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DIA and DA Solitary Waves in Adiabatic Dusty Plasma.

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DIA AND DA SOLITARY WAVES IN ADIABATIC DUSTY PLASMA

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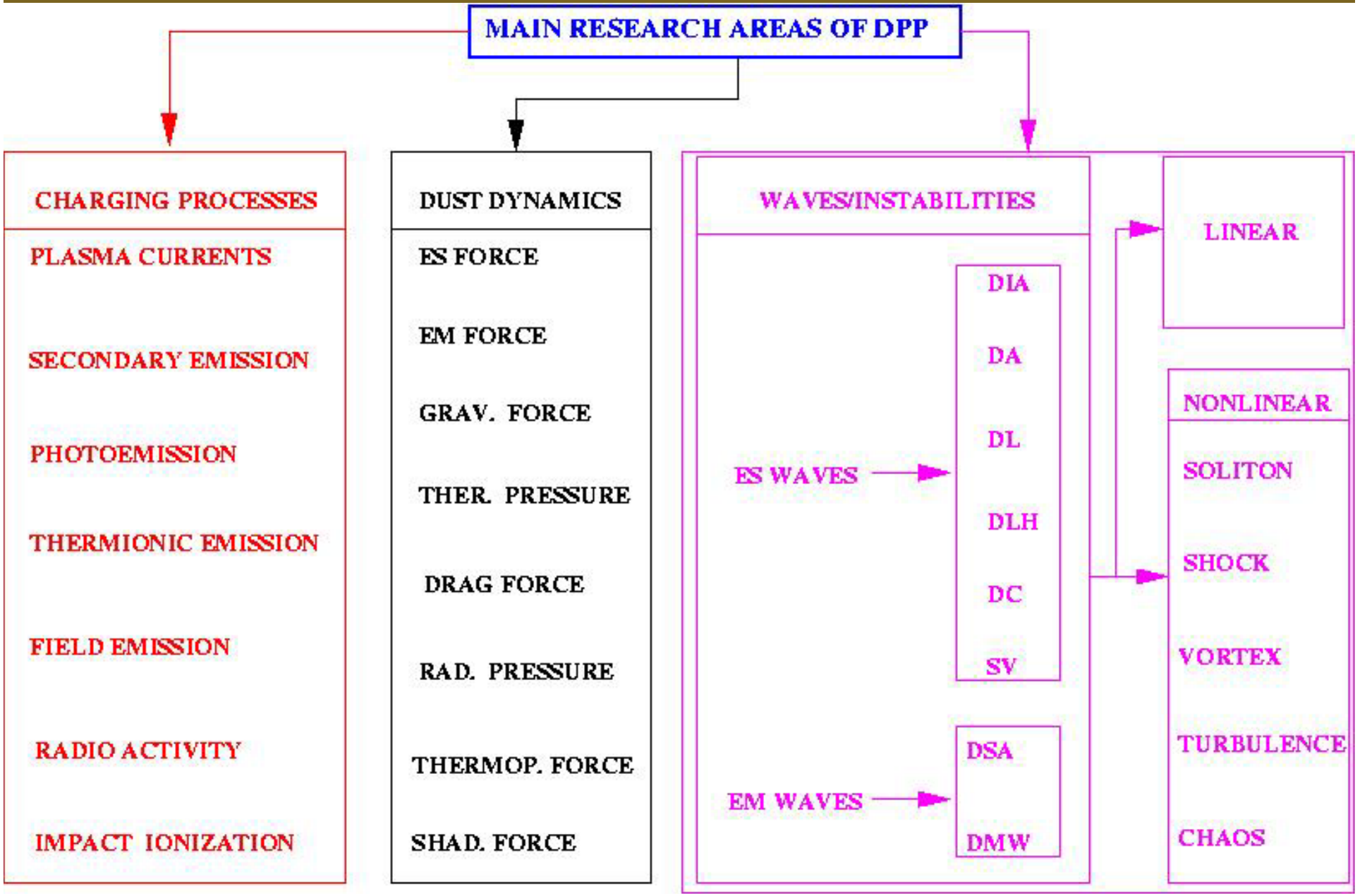
Introduction



❑ DIA and DA Waves

Two Normal Electro-acoustic Modes in Dusty Plasmas:

