The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

1953-44

International Workshop on the Frontiers of Modern Plasma Physics

14 - 25 July 2008

Transport and mixing in plasma turbulence.

J.J. Rasmussen

Risoe National Lab. for Sustainable Energy
Technical University of Denmark
Roskilde
Denmark

Transport and mixing in plasma turbulence

Jens Juul Rasmussen

V. Naulin, O.E. Garcia, A.H. Nielsen,

R. Basu, M. Priego, J. Gavnholt, …

Association EURATOM - Risø DTU
Plasma Physics and Technology, Bldg. 128,
DK-4000 Roskilde, Denmark

jens.juul.rasmussen@risoe.dk

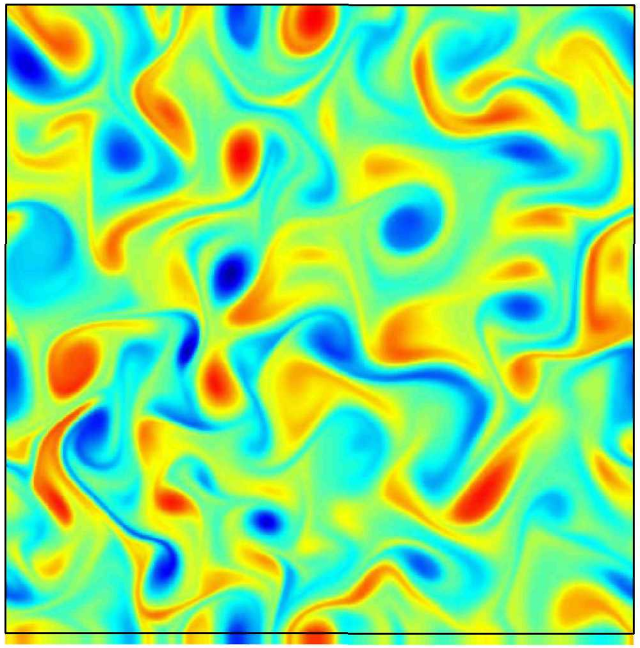
Outline

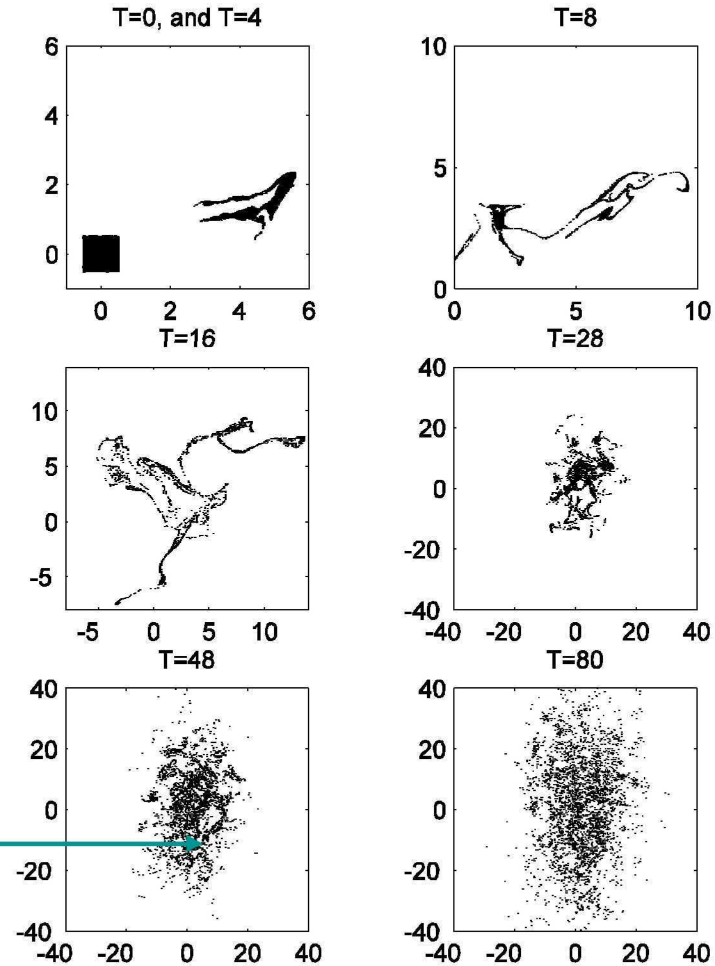
Motivation: In magnetically confined plasmas turbulent – anomalous – transport is the dominant mechanism for transport of mass, energy and momentum! (Balescu, *Aspects of Anomalous Transport in Plasmas,* CRC Press 2005)

- Turbulent dispersion and mixing of (passive) particles:
  - in drift wave turbulence
  - in global interchange turbulence in the edge/scrape-off-layer, SOL
- Relation between the passive particle diffusion and bulk plasma transport – the turbulent density flux
- Passive tracer particles/fields are used to model impurity transport
- Inertia effects; pinching and clustering in 2D drift wave turbulence
- Particle mixing and transport in strongly intermittent turbulence in SOL: Curvature pinch in inhomogeneous magnetic fields.

Particle dispersion in plasma turbulence

Vorticity





Drift-wave turbulence

Hasegawa Wakatani Equations

PRL **50,** 682 (1983)

Particles convected by fluctuating ***ExB*** velocity

Basu et al Phys. Plasmas **10**, 2696 (2003);

Comm.Nonlinear Sci. Numer. Simul. **8,** 477 (2003)

Dynamics of passive tracers

- Investigation of mixing and diffusion properties of turbulence – provides a diffusion coefficient – turbulent flux via Fick's law.
- Passive tracers or passive fields are also widely applied for modelling impurity ion transport in plasmas :: impurities are all materials besides the bulk plasma species; but here is does not include dust particles, only impurity ions are treated!
- Impurities, e.g., originate from sputtering off plasma facing components, PFC
- **Passive tracers/fields do not contribute to charge neutrality and do not dynamically react back on the turbulence**
- **Severe condition**: impurity density should be much smaller than density of bulk ions and contribution to quasi-neutrality condition much smaller than each of the contributing terms. (Naulin et al Phys. Scripta **T122**, 129 (2006)).

Passive scalar dynamics is a classical problem in fluid dynamics

Falkovich et al. Rev. Mod. Phys. **73**, 913 (2001);

Warhaft, Ann. Rev. Fluid Mech. **32**, 203 (2000)

Tracing particles in the turbulence

Up to 100.000 particles are adverted in resistive drift-wave turbulence – Hasegawa-Wakatani (PRL 1983) model 2D.

$$\vec{x}(t) = \vec{x}_0 + \int_0^t \vec{v}(\vec{x}(t'), t')dt$$

Principal component of $\vec{v} = (u, v)$ is the $E \times B$-velocity, $\vec{v}_E$:: Ideal inertia-less particles.

Inertial effects : adding the polarization drift, $\vec{v}_p$

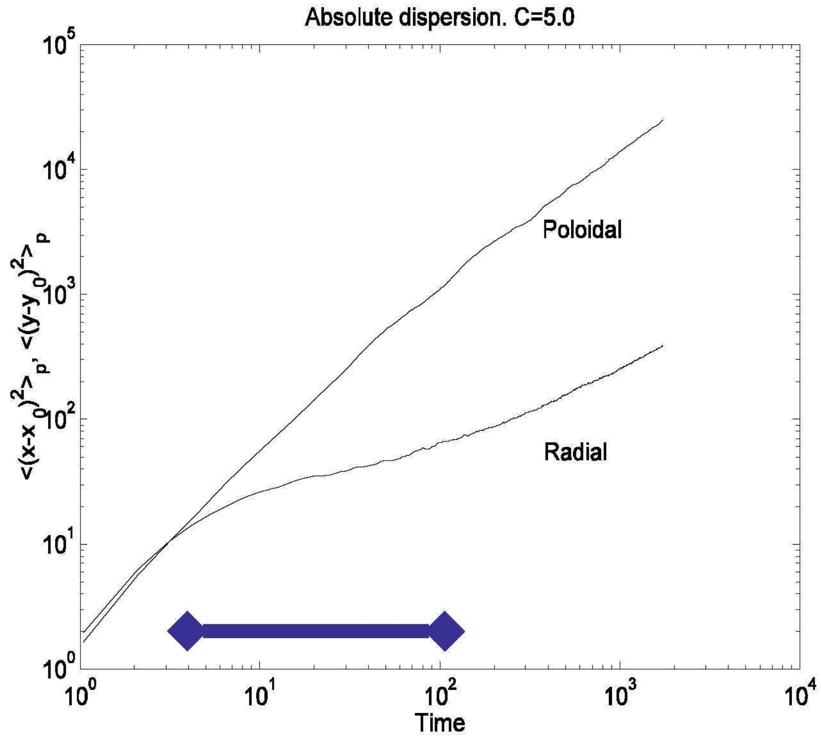
$$\vec{v}_p = -\zeta \left(\frac{\partial}{\partial t} + (\vec{v}_E \cdot \nabla) \right) \nabla\varphi$$

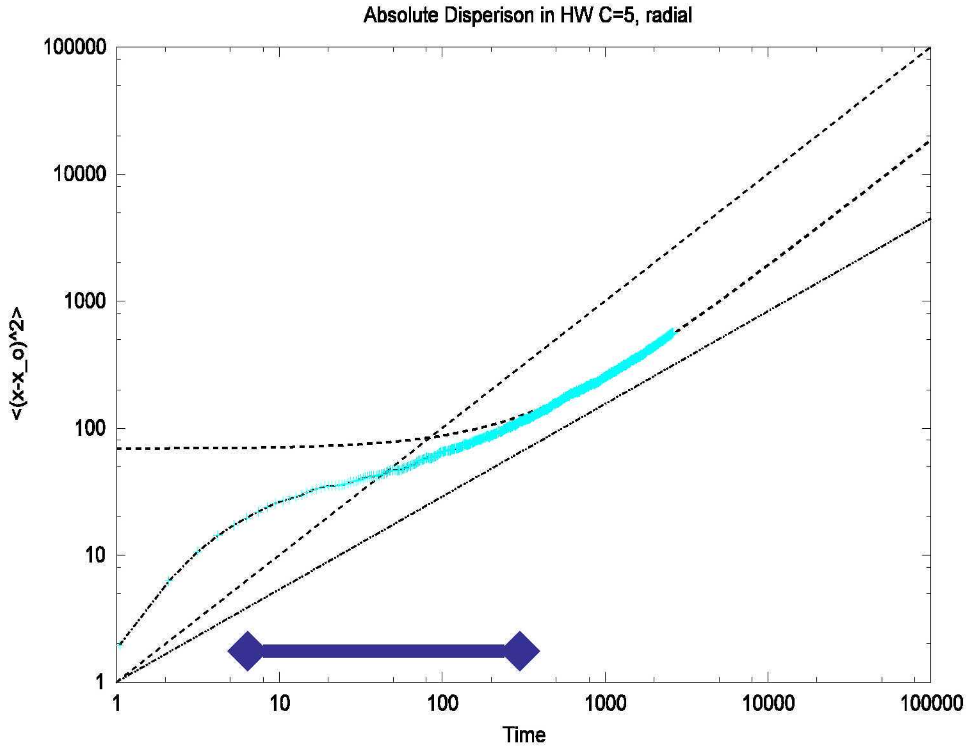
$\zeta = \frac{eM}{qm_i} \frac{\rho_s}{L_n}$

Important for heavier impurities!

