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Shear Flow Effects on the Excitation and Evolution of Neoclassical Tearing Modes.

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Shear Flow Effects on the Excitation and Evolution of Neoclassical Tearing Modes

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Tokamak Instabilities

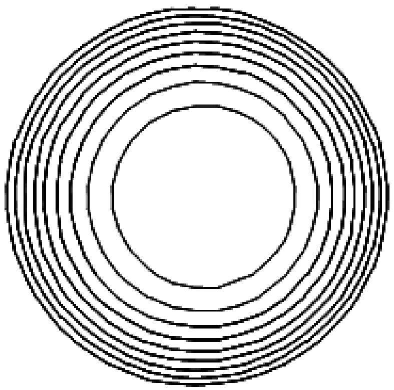
Magnetic field perturbation:

$$\vec{b} = \nabla \varphi \times \nabla \tilde{\psi}$$

$$\tilde{\psi}(\varphi, \vartheta, r) = \tilde{\psi}_0(r) \cdot e^{\, i(n\varphi - m\vartheta)}$$

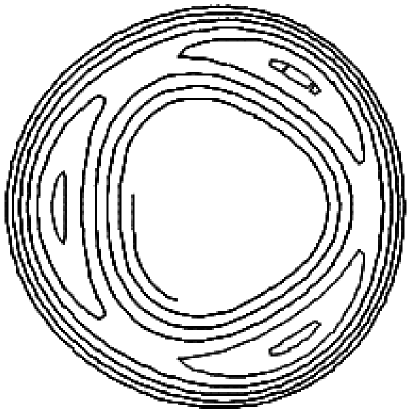
$q(r) = \frac{m}{n}$ – inside the plasma $(q^* > \frac{m}{n})$

$q(r) = \frac{m}{n}$ – outside the plasma $(q^* < \frac{m}{n})$



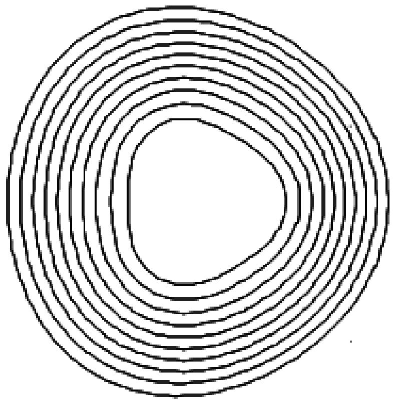
Unperturbed Magnetic Surfaces

▶



Internal (Tearing) Modes

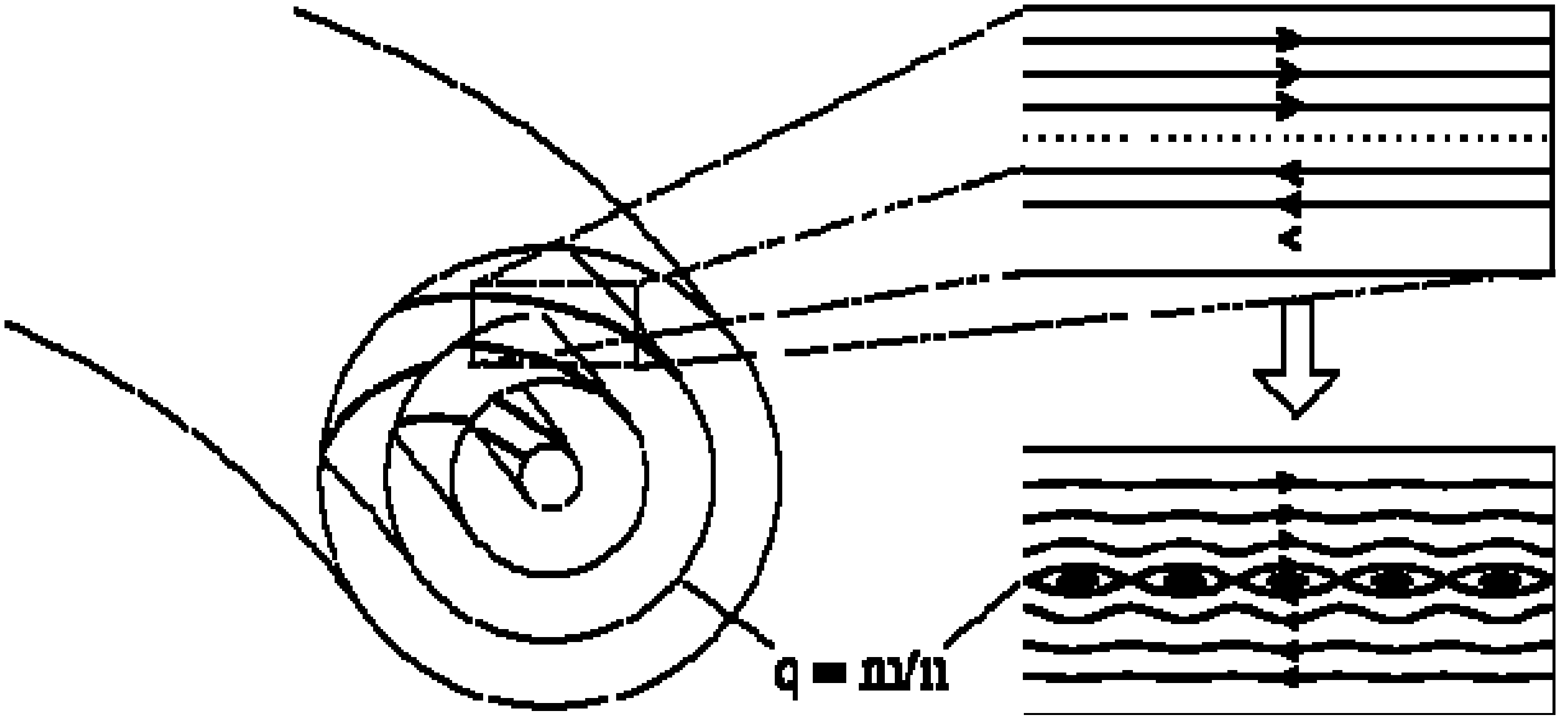
or



External Kink Modes

Tearing Modes and Magnetic Reconnection

k•B=0



``Tearing'' of a current sheet

Classical Tearing Modes

- **Asymptotic theory**- uses two regions of the plasma
  - Outer region - marginal ideal MHD - kink mode
  - Inner region - include effects of inertia, resistivity, nonlinearity, viscosity etc.
- Matching between inner and outer region

$$\frac{1}{2}\Delta'\psi_1 = \mu_0 R\int_{-\infty}^{\infty} d\rho \oint \frac{d\alpha}{2\pi}\cos(m\alpha)J_{\|},$$

- Linear theory : $\gamma \sim (\Delta')^{4/5} S^{-3/5}$

Classical TM - contd.

- Near mode rational surface **k·B = 0** ,

  $B_0 = B(r=r_s) - B_\theta(nq'/m)(r-r_s)\alpha$, $\alpha = \theta - (n/m)\varsigma$

  $\delta B = \delta B_r \sin(m\alpha)\,\mathbf{r}$

- Leads to the formation of a **magnetic island**
- Island width $w = 4(\delta B_r\, r_s / B_\theta\, nq')^{1/2}$
- when w > resonant layer thickness - nonlinear effects important
- Nonlinear Evolution - Rutherford regime

$$\frac{dw}{dt} \approx \eta\Delta'$$

$\Rightarrow$ w α t

- The form of the Rutherford equation can be traced to the form of Ohm's Law which governs the inner region solution, e.g.

$$\boxed{E_{\|} = \eta J_{\|}} \qquad E_{\|} \sim -\frac{\partial A_{\|}}{\partial t} \qquad J_{\|} \sim -\nabla^2 A_{\|}$$

$$\Downarrow \qquad \Downarrow$$

$$\frac{d\delta B}{dt} = \eta \frac{\Delta'}{w} \delta B \qquad \Rightarrow \qquad \boxed{\frac{dw}{dt} \approx \eta \Delta'}$$

- In high temperature tokamaks neoclassical effects need to be retained

What are NTMs?

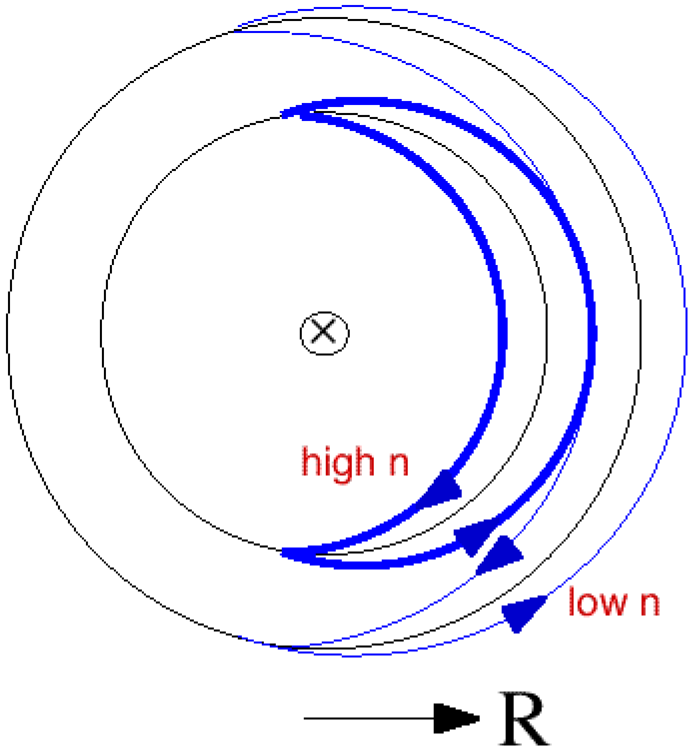
- NTMs are relatively large size **magnetic islands** that develop slowly at mode rational surfaces with low (m,n) mode numbers in **high temperature tokamak** plasmas.

- Like the classical TMs they are current driven but the current source is the **bootstrap current** - a neoclassical (toroidal geometry driven) source of free energy.

- They limit the attainable β in a tokamak to values well below the ideal MHD limit - hence they are a **major concern** for all reactor grade machines i.e. long pulse (steady state) devices.

BOOTSTRAP CURRENT

Projection into a poloidal plane



generated by trapped particles:

example: banana particles

- electrons drift from flux surfaces due to the ∇B-drift
- electrons with low parallel velocity are trapped in the toroidal mirror
  $\Rightarrow$ **banana orbits**
- at the intersection of 2 banana orbits a net current results due to the density gradient
- passing particles exchange momentum with trapped particles
  $\Rightarrow$ **bootstrap current**

similar: helically trapped particles

Modified Ohm's Law

$$< E_{\|} > \; = \; \eta J_{\|} + \frac{1}{neB} < B \cdot \nabla \cdot \pi_{\| e} >$$

$\Downarrow$

Bootstrap current

$\Uparrow$

$$\frac{1}{neB} < B \cdot \nabla \cdot \pi_{\| e} > \; \approx \; \frac{\mu_e}{\nu_e} \frac{1}{B_\theta} \frac{dp}{dr} + \eta \frac{\mu_e}{\nu_e} J_{\|}$$

Electron viscous stress which describes damping of poloidal electron flows - new free energy source.

Dependence on pressure gradient, also fraction of trapped particles

Modified Rutherford Equation

$$\frac{dw}{dt} = \frac{\eta}{\mu_0}(\Delta' + \frac{D_{nc}}{w})$$

where $$D_{nc} = -\sqrt{\epsilon}\,\frac{2\mu_0}{B_\theta^2}p'\frac{q}{q'}k_0$$

$p'q' < 0, \quad D_{nc} > 0$ **Unstable for normal tokamak operation**

$p'q' > 0, \quad D_{nc} < 0$ **Stable in reversed shear regions**

- Can be unstable for $\Delta' < 0 \Rightarrow$ $$w_{sat} = \frac{D_{nc}}{-\Delta'} \approx \frac{r_s\beta_\theta}{m}$$

- for small islands $$w \sim \sqrt{\eta t}$$

Finite perpendicular thermal conductivity effect

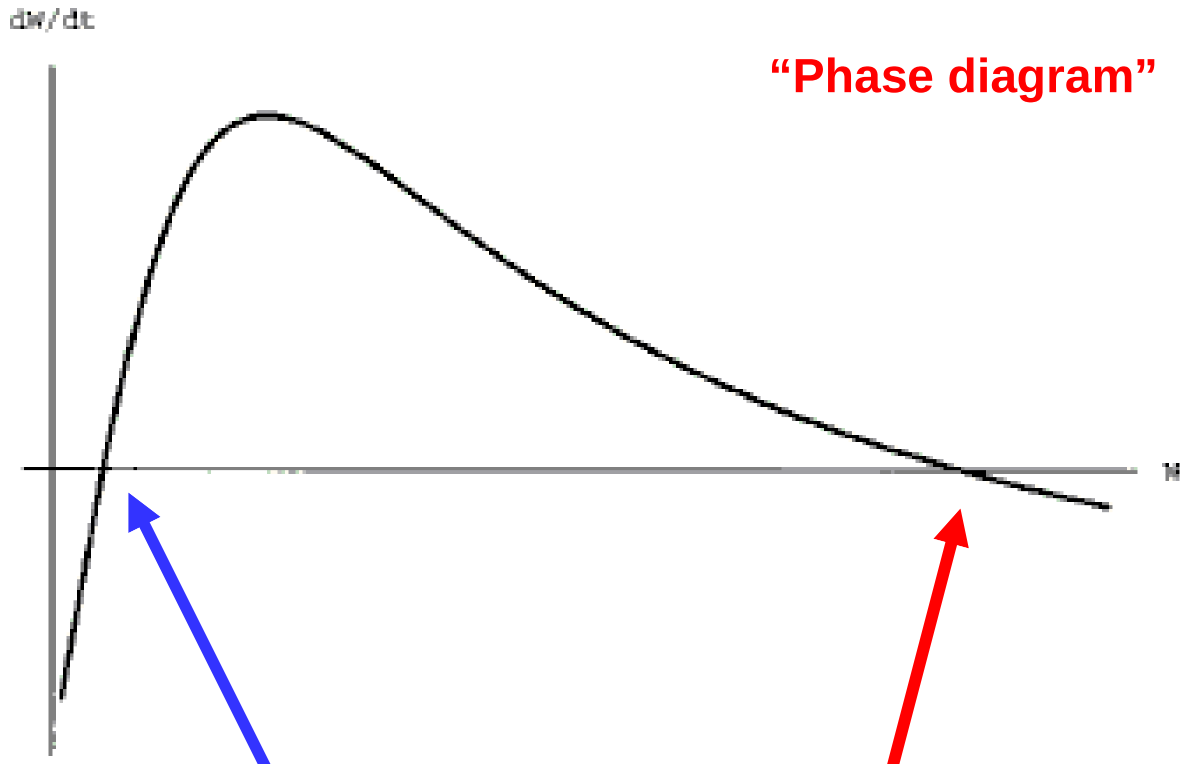
$$\frac{dw}{dt} = \frac{\eta}{\mu_0}(\Delta' + D_{nc}\frac{w}{w^2 + w_c^2})$$

$$w_c \sim \left(\frac{\chi_\perp}{\chi_\parallel}\right)^{1/4} \sqrt{\frac{q^2 R}{m q'}}$$

Threshold - "seed" – island size

$$w_{seed} = -\frac{\Delta' w_c^2}{D_{nc}}$$

NTM characteristics



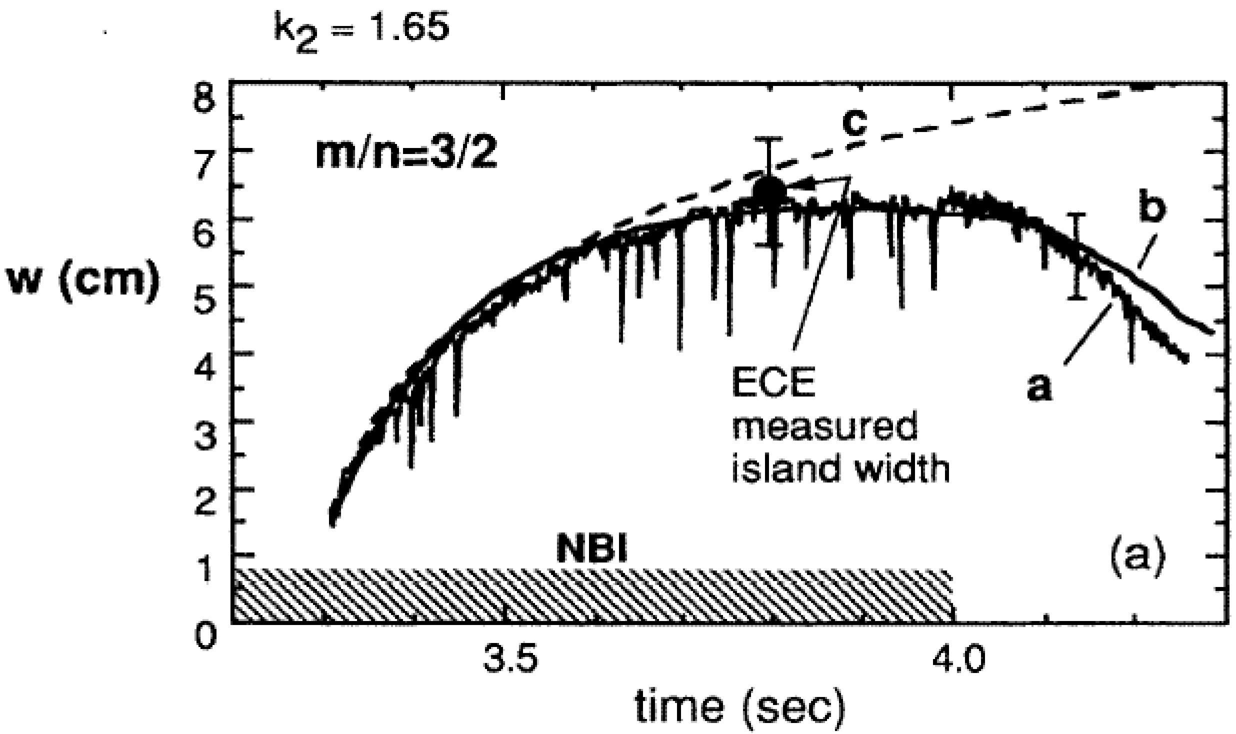
"Phase diagram"

"seed" island necessary for growth
– so NTM is a nonlinear mode
"subcritical instability"

Saturation width proportional to β_θ - hence limits plasma pressure

How is the seed island created?

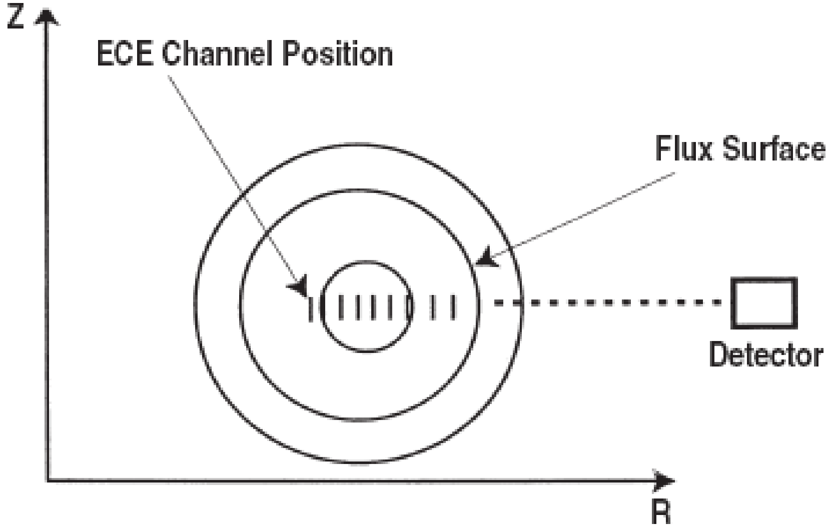
TFTR



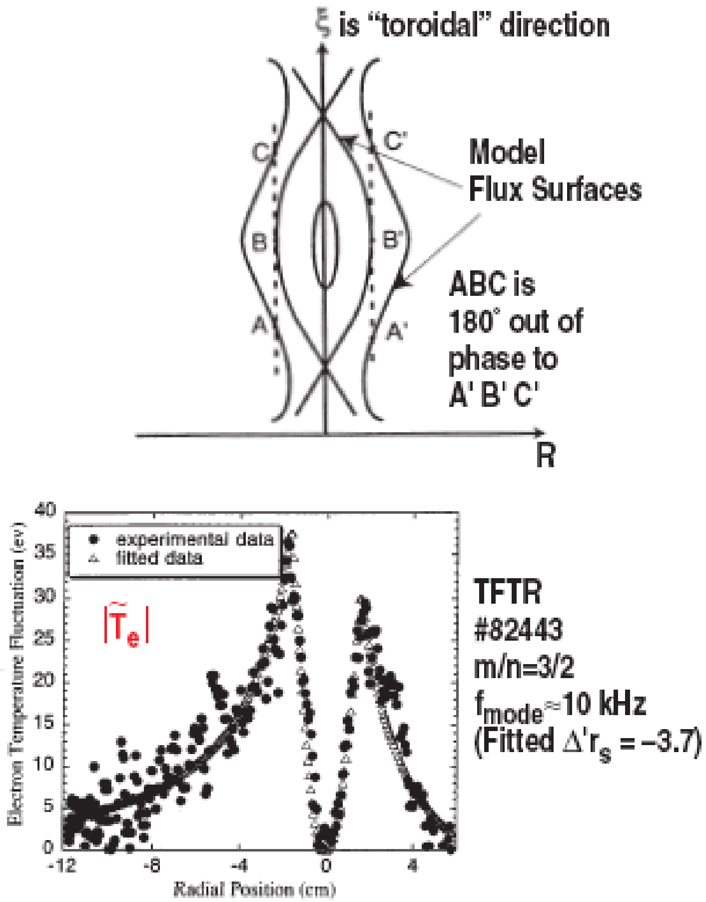
Comparison of “measured” island widths with Rutherford model estimates.

Island Structure Can be Measured by Electron Cyclotron Emission of T_e Fluctuation Radial Profile

- Magnetic surface distortion
  - ★ leads to T_e fluctuation



(Y. Nagayama et al., 1990)



(C. Ren, et., 1998)

w also measured by magn. Probes:

$$w = 4\sqrt{\frac{q\psi}{q'B_{\theta_s}}} = 4\sqrt{\frac{R_0 q}{B_0 s}\rho_s^m \delta B_{\theta,mn,edge}}.$$

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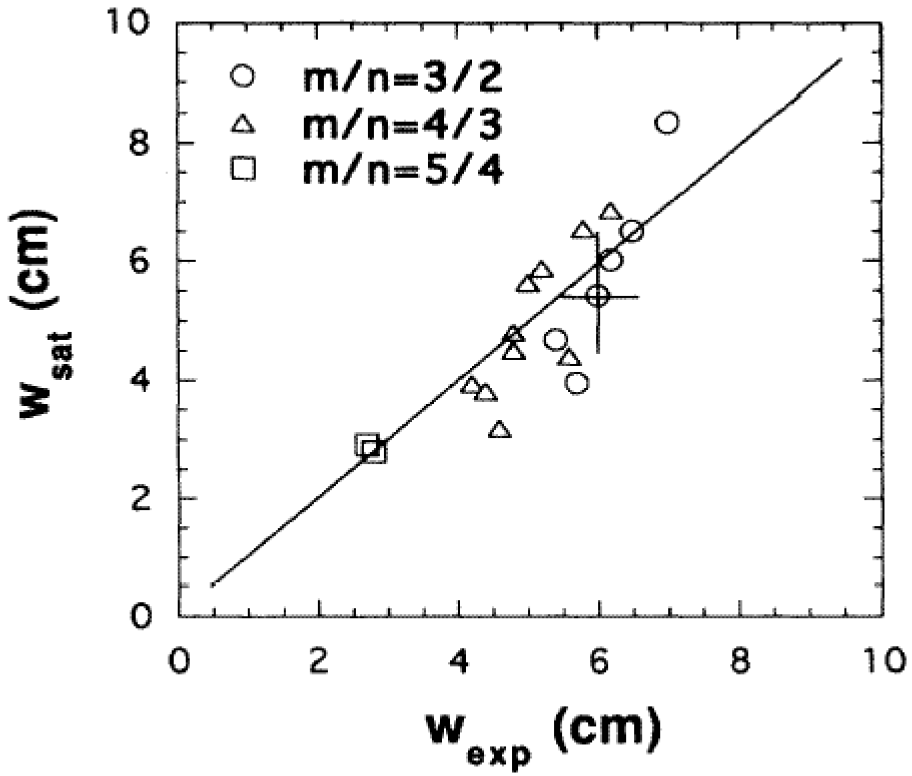
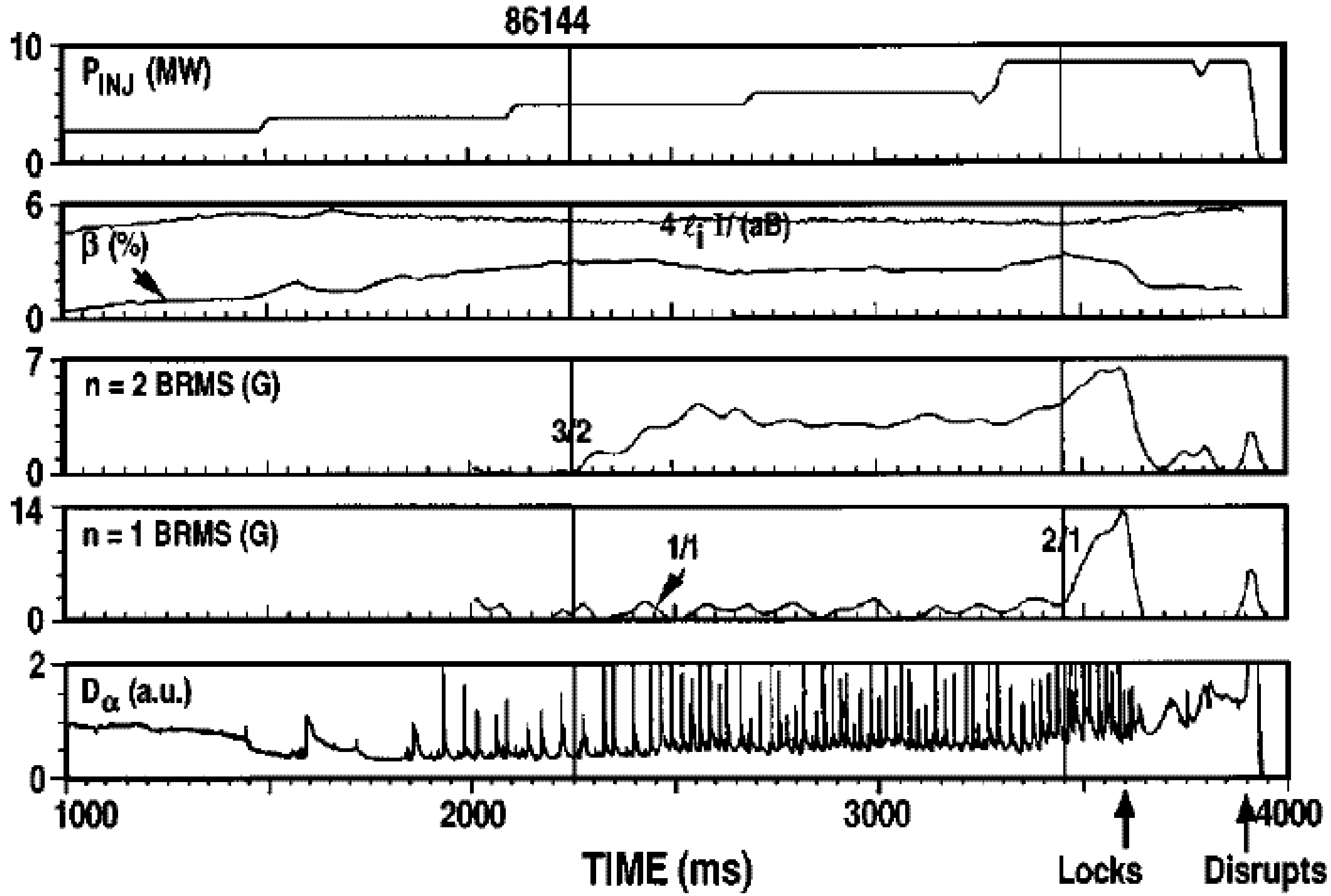
TFTR



Theory - experiment comparison of saturated island widths

D- III- D observations



A 3/2 mode is excited at t=2250 - saturates beta; at t=3450 a 2/1 mode grows to large amp, locks and disrupts. Ideal beta limit is 3.4

[O. Sauter et al, PoP 4 (1997) 1654]

ITER

Island size would be about 60 to 70 cms at q=2 surface

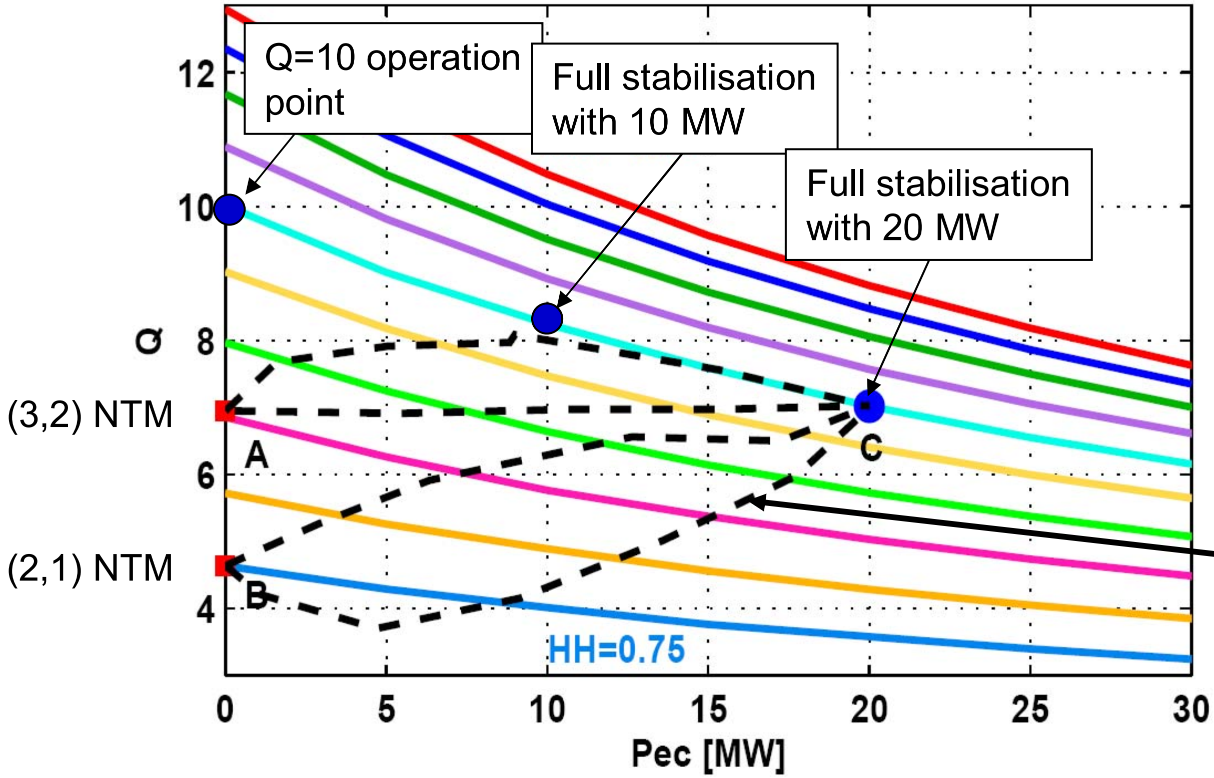
Would seriously compromise performance of ITER

A control scheme using ECCD has been planned

Many other factors can compromise the effectiveness of the control scheme

Some unresolved issues: size of seed island, fast particle interactions, **plasma rotation**

ITER NTMs stabilisation goals



ITER burn curves with ECCD at q=3/2 (A) and q=2 (B)

(O. Sauter, H. Zohm, EPS 2005)

HH = 1.0

Sketch of paths with partial stabilisation

Impact on Q in case of continuous stabilisation (worst case):

- Q drops from 10 to 5 for a (2,1) NTM and from 10 to 7 for (3,2) NTM
- with 20 MW needed for stabilisation, Q recovers to 7, with 10 MW to Q > 8
- note: if NTMs occur only occasionally, impact of ECCD on Q is small

Flows in tokamaks and their possible impact?

- Flows (particularly in the toroidal direction) can arise in a tokamak from unbalanced neutral beam injection (for heating)

- There is also evidence of spontaneous rotation arising during RF heating

- Such flows can influence both outer layer and inner layer dynamics for resistive modes including NTMs

- They can also bring about changes in linear coupling mechanisms such as toroidal coupling between harmonics.

- Past nonlinear studies – mainly numerical – and often limited to simple situations (e.g. poloidal flows, non-self consistent) reveal interesting effects like oscillating islands, distortion in eigenfunctions etc. for classical tearing modes

Refs: Chen &Morrison, '92, 94; Bondeson & Persson, '86,'88,'89; M.Chu,'98

Dewar & Persson, '93; Pletzer & Dewar, '90,'91,'94;

Experimental Evidence of Flow effects on NTMs

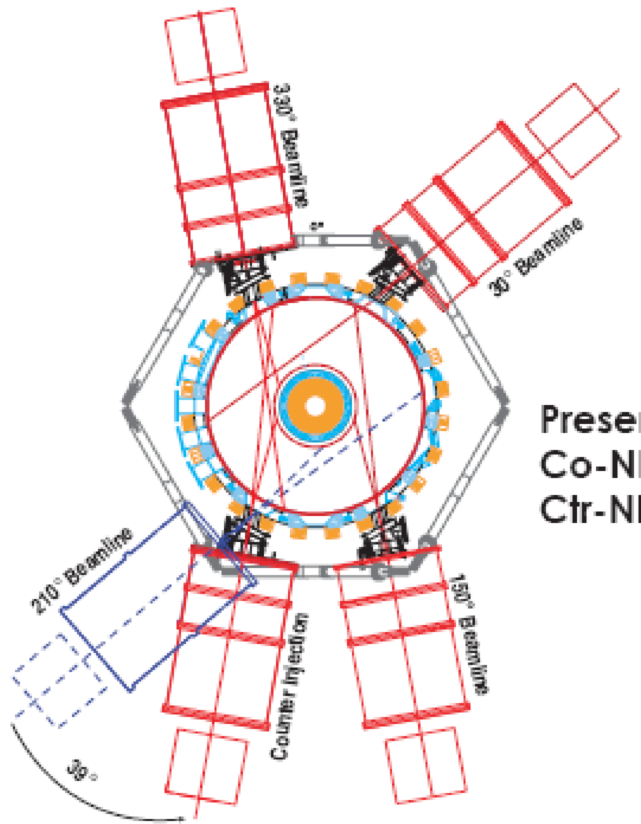
- **D-IIID**
- **JET**

- **Near-toroidal beams inject energy and momentum**
  - ★ net torque varied by ratio of co to counter beams

- **Changes in tearing mode saturated amplitude observed**

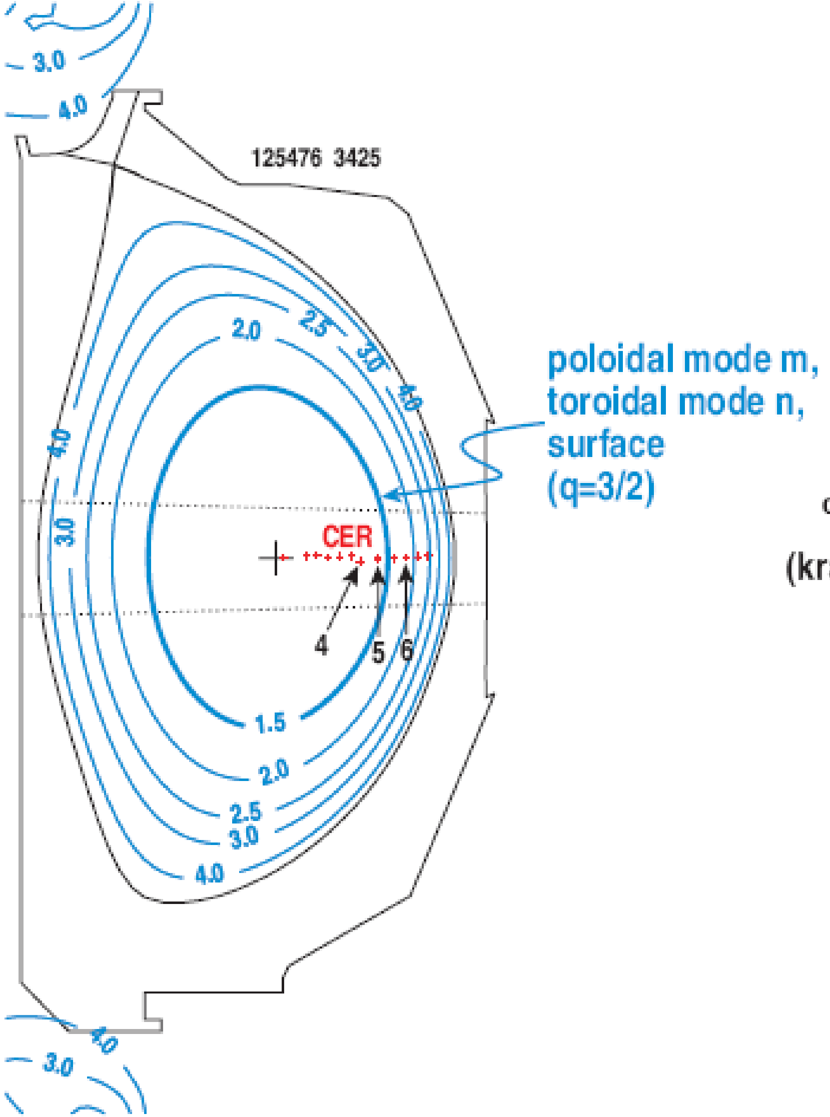
  •hybrid scenario
  •sawteething, ELMy H-mode

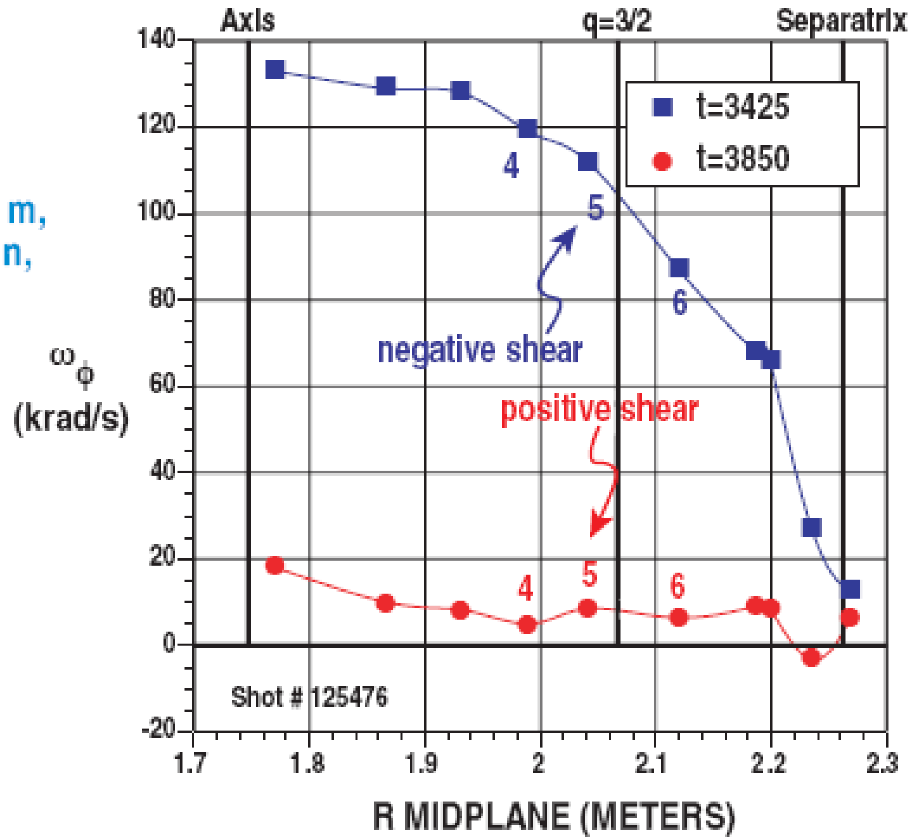
Plan View of DIII–D Tokamak



Present capability:
Co-NBI 12.5 MW
Ctr-NBI 5 MW

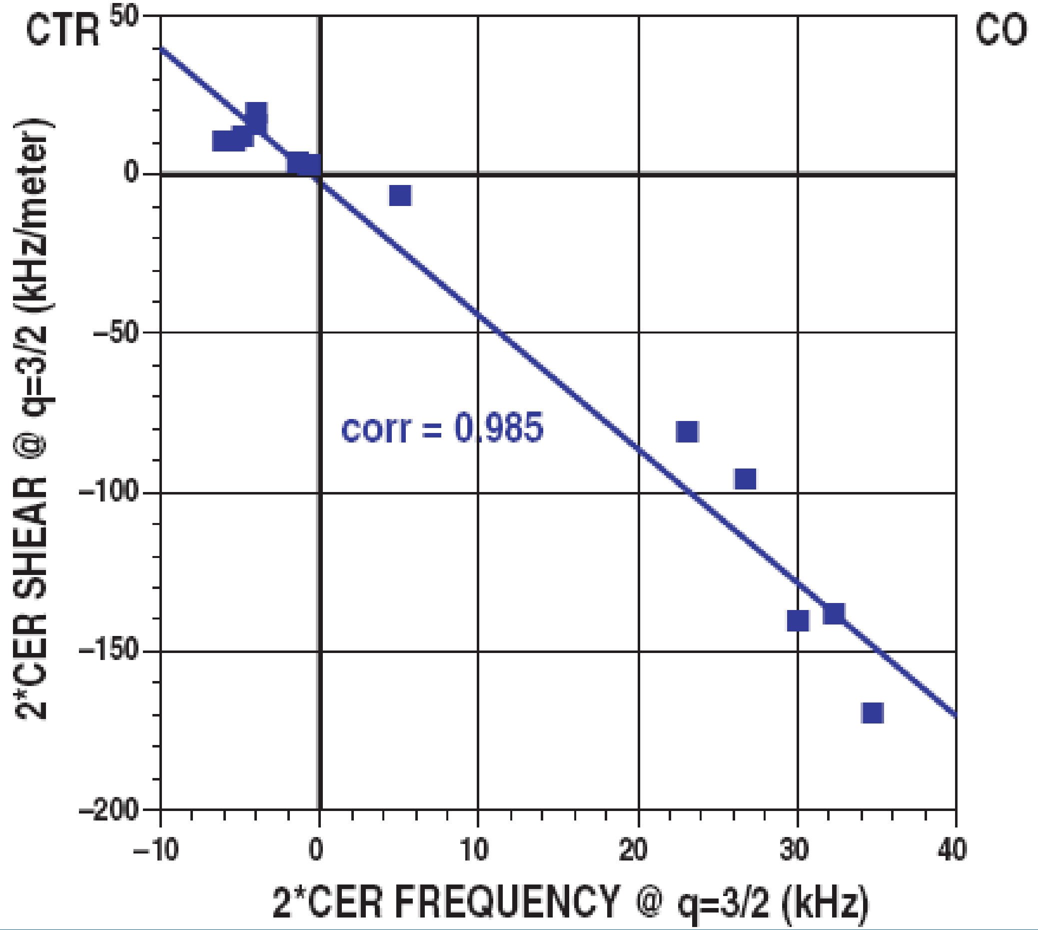
Plasma Rotation Measured by Charge Exchange Recombination of CVI Line



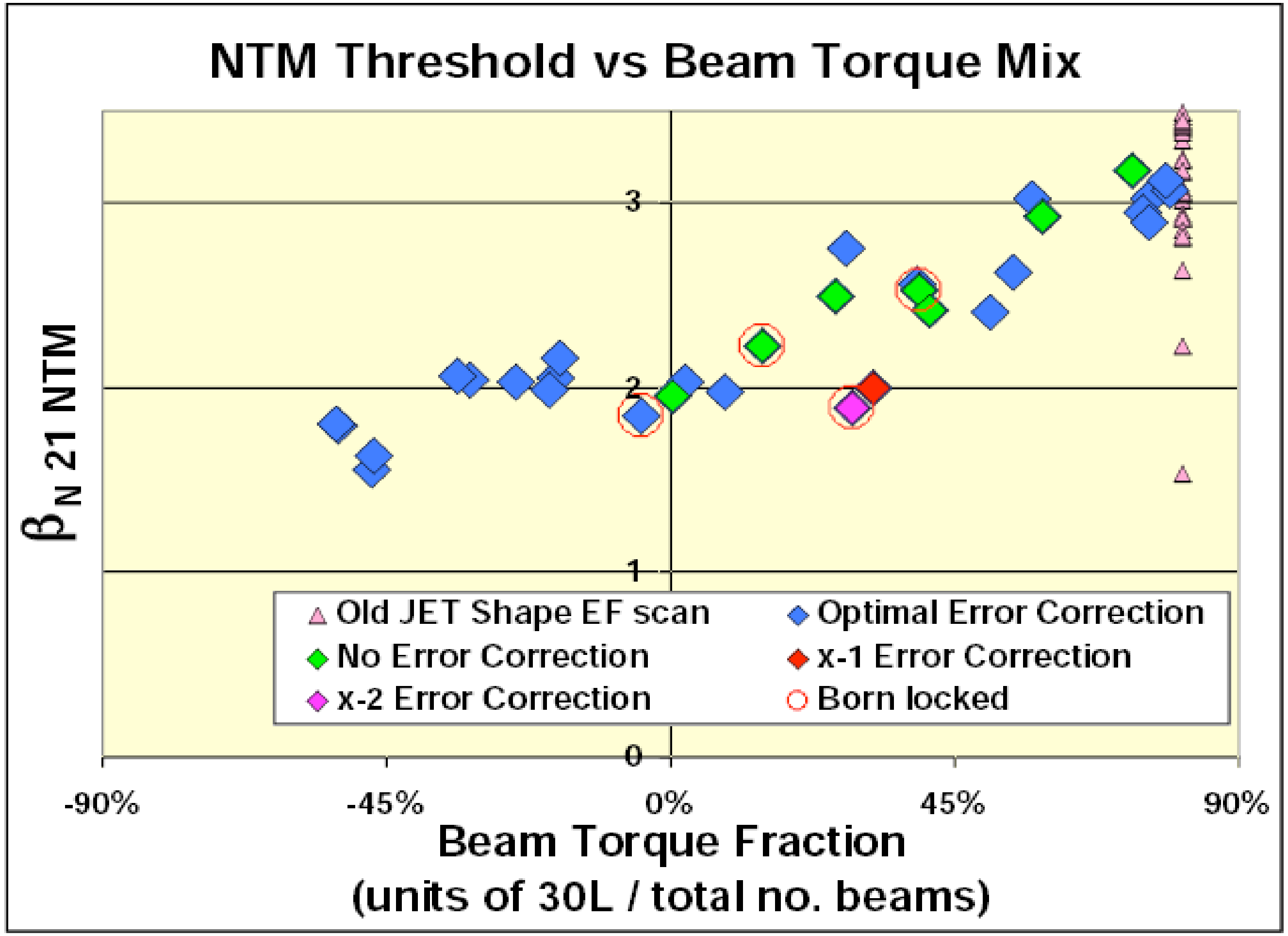


Rotation Shear is Well Correlated with Rotation at q=3/2 in Sawteething Plasmas

- **Unfortunately, one can not separate these, yet, experimentally**
  - ★ $L_{\omega_\phi} \equiv -\omega_\phi/(d\omega_\phi/dR) \approx 0.24$ m or $L\omega_\phi/r_s \approx 0.66$

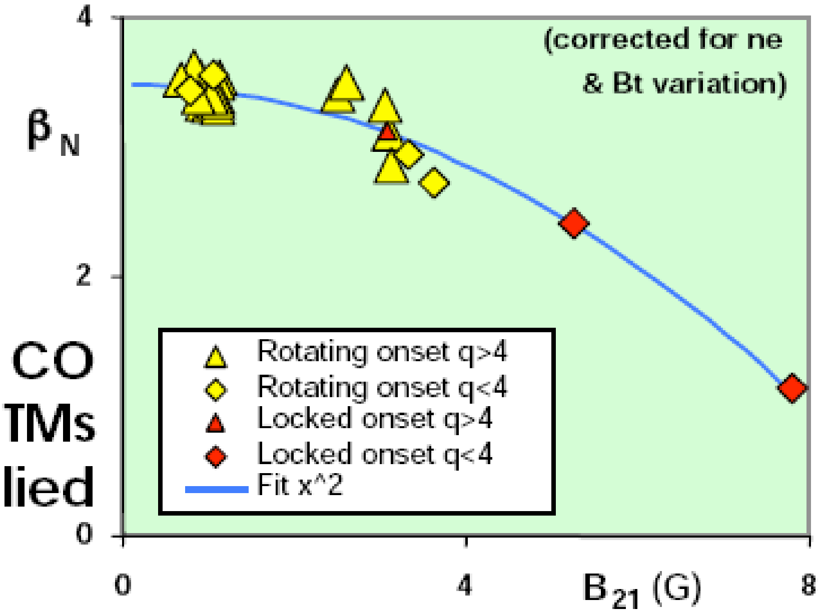


β ramps at fixed co:counter ratio

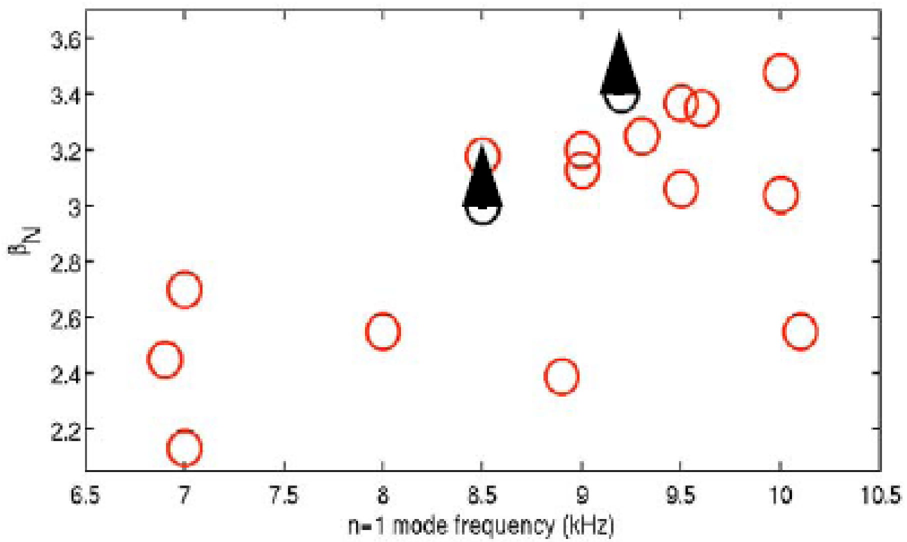


- **Clear trend towards lower 2/1 NTM β threshold as rotation balances**
  - Suggests thresholds may be lower in ITER

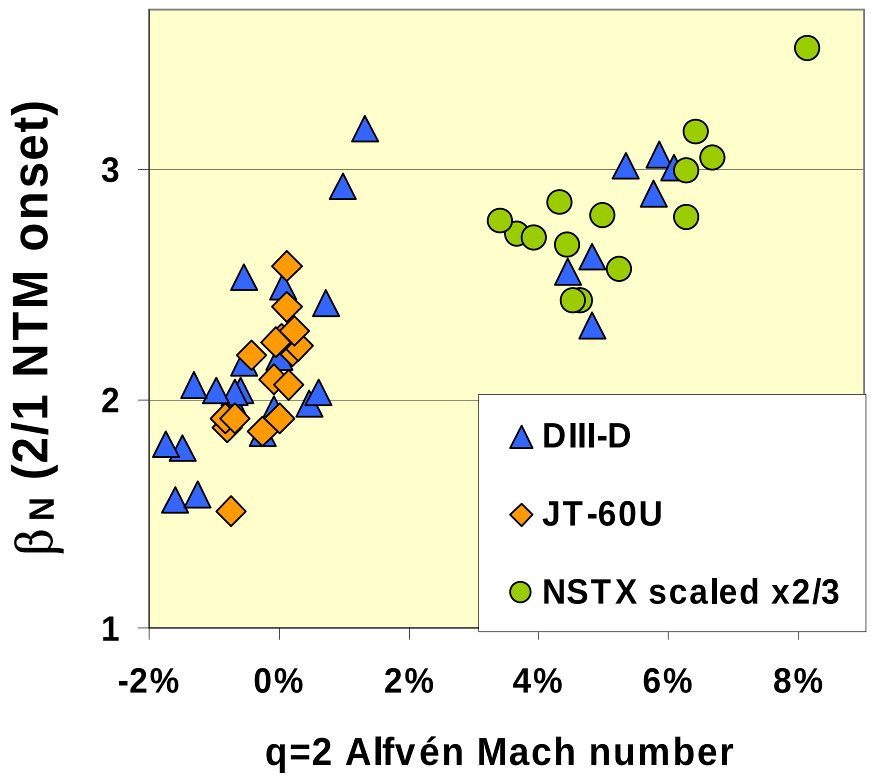
ALL CO
DIII–D 2/1 NTMs
with EF applied



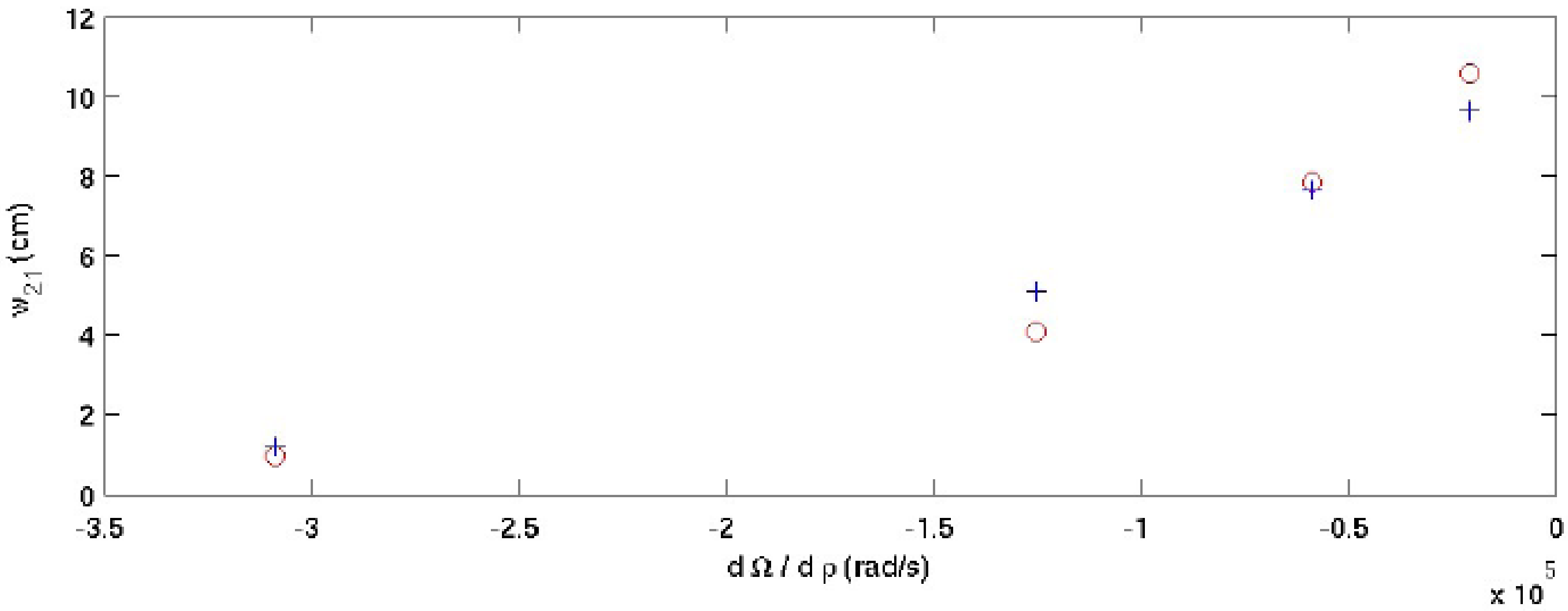
JET 3/2 NTMs: NB momentum scan



Experimental evidence of flow effects on NTM onset



Experimental evidence of flow effects on NTM saturation



How is flow affecting the stability properties of NTMs?

- Is it changing the inner layer dynamics?
- Is it affecting the outer layer dynamics?
- Is it changing toroidal coupling properties?
- What is the role of flow shear? – does the sign matter?

Island equation with sheared flow

Pressure/curvature **Neoclassical current**

$$0.41\frac{\partial W}{\partial t} = D_R^{neo}\left[\frac{\Delta_c'}{4} - \frac{19.5}{W}\frac{\epsilon L_s^2}{B_0^2}\frac{\partial p(0)}{\partial \psi} + 0.58\frac{\sqrt{\epsilon}\beta_\theta \frac{L_q}{L_p}}{W}\frac{W^2}{W^2+W_\chi^2}\right.$$

$$\left. + \frac{L_s^2}{k_\theta^2 v_A^2}\left(2.3\frac{(\omega-\omega_E)(\omega-\omega_E-\omega_*)}{W^3} + 0.24\frac{{\omega_E'}^2}{W}\right) - 0.77\frac{L_s}{k_\theta v_A}\frac{\bar{v}_{||0}}{v_A}\frac{\omega_E'}{W}\right]$$

differential flow **flow shear**

polarization current

drift freq, $\omega_E = k_\theta c \Phi_0'(r=r_s)/B_0$; flow shear, $\omega_E' = k_\theta c \Phi_0''(r=r_s)/B_0$

$\bar{v}_{||0}$ = average value of equilibrium parallel flow

Island saturation width determined by balance between the Δ' term and the bootstrap contribution

$$W_{sat} \sim \frac{\beta_\theta}{(-\Delta')} \frac{L_q}{L_p}$$

Experimental evidence suggests that β_θ and $\frac{L_q}{L_p}$ do not change significantly with changing flow

So something is happening to Δ'

What is the dependence of Δ' on flow shear?

Heuristic Model

- rotation shear provides additional drive to alter field line pitch
- can increase or decrease field line bending energy and thereby change Δ'

$$\Delta' r_s = C_1 + C_2 \left(-\frac{d\omega_\phi}{dR} L_s \tau_A \right)$$

Simplest empirical form

Can one see this scaling from theoretical models ?

- RMHD code
- Newcomb eqn. with flow

Code NEAR

- NEAR – fully nonlinear toroidal code that solves a set of RMHD eqns. and contains neoclassical viscous terms as well as toroidal flow

- Has been benchmarked to reproduce linear (classical) tearing mode dynamics as well as nonlinear saturated behaviour

- It has also reproduced well the dynamics of NTMs – e.g. threshold dynamics, scaling with β_p , island saturation etc.

- Have examined the scaling of $\Delta^{/}$ with toroidal flow shear for classical tearing modes

Model Equations (GRMHD)

$$\frac{\partial \Psi}{\partial t} - (\boldsymbol{b}_0 + \boldsymbol{b}_1) \cdot \nabla \phi_1 - \boldsymbol{b}_1 \cdot \nabla \phi_0 = \eta \tilde{J}_{||} - \frac{1}{ne} \boldsymbol{b}_0 \cdot \nabla \cdot \boldsymbol{\Pi}_e$$

bootstrap current

$$\nabla \cdot \left(\frac{\rho}{B_0} \frac{d}{dt} \frac{\nabla \phi_1}{B_0} \right) + (\boldsymbol{V}_1 \cdot \nabla) \left(\nabla \cdot \left(\frac{\rho}{B_0} \frac{\nabla \phi_0}{B_0} \right) \right) = (\boldsymbol{B}_0 \cdot \nabla) \frac{\tilde{J}_{||}}{B_0} + (\boldsymbol{B}_1 \cdot \nabla) \frac{J_{T||}}{B_0} + \nabla \cdot \frac{\boldsymbol{B}_0 \times \nabla p_1}{B_0^2} + \nabla \cdot \frac{\boldsymbol{B}_0}{B_0^2} \times \nabla \cdot \boldsymbol{\Pi}$$

GGJ

$$\frac{dp_1}{dt} + (\boldsymbol{V}_1 \cdot \nabla) p_0 + \Gamma p_T \nabla \cdot \boldsymbol{V}_1 = (\Gamma - 1) \left[\eta J_{T||}^2 - \boldsymbol{\Pi} : \nabla \boldsymbol{V} + \boldsymbol{\Pi_e} : \nabla \frac{\boldsymbol{J}}{ne} - \nabla \cdot \boldsymbol{q} \right]$$

heat flow

$$\rho \frac{d\widetilde{V}_{||}}{dt} + (\boldsymbol{V}_1 \cdot \nabla) V_{||0} = -\boldsymbol{b}_0 \cdot \nabla p_1 - \boldsymbol{b}_1 \cdot \nabla p_T - \boldsymbol{b}_0 \cdot \nabla \cdot \boldsymbol{\Pi}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{V} \cdot \nabla$$

$$\boldsymbol{V} = \Omega(\psi) R^2 \boldsymbol{\nabla}\zeta + \boldsymbol{V}_1 = \frac{\boldsymbol{B}_0 \times \nabla\Phi_0}{B_0^2} + V_{0||}\boldsymbol{b}_0 + \frac{\boldsymbol{B}_0 \times \nabla\Phi_1}{B_0^2} + \tilde{V}_{||}\boldsymbol{b}_T$$

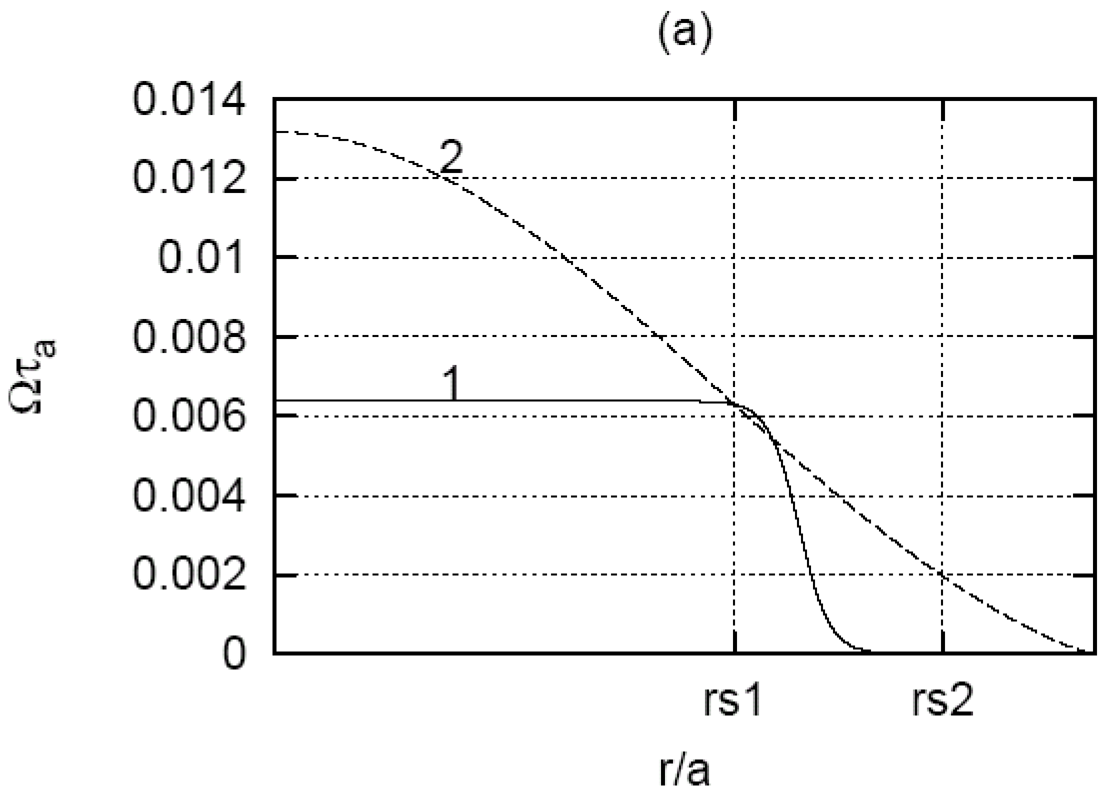
Equilibrium flow

- **Neoclassical closure**

$$\vec{\nabla} \cdot \Pi_s = \rho_s \mu_s \left\langle B^2 \right\rangle \frac{\vec{V}_s \cdot \vec{\nabla}\Theta}{\left(\vec{B} \cdot \vec{\nabla}\Theta\right)^2} \vec{\nabla}\Theta,$$

- appropriate for long mean free path limit
- reproduces poloidal flow damping
- gives appropriate perturbed bootstrap current

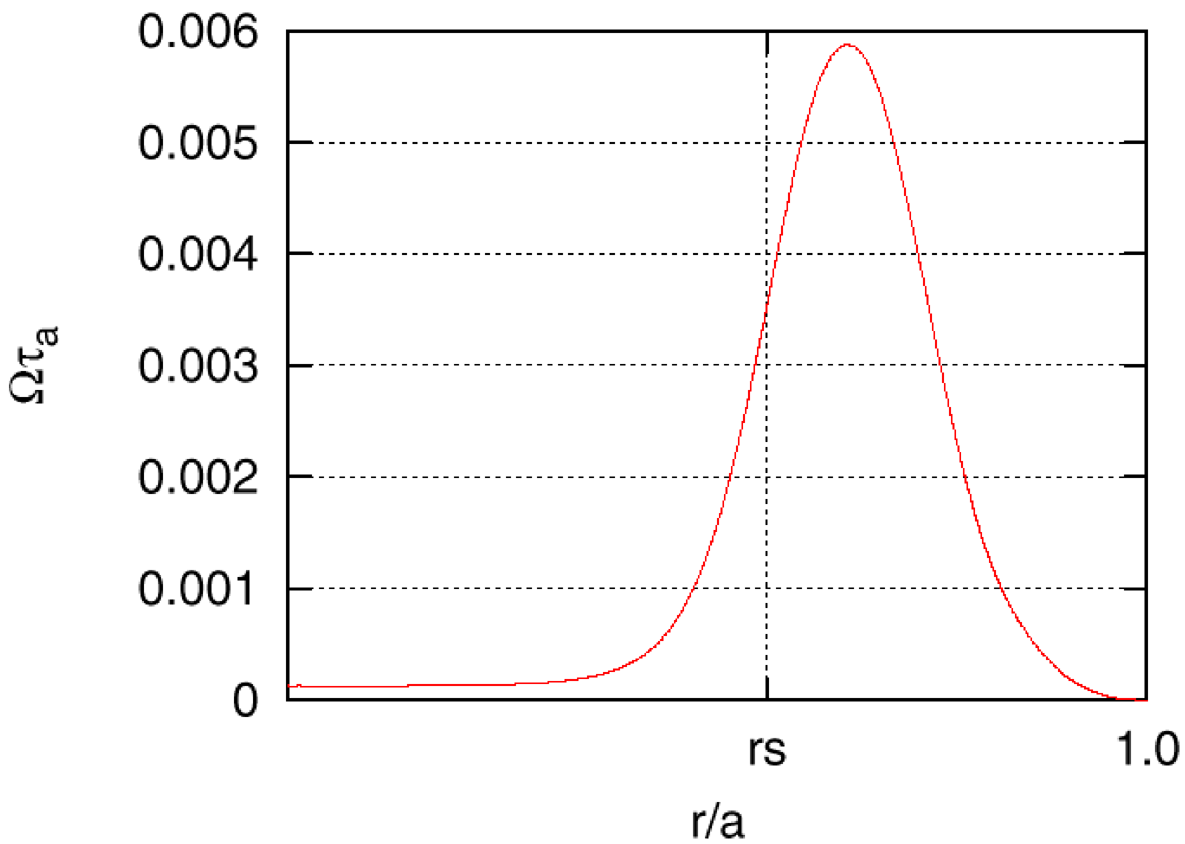
Toroidal flow profiles



1- differential flow

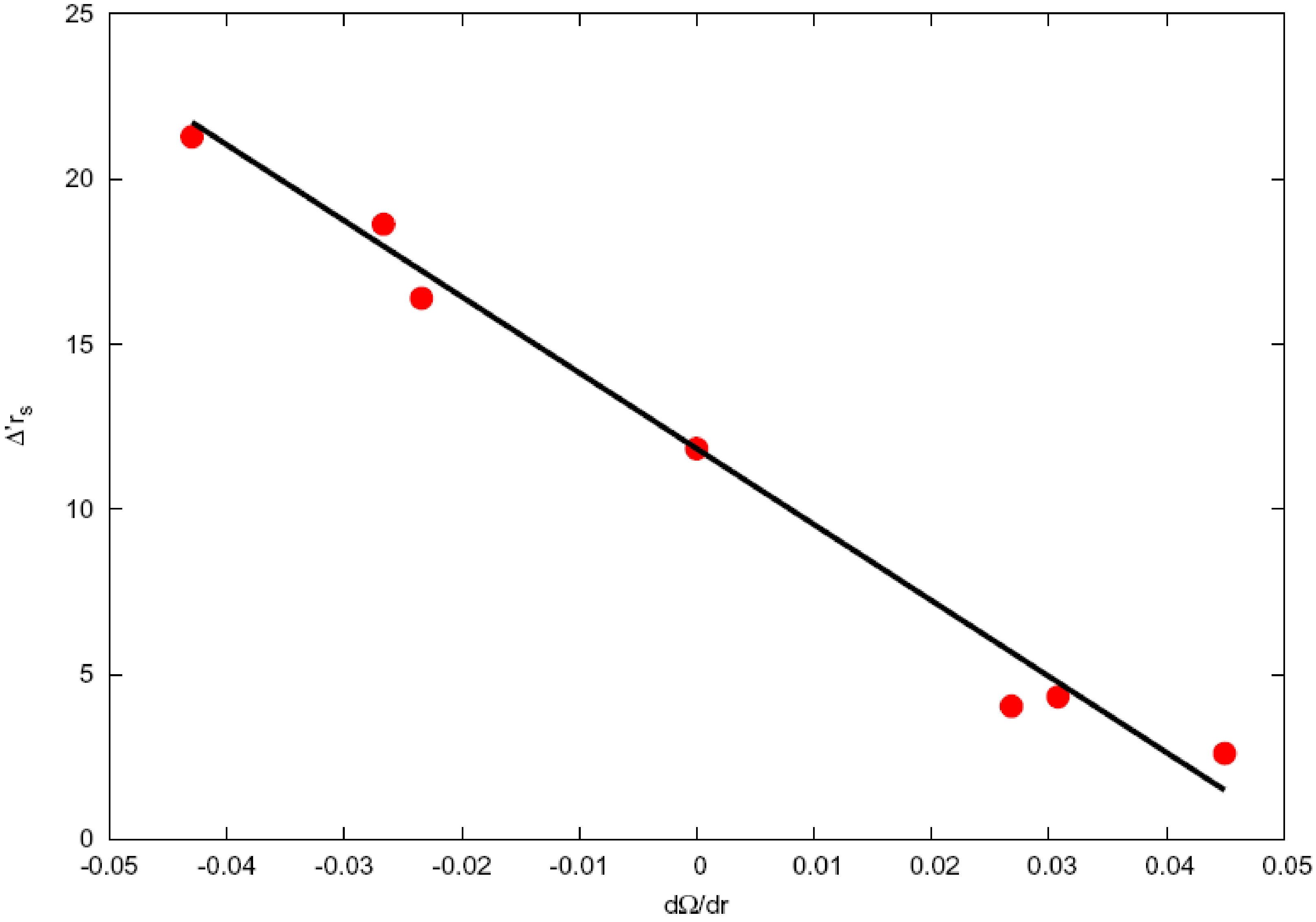
2- sheared flow

Profile with positive flow shear at (2,1) surface



- Looked at single helicity mode dynamics

Results from NEAR



Newcomb Equation with sheared flow:

$$H\frac{d^2\psi}{dr^2} + \left(\frac{dH}{dr} + h_f\right)\frac{d\psi}{dr} - \left[\frac{g}{F^2} + \frac{g_f}{F^2} + \frac{1}{F}\frac{d}{dr}\left(H\frac{dF}{dr}\right)\right]\psi = 0$$

h_f and **g_f** are additional contributions due to flow

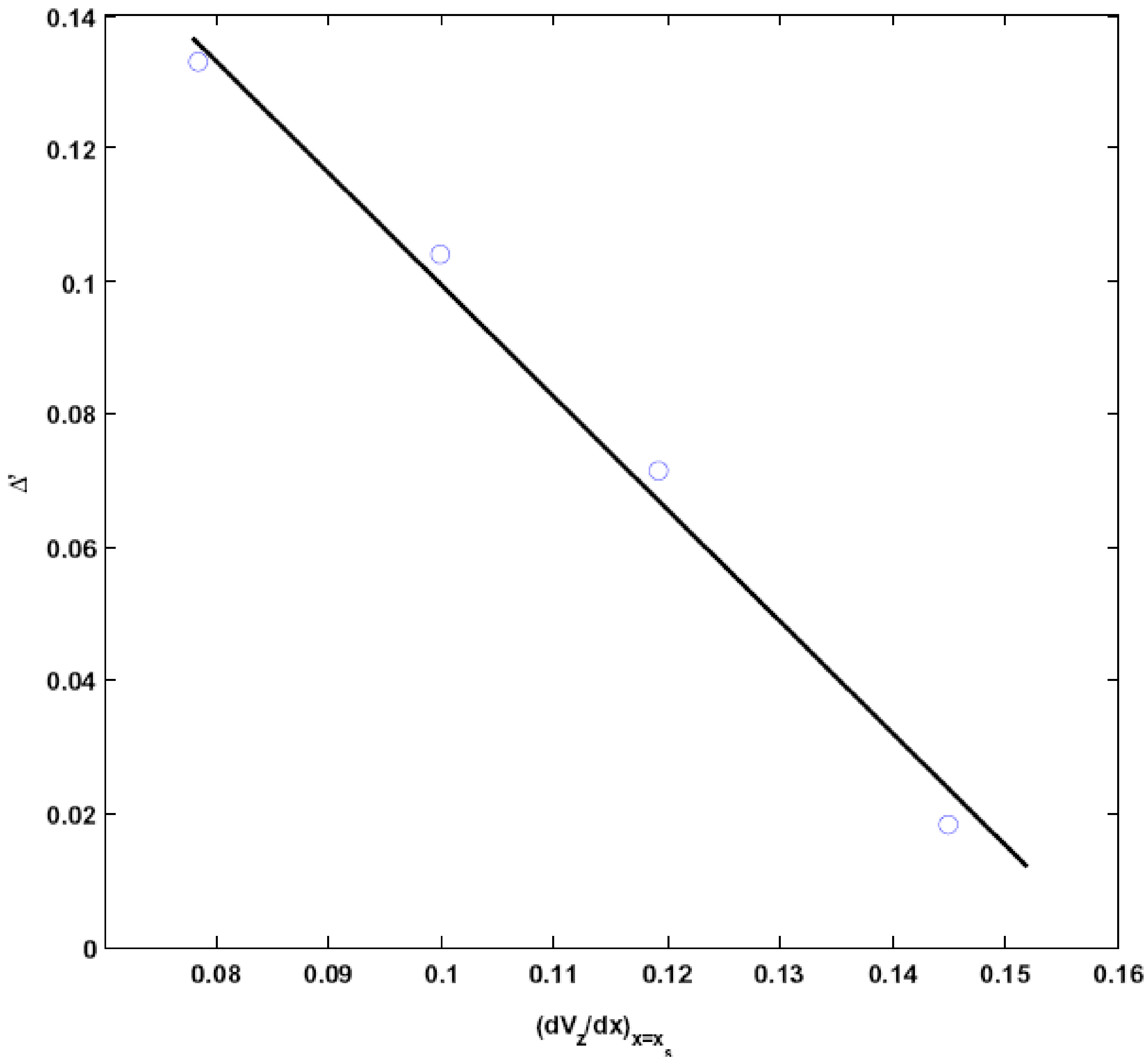
- **Limit: h_f , g_f → 0,** Furth, Rutherford, Selberg equation

*[Phys. Fluids **16**, 1054 (1973)]*

- **Limit: slab geometry, (1/r) → 0, d/dr → d/dx, m/r ✸ k_y**

Chen-Morrison Equation *[Phys. Fluids B **2**, 495 (1990)]*

$$\Delta' = -\frac{1}{r_s\psi_s^2}\int_0^a \left[\left(\frac{d\psi}{dr}\right)^2 + \left\{\frac{g}{HF^2} + \frac{1}{HF}\frac{d}{dr}\left(H\frac{dF}{dr}\right) - \frac{2m^2k_z^2}{(k_z^2r^2+m^2)^2} + \frac{g_f}{HF^2} + \frac{1}{2r}\frac{d}{dr}\left(\frac{rh_f}{H}\right)\right\}\psi^2\right] r\,dr$$


0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Δ'
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
(dVZ/dx)x=xs

Conclusions and Future Work

- Strong experimental evidence for toroidal shear flow induced modification of NTM threshold β and saturated island size

- Main effect appears to arise from change in Δ'

- Heuristic model and empirical fitting gives linear scaling of Δ' with flow gradient

- Preliminary investigations with resistive MHD code NEAR and Newcomb equation analysis supports above scaling

- Necessary to carry out better numerical investigations e.g. using PEST3 or other codes

- Need analytic modeling for better understanding of the underlying physics