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**Filamentation instability of counterpropagating charged particle  
beams: Statistical properties**

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beams: Statistical properties

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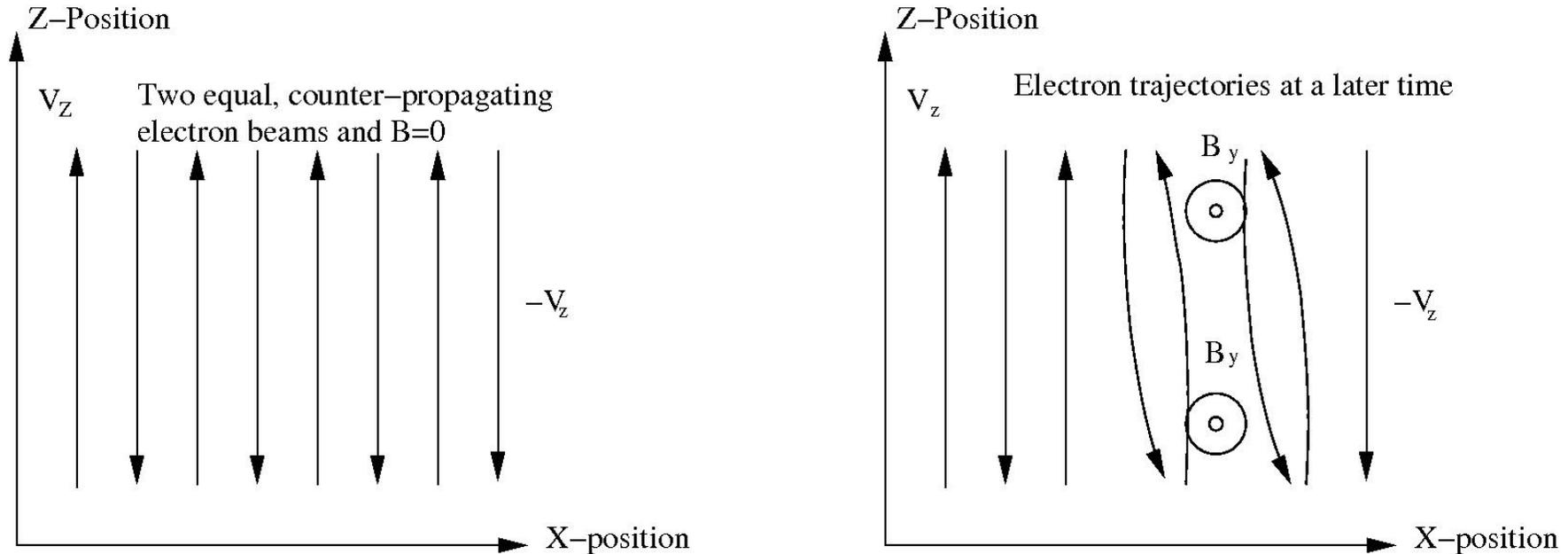
# Overview

- Consider two counterstreaming beams of electrons.
- The electrostatic two-stream instability grows, if the beams move with a nonrelativistic relative speed  $v_b$ .
- *Cool* electron beams of infinite extent: Bulk electron density  $n_p$ , beam electron density  $n_b$  and  $n_i = n_p + n_b$  is the density of the immobile ions.
  - $v_b \approx c$  and  $n_b \ll n_p$  : Mixed modes grow fastest.
  - $v_b \approx c$  and  $n_b \approx n_p$  : Filamentation modes win.

# Overview

- Physical mechanism of the filamentation instability
- *1D particle-in-cell simulation*: Large domain, obtain size distribution of filaments just before mergers start.
- *2D particle-in-cell simulation*: Allows for filament mergers, obtain time-evolution of filament's mean size
- Conclusion
  
- *Thanks to*: Deutsche Forschungsgemeinschaft, Vetenskapsrådet
- *Co-workers*: P. K. Shukla, G. Rowlands, I. Lerche and L.O.C. Drury

# The filamentation mechanism



- Initially, two symmetric beams counter-propagate and the current cancels  $\rightarrow$  No net current and no magnetic field.
- A small magnetic field perturbation in z-direction separates the electrons moving in opposite directions  $\rightarrow$  A net current grows, that amplifies the magnetic field.