Inertia effects make the advection velocity compressible!

Particle dispersion





The mean square particle displacement radially and poloidally.
x - radial direction,
y – poloidal direction

The radial particle displacement.
Fit: $At^\beta + B$ for $t > 400$; $\beta = 1$.
Asymptotically normal diffusion!

Diffusion coefficient $D_x = \langle X^2(t)\rangle/2t$

Particle trapping in moving vortical structures:

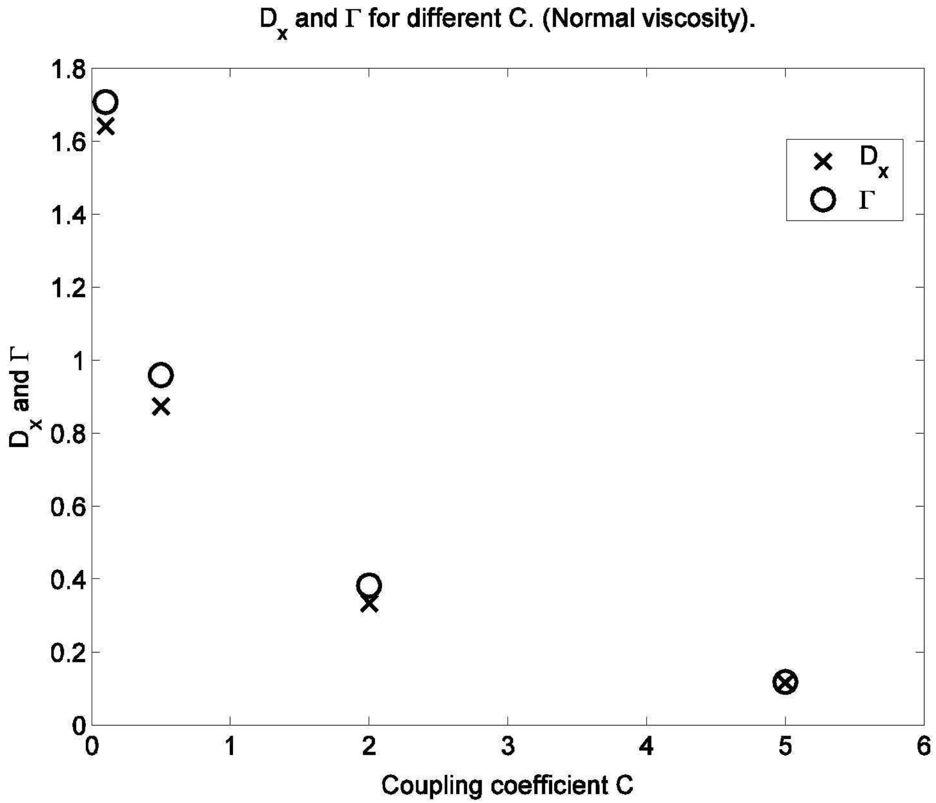
Diffusion coefficient and flux

Particle density flux:

$\Gamma = n\, v_x = n\, v_{ExB}$

Fick's law:

$\Gamma = - D\, d_x n_0$



Comparison between D_x for tracer particles and D from the flux Γ; $D = - \Gamma / d_x n_0$

For the present case – HWe, fluctuations around a frozen background profile - passive particle diffusion really mimics bulk plasma transport!

Evolution of impurities as a passive field

The impurities are treated as a passive scalar advected by the turbulent fluctuations, i.e., the the impurities do not act back on the turbulence or the background plasma profile.

$$\partial_t n_{imp} + \nabla \cdot (\vec{v} n_{imp}) = \mu \nabla^2 n_{imp}$$

The influence of inertia enters via the polarization.

$$(\partial_t + \mathbf{v}_E \cdot \nabla) n_{imp} = \zeta \nabla \cdot (n_{imp} (\partial_t + \mathbf{v}_E \cdot \nabla) \nabla \phi) + \mu \nabla^2 n_{imp}.$$

Restriction $n_{imp} \ll n$!

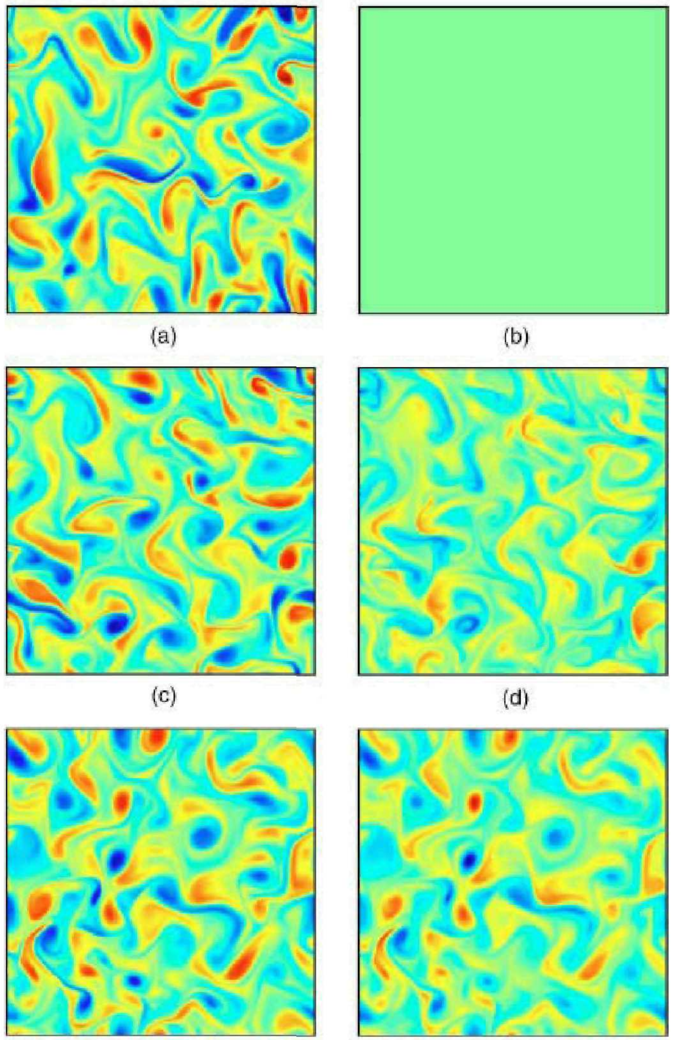
Lagrangian invariant: $(\partial_t + \mathbf{v}_E \cdot \nabla)(\ln n_{imp} - \zeta \omega) \approx 0$

Turbulent mixing will homogenize the Lagrangian invariant :

$\ln n_{imp} - \zeta \omega \approx const.$

The initially homogeneous impurity density field will granulate.

Clustering/aggregation of inertial impurities


(a) (b)
(c) (d)
(e) (f)

Vorticity | Impurity density fluctuations

The impurity equation may be written as:

$$D_t(\ln n_{imp} - \zeta\omega) = o(\zeta^2)$$

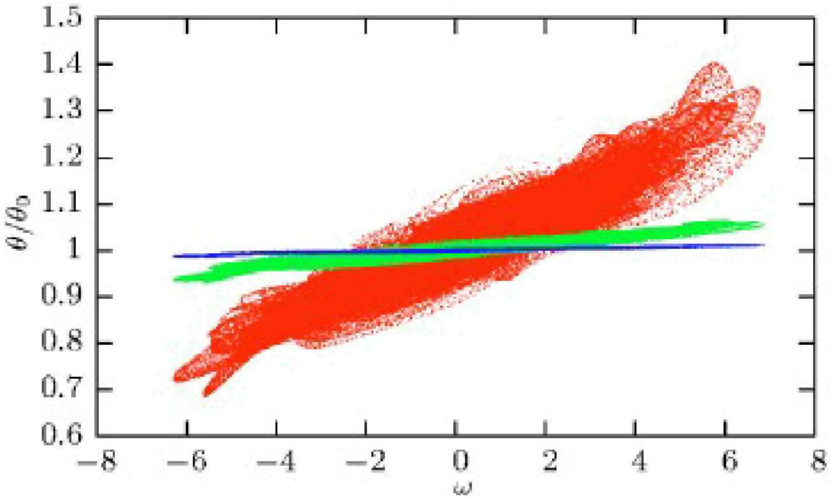
$$n_{imp}/n_{imp0} \sim \zeta\omega$$

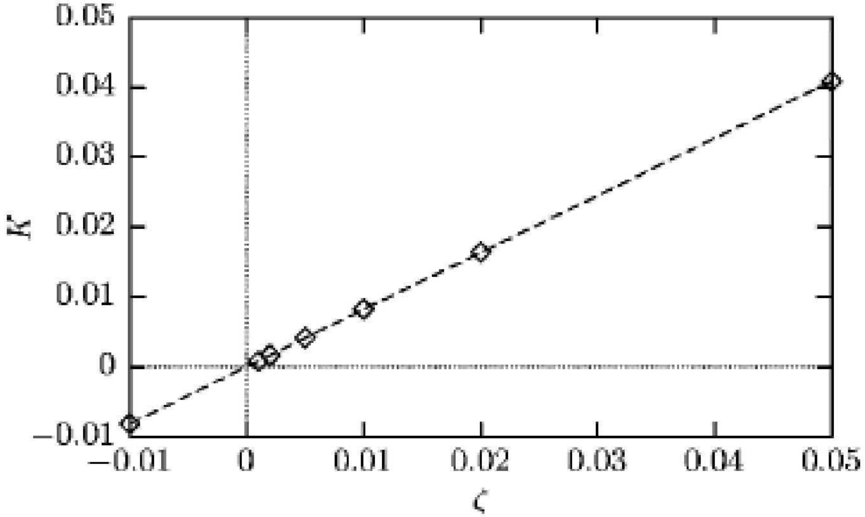
Positive impurities (ς > 0) (this case) cluster in positive vortices

