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Quantum Plasmas – Recent Developments

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Collaborators

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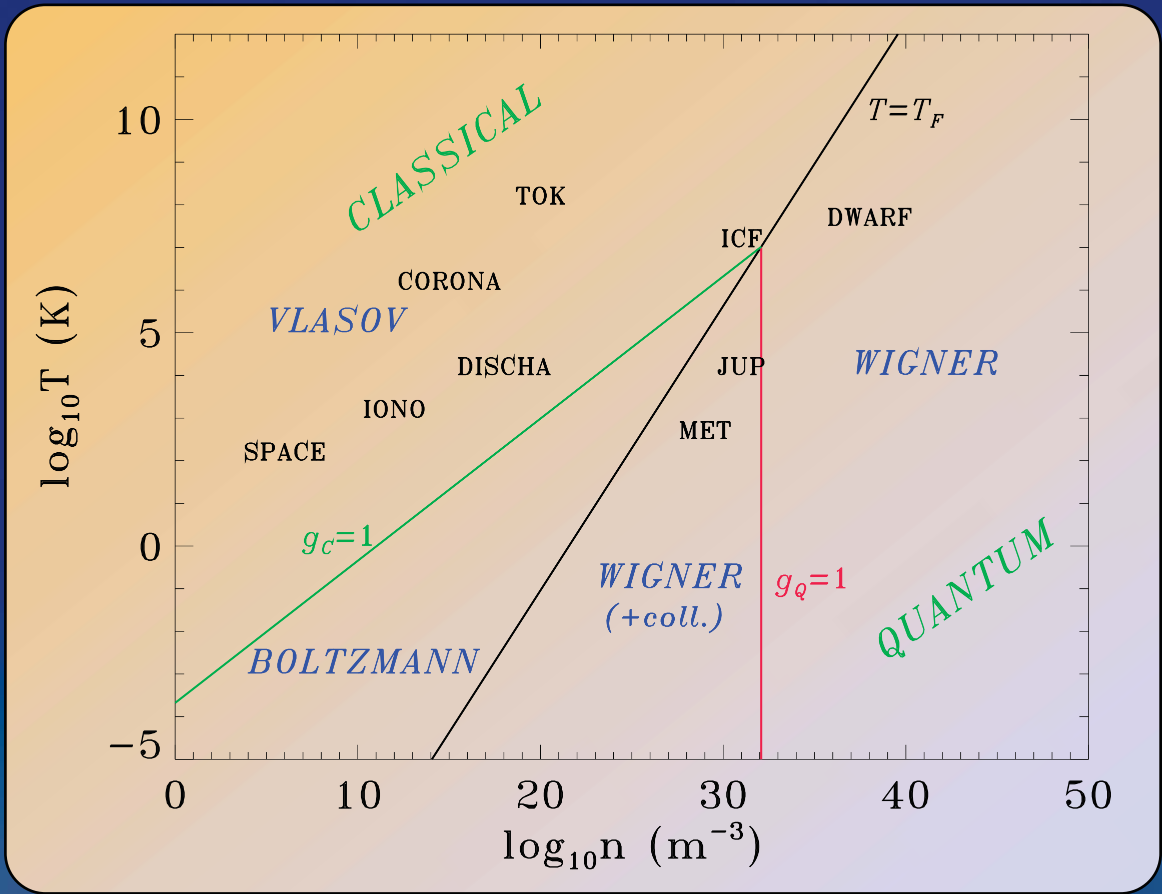
- P. K. Shukla (Ruhr-U. Bochum, also S. Ali)
- G. Manfredi (U. Strasbourg)
- C. S. Liu (U. Maryland)

Overview

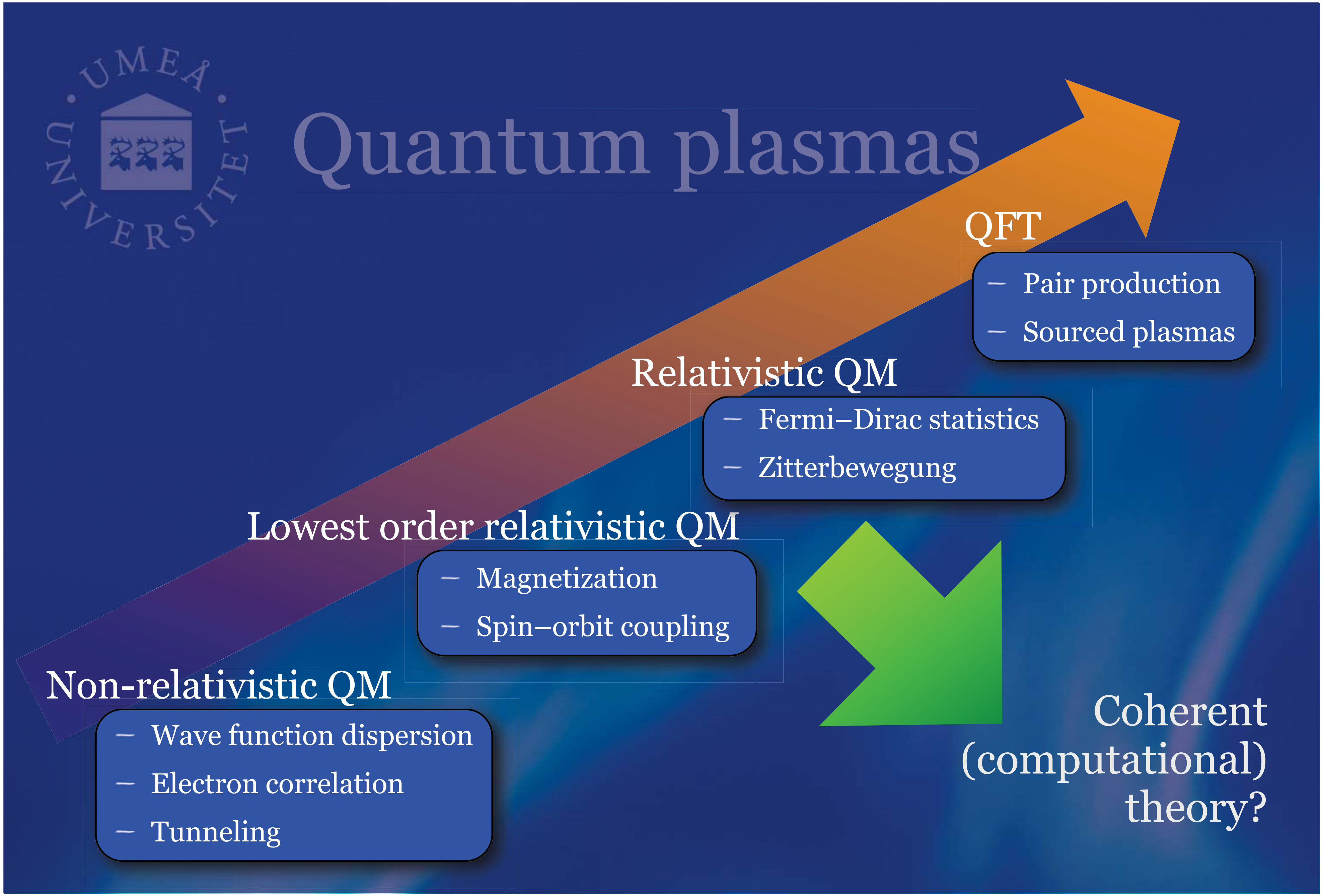
- Why look at quantum plasma effects?
- Schrödinger’s description.
- Non-relativistic single electron dynamics.
- Paramagnetic electrons.
- From micro to macro physics.
- MHD regime.
- Conclusions - what the future might bring.

Quantum plasmas



From G. Manfredi, Fields Inst. Comm **46**, 263 (2005)

Quantum plasmas


UMEÅ UNIVERSITET
Quantum plasmas
QFT
– Pair production
– Sourced plasmas
Relativistic QM
– Fermi–Dirac statistics
– Zitterbewegung
Lowest order relativistic QM
– Magnetization
– Spin–orbit coupling
Non-relativistic QM
– Wave function dispersion
– Electron correlation
– Tunneling
Coherent (computational) theory?

Quantum plasmas

- Manifold applications (Pines, 1961; Kremp et al. 2005)
  - Condensed matter systems (Gardner, SIAM J. Appl. Math., 1994).
  - Astrophysical environments (Harding & Lai, Rep. Prog. Phys., 2006).
  - Ultracold plasmas (Rydberg states) (Li et al., PRL, 2005).
  - Nanostructured materials (Craighead, Science, 2000).
  - Laser-plasmas (Glenzer et al., PRL, 2007).
  - Spintronics.
- Interesting fundamental aspects of matter dynamics.
- Quantum to classical transition?
- Collective quantum systems.

Schrödinger description

Electron properties described by complex scalar wavefunction ψ ($|\psi|^2$ = probability)

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

where we have the Hamiltonian operator

$$H = -\frac{\hbar^2}{2m_e}\nabla^2 - e\phi$$

and ϕ is the *external* electrostatic potential and e being the magnitude of the electron charge.

Schrödinger description

- Nice approach: allows for easy generalizations, new interactions can be incorporated in Hamiltonian.
- Microscopic equations of motion

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{1}{i\hbar}[F, H]$$

for some operator F, $[F, H]$ Poisson brackets.

Example:

$$\boldsymbol{v} \equiv \frac{d\boldsymbol{x}}{dt} = [\boldsymbol{x}, H] = \frac{\boldsymbol{p}}{m_e}$$

for previous scalar electron description.

Schrödinger description

> Quantum statistical dynamics

$$W(t,\boldsymbol{x},\boldsymbol{p}) = \frac{1}{(2\pi\hbar)^3}\int d^3y\, \exp(i\mathbf{p}\cdot\boldsymbol{y}/\hbar)\langle\psi^*(t,\boldsymbol{x}+\boldsymbol{y}/2)\psi(t,\boldsymbol{x}-\boldsymbol{y}/2)\rangle$$

$$\left(\frac{\partial}{\partial t}+\frac{\boldsymbol{p}}{m}\cdot\nabla\right)W+\frac{2e}{\hbar}\phi\sin\left(\frac{\hbar}{2}\overleftarrow{\nabla}\cdot\overrightarrow{\nabla}_p\right)W=0$$

> Fluid moments from quantum kinetic equation, or from summing up particle contributions in Madelung picture.

Schrödinger description

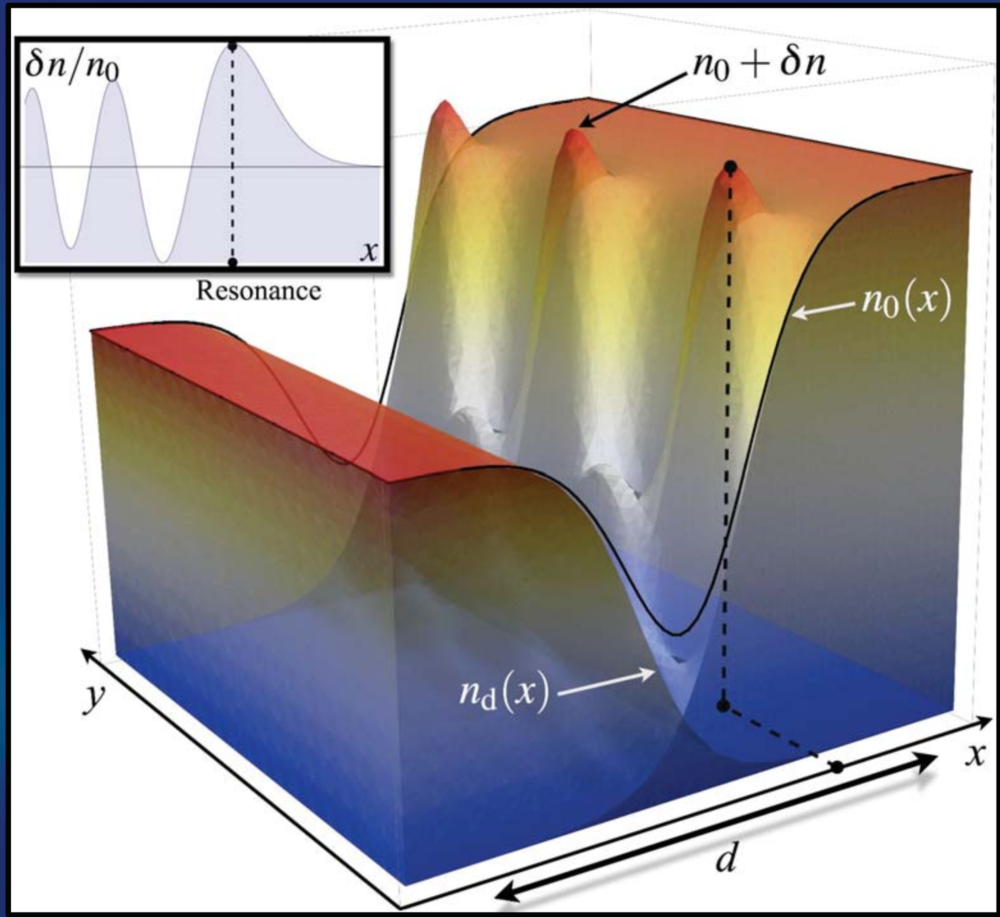
- Quantum pressure (gradient of Bohm-de Broglie potential)

$$m\frac{d\boldsymbol{v}}{dt} \sim \frac{\hbar^2}{2m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right)$$

- Gives higher order dispersion (spreading of electronic wave function).

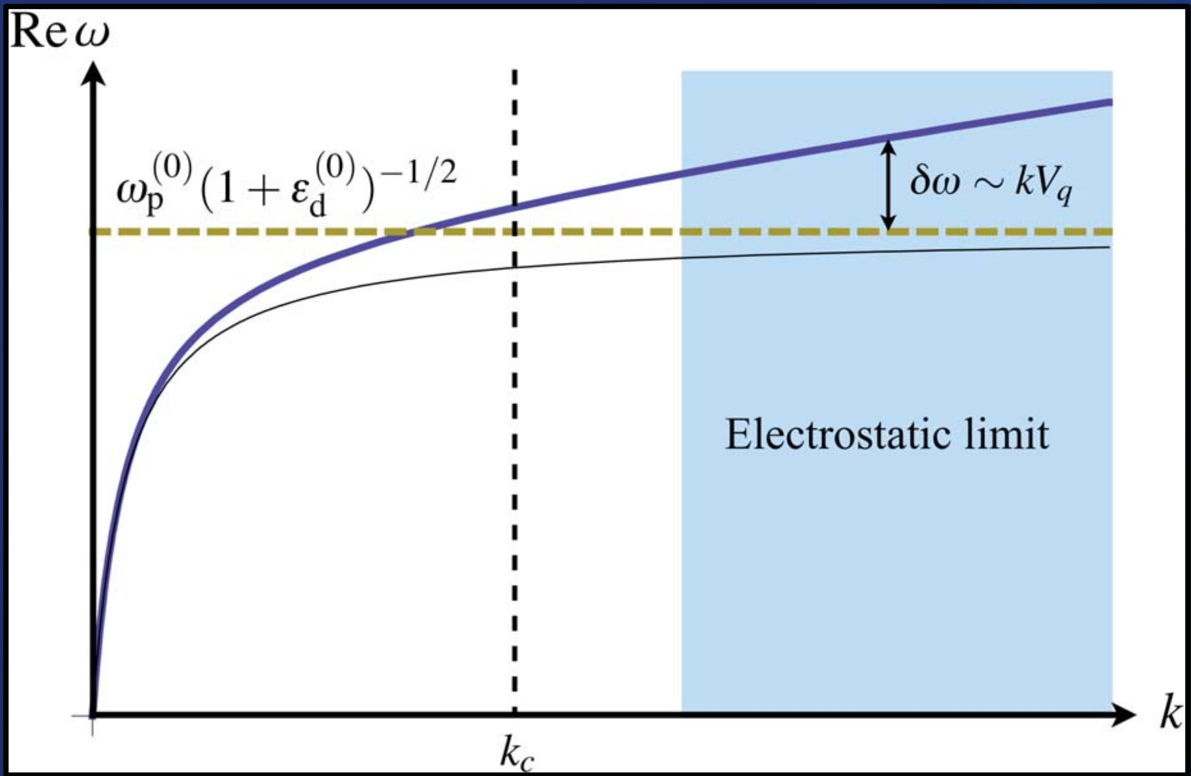
Plasmonic devices

- *Surface plasmon polaritons* propagating on conductor surface.
- Classically, surface is sharp.
- Broadening of surface layer due to quantum dispersion.
- Finite width gives damping!



Plasmonic devices

$$\omega \approx \frac{\omega_{\mathrm{p}}^{(0)}}{(1+\epsilon_{\mathrm{d}}^{(0)})^{1/2}}\left[1+(0.6+2i)\left(\frac{\hbar k^2}{m\omega_{\mathrm{p}}^{(0)}}\right)^{1/2}\right] \Rightarrow$$

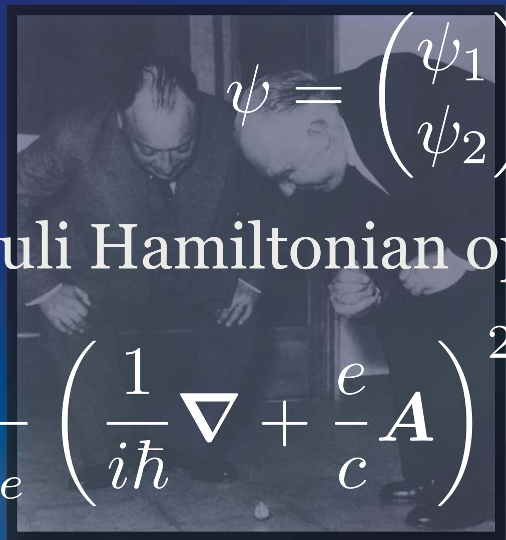


$$\delta_{\mathrm{SP}} \approx \left(\frac{\lambda}{100\,\mathrm{nm}}\right)^4 \mu\mathrm{m}$$

$$\boxed{\begin{array}{c}\lambda \sim 30\,\mathrm{nm}\\ \Downarrow \\ \delta_{\mathrm{SP}} \sim 10\,\mathrm{nm}\end{array}}$$

Spin contributions

Electron properties described by complex *spinor* wavefunction (spin degrees of freedom)



$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

and the Pauli Hamiltonian operator

$$H = \frac{1}{2m_e}\left(\frac{1}{i\hbar}\boldsymbol{\nabla} + \frac{e}{c}\boldsymbol{A}\right)^2 + \mu_B \boldsymbol{B}\cdot\boldsymbol{\sigma} - e\phi$$

Here $\boldsymbol{A}$ is the vector potential, $\boldsymbol{\sigma}$ is the spin operator, $\mu_B = e\hbar/2m_e$ is the Bohr magneton.

Quantum equations of motion for electrons

$$\boldsymbol{v} = \frac{1}{m_e}\left(\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}\right)$$

$$m_e \frac{d\boldsymbol{v}}{dt} = -e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) - \frac{2\mu_B}{\hbar}\boldsymbol{\nabla}(\boldsymbol{B} \cdot \boldsymbol{S})$$

$$\frac{d\boldsymbol{S}}{dt} = \frac{2\mu_B}{\hbar}\boldsymbol{B} \times \boldsymbol{S}$$

where $\boldsymbol{S} = (\hbar/2)\boldsymbol{\sigma}$.

Multiparticle theory

- For spin systems: Decompose spinor wave function $\psi_\alpha = \sqrt{n_\alpha}\,\exp(iS_\alpha/\hbar)\varphi_\alpha$, φ_α unit spinor .
- Electron fluid equations (not complete!)

$$m_e\left(\frac{\partial}{\partial t} + \boldsymbol{v}_e \cdot \boldsymbol{\nabla}\right)\boldsymbol{v}_e = -e\left(\boldsymbol{E} + \boldsymbol{v}_e \times \boldsymbol{B}\right) - \frac{\boldsymbol{\nabla} p_e}{n_e} - \frac{2\mu_B}{\hbar} S_a \boldsymbol{\nabla} B^a$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla}\right)\boldsymbol{S} = \frac{2\mu_B}{\hbar}\boldsymbol{B} \times \boldsymbol{S} + \text{thermal and nonlinear spin terms}$$

Maxwell's equations

Due to the intrinsic magnetization, given by

$$\boldsymbol{M} = -\frac{2\mu_B n_e}{\hbar}\boldsymbol{S},$$

Ampère's law is modified according to

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0(\boldsymbol{j} + \boldsymbol{\nabla} \times \boldsymbol{M}) + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}.$$

Gives dynamic spin contribution to Maxwell's equations.

