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Physics behind Modern Gyrokinetic Theory, Simulation, and Comparisons to Tokamak Experiments.

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P.H. Diamond, G. Rewoldt, W. Wang, Z. Lin,…
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Outline

Properties of Tokamak Core Turbulence

Nonlinear Gyrokinetic Description

Some Examples in Tokamak Confinement Physics:

- Role of Zonal Flows
- Turbulence Spreading

Emphasis:

Conservation Laws

Study of New Physics Mechanisms

Synergism among theory, simulation, and experiment

Outline

Properties of Tokamak Core Turbulence

Nonlinear Gyrokinetic Description

Some Outstanding Confinement Physics Issues:

- Role of Zonal Flows
- Turbulence Spreading

Microinstabilities in Tokamaks

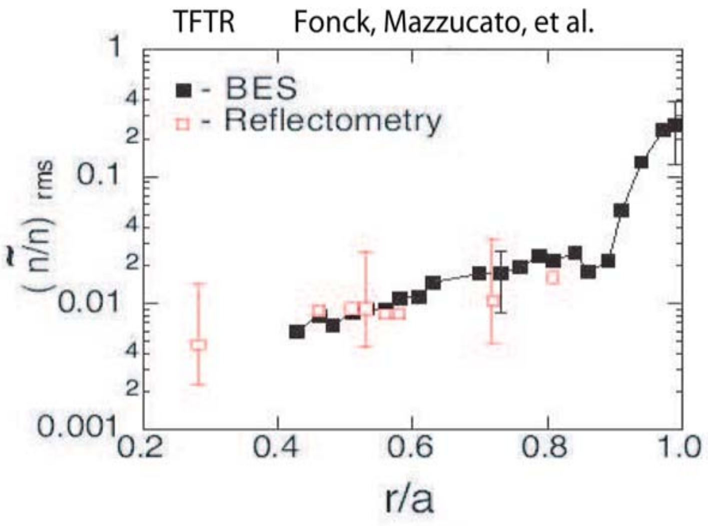
- Tokamak transport is usually anomalous, even in the absence of large-scale MHD

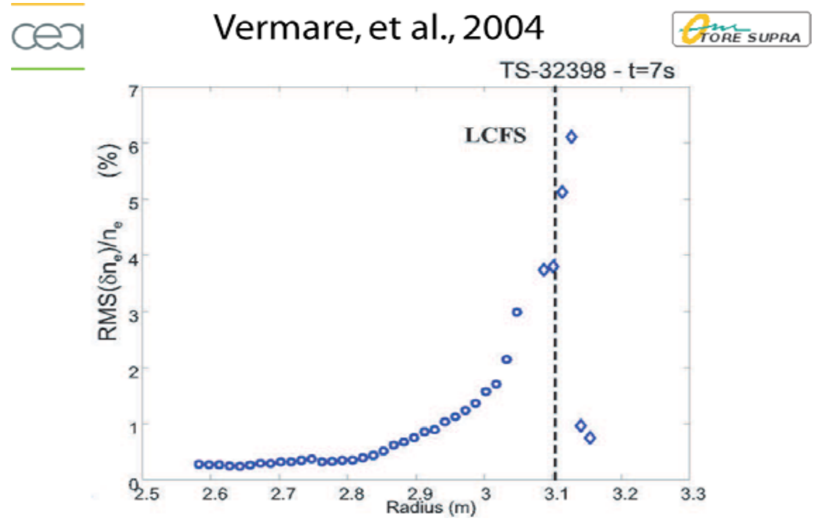
- Caused by small-scale collective instabilities driven by gradients in temperature, density, …

- Instabilities saturate at low amplitude due to nonlinear mechanisms

- Particles **E** x **B** drift radially due to fluctuating electric field

Amplitude of Tokamak Microturbulence



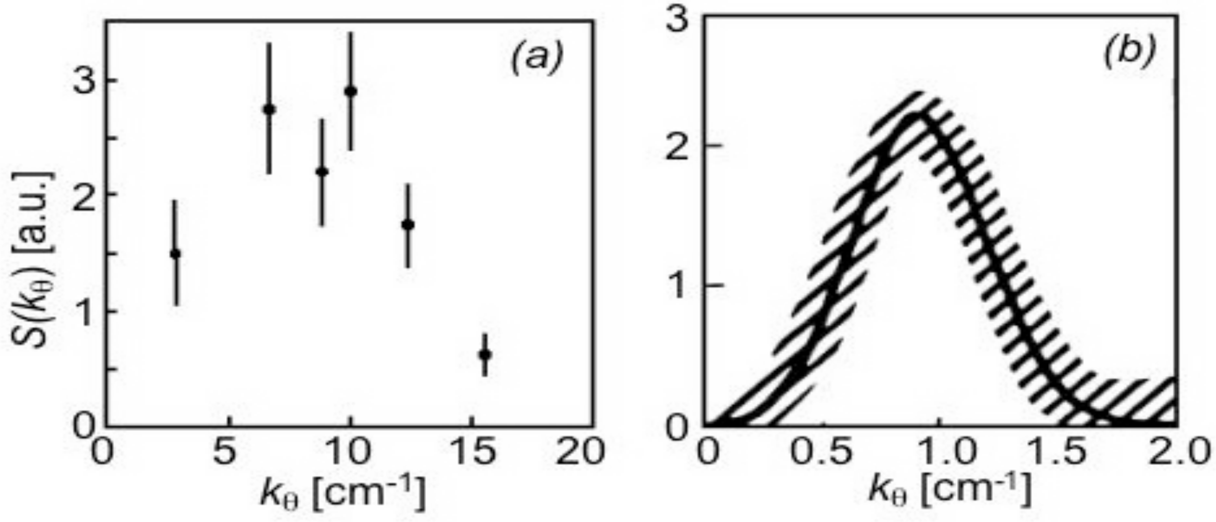


• Relative fluctuation amplitude $\delta n / n_0$ at core typically less than 1%

• At the edge, it can be greater than 10%

• Confirmed in different machines using different diagnostics

k-spectra of tokamak micro-turbulence



$k_\theta\ \rho_i$ ~ 0.1 - 0.2

-from Mazzucato et al., PRL '82 (μ-wave scattering on ATC)
Fonck et al., PRL '93 (BES on TFTR)

-similar results from

TS, ASDEX, JET, JT-60U and DIII-D

Properties of Tokamak Core Microturbulence

- $\delta n / n_0 \sim 1\%$
- $k_r \rho_i \sim k_\theta \rho_i \sim 0.1 - 0.2$
- $k_{||} < 1/qR << k_\perp$: Rarely measured
- $\omega - \mathbf{k} \cdot \mathbf{u}_E \sim \Delta\omega \sim \omega_{*pi}$:

Broad-band $\Rightarrow$ Strong Turbulence

Sometimes Doppler shift dominates in rotating plasmas

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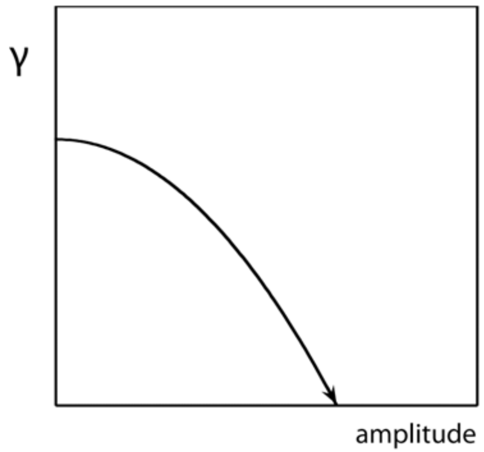
Electrostatic Microinstabilities in Tokamaks

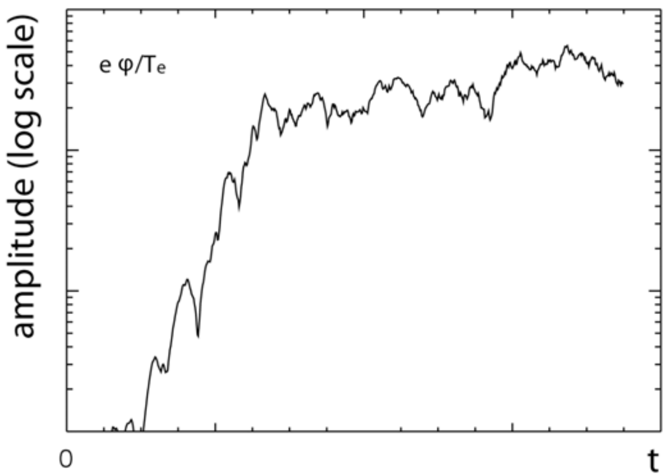
Classification: Free energy	Spatio-temporal Scales (wavelength, frequency direction, rough mag.)	Accessibility Mechanism for Instability
Trapped Ion Mode n, T_e, (ITG-TIM) T_i	$\sim \rho_\theta$ $\sim \omega^*_e$	Trapped ion precession resonance (coll-less) Collisions btwn trapped and passing ions (dissipative)
Ion Temp. Grad. Mode T_i	$> \rho_i$ $< \omega^*_{pi}$	Bad curvature or Negative compressibility
Trapped Electron Mode n or T_e	$\sim \rho_i$ $< \omega^*_e$	Trapped electron precession resonance (coll-less) Collisions btwn trapped and passing e^-s (dissipative)
Electron Temp. G Mode T_e	$> \rho_e$ $< \omega^*_{pe}$	Bad curvature or Negative compressibility

Heuristic Estimation of Diffusion Coefficient

$$\gamma = \gamma_{lin} - k_{\perp}^2 D_{turb} \rightarrow 0$$

• Nonlinear coupling induced dissipation leads to saturation (B. Kadomtsev '65)

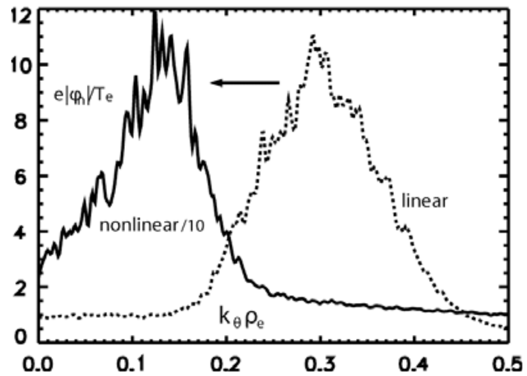




• $D_{turb} \sim \gamma_{lin} / k_{\perp}^2 \sim (v_{Ti} / a)\, \rho_i^2 \sim (\rho_i/a)\,(cT_i/eB)$: GyroBohm scaling; since $\gamma_{lin} \propto \omega_* \propto (v_{Ti}/a)$, and $k_{\perp} \propto \rho_i^{-1}$.

