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Patterns of sound radiation behind point-like charged obstacles in plasma flows.

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Patterns of sound radiation behind point-like charged obstacles in plasma flows

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Basic form of the dielectric function with isothermal electrons and cold ions:

$$\varepsilon(\omega,k)=1+\frac{1}{(k\lambda_D)^2}-\left(\frac{\Omega_{pi}}{\omega}\right)^2$$

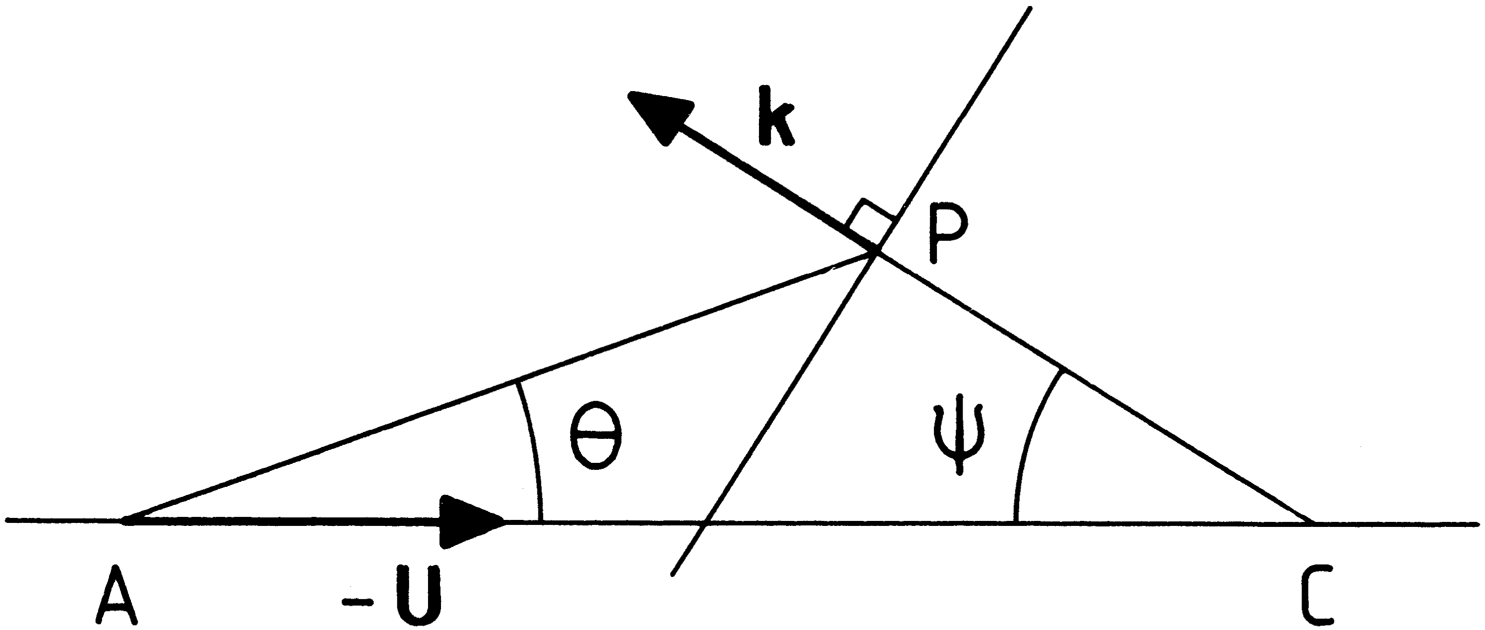
Dispersion relation, $\varepsilon(k,\omega) = 0$:

$$\omega(k)=\frac{C_s\,|\,k\,|}{\sqrt{1+(k\lambda_{De})^2}}$$

Basic assumtions:

1. All the relevant waves can be described by the (normalized) dispersion relation $\omega(k) = |k|/(1+k^2)$, as found before.
2. The wavepattern has reached a stationary stage.
3. We consider only the ”far field”, which can be described by geometrical optics.

Definitions of angles and directions:



In a moving frame:

$$\Omega(\mathbf{k}) = \omega(\mathrm{k}) - \mathbf{U}\cdot\mathbf{k}$$

The group velocity for the isotropic case
$\omega = \omega(|k|)$:

$$\nabla_k \Omega(\mathbf{k}) = \mathbf{u}_\mathrm{g} - \mathbf{U} = \mathrm{u}_\mathrm{g}\frac{\mathbf{k}}{\mathrm{k}} - \mathbf{U}$$

$$\nabla_k \Omega - component\ parallel\ to\ AC = u_g \cos(\psi) - U$$

$$\nabla_k \Omega - component\ perpendicular\ to\ AC = u_g \sin(\psi)$$

The direction of the group velocity $\nabla_k \Omega(\mathbf{k})$ must be parallel to *AP*. We then find:

$$\tan(\theta) = \frac{u_g \sin(\psi)}{U - u_g \cos(\psi)}$$

We have not yet used the form of $\omega = \omega(k)$, only its isotropy.

Use $u_g = 1/(1+k^2)^{3/2}$.
Also, for given U we have the relation between k and ψ:

$$\Omega(k,\psi) = \omega(k) - Uk\cos(\psi) = 0 \quad \textit{gives}$$

$$\cos(\psi) = \frac{1}{U\sqrt{1+k^2}}$$

We find:

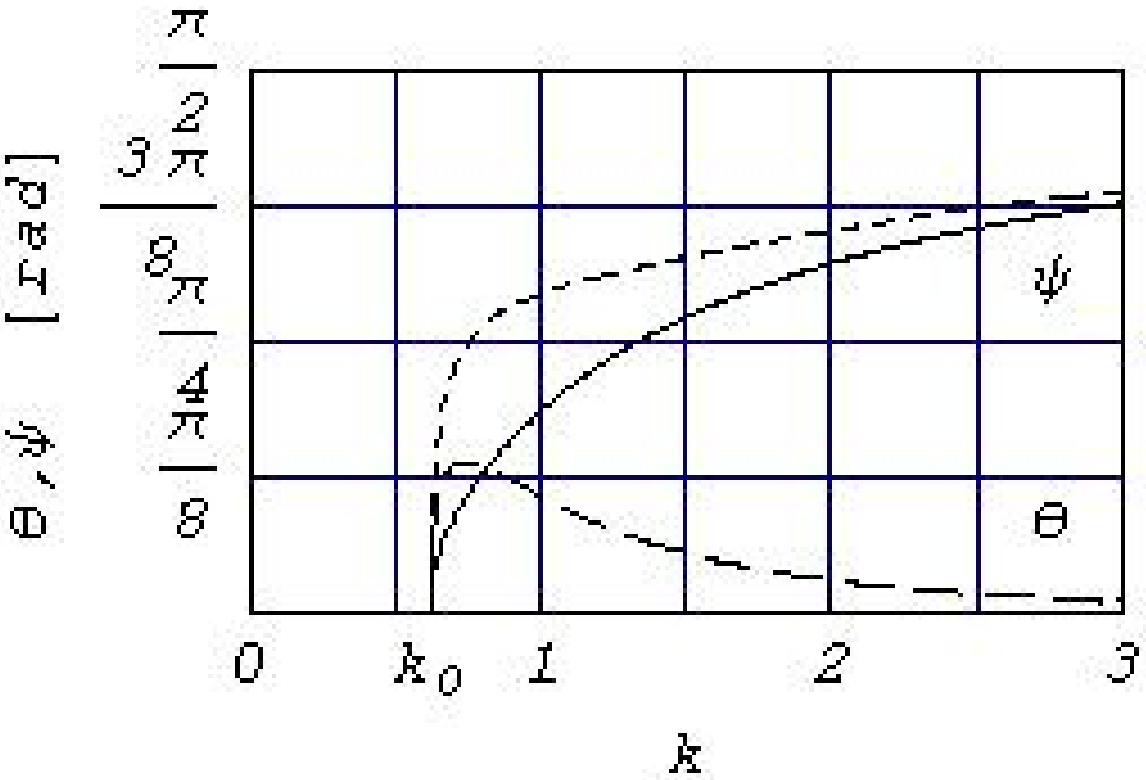
$$\tan(\theta) = \frac{\sqrt{U^2(1+k^2)-1}}{U^2(1+k^2)^2-1}$$

For $U < 1$ (i.e. Mach number < 1) we have a limiting minimum wavenumber (by $\cos(\Psi) = 1/U\sqrt{(1+k^2)} < 0$)

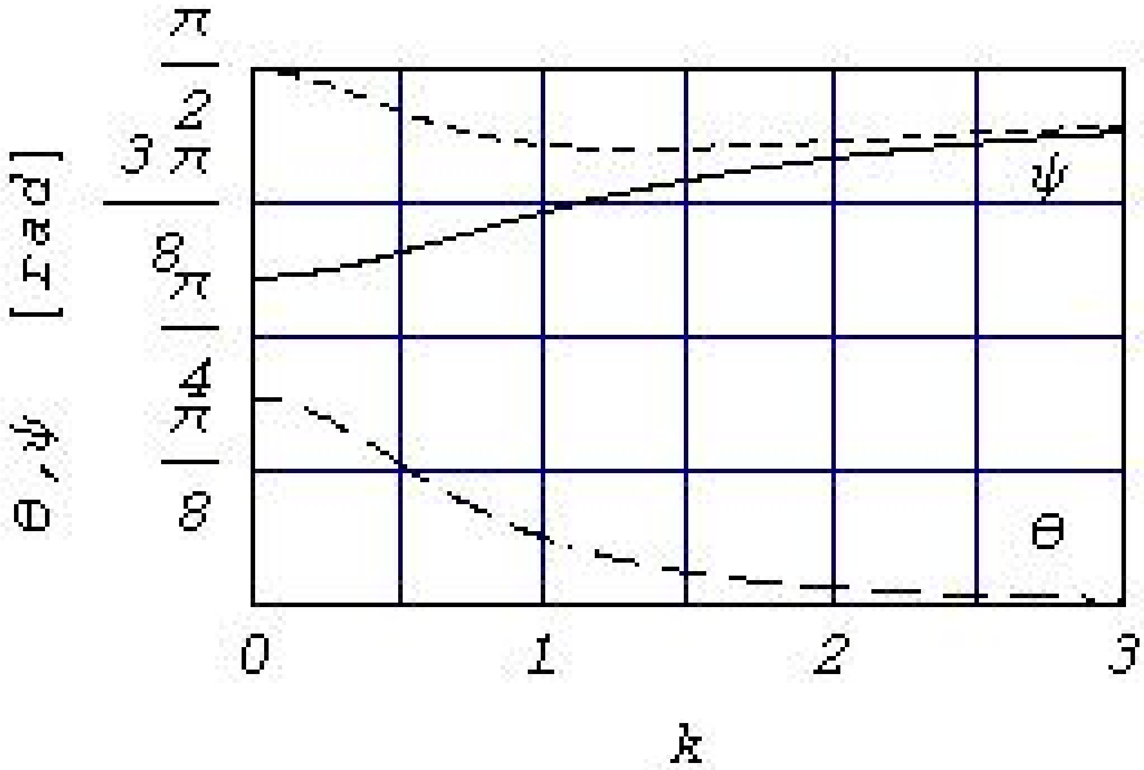
$$k_0 = \frac{\sqrt{1-U^2}}{U} \quad .$$