MHD regime

Single fluid dynamics (for lowest order coherent spin)

$$\rho \frac{d\boldsymbol{v}}{dt} = -\boldsymbol{\nabla}\left(\frac{B^2}{2\mu_0} - \boldsymbol{M}\cdot\boldsymbol{B}\right) + \boldsymbol{B}\cdot\boldsymbol{\nabla}\left(\frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M}\right) - \boldsymbol{\nabla}p + \frac{\hbar^2\rho}{2m_e m_i}\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times\left\{\boldsymbol{v}\times\boldsymbol{B} - \frac{[\boldsymbol{\nabla}\times(\boldsymbol{B}-\mu_0\boldsymbol{M})]\times\boldsymbol{B}}{e\mu_0 n_e} - M_a\boldsymbol{\nabla}B^a\right\}$$

For high densities /low temperature: only electrons above Fermi level contribute to magnetization; implies reduced magnetization.

Model magnetization using Brillouin function for spin-1/2 particles (for long enough time scales)

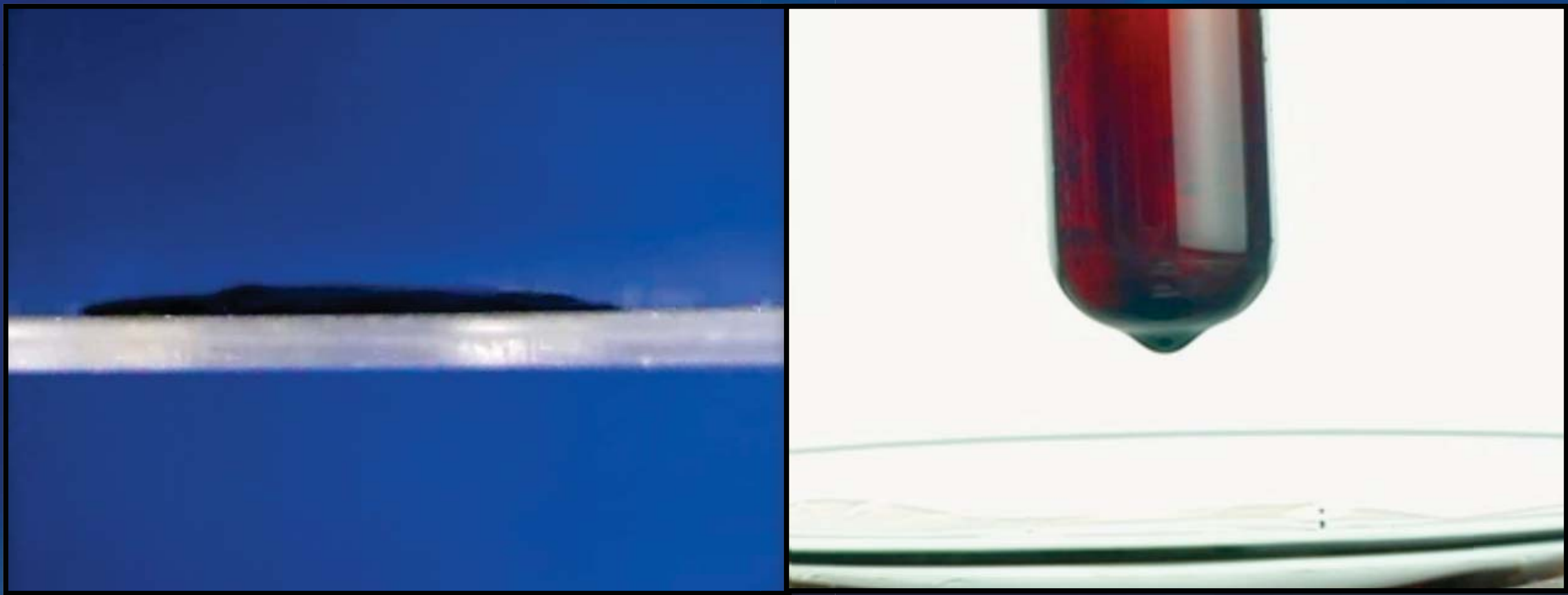
$$\boxed{\boldsymbol{M} = \mu_B n_e \tanh x\, \hat{\boldsymbol{B}}}$$

where the Zeeman energy $x = \mu_B B/k_B T_e$ gives the degree of alignment through the Brillouin function. For high temp., magnetization $\longrightarrow$ 0.

Instabilities and ferrofluids

Ferrofluids   Nanostructured paramagnetic fluids. Formalism similar to the above applicable.

Normal field instability - saturated by gravity and surface tension (Cowley & Rosensweig 1967).



http://mrsec.wisc.edu/Edetc/cineplex/ff/text.html

Spin kinetics

Spin dependent distribution function: $f = f(t, \boldsymbol{x}, \boldsymbol{v}, \boldsymbol{s})$

$$\frac{\partial f}{\partial t} + \frac{d\boldsymbol{x}}{dt} \cdot \nabla_x f + \frac{d\boldsymbol{v}}{dt} \cdot \nabla_v f + \frac{d\boldsymbol{s}}{dt} \cdot \nabla_s f = 0$$

(see also Cowley et al. Phys. Fluids 1986; Kulsrud et al. Nucl. Fusion, 1986)

Quantum equations of motion gives semiclassical dynamics.

See Gert Brodin's talk.

Spin kinetics

Formal structure: Wigner matrix from density matrix

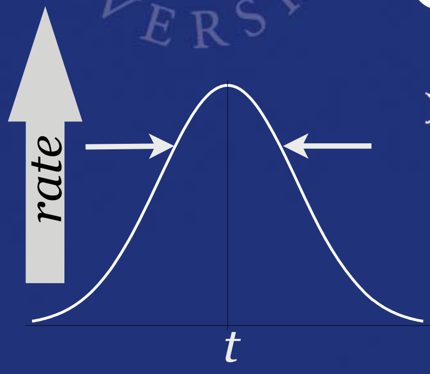
$$W_{ab}(t, \boldsymbol{x}, \boldsymbol{p}) = \frac{1}{(2\pi\hbar)^3} \int d^3y \, \exp(i\boldsymbol{p} \cdot \boldsymbol{y}/\hbar) \rho_{ab}(t, \boldsymbol{x} + \boldsymbol{y}/2, \boldsymbol{x} - \boldsymbol{y}/2)$$

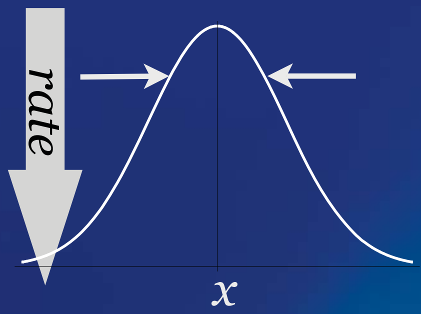
Distribution function defined by

$$f(t, \boldsymbol{x}, \boldsymbol{p}, \boldsymbol{s}) = \tfrac{1}{2}(1 + \boldsymbol{s} \cdot \boldsymbol{\sigma})_{ab} W_{ab}$$

Evolution determined by governing equation for density matrix. Lowest order terms gives semiclassical kinetic equation. Generalized spin Wigner equation (J. Zamanian et al., submitted; Arnold & Steinrück, ZAMP, 1989).

Issues in relativistic quantum plasmas





- Pair production: "Schwinger mechanism" for fields with spatial *and* temporal variation.
    - Temporal compression: *increased* production rate.
    - Spatial compression: *lower* production rate.
    - Laser fields → production rate unknown.
- Strong fields + relativistic plasma particles: can computational models be developed?
- Quantum field theoretical models (Melrose, 2008)?

Gigagauss laboratory fields

- Currently, gigagauss laboratory fields generated in solid-laser interactions.
- Look for quantum plasma effects:
  - Landau quantization.
  - Spin effects.
  - ...

Conclusions

- New important effects appear from collective quantum domain.
- Wide ranging possibilities for applications.
  - Nanomaterials.
  - Astroplasmas.
  - Ultracold plasmas.
- Interesting future possibilities:
  - Theoretical development: Dense, relativistic plasmas using computationally viable models.
  - Solitons and plasmonics.