# PIC simulations of filamentation instability

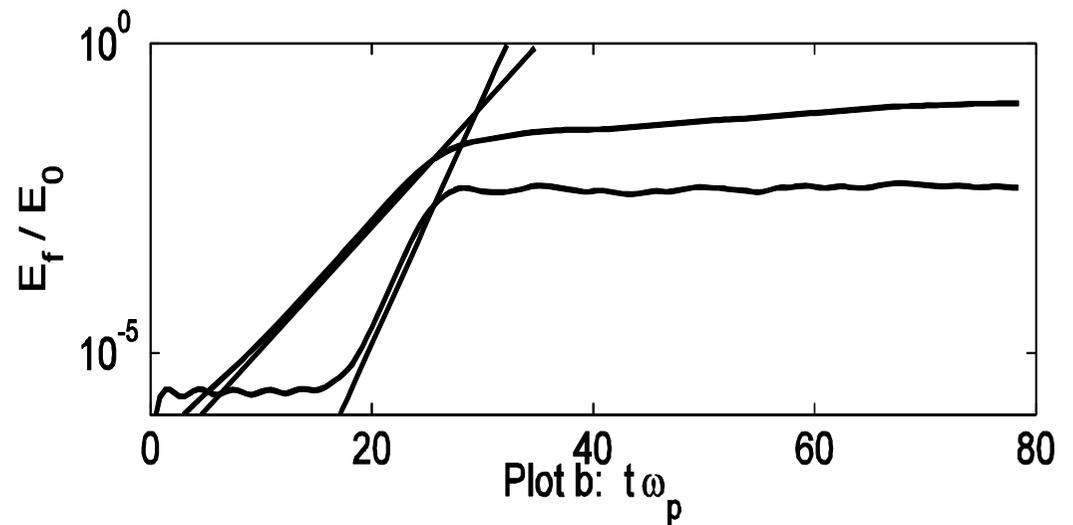
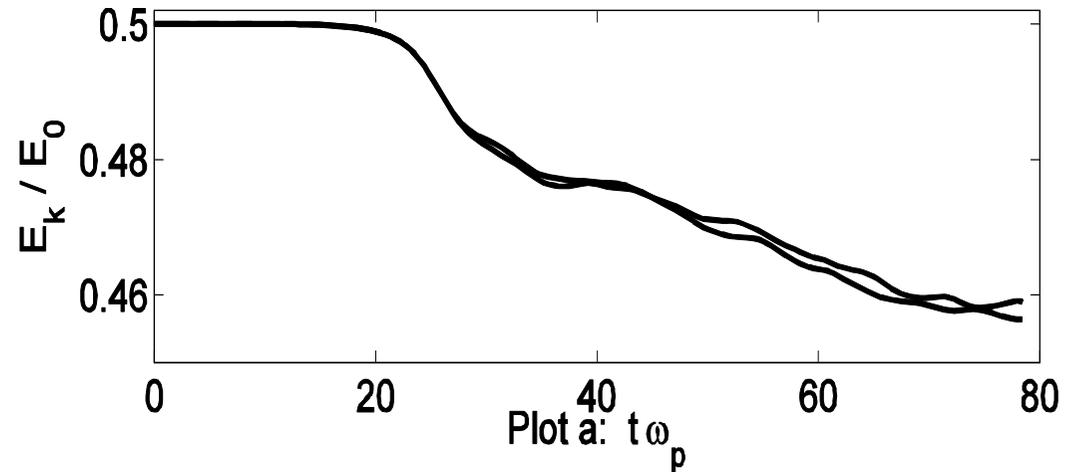
- Counterstreaming beams with velocity  $\pm v_b \mathbf{z}$
- Filamentation instability grows fastest if  $k_z = 0$
- Magnetic / electric field growth in  $x$ - $y$  plane
- Modulation in  $x$ - $y$  of current  $j_z$  and charge  $\rho$
- *Morse and Nielson, Phys Fluids 14, 830, 1971*  
*1D simulation: (x) 2D simulation: (x,y)*
- Now we can redo this and model *many* flux tubes to get some of their statistical properties.