Negative impurities (ς > 0) will cluster in negative vortices

Priego et al Phys. Plasmas **12**, 062312 (2005)

Impurity density and vorticity





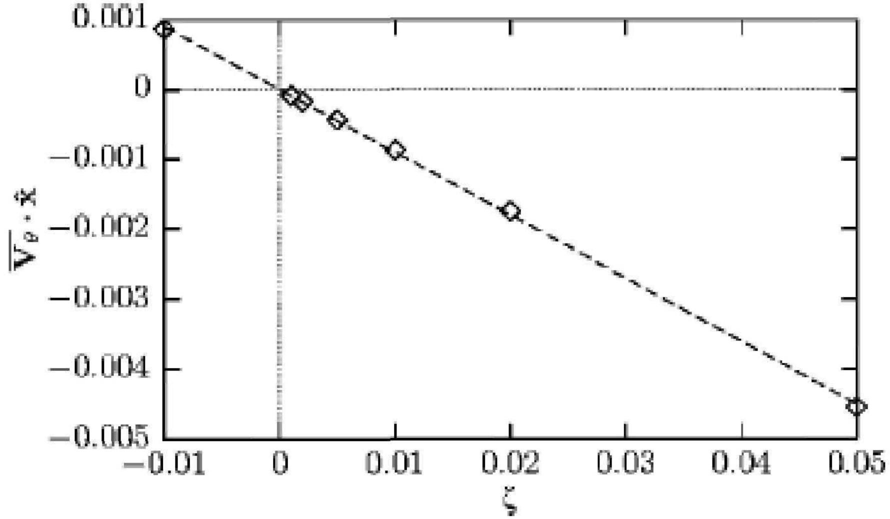
Scatter plot of impurity density and vorticity, $\zeta = 0.05$, $\zeta = 0.01$, and $\zeta = 0.002$.

Linear regression: $\theta/\theta_0 = 1 + K\omega$; $K = 0.82\zeta$ $(\theta \equiv n_{imp})$

Priego et al Phys. Plasmas **12**, 062312 (2005)

Impurity pinch

Finite inertia also introduce a pinch effect: the (positive) impurities are transported up the density gradient – negative pinch velocity



Specific properties of the HWe ?

Turbulence and transport in the edge/SOL

In the edge/scrape-off-layer (SOL) region turbulence and transport is strongly intermittent and characterized by:

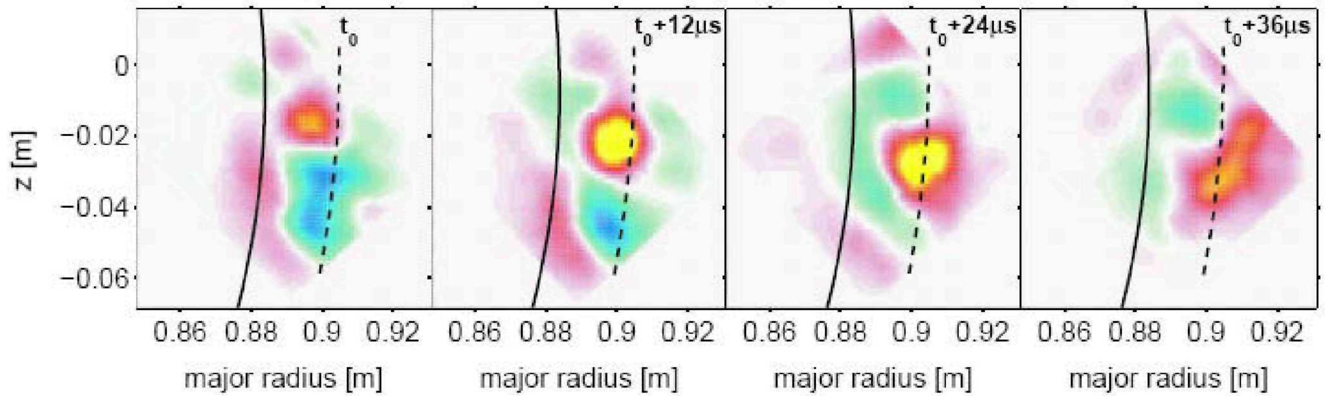
- **large-amplitude, radially propagating blob-like structures of particles and heat, generated close to the last closed flux surface (LCFS),**
- **resulting in asymmetric conditional wave forms, and skewed and flattened PDFs with broad tails**
- **results in localized power loads at PFCs.**

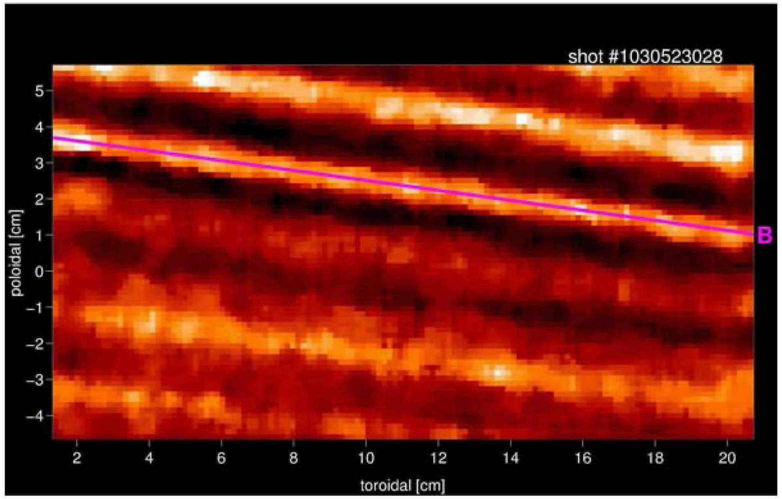
Observed under a variety of conditions (linear to tororoidal devices) :
see, e.g., Zweben Phys. Fluids **28** 974 (1985); Antar *et al*. PoP **10,** 419 (2003); Boedo *et al* PoP **10**, 1670 (2003); Zweben *et al*. Nucl. Fus. **44**, 134 (2004); Grulke *et al* PoP **13**, 012306 (2006); Garcia *et al.* PPCF **48**, L1 (2006). S.J. Zweben *et al.* PPCF **49,** S1 (2007)

Blob propagation in Alcator, C-Mod

OBSERVATIONS OF DENSITY BLOBS AT THE OUTBOARD MIDPLANE OF ALCATOR C-MOD (D® - LIGHT) O. GRULKE ET AL. POP 13, 012306 (2006).

Review: S.J. Zweben *et al.* PPCF 49, S1 (2007)





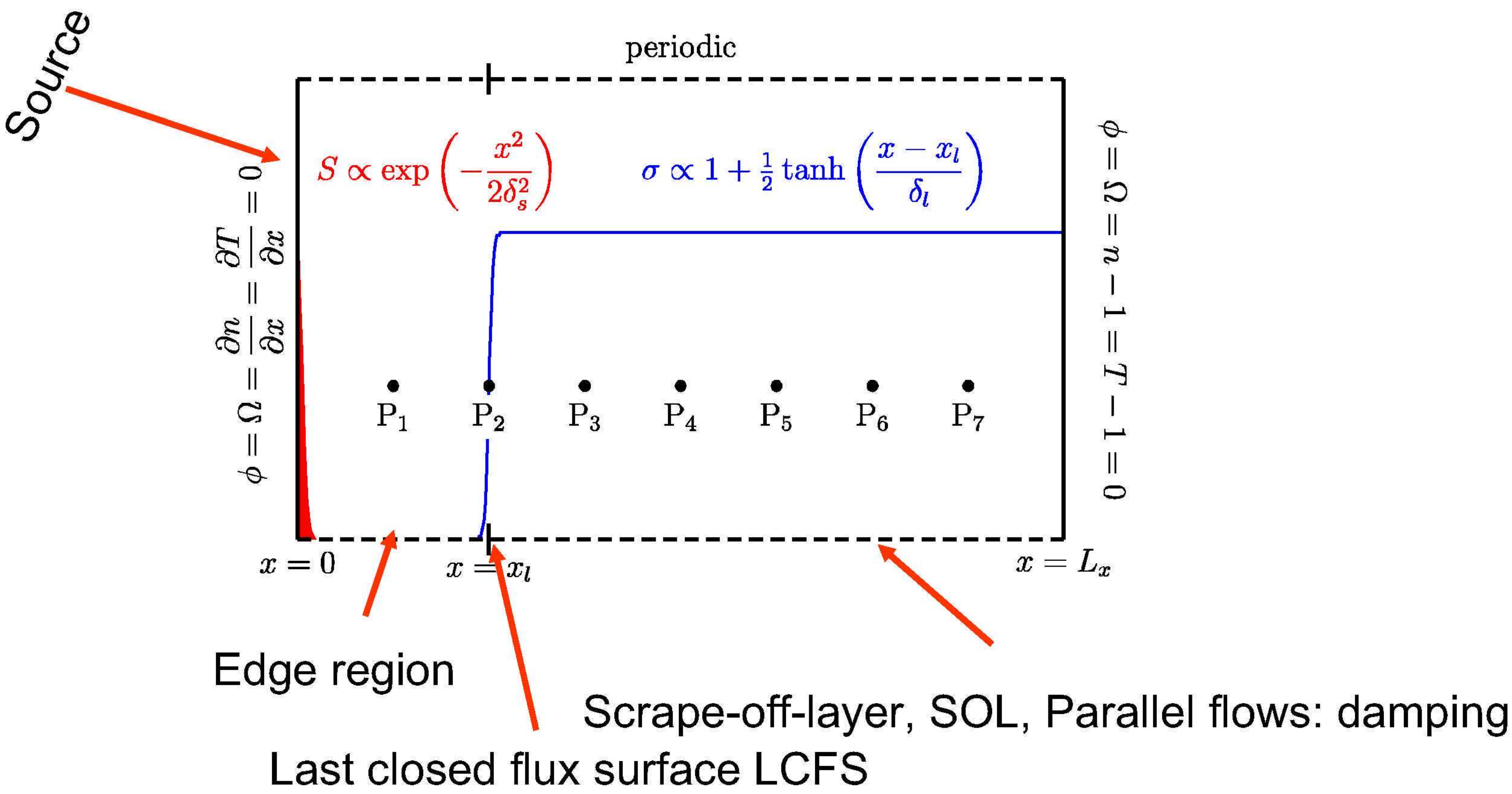
Inferred observations in the poloidal/toroidal direction