We have no such wavenumber for the supersonic case, $U > 1$

$U = 0.85$

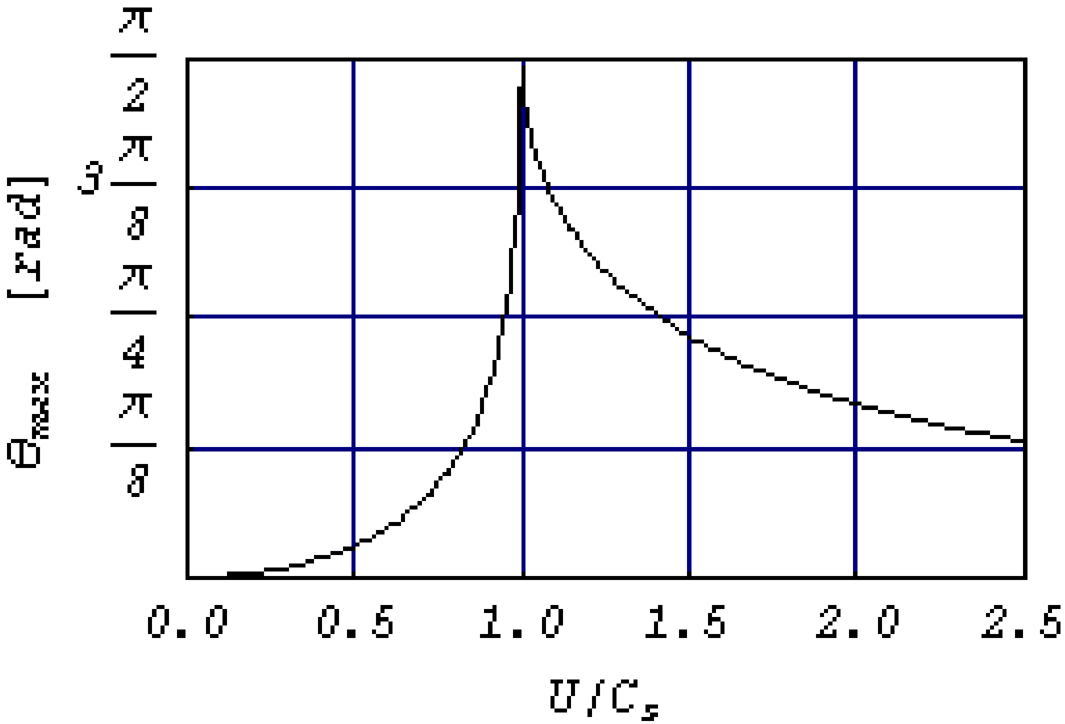

$\frac{\pi}{2}$
$\frac{3\pi}{8}$
$\frac{\pi}{4}$
$\frac{\pi}{8}$
Θ, ψ [rad]
ψ
Θ
0
k_0
1
2
3
k

$U = 1.75$



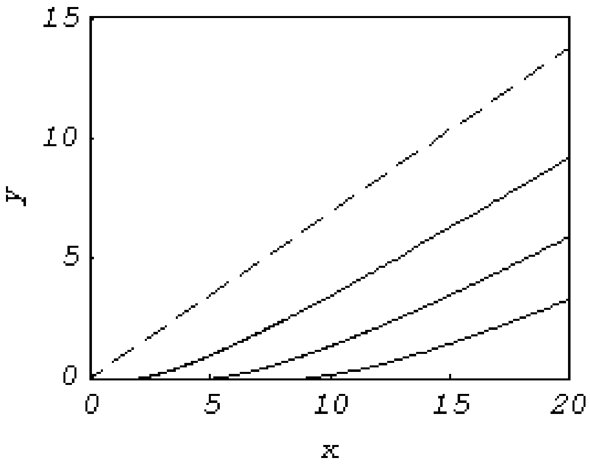
$\frac{\pi}{2}$
$\frac{3\pi}{8}$
$\frac{\pi}{4}$
$\frac{\pi}{8}$
Θ, ψ [rad]
0
1
2
3
k
ψ
Θ

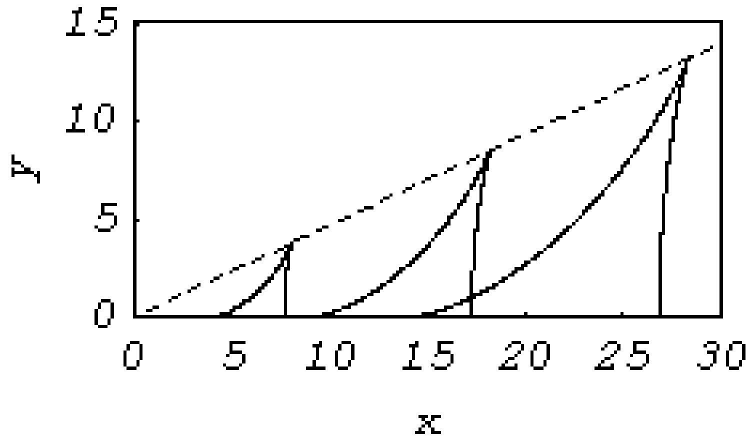
The maximum opening angle θ_{max} in the Mach cone, for varying U:



Phase-fronts determined by:

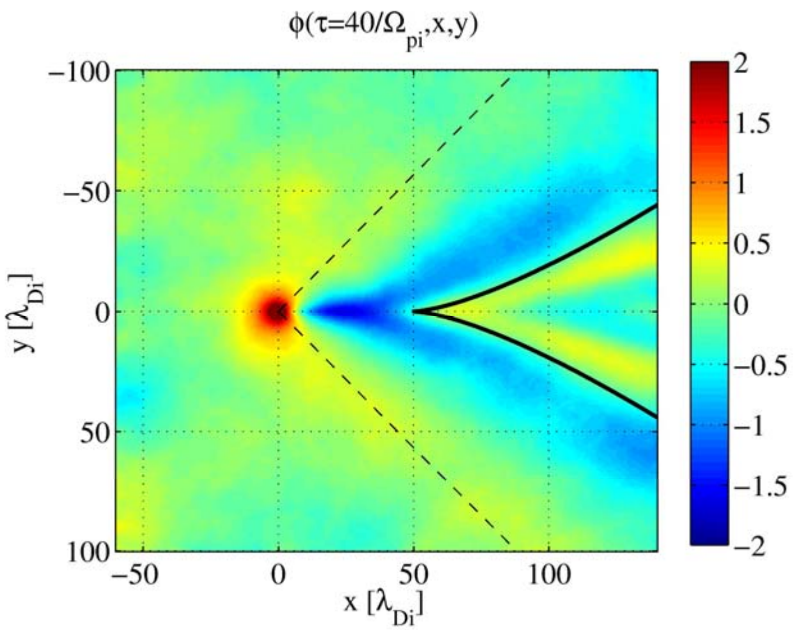
$$\frac{dr}{k_\theta} = \frac{r d\theta}{-k_r} \quad giving \quad \frac{dr}{d\theta} = r\tan(\theta + \psi)$$

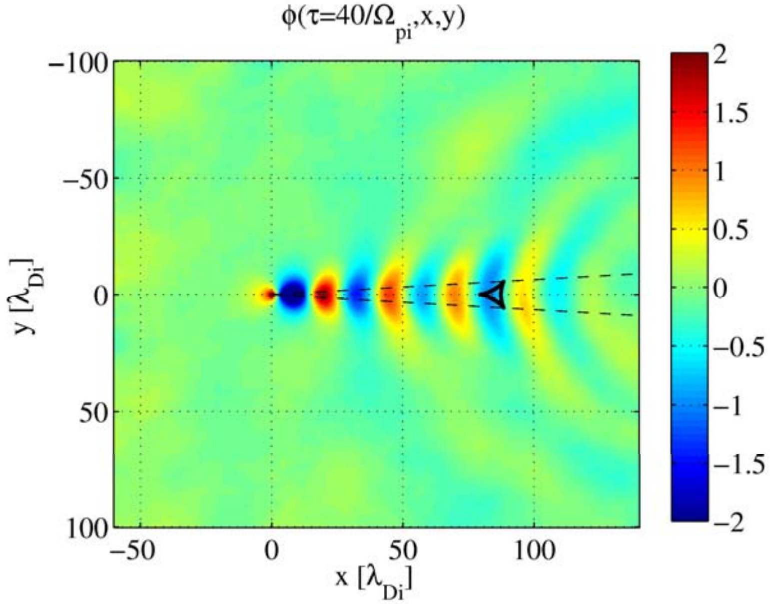


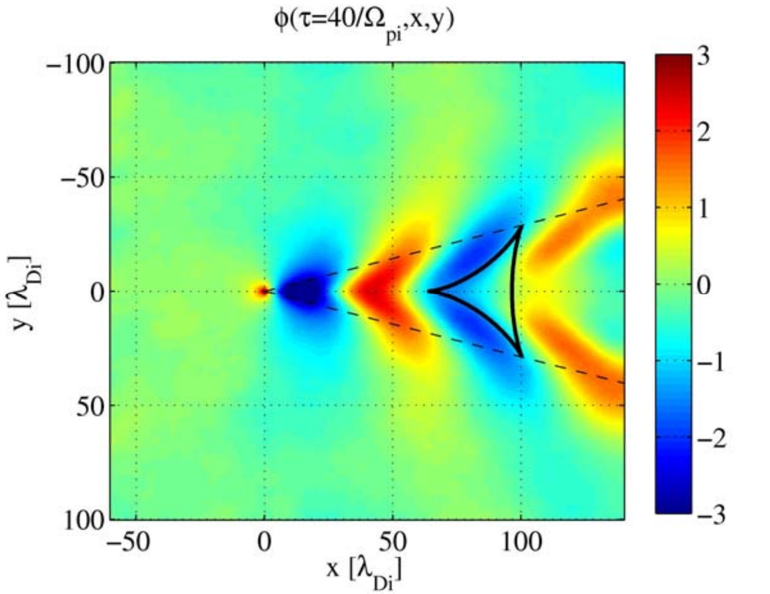


NB. No distinction here between two or three spatial dimensions! Also, the same results are found for moving single charges or dipoles, for instance.

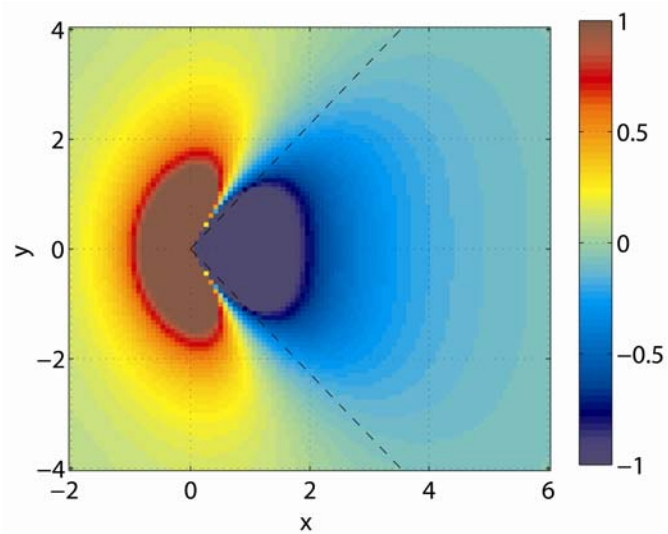
U = 1.33, 0.75, 0.42

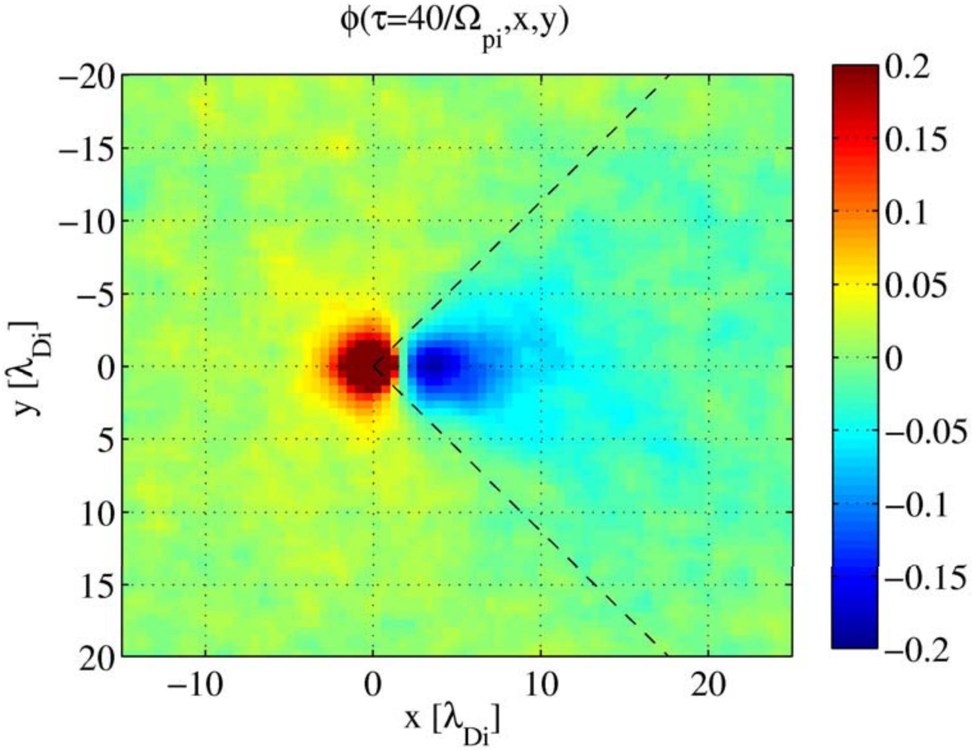

ϕ(τ=40/Ω_pi,x,y)
y [λ_Di]
x [λ_Di]


ϕ(τ=40/Ω_pi,x,y)
y [λ_Di]
x [λ_Di]

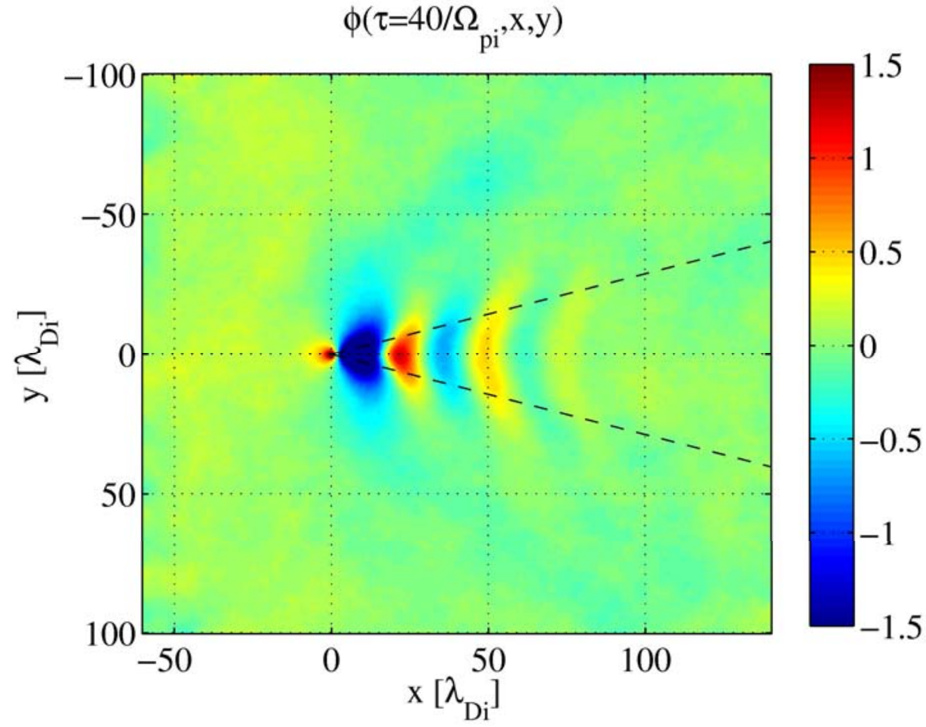

ϕ(τ=40/Ω_pi,x,y)
y [λ_Di]
x [λ_Di]

Importance of ion Landau damping, here with $T_i = T_e$, assuming quasi-neutrality in the analysis:





Limiting case of fluid vs. kinetic model?
Here $T_e = 25\ T_i$.



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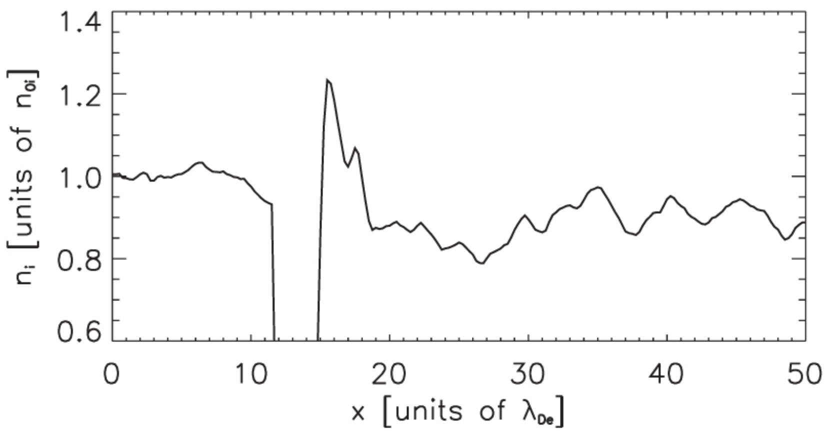
Ion focusing:



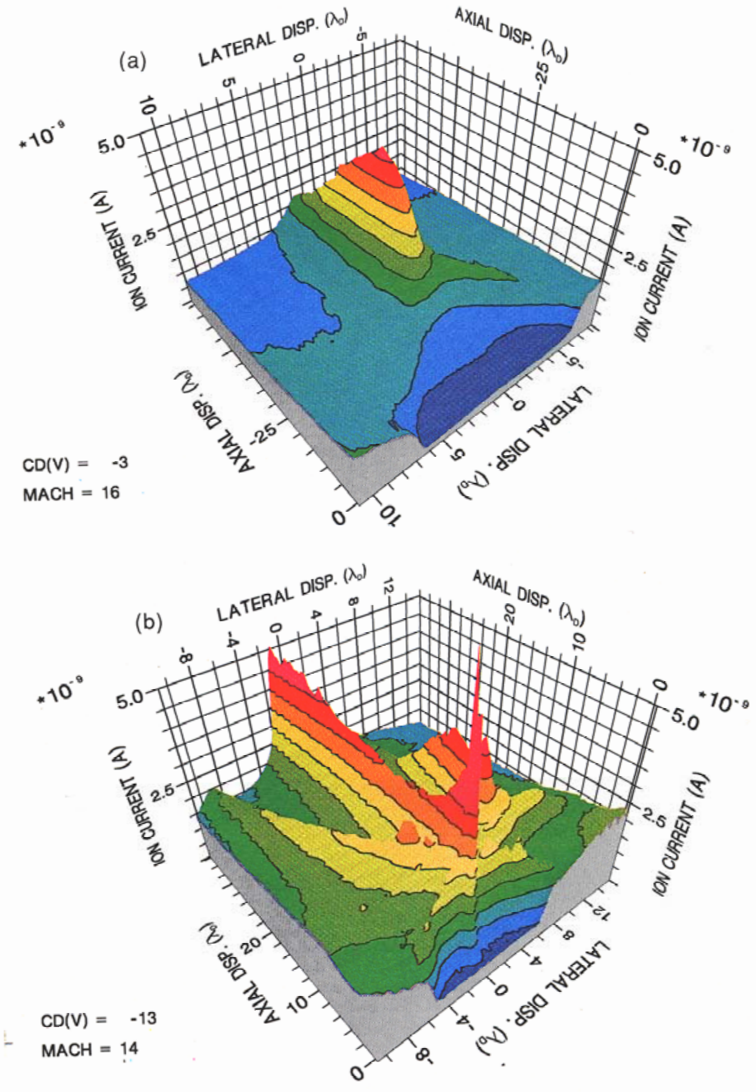
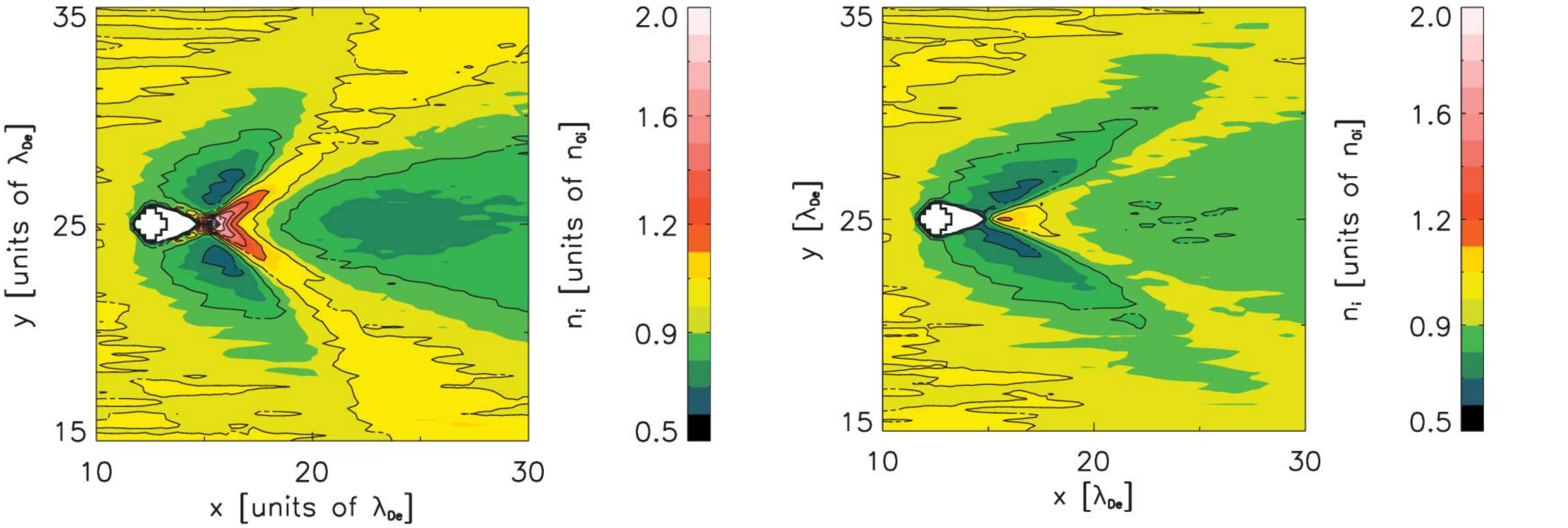


Fig. 9. Comparison between the plasma wake regions behind two circular discs at different potentials. The applied potentials are −3 V in the upper part and −13 V in the lower part (∼same Mach number)

$T_e = 80\ T_i$. Insulating and conducting materials, here with $U = 2.5\ C_s$.



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Thank you for your attention!