# Simulation setup

- Initially  $E = 0$ ,  $B = 0$
- *1D simulation*
- $v_b = 0.3 c$  and  $v_b / v_{th} = 18 \rightarrow$  Beams are cool.
- Box length: 444 (6600) electron skin depths  $c / \omega_p$
- *2D simulation*
- $v_b = 0.7 c$  and  $v_b / v_{th} = 5 \rightarrow$  Beams are warm
- Quadratic box with side length  $100 c / \omega_p$

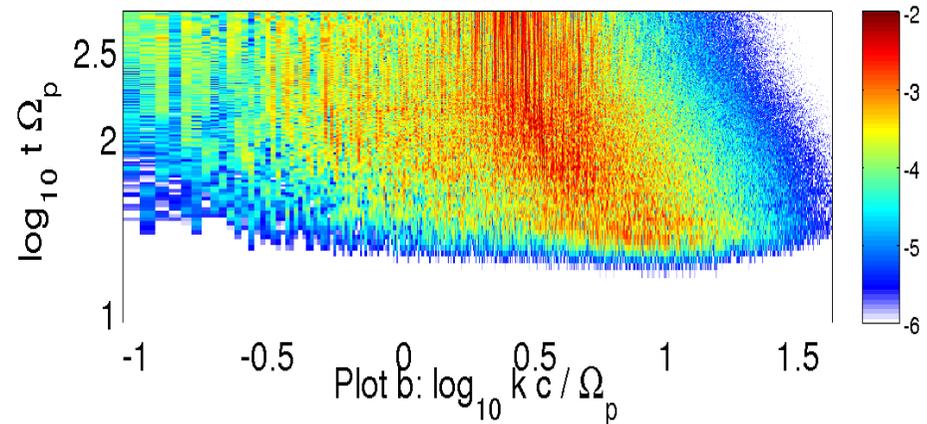
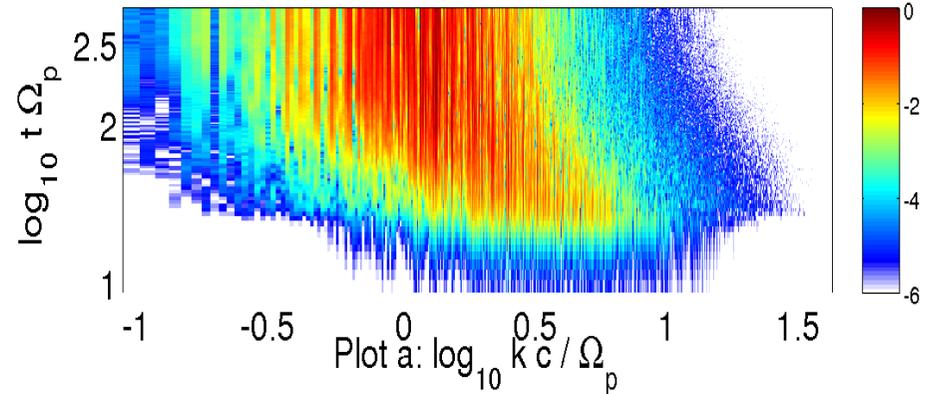
# Field / particle energies in 1D

- Upper plot: Kinetic energy of each beam
  - Lower plot: Energy in  $B_y$  and in  $E_x$
  - Electric energy grows *twice* as fast as the magnetic, suggesting  $E_x(t) \sim B_y (dB_y / dx)$
- Quenching by magnetic pressure gradient



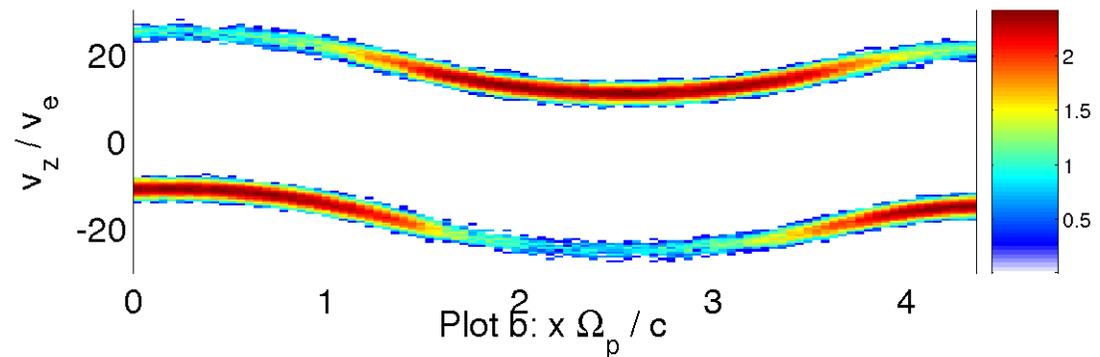
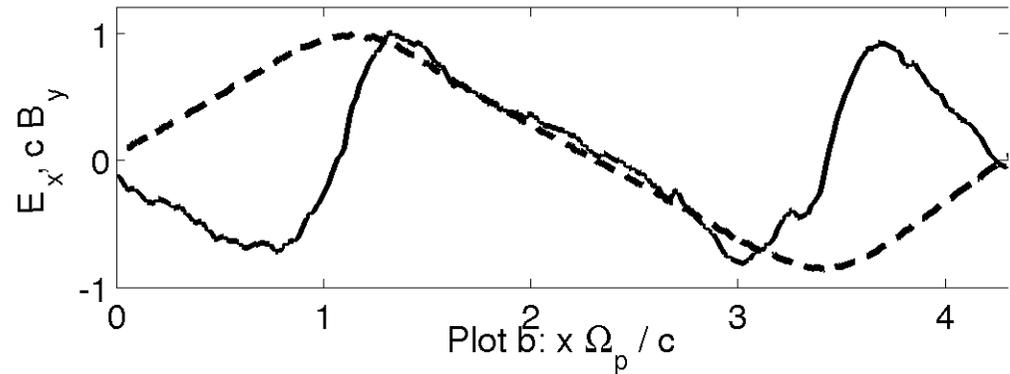
# Stalling of 1D filamentation

- $B_y^2(k_x, t)$  grows at high  $k_x$  and shifts to lower wavenumbers in time
- $E_x^2(k_x, t)$  grows later and at higher  $k_x$
- The late power spectra are time-stationary
- The time-stationarity reflects the stalling of the electron distribution



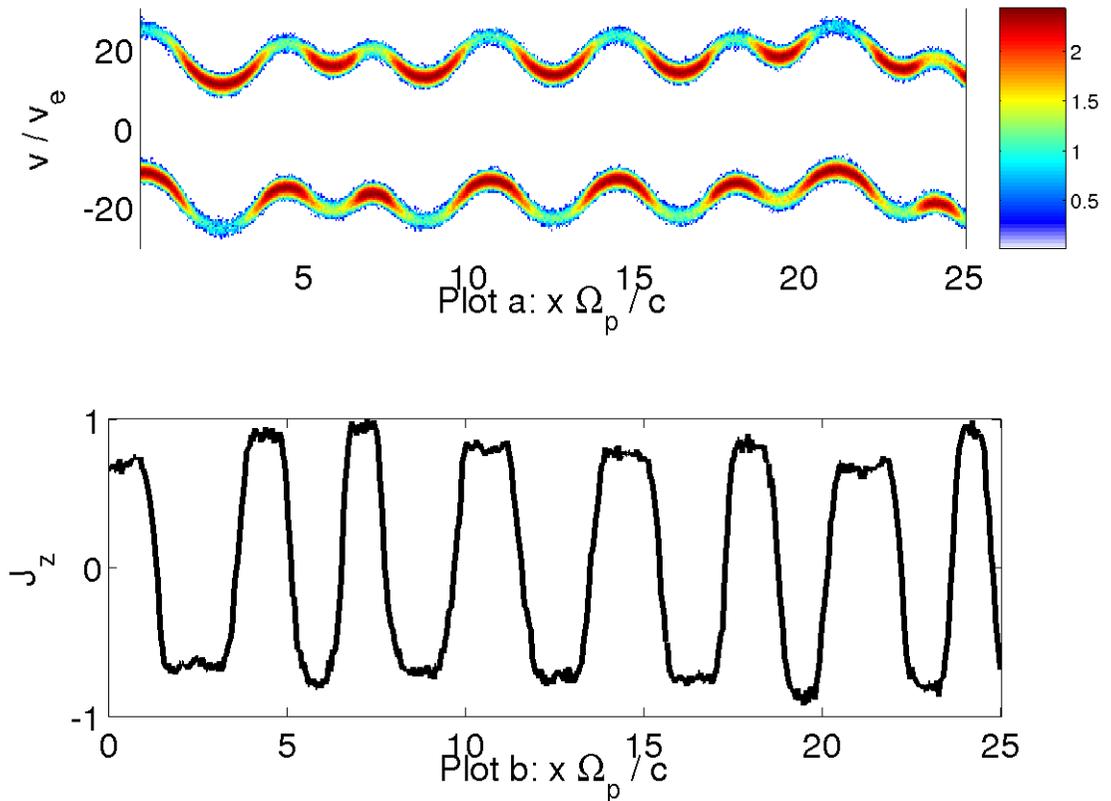
# Field distribution

- Dashed line:  $cB_y$
- Solid line:  $5E_x$
- Steady state involves field distributions with a constant slope
- Fourier transform of a constant slope gives power-law



