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Test particles, test modes & drift turbulence.

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Test particles, test modes

&

drift turbulence

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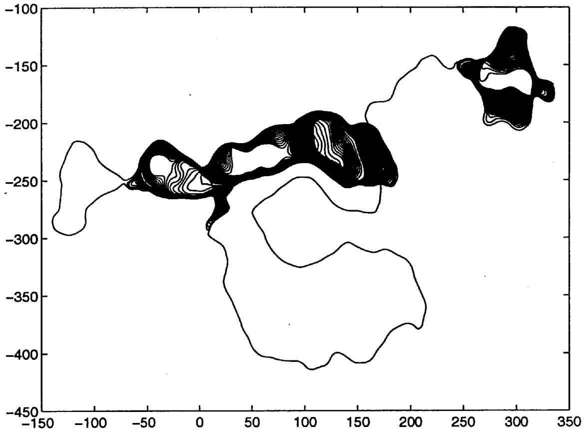


The ExB stochastic drift in turbulent plasmas can determine

trajectory trapping or eddying

↓

multiscale structures of trajectories

We present:

➢ ***analytical methods*** **for determining the statistical characteristics of the trajectories and of the trajectory structures;**

➢ ***the effects of the trajectory structures*****:**

- **on the transport coefficients;**
- **on turbulence (test modes on turbulent plasmas for drift instability).**

Content

1) Test particle statistics in stochastic potential

- the constraints
- semi-analytical statistical methods
- velocity on structure method (VS)
- the probability of displacements and vortical trajectory structures
- perturbed structures and transport coefficients (anomalous regimes)
- effect of average flows and of potential drift

2) Test modes in turbulent plasma

1) Test particle statistics in stochastic potential

We consider a turbulent plasma with given (measured) statistical characteristics of the stochastic potential and study test particle motion

Non-linear stochastic equation:

$$\boxed{\frac{d\vec{x}(t)}{dt} = \vec{v}(\vec{x}(t), t), \quad \vec{x}(0) = 0, \quad \vec{v}(\vec{x}, t) = -\frac{\nabla \phi \times \vec{e}_z}{B}}$$

where $\vec{v}(\vec{x}, t)$ is a stationary and homogeneous stochastic velocity field with Gaussian distribution and known spectrum or ***Eulerian correlation*** **(EC):**

$$\boxed{E(\vec{x},t) = \left\langle \phi(\vec{x}_1,t_1)\phi(\vec{x}_1+\vec{x},t_1+t) \right\rangle = \Phi^2 f\left(\frac{\vec{x}}{\lambda_c}, \frac{t}{\tau_c}\right),}$$

$$E_{ij}(\vec{x},t) \equiv \left\langle v_i(\vec{x}_1,t_1) v_j(\vec{x}_1+\vec{x},t_1+t) \right\rangle = -\left(\frac{\Phi}{\lambda_c}\right)^2 \varepsilon_{im}\varepsilon_{jn}\partial_m\partial_n f$$

where Φ is the amplitude , λ_c the correlation length, τ_c the correlation time.

The Kubo number : $$K = \frac{V\tau_c}{\lambda_c} = \frac{\tau_c}{\tau_{fl}}, \quad \tau_{fl} = \frac{\lambda_c}{V}, \quad V = \frac{\Phi}{\lambda_c}$$

(describes the ***decorrelation due to time variation*** of the stochastic field).

The stochastic velocity ***v**(**x**,t)* is the ***nonlinear kernel*** of the test particle models to which one can add ***collisions, average velocity, parallel motion, etc.***

They represent decorrelation mechanisms (each introduces a dimensionless quantity similar to K).

To determine: The statistical properties of the trajectories:

the probability of displacements (pdf) **P(x,t)**

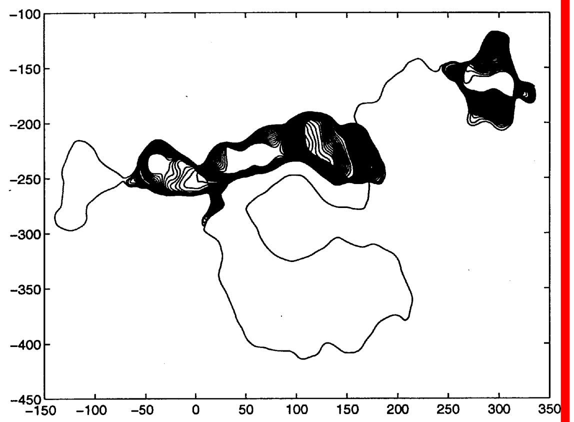
$$\left\langle x^2(t)\right\rangle = \int P(x,t)\, x^2\, dx,$$

$$D(t) \equiv \frac{1}{2}\frac{d\left\langle x^2(t)\right\rangle}{dt}, \quad D = \frac{\left\langle x^2(\tau_c)\right\rangle}{\tau_c}$$

Lagrangian velocity correlation **(LVC) *:*** $L_{ij}(t) \equiv \left\langle v_i(0,0)\, v_j(\vec{x}(t),t)\right\rangle$

$$\left\langle x^2(t)\right\rangle = 2\int_0^t (t-t')\, L_{xx}(t')\, dt', \qquad D(t) = \int_0^t L_{xx}(t')\, dt',$$

There are two strong constraints for the statistical methods:



(A) The invariance of the potential for the static case

(Hamiltonian equation of motion)

$$\frac{d\phi(\vec{x}(t),t)}{dt} = \cancel{v_i(\vec{x}(t),t)\frac{\partial\phi(\vec{x}(t),t)}{\partial x_i}} + \frac{\partial\phi(\vec{x}(t),t)}{\partial t} = \frac{\partial\phi(\vec{x}(t),t)}{\partial t}$$

- ***static case*** ($K=\infty$): invariance of the potential and ***permanent trapping*** on the contour lines;
- ***slowly varying potential*** (**K > 1**): approx. invariance of the potential and ***temporary trapping***
- potential with ***fast time variation*** (**K < 1**): no trapping.

(B) The *statistical invariance* of the Lagrangian velocity due to $\nabla\cdot\vec{v}(\vec{x},t)=0$

$$P\left[\vec{v}(\vec{x}(t),t)\right] = P\left[\vec{v}(\vec{x}(0),0)\right] = P\left[\vec{v}(\vec{x},t)\right]$$

Semi-analytical statistical methods

The existing analytical methods
(Corrsin Approximation, DIA, functional integration) are not compatible with the conditions (A) and (B). They do not describe trapping and lead to diffusive transport in in the static potential

❑ *The decorrelation trajectory method* **(DTM)** *1998*

(M.Vlad, F. Spineanu, J. H. Misguich, R. Balescu, “Diffusion with intrinsic trapping in 2-d incompressible stochastic velocity fields”, **Physical Review E 58** (1998) 7359)

- DTM is based on a set of simple (deterministic) trajectories determined from the Eulerian correlation EC of the stochastic potential.
- main consequences of condition (A) are fulfilled, condition (B) is not;
- ***Main physical result***: ***trapping produces memory effects*** ***(long-time correlation of the Lagrangian velocity)*** and subdiffusive transport for static potential.

❑ ***The nested subensemble method (NSM)*** *2004*

(M. Vlad, F. Spineanu, "Trajectory structures and transport", **Physical Review E 70** (2004) 056304(14))

- NSM is a systematic expansion based on dividing the space of realizations of the stochastic potential in nested subensembles. NSM yields detailed statistical information.
- all consequences of (A) are fulfilled, condition (B) is improved (but not enough in order 2)
- ***Main physical result***: ***trapping determines coherence*** in the stochastic motion and ***quasi-coherent trajectory structures***.

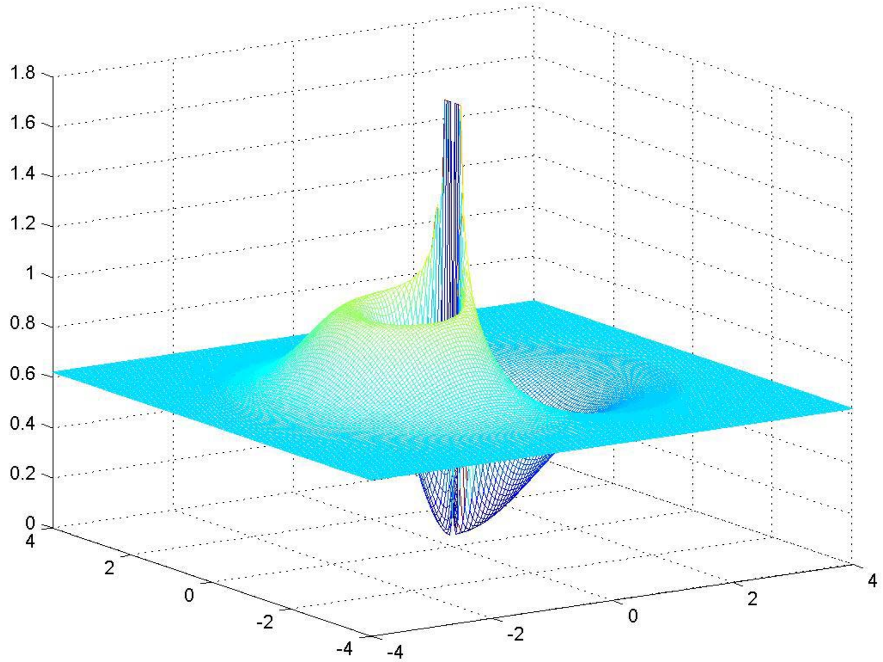
❑ ***The velocity on structure method (VS)*** *2008*

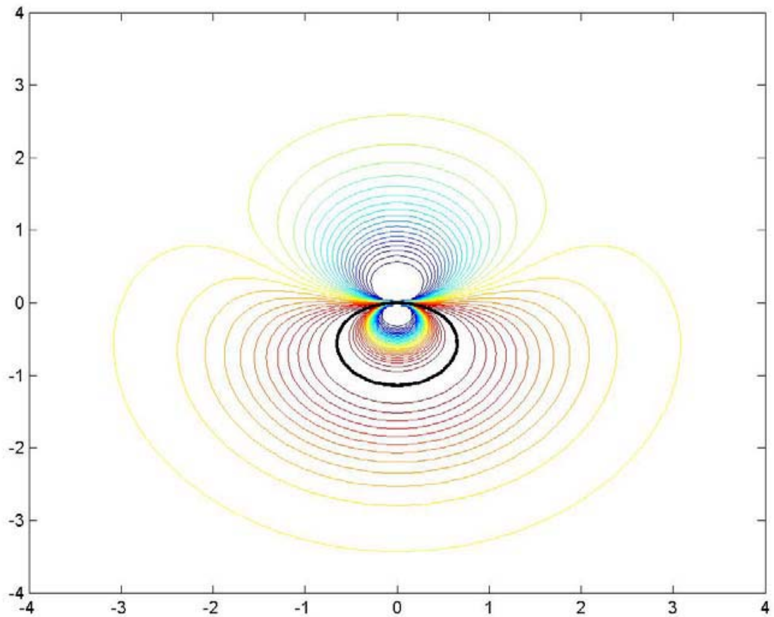
- ***Both conditions (A) and (B) are respected***;
- Trajectory structures are confirmed and better described
- ***Main physical result***: ***the distribution of displacements until decorrelation is determined*** (the 1-step pdf).

It is strongly non-Gaussian in the trapping regime **(K>>1)**.

- ***The main ideas of the VS:***
  - the contour lines of the potential with the same value ϕ form ***geometrical structures***.

The (conditional) probability that the potential in a point x is ϕ, given that it has the same value in x=0 and a fixed orientation of the gradient:





The maximum of the probability is on the contour line ϕ of the average potential;
The size of the structure increases when ϕ decreases.

-The velocity is always tangent to the contour line of the potential;

- the Eulerian velocity on the contour lines with the same ϕ has the same probability as in the whole set o realizations $\longrightarrow$ the characteristics of the 1-dimensional motion are the same for all structures.

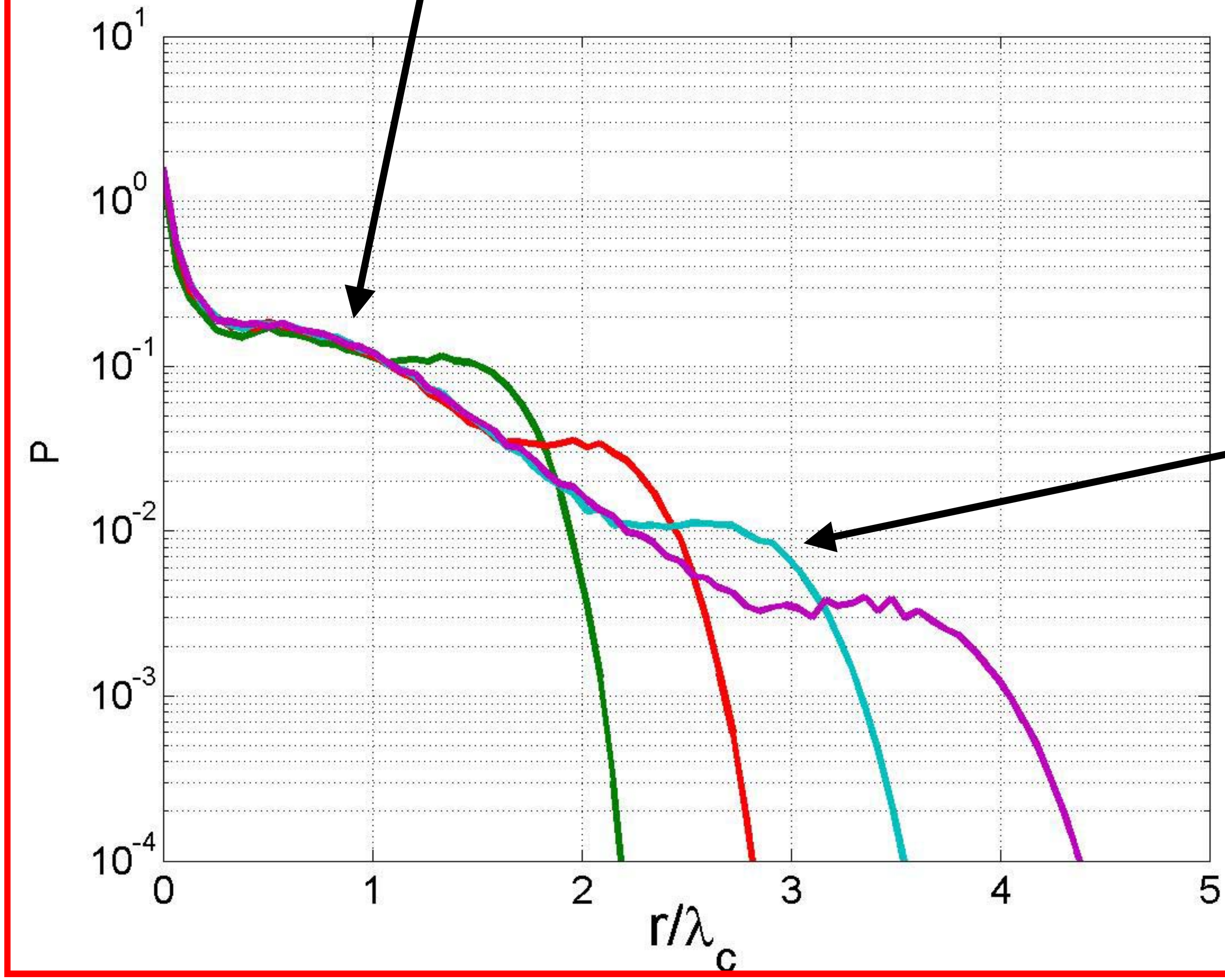
- the motion along the structure (along the contour line of the average potential) is a “standard” stochastic motion with positive Gaussian velocity. The correlation and the average displacement are determined by Corrsin approximation (integral equation solved by iterations).
This motion eventually leads to *uniform distribution on the structure*.

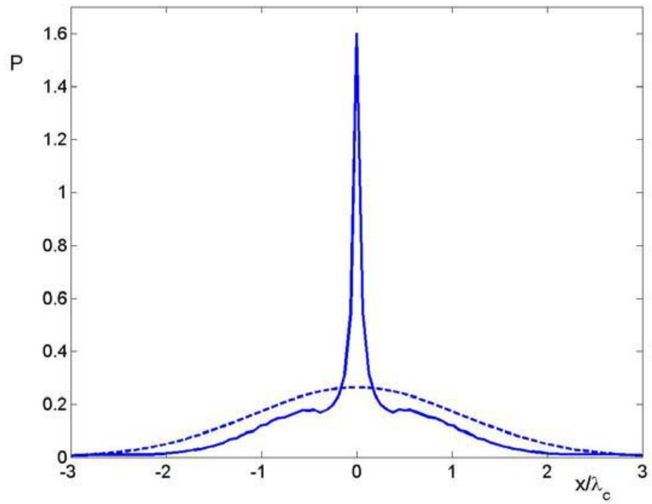
- Thus, the ***geometrical contour line structures*** become ***trajectory structures*** in a time that depends on ϕ as a decreasing function. Thus the formation time of a structure increases when the size of the structure increases.

➢ The pdf of the displacements is obtained by summing the contribution of all values of ϕ

The probability of displacements for static potential

Time invariant part formed by trajectory structures of different scales (vortical motion)





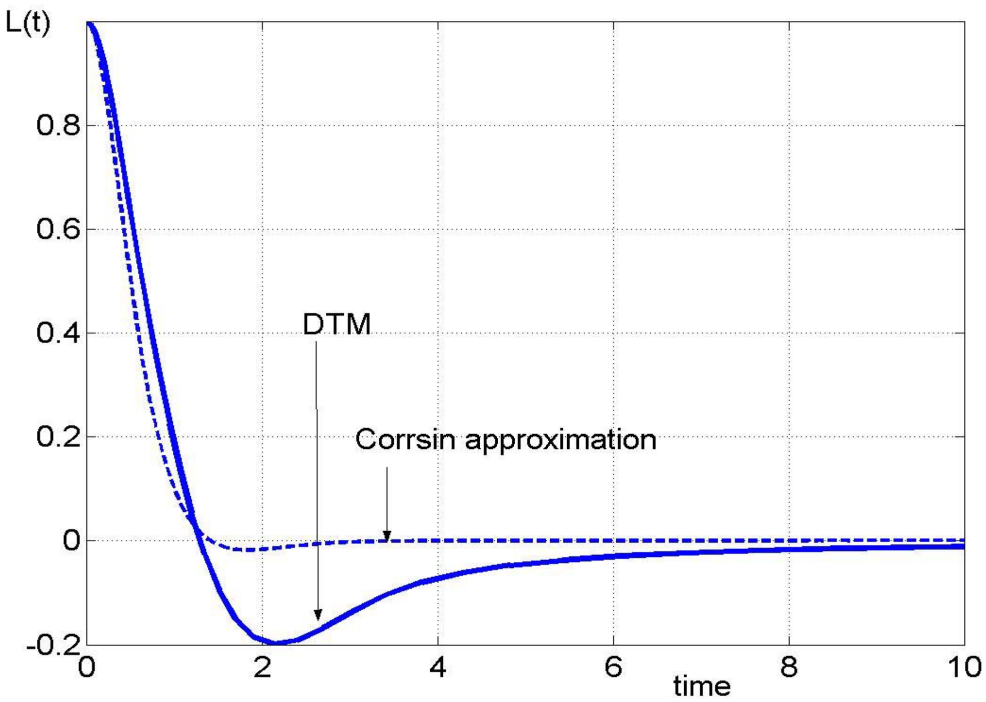
Moving maximum of trajectories that are not in a structure at that time (radial motion). Decreasing number of free particles.

