The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1953-33

International Workshop on the Frontiers of Modern Plasma Physics

14 - 25 July 2008

MHD turbulence and scaling connections with a wider class of nonlinear phenomena.

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MHD turbulence and scaling- connections with a wider class of nonlinear phenomena

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- Turbulence, MHD turbulence and (formal) dimensional analysis
- Scaling and physics- examples from the solar wind
- How general is the concept of a Reynolds number?
- What turbulence does/ does not have in common with idealized avalanching systems (SOC)

more details in Chapman et al, GRL 2007, arXiv:0707.3958

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Universality- an example

Pendulum

$$F = mg, F_t = mg\sin\theta, a_t = l\frac{d^2\theta}{dt^2}$$

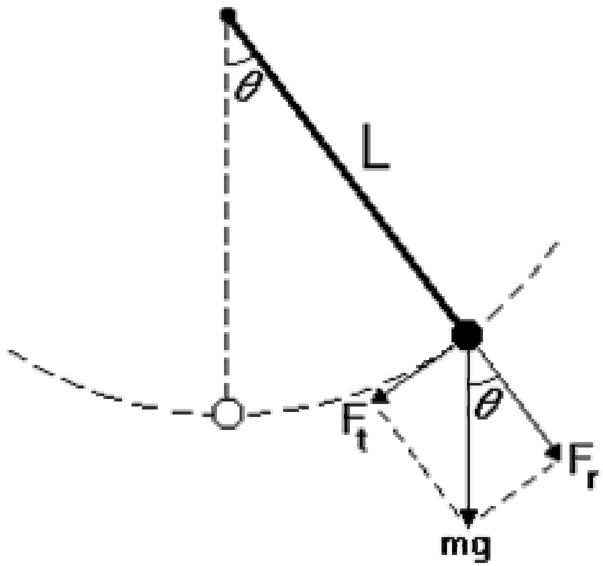
$$F_t = ma_t; \frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta = -\omega^2\frac{\partial V}{\partial\theta}$$

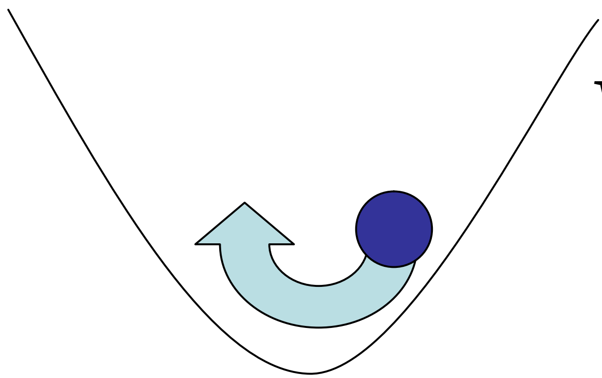
$$V(\theta) = 1 - \cos(\theta) \sim \frac{\theta^2}{2} + \ldots$$

same behaviour at

any local minimum in $V(\theta)$

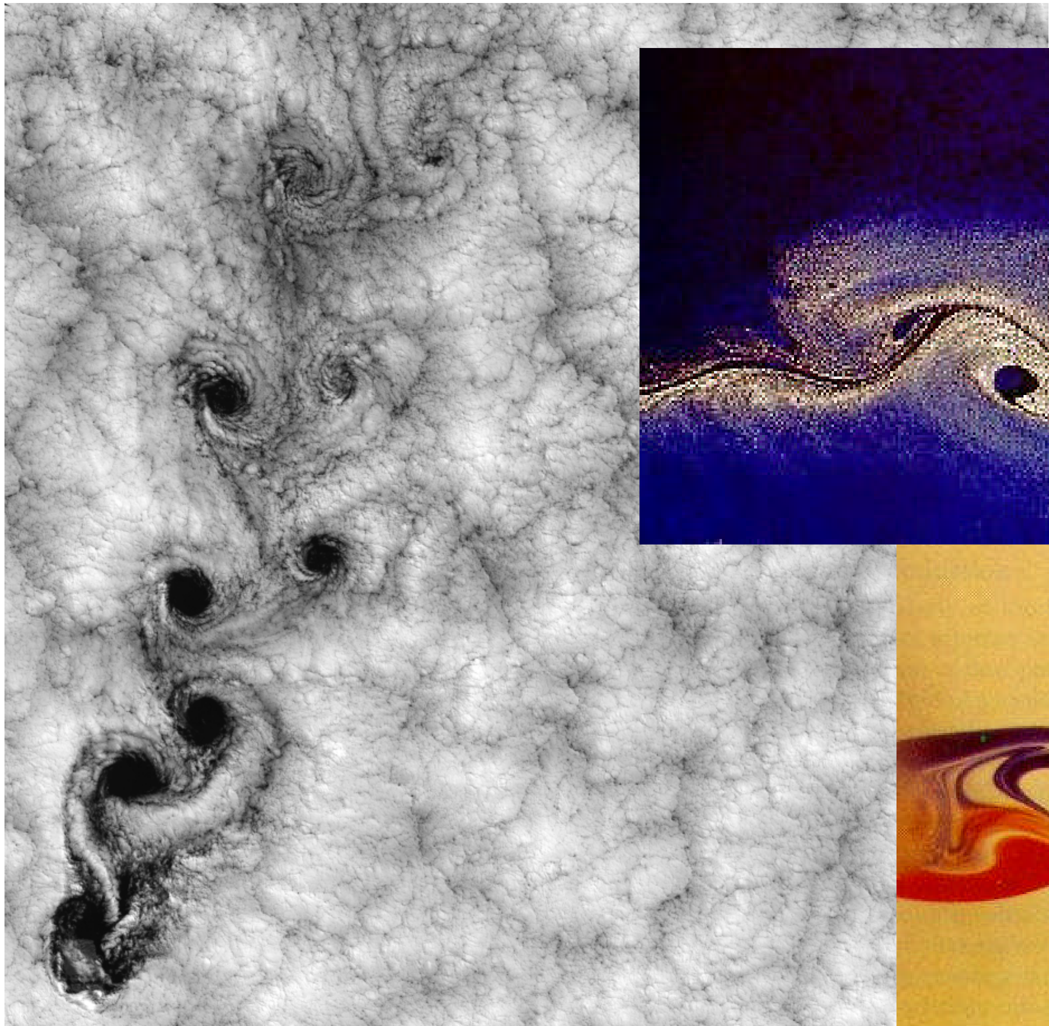
(insensetive to details)

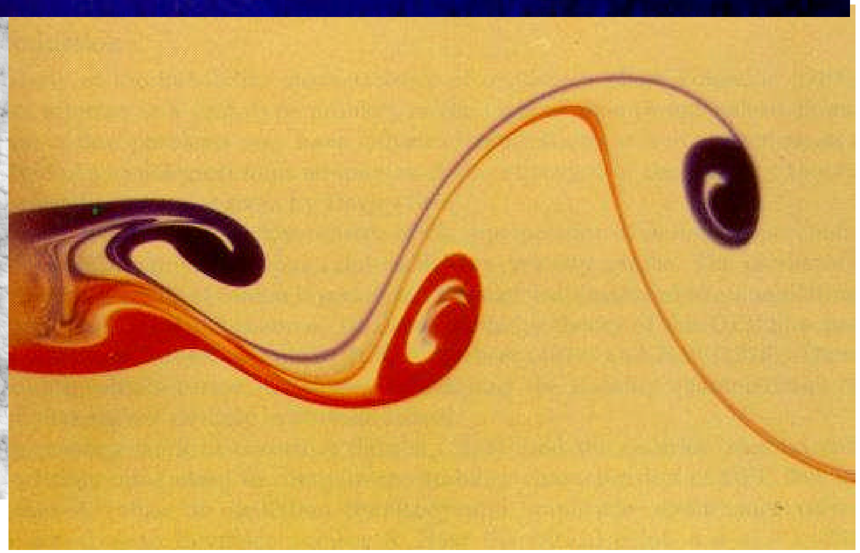




$V(\theta)$

Similarity in action…





Similarity and universality

- Different systems, same physical model
- The same function (suitably normalized) can describe them
- This function is universal (the details do not matter)
- The values of the normalizing parameters are not universal
- How can we find the physical model (solution)?
- Particularly useful in nonlinear systems which are 'hard' to solve – i.e. turbulence!
- 'Classical' inertial range turbulence- self similarity, intermittency…

Quantifying scaling/turbulence

structures on many length/timescales.

Reproducible, predictable in a *statistical* sense.

look at (time-space) differences:

$y(t,\tau) = x(t+\tau) - x(t)$

for all available t_k of the timeseries $x(t_k)$

test for statistical scaling i.e

structure functions $S_p(\tau) = \langle | y(t,\tau) |^p \rangle \propto \tau^{\zeta(p)}$

we want to measure the $\zeta(p)$

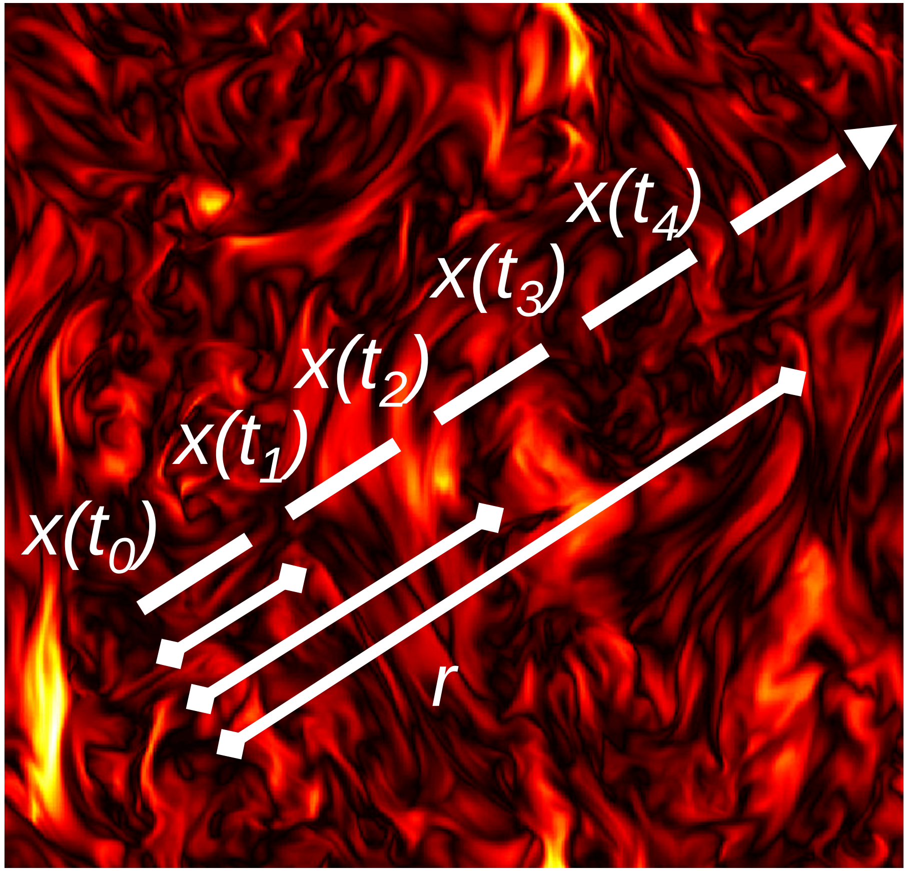
fractal (self- affine) $\zeta(p) \sim \alpha p$

multifractal $\zeta(p) \sim \alpha p - \beta p^2 + \ldots$

would like $\langle | y(t,\tau) |^p \rangle = \int_{-\infty}^{\infty} | y |^p \; P(y,\tau) dy$

finite system/data! conditioning- an estimate is:

$\langle | y |^p \rangle = \int_{-A}^{A} | y |^p \; P(y,\tau) dy$ where $A \sim [10-20]\sigma(\tau)$



Some Phenomenology....Kolmogorov vz MHD scaling

velocity difference $d_r v = v(l+r) - v(l)$, energy transfer rate $\varepsilon_{\mathrm{r}} \sim \dfrac{d_r v^2}{T}$

Kolmogorov: simply have T as the eddy turnover time $T \sim r/d_r v$ so that $\varepsilon_{\mathrm{r}} \sim \dfrac{d_r v^3}{r}$

MHD: now T is due to (say) Alfvenic collisions $T \sim \dfrac{r}{d_r v}\left(\dfrac{v_0}{d_r v}\right)^{\alpha}$ giving $\varepsilon_{\mathrm{r}} \sim \dfrac{d_r v^{3+\alpha}}{r}$

intermittency $\left\langle \varepsilon_r{}^p \right\rangle \sim \bar{\varepsilon}^p \left(r/L\right)^{\tau(p)}$

$\Rightarrow$ Kolmogorov: $\left\langle d_r v^p \right\rangle \sim r^{p/3}\bar{\varepsilon}^{p/3}\left(L/r\right)^{\tau(p/3)} \sim r^{\zeta(p)}$

$\Rightarrow$ MHD: same with $p/3 \to p/(3+\alpha)$ intermittency free $E(k) \sim \left\langle dv^2 \right\rangle/k \sim k^{-(5+\alpha)/(3+\alpha)}$

$\left\langle \varepsilon_r \right\rangle = \bar{\varepsilon}$ independent of r (steady state) so $\tau(1) = 0$ and $\zeta(\alpha+3) = 1$

what is α?

Kolmogorov Obukhov (1941) hydrodynamic: $\alpha = 0$

Irosnikov Kraichnan (1964) weak isotropic MHD $\alpha = 1$,

Goldreich Sridhar (1994-5) strong MHD $\alpha_{\perp} = 0$

Boldyrev (2005) strong, background field anisotropic MHD $\alpha_{\perp} = 1$

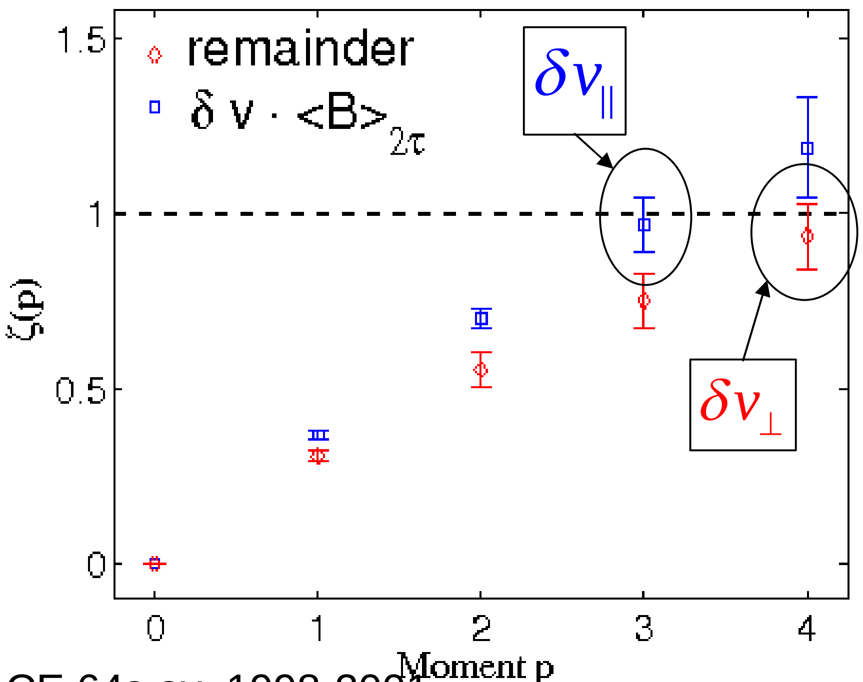
Velocity fluctuations parallel and perpendicular to the *local* B field direction

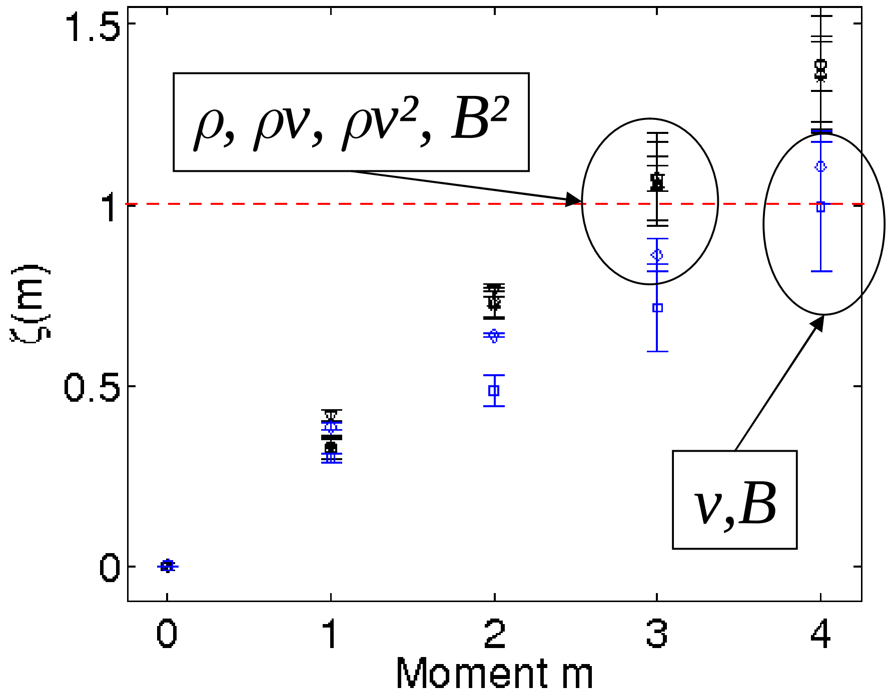
Exponents $\zeta(p)$ for $<|\delta v_{\|,\perp}|^p>\sim\tau^{\zeta(p)}$ for

$\delta v_{\|}=\delta\mathbf{v}.\hat{\mathbf{b}}$ and its remainder $\delta v_{\perp}=\sqrt{\delta\mathbf{v}\bullet\delta\mathbf{v}-\left(\delta\mathbf{v}.\hat{\mathbf{b}}\right)^2}$

$\zeta(3+\alpha)=1$ determines phenomenology

$\overline{\mathbf{B}}=\mathbf{B}(t)+\ldots+\mathbf{B}(t+\tau'),\ \hat{\mathbf{b}}=\frac{\overline{\mathbf{B}}}{|\overline{\mathbf{B}}|}$,here $\tau'=2\tau$ and $\delta\mathbf{v}=\mathbf{v}(t+\tau)-\mathbf{v}(t)$



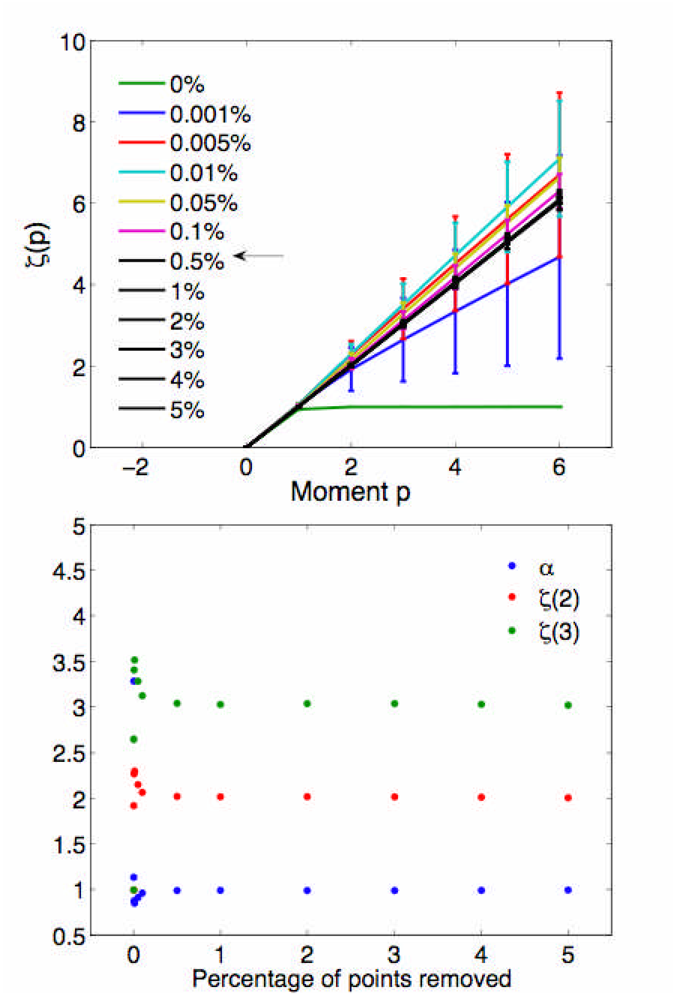


ACE 64s av. 1998-2001

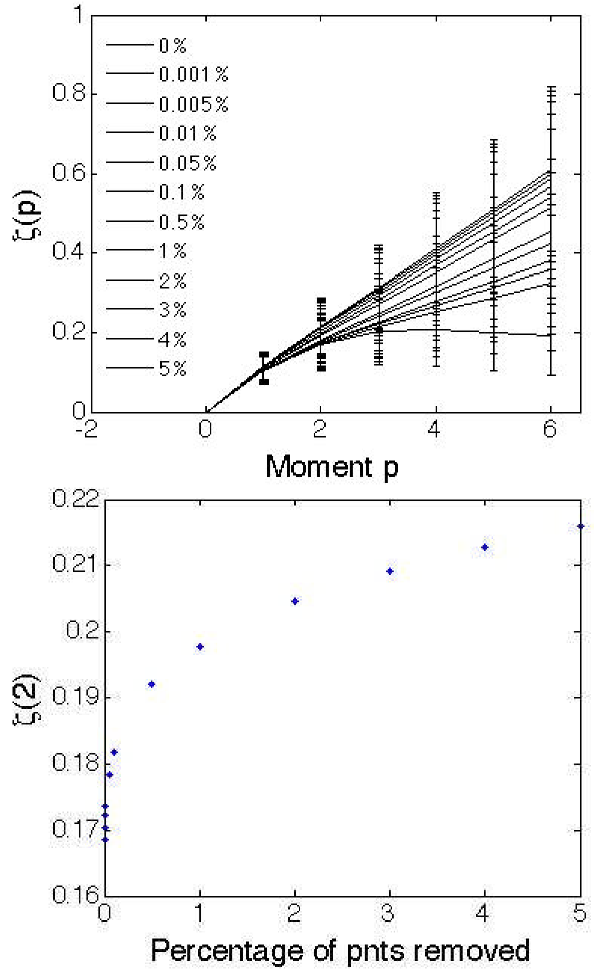
Chapman et al GRL (2007), see also Hnat et al PRL(2005), Kiyani et al PRL(2007)

Distinguishing self- affinity (fractality) and multifractality

Levy flight -- Fractal



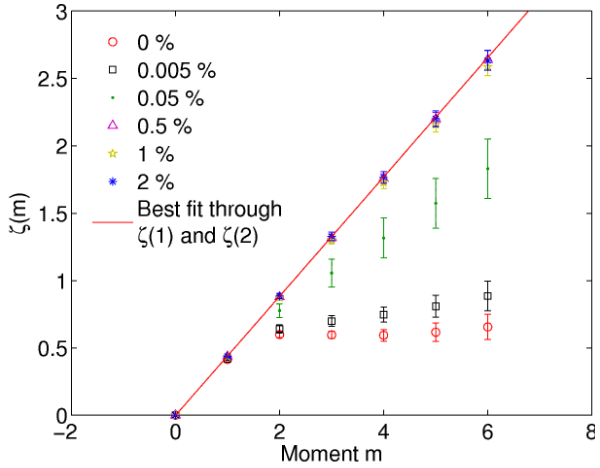
P-model -- Multifractal



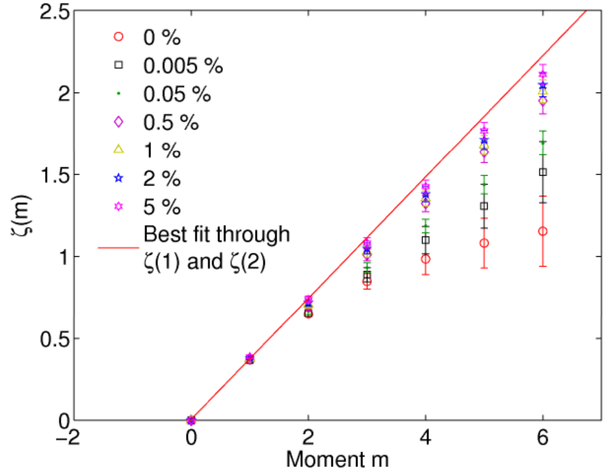
Kiyani et al, PRL (2007)

Solar cycle variation WIND -- $|B|^2$

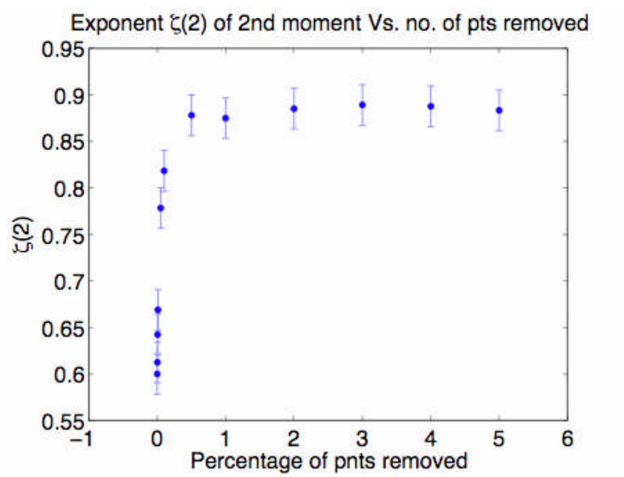
2000 - Solar max

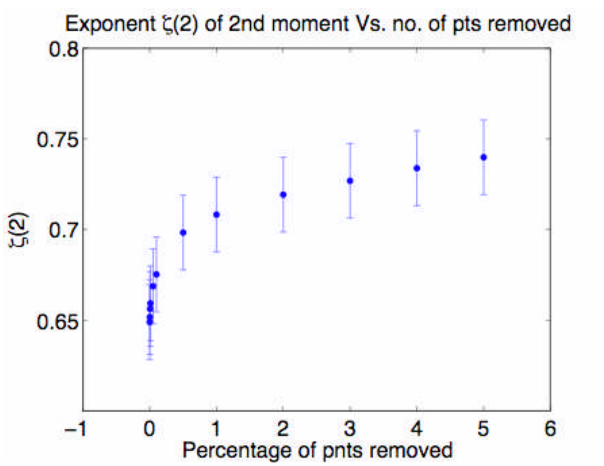


1996 - Solar min



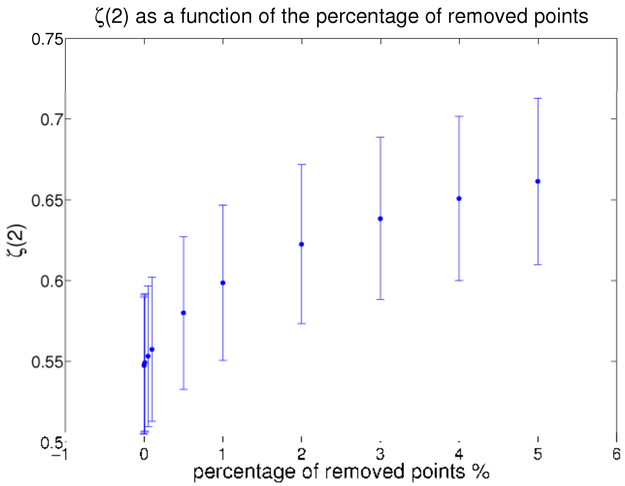
Fractal signature ‘embedded’ in (multifractal) solar wind inertial range turbulence -coincident with complex coronal magnetic topology



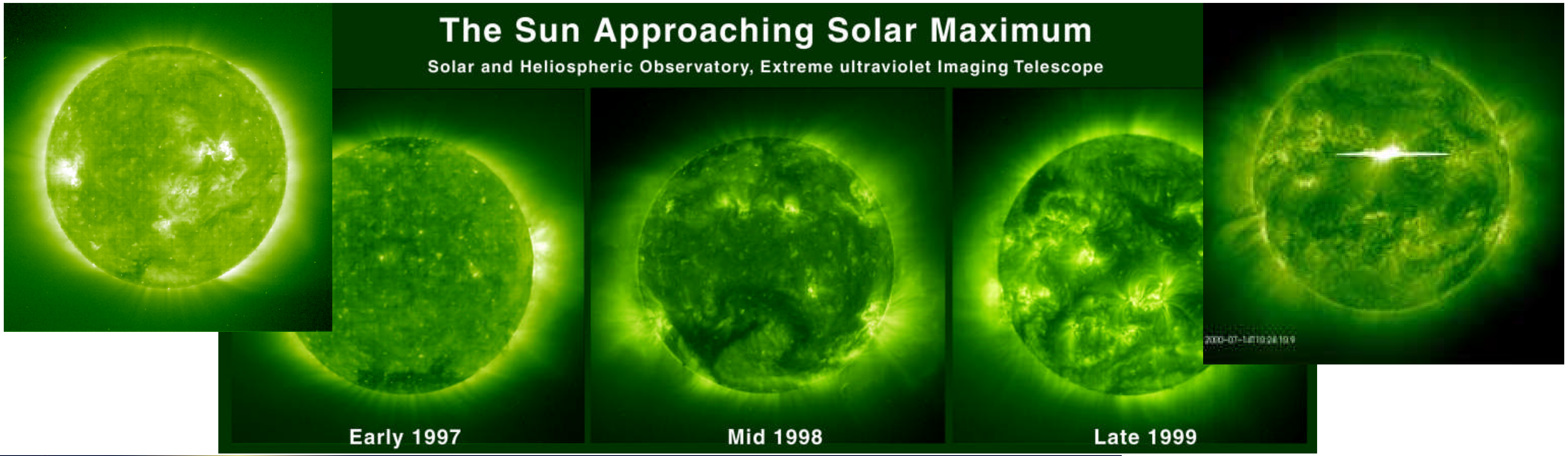


Kiyani et al, PRL (2007), Hnat et al, GRL, (2007)

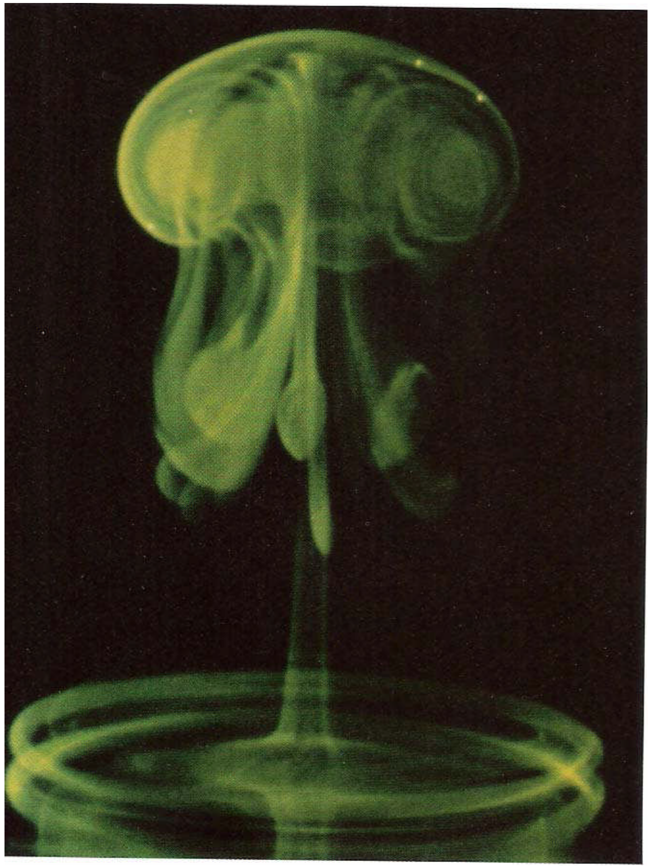
ULYSSES- north polar pass at solar minimum



ULYSSES 60s averages
July-Aug 1995, ~8.5×10^4 points, selected as a quiet interval
-Multifractal
-Fractality coincides with topologically complex coronal fields?



Similarity in action…



Peck and Sigurdson, A Gallery of Fluid Motion, CUP(2003)

Buckingham π theorem

System described by $F(Q_1...Q_p)$ where $Q_{1..p}$ are the relevant macroscopic variables

F must be a function of dimensionless groups $\pi_{1..M}(Q_{1..p})$

if there are R physical dimensions (mass, length, time etc.)

there are $M = P - R$ distinct dimensionless groups.

Then $F(\pi_{1..M}) = C$ is the general solution for this universality class.

To proceed further we need to make some intelligent guesses for $F(\pi_{1..M})$

See e.g. *Barenblatt, Scaling, self - similarity and intermediate asymptotics, CUP, [1996]*

also *Longair, Theoretical concepts in physics, Chap 8, CUP [2003]*

Example: simple (nonlinear) pendulum

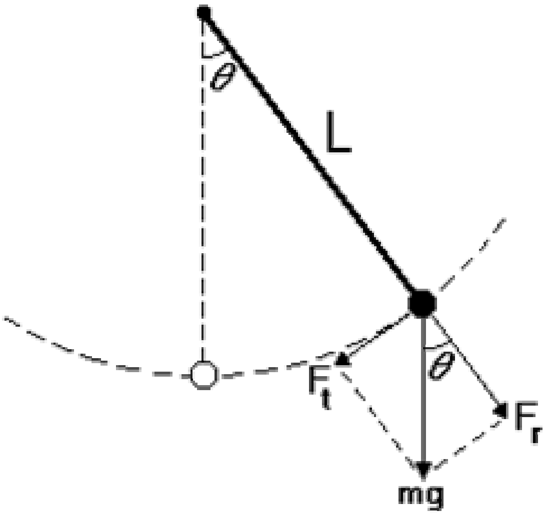
System described by $F(Q_1 ... Q_p)$ where Q_k is a macroscopic variable

F must be a function of dimensionless groups $\pi_{1..M}(Q_{1..p})$

if there are R physical dimensions (mass, length, time etc.) there are $M = P - R$ dimensionless groups

Step 1: write down the relevant macroscopic variables:

variable	dimension	description
θ_0	–	angle of release
m	$[M]$	mass of bob
τ	$[T]$	period of pendulum
g	$[L][T]^{-2}$	gravitational acceleration
l	$[L]$	length of pendulum



Step 2: form dimensionless groups: $P = 5, R = 3$ so $M = 2$

$\pi_1 = \theta_0, \pi_2 = \dfrac{\tau^2 l}{g}$ and no dimensionless group can contain m

then solution is $F(\theta_0, \tau^2 l/g) = C$

Step 3: make some simplifying assumption: $f(\pi_1) = \pi_2$ then the period: $\tau = f(\theta_0)\sqrt{l/g}$

NB $f(\theta_0)$ is universal ie same for all pendula-

we can find it knowing some other property eg conservation of energy..

Example: fluid turbulence, the Kolmogorov '5/3 power spectrum'

System described by $F(Q_1 ... Q_p)$ where Q_k is a macroscopic variable

F must be a function of dimensionless groups $\pi_{1..M}(Q_{1..p})$

if there are R physical dimensions (mass, length, time etc.) there are $M = P - R$ dimensionless groups

Step 1: write down the relevant variables (incompressible so energy/mass):

variable	dimension	description
$E(k)$	$[L]^3[T]^{-2}$	energy/unit wave no.
ε_0	$[L]^2[T]^{-3}$	rate of energy input
k	$[L]^{-1}$	wavenumber

Step 2: form dimensionless groups: $P = 3, R = 2,$ so $M = 1$

$$\pi_1 = \frac{E^3(k)k^5}{\varepsilon_0^2}$$

Step 3: make some simplifying assumption:

$F(\pi_1) = \pi_1 = C$ where C is a non universal constant, then: $E(k) \sim \varepsilon_0^{2/3} k^{-5/3}$

Buchingham π theorem (similarity analysis)

universal scaling, anomalous scaling

System described by $F(Q_1 ... Q_p)$ where Q_k is a relevant macroscopic variable

F must be a function of dimensionless groups $\pi_{1..M}(Q_{1..p})$

if there are R physical dimensions (mass, length, time etc.) there are $M = P - R$ dimensionless groups

Turbulence:

variable	dimension	description
$E(k)$	$L^3 T^{-2}$	energy/unit wave no.
ε_0	$L^2 T^{-3}$	rate of energy input
k	L^{-1}	wavenumber

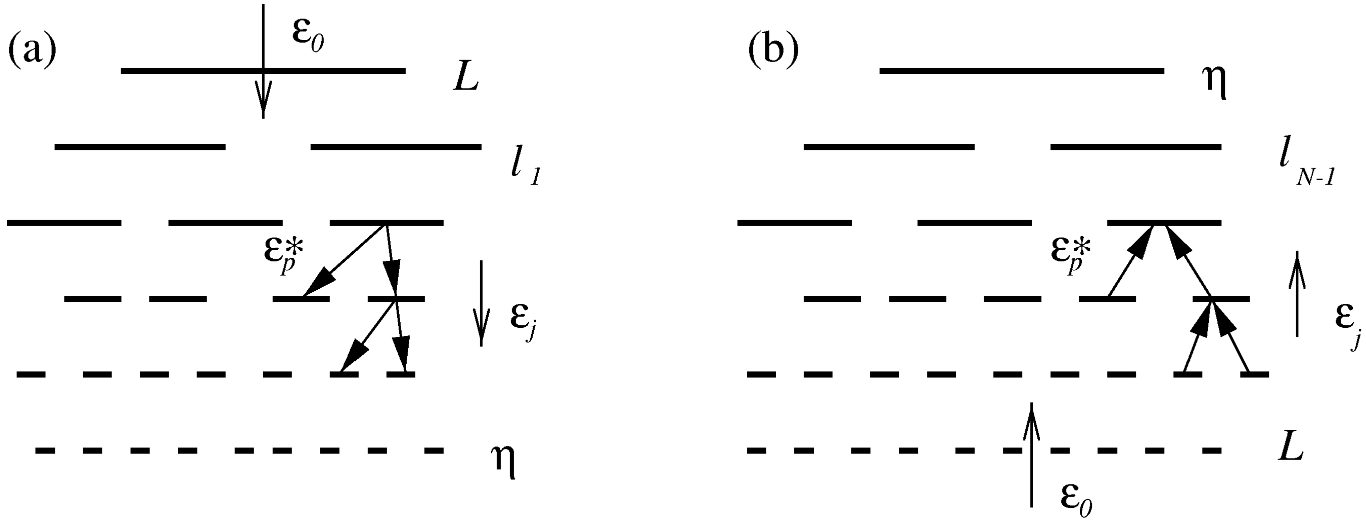
$$M = 1, \pi_1 = \frac{E^3(k)k^5}{\varepsilon_0^2}, E(k) \sim \varepsilon_0^{2/3} k^{-5/3}$$

introduce another characteristic speed....

variable	dimension	*description*
$E(k)$	$L^3 T^{-2}$	energy/unit wave no.
ε_0	$L^2 T^{-3}$	rate of energy input
k	L^{-1}	wavenumber
v	$L T^{-1}$	characteristic speed

$$M = 2, \pi_1 = \frac{E^3(k)k^5}{\varepsilon_0^2}, \pi_2 = \frac{v^2}{Ek} \text{ let } \pi_1 \sim \pi_2^{\alpha}, E(k) \sim k^{-(5+\alpha)/(3+\alpha)}$$

Turbulence and ‘degrees of freedom’



- System is driven on one lengthscale (L) and dissipates on another (η) –forward cascade
- Inverse cascade- same thing, just the other way around
- System has many degrees of freedom i.e. structures on many lengthscales (eddies here)
- System is scaling- structures, processes can be rescaled to ‘look the same on all scales’
- These structures transmit some dynamical quantity from one lengthscale to another that is, over all the d.o.f.
- There is conservation of flux of the dynamical quantity- here energy transfer rate
- Steady state (not equilibrium) means energy injection rate balances energy dissipation rate on the average

Homogeneous Isotropic Turbulence and Reynolds Number

Step 1: write down the relevant variables:

variable	dimension	description
L_0	$[L]$	driving scale
η	$[L]$	dissipation scale
U	$[L][T]^{-1}$	bulk (driving) flow speed
ν	$[L]^2[T]^{-1}$	viscosity

Step 2: form dimensionless groups: $P = 4, R = 2,$ so $M = 2$

$\pi_1 = \frac{UL_0}{\nu} = R_E, \pi_2 = \frac{L_0}{\eta}$ and importantly $\frac{L_0}{\eta} = f(N)$, where N is no. of d.o.f

Step 3: d.o.f from scaling ie $f(N) \sim N^\alpha$ here $\frac{L_0}{\eta} \sim N^3$,or $N^{3\beta}$ or $\frac{L_0}{\eta} \sim \lambda^{N/3}$ or ...

Step 4: assume steady state and conservation of the dynamical quantity, here energy...

transfer rate $\varepsilon_r \sim \frac{u_r^3}{r}$, injection rate $\varepsilon_{inj} \sim \frac{U^3}{L_0}$, dissipation rate $\varepsilon_{diss} \sim \frac{\nu^3}{\eta^4}$ - gives $\varepsilon_{inj} \sim \varepsilon_r \sim \varepsilon_{diss}$

this relates π_1 to π_2 giving: $R_E = \frac{UL_0}{\nu} \sim \left(\frac{L_0}{\eta}\right)^{4/3} \sim N^\alpha, \alpha > 0$ thus N grows with R_E

Generalize the idea of a Reynolds Number

... a control parameter for the onset of 'disorder'

(turbulence, burstiness)

The above is true for other systems with:

$P = 4, R = 2 \ (L, T), \ \text{so} \ M = 2$

$\pi_1 = R_E$ the Reynolds Number

$\pi_2 = f(N)$ where N is the number of degrees of freedom

flux of some dynamical quantity is conserved- steady state

scaling so $f(N) \sim N^{\alpha}$

gives $\pi_1 = f(\pi_2)$ or $R_E = f(N)$

Avalanching systems and scaling behaviour

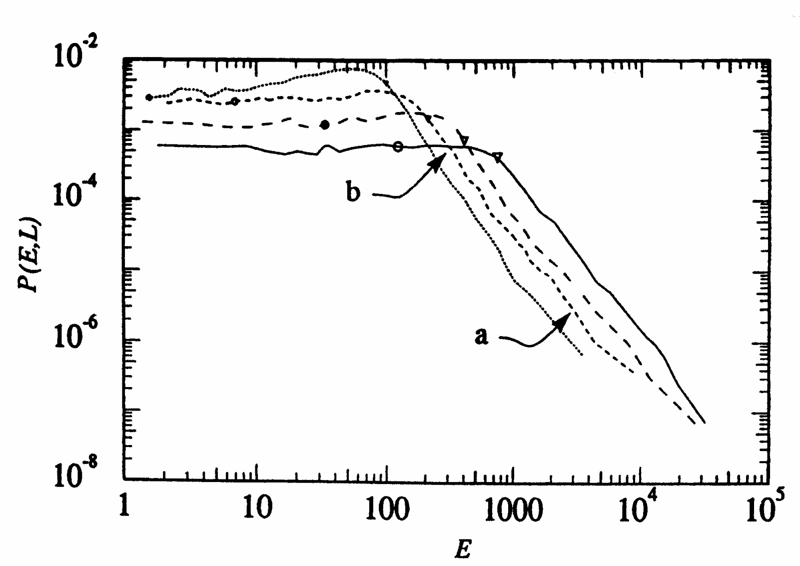
Avalanche models: add grains slowly, redistribute only if local gradients exceeds a critical value

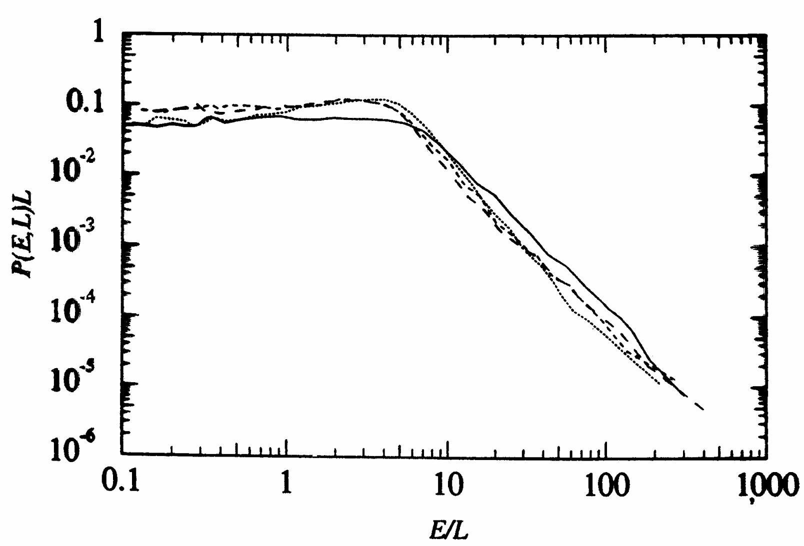
Suggested as a model for bursty transport and energy release in plasmas- solar corona, magnetotail, edge turbulence in tokamaks (L-H), accretion disks

Avalanching systems

- Threshold for avalanching
- Avalanches are much faster than feeding rate
- Avalanches on all sizes, no characteristic size
- Feeding rate=outflow rate on average only
- System moves through many metastable states- rather than toward an equilibrium

Statistics of avalanches (rice)





Shown: Statistics of energy dissipated per avalanche

- Power law- no characteristic event size: scaling
- ‘finite size scaling’- Normalize to the size of the box

Frette et al, Nature (1996)

- Dynamical quantity- rice
- Flux is conserved
- d.o.f. are the possible avalanche (sizes/topplings)

Avalanche model (Self Organized Criticality and all that...)

Step 1:

variable	dimension	description
L_0	$[L]$	system size
δl	$[L]$	grid size
h	$[S][T]^{-1}$	average driving rate per node
ε	$[S][T]^{-1}$	system average dissipation/loss

Step 2: form dimensionless groups: $P=4, R=2,$ so $M=2$

$$\pi_1 = \frac{h}{\varepsilon} = R_A, \pi_2 = \frac{L_0}{\delta l} = f(N)$$ where N is no. of d.o.f.

Step 3: d.o.f from scaling ie $f(N) \sim N^{\alpha}, N \sim \left(\frac{L_0}{\delta l}\right)^{\alpha}$ with Euclidean dimension $D \geq \alpha > 0$

Step 4: assume steady state and conservation of the dynamical quantity, here sand...S

conservation of flux of sand gives $h \times (\text{no of nodes}) \sim \varepsilon$

so $h\left(\frac{L_0}{\delta l}\right)^{D} \sim \varepsilon$ this relates π_1 to π_2 giving $R_A = \frac{h}{\varepsilon} \sim \left(\frac{\delta l}{L_0}\right)^{D} \sim N^{-\alpha D}$

this is in the opposite sense to fluid turbulence, N is maximal when $R_A \to 0$

How is SOC different to turbulence? consider...

Intermediate driving (or what happens as we change $R_A \sim h/\varepsilon$):

Suggest two conditions for avalanching transport:

$h\delta t \ll g\delta l$ - takes many timesteps δt to make a cell go unstable

$h\delta t \ll g\delta l \left(\frac{L_0}{\delta l} \right)^D$ -takes many timesteps to swamp the system

where g is average critical gradient, D is Euclidean dimension.

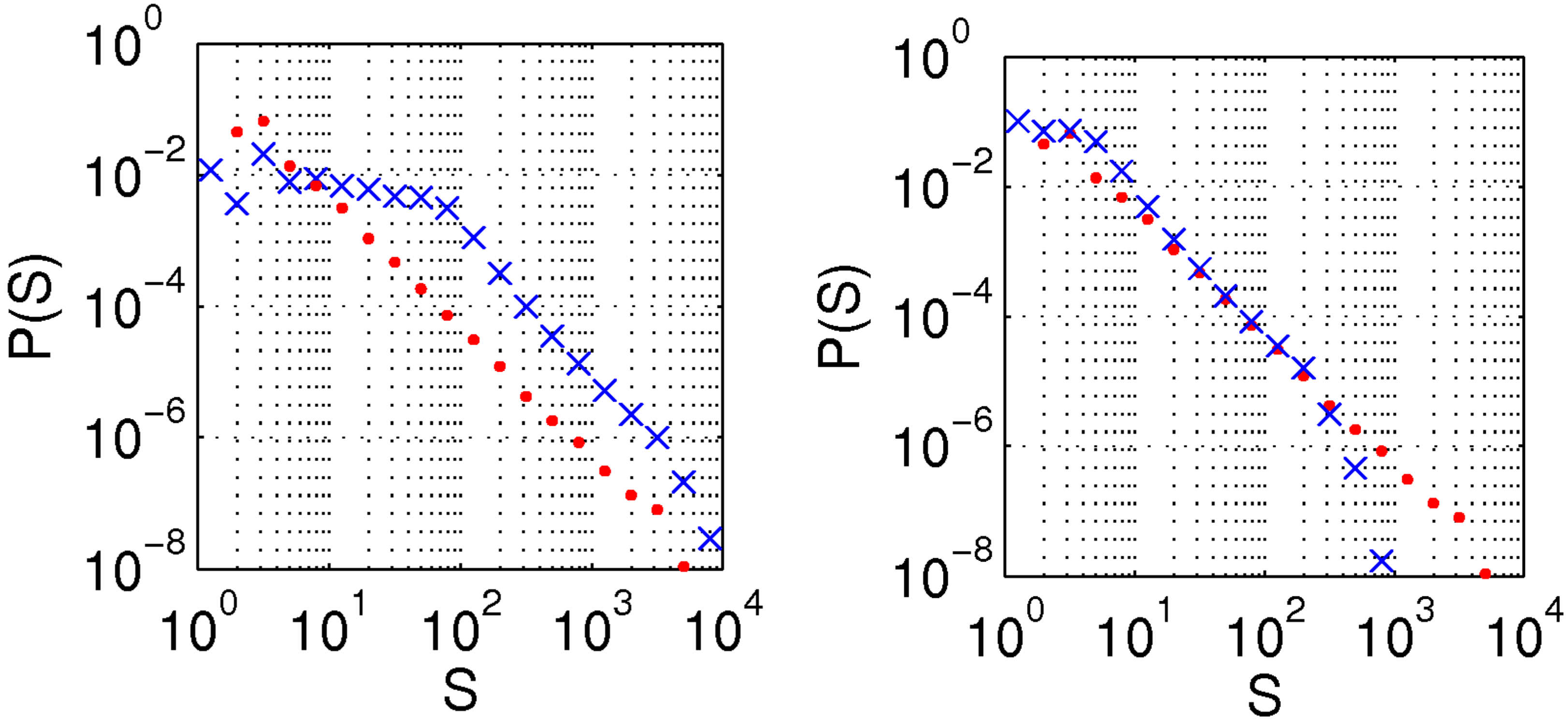
These are both satisfied for SDIDT $(h \to 0, \varepsilon \to 0)$

If $L_0 \gg \delta l$ we can consider intermediate behaviour $gL_0 \gg h\delta t > g\delta l$

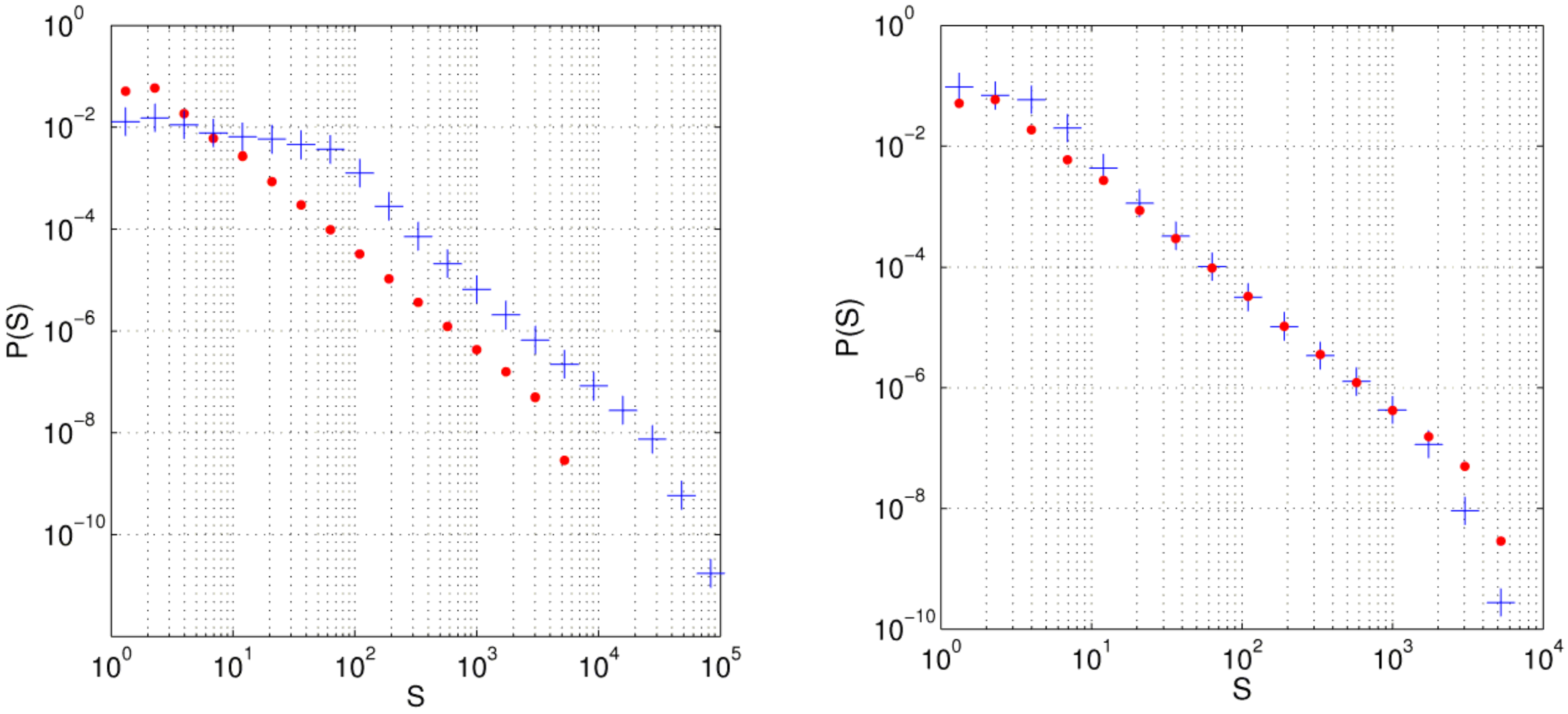
where the smallest avalanches are swamped, but large avalanches persist.

Corresponds to:

reducing the available d.o.f. by increasing h, and hence R_A



Two runs of the BTW (Bak et al, PRL, [1987]) sandpile driven at the ‘top’ corner formed by two adjacent closed boundaries, the other boundaries are open. The box is 100x100 and *h* is 4 (●) and 16 (**X**). Left: raw results; Right: the *h*=16 run is rescaled S→S/16.

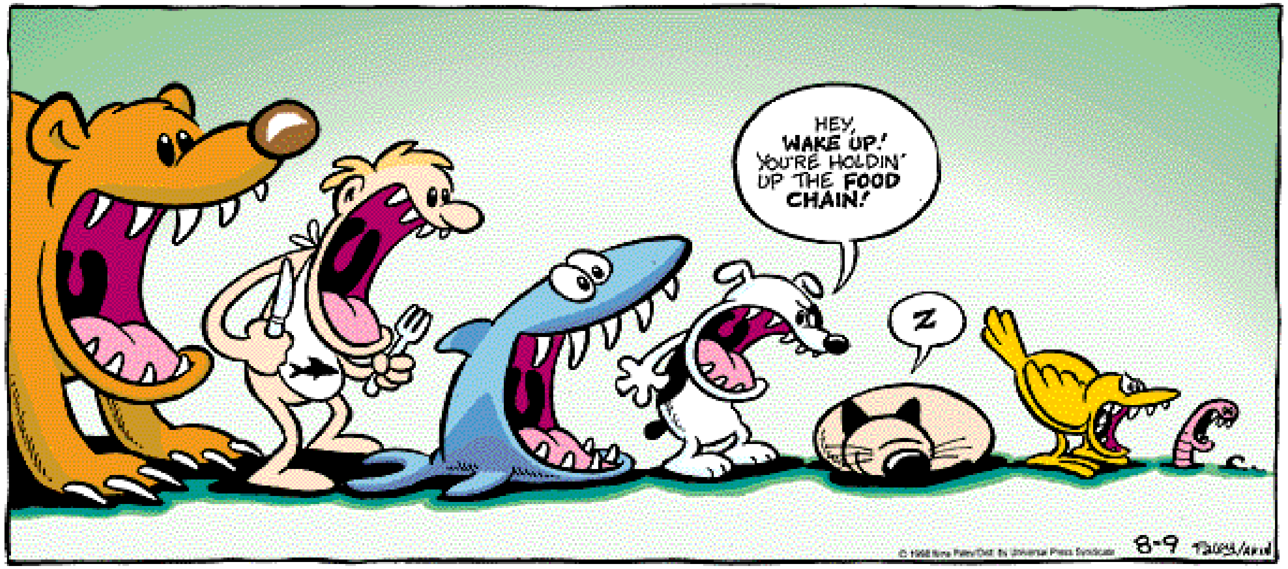


Two runs of the BTW (Bak et al, PRL, [1987]) sandpile
Box 100×100, $h=4$ (●); box 400×400 and $h=16$ (**X**).
Left: raw results; Right: the $h=16$ run is rescaled $S \rightarrow S/16$.
$h=16$, 400×400 run has same scaling, dynamic range as $h=4$, 100×100

To Conclude..

- Scaling- a manifestation of universal behaviour of disordered systems
- Intermittency free scaling in MHD turbulence
- Outlined a general framework for identifying a Reynolds number *R*
- *R* is the control parameter for a broad range of systems that are many coupled d.o.f., driven, dissipating and on average in steady state
- Scaling, flux conservation relates the Reynolds number to the number of d.o.f.
- Discussed avalanche models for bursty dynamics and turbulence
- Avalanche models- maximal d.o.f. (SOC) when $R \rightarrow 0$, *in the opposite sense to fluid turbulence,* crossover to laminar flow as we increase *R* but if the system is large enough, we still see 'SOC' over a range of *R*- so applicable to real systems
- Speculate that there are applications elsewhere- level of complexity of ecosystems, of individual organisms, of organizations…

A Reynolds number for ecosystems



- d.o.f. are 'meta- species' i.e. any (interchangeable) species that occupies a particular niche in the web
- Species all linked by predation/consumption which processes some dynamical quantity (energy, biomass..)
- System driven by 'bottom' species introducing energy/biomass and top predators removing it
- It does not matter what the dynamical quantity is as long as we can conserve flux
- still ok if there are losses i.e. a fixed fraction is passed from one species to the next, or if there is recycling (bottom species feeding off dead top predators)
- Steady state: timescale over which we change R_B is slow compared to timescale for d.o.f. to propagate the dynamical quantity through the web (recycling time)

Velocity fluctuations parallel and perpendicular to the *local* B field direction

Exponents $\zeta(p)$ for $<\left|\delta v_{\parallel,\perp}\right|^{p}>\sim\tau^{\zeta(p)}$ for

$\delta v_{\parallel}=\delta\mathbf{v}.\hat{\mathbf{b}}$ and its remainder $\delta v_{\perp}=\sqrt{\delta\mathbf{v}\bullet\delta\mathbf{v}-\left(\delta\mathbf{v}.\hat{\mathbf{b}}\right)^{2}}$

$\overline{\mathbf{B}}=\mathbf{B}(t)+\ldots+\mathbf{B}(t+\tau'),\ \hat{\mathbf{b}}=\frac{\overline{\mathbf{B}}}{\left|\overline{\mathbf{B}}\right|}$, here $\tau'=2\tau$ and $\delta\mathbf{v}=\mathbf{v}(t+\tau)-\mathbf{v}(t)$