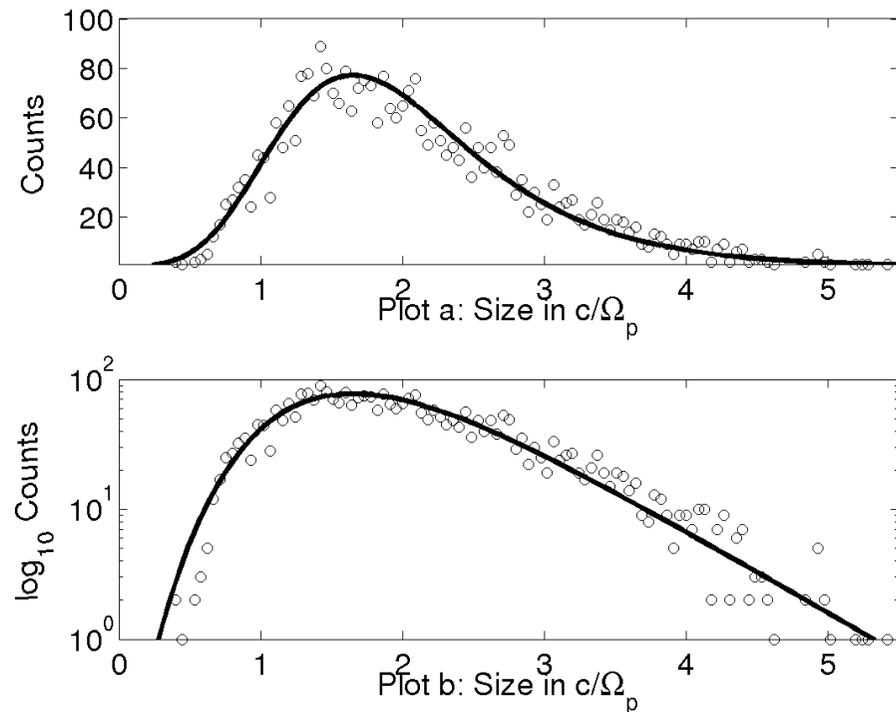
# Current distribution

- Upper plot: 10-log of electron phase space density.
- Lower plot:  $J_z$  current normalized to peak
- Zero crossings are filament boundaries



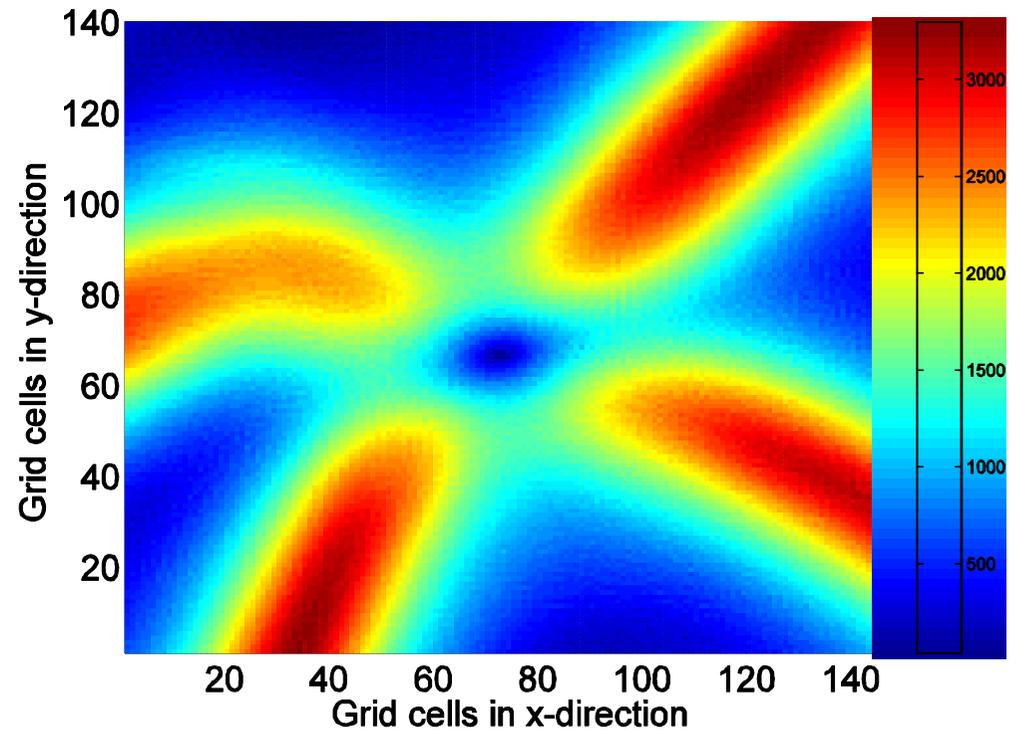
# Gumbel distribution $\sim \exp(-u-\exp[u])$