The diffusion coefficient for static potential:

$$D(t) \cong \frac{\left\langle x^2(t) \right\rangle}{t}, \quad \left\langle x^2(t) \right\rangle = \int [P_{tr}(x) + P_f(x,t)] x^2 dx$$

constant at large t

$\approx n_f(t) V^2 t^2$



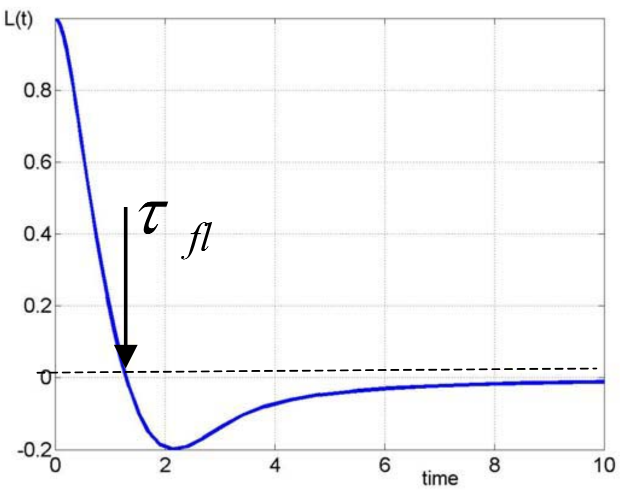
Decay of D(t) due to trapping

Correlation of the Lagrangian velocity with long negative tail that exactly compensates the positive part at small time delay

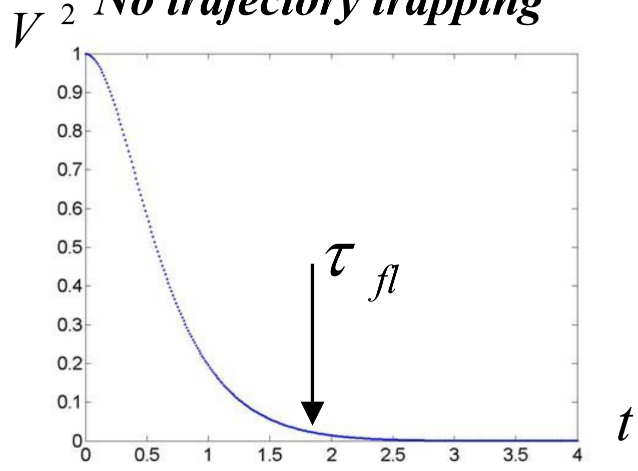
→ ***Subdiffusive transport***

Effect of a perturbation = decorrelation mechanism characterized by a time τ_d

Trajectory trapping



No trajectory trapping



- ***Strong decorrelation mechanism with small decorrelation time*** $\tau_d < \tau_{fl}$

$$D \downarrow \quad when \quad \tau_d \downarrow, \quad D \approx V^2 \tau_d$$

- ***Weak decorrelation mechanism with large decorrelation time*** $\tau_d > \tau_{fl}$

* The negative part of the LVC is cut out;
Anomalous diffusion regime (increased diffusion at stronger decorrelation)

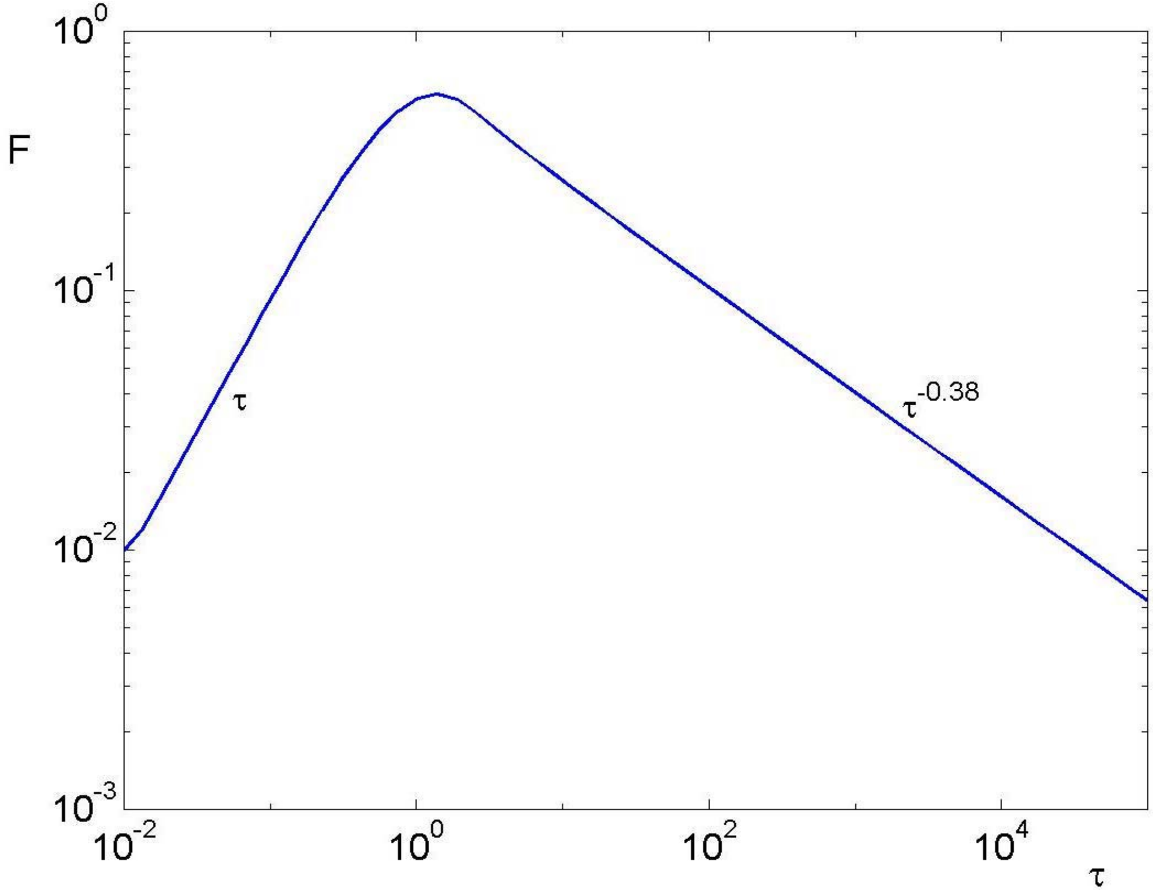
$$D \uparrow \quad when \quad \tau_d \downarrow$$

* The LVC is not influenced;
Transport coefficient independent on τ_d and stable to such perturbations.

The diffusion coefficient for time dependent potentia (finite K):

- is obtained from the corresponding frozen potential :

$$D(\tau_c) = \frac{\left\langle x^2(\tau_c) \right\rangle}{\tau_c} = D_B F(\tau_c)$$



$$D(K) = D_B F(K),$$

$$D_B = \frac{\Phi}{B}$$

Decay of D(K) due to trapping

(weaker from the VS than the DTM)

➢ **The effect of parallel motion is** similar with that of time variation of the potential: the correlation of the potential decays (is destroyed) because particles move out of the correlated zone along the magnetic field.

Effective Kubo number

$$K_{eff} = \frac{K K_{II}}{K + K_{II}}, \quad K_{II} = \frac{\tau_{II}}{\tau_{fl}}, \quad \tau_{fl} = \frac{\lambda_{II}}{v_{II}}$$

$$K_{II} << 1 \rightarrow K_{eff} << 1$$

➢ **The effect of collisions:** diffusion of the potential correlation.
E(x) is not destroyed but spreaded due to collisional motion.

Effective Kubo number

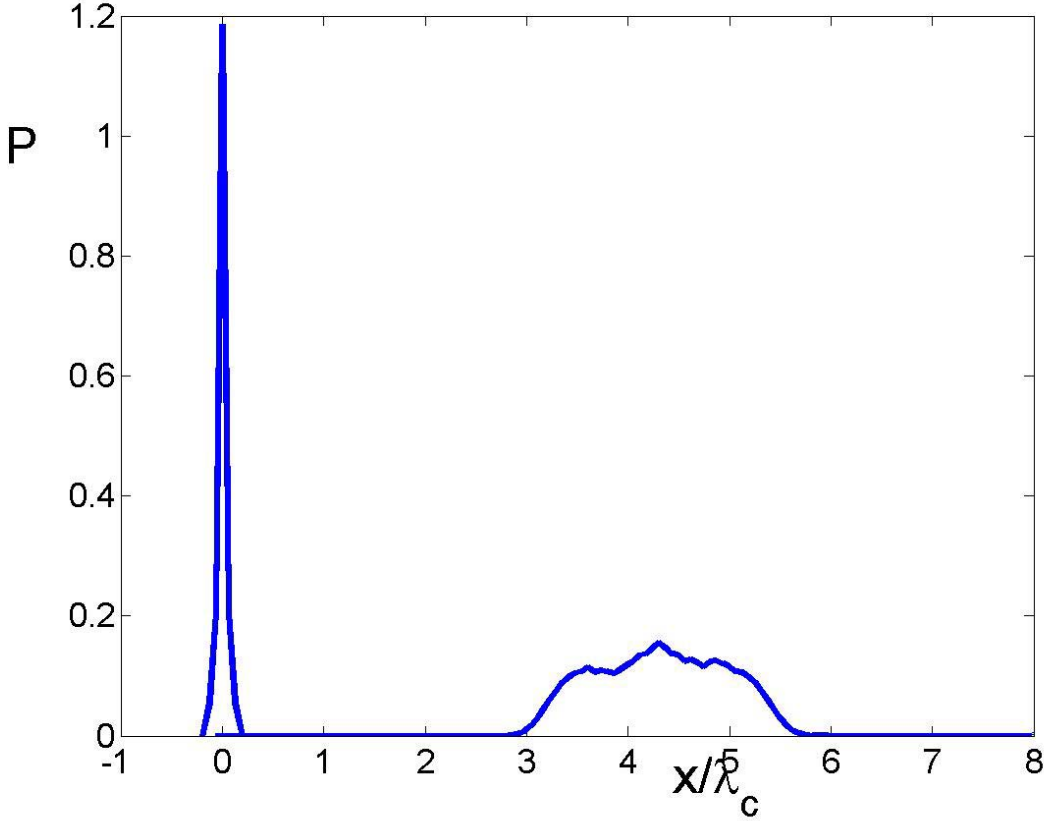
$$K_{eff} = \frac{K}{\left(1 + 2\chi K\right)^{3/2}}, \quad \chi = \frac{D_0}{\beta}$$

$$\overline{V}_d = \frac{V_d}{V}$$

➢ **The effect of an average velocity:**

Strips of open contour lines of the biased potential appear (along the average velocity)

For $\overline{V}_d > 1$ all the contour lines are opened and there is no trapping.



Trapping for $\overline{V}_d < 1$ and motion on the opened contour line with larger velocity

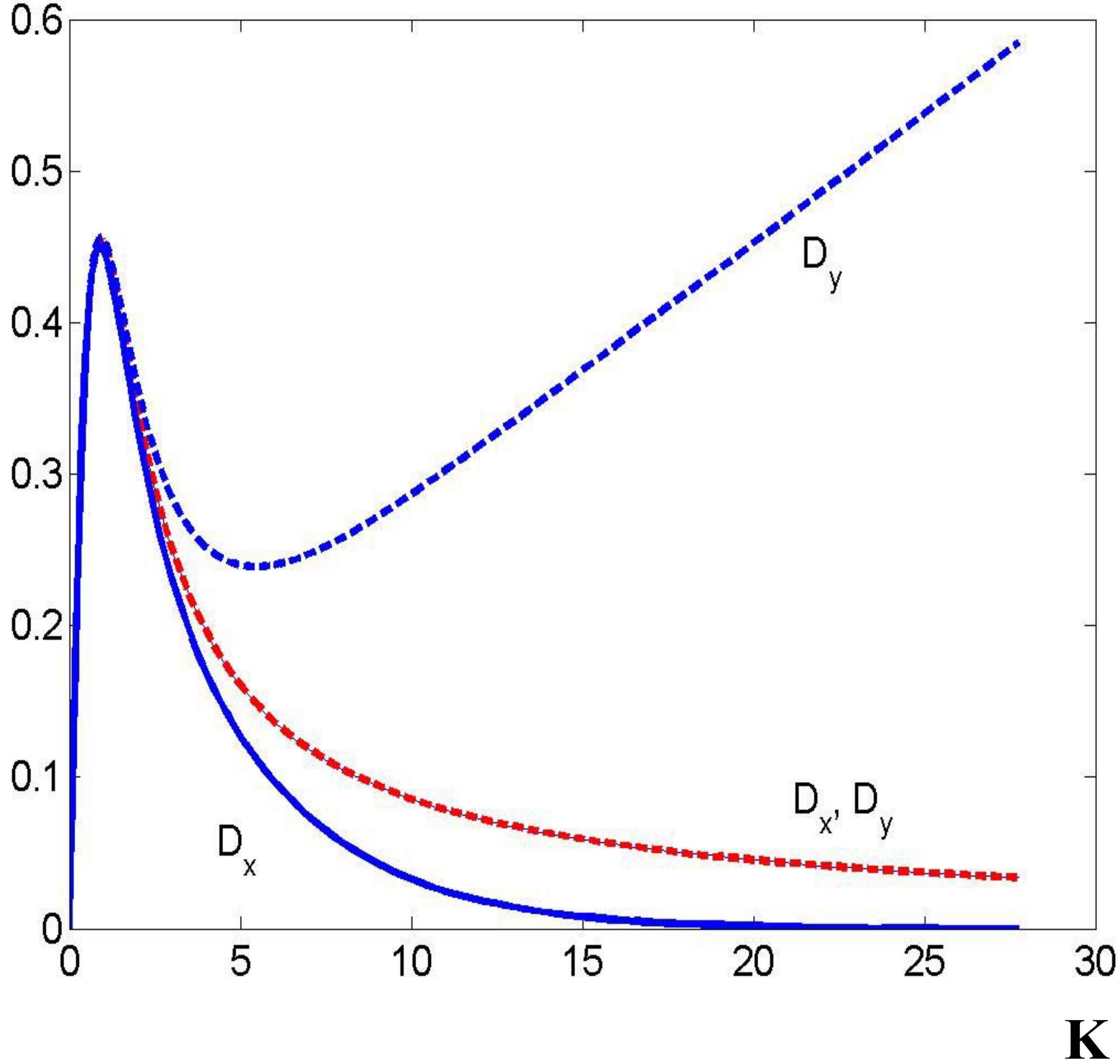
$$\langle x(t) \rangle = V_d t$$

(statistical invariance of the Lagrangian velocity).

$$\frac{n_f \langle v(x(t)) \rangle}{n} = V_d$$

$$\rightarrow \langle v(x(t)) \rangle > V_d$$

Very large amplification of the diffusion coefficient along the average velocity and strong decrease of the diffusion in the perpendicular direction ***at large*** **K**



$\overline{V}_d = 0.3$

The general conclusion of all this studies of test particles stochastic fields:

Trapping **determines:**

memory effects,

anomalous transport,

non-Gaussian distribution,

high degree of coherence

Trapping exists when K > 1 and, if there are other components of the motion, they are weak perturbations (collisions, average flow, …)

$$\chi = D_0 / V\lambda_c < 1, \quad V_d / V < 1, \ldots$$

2) Test modes in turbulent plasma

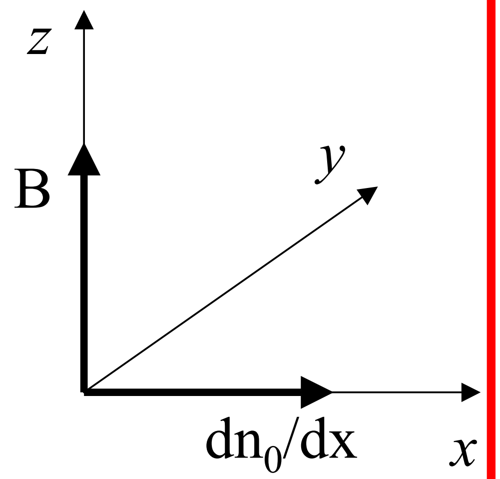
- We consider a turbulent plasma with given (measured) statistical characteristics of the stochastic potential and study linear test modes

- ***AIM: to find the effect of trajectory trapping on test modes***

The growth rate and frequency of the test modes are determined as function of the statistical characteristics of the background turbulence

- ***Drift instability*** in cartezian slab geometry, with constant magnetic field and density gradient.

For smooth density (quiescent plasma):
- The instability is produced by the combined effect of resonant electrons and finite Larmor radius (FLR) of the ions;
- develops turbulence, inverse cascade and vortical structures.



$$\omega = \omega_{*e} \frac{\Gamma_0}{2-\Gamma_0}, \quad \gamma = \sqrt{\pi}\, \frac{\omega(\omega_{*e}-\omega)}{2-\Gamma_0} \frac{1}{|k_z| v_{Te}}, \quad \Gamma_0 = \Gamma_0\left(\frac{k_\perp^{\ 2} \rho_{Li}^{\ 2}}{2} \right)$$

$$\omega_{*e} \equiv k_y V_{*e}, \quad V_{*e} = \frac{T}{e n_0 B} \frac{d n_0}{dx}$$

The formal solution of the Vlasov equation for an initial perturbation with $\varphi_0(\vec{x}, z)$ obtained with the characteristic method (integration along trajectories):

$$n^e(\vec{x},z,t) = n_0(x)\left[1 + \frac{e\phi(\vec{x},z,t)}{T} - \frac{e}{T}\int dv_z F_M(v_z)\int_0^t d\tau\left(\partial_\tau - V_*\partial_y\right)\phi\left(\vec{x}^e(\tau), z - v_z(t-\tau), \tau\right)\right]$$

$$n^i(\vec{x},z,t) = n_0(x)\left[1 + \frac{e\phi(\vec{x},z,t)}{T}\right] +$$

$$n_0(x)\frac{e}{T}\int d^2v\, F_M(v)\int_0^t d\tau\left\{V_*\partial_y\overline{\phi}\left(\vec{x} - \vec{\rho} + \delta\vec{\zeta}(\tau), z, \tau\right) + \right.$$

$$\left.\left(\frac{\varepsilon_{ij}}{B}\partial_j\overline{\phi}\left(\vec{x} - \vec{\rho} + \delta\vec{\zeta}(\tau), z, \tau\right)\partial_i - \partial_t\right)\phi\left(\vec{x} + \delta\vec{\zeta}(\tau), z, \tau\right)\right\}$$

$$n^e(\vec{x},z,t) = n^i(\vec{x},z,t) \longrightarrow \varphi(\vec{x},z,t)$$

➢ **the solution in the zero Larmor radius limit:**

$$\int_0^t d\tau \left(V_* \partial_y - \partial_t\right) \phi\left(\vec{x} + \delta\vec{\zeta}(\tau), z, \tau\right) \longrightarrow \varphi(\vec{x}, z, t) = \varphi_0(\vec{x} + V_* t, z)$$

The time evolution of the potential is its displacement with the diamagnetic velocity $\longrightarrow$ Kubo number in drift type turbulence is large (even in quasilinear case)
The existence of trapping is determined by the amplitude of the ExB drift: it appears when V is larger than the diamagnetic velocity.

$V_d = V_* / V$ ***Trapping parameter for drift type turbulence***

QL: $\tau_{II}^{e} << \tau_* < \tau_{fl} << \tau_c < \tau_{II}^{i}$

NL: $\tau_{II}^{e} << \tau_{fl} < \tau_* << \tau_{II}^{i} < \tau_c$

$$\tau_* = \lambda_c / V_* = 2\pi / \omega_*$$

Thus:

- ***trapping and the nonlinear effects are important for drift type turbulence;***
- ***nonlinear effects appear if*** $V_d < 1$ ***for ions***
- ***electrons have quasiliniar statistics***

Modes on turbulent plasma:

Effect of ion trajectory structures

Nonlinear effect of the diamagnetic velocity

$$\omega = \omega_{*e} \frac{\Gamma_0}{2-\Gamma_0} \exp\left(-\frac{1}{2} k_i^{\,2} S_i^{\,2}(K) \right)$$

$$\gamma = \sqrt{\pi}\, \frac{\omega(\omega_{*e}-\omega)}{2-\Gamma_0} \frac{1}{|k_z| v_{Te}} - k_y^{\,2} D_y \cos(\omega\tau_e) - k_i k_j R_{ij} \frac{\omega}{k_2}$$

$$R_{ij}(\tau,t) = \int_{\tau}^{t} d\theta' \int_{-\infty}^{\tau-\theta'} d\theta\, M_{ij}(|\theta|)$$

$$M_{ij}(|\theta|) = \left\langle v_i(0,0) \frac{\partial}{\partial x_2} v_j(\vec{x}(\theta),\theta) \right\rangle$$

$S_i(K) =$ average size of the trapped trajectory

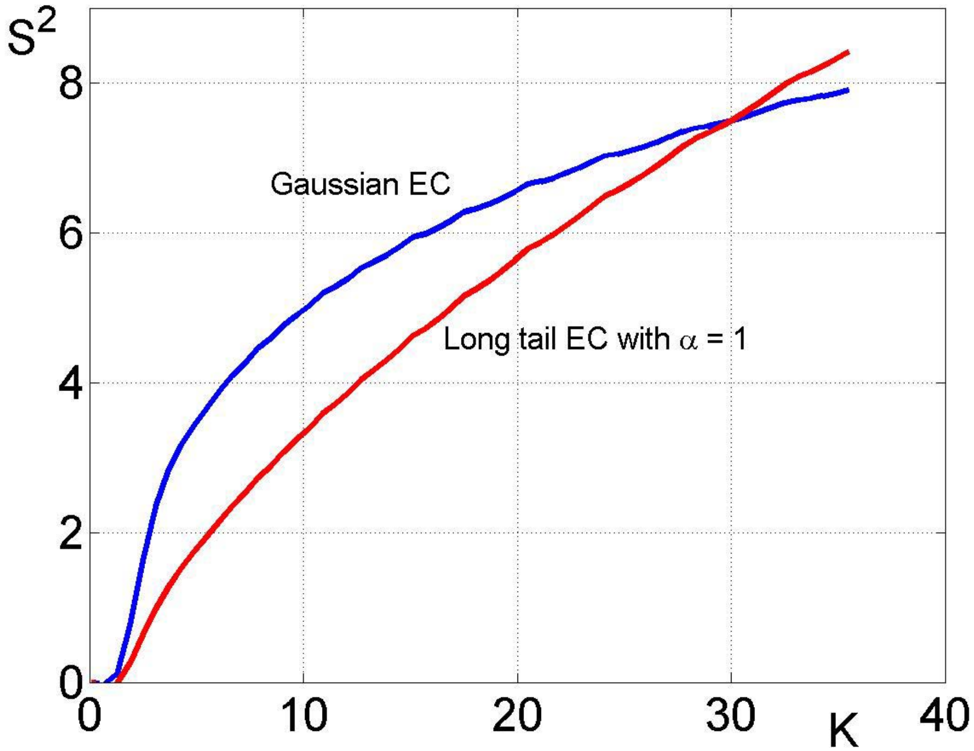
Damping term due to the random part of ion motion (similar with Dupree, but includes the influence of trapping)

Potential with slow time variation **K >1** (*temporary trajectory trapping*)

The large structures of trajectories are destroyed due to time variation, but smaller ones that have the formation time smaller than the correlation time still exist.

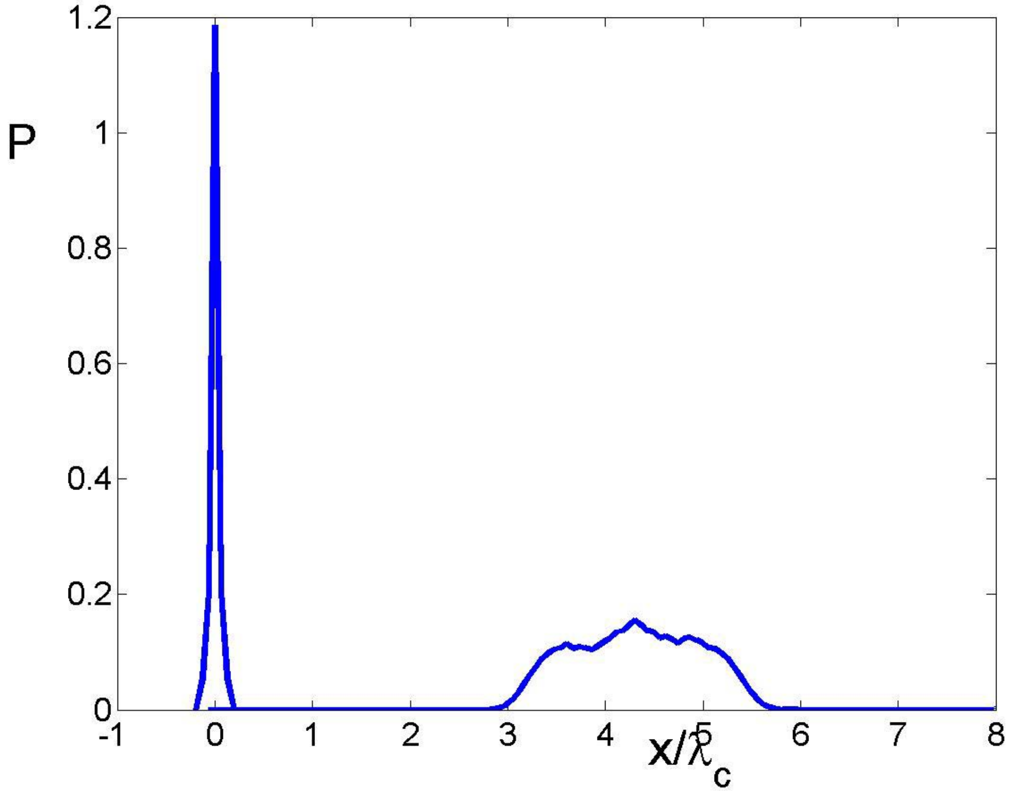
The average size of the trapped trajectories is zero for K < 1 and increases with the increase of K, as a power law at large K

$S^2(K) \approx K^\nu$ for K>>1, where ν depends on the EC $(\nu = 0.37, \quad 0.70)$



Moving stochastic potential: trapped particles move with V_d and free particles move in opposite direction $\longrightarrow$ $\langle x(t) \rangle = 0$ and large $\langle x^2(t) \rangle$

(because of this separation)



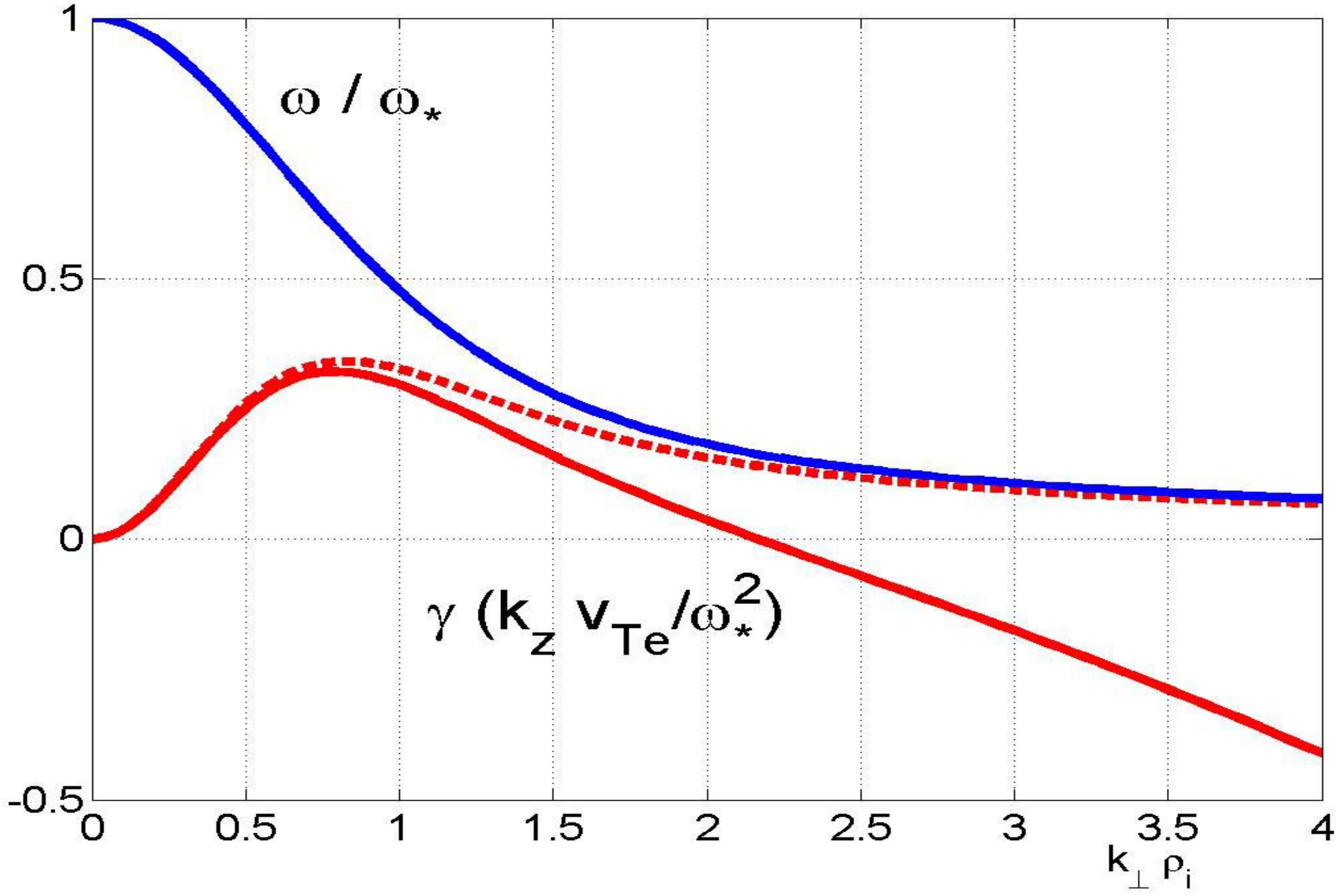
$\longleftarrow$ **0** $\longrightarrow$

V_d $\qquad$ $V' > V_d$

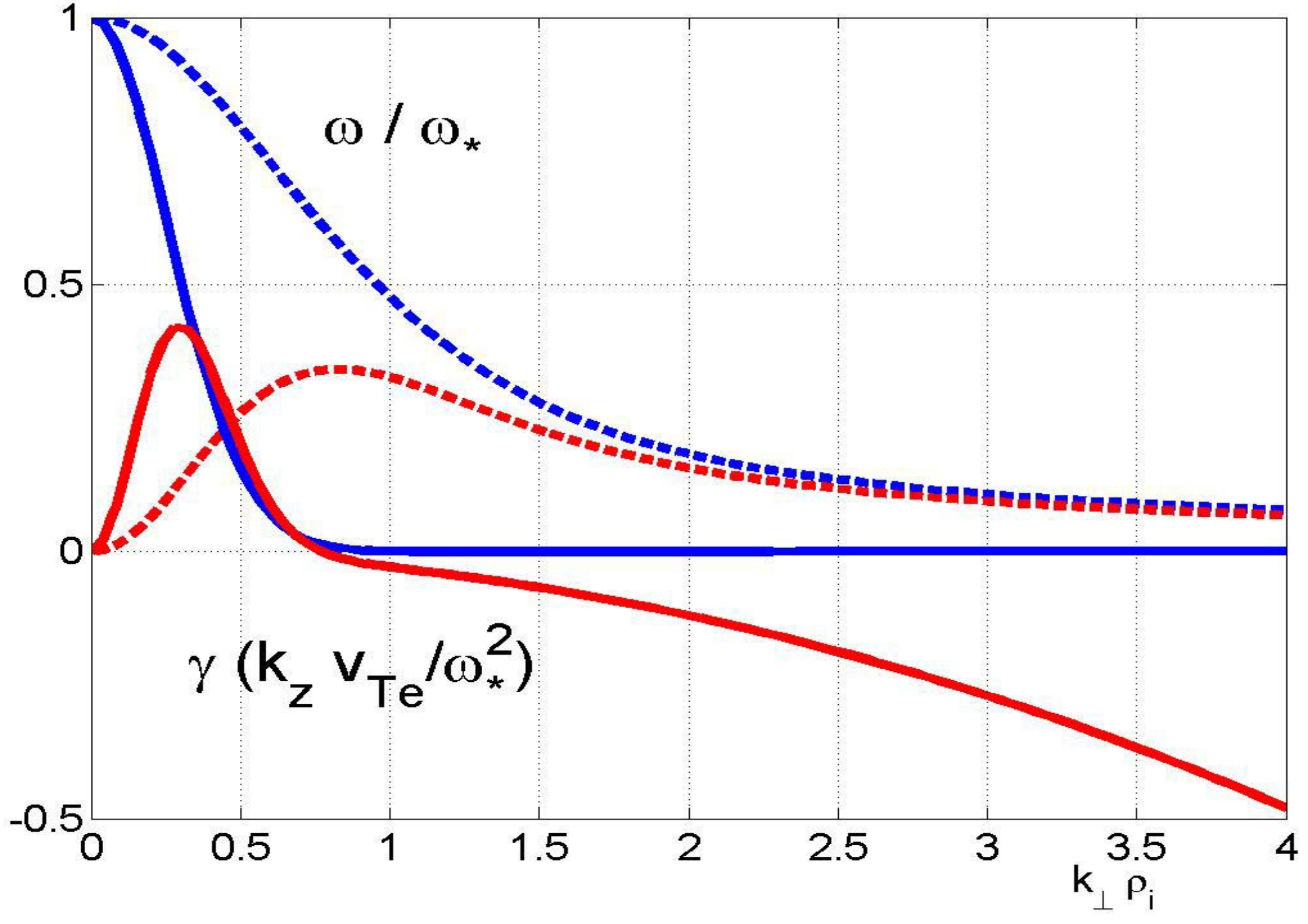
K < 1

Damping of the test modes at large k

(resonance broadening - Dupree)



K >> 1 ***Destabilization of the small wave numbers and narrow spectrum. The inverse cascade appears as the displacement of the unstable range toward small k due to trajectory trapping.***



Conclusions

- ***Trapping determines a strong influence on test modes on turbulent plasmas producing large scale potential cells.***