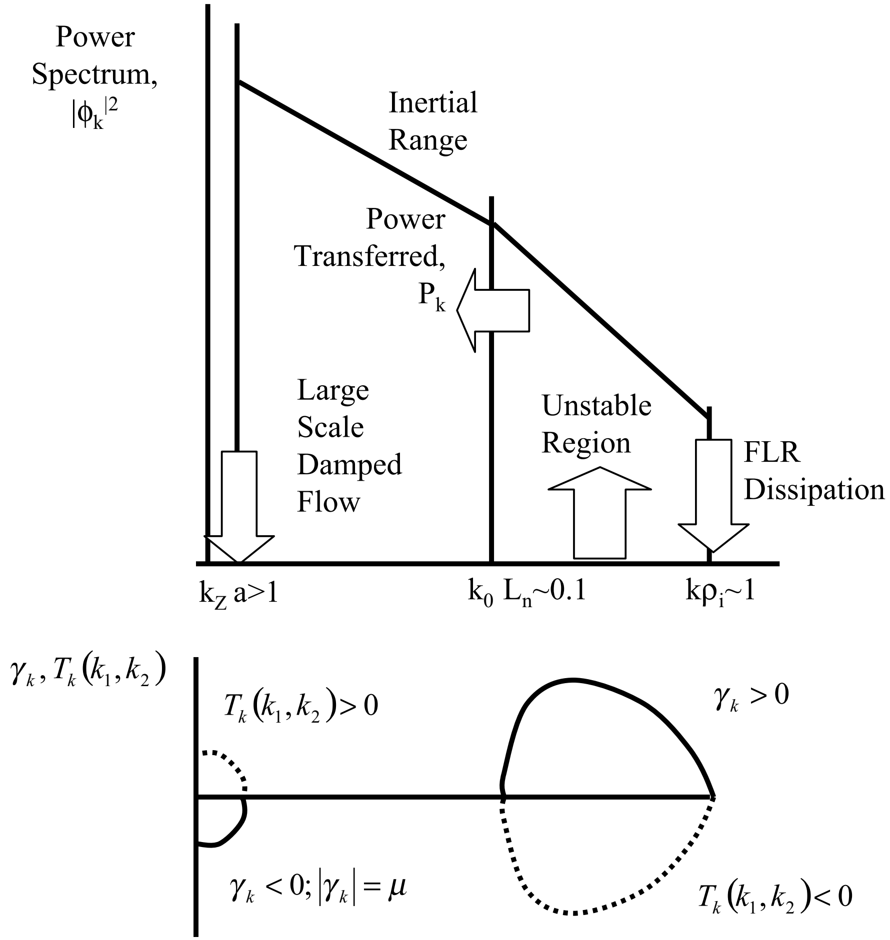
F.T., Write as KE, and Average Energy Eqn over Z-flow scales:

NL Energy Transfer

$$\frac{1}{2} \frac{\partial \langle u_{\theta_Z}{}^2 (\mathbf{k}_Z) \rangle}{\partial t} - P_{k_Z}^{turb} = -\mu \langle u_{\theta_Z}{}^2 (\mathbf{k}_Z) \rangle$$

where $$P_{k_Z}^{turb} = \sum_{\substack{\mathbf{k}_1 \mathbf{k}_2 \\ \mathbf{k}_Z = \mathbf{k}_1 \pm \mathbf{k}_2}} \boxed{\mathrm{Re} \langle u_{\theta_Z}{}^* (\mathbf{k}_Z) (\tilde{\mathbf{u}}(\mathbf{k}_1) \bullet \nabla) \tilde{u}_\theta (\mathbf{k}_2) \rangle}$$

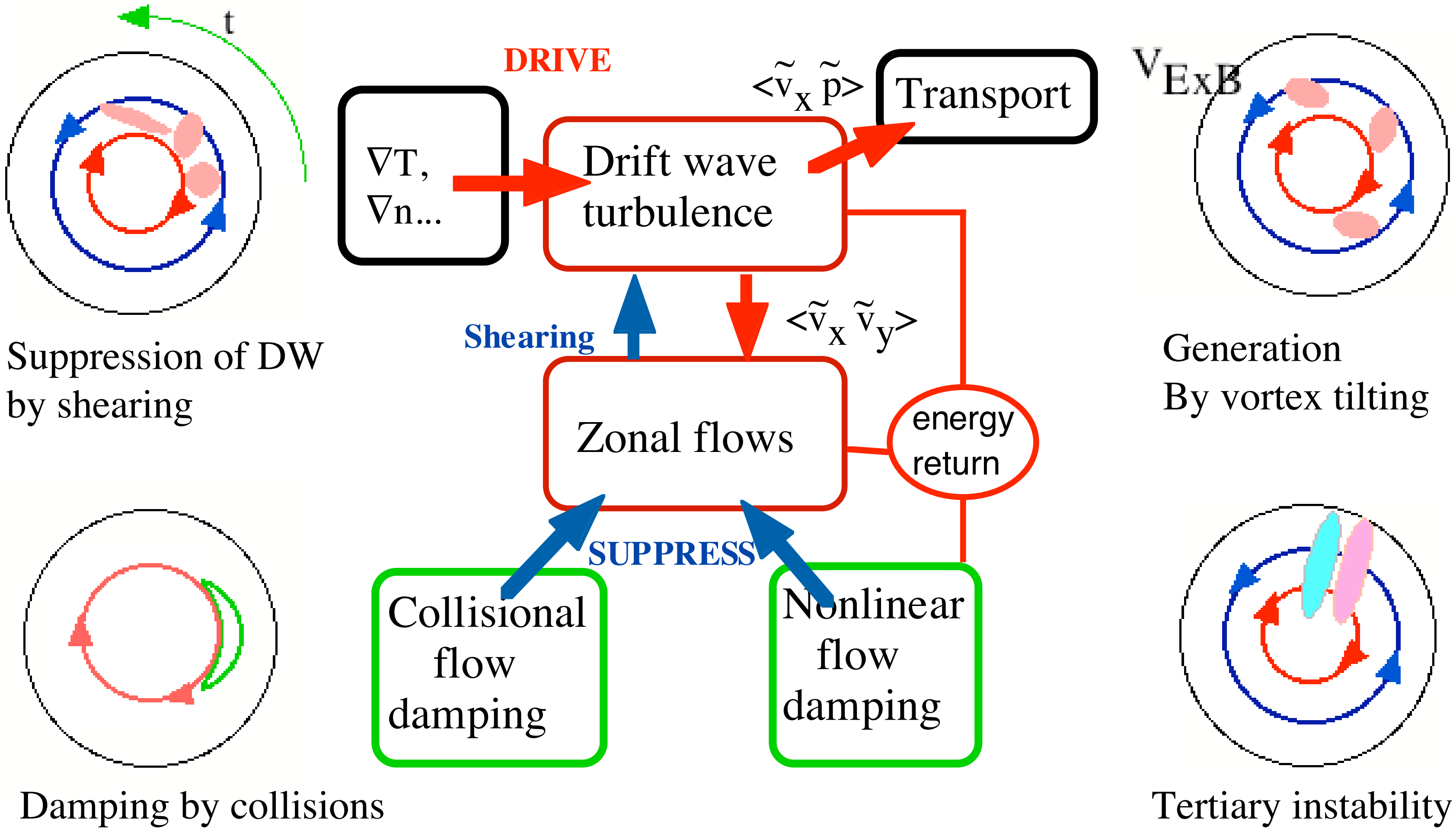
Flow Generation from Turbulence: Fourier Space



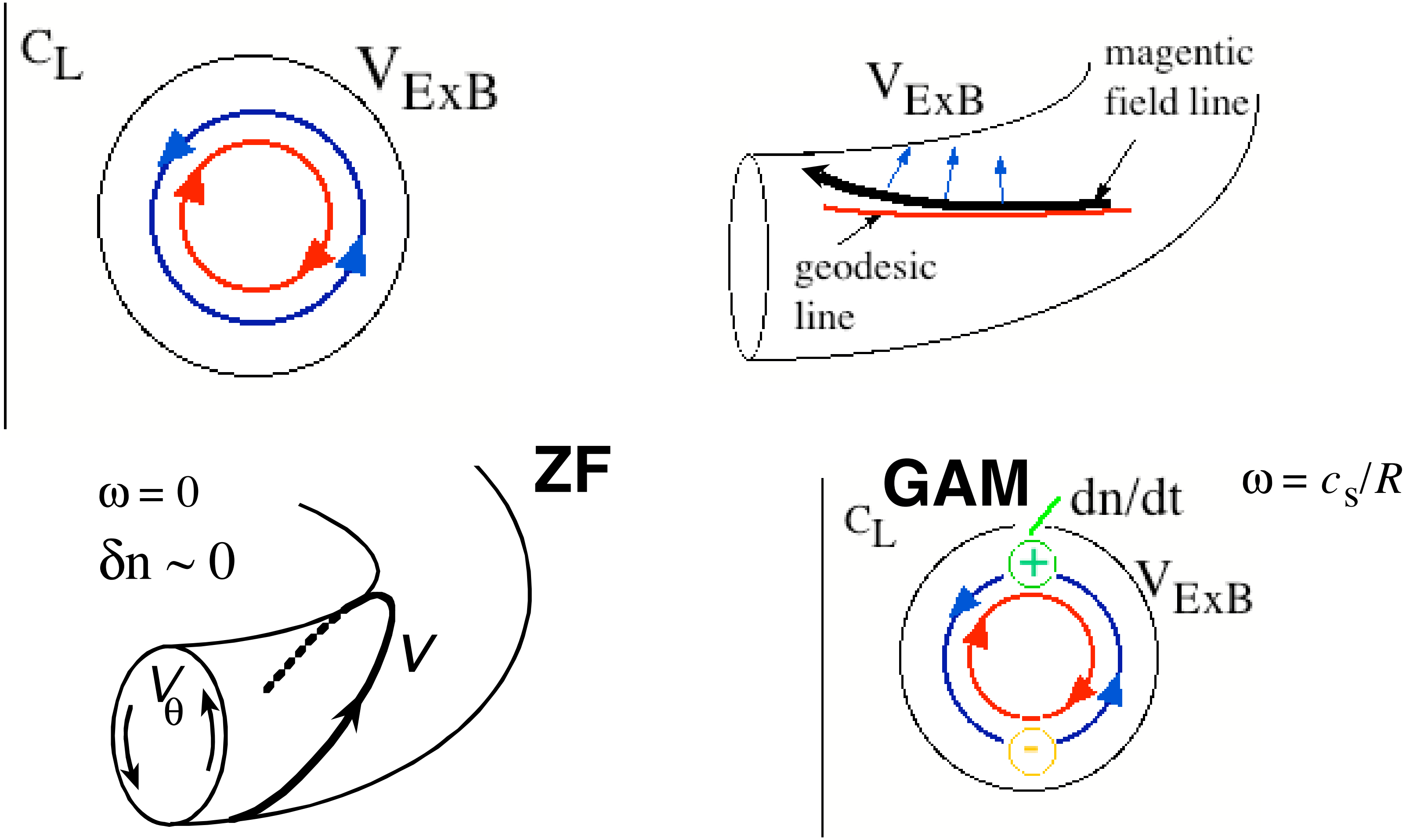
- Free Energy Source Releases Energy On One Scale
- Nonlinear Energy Transfer Moves Energy to Dissipation Region
- Shear Flows Develop Via Transfer of Energy to LARGE SCALES (small k)

Schematic of NONLINEAR drift turbulence-zonal Flow interactions

Ref: Itoh APS 2005



Large Scale Sheared Flows Can Develop



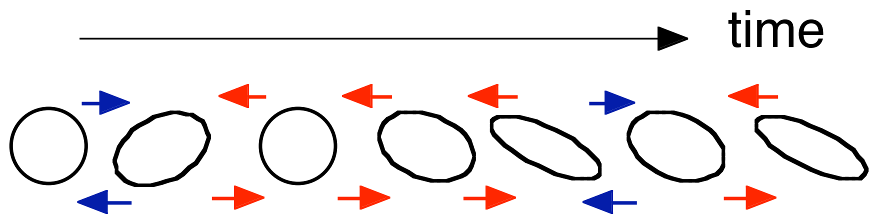
New feature: geodesic acoustic coupling (GAC)

B D Scott 2003

Impact of ZFs on Turbulence

Ref: Itoh APS 2005

Random stretching of DW eddies



$$\left\langle \delta k_r^2 \right\rangle \sim D_k t$$

$$D_k \sim k_\theta^2 \left\langle \tilde{V}_{ZF}'^{\,2} \right\rangle \tau_c$$

k_r^2 of DW packet ⬆

$$\omega_k = \frac{k_\theta V_d}{1 + k_\perp^2 \rho_s^2}$$

DW energy $W_k = \omega_k N_k$ ⬊

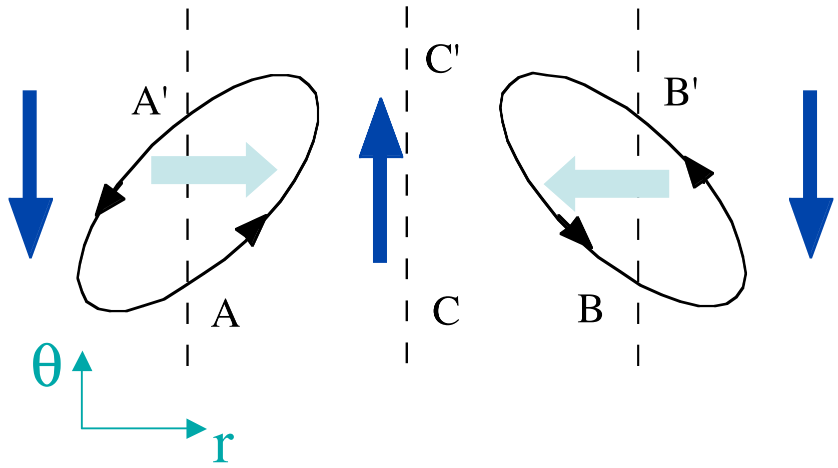
Energy for ZFs excitation is extracted from DWs

Note: Conservation energy between ZF and DW

Generation Mechanism

Ref: Itoh APS 2005

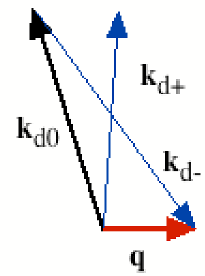
(1) Tilt of convection cell by a sheared flow



(2) Modulational Instability

$$\boxed{\phi_{ZF}} \;\; \boxed{\phi_d} \;\; \phi_+ \;\left(k_+ = k_x + q_r\right) \;\; \boxed{\phi_d} \;\; \boxed{\delta\phi_{ZF}} \propto \phi_d^2\,\phi_{ZF}$$

$$\phi_- \;\left(k_- = k_x - q_r\right)$$



2

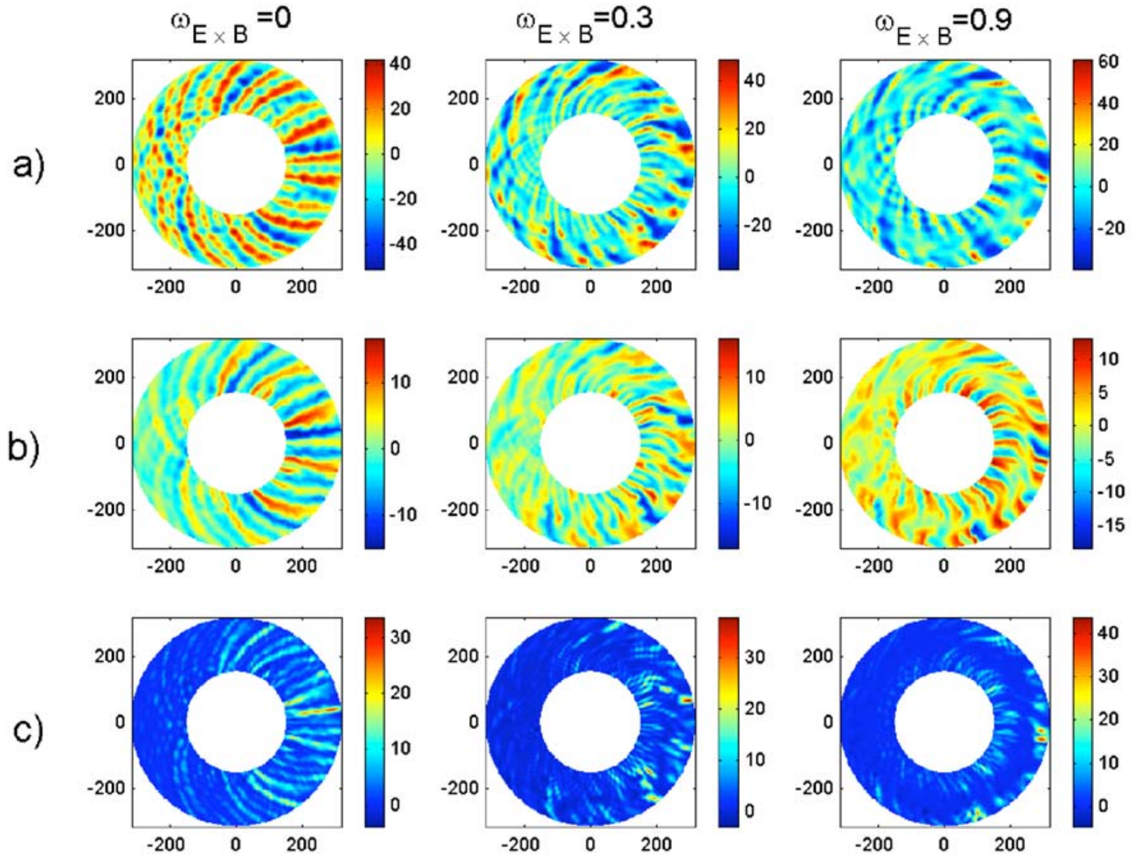
External shear flow breaks streamers

RBM *[Beyer et al, 00]*

Potential

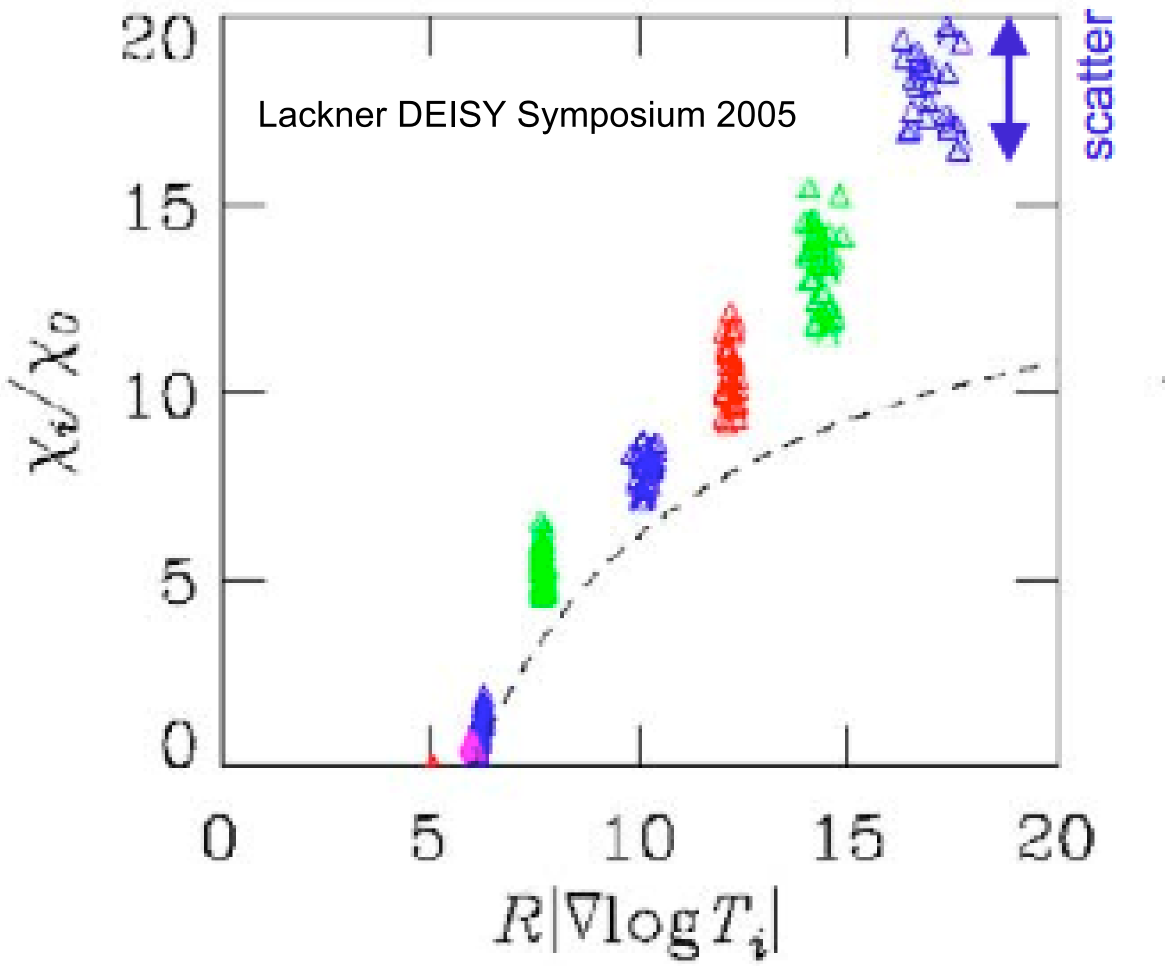
Pressure

Flux



Increasing velocity shear V_E'

Implication: Plasma Predicted to Sit At/Near Marginal Stability



Questions of Interest

- What Drives the Turbulence (I.e. Free Energy Source & Underlying Instability)?
- What are Spatio-temporal Scales of Turbulence?
- How Do Nonlinear Interactions Lead to Observed Spatio-temporal Scales?
- What does resulting flux v gradient curve look like ?

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Linear Signal Processing Gives Average Spatio-temporal Scales of TIME STATIONARY Turbulence

Fourier Transform:

$$f(t) = \sum_{\omega} f(\omega)\exp(i\omega t)\Delta\omega \qquad f(\omega) = \frac{1}{2\omega}\sum_{t} f(t)\exp(-i\omega t)\Delta t$$

Correlation Functions & Power Spectra:

$$C_{auto}(\tau) = \frac{1}{N_{ens}}\sum_{n=1}^{N_{ens}}\frac{1}{T}\int_{-T/2}^{T/2}\tilde{f}_i(t)\tilde{f}_i(t+\tau) \qquad P_{auto}(\omega) = \frac{1}{N_{ens}}\sum_{n=1}^{N_{ens}}\left|\tilde{f}_i(\omega)\right|^2$$

$$C_{crs}(\tau) = \frac{1}{N_{ens}}\sum_{n=1}^{N_{ens}}\frac{1}{T}\int_{-T/2}^{T/2}\tilde{f}_i(t)\tilde{g}_i(t+\tau) \qquad P_{crs}(\omega) = \frac{1}{N_{ens}}\sum_{n=1}^{N_{ens}}\tilde{f}_i(\omega)\tilde{g}_i^{*}(\omega)$$

Inter-relation Between Two Signals: coherence and crossphase

$$\gamma_{12}(\omega) = \frac{\left|P_{crs}(\omega)\right|}{\sqrt{P_1 P_2}}, \ \tan\phi(\omega) = \frac{\mathrm{Im}(P_{crs}(\omega))}{\mathrm{Re}(P_{crs}(\omega))}$$

Nonlinear Energy Transfer Comes from 3rd Order Spectrum:

Example: Collisional Drift Turbulence Model (Hasegawa-Wakatani 1983)

$$\frac{\partial W^n}{\partial t} = \frac{\Gamma_r}{L_n} + T^n + C_1\left(W^n - \left\langle n^* \phi \right\rangle\right)$$

$$\frac{\partial W^\phi}{\partial t} = T^\phi - C_2\left\langle \left|\nabla_\perp^2 \phi\right|^2 \right\rangle - C_3 W^\phi$$

Internal and Kinetic Energies Defined As

$$W^n = \sum_m \left|n_m(r,t)\right|^2$$

$$W^\phi = \sum_m \left|\nabla_\perp \phi_m(r,t)\right|^2$$

Nonlinear Energy Transfer From Cross-Bispectrum:

$$T_\theta^n(r,f,f') = -\mathrm{Re}\left\langle \tilde{n}^*(r,f)\tilde{v}_\theta(r,f-f')\frac{1}{r}\frac{\partial \tilde{n}}{\partial \theta}(r,f') \right\rangle$$

$$T_{ZF}^\phi(r,f,f') = -\mathrm{Re}\left\langle V_{ZF}{}^*(r,f)\tilde{v}_\theta(r,f-f')\frac{1}{r}\frac{\partial \tilde{v}_r}{\partial \theta}(r,f') \right\rangle$$

Rate and Direction of Energy Transfer Determined by BiSpectrum

Bi-spectrum Defined As

$$B(\omega_1,\omega_2)=\left\langle X(\omega_1)X(\omega_2)X^*(\omega_1+\omega_2)\right\rangle$$

Degree of Phase Coherence Determined by BiCoherence

$$\hat{b}^2(\omega_1,\omega_2)=\frac{B(\omega_1+\omega_2)}{\left\langle\left|X(\omega_1)X(\omega_2)\right|^2\right\rangle\left\langle\left|X^*(\omega_1+\omega_2)\right|^2\right\rangle}\quad;\quad 0<\hat{b}<1$$

BiPhase Determines Phase Delay Between Interacting Waves:

$$\Theta(\omega_1,\omega_2)\equiv Tan^{-1}\left\{\frac{\mathrm{Im}\left[B(\omega_1,\omega_2)\right]}{\mathrm{Re}\left[B(\omega_1,\omega_2)\right]}\right\}\quad;\quad -\pi<\Theta<\pi$$

Energy Transfer Direction and Rate:

$$\boxed{\mathrm{Re}\left[B(\omega_1,\omega_2)\right]=\left|B(\omega_1,\omega_2)\right|\cos\left[\Theta(\omega_1,\omega_2)\right]}$$

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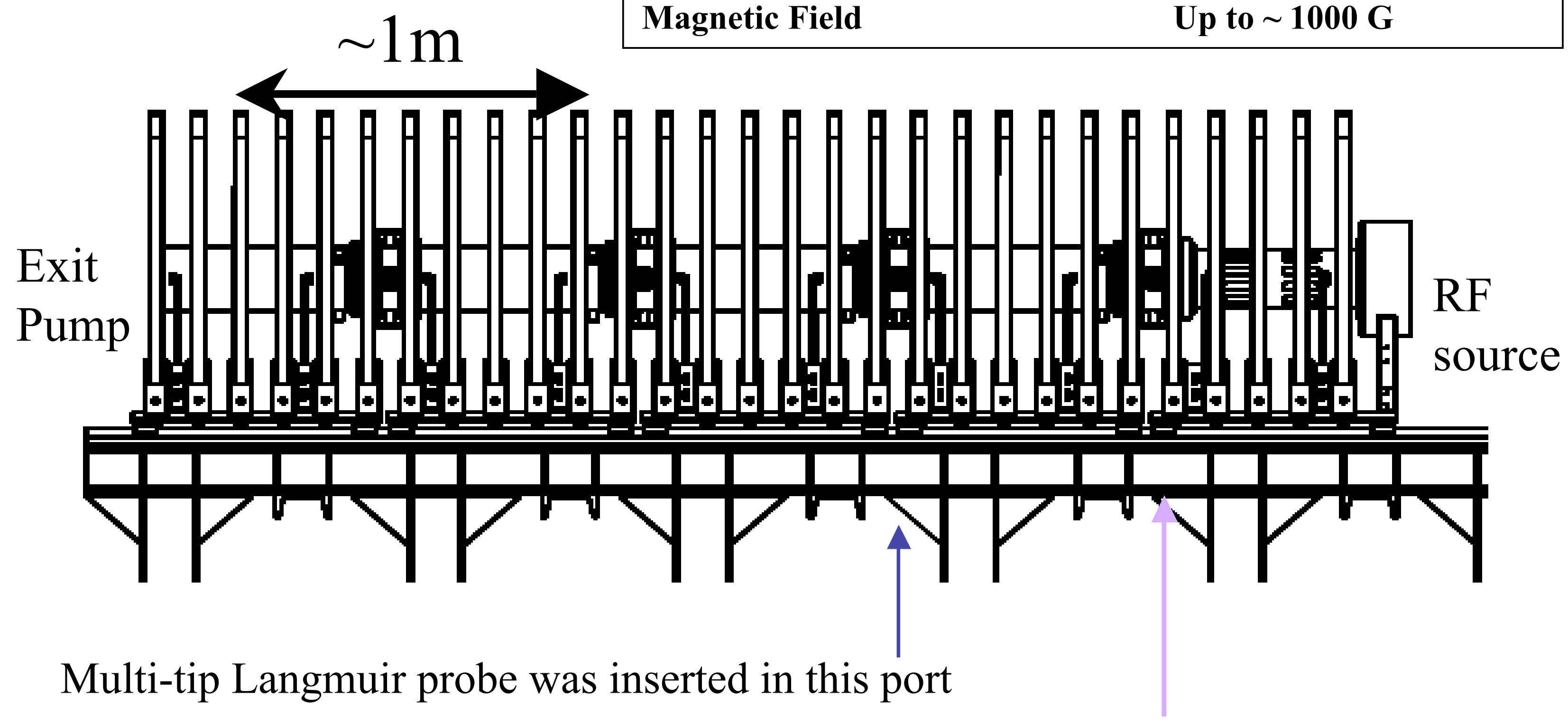
Controlled **S**hear **D**e-Correlation **Ex**periment (**CSDX**)

CSDX parameter	**Typical value**
Gas pressure	**0.5 - few mTorr**
T_e	**~ 3 eV**
T_i	**~ 0.7 eV**
n_e	**1-10 x 10^{12} cm^{-3}**
Source (Helicon 13.56 MHz)	**1500W (typically)**
Magnetic Field	**Up to ~ 1000 G**

m=0 Helicon Plasma Source



Center for Energy Research

G.R. Tynan, Int'l Workshop on Frontiers in Plasma Physics, July 2008, ICTP Triest, Italy

UCSD Jacobs | Mechanical and Aerospace Engineering

Dimensionless Scales in CSDX

Length Scales

$$\rho_i / L_n \sim 0.1$$

$$\rho_S / L_n \sim 0.5$$

$$\lambda_e^{mfp} / L_{\|} \sim 0.05 - 0.1$$

Collision Rates

$$\nu_e / \Omega_{C_e} \sim 0.005$$

$$\nu_{ii} / \Omega_{Ci} \sim 1$$

$$\nu_{i0} / \Omega_{Ci} \sim 0.01$$

Parallel Dissipation Rate

$$\frac{k_{\|}^2 C_s^2}{\nu_e \Omega_{Ci}} \sim 1$$

Non-ambipolar G.C. Drifts

Polarization Drift

$$\omega / \Omega_{C_i} \sim 0.1 - 0.3$$

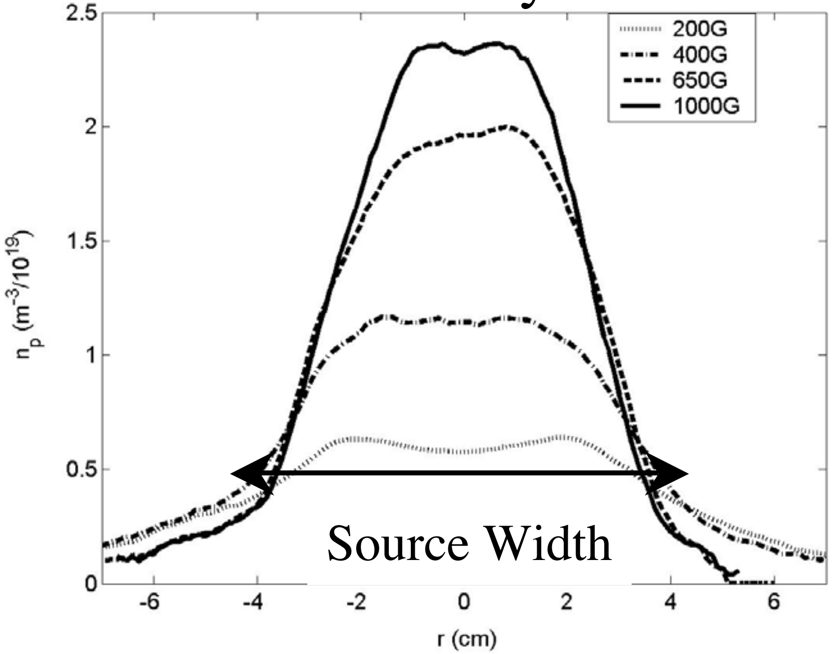
Ion-Ion Collisional Drift

$$\frac{\mu_{ii}}{L_{\perp}^2 \Omega_{ci}} \approx 0.01 - 0.1$$

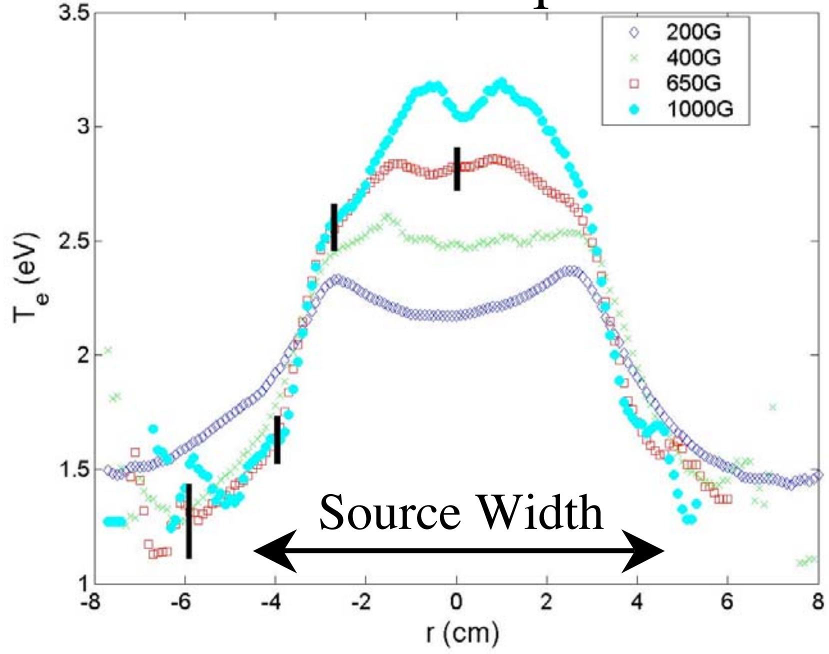
DEVELOPMENT OF DRIFT TURBULENCE FROM LINEAR DRIFT WAVES

Equilibrium Profiles Evolve as B Field Increases in CSDX Helicon Plasma

Density

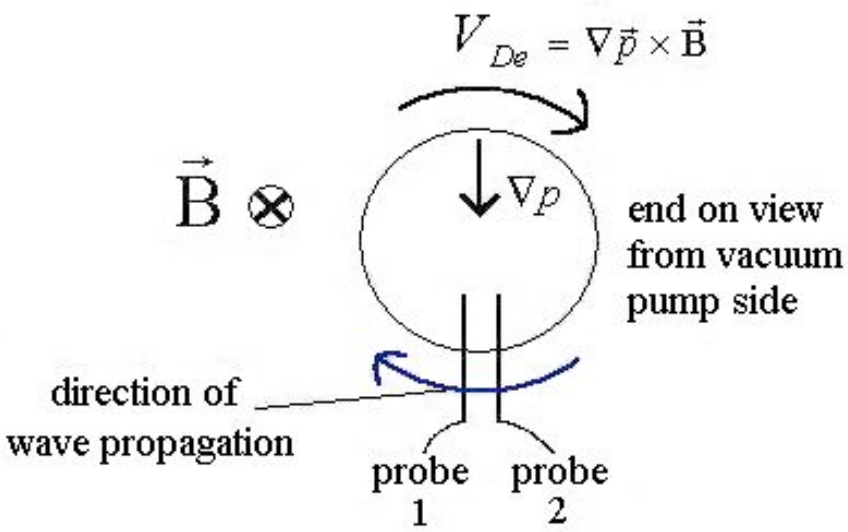


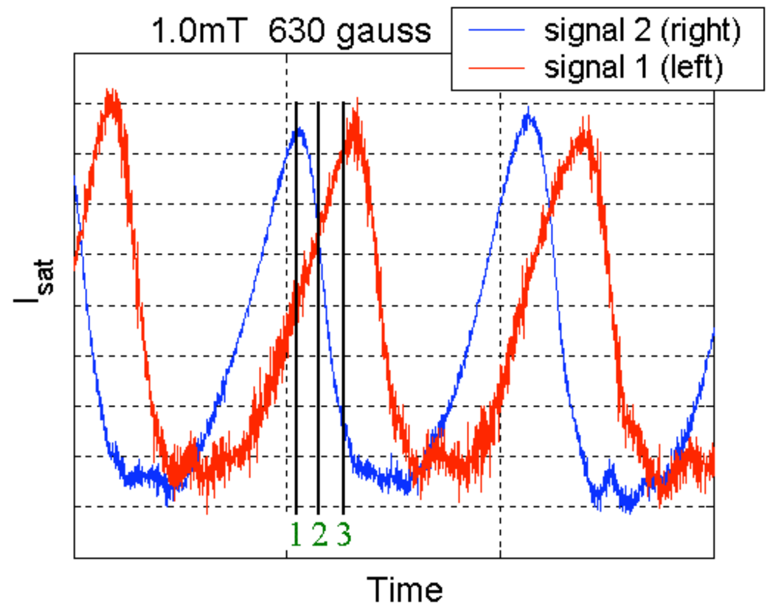
Electron Temperature

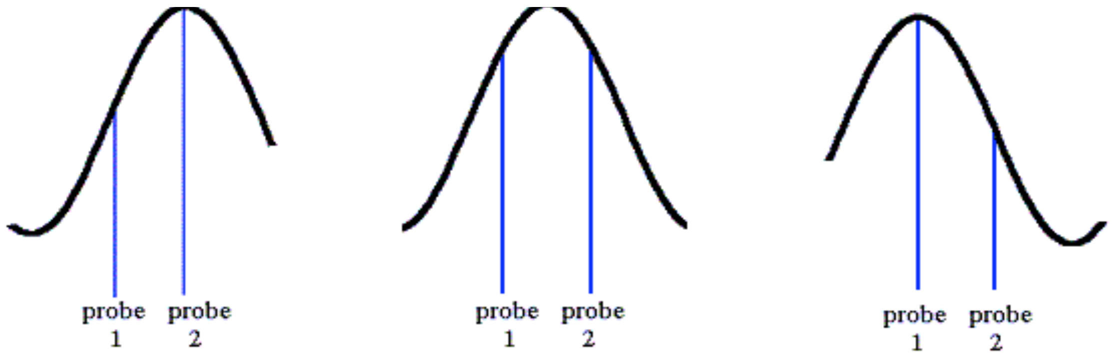


Burin et al, May 2005 PoP

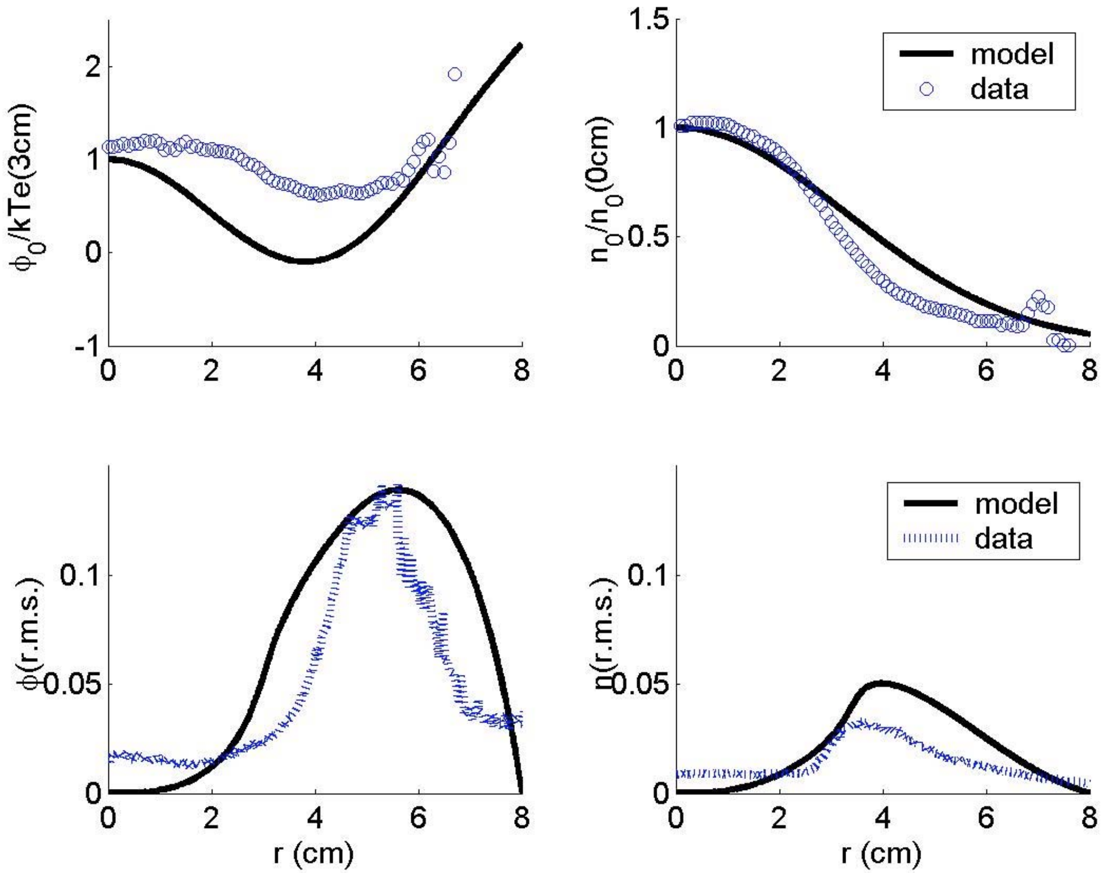
Wave at Onset Consistent w/ Collisional Drift Wave





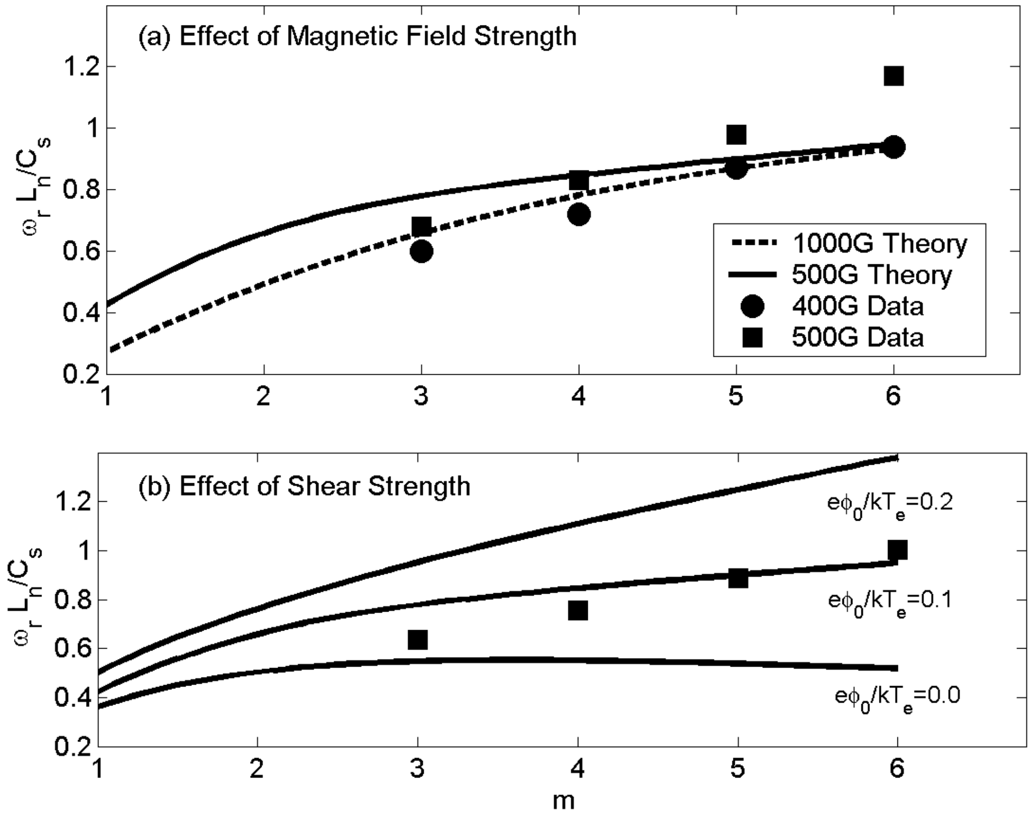


Linear Eigenmodes of DW w/ Flow Shear Match Observations Near Onset



Burin et al, May 2005 PoP

Drift Wave Dispersion Agrees with Linear Theory



Burin Phys. Plasmas 2005

EVIDENCE FOR INVERSE ENERGY TRANSFER DURING DEVELOPMENT OF TURBULENCE

Development of Wave-Wave Interactions

Keep Convective Derivative (Nonlinear(!)) in Momentum Equation, e.g. θ component:

$$\frac{\partial \tilde{u}_\theta}{\partial t} + \frac{\partial \tilde{u}_r \tilde{u}_\theta}{\partial r} = -\nu_{damp} \tilde{u}_\theta$$

F.T., Write as KE, and Average Energy Eqn for wavenumber k:

$$\frac{1}{2}\frac{\partial \left\langle u_\theta{}^2(\mathbf{k})\right\rangle}{\partial t} - P_k^{turb} = -\mu \left\langle u_\theta{}^2(\mathbf{k})\right\rangle$$

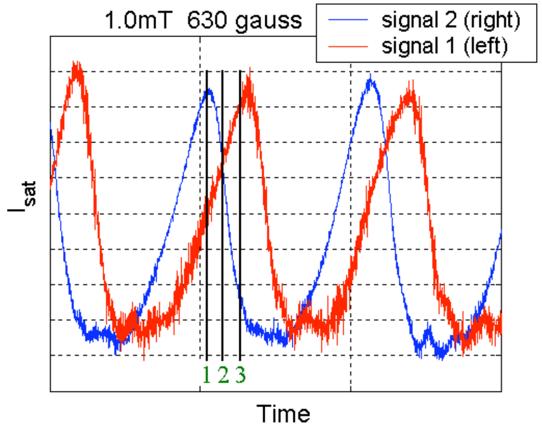
where $$P_{k_Z}^{turb} = \sum_{\substack{\mathbf{k}_1\mathbf{k}_2 \\ \mathbf{k}=\mathbf{k}_1+\mathbf{k}_2}} \boxed{\mathrm{Re}\left\langle u_\theta{}^*(\mathbf{k})\left(\tilde{\mathbf{u}}(\mathbf{k}_1)\bullet\nabla\right)\tilde{u}_\theta(\mathbf{k}_2)\right\rangle}$$

<u>Represents NL Energy Transfer due to Wave-Wave Interaction</u>

Local k-spectra Are Constructed from 2-Point Measurements

Ref: J.M. Beall et al, J. App. Phys. 1982

Measure Fluctuations at 2 Points:

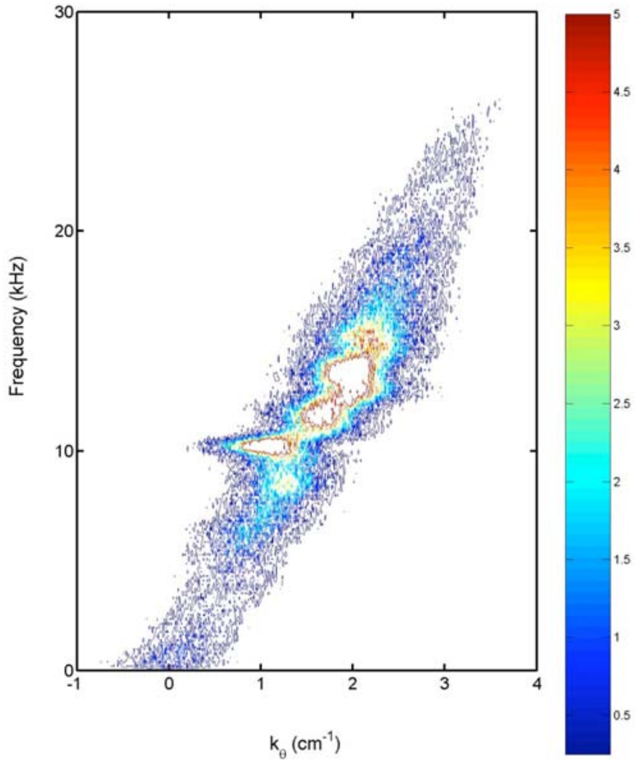


Find Phase Delay due to Propagation

$$\delta\vartheta = Tan^{-1}\frac{\mathrm{Im}(\langle X_1(f)X_2^*(f)\rangle)}{\mathrm{Re}(\langle X_1(f)X_2^*(f)\rangle)}$$

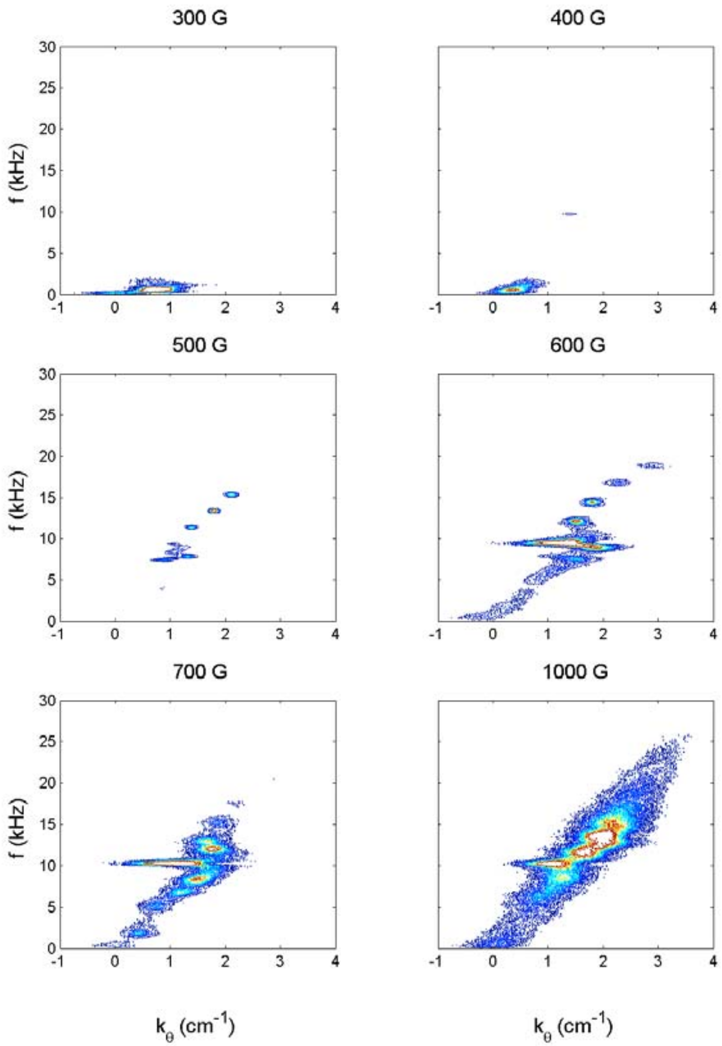
Local Wavenumber $k_{\theta_{local}} = \dfrac{\delta\vartheta}{\delta x}$

Build-up Local k-spectra from Multiple Realizations:



Burin et al, May 2005 PoP

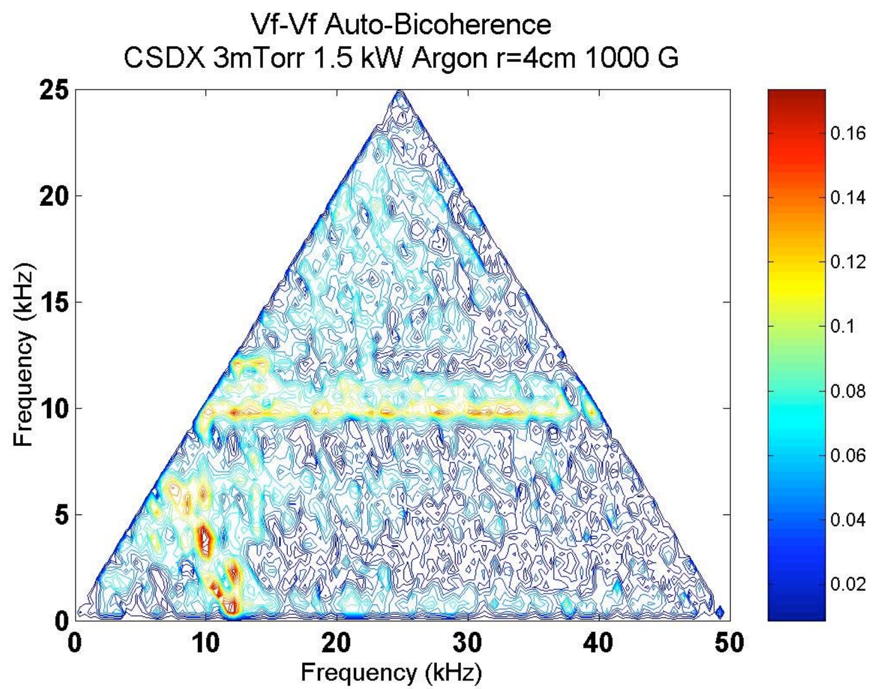
Evolution of $(\phi_f / kT_e)^2$ Power Spectrum with Increasing Magnetic Field



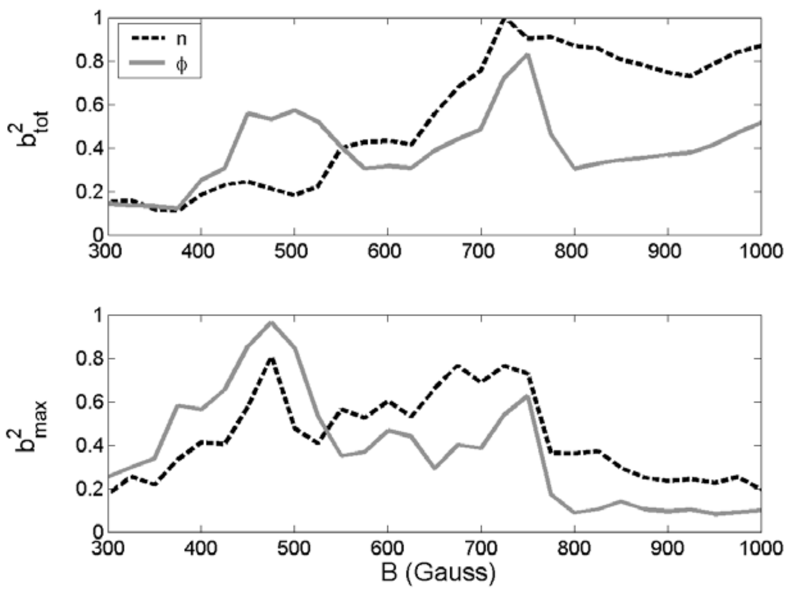
- Coherent Drift Waves Appear at ~400G
- Harmonics Develop As B Increases
- Coherent Modes at Intermediate B
- Broadband Spectra at 1kG

Burin et al, May 2005 PoP

Intensity of 3-wave Interactions Increases as Turbulence Develops



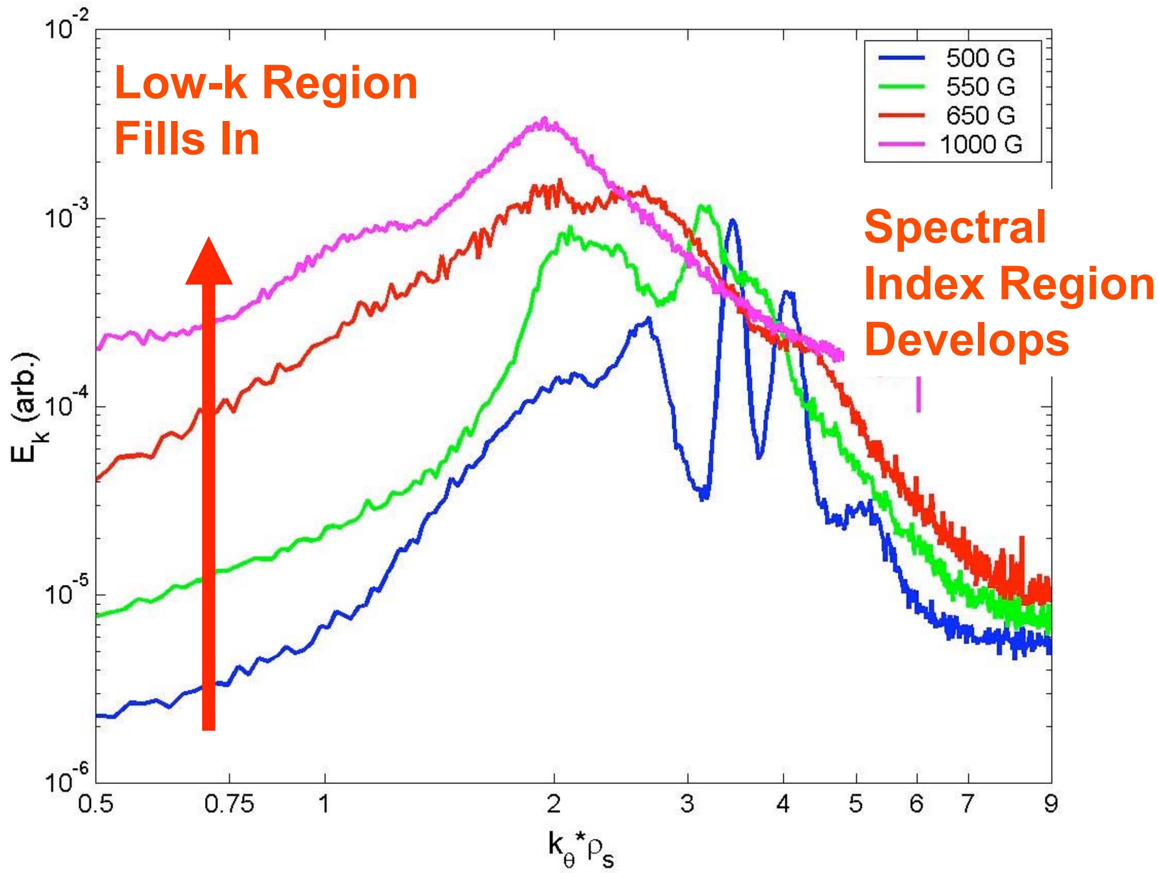
Tynan, PoP 2004



Burin, PoP 2005

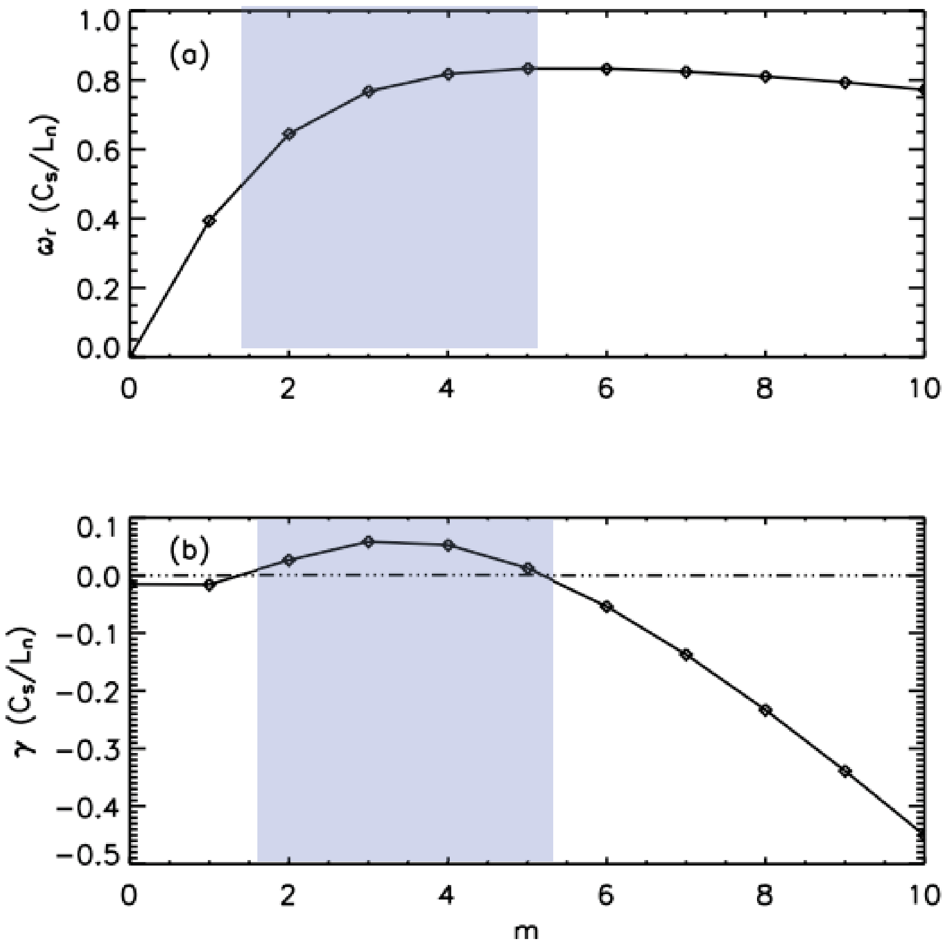
Evolution of Energy Spectrum w/ Magnetic Field

$$E_{k_\theta} = \left(\frac{n_{k_\theta}}{n}\right)^2 + k_\theta^2\left(\frac{e\phi_{k_\theta}}{k_B T_e}\right)^2$$



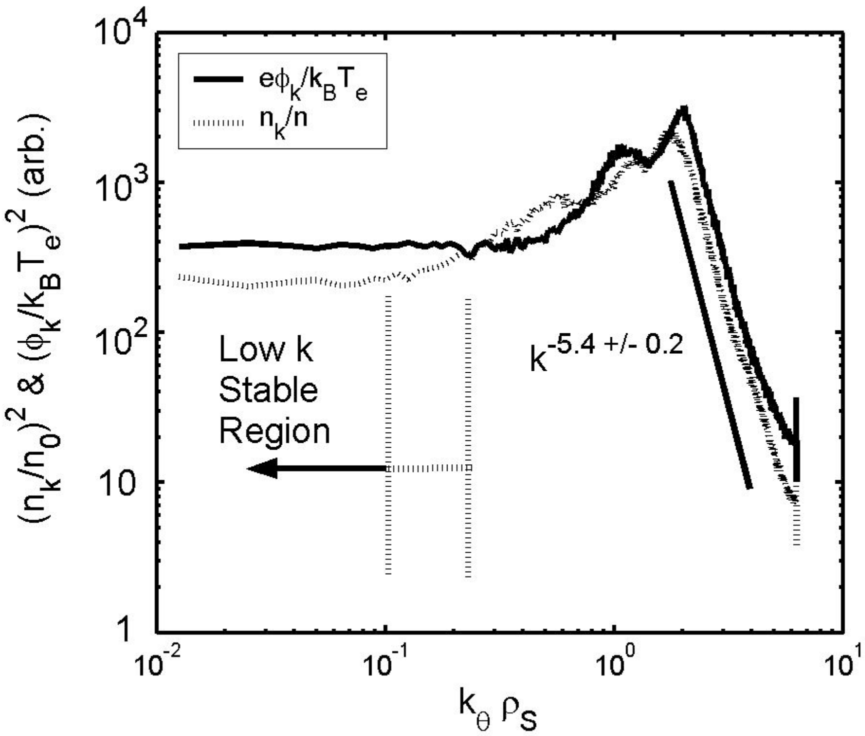
Burin et al, May 2005 PoP

Real and Imaginary Frequencies of Linear Eigenmodes



Free Energy Sources Are Known ➔ Can Find Linearly Unstable Region

- Include grad-P, Vshear Free Energy Sources
- Include Neutral Flow Drag (Effective at high k), FLR Damping
- Find Stable & Unstable Regions



Local Wavenumber

Implies Energy MUST Be Transferred Into Low-k Region Via Nonlinear Processes

Tynan et al Nov 2004 PoP

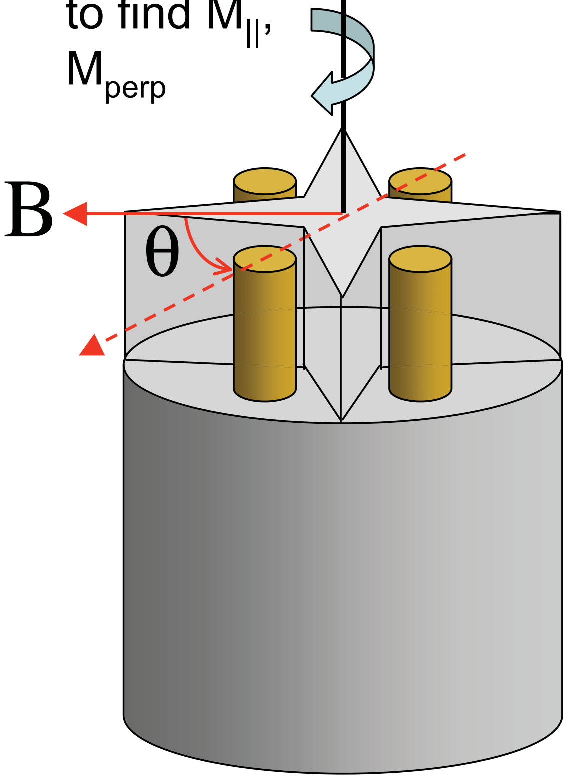
Conclusions from Laboratory Plasma Experiment

- Drift Waves Develop if Grad-P is High Enough and Damping Small Enough
- Coherent Waves Obey Linear Dispersion Relation
- Weakly Dispersive Waves Allow NONLINEAR 3-Wave Coupling to Begin
- Nonlinear Energy Transfer Re-arranges Spectrum
- Broad Spectrum Emerges & Shows Indicates of INVERSE ENERGY TRANSFER…
- PART II… EMERGENCE OF ORDERED FLOWS OUT OF TURBULENCE, CONFINEMENT DEVICE RESULTS, BIFURCATIONS,…

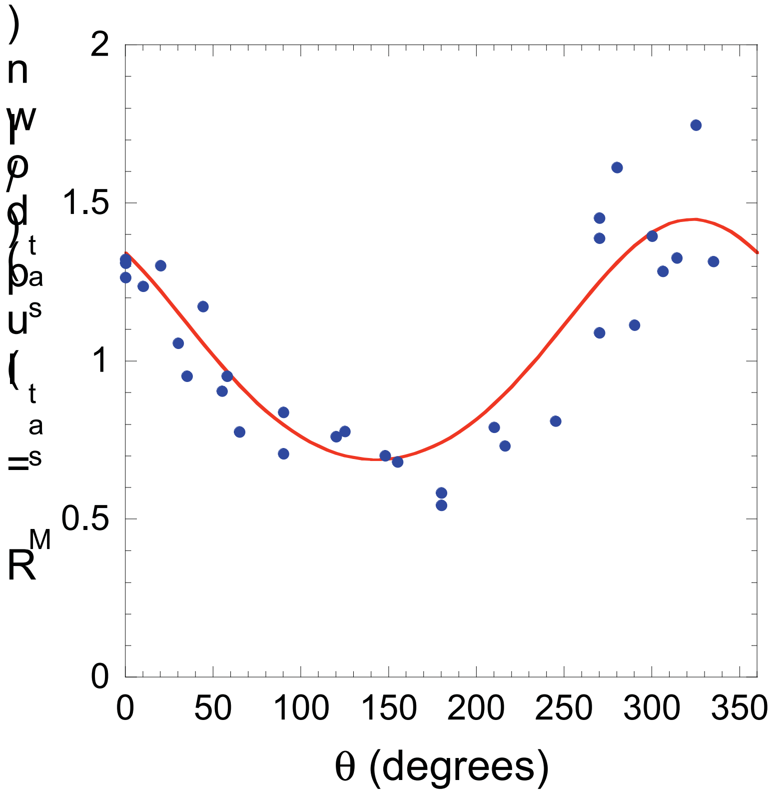
EVIDENCE FOR EXISTENCE OF SHEAR LAYER SUSTAINED BY TURBULENT REYNOLDS STRESS

Mach Probe Measures V_θ, Vz Profiles

Rotate about probe axis to find $M_{||}$, M_{perp}

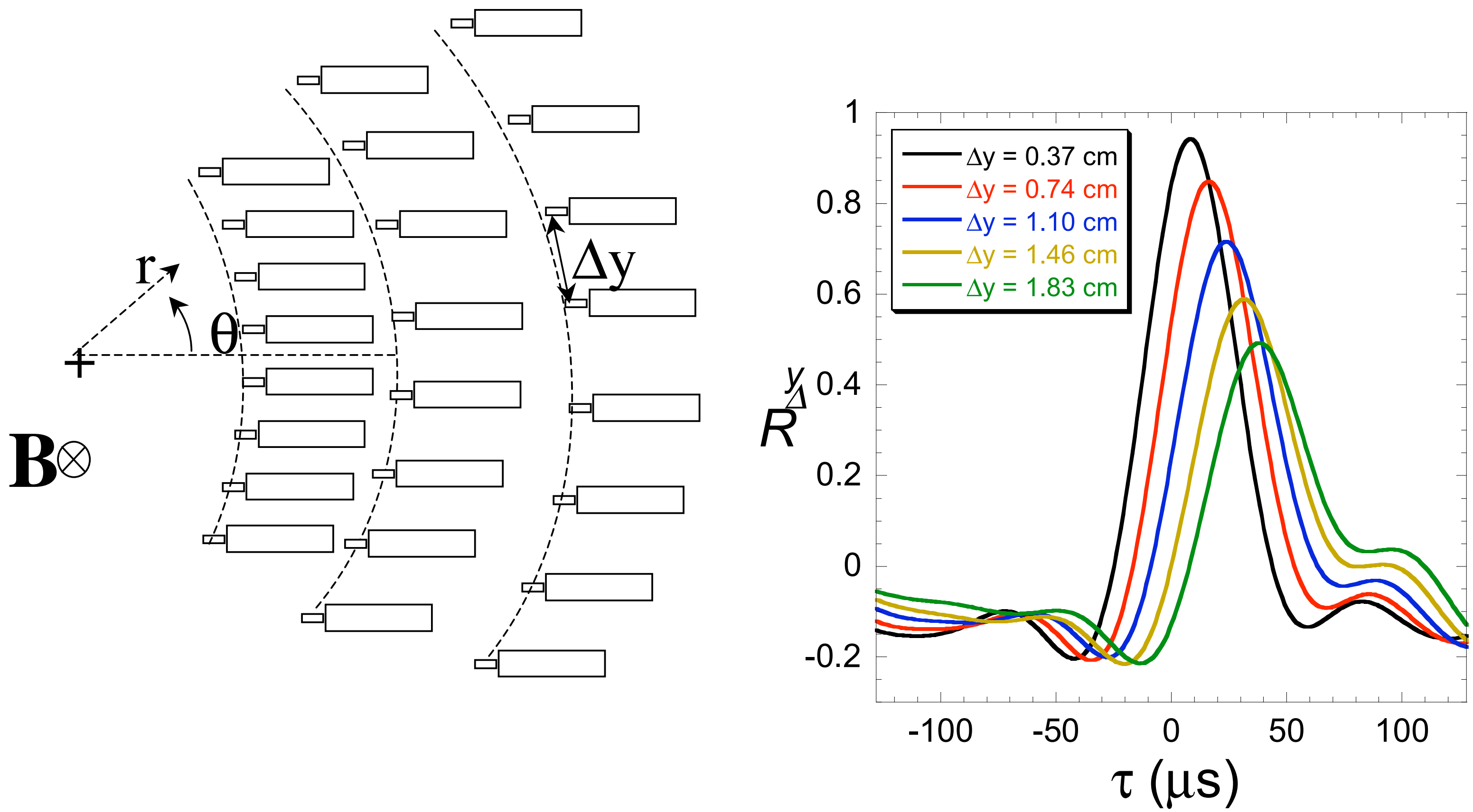


R_M = exp[K sinΔα/Δα (M_{par} cosθ + M_{perp} sinθ)]



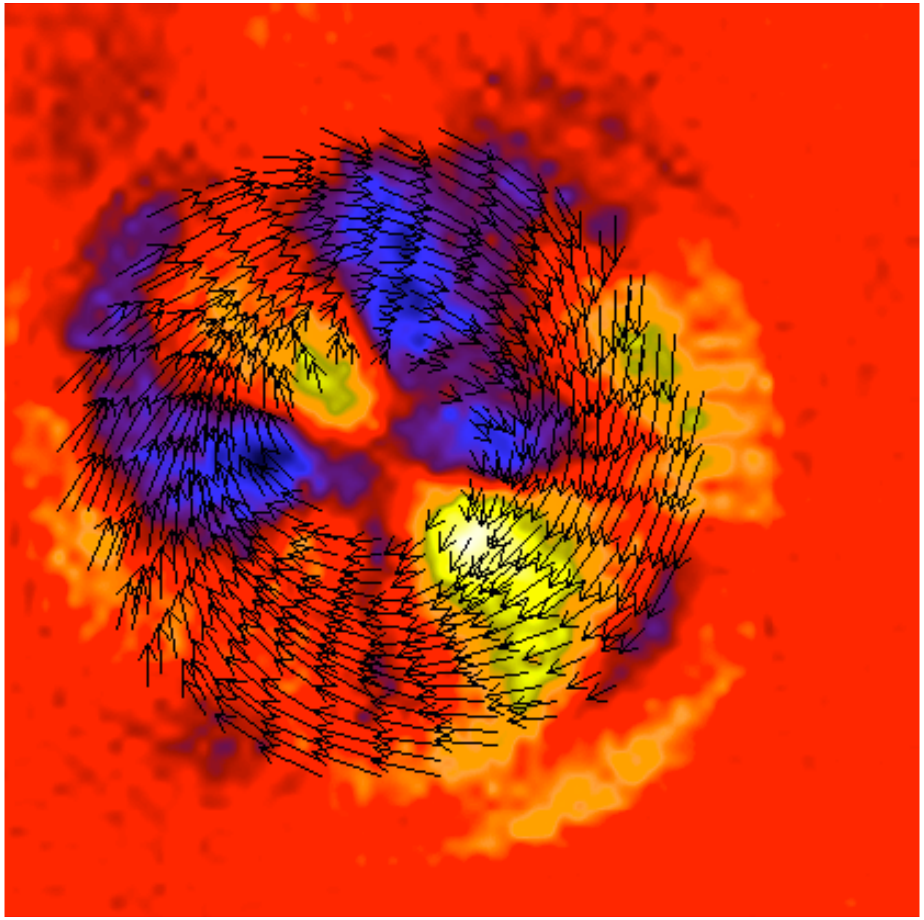
Shikama *et al*., Phys. Plasmas (2005),
Van Goubergen *et al*., PPCF 41 L17 (1999),
Gunn *et al*., Phys. Plasmas (2001)

Observe Fluctuation Propagation Speed w/ Multipoint Probe Array or Fast Imaging

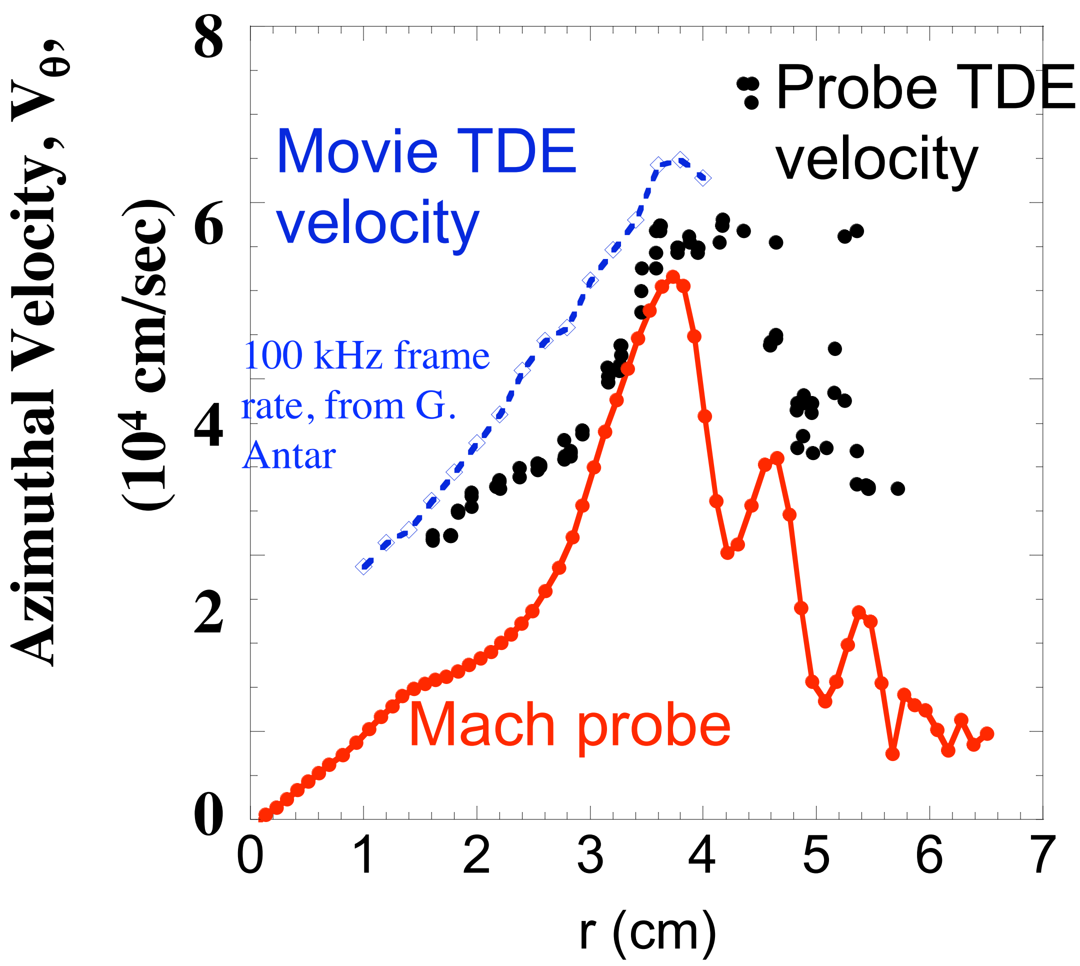


Direct Imaging of Turbulence Density Fluctuations and Shear Flow

We Can Also Infer Flowfield from Motion of Density Perturbations



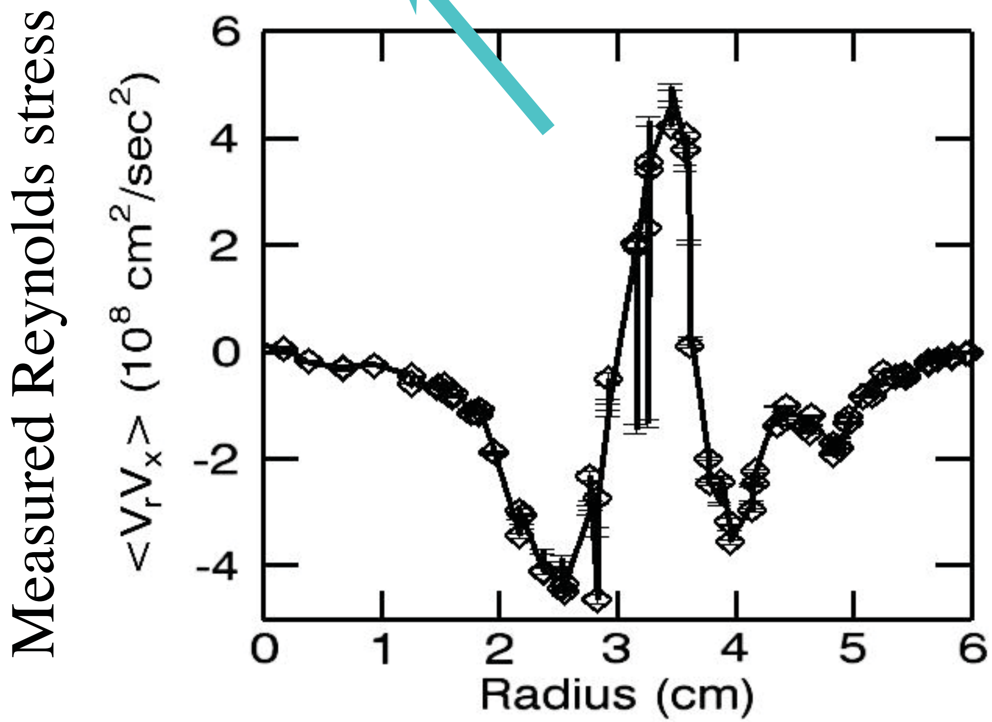
Independent Measurements of Shear Layer



Sound speed
$c_s = (T_e/M_i)^{1/2}$
$= 2.8 \times 10^5$ cm/s

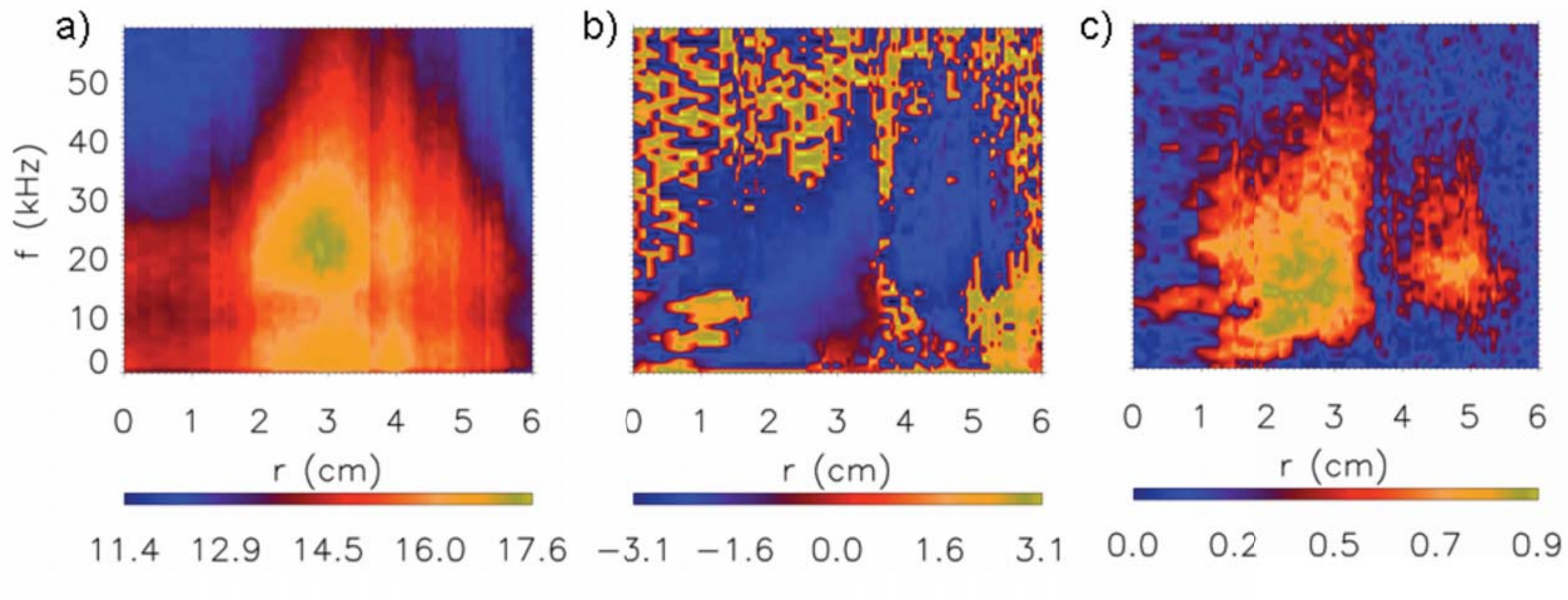
Use Measured Reynolds Stress in Azimuthal Momentum Balance & Solve for V Profile

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left\langle \tilde{v}_r \tilde{v}_\theta \right\rangle\right) = -\nu_{io}\overline{V}_{i\theta} + \mu_{ii}\nabla^2\overline{V}_{i\theta}$$



Tynan et al April 2006 PPCF Holland et al, in press, PRL

Spectral properties of Reynolds stress



(a) Absolute magnitude of the power spectrum of the turbulent Reynolds stress. Color scale is logarithmic (base 10)

(b) Cross-phase $\alpha_{\delta v_r, \delta v_\theta}$ between radial and azimuthal turbulent velocity fields

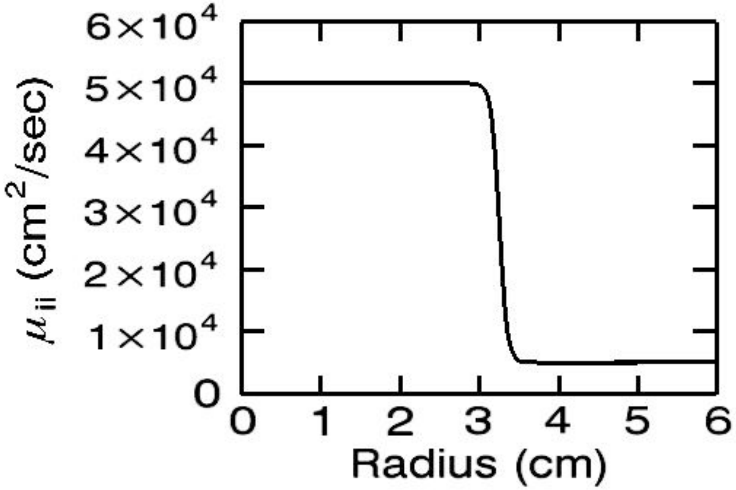
(c) Squared cross-coherence $\gamma^2_{\delta v_r, \delta v_\theta}$ between radial and azimuthal turbulent velocity fields .

Estimate Dissipation from Measurements

Measure:

$$\int_{-a}^{a} T_i dl = 0.7eV$$

$$\int_{-a}^{a} T_{gas} dl = 0.4eV$$



Assume:

$$T_i(0) > T_i(a)$$

$$T_{gas}(a) = T_{wall}$$

$$\mu_{ii} = \frac{3}{10}\rho_i^2 \nu_{ii}$$

$$P_{gas} = n_{gas} T_{gas} = const$$

$$\mu_{ii} \propto n_i T_i^{1/2}$$

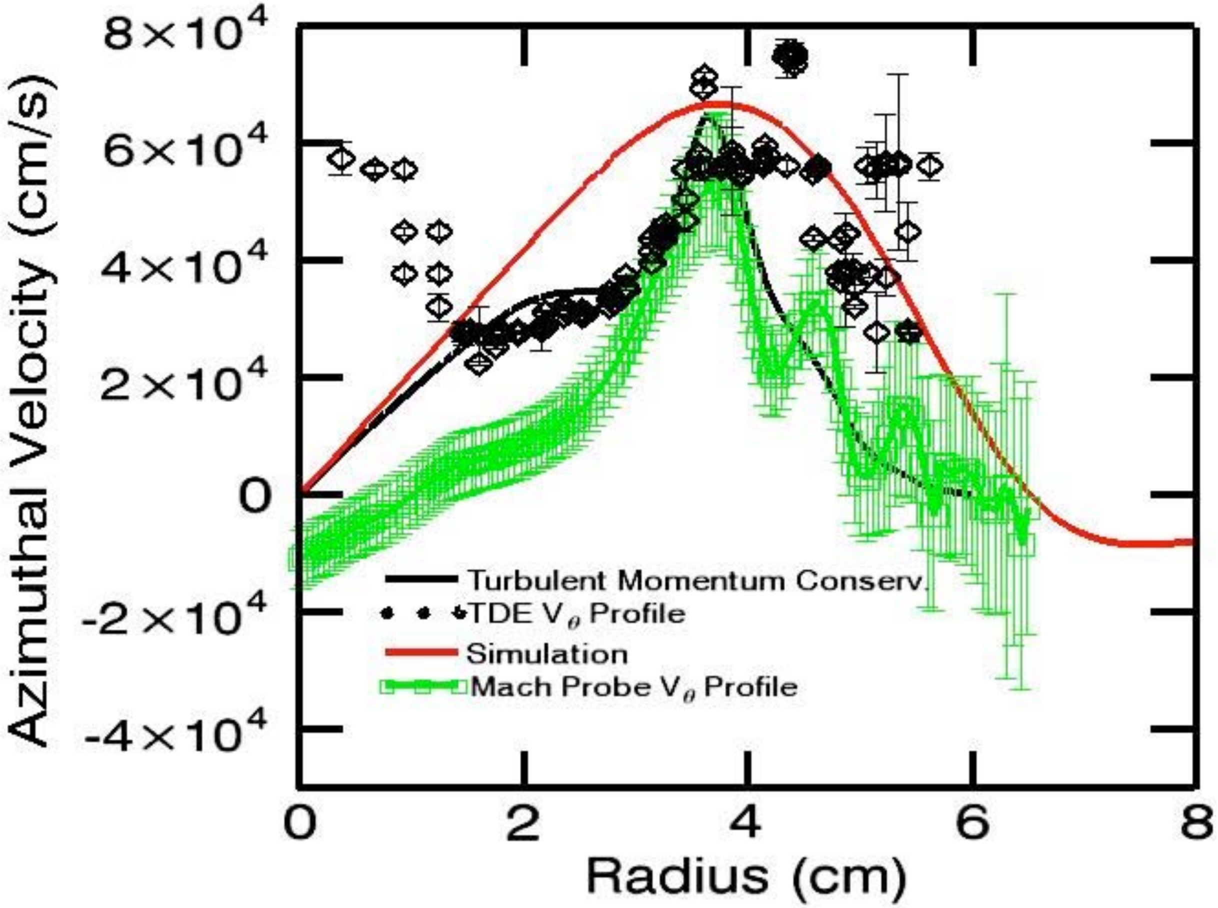
$$\bar{\mu}_{ii} \approx 4 \times 10^4 \, cm^2 / \sec$$

$$\mu_{ii}(0) > \mu_{ii}(a)$$

$$\nu_{i0} \sim 6 \times 10^3 \sec^{-1}$$

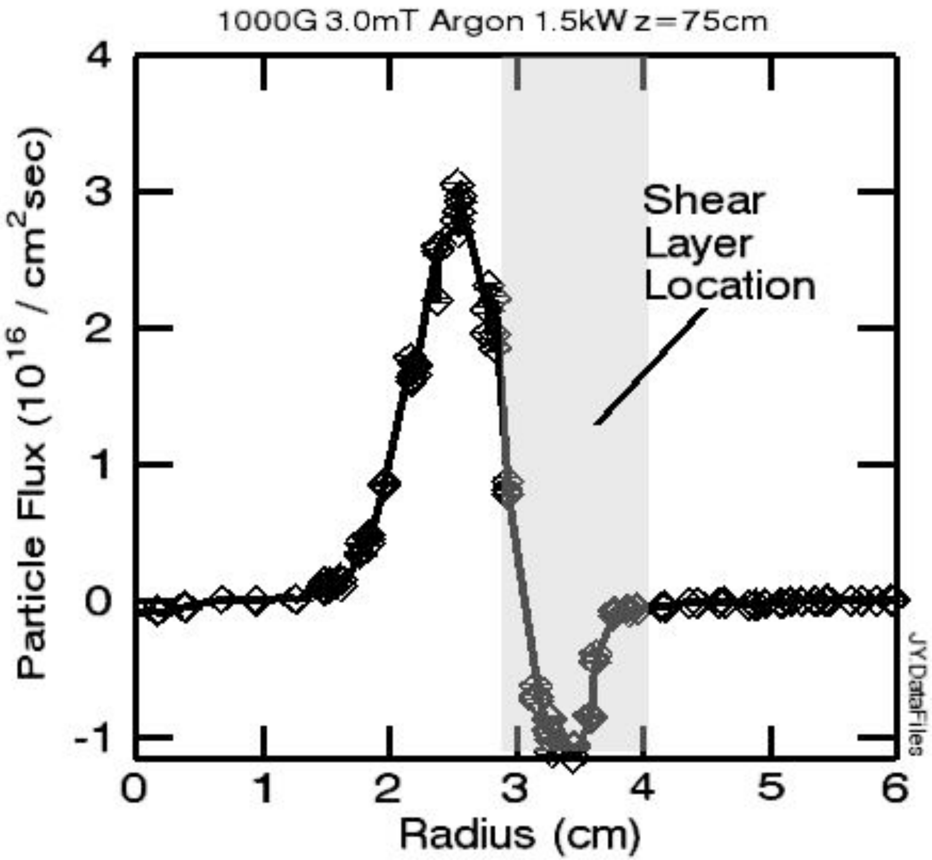
Tynan et al, April 2006 PPCF, Holland et al, in press, PRL

Measured Velocity Profile Consistent with Turbulent Momentum Balance



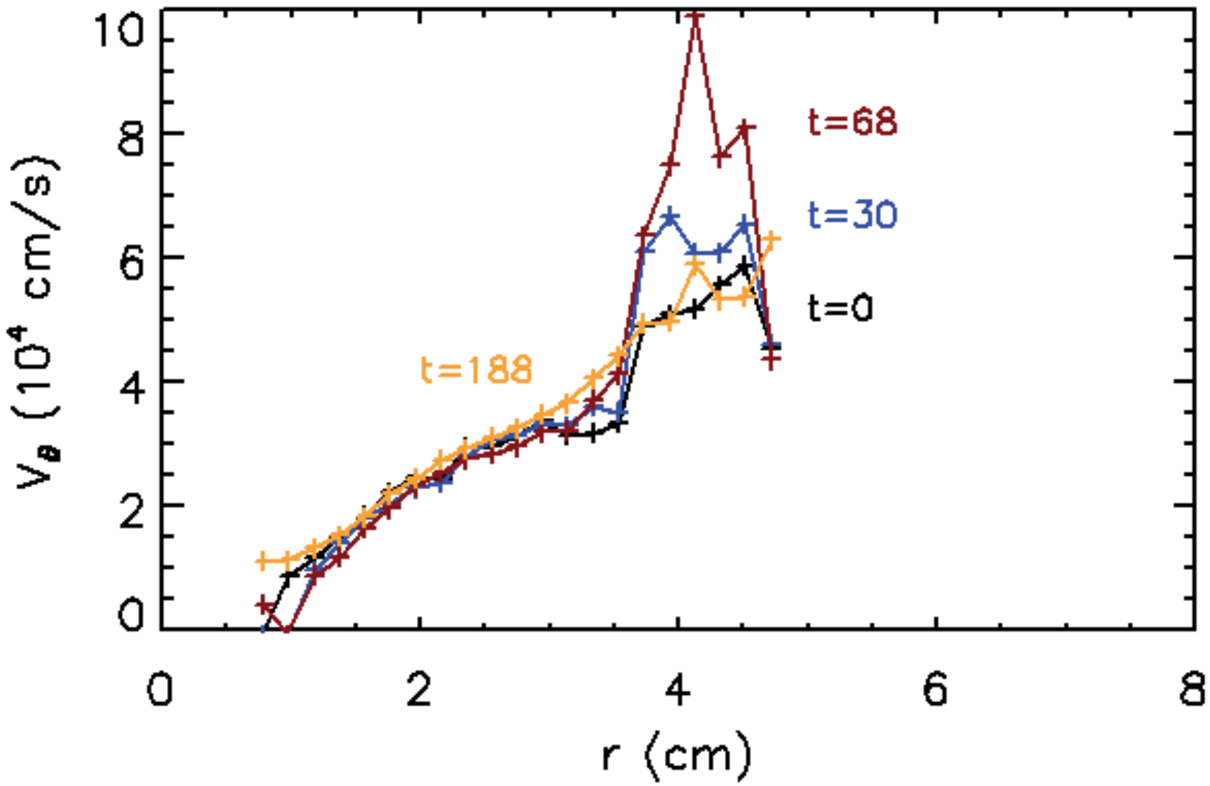
Tynan et al, April 2006 PPCF, , Holland et al, PRL 2006

No Turbulent Particle Transport Across Shear Layer



Tynan et al, April 2006 PPCF, Holland et al, In press, PRL

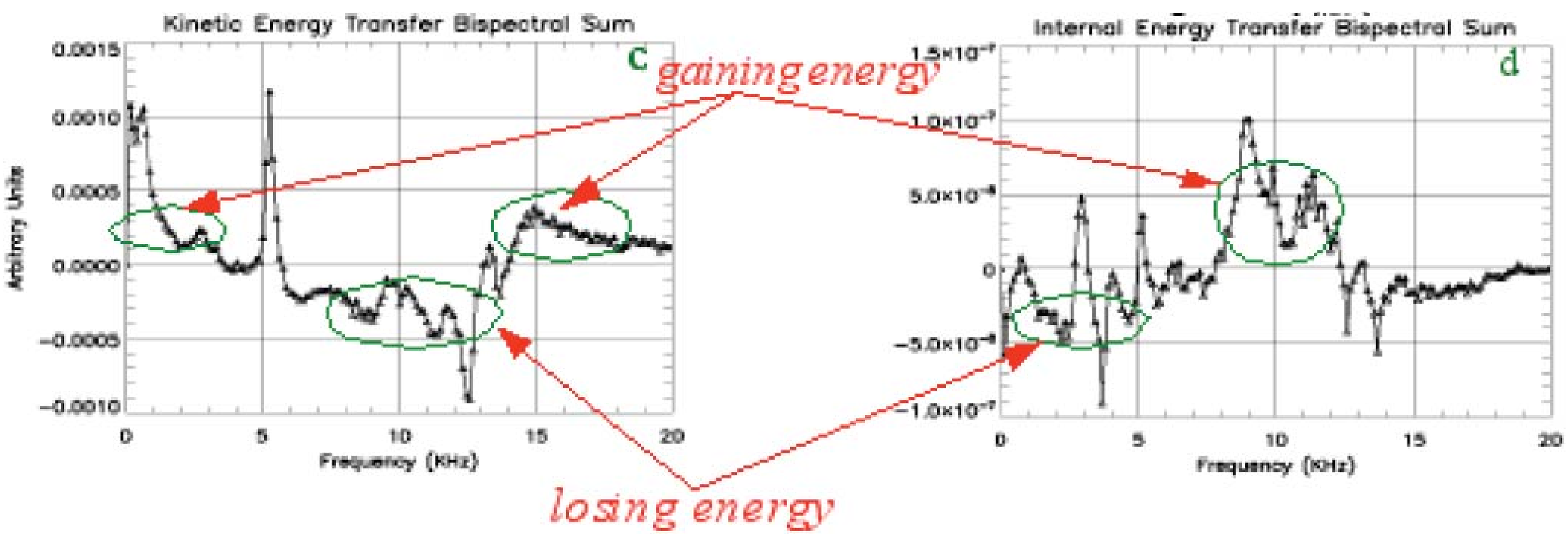
Fast-frame imaging showing the dynamics of the shear layer



Net Nonlinear Energy Transfer into/out-of Frequency f

$$T_u \equiv \left\langle -\mathrm{Re} \sum_{\substack{\omega_1,\omega_2 \\ \omega=\omega_1+\omega_2}} (\hat{z}\times\nabla_\perp\phi^*_\omega)\cdot[(\hat{z}\times\nabla_\perp\phi_{\omega_1}\cdot\nabla_\perp)(\hat{z}\times\nabla_\perp\phi_{\omega_2})] \right\rangle$$

$$T_n \equiv \left\langle -\mathrm{Re}[\sum_{\substack{\omega_1,\omega_2 \\ \omega=\omega_1+\omega_2}} n^*_\omega(\hat{z}\times\nabla_\perp\phi_{\omega_1}\cdot\nabla_\perp)n_{\omega_2}] \right\rangle$$



Shear Layer Formation in Collisional Drift Turbulence Simulations

Nonlinear Drift Turbulence Simulation

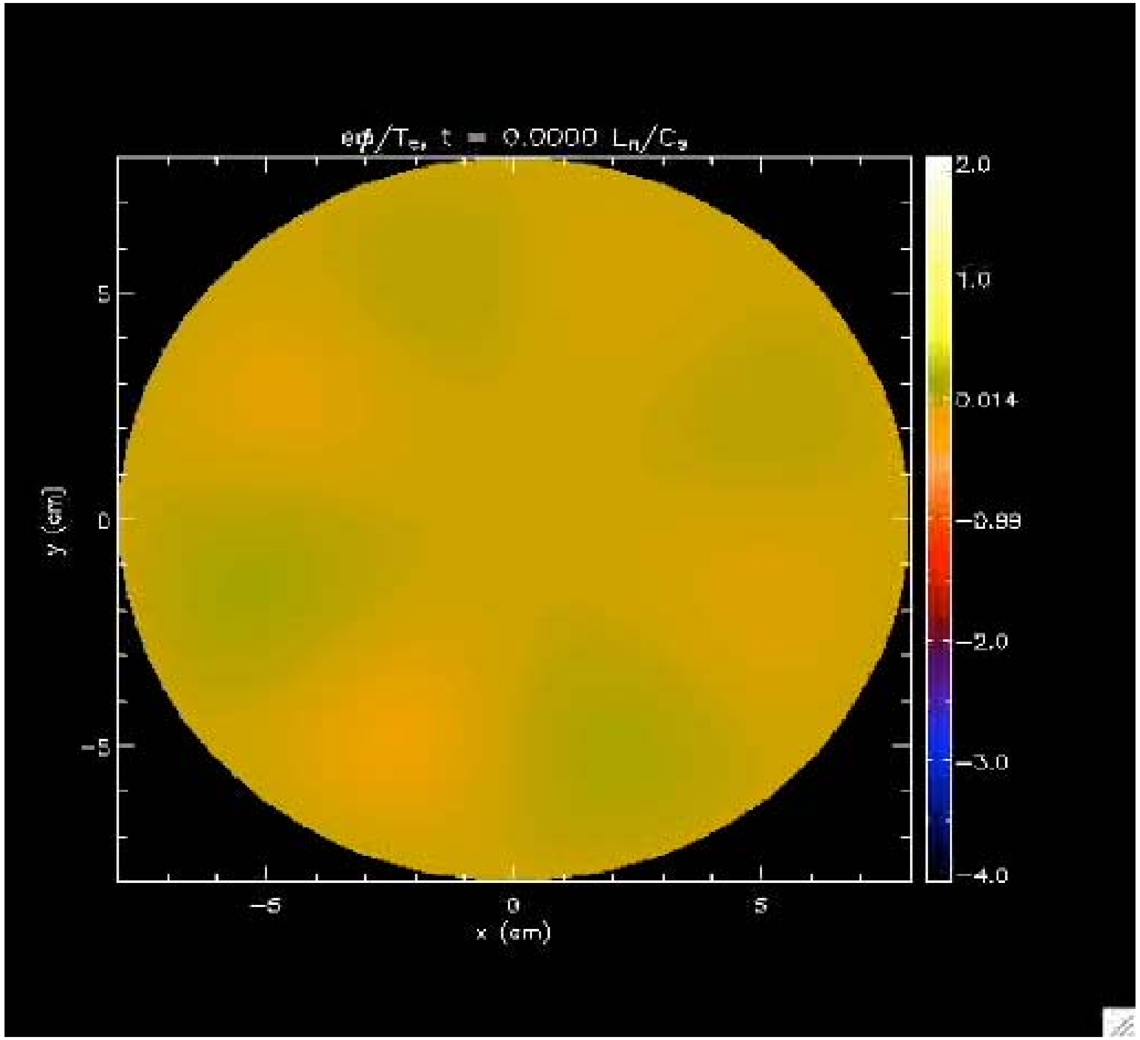
- (2D) Hasegawa-Wakatani model in cylindrical geometry.
  – Includes ion-neutral flow damping effect ν, **neglects** nonlocal (finite ρ_s / L_n) terms, **fixed parallel wavenumber.**

$$\left(\frac{\partial}{\partial t}+\vec{V}_{E\times B}\cdot\vec{\nabla}\right)n+\frac{V^*}{r}\frac{\partial n}{\partial\theta}+\frac{k_{\|}^2 v_{the}^2}{\omega^* v_e}\left(n-\phi\right)=D_n\nabla_\perp^2 n$$

$$\left(\frac{\partial}{\partial t}+\vec{V}_{E\times B}\cdot\vec{\nabla}\right)\nabla_\perp^2\phi+\frac{k_{\|}^2 v_{the}^2}{\omega^* v_e}\left(n-\phi\right)=\nu\nabla_\perp^2\phi+\mu\nabla_\perp^4\phi$$

  – Parameters used reflect best estimates for average CSDX values:
    - ρ_s = 1 cm, L_n = 2 cm, $\omega_{\|}$ = 1, $\nu = 0.03 C_s/L_n$,
    - $D_n = 0.01\ \rho_s^2\ C_s/L_n$, $\mu = 0.4\ \rho_s^2\ C_s/L_n$
  – Advances eqns by combination of 2nd order RK and implicit treatment of diffusive terms (conserves energy to within 1%).

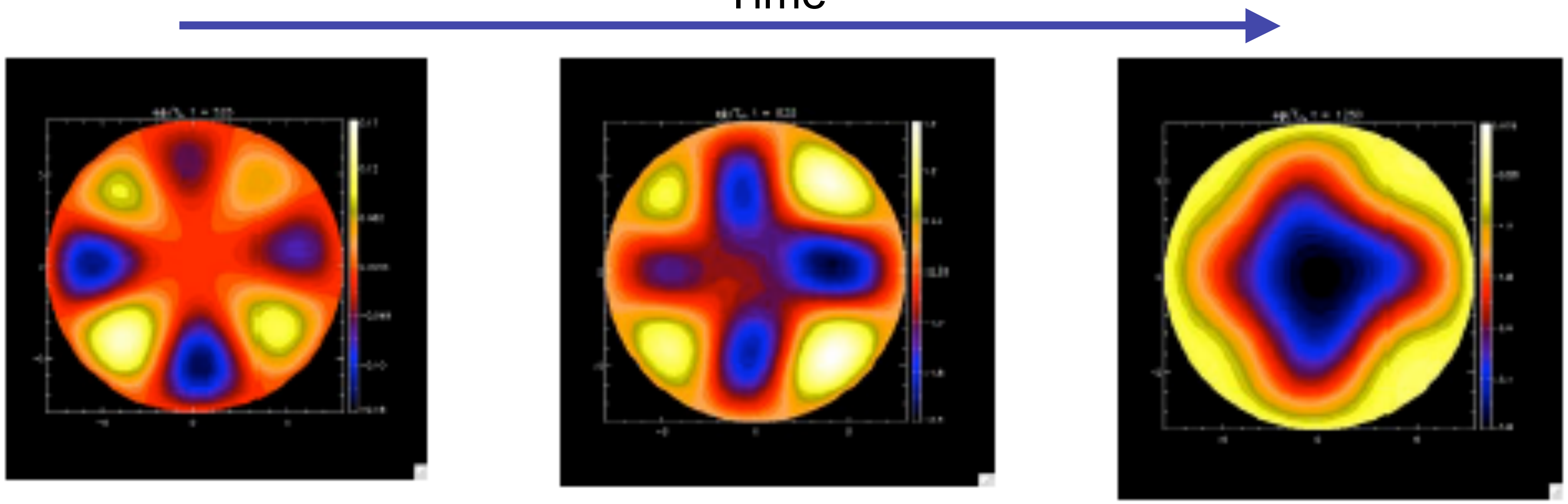
Zonal Flow Forms from Vortex Merging



Simulations Show Zonal Flow Formation Vortex Merging

Iso-Potential Contours:

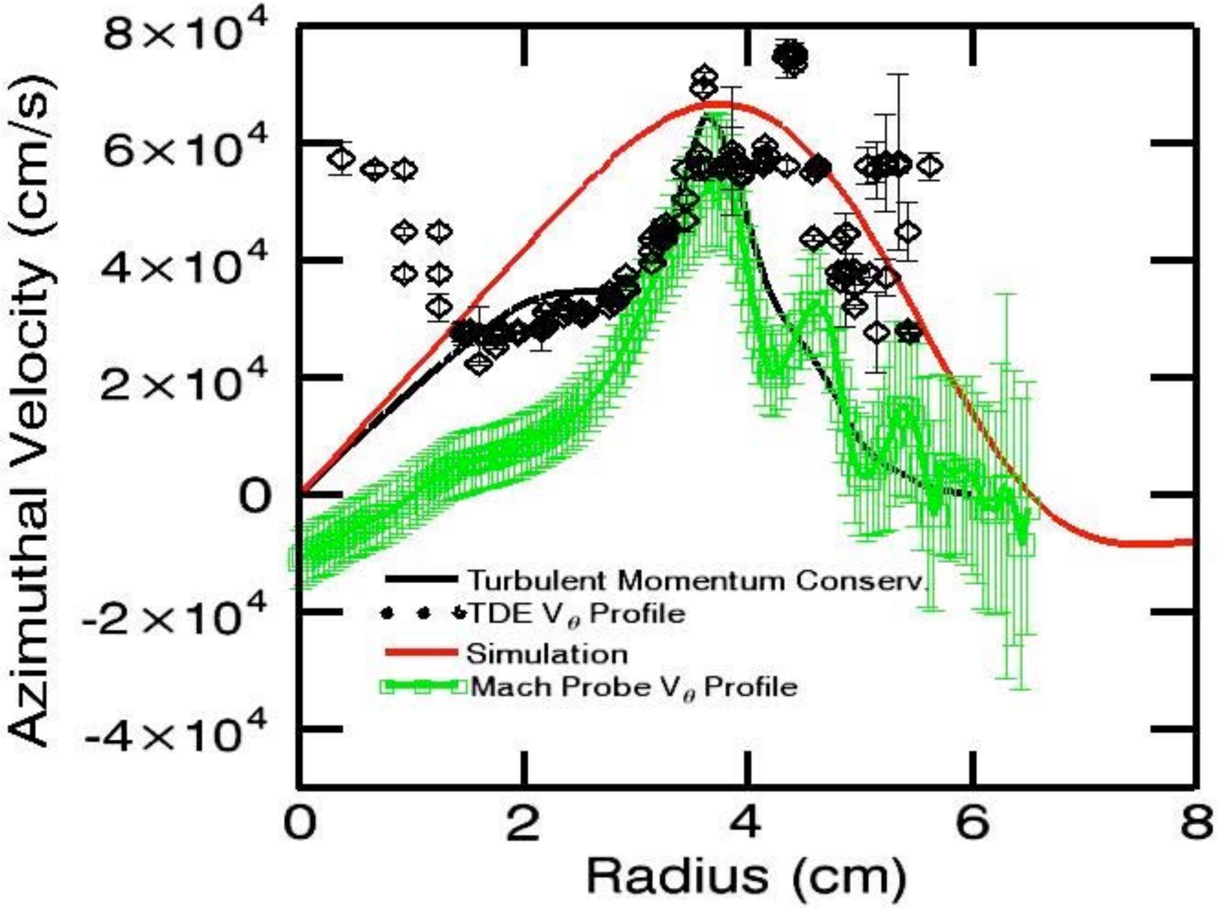
Time



– Simulation uses 64 x 64 pts, results insensitive to changes in D_n, ν, μ

– Changing L_n to 10 cm does not qualitatively affect results

Measured Velocity Profile Consistent with Turbulent Momentum Balance

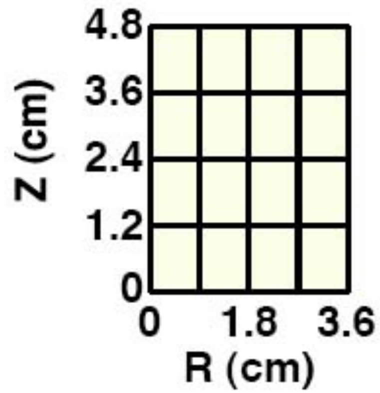


Tynan et al April 2006 PPCF

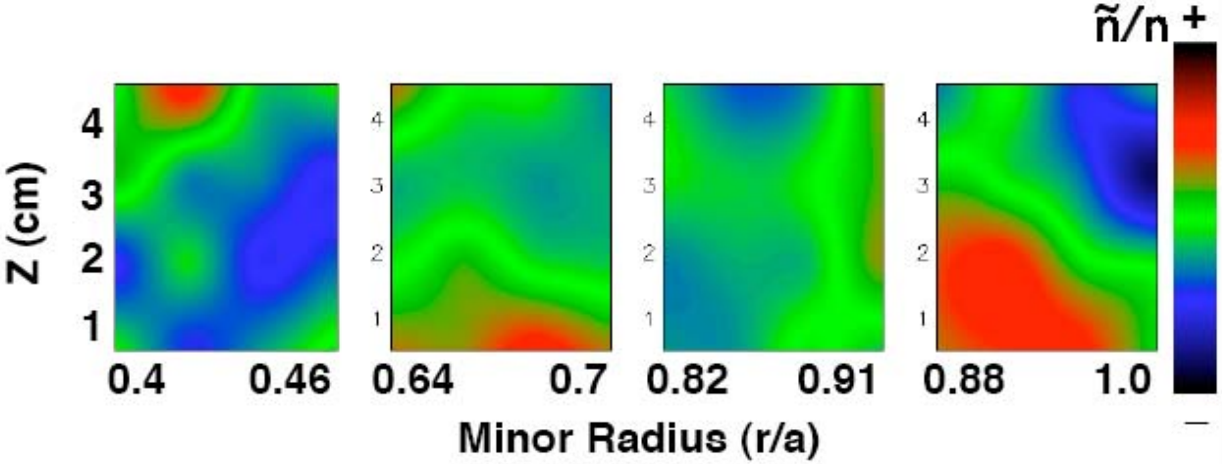
Evidence for Modulational Instability Behavior in Lab Plasma DW Turbulence-ZF Interactions

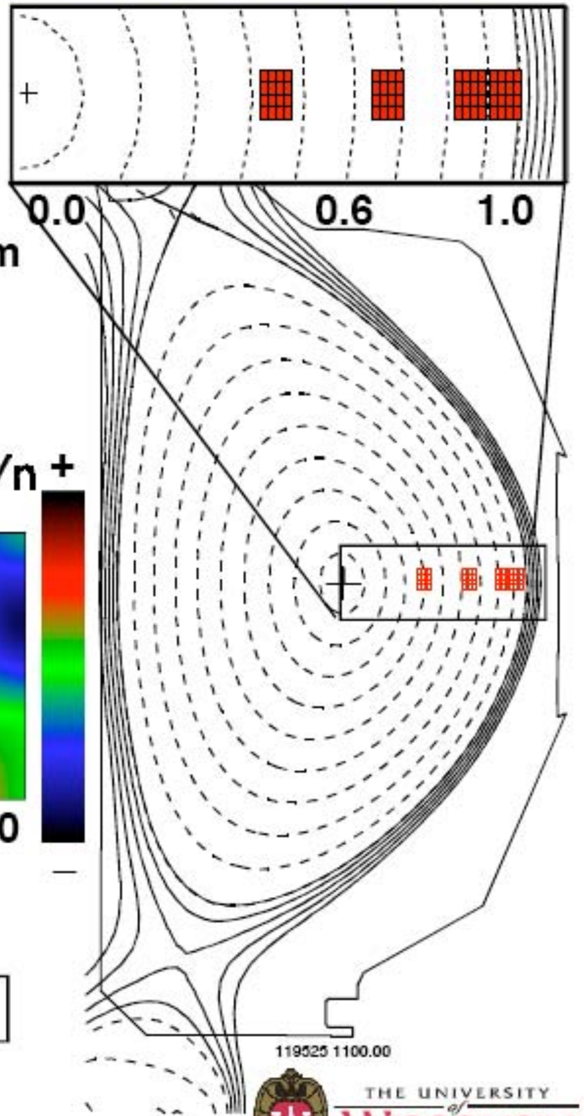
Turbulence Measurements in Tokamak Core Region

Visualizations of Core Plasma Turbulence Obtained with High-Sensitivity BES System Across much of Minor Radius



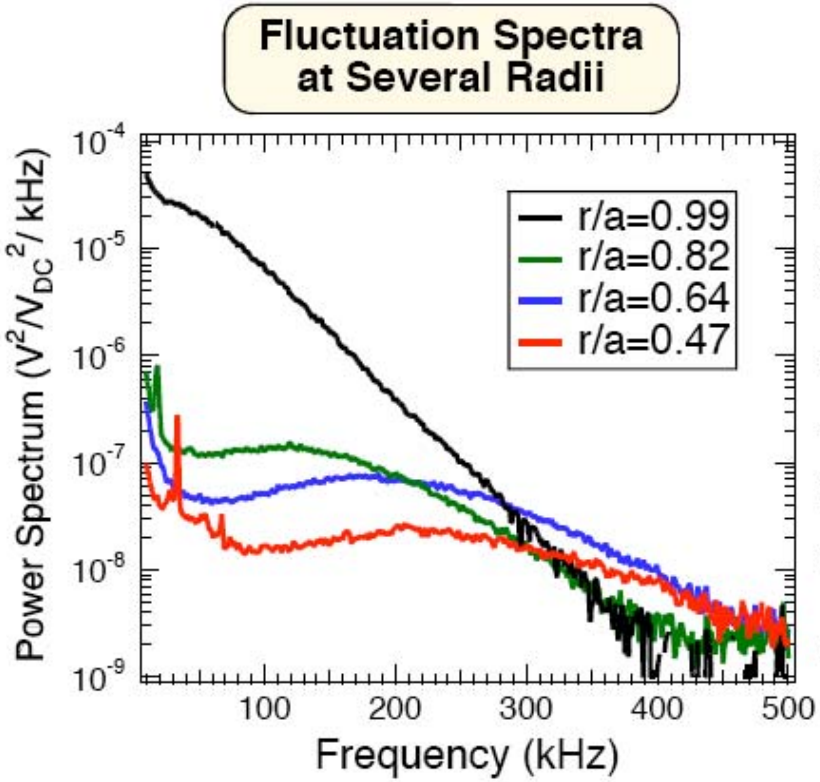
- 1 MHz sampling
- L-mode discharges (1.2 MA, 2.0 T, 5 MW NBI)
- Frequency-filtered per spectrum
- 2D spline
- 10^5 time dilation (5 μs = 1 sec)

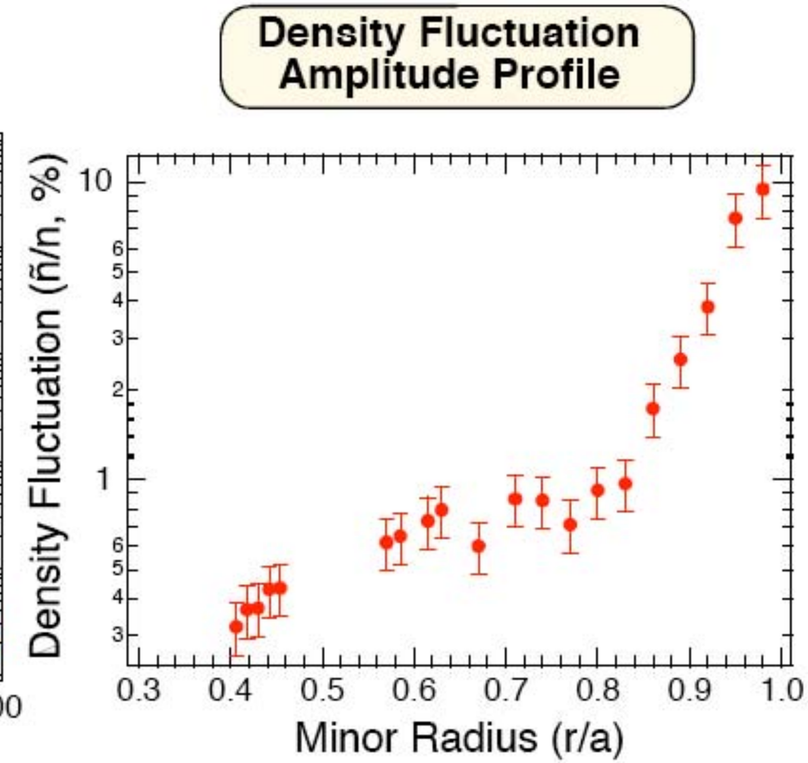




ñ/n	0.5%	0.75%	0.9%	2-8%

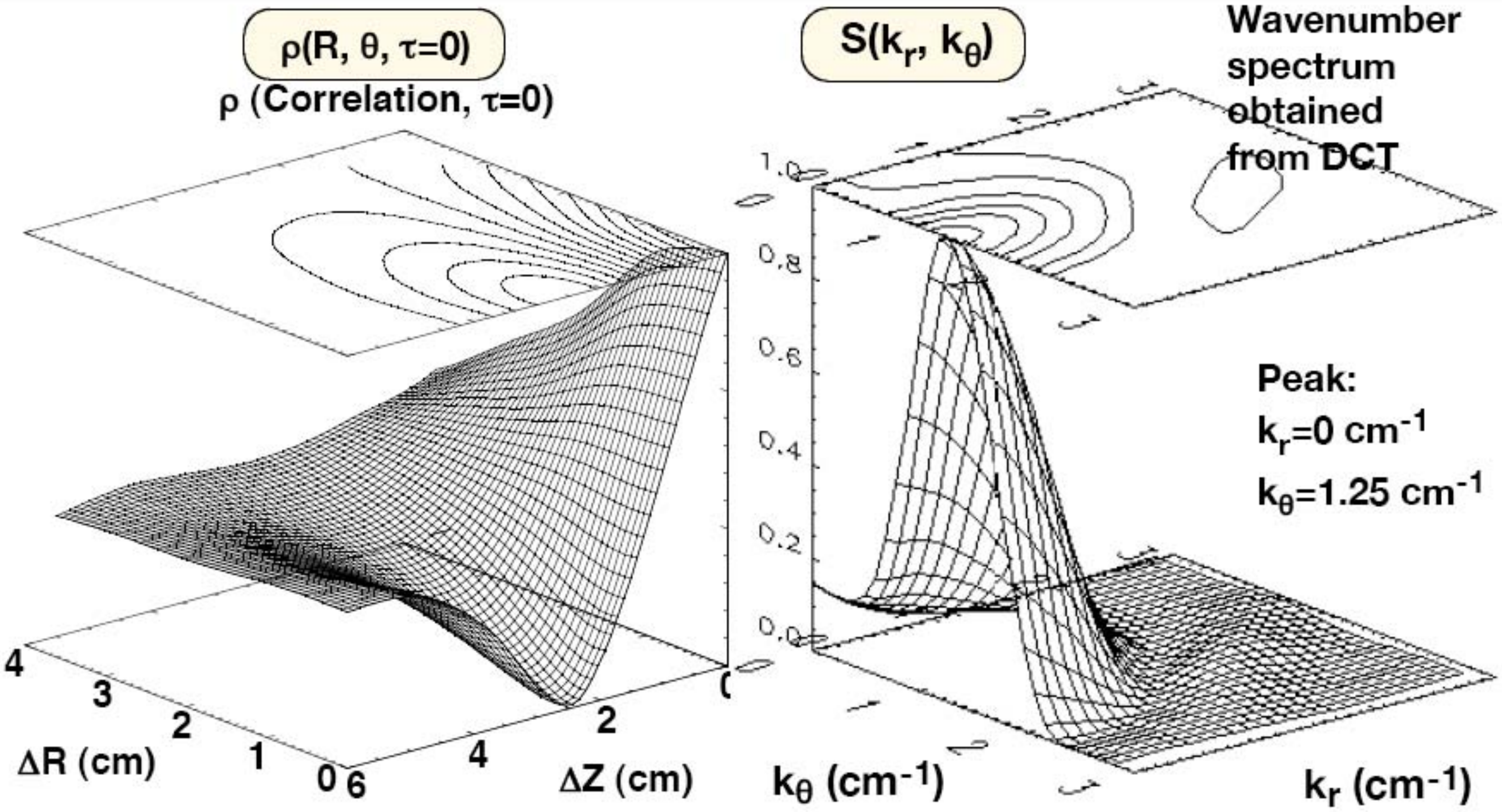
Fluctuation Spectra and Amplitude Vary Strongly with Radius





- **Density fluctuation amplitude in L-mode discharges shows wide dynamic range across plasma radius**
- **Spectra strongly Doppler-shifted to higher frequency towards core**

2D Correlation and $S(k_r, k_\theta)$ Spectra Confirm Spatial Asymmetry



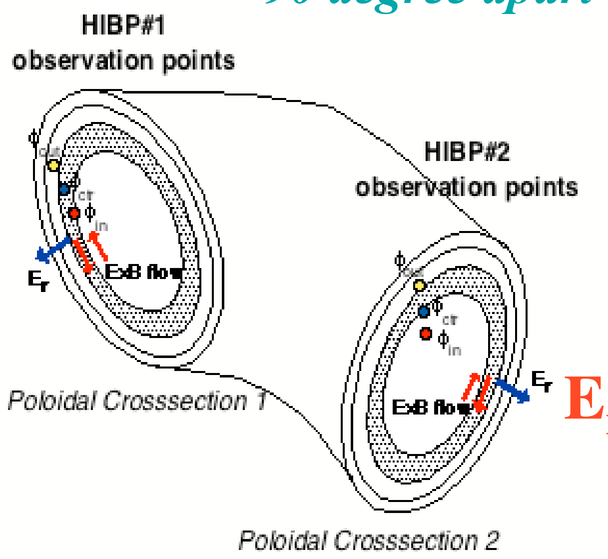
- Exhibits radially decaying, poloidally wavelike structure, $L_{c,\theta} > L_{c,r}$
- Wavenumber spectra can be compared with turbulence simulations

Existence of Zonal Flows & GAMs in Confined Plasmas

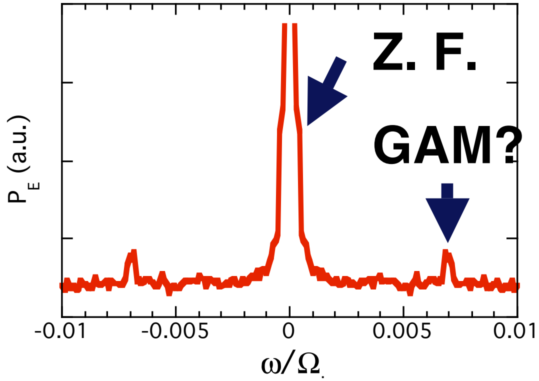
Zonal flows really do exist !

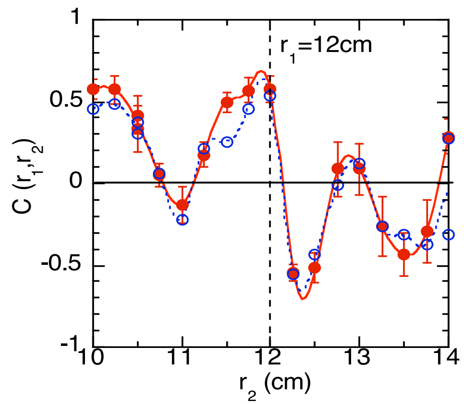
CHS Dual HIBP System

90 degree apart



$E_r(r,t)$





Radial distance

$E_r(r,t)$ { High correlation on magnetic surface,
Slowly evolving in time,
Rapidly changing in radius.

Fujisawa, PRL 2004

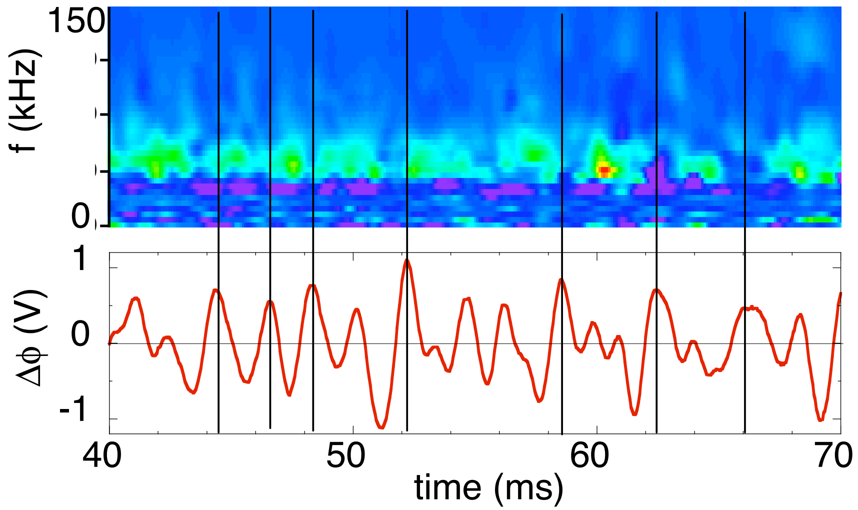
Suppression of transport by ZF

Nonlinear Interaction (3)

HIBP on CHS

$$\Gamma(\omega) = \frac{1}{B}\left\langle \tilde{E}_{\theta,\omega}\,\tilde{n}_{\omega}\right\rangle$$

ZF



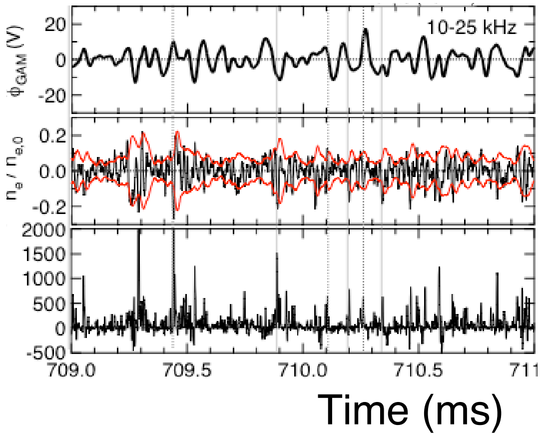
Fujisawa, PPCF 2006

Regulation of transport by GAMs

GAMs

DW

$\Gamma(t)$



HIBP on JFT-2M

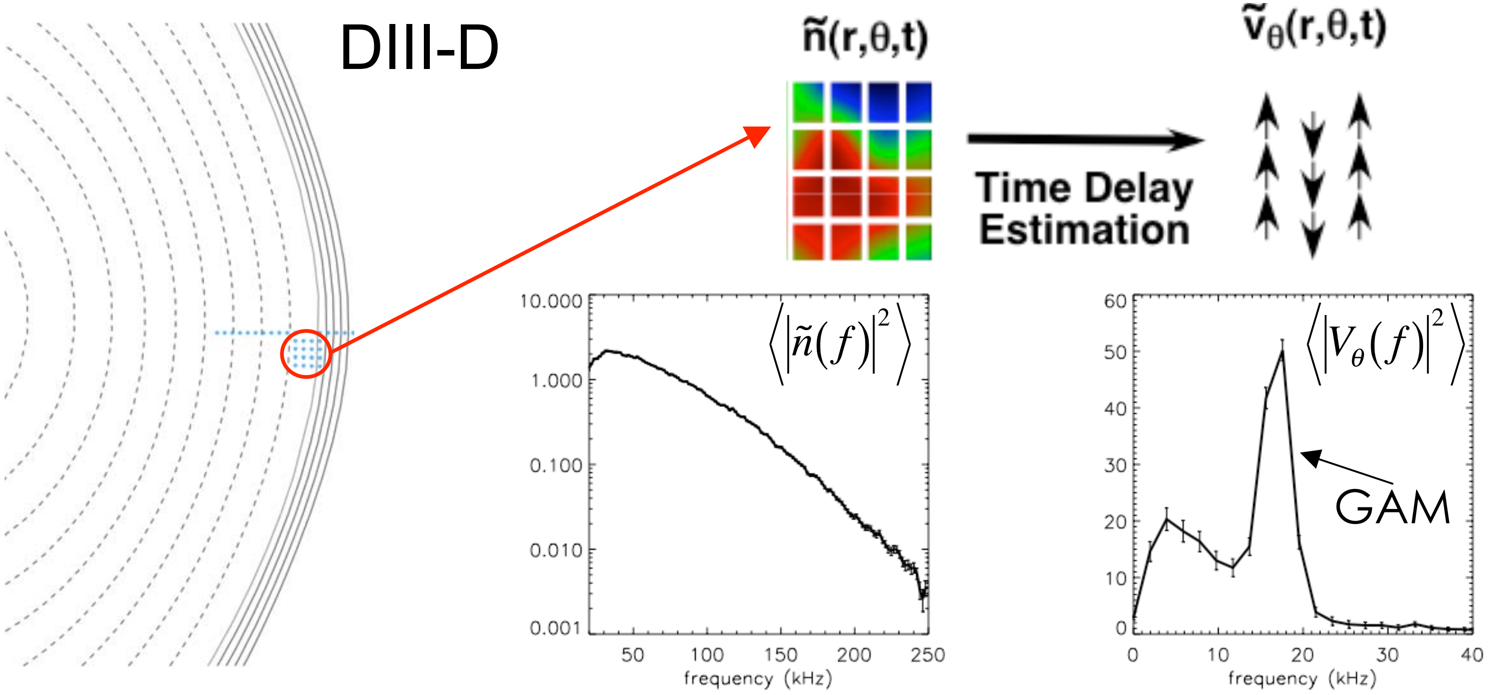
Modulation of envelope and transport by GAMs were confirmed.

Ido, submitted to NF

GAM-ZF Observed in Edge Region

- BES measures localized, long-wavelength ($k_\perp \rho_i < 1$) density fluctuations
  - Can be radially scanned shot to shot to measure turbulence profiles
  - Recent upgrades allow for BES to measure core fluctuations

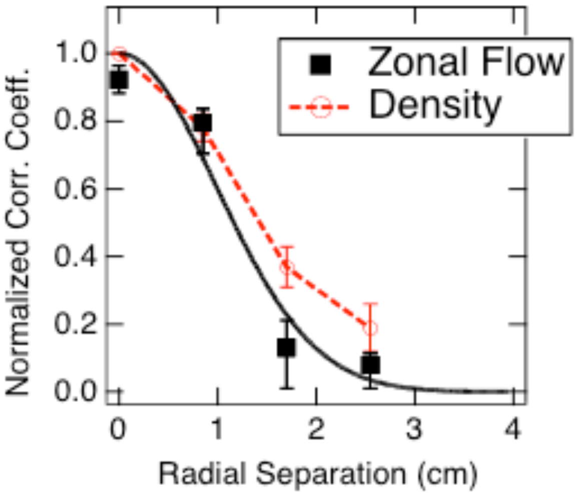
- Time-delay estimation (TDE) technique uses cross-correlations between two poloidally separated measurements to infer velocity

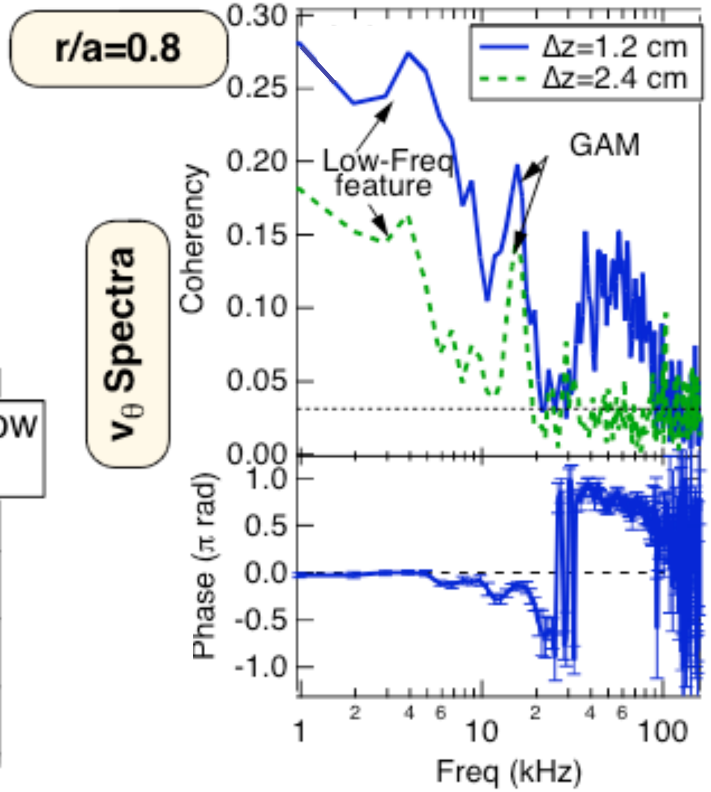


McKee PoP 2001

Measured V_θ Spectra Exhibit Signatures of Both ZMF Zonal Flows and GAMs

- Spectra indicate broad, low-frequency structure with zero measurable poloidal phase shift
  - Consistent with low-m (m=0?)
  - Peaks at/near zero frequency
- GAM also clearly observed near 15 kHz
- ZMF zonal flow has radial correlation length comparable to underlying density fluctuations
  - Necessary for effective shearing of turbulence

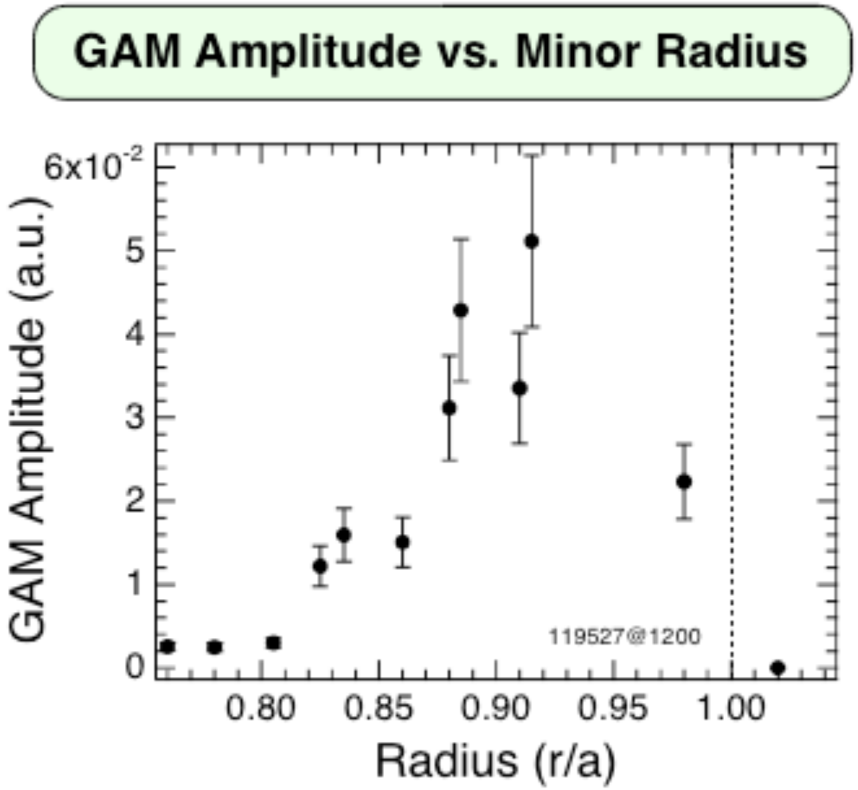




Gupta *et al.*, PRL **97** 125002 (2006)

GAMs Observed to Peak in Plasma Edge

- GAM velocity oscillation amplitude peaks near r/a ~ 0.9 - 0.95
  - Decays near separatrix
  - Decays inboard, still detectable to r/a ~ 0.75
  - Consistent with HIBP measurements on JFT-2M (Ido *et al.*, PPCF 2006)
- ZMF zonal flows not observed for r/a > ~ 0.9, but do increase towards core
  - Harder to quantify radial dependence because of broad spectral characteristics

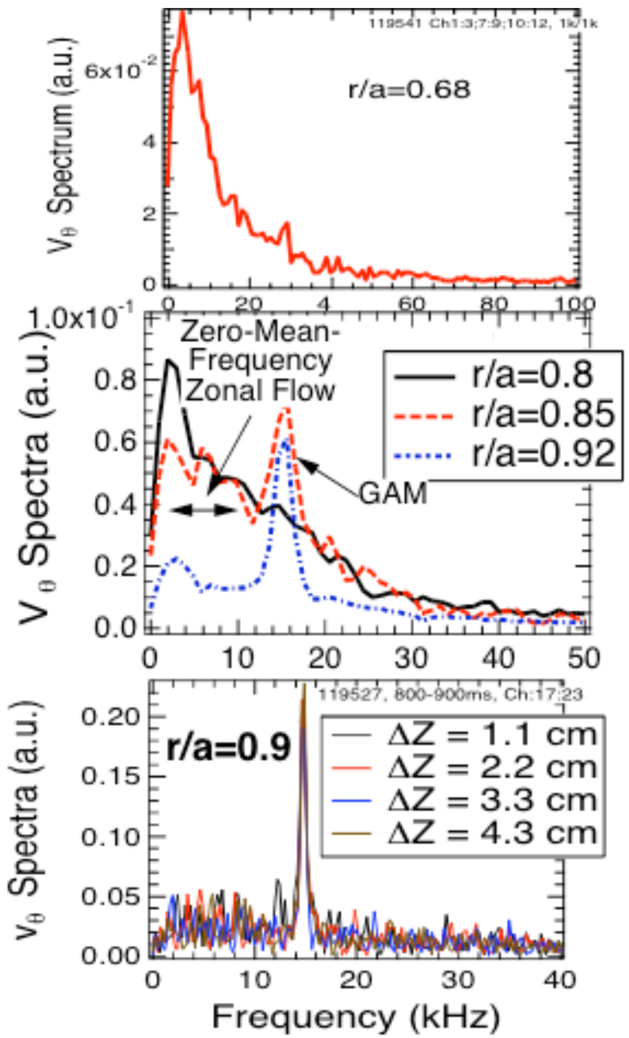


McKee *et al.*, PPCF 2006

Observe Transition from ZMF-Dominated Core to GAM-Dominated Edge

- Velocity spectra show broad ZF spectrum for r/a < 0.8 → ZMF flow
- Superposition of broad spectrum and GAM peak near r/a = 0.85
- GAM dominates for r/a > 0.9
- Consistent with theory/simulation expectations that GAM strength increases with *q*
  - Increase in GAM strength with q_{95} also observed (McKee *et al.*, PPCF 2006)
- GAM is highly coherent, with correlation time τ_{GAM} > 1 ms, two orders of magnitude larger than turbulence decorrelation τ_{turb} ~ 10 μs
  - Indicates GAM is "slow" relative to edge turbulence timescales, and so can effectively interact with turbulence (Hahm *et al.*, PoP '99)

McKee IAEA 2006

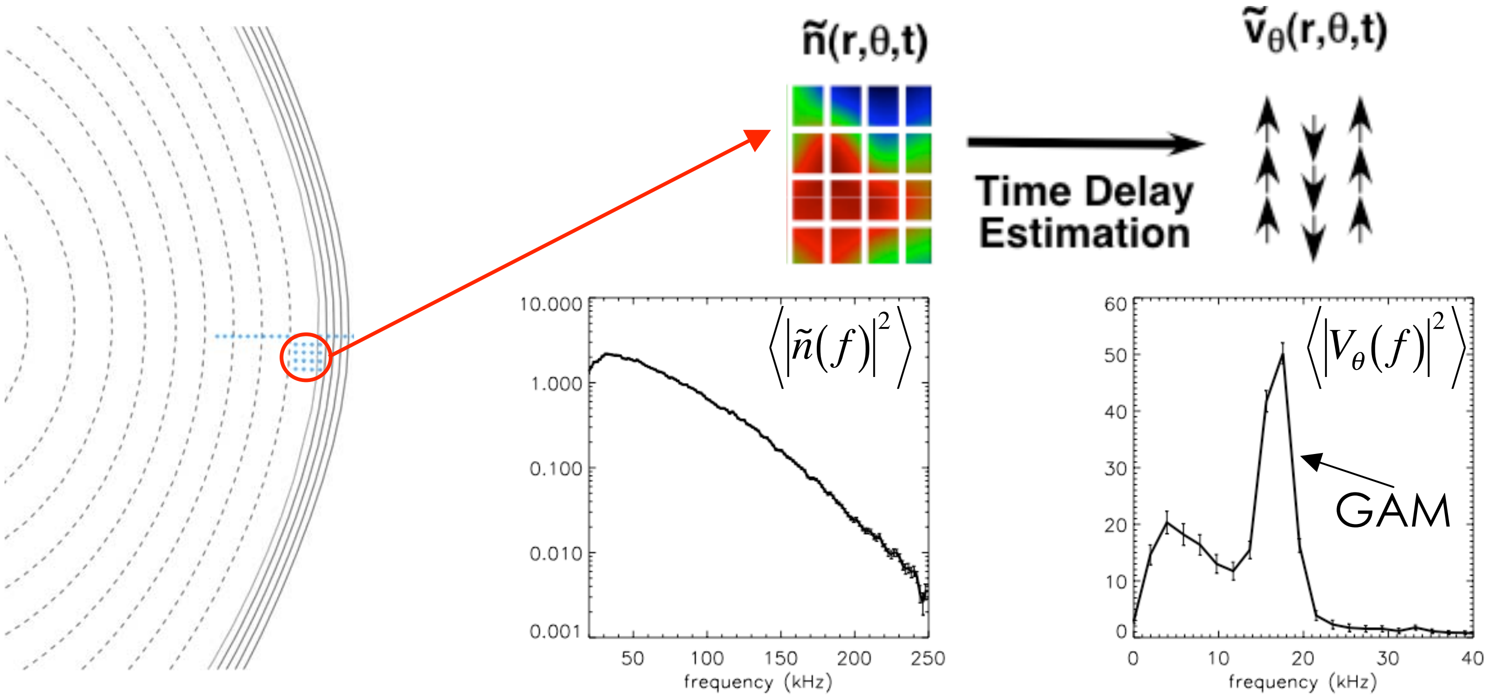


Measuring Nonlinear Effect of Zonal Flows on Turbulence in a Tokamak

BES System Configured to Provide Zonal Flow Measurements Over Large Fraction of Plasma

- BES measures localized, long-wavelength ($k_\perp \rho_i < 1$) density fluctuations
  - Can be radially scanned shot to shot to measure turbulence profiles
  - Recent upgrades allow for BES to measure core fluctuations

- Time-delay estimation (TDE) technique uses cross-correlations between two poloidally separated measurements to infer velocity



Measuring Nonlinear Energy Transfer in Tokamaks

$$T_n^Y(f',f) = -\mathrm{Re}\left\langle n^*(f) V_y(f-f') \frac{\partial n}{\partial y}(f') \right\rangle$$

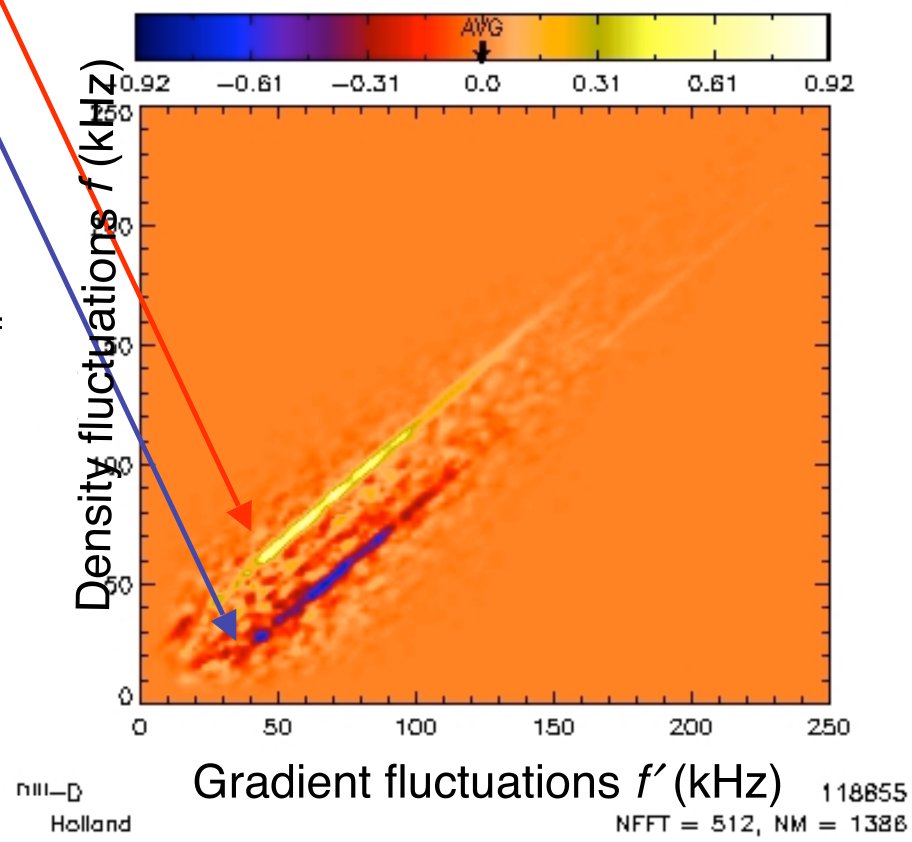
- $T_n^Y(f',f)$ measures the transfer of energy between density fluctuations at f and poloidal density gradient fluctuations at f' at a specific spatial location due to poloidal convection
  - A positive value of T_n^Y indicates that $n(f)$ is gaining energy from $\partial n/\partial y(f')$
  - A negative value indicates that $n(f)$ is losing energy to $\partial n/\partial y(f')$

GAMs Drive Clear Forward Transfer of Internal Energy in Frequency Space

- $T_n^Y(f',f)$ clearly shows that fluctuations at $f = f' + f_{GAM}$ gain energy, while those at $f = f' - f_{GAM}$ lose energy
- Density fluctuations gain energy from lower frequency gradient fluctuations, and lose energy to higher frequency gradient fluctuations
- Simple picture: energy moves between n, $\partial n/\partial y$ to high f in "steps" of f_{GAM}

  Net transfer of energy to high f!
- Demonstrates that the convection of density fluctuations by the GAM leads to a cascade of internal energy to high f

$$T_n^Y(f',f) = -\mathrm{Re}\left\langle \tilde{n}^*(f) V_y(f-f') \frac{\partial \tilde{n}}{\partial y}(f') \right\rangle$$



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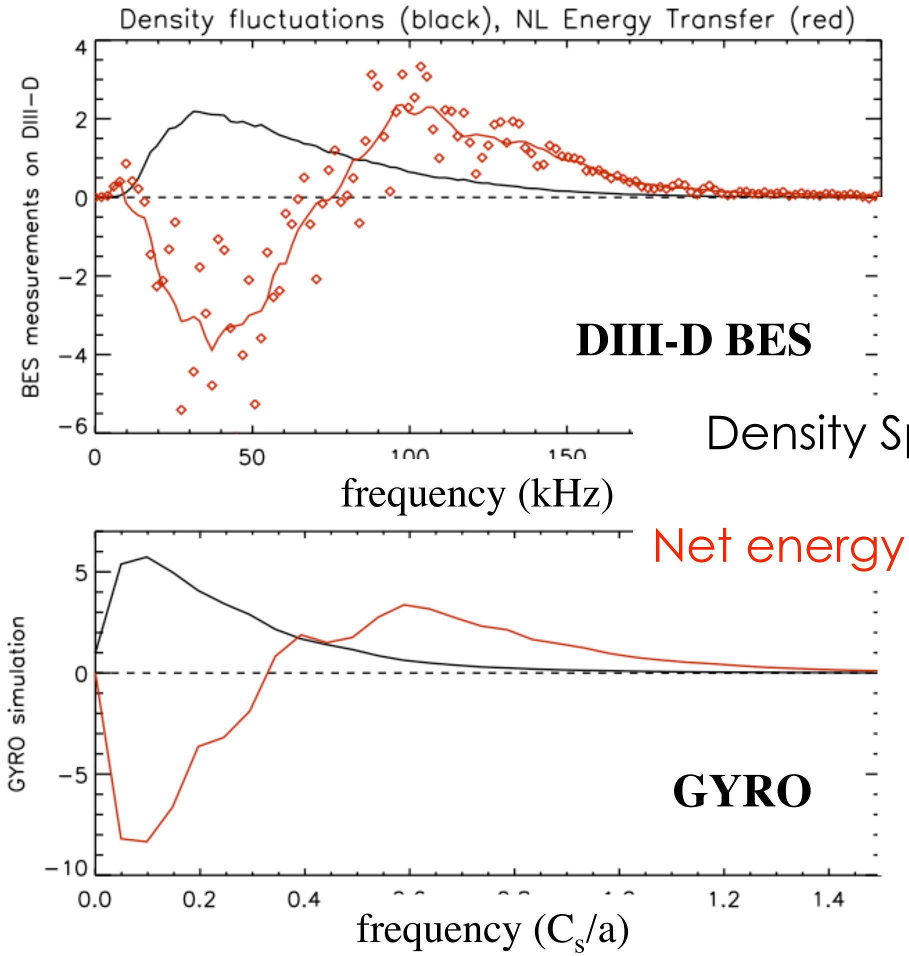
GAMs Nonlinearly Transfer Density Fluctuation Power from f<75kHz region to f>75kHz region



Density Spectrum $\langle | \delta n(f) |^2 \rangle$

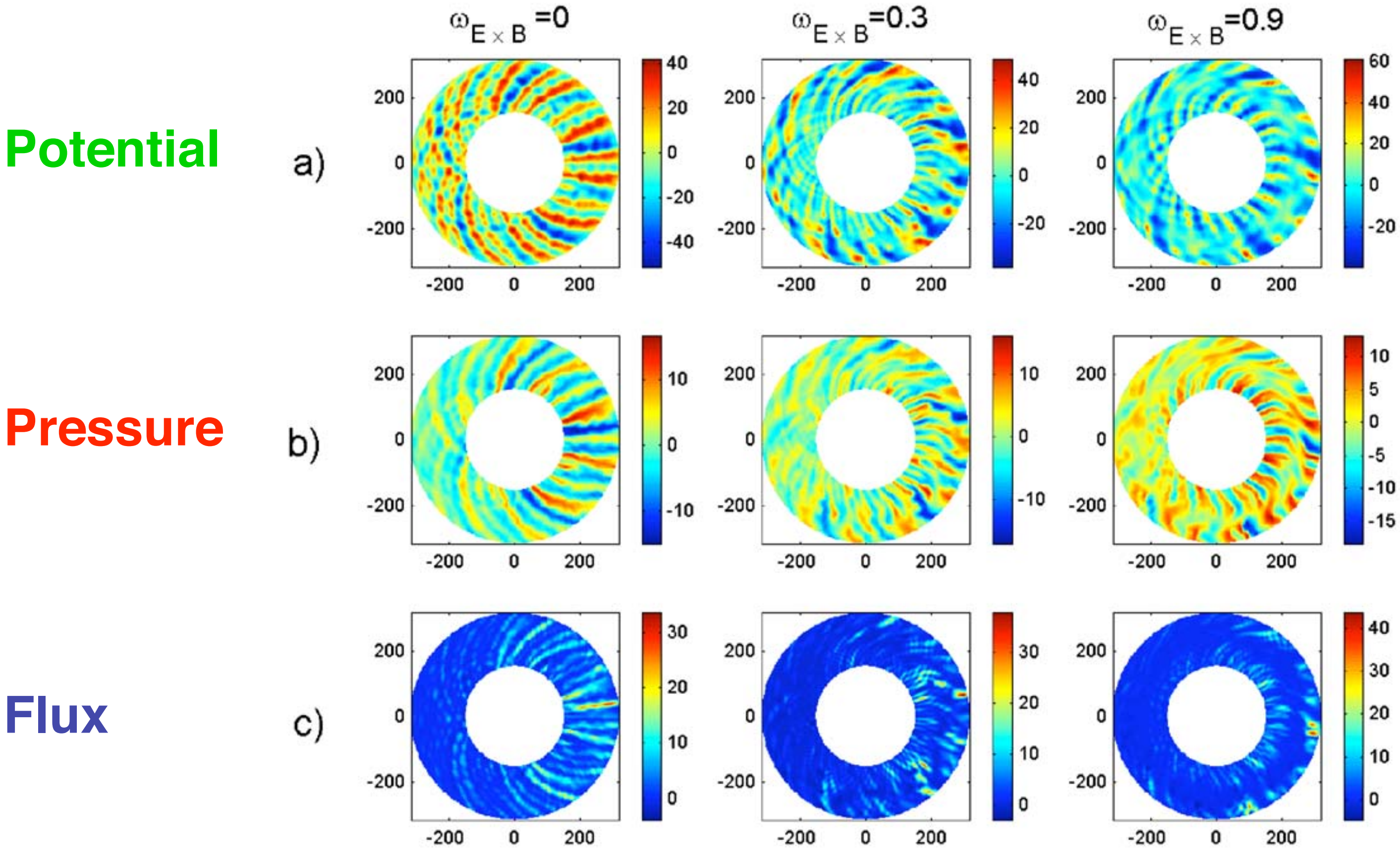
Net energy transfer $\int df' \, T_n^Y(f', f)$

Implications for Transport

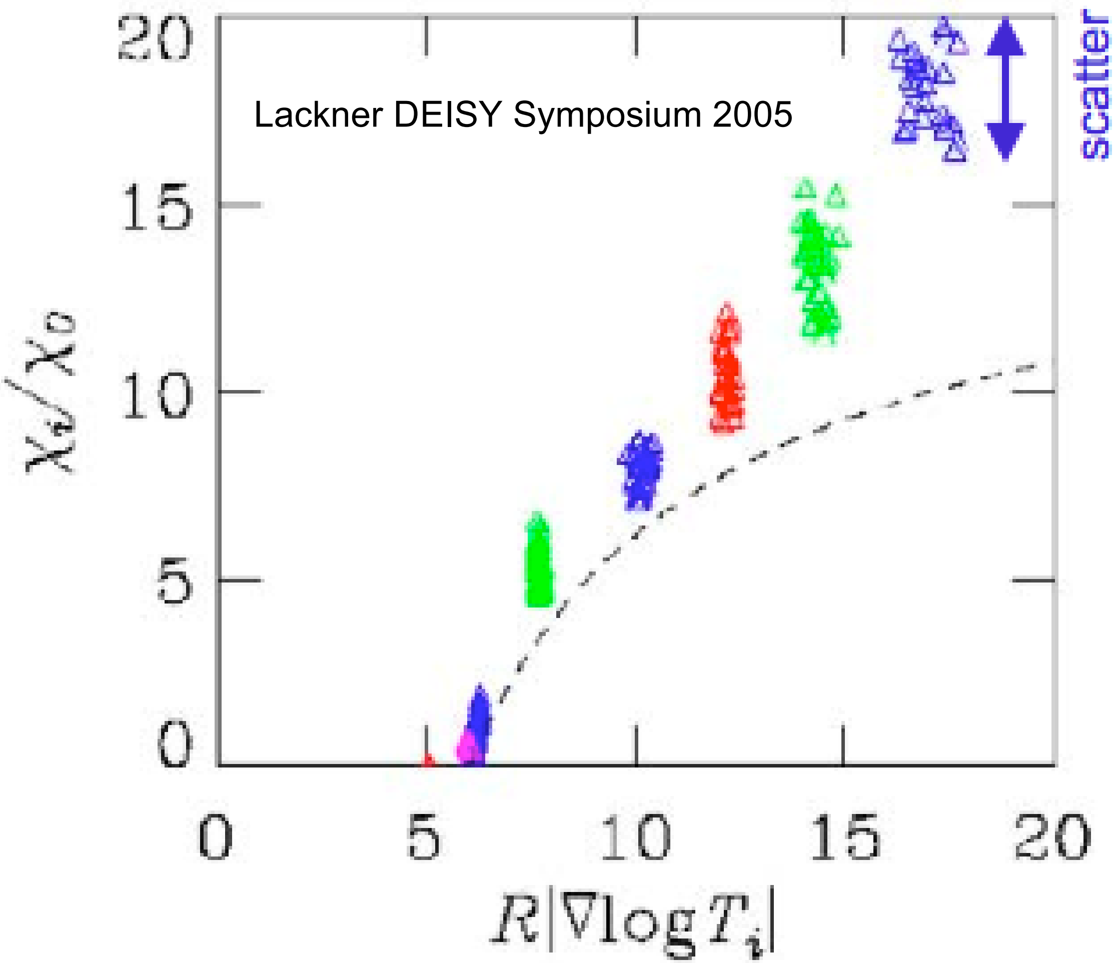
2

External shear flow breaks streamers

RBM *[Beyer et al, 00]*



Implication: Plasma Predicted to Sit At/Near Marginal Stability



Experiments Confirm Essential Elements of Nonlinear Turbulence-Zonal Flow Interactions

- Inverse Energy Transfer Exists in Drift Turbulence
- Zonal Flow Can Be Self-consistent w/ Turbulent R/S
- Particle Transport Across Shear Layer Inhibited
- Fluid-based Turbulence Simulations are Consistent w/ CSDX Experiments
- Kyushu LMD Experiments Show Inverse Kinetic Energy Transfer
- Zonal Flows (ZMF and GAMs) Exist in Confinement Devices
- They Regulate Spatial Scale of Turbulence
- They Regulate Cross Field Particle Flux

Open Questions

- Can We Measure the NL Kinetic Energy Transfer?
- Can We Show Self-consistent Picture of Linear Instability, NL Energy Transfer, and Saturated Spectra?
- Is the ZF Really Driven by Turbulence in Hot Fusion-grade Plasmas?
- Do Reynolds-stress Driven Flows Trigger Transport Barrier Formation?
- Is the NL Turbulence-ZF Interaction Consistent with Observed Critical Gradient Behavior ?

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- All the papers in the April 2006 special issue of Plasma Physics and Controlled Fusion.

Homework: Analyzing Phase coherence nonlinear interactions Numerically using Digital Signal Processing:

1) Generate Test Signal:

$$x(t) = \sin(\omega_1 t + \theta_1) + \sin(\omega_2 t + \theta_2) + \frac{3}{4}\sin(\omega_3 t + \theta_3) + \frac{1}{4}\sin(\omega_4 t + \theta_4)$$

$$\omega_3 = \omega_1 + \omega_2 \,;\, \omega_4 = \omega_1 - \omega_2 \quad \theta_1 : [0, 2\pi]\ random;\ \theta_2 : [0, 2\pi]\ random$$

And consider three different cases:

a) $\theta_3 \in [0, 2\pi]\, random \quad \theta_4 = \theta_1 \pm \theta_2$

b) $\theta_3 = \theta_1 \pm \theta_2 \quad \theta_4 \in [0, 2\pi]\, random$

c) $\theta_3, \theta_4 \in [0, 2\pi]\, random$

<u>Show that the bicoherence measures the degree of phase coherency between interacting triplets:</u>

BiCoherence is defined as

$$\hat{b}^2(\omega_1,\omega_2) = \frac{B(\omega_1+\omega_2)}{\left\langle \left| X(\omega_1) X(\omega_2) \right|^2 \right\rangle \left\langle \left| X^*(\omega_1+\omega_2) \right|^2 \right\rangle} \quad ; \quad 0 < \hat{b} < 1$$

Where the Bi-spectrum Defined As

$$B(\omega_1,\omega_2) = \left\langle X(\omega_1) X(\omega_2) X^*(\omega_1+\omega_2) \right\rangle$$

X(ω) denotes the fourier transform of x(t), and <> denotes
An ensemble average

Homework Question

Suppose the mean pressure gradient driving turbulence is constant, and the turbulence is coupled to a zonal flow such as discussed in this talk. How does the turbulence amplitude respond to an increase or decrease in the damping rate of the zonal flow?

Hint for HW Problem: Self-regulation - Predator-prey model, Malkov 2001

DW $\frac{\partial}{\partial t}\mathcal{E}_d = \gamma_L \mathcal{E}_d - \gamma_2 \mathcal{E}_d^{\,2} - \alpha V_{ZF}^2 \mathcal{E}_d$

ZF $\frac{\partial}{\partial t} V_{ZF}^2 = -\gamma_{damp} V_{ZF}^2 + \alpha V_{ZF}^2 \mathcal{E}_d$

DW amplitude is influenced by ZF damping.