- Measure distance between zero-crossings.
- Count how many filaments have a given size
- The solid lines show the Gumbel distribution
- The circles give the number of filaments with a given size

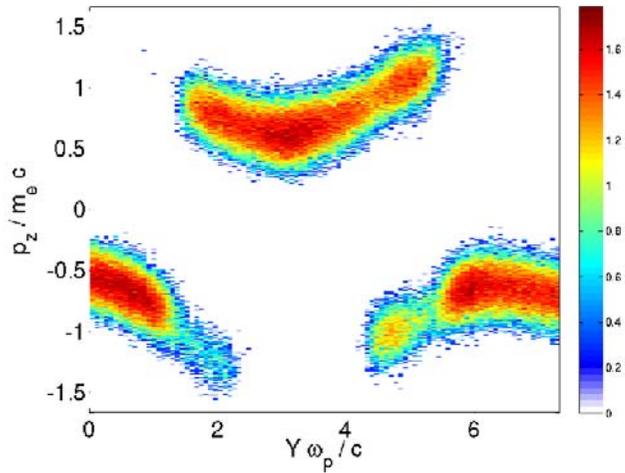


# We go from 1D to 2D

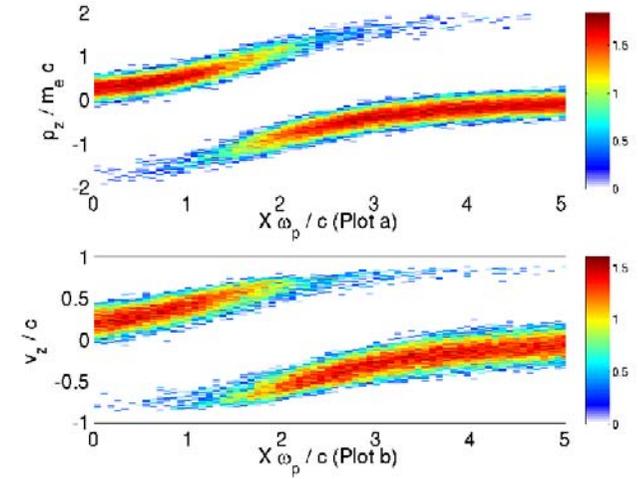
- Two flux tubes with electrons flowing in opposite directions repel each other
- 1D: The system locks up
- 2D: Flux tubes can go around each other
- 2D: Magnetic fields can reconnect, by which the flux tubes merge



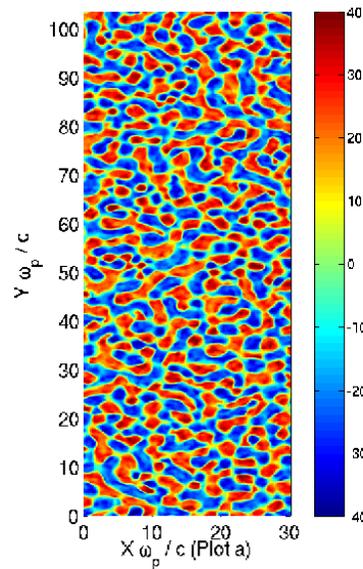
# 2D: Enabling flux tube merging



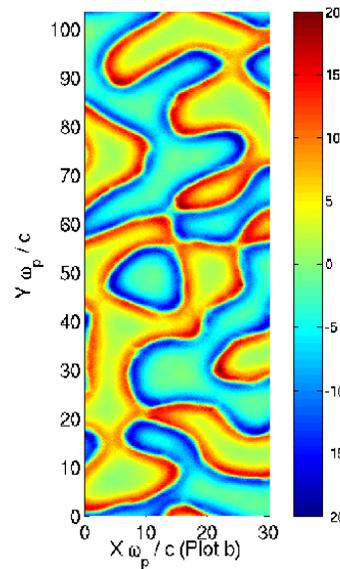
Phase space plots cut the distribution  $J_z(x,y)$  below along one direction



Early time



Late time

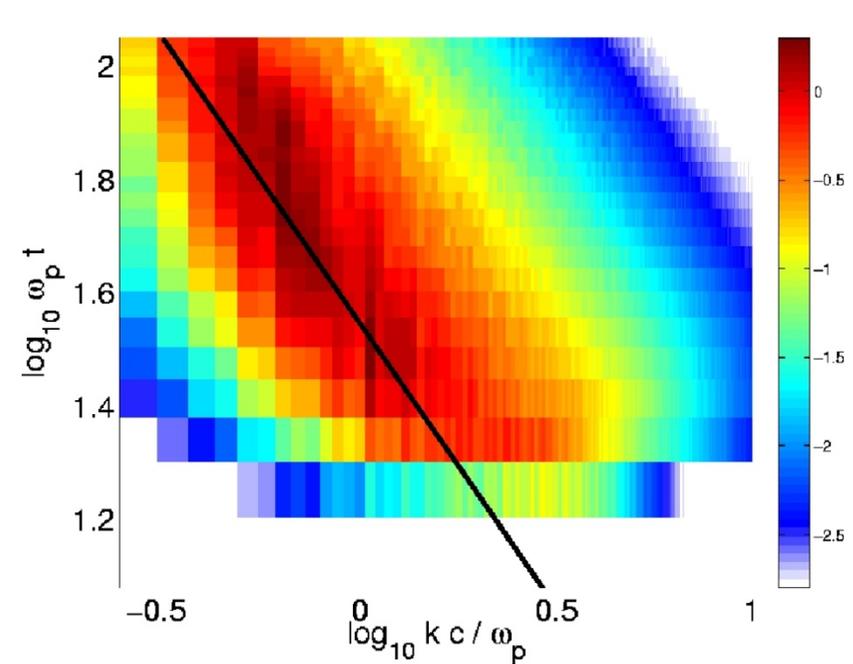
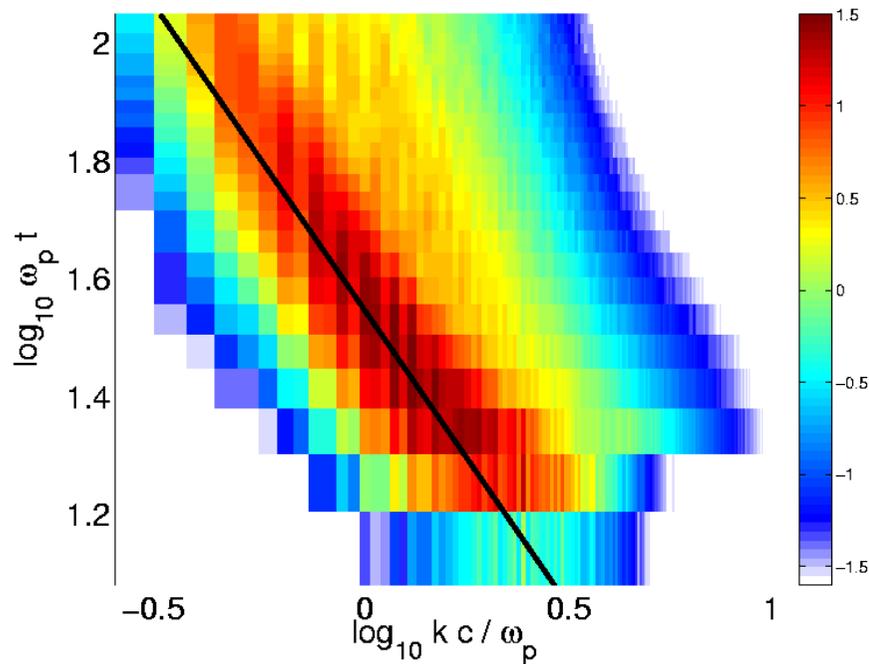


# Change of characteristic size

- The coherence length is difficult to quantify in x-y.
- Thus: 2D Fourier transform of  $C(x,y)$  over x,y
- Get the power spectrum  $C_z^2(k_x, k_y)$ ,  $C_{x+iy}^2(k_x, k_y)$
- The system is symmetric around the z-direction  
→ Power spectrum is integrated over the azimuth
- We obtain  $C_z^2(k)$  and  $C_{x+iy}^2(k)$  for each simulation time slice

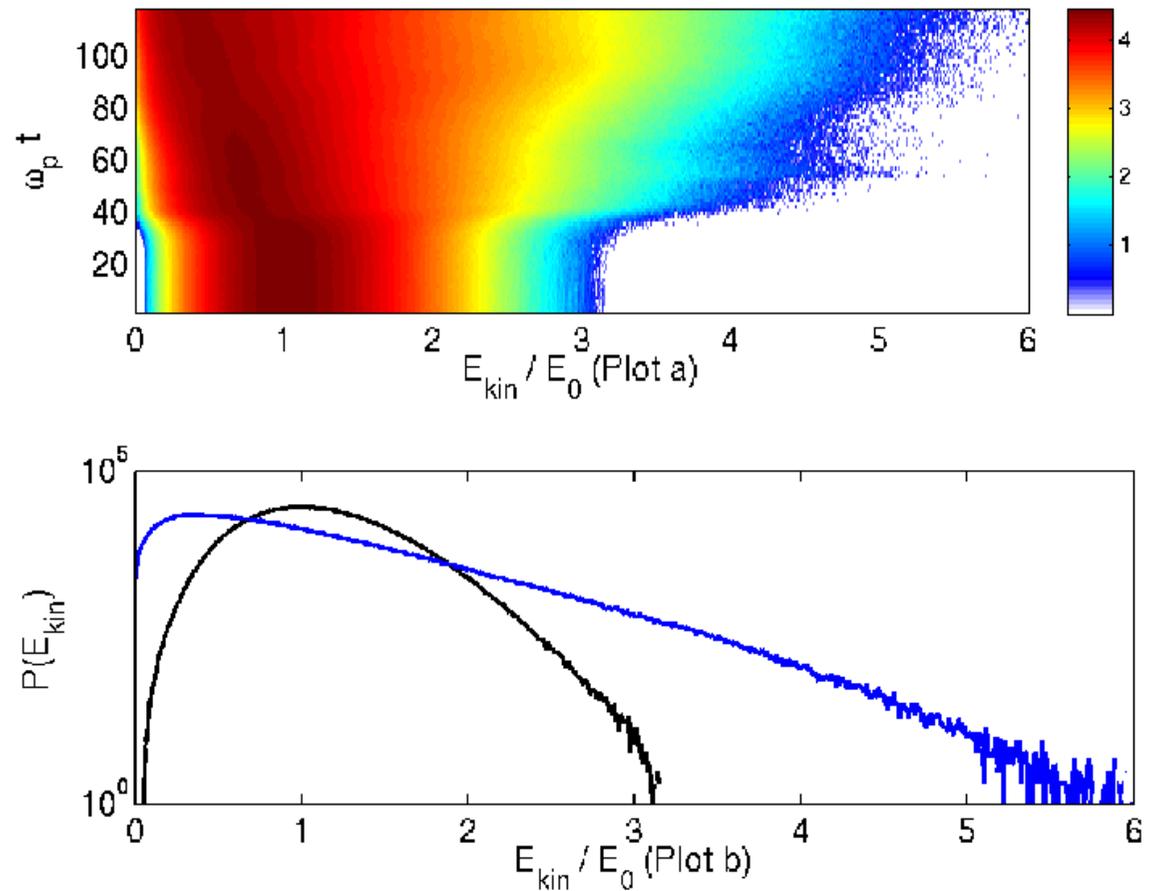
# Filament size distribution

- The size is initially about an electron skin depth
- The peak power goes to lower wavenumbers as  $k \sim t^{-1}$
- This resembles the vortex size evolution in fluid turbulence.
- Left: spectrum of  $C_z$     Right: spectrum of  $C_{x+iy}$



# Electron acceleration

- We compare initial electron energy distribution against final one.
- The peak electron energy increased by a few times.
- No substantial electron acceleration



# Summary

- The filamentation instability generates magnetic fields through current separation
- The electric fields driven by the magnetic pressure gradient quench its growth
- The filament size distribution upon saturation decreases exponentially with increasing size
- Mergers let the filament size increase linearly with time. The size is limited by the number of mergers
- In 3D secondary instabilities may show up, destroying the flux tubes