Spectral Transfer in $k_\perp$

- γ_{lin} usually peaks at high **k**
- Spectrum peaks at lower **k** at nonlinear saturation



from Lin *et al*., IAEA/TH/8-4 (2004)

→**Dual cascade of Hasegawa-Mima type system** or

→**Compton Scattering (Nonlinear Landau Damping)**

Sagdeev and Galeev, *Nonlinear Plasma Theory* (1969)

Chen *et al*., Phys. Rev. Lett. **39**, 754 (1977)

Similon and Diamond, Phys. Fluids **27**, 916 (1984)

Hahm and Tang, Phys. Fluids B **3**, 989 (1991)

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Some Outstanding Confinement Physics Issues:

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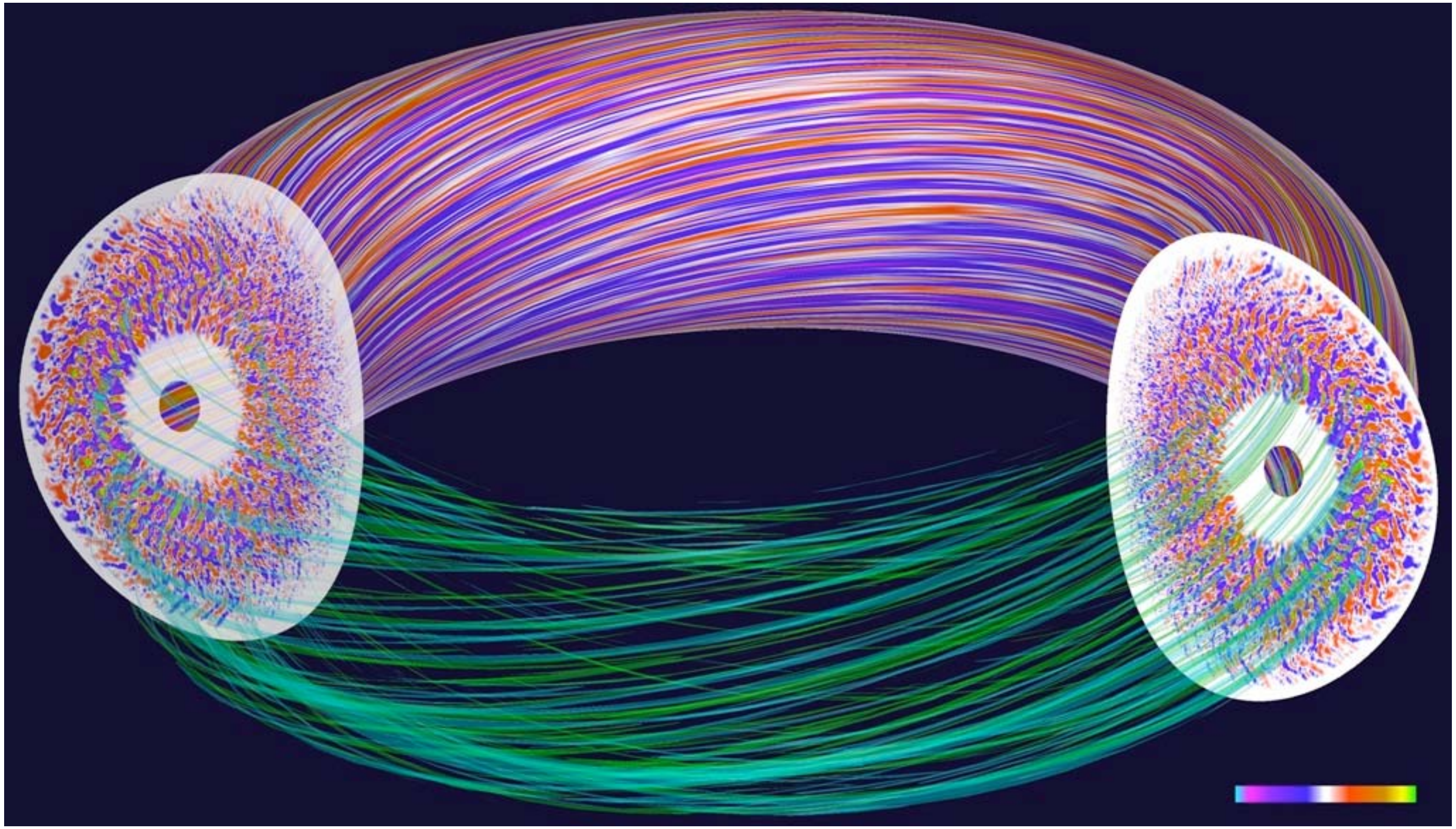
L’aspect Cinématique de la Théorie Gyrocinétique



GTS simulation of ITG Turbulence: S. Ethier, W. Wang et al.,

Standard Nonlinear Gyrokinetic Ordering I.

Frieman and Chen, Phys. Fluids 1982

Minimum number of ordering assumption

- $\omega/\Omega_i \sim k_{||}/k_\perp \sim \varepsilon_{k,\omega} << 1$; from spatio-temporal scales of fluctuations
- $k_\perp \rho_i \sim 1$ for generality:
Short wavelength modes (with higher γ_{lin}) can affect
the modes at NL peak ($k_\perp \rho_i \sim 0.1 \sim 0.2$) through NL coupling.
$\rightarrow \omega \sim k_{||} v_{Ti}$ for wave-particle resonance

i.e., Landau damping

- $\delta f/f_0 \sim e\delta\phi/T_e \sim 1/k_\perp L_p \sim \varepsilon_\phi << 1$; from small relative fluctuation amplitude

  - $k\, e\delta\phi/T_e \sim 1/L_p$: **ExB** Nonlinearity ~ Linear Drive

  - $\delta n/n_0 \sim \rho/L \sim$ roughly experimental values.

Standard Nonlinear Gyrokinetic Ordering II.

- While the physics origins of $\varepsilon_{k,\omega}$ and ε_{ϕ} are different, the maximal ordering for NL GK corresponds to $\varepsilon_{k,\omega} \sim \varepsilon_{\phi}$

- $\varepsilon_{k,\omega} >> \varepsilon_{\phi}$ leads back to the Linear Gyrokinetics:

Taylor-Hastie, Plasma Phys. **10**, 419 '68
Rutherford-Frieman, Phys. Fluids **11**, 569 '68
Tang, Nuclear Fusion **18**, 1089 '78
Antonsen-Lane, Phys. Fluids **23**, 1205 '80
Horton, Rev. Mod. Phys **71**, 735 '99

- With $\varepsilon_{k,\omega} << \varepsilon_{\phi}$, one cannot recover the linear dispersion relation of instabilities:

Self-sustained Turbulence, BS from **BDS**

Scott, Phys. Rev. Lett. **65**, 3289 ‘90

Conventional Nonlinear Gyrokinetic Equation

[eg., Frieman and Chen, Phys. Fluids 1982]

- Foundations of Tokamak Nonlinear Kinetic Theory
  for analytic applications, ballooning codes...
- Number of assumptions minimum
- Based on direct gyro-phase average of Vlasov equation
  Lots of algebra and book keeping
- Direct expansions in ε: Self-consistent up to $O(\varepsilon^2)$ →

Should be fine for linear phase and saturation due to **E**x**B** nonlinearity

- Velocity space nonlinearity: $\nabla_{||}\, \delta\phi\, \partial_{v||}\delta f \sim O(\varepsilon^3)$

Energy, phase space volume **not** conserved.

- May not be able to describe long term behavior accurately

Topic of Current Research: [Villard, Hatzky, Sorge, Lee, Wang, Ku]

→ Physics responsible for the difference?

Conventional Nonlinear GK Derivation:Heuristic

- Transforming to guiding center variables, R = x + ρ, $\mu = v_\perp^2/2B$, $\mathbf{v} = v_{||}\mathbf{b} + (\mathbf{e}_1 \cos\theta + \mathbf{e}_2 \sin\theta)$, one can write the Vlasov equation as

$$\frac{\partial}{\partial t} f + \mathrm{v}_{\|}\mathbf{b}\cdot\frac{\partial}{\partial \mathbf{R}} f + \frac{\mathbf{E}\times\mathbf{b}}{\mathbf{B}}\cdot\frac{\partial}{\partial \mathbf{R}} f + (q/m)E_{\|}\frac{\partial}{\partial \mathrm{v}_{\|}} f - \Omega\frac{\partial}{\partial\theta} f = 0$$

- Since $\Omega >> \omega$, to the lowest order $\Omega(\partial/\partial\theta)f=0$

- Writing $f = \langle f \rangle + \tilde{f}$, with $f = \langle f \rangle >> f$ in which $\langle \cdots \rangle$ indicates gyrophase average,

$$\frac{\partial}{\partial t}\langle f\rangle + \mathrm{v}_{\|}\mathbf{b}\cdot\frac{\partial}{\partial \mathbf{R}}\langle f\rangle + \frac{\mathbf{E}\times\mathbf{b}}{\mathbf{B}}\cdot\frac{\partial}{\partial \mathbf{R}}\langle f\rangle + (q\tilde{/}m)E_{\|}\frac{\partial}{\partial \mathrm{v}_{\|}}\langle f\rangle - \Omega\frac{\partial}{\partial\theta} f = 0$$

which is a solubility condition for $\langle f \rangle$.

- Gyro-phase averaging, one gets an electrostatic NL GK equation in a uniform B field:

$$\frac{\partial}{\partial t}\langle f\rangle + \mathrm{v}_{\|}\mathbf{b}\cdot\frac{\partial}{\partial \mathbf{R}}\langle f\rangle + \frac{\langle\mathbf{E}\rangle\times\mathbf{b}}{\mathbf{B}}\cdot\frac{\partial}{\partial \mathbf{R}}\langle f\rangle + (q/m)\langle E_{\|}\rangle\frac{\partial}{\partial \mathrm{v}_{\|}}\langle f\rangle = 0$$

- Frequency-wave number expansion and amplitude expansion,and geometric expansion (if it were included) are all lumped together in this procedure. If one modifies an ordering, needs to do the derivation all over again

Nonlinear Gyrokinetics for Large Scale Computation

• Direct simulation of actual size fusion plasmas in realistic geometry using the primitive nonlinear plasma equations (Vlasov-Maxwell), is far beyond the computational capability of foreseeable future.

• For turbulence problems in fusion plasmas, the temporal scales fluctuations much longer than the period of a charged particle's cyclotron motion, while the spatial scales and gyro-orbits are much smaller than the macroscopic length scales: → details of the charged particle's gyration motion are not of physical interest → Develop reduced dynamical equations which capture the essential features

• After decoupling of gyro-motion, gyrokinetic equation describes evolution of gyro-center distribution function, independent of the gyro-phase, θ, defined over a five-dimensional phase space ($\mathbf{R}$, $v_{||}$, μ). →
save enormous amounts of computing time by having a time step greater than the gyro-period, and by reducing the number of dynamical variables.

• In gyrokinetic approach, gyro-phase is an ignorable coordinate, magnitude of the perpendicular velocity enters as a parameter in terms of an adiabatic invariant μ

• Nonlinear gyrokinetic equations are now widely used in turbulence simulations.

Modern Nonlinear Gyrokinetics

- Starting from the original Vlasov-Maxwell system (6D), pursue **"Reduction of dimensionality"** for both computational and analytic feasibility.

- Keep intact the underlying symmetry/conservation of the original system.

- Perturbation analysis consists of near-identity coordinate transformation which **"decouples"** the gyration from the slower dynamics of interest in the single particle Lagrangian, rather than a direct "gyro-phase average" of Vlasov equation.

- This procedure is reversible:
The gyro-phase dependent information can be recovered when it is needed.

Phase Space Lagrangian Derivation of Nonlinear Gyrokinetics

[since Hahm, PF **31**, 2670 '88, followed by Brizard, Sugama,…]

- **Conservations Laws are Satisfied.**
- Various expansion parameters appear at different stages

→Flexibility in variations of ordering for specific application

- Guiding center drift calculations in equilibrium field **B**:

Expansion in $\delta_B = \rho_i / L_B \sim \rho_i / R$.

- Perturbative analysis consists of near-identity transformations to new variables which remove the gyro-phase dependence in perturbed fields $\delta\mathbf{A}(\mathbf{x})$, $\delta\phi(\mathbf{x})$ where $\mathbf{x} = \mathbf{R} + \rho$:

Expansion in $\varepsilon_\phi = e[\delta\phi - (v_{||}/c)\delta A_{||}]/T_e \sim \delta B_{||}/B_0$.

- Derivation more transparent, less amount of algebra

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Single Particle Phase Space Lagrangian

[Littlejohn, Cary '83,...]

- Fundamental 1-form (phase space Lagrangian in non-canonical variables)

$$\gamma \equiv (e\mathbf{A}(\mathbf{x}) + m\mathbf{v}) \cdot d\mathbf{x} - (m/2)v^2 dt$$

- Transformation to guiding center variables: $\mathbf{x} \equiv \mathbf{R} + \rho$, $\mu \equiv v_\perp^2/2\Omega$, $\theta \equiv tan^{-1}(\frac{\mathbf{v}\cdot\mathbf{e_1}}{\mathbf{v}\cdot\mathbf{e_2}}), \ldots$
- The zero-th order phase space Lagrangian for guiding center:

$$\gamma_0 = (e\mathbf{A}(\mathbf{R}) + mv_{\|}\mathbf{b}(\mathbf{R})) \cdot d\mathbf{R} + \frac{\mu B}{\Omega} d\theta - H_0 dt$$

angle variable θ is ignorable
action is an adiabatic invariant μ

$$H_0 = \mu B + (m/2)v_{\|}^2$$

Euler-Lagrange Equation

- From variation of phase space Lagrangian:

$$\frac{d\theta}{dt} = \Omega, \quad \frac{d\mu}{dt} = 0$$

  Decoupling of gyromotion, adiabatic invariant

- $$-e\mathbf{B}^*\times\frac{d\mathbf{R}}{dt} - m\mathbf{b}\frac{dv_\parallel}{dt} = \mu\nabla B$$

  where $\mathbf{B}^* \equiv \nabla\times(\mathbf{A} + \frac{m}{e}v_\parallel\mathbf{b}) = \mathbf{B} + \frac{m}{e}v_\parallel\nabla\times\mathbf{b}$

- Decompose via $\mathbf{b}\times$ and $\mathbf{B}^*$, to get

$$\frac{d\mathbf{R}}{dt} = v_\parallel\frac{\mathbf{B}^*}{B^*} + \frac{\mu}{e}\frac{\mathbf{b}}{B^*}\times\nabla B,$$

  and

$$\frac{dv_\parallel}{dt} = -\frac{\mu}{m}\frac{\mathbf{B}^*}{B^*}\cdot\nabla B$$

More on Guiding Center Drift

- Frequently asked question:
  "**Where is the curvature drift?**"
  Using an identity $\mathbf{B}^* = B^*\mathbf{b} + \frac{m}{e}v_{\|}\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b}$:

$$\frac{d\mathbf{R}}{dt} = v_{\|}\frac{B^*\mathbf{b} + \frac{m}{e}v_{\|}\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b}}{B^*} + \frac{\mu}{e}\frac{\mathbf{b}}{B^*}\times\nabla B$$

- Infrequently asked question: "**Do conventional guiding center drifts conserve energy?**"

$$\frac{d\mathbf{R}}{dt} = v_{\|}\mathbf{b} + \mathbf{v}_{curv} + \mathbf{v}_{gradB}, \quad \frac{dv_{\|}}{dt} = -\frac{\mu}{m}\mathbf{b}\cdot\nabla B$$

  do *not* conserve energy exactly, while our E-L eqns do.
- $\mathbf{B}^*$ is a manifestation of Hamiltonian structure
- B^* is the density of phase-volume, $d^6\mathbf{Z} = B^* d\mu d\theta dv_{\|} d^3\mathbf{R}$

Usual Practice Violates Conservation Laws

– Frequently committed mistakes in simulation community:

- *"I have a freedom to ignore small terms!"*
  e.g., $B* \to B$
  Error in the phase density volume: Artificial dissipation
- *"I am using equations from a text book."*

$$\frac{d\mathbf{R}}{dt} = v_{\|}\mathbf{b} + \mathbf{v}_{curv} + \mathbf{v}_{gradB}, \quad \frac{dv_{\|}}{dt} = -\frac{\mu}{m}\mathbf{b}\cdot\nabla B$$

  Error in single particle kinetic energy
- VERDICT: Did Little Physics out of Advanced Theory

- SENTENCE: Life-time Community Service

Lie Perturbative Analysis I.

[from Hahm, PF **31**, 2670 '88]

- Consider electrostatic fluctuation only (for illustration): $\delta\phi(\mathbf{x}) = \delta\phi(\mathbf{R} + \rho)$
- While gyromotion has been decouple in the zero-th order phase space Lagrangian, it appears again in the perturbation. Since it is $O(\epsilon_\phi)$, we can remove it via *near-identity, phase-space preserving* Lie transform.
- In addition to zero-th order γ_0, $\gamma_1 = -e\delta\phi(\mathbf{R} + \rho)dt$
- Perform Lie-perturbation:

$$\Gamma_1 = \gamma_1 - L_1\gamma_0 + dS_1$$

where $(L_1\gamma)_\mu = g_1^\nu(\frac{\partial\gamma_\mu}{\partial z^\nu} - \frac{\partial\gamma_\nu}{\partial z^\mu})$, transformation of 1 form

Lie Perturbative Analysis II.

- One can choose the gauge function S_1 and the *generator* $\mathbf{g}_1$ such that the gyrophase is removed from Γ_1
- $\Omega\frac{d}{d\theta}S_1 - \frac{\partial S_1}{\partial t} - \frac{d\mathbf{R}}{dt}\cdot\nabla S_1 - \frac{dv_\parallel}{dt}\frac{\partial}{\partial v_\parallel}S_1 = \frac{e}{\Omega}(\delta\phi - <\delta\phi>)$
- Using $\epsilon_{k,\omega} << 1$, we obtain
  $dS_1 = \frac{e}{\Omega}(\delta\phi - <\delta\phi>)d\theta$
- $$\Gamma_1 = -e <\delta\phi> dt$$
  where $<...>$ is the gyrophase average $\frac{1}{2\pi}\int(...)$
- Note that *decoupled gyrophase information* is kept in S_1 and $\mathbf{g}_1$ to be used later when necessary.

Lie Perturbative Analysis III.

- Now, $\Gamma = \Gamma_0 - e < \delta\phi > dt$,
  $H = H_0 + H_1 = \mu B + (m/2)v_\parallel^2 + e < \delta\phi >$
- Euler-Lagrange Equation

$$\frac{d\mathbf{R}}{dt} = v_\parallel \frac{\mathbf{B}^*}{B^*} + \frac{\mathbf{b}}{B^*} \times (\frac{\mu}{e}\nabla B + \nabla < \delta\phi >),$$

  and

$$\frac{dv_\parallel}{dt} = -\frac{1}{m}\frac{\mathbf{B}^*}{B^*} \cdot (\mu\nabla B + e\nabla < \delta\phi >)$$

- $\mathbf{B}^*$ correction in the last term crucial for momentum pinch
- The second order perturbation in $\epsilon_\phi \sim \rho/L_p$ is necessary for energy conservation.

Gyrokinetic Vlasov-Poisson System

- With Euler-Lagrange Eqns, Gyrokinetic Vlasov equation for gyrocenter distribution function $F(\overline{R}, \overline{\mu}, \overline{v}_{\|}, t)$ is:

$$\frac{\partial F}{\partial t} + \frac{d\overline{R}}{dt} \cdot \overline{\nabla} F + \frac{d\overline{v}_{\|}}{dt} \frac{\partial F}{\partial \overline{v}_{\|}} = 0$$

  Note reduction of dimensionality achieved by $\frac{\partial F}{\partial \theta} = 0, \frac{d\overline{\mu}}{dt} = 0$
- Self-consistency is enforced by the Poisson's equation. Debye shielding is typically irrelevant, one must express the ion particle density $n_i(\mathbf{x})$ in terms of the gyrocenter distribution function $F(\overline{R}, \overline{\mu}, \overline{v}_{\|}, t)$
- Lee [PF **26**, 556 '83] has identified the *polarization density* (in addition to the guiding center density). It was a key breakthrough in advances in GK particle simulations.

$$\delta n_i(\mathbf{x}) = \delta n_{gc} + \rho_i^2 \nabla_\perp \cdot N_0 \nabla_\perp (e\delta\phi / T_i)$$

Pullback Transformation

- Widespread Misconception:
  "Gyrokinetic theory throws away the gyrophase dependent part of F."
- The gyrophase dependent information is kept in the gauge function S_1 or a generator $\mathbf{g}_1$.
- This can be used *reversibly* whenever one wants to calculate a quantity in the particle frame from the gyrocenter distribution function.

$$\int d^6\overline{Z}\ (T_G^* F(Z))\, K(\overline{R})\delta^3(\overline{R} - \mathbf{x} + \overline{\rho}) \to K(x)$$

- Examples include the polarization density, diamagnetic current, and other quantities related to finite Larmor radius effects.

Polarization Density

- More systematic derivation of GK Poisson's eqn started since Dubin *et al.*,[PF **26**, 3524 '83] via *pullback* transformation:

$$\nabla^2\delta\phi = -4\pi e[\int d^6\overline{Z}\ (T_G^*\delta f)\,\delta^3(\overline{R}-\mathbf{x}+\overline{\rho}) - \delta n_e(\mathbf{x},t)],$$

  where

$$T_G^*\delta f \equiv \delta f + \left(\frac{\partial S_1}{\partial\overline{\theta}}\right)\frac{\partial F_0}{\partial\overline{\mu}} + [\frac{1}{\Omega}\left(\overline{\nabla}S_1\right)\times\mathbf{b}]\cdot\overline{\nabla}F_0$$

- Contribution to the ion particle density which involves S_1 is the general form of polarization density. After linearization,

$$\{k^2\lambda_{Di}^2\}\frac{e\delta\phi_{\mathbf{k}}}{T_{i\perp}}n_0 + \{1-\Gamma_0(b)\}\frac{e\delta\phi_{\mathbf{k}}}{T_{i\perp}}n_0 = \delta\overline{N}_{i\mathbf{k}} - \delta n_{e\mathbf{k}}$$

- It is obvious that the *polarization density* statisfies

$$\frac{\partial}{\partial t}\delta n^{pol} + \frac{\partial}{\partial\mathbf{x}}\cdot n_0\mathbf{v}^{pol} = 0$$

Conservation of Energy
and Phase-Space Volume

- It is straight-forward to show the Liouville's theorem:

$$\overline{\nabla}\cdot\left(B_{\parallel}^{*}\frac{d\overline{R}}{dt}\right)+\frac{\partial}{\partial\overline{v}_{\parallel}}\left(B_{\parallel}^{*}\frac{d\overline{v}_{\parallel}}{dt}\right)=0$$

- The invariant energy for GK Vlasov-Poisson system is obtained by transforming the energy constant of the original Vlasov-Poisson system [Dubin *et al.*,'83]

$$E=\int d^6\mathbf{Z}\,F_i(\mu B+\frac{M}{2}v_{\parallel}^2)+\int d^6\mathbf{z}\,f_e(\mathbf{z})\frac{1}{2}m_e v^2$$

$$+\frac{1}{8\pi}\int d^3\mathbf{x}\,|\mathbf{E}|^2+\frac{e^2}{2\Omega}\int d^6\mathbf{Z}\,F_i\left(\frac{\partial}{\partial\mu}\langle\delta\tilde{\phi}^2\rangle+\frac{1}{\Omega}\langle\nabla\delta\tilde{\Phi}\times\mathbf{b}\cdot\nabla\delta\tilde{\phi}\rangle\right)$$

Note that the sloshing energy (last term) can be obtained from perturbation up to $O(\epsilon_{\phi}^2)$.

Extensions to Edge

[for core transport barriers → Hahm, Phys. Plasmas **3**, 4658, '96]

Expansion in $\epsilon_B \sim \rho_i/L_E \sim \frac{B_\theta}{B}$:

- From $\rho_{ip} \sim L_P \sim L_E$,
  $u_E \sim u_{*i} \sim \frac{\rho_i}{L_p} v_{Ti}$, $\frac{e\Phi^{(0)}}{T_e} \sim 1$.
- $|S-1| \sim 1$ (banana orbit distortion), $\frac{\omega_E}{\Omega_i} \sim \epsilon_B^2$ (circular gyro-orbit)
  where $\omega_E \equiv \frac{(RB_\theta)^2}{B} \frac{\partial}{\partial \psi} (\frac{E_r}{RB_\theta})$ [Hahm-Burrell, PoP '95]
  $S \simeq 1 + (\frac{B}{B_\theta})^2 \frac{\omega_E}{\Omega_i}$ [Hinton-Kim, Furth-Rosenbluth, Shaing,...]
- The zero-th order phase space Lagrangian

$$\gamma_0 \equiv (e\mathbf{A} + m\mathbf{u}_E + m v_\| \mathbf{b}) \cdot d\mathbf{R} + \frac{\mu B}{\Omega} d\theta - H_0 dt$$

with a guiding-center Hamiltonian

$$H_0 = e\Phi + \mu B + (m/2)(v_\|^2 + u_E^2) + \frac{\mu B}{2\Omega} \mathbf{b} \cdot \nabla \times \mathbf{u}_E.$$

Summary

- Modern Nonlinear Gyrokinetic Theory has provided a firm theoretical foundation for recent remarkable advances in gyrokinetic simulations and associated theories.
- Its elegance and relative simplicity have contributed to deeper understanding of the gyrokinetic system, not only improving treatment of familiar ones, but also indentification of novel physics effect.
- Significant example: Turbulent Convective Pinch of Toroidal Momentum
- It should be useful for even more complicated systems where several expansion parameters exist.

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References on Nonlinear Gyrokinetic Theory I.

- Theoretically-oriented Recent Review

  Brizard and Hahm, Rev. Mod. Phys. **79**, 421 '07
- Pioneering paper on conventional NL GK

  Frieman and Chen, PF **25**, 502 '82
- NL GK for particle simulation:

  Lee, PF **26**, 556 '83
- Proto-type Modern NL GK using Hamiltonian method:

  Dubin, Krommes, Oberman, and Lee, PF **26**, 3524 '83

  Hagan and Frieman, PF **28**, 2641 '85

  Yang and Choi, Phys. Lett. A **108**, 25 '85 (Electrostatic)

  Hahm, Lee, and Brizard, PF **31**, 1940 '88

References on Nonlinear Gyrokinetic Theory II.

- Modern NL GK using phase-space Lagrangian Lie perturbation method:

  Hahm, PF **31**, 2670 '88 (General geometry, electrostatic)

  Brizard, J. Plasma Phys. **41**, 541 '89

  (General geometry, electromagnetic)

- NL GK for strongly rotating plasmas:

  Hahm, PF-B **4**, 2801 '92 (in slab)

  Brizard, PoP **2**, 459 '95 (in terms of toroidal rotation)

  Hahm, PoP **3**, 4658 '96 (in terms of E_r)

- Energy conservation theorem:

  Brizard, PoP **7**, 4816 '00

  Sugama, PoP **7**, 466 '00 (introduction of field theory)

References on Topics related to Modern NL GK using phase-space Lagrangian Method

- Bounce-averaged Nonlinear Kinetic equation

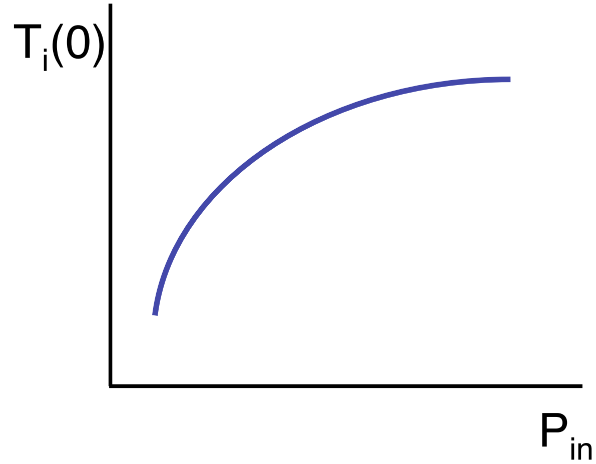
  Fong and Hahm, PoP **6**, 188 '99 (electrostatic)

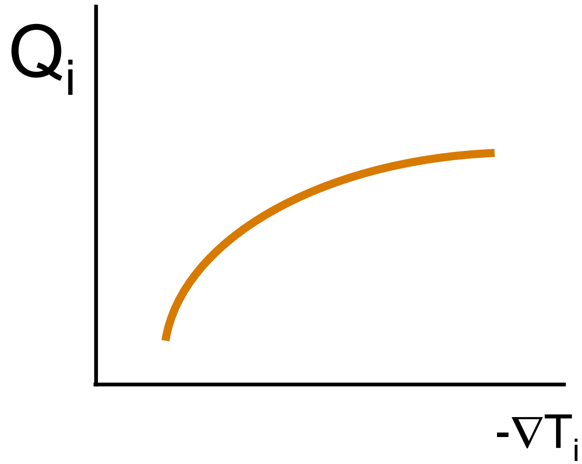
  Brizard, PoP **7**, 3238 '00 (electromagnetic)

- High frequency linear gyrokinetic theory:

  Qin and Tang, PoP **11**, 1052 '04 (recovery of compressional Alfven wave, elucidation of differential geometrical meaning of pullback transformation)

Flux-Gradient Relation





Generalization of Fick's Law:

$$\begin{pmatrix} Q_i \\ Q_e \\ \Gamma \\ \Gamma_\phi \end{pmatrix} = -\begin{bmatrix} \chi_i & \cdots & \cdots & \cdots \\ \cdots & \chi_e & \cdots & \cdots \\ \cdots & \cdots & D & \cdots \\ \cdots & \cdots & \cdots & \chi_\phi \end{bmatrix} \begin{pmatrix} \nabla T_i \\ \nabla T_e \\ \nabla n \\ \nabla U_\phi \end{pmatrix}$$

Outline

Properties of Tokamak Core Turbulence

Nonlinear Gyrokinetic Description

Some Outstanding Confinement Physics Issues:

Ion Thermal Transport with Zonal Flows

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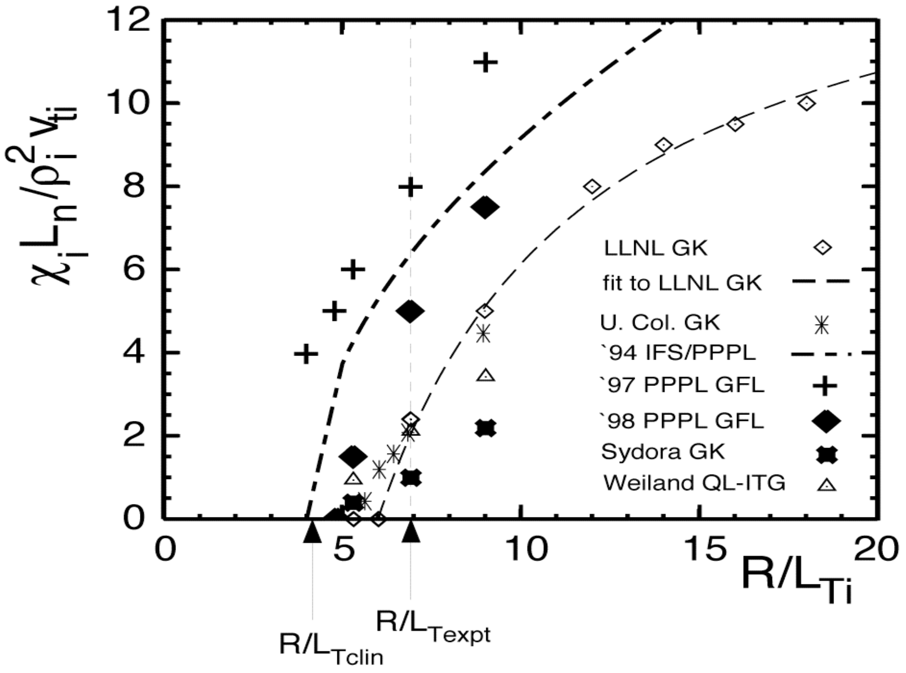
Ion Thermal Transport

* Better understood, compared to other transport channels
* In Aux-heated plasmas, typically $\chi_i >> \chi_{i,Neo}$ (cf. ITBs)
* Ion Temperature Gradient (ITG) Turbulence: Best Candidate
* With recent advances in gyrokinetic codes, simulation results begin to converge for simple cases, not only in numbers, but also in underlying physics.

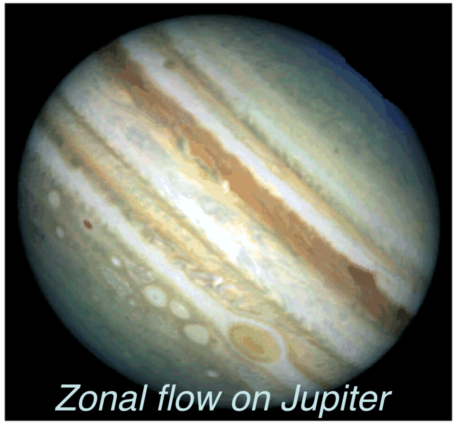
The effective upshift of onset condition for large ion heat flux is caused by Zonal flows

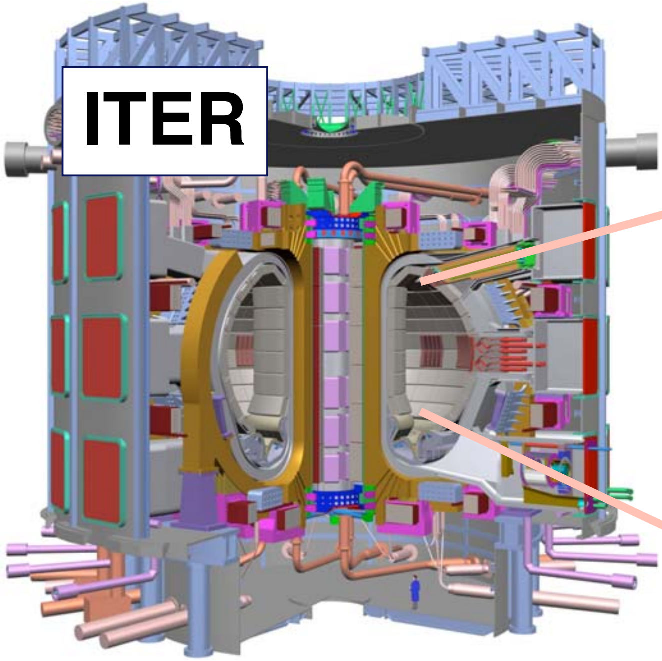
[Dimits et al., Phys. Plasmas, 2000] from Cyclone project

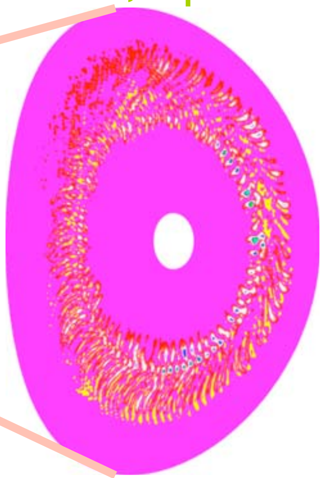


What is a zonal flow?

Made in Japan, edited in USA







From GTS

Drift Waves

↓

Drift waves + Zonal flows

Paradigm Change

ZFs are "modes", but:

1. No direct radial transport
2. No linear instability
3. Turbulence driven

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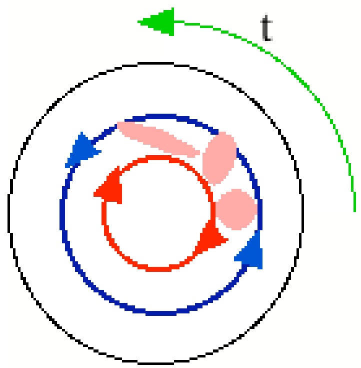
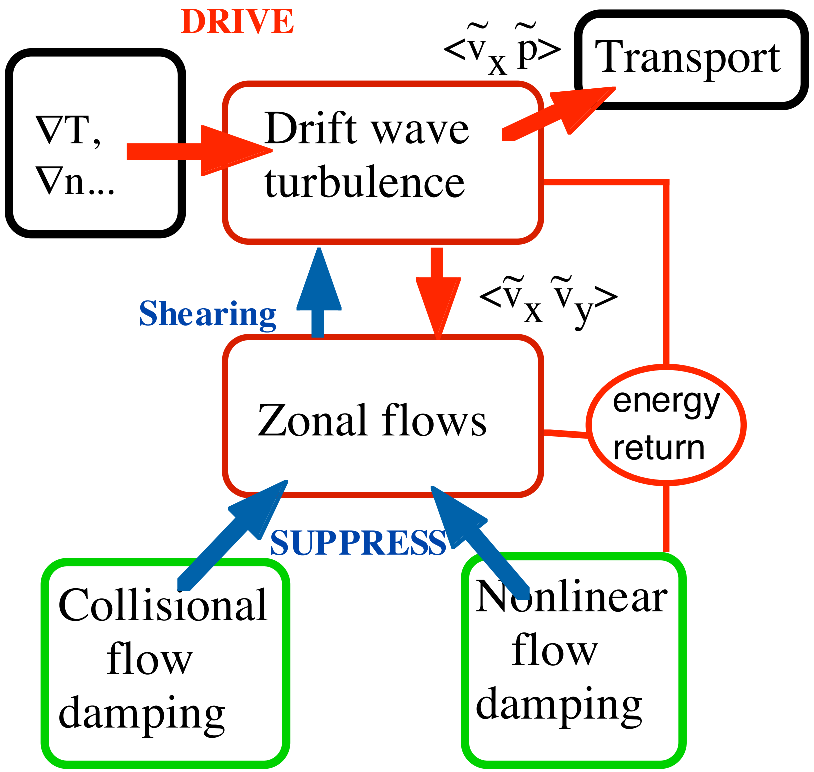
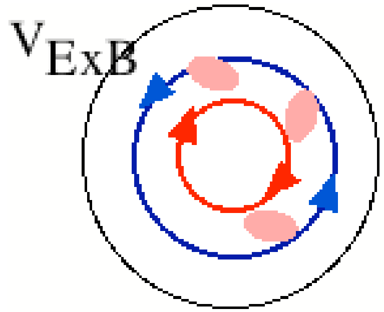
Basic Physics of a Zonal Flow

from Diamond,Itoh,Itoh, and Hahm, “Zonal Flows in Plasma-a Review” PPCF ‘05

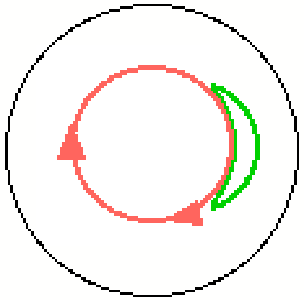


Suppression of DW by Shearing

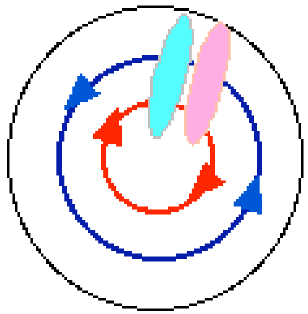




Generation by Vortex Tilting



Damping by Collisions



Role of E x B Shear in Reducing Turbulence

- ExB shear decorrelation in cylinder [Biglari-Diamond-Terry, PF-B '90]

$$\omega_E > \Delta\omega_T$$

- Turbulence quenching in gyrofluid simulation [Waltz-Kerbel-Milovich, PoP '94]

$$\omega_E > \gamma_{lin}$$

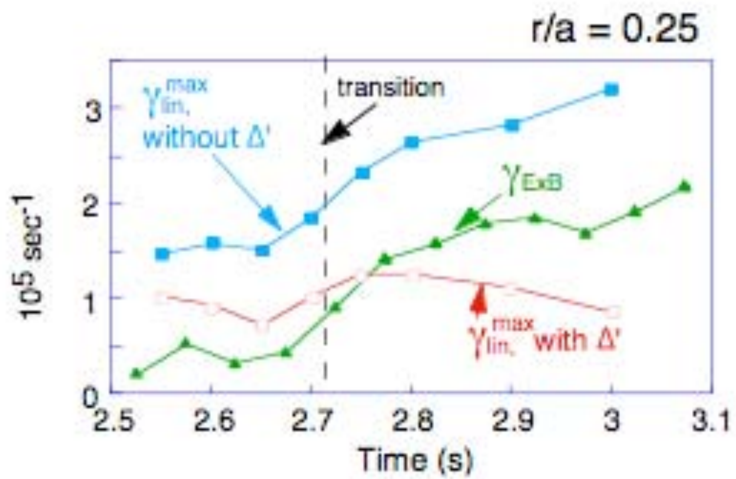
- ExB Shearing Rate in **General Toroidal Geometry** [Hahm-Burrell, PoP '95]

$$\omega_E = \frac{\Delta r_0}{\Delta \ell_\perp} \frac{(RB_\theta)^2}{B} \frac{\partial}{\partial \psi} \left(\frac{E_r}{RB_\theta} \right)$$

- Made possible by developments of experimental diagnostics for E_r and B_θ (MSE, CHERS, …)

- Useful Rule of Thumb for Indication of the importance of ExB shear

- Widely used for experimental results analysis (TFTR, DIII-D, JET, JT60-U, AUG, TEXTOR, NSTX, MAST, LHD, W7-AS,…)
with linear growth calculations via
FULL [Rewoldt,…]
Gryffin [Beer, …]
GS2 [Kotschenreuther,…]
Synakowski et al., PoP '97 ----->



Key Physics Mechanisms behind Size Scaling

- **Global Toroidal ITG eigenmode**

[Horton-Choi-Tang, PF '81] [Cowley-Kulsrud-Sudan, PF B' 91]

[Romanelli-Zonca, PF B' 93][Parker-Lee-Santoro, PRL'93]

Bohm Scaling ?

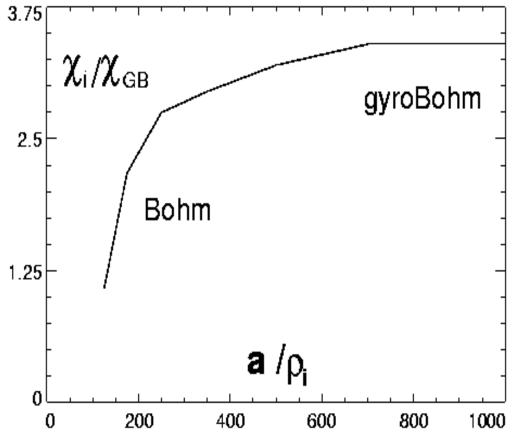
- **Self-regulation by Zonal Flows**:

[Cast of Thousands][Review: Diamond et al., PPCF '05]

GyroBohm scaling ?

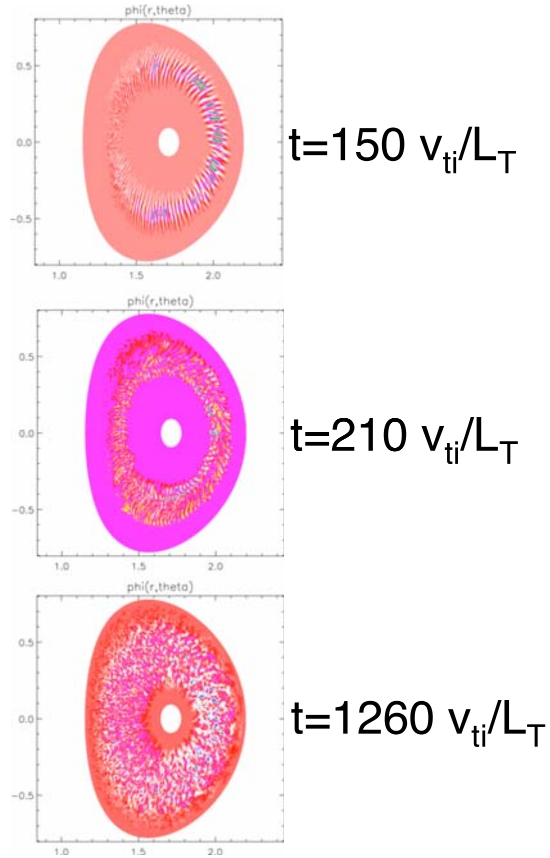
- **Turbulence spreading**

[Garbet et al., NF 94, PoP 08]
[Hahm, Diamond, Lin, et al., PPCF '04, PoP'05]
[Gurcan, Diamond, Hahm et al, PoP '05, 06]
[Naulin et al., PoP '05] [Waltz et al., PoP '05],…



Deviation from GyroBohm ?

[Lin, et al., PRL '02]



Density fluctuations from a GTS simulation of a shaped plasma with typical DIII-D core parameters
[Wang, Hahm, Lee *et al.,* PoP `07]

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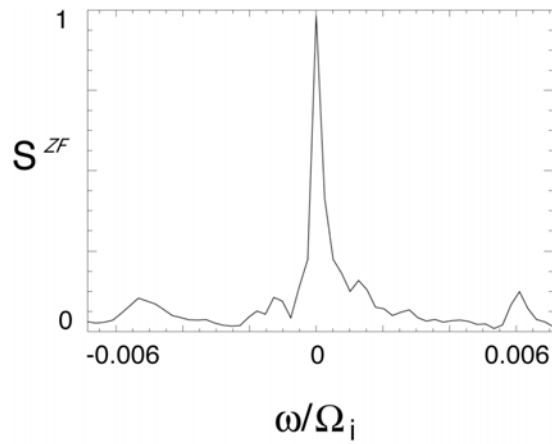
Characterization of Zonal Flow Properties Motivated Experimental Measurements

[Hahm, Burrell, Lin, Nazikian, Synakowski, PPCF '00]

From GTC simulations:

- n=0, m=0, broad k_r, potential fluctuation
- Broad-band zero-freq ZF & GAM
- Properties of associated density fluctuations

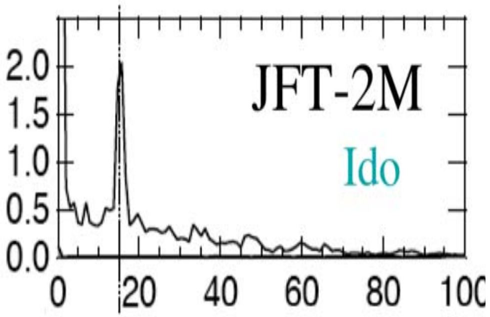


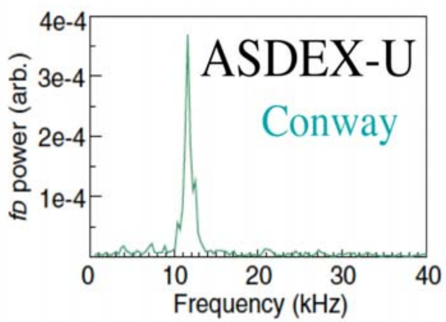
Experiments:

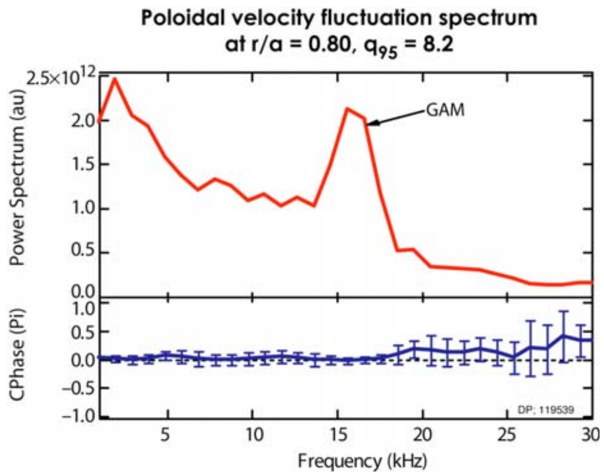
DIII-D (BES, PCI, Langmuir Probe)

TEXT, JFT-2M, JIPPT-IIU (HIBP),

AUG, (Doppler Reflectometry)







McKee, et al., BES on DIII-D [presented by Burrell, IAEA-TM, Sept. '05]

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E x B Shearing by time-dependent Zonal Flow

[Hahm, Beer, Lin, et al., PoP '99]

- Gyrofluid Simulations observed that instantaneous $\omega_E(t) >> \gamma_{\text{lin}}$ while turbulence was at L-mode level and transport was anomalous.

- Effective E x B shearing rate has been analytically derived to take into account the time dependence of zonal flows

- From Gyrofluid simulation data analysis, has been observed: $\omega_E^{\text{eff}} \sim \gamma_{\text{lin}}$

-- Shearing due to GAM is predicted to be ineffective for core turbulence.

- Gyrokinetic simulations demonstrated broadening of k_r of ITG turbulence (a symptom of eddy breaking-up) due to zonal flows quantitatively.

-- On the other hand, GAMs can affect edge turbulence, not only by shearing, but also by reducing zonal flows

[B.D. Scott, New Journal of Phys. (2005)]

Ion Thermal Transport scales with I_p in NSTX and Tokamaks

While not inconsistent with neoclassical theory in NSTX,…

Zonal flow characteristics depend on q values:

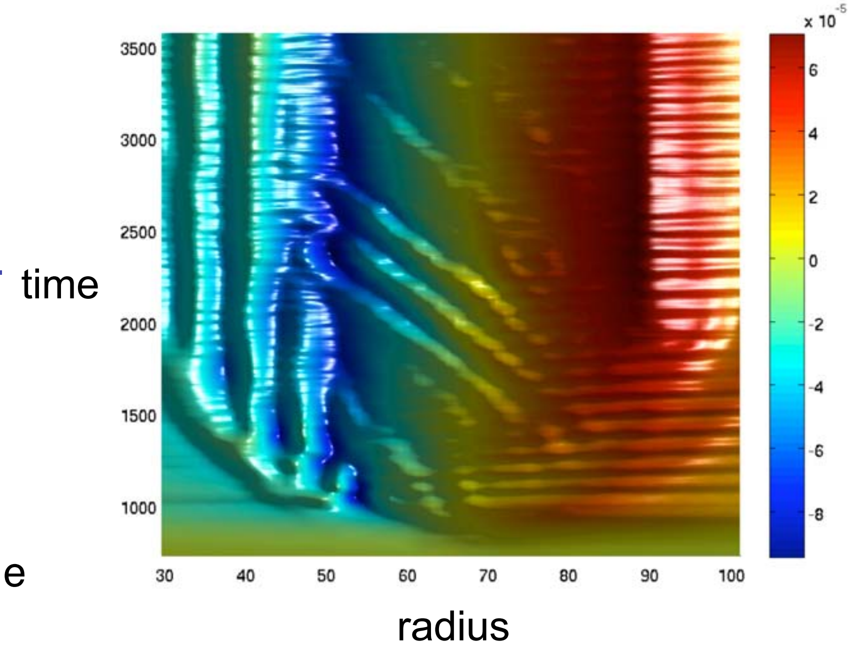
GAMs can exist only in high q region.
In low q region, Stationary Zonal Flows persists.

GAMs are less effective in reducing turbulence than Stationary ZFs, due to its high frequency
[Hahm, Beer, Lin et al., Phys. Plasmas '99]

Transport is consequently lower for lower q value
[Miyato et al., IAEA '04, Angelino et al., PPCF '06]

Can Experimental Relevance be tested on DIII-D with BES,…?
on NSTX
with poloidal CHERS, Reflectometry,… ?

and on TS ?



Stationary Zonal Flows

GAM

GTS simulation by W. Wang

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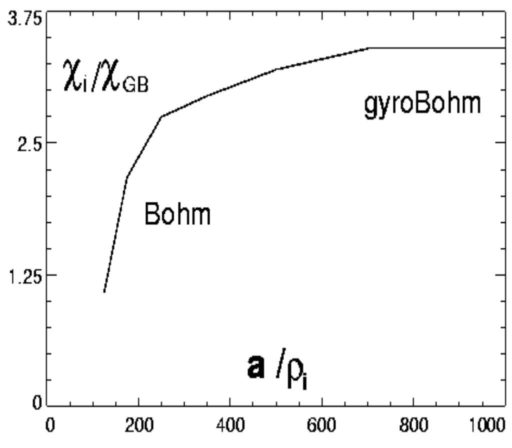
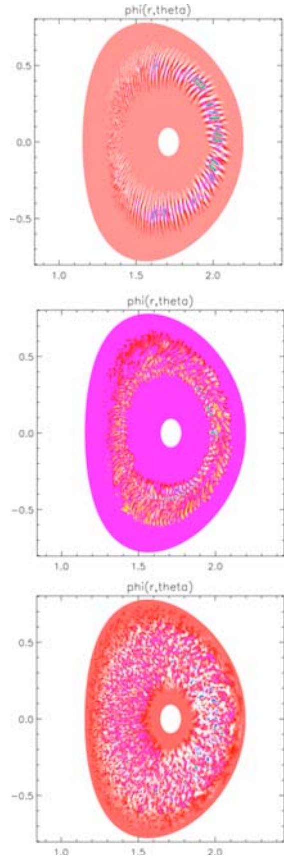


Deviation from GyroBohm ?

[Lin, et al., PRL '02]



$t=150\ v_{ti}/L_T$

$t=210\ v_{ti}/L_T$

$t=1260\ v_{ti}/L_T$

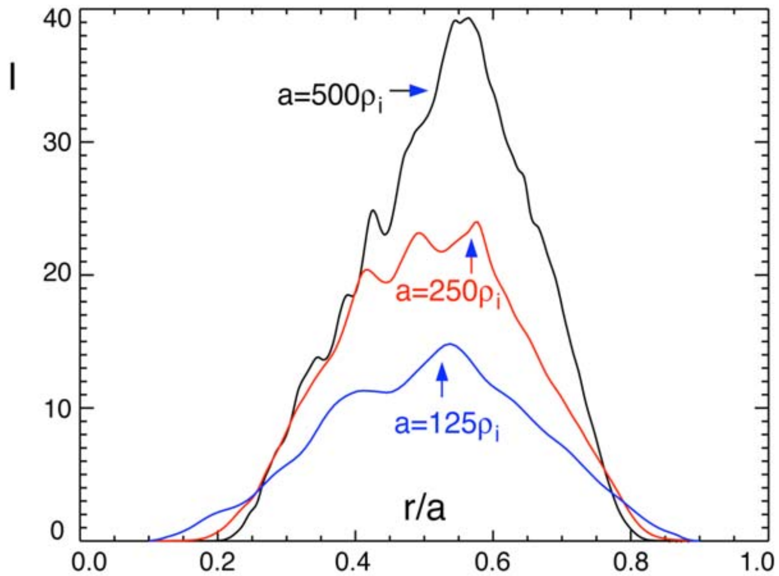
Density fluctuations from a GTS simulation of a shaped plasma with typical DIII-D core parameters
[Wang, Hahm, Lee *et al.,* PoP `07]

Deviation from GyroBohm Scaling due to Turbulence Spreading

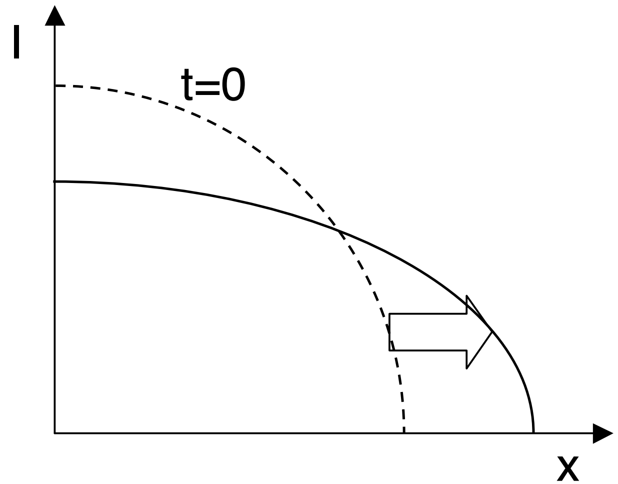
[Hahm,Diamond,Lin,Itoh,Itoh, PPCF '04]

- Garbet et al., '94 discussed spreading in the context of transient transport.
- Widely observed in many global gyrokinetic simulations [Sydora, Parker,Lin,…]
- Theoretical Research motivated by GK simulations reporting deviation from "gyroBohm" scaling for moderate system size

[Lin, Ethier, Hahm, Tang, PRL '02]



Range of fluctuation spreading into linearly stable zone: GK simulation: $\Delta \approx 25\ \rho_i$



Nonlinear diffusion model: $\Delta \approx 18\ \rho_i$

Turbulence Spreading from Edge to Core

[Hahm,Diamond,Lin,Rewoldt,Gurcan,Ethier, PoP '05]

- A model nonlinear diffusion equation

$$\frac{\partial}{\partial t} I = \gamma(x) I - \alpha I^2 + \chi_0 \frac{\partial}{\partial x}\left(I \frac{\partial}{\partial x} I \right)$$

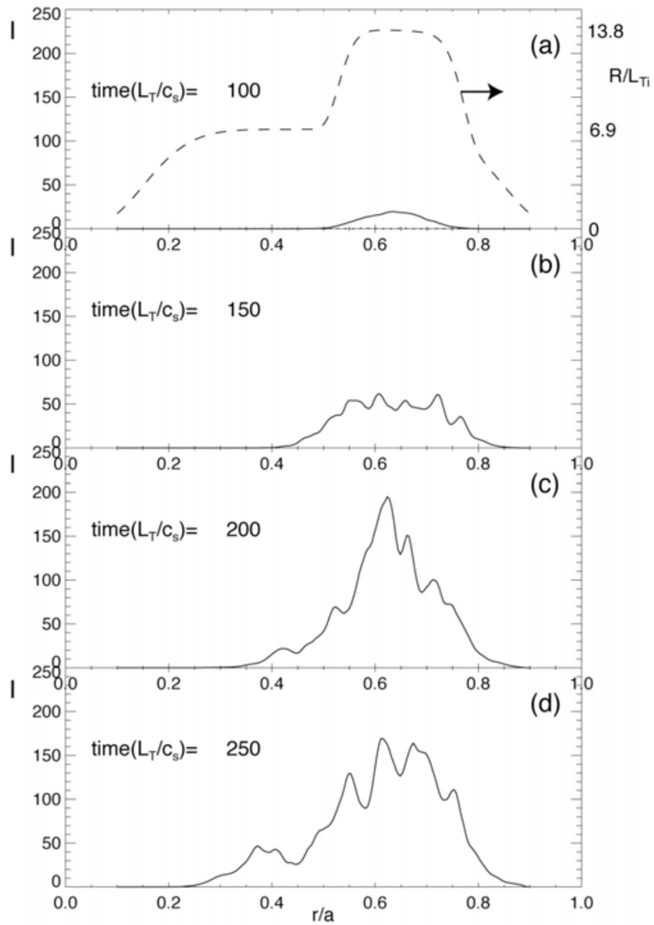
- **Ballistic Front Propagation a la** Fisher-Kolmogorov [Gurcan, Diamond, PoP'06]

$$U_x = \gamma^{1/2} \times \left(\frac{\chi_0 I}{2} \right)^{1/2}$$

- From simulation, initial turbulence growth at the edge is followed by ballistic front propagation into core

- Front speed increases with R/L_T

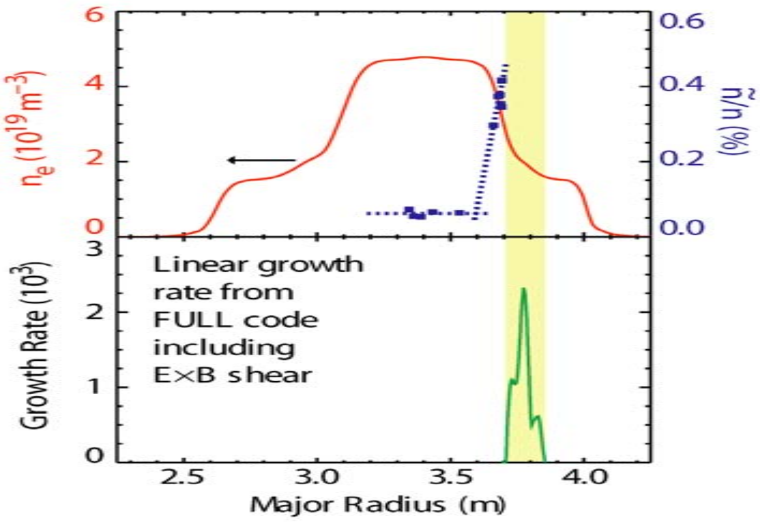
---> Fast response of core after L-H transition [JET, JT-60U, DIII-D…]?



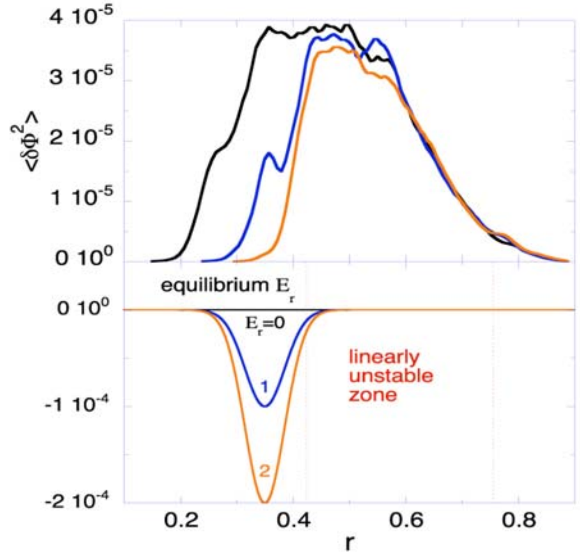
Turbulence Spreading through a Transport Barrier

- Often, transport is anomalous where pressure gradient is weak and linear instabilities are absent
e.g., NSTX [Kaye et al., NF '07]

- Non-zero fluctuations and anomalous transport observed in linearly stable zone of JT-60U reversed shear plasma
  [Nazikian et al.,PRL '05]

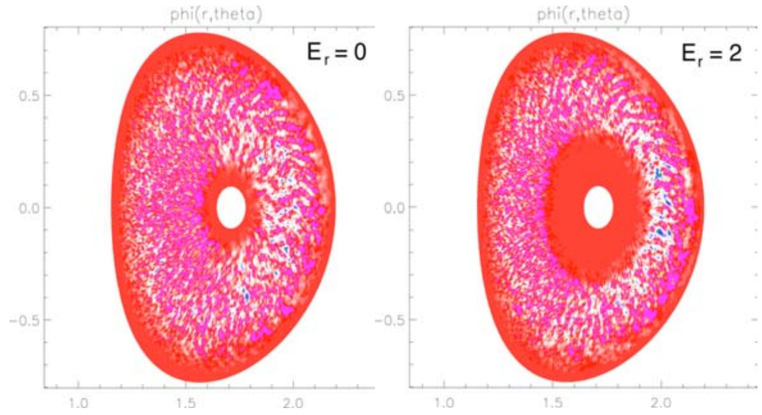


From GTS simulation [W.Wang et al.,] and simple analytic theory, study effects of E x B flow shear on turbulence spreading by placing E x B shear layer next to linearly unstable zone as a barrier

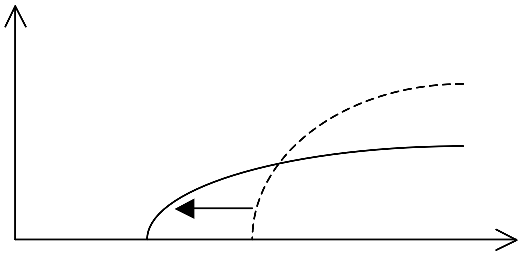


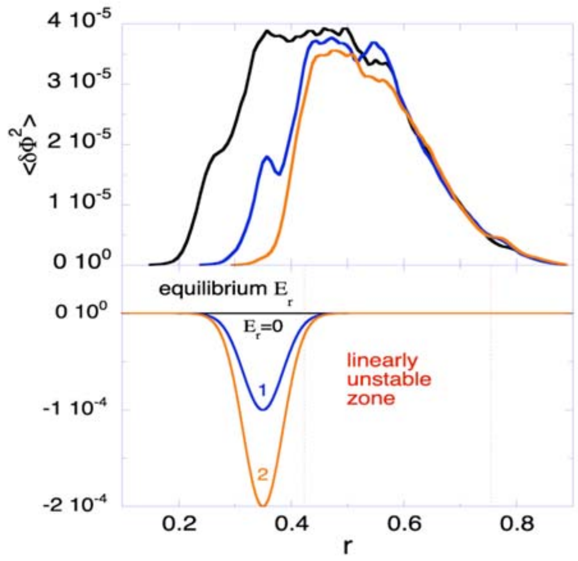
Turbulence spreading is reduced by E x B shear

- GTS nonlinear simulation of ITG turbulence exhibits significant turbulence spreading into the linearly stable zone
  [Wang et al., PoP 14, 072306 '07]



With ExB shear, spreading extent is reduced as expected from nonlinear diffusion model
[Hahm, Diamond, Lin et al., PPCF 46, A323 '04]:





$\Delta / V_s \sim 1/|\gamma|' \Delta \implies \Delta \sim (V_s / |\gamma'|)^{1/2}$

ω_{ExB} increases $|\gamma'|$

Look for ω_{ExB} effects on V_s

Spreading Speed is controlled by Local Value of ω_{ExB}

- Spatio-temporal evolution of front propagation:

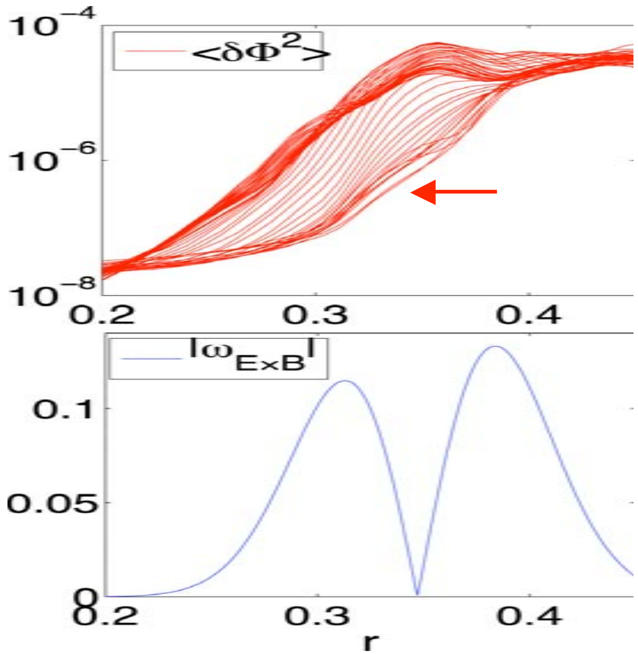
V_s increases with fluctuation intensity
[Hahm, Diamond, Lin et al., PPCF '04
Gurcan, Diamond, Hahm, PoP '05]

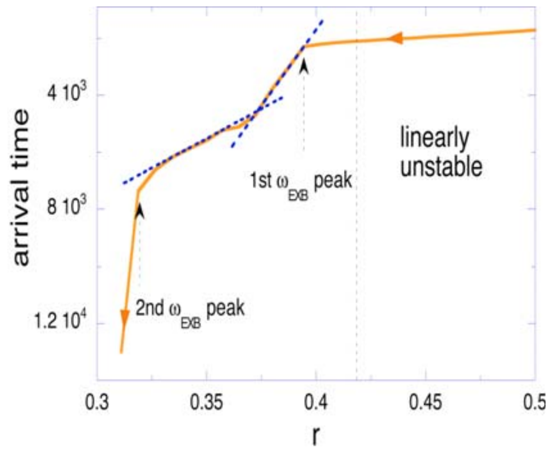
ω_{ExB} (not E_r) reduces fluctuation intensity locally

⇨ ω_{ExB} decreases V_s locally

$\Delta \sim (V_s / |\gamma'|)^{1/2}$

Rough agreement in trend





Rotation Plays a Central Role in Magnetic Confinement