Magnetic field line

Radial velocity: 0.05 – 0.1 of the sound speed

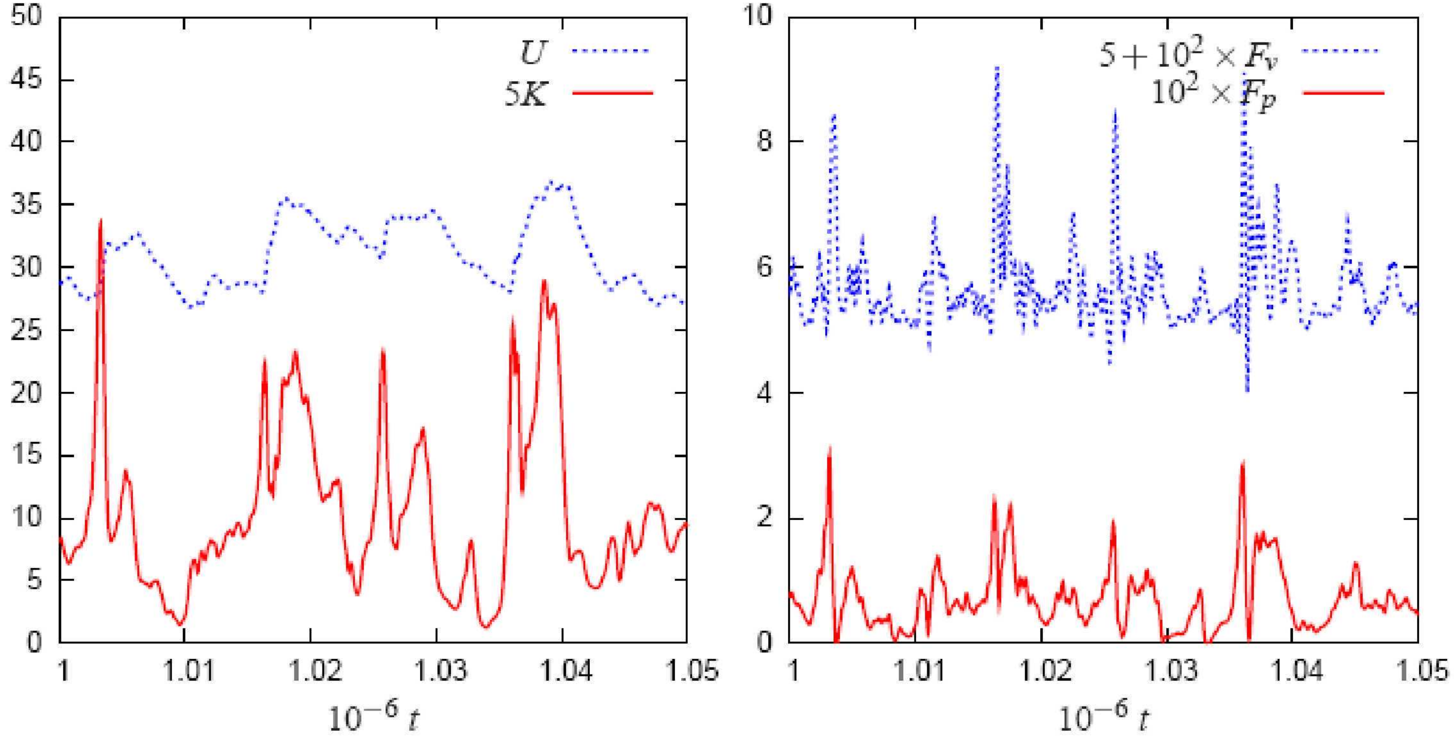
Simulations of Edge-SOL

Risø ESEL code: interchange dynamics at the outboard midplane of a toroidally magnetized plasma. B-field gradient and curvature. Global evolution.



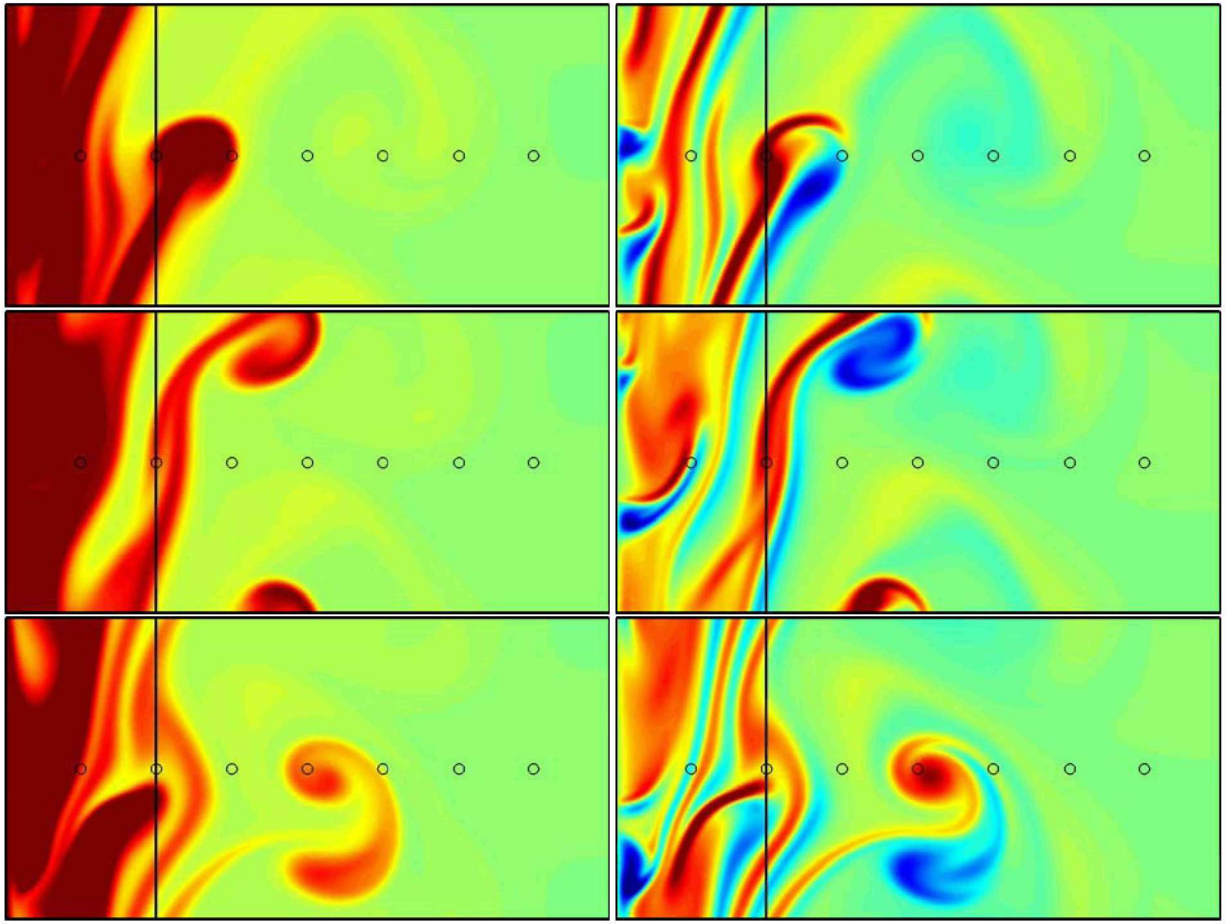
Garcia et al, PRL **92**, 165003 (2004); Phys. Plasmas **12**, 062309 (2005); Phys. Scripta **T122**, 89 (2006)

Energetics and energy transfer



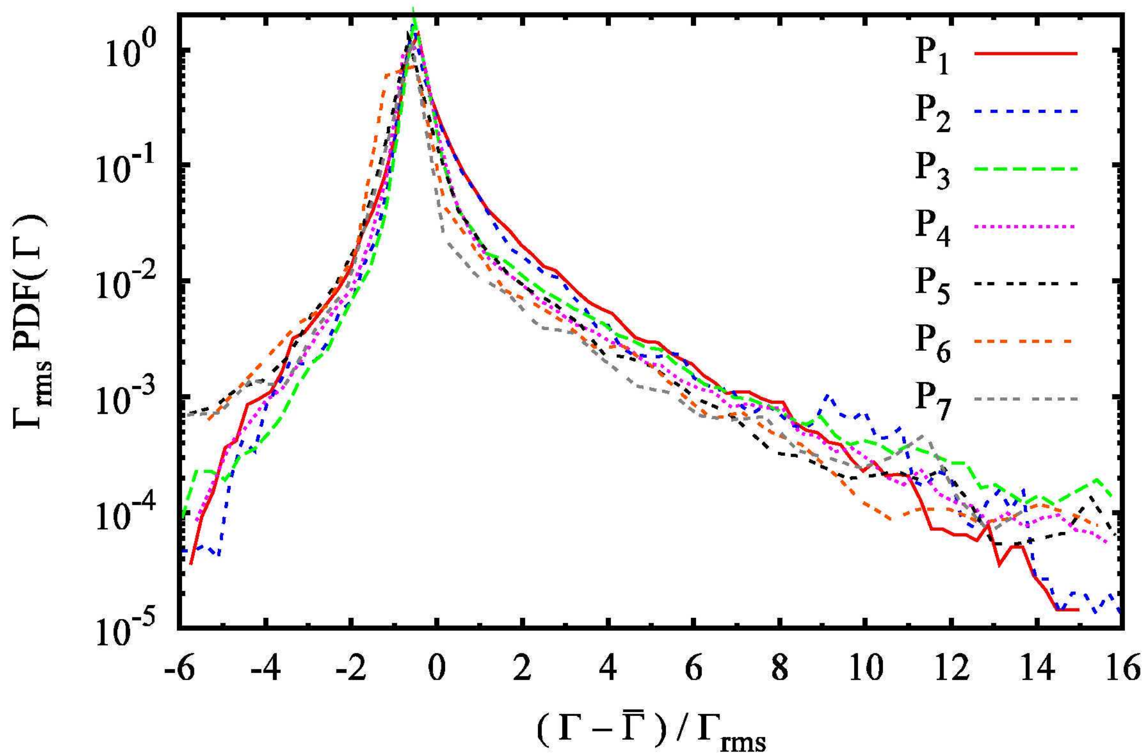
Bursting : Kinetic energy contained by the mean, U, and fluctuating, K, motions.
The collective energy transfer terms F_p and F_v.

Spatial structure during a burst



Formation and propagation of density blob.
Particle density (left) and vorticity (right) during a burst ($\Delta t = 500$), radial blob velocity < $0.02c_s$.

Particle density flux



Re-scaled PDF of particle density flux, $\Gamma = (n - \overline{n})v_x$, measured at the probes, P_i.

Exponential tails: flux dominated by strong bursts.

Dynamics of impurity ions

The passive tracer particles model impurity dynamics, in the limit of no back-reaction on the plasma dynamics:

Impurity density is much lower than the plasma particle density.
(Naulin PRE '05; Priego *et al* PoP '05; Naulin *et al* Physica Scripta '06)

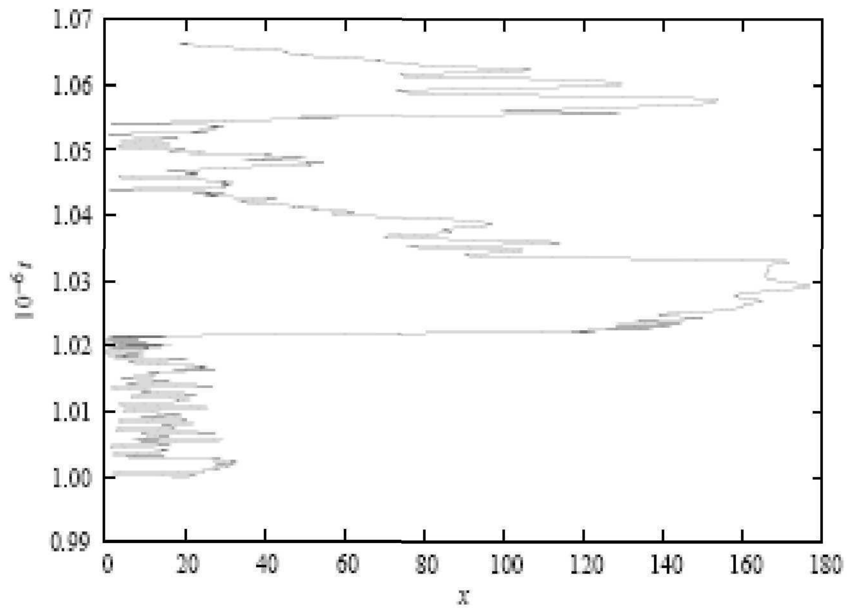
Particles are advected as:

$$\frac{d\vec{x}}{dt} = \vec{v} = \frac{1}{B(x)}\,\hat{z} \times \nabla\phi$$

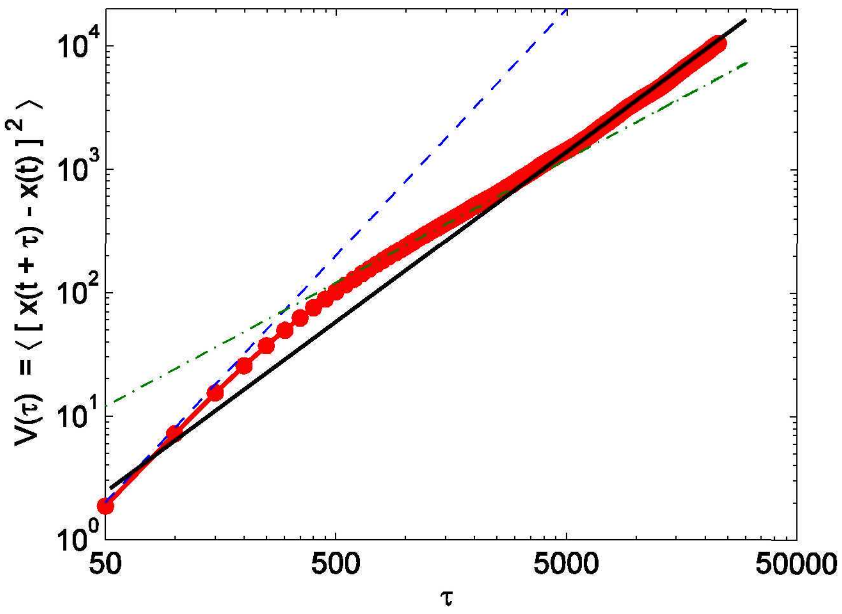
Finite inertia effects are neglected; $\vec{v}$ is compressible due to the spatial dependence of $B(x)$

Garcia *et al* EPS 2005

Particle dynamics

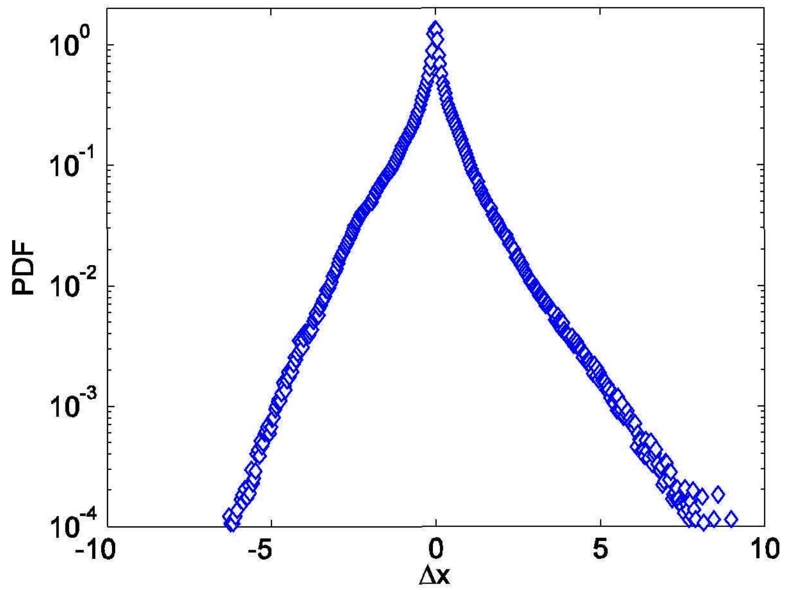


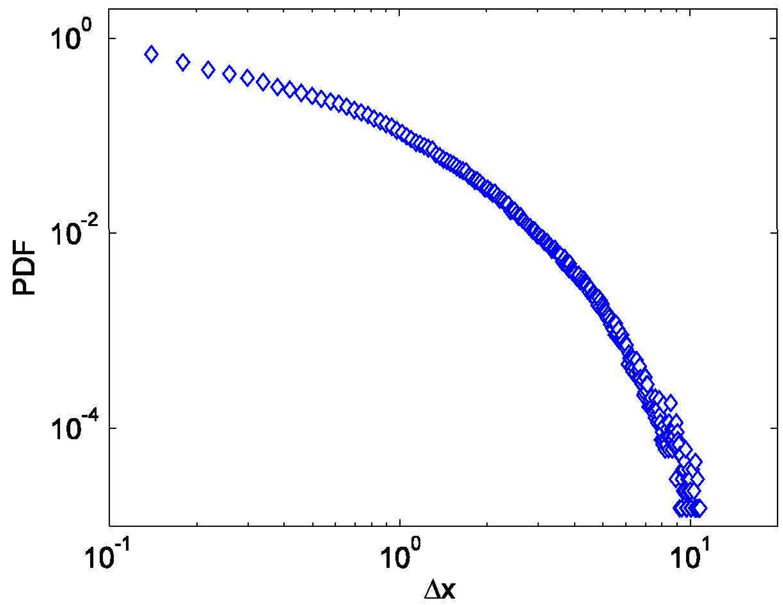
Trajectory of a test particle
released inside LCFS



Variogram of the particle motion,
$- - - \ \tau^2$; $- - - \ \tau$; —— $\tau^{1.4}$

Step size PDF



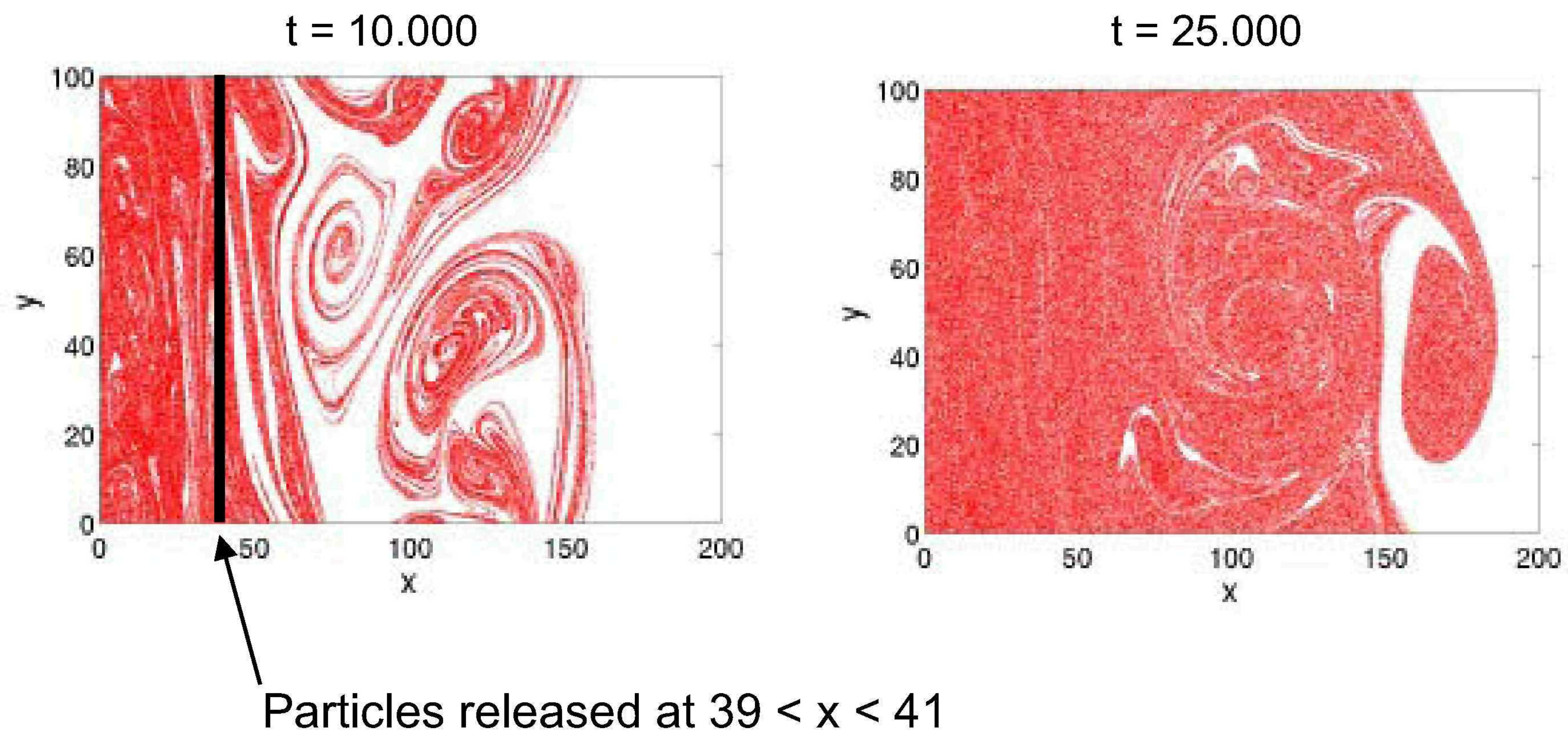


PDF of the radial displacement, Δx, over $\Delta t = 50$; all particles. $\langle \Delta x \rangle = -0.08$, standard deviation, $\sigma = 1.02$, skewness, $S = 0.4$, and kurtosis, $K = 10.7$.

Broad exponentially decaying tails.

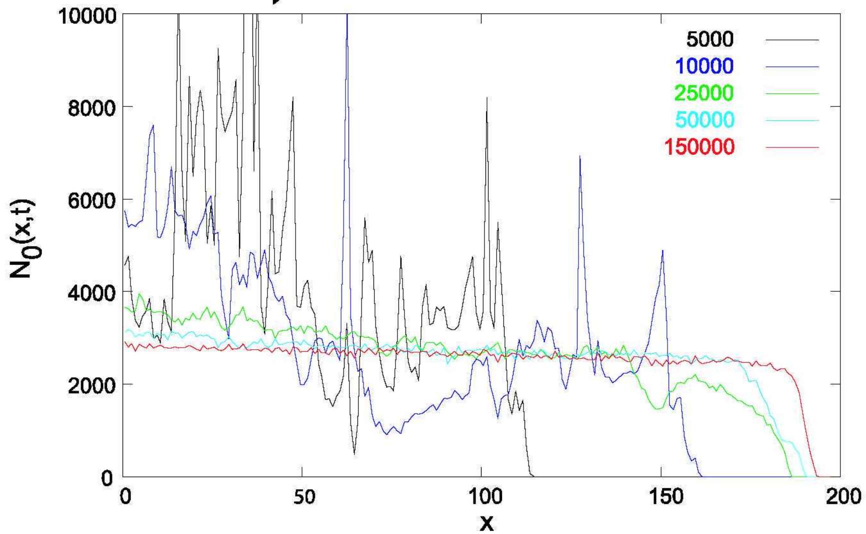
Long steps are almost equally probable in both in- and outgoing directions.

Particle dispersion

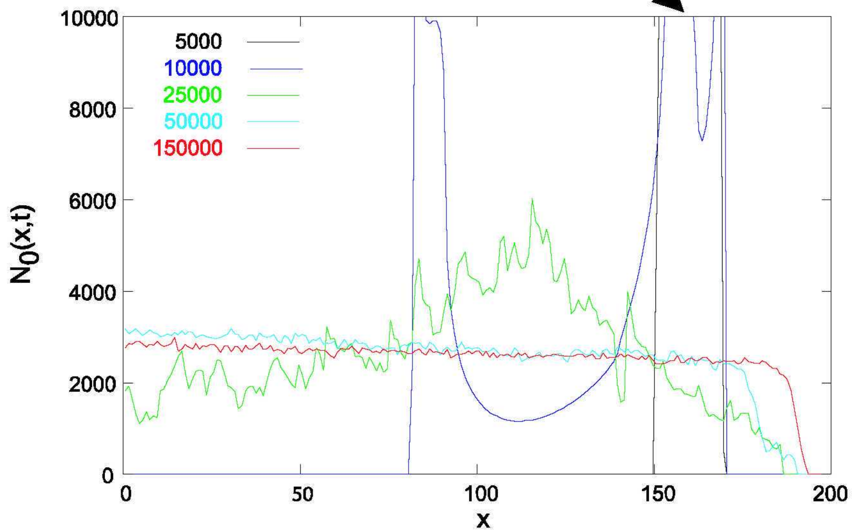


Evolution of the impurity density

Released in 39 < x < 41



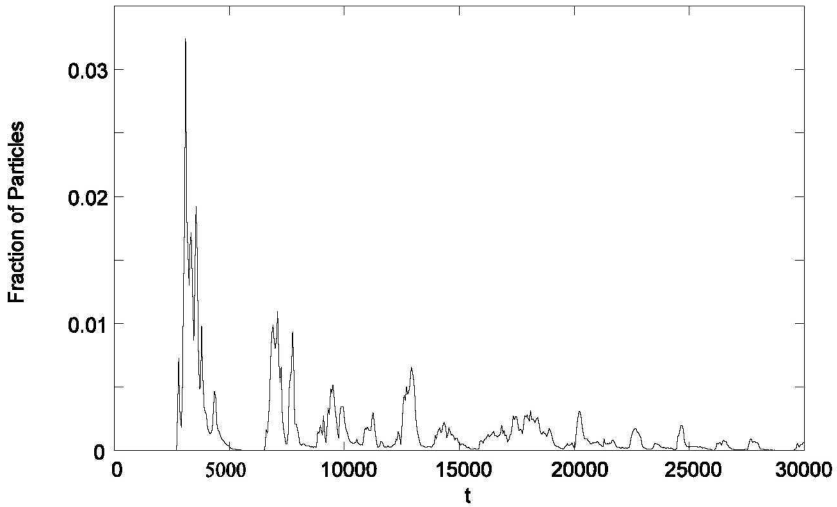
Released in 159 < x < 161



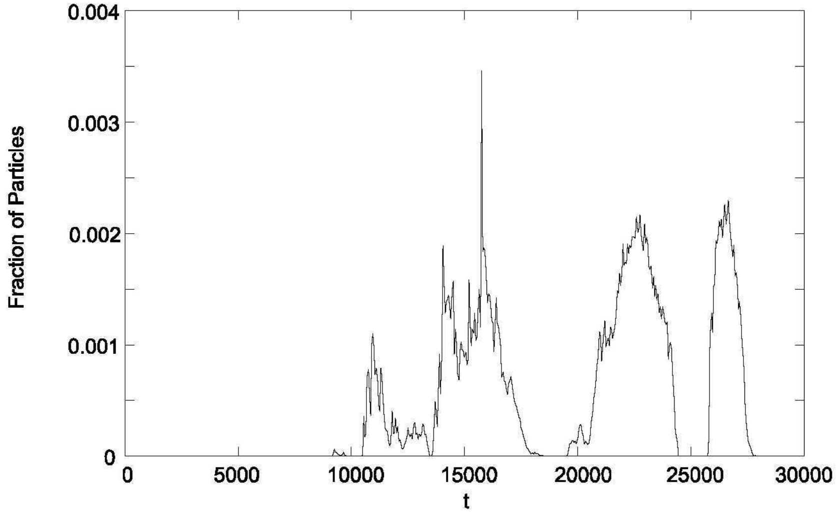
Evolution of the impurity/tracer particle density N_0 averaged over y.

Arrival times

Plane at x = 80



Plane at x = 160



The relative number of particles passing through a radial plane versus time; first passage. Particles released inside LCFS, 39 < x < 41.

Velocity of the front of the particles > $0.02c_s$, typical blob speed.

Evolution of the impurity density

Density profile $N_0(x) \propto B(x)$ independent of release position.

The transport is not "Fickian" diffusion. It can be described by an effective pinch:

$$\left(\frac{\partial}{\partial t} + \frac{1}{B}\,\hat{z} \times \nabla\phi \cdot \nabla\right)\frac{N}{B} = 0,$$

N/B is a Lagrangian invariant: Effective turbulent mixing: N/B uniformly distributed in space.

Uniform distribution within few burst times

Impurities are effectively mixed by the turbulence in the SOL within a few burst periods. Even if originating far out in the SOL they will quickly penetrate across the LCFS into the edge plasma. Corresponding to the so-called inward (curvature) pinch.

Summary

- Dynamics of passive particles in turbulence:
- Diffusion coefficient mimics bulk plasma transport for a "fluctuation" model
- Modelling impurity transport by passive particles:
- Clustering/aggregation of inertial impurities and "inertial pinch"
- Edge/SOL turbulence and transport in a magnetically confined plasmas is bursty/intermittent with broad tailed PDFs and is not diffusive in the Fickian sense.
- No parametrized diffusion type equation: Transport characterisation calls for a universal PDF
- Impurities are effectively mixed in SOL and penetrates the LCFS.
- Impurity pinch: curvature pinch

EXTRAS

Classical Particle Dispersion

Single particle dispersion:

G.I. Taylor 1915

Stationary, homogeneous turbulent flows:

$$R^2(t) \equiv \langle(\vec{r}(t) - \vec{r}(t_0))^2\rangle$$

$$R^2(t) = 2\langle v^2\rangle \int_0^t (t-\tau) C_L(\tau) d\tau$$

Lagrangian integral time scale; $\tau_L = \int_0^\infty C_L(\tau) d\tau$

$$C_L(t) = \langle\vec{v}(\tau + t)\cdot\vec{v}(\tau)\rangle / \langle\vec{v}^2(\tau)\rangle$$

Two limits:

$$t \ll \tau_L \, : \, R^2(t) = \langle v^2\rangle t^2$$

$$t \gg \tau_L \, : \, R^2(t) = 2Dt$$

Diffusion coefficient: $D = \langle v^2\rangle\tau_L$

General: $R^2(t) \propto t^\alpha$

$\alpha > 1$ **Superdiffusion** $\alpha < 1$ **Subdiffusion**

Fick's law:

$D^{eff} = \Gamma_0/\nabla n_0$, with normalizations $D^{eff} = \Gamma_0$.

Particle dispersion in drift wave turbulence

Particle dispersion in 2D drift wave turbulence

Hasegawa-Wakatani equations (HWE): the resistive drift wave instability (PRL 50, 682 (1983)):

$$\partial_t n + \partial_y \varphi + \{\varphi, n\} = -C\,(n - \varphi) + \mu_n \nabla^2 n$$

$$\partial_t \nabla^2 \varphi + \{\varphi, \nabla^2 \varphi\} = -C\,(n - \varphi) + \mu_\varphi \nabla^4 \varphi$$

$1/C = 1/k_\parallel^2 L_\parallel^2 \; L_\parallel = (L_n T_e / m_e c_s \nu_{ei})^{1/2}$

$\{\varphi, \psi\} \equiv \hat{z} \times \nabla\varphi \cdot \nabla\psi = \frac{\partial\varphi}{\partial x}\frac{\partial\psi}{\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial\varphi}{\partial y}$

$u = -\frac{\partial\varphi}{\partial y}; \; v = \frac{\partial\varphi}{\partial x}$

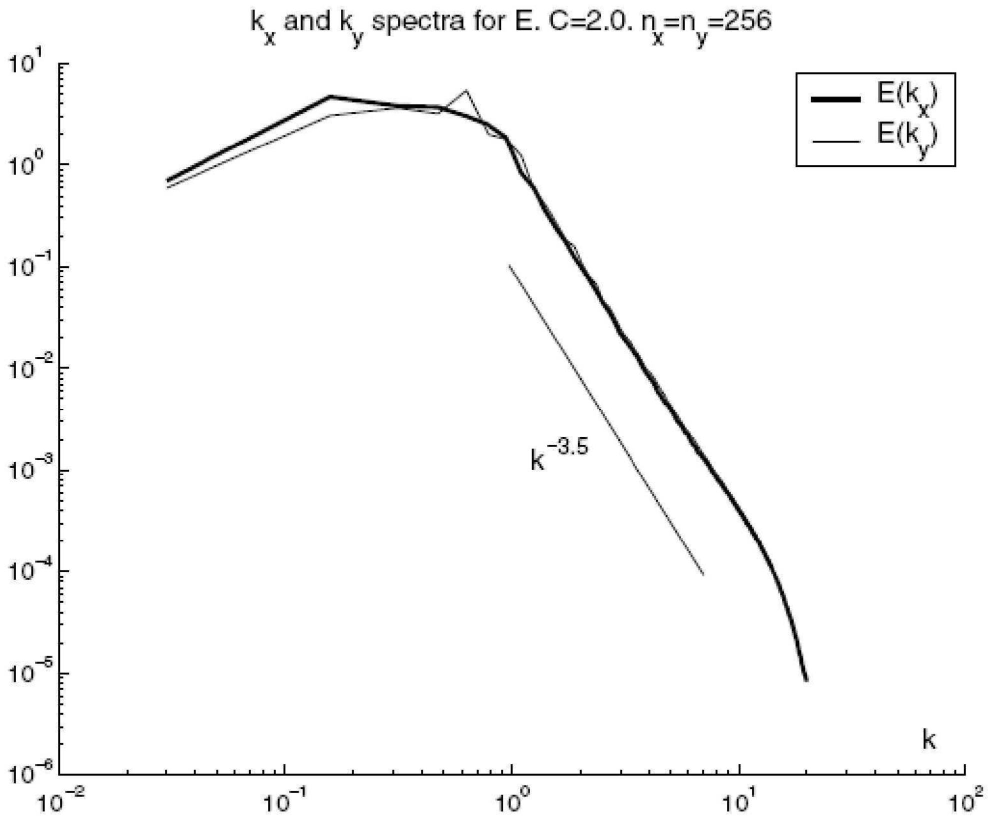
Normalization: $\rho_s = c_s/\Omega_i$ for lengths; L_n/c_s for the times;
$c_s = \sqrt{T_e/m_i}$; $L_n = |(\nabla n_0(x)/n_0(x))^{-1}|$
$(T_e/e)\,(\rho_s/L_n)$ for potential $n_0\rho_s/L_n$ for density; $\mu_n = \mu_\varphi = \mu$.

Naulin et al. Phys. Plasmas 6,4575 (1999)

Basu et al Phys. Plasmas **10**, 2696 (2003);

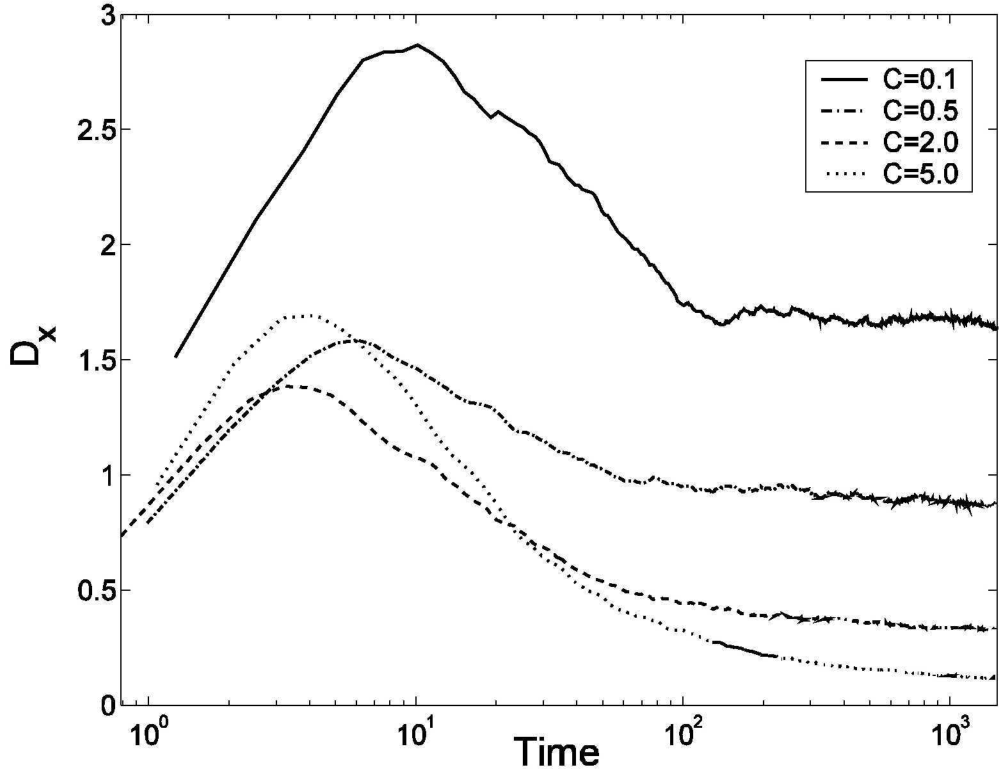
Comm.Nonlinear Sci. Numer. Simul. **8,** 477 (2003)

Energy spectrum



Spectrum is isotropic

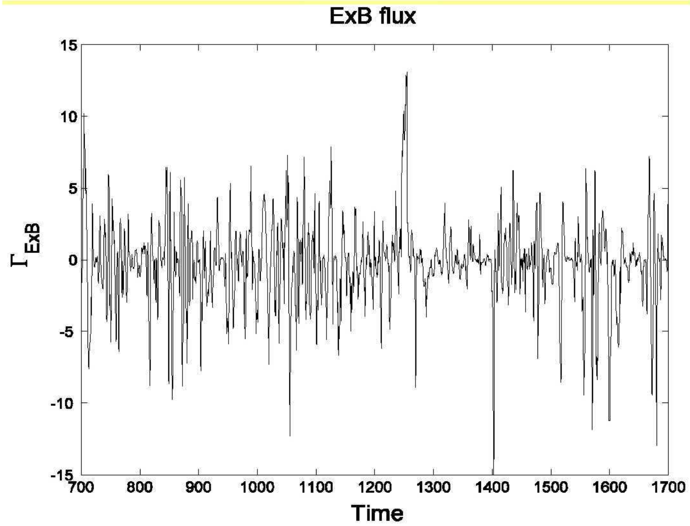
Running diffusion coefficient



Running diffusion coefficient: $D_x = \langle X^2(t)\rangle/2t$, for varying adiabaticity parameter C.

Turbulent particle density flux

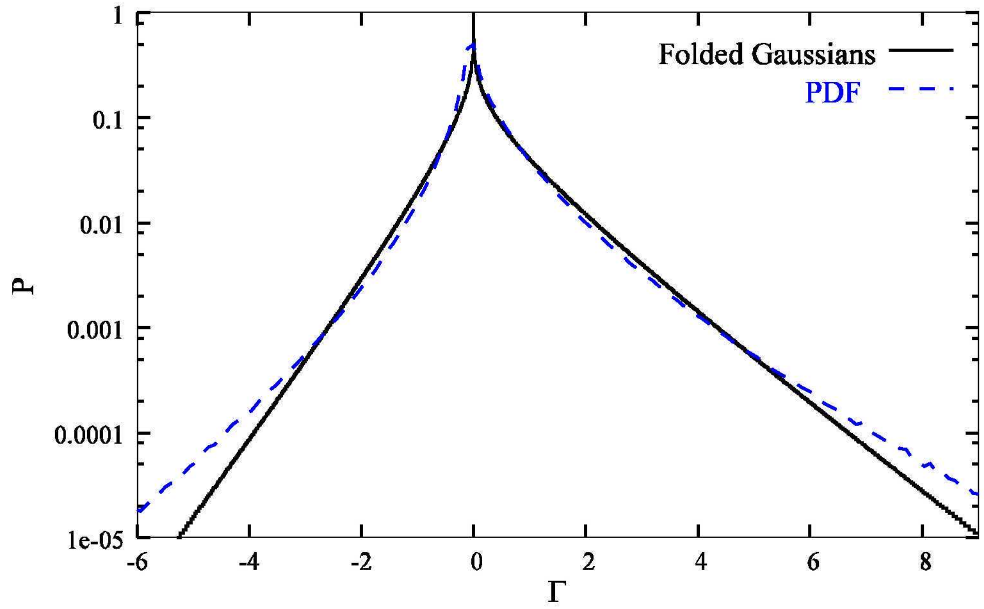
Particle density flux:
$\Gamma = n\, v_x = n\, v_{ExB}$



Bursty flux !

Naulin *et al* PLA **321** 355 (2004)

Flux PDF



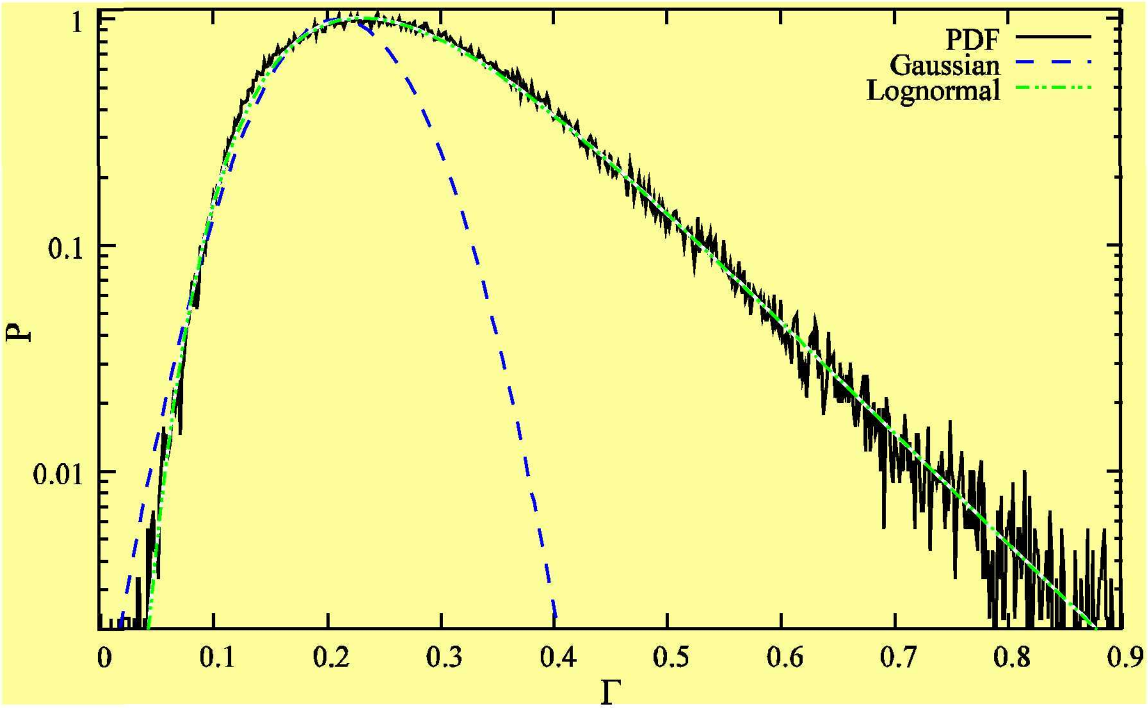
CORRELATED GAUSSIANS PDFS FOR V_X AND N

$$p_G = \frac{1}{\pi}\frac{\sqrt{1-\gamma^2}}{\sigma_n \sigma_{v_x}} K_0\left(\frac{|\Gamma|}{\sigma_n\sigma_{v_x}}\right)\exp\left(-\gamma\frac{\Gamma}{\sigma_n\sigma_{v_x}}\right)$$

γ is the correlation: $\gamma = -\frac{\langle v_x n\rangle}{\langle v_x^2\rangle^{1/2}\langle n^2\rangle^{1/2}} = \cos\alpha.$

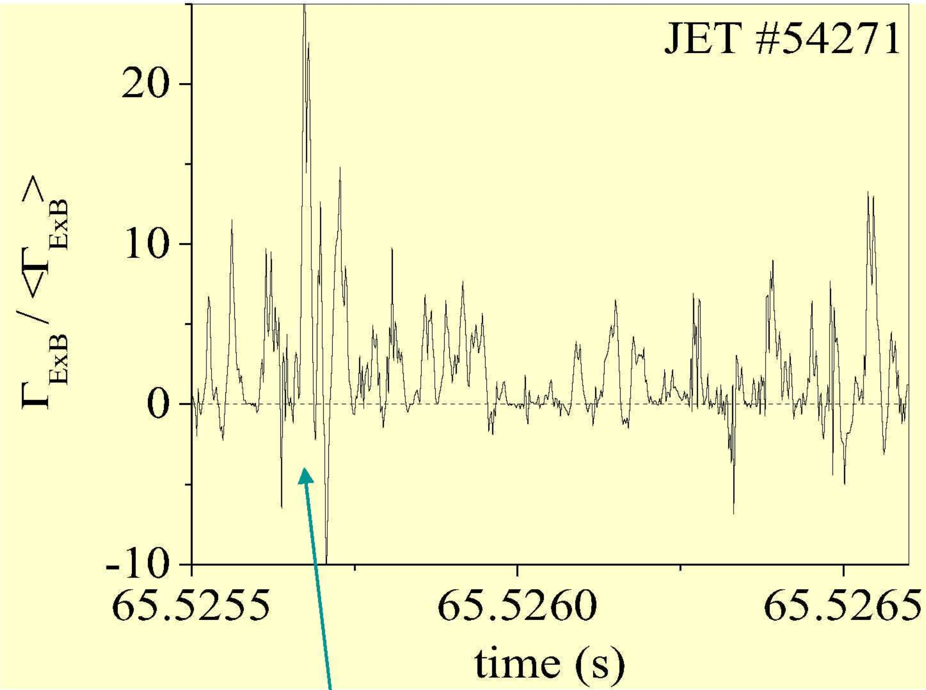
Turbulent particle density flux

Particle density flux:
flux surface averaged

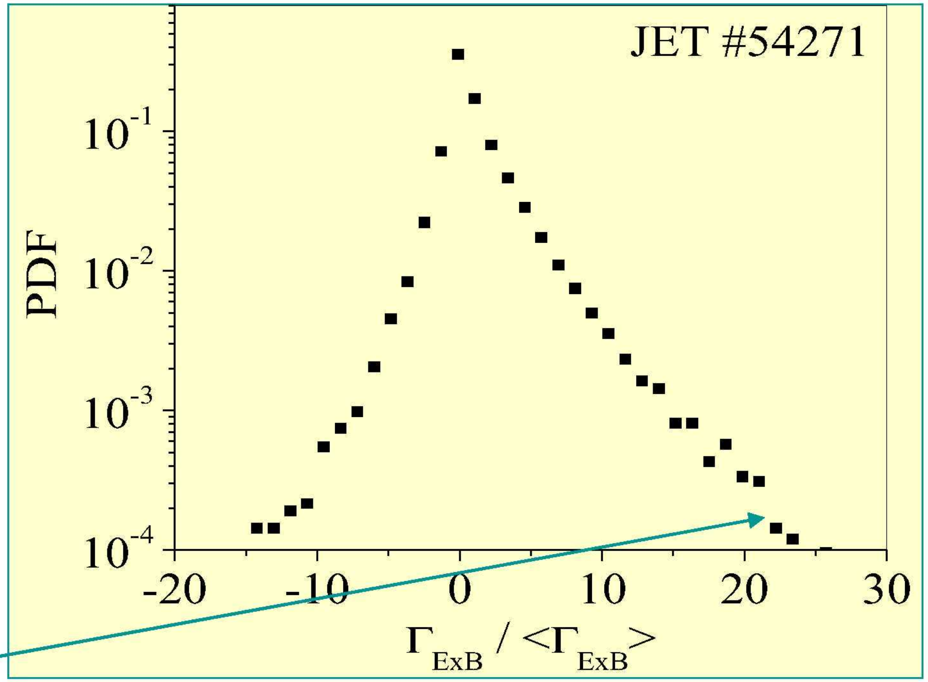


The probability distribution function for the plasma flux across the magnetic field is strongly ***non-Gaussian***, i.e., strong bursts are dominating!

Edge/SOL turbulence transport in JET



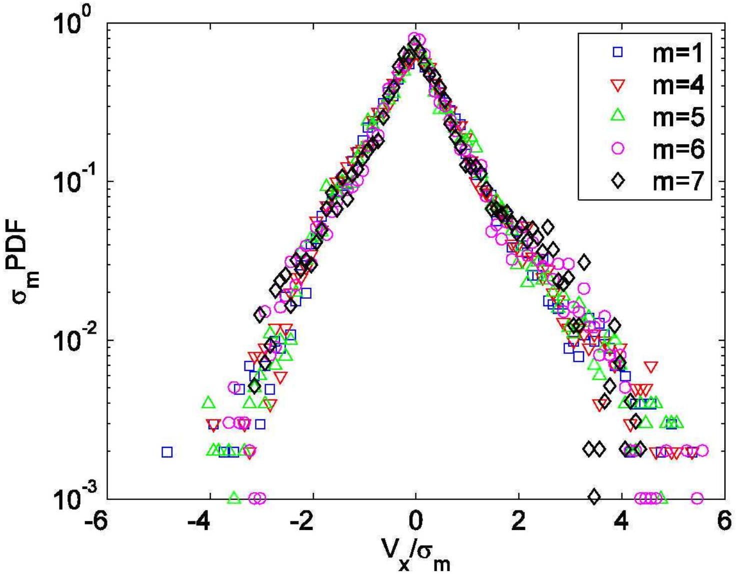
Pdf of flux

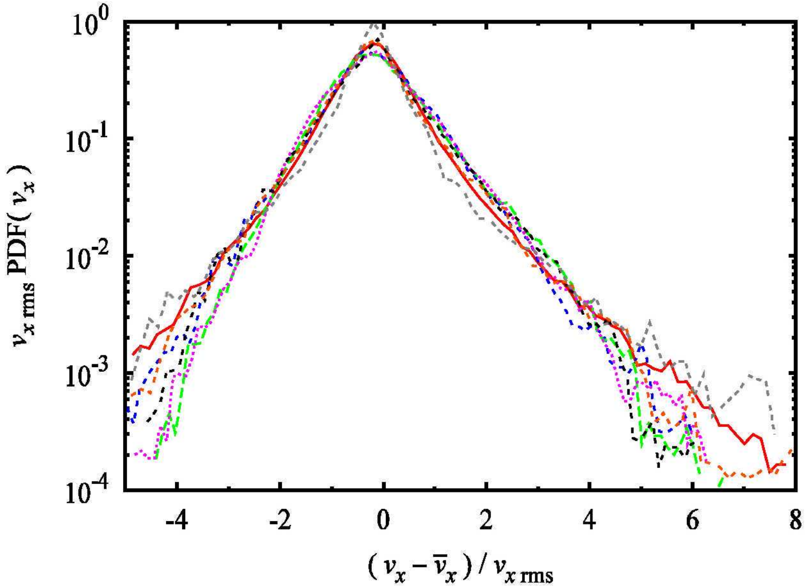


Large intermittent burst

B. Gonçalves: 11th European Fusion Physics Workshop, Heraklion, Crete 8-11 Dec. 2003

Radial velocity PDF





Re-scaled PDF of the radial particle velocity coarse grained over time intervals $\Delta t = 50 \cdot 2^{m-1}$; particles released inside LCFS

Re-scaled PDF of the turbulent radial ExB-velocity recorded at the probes P_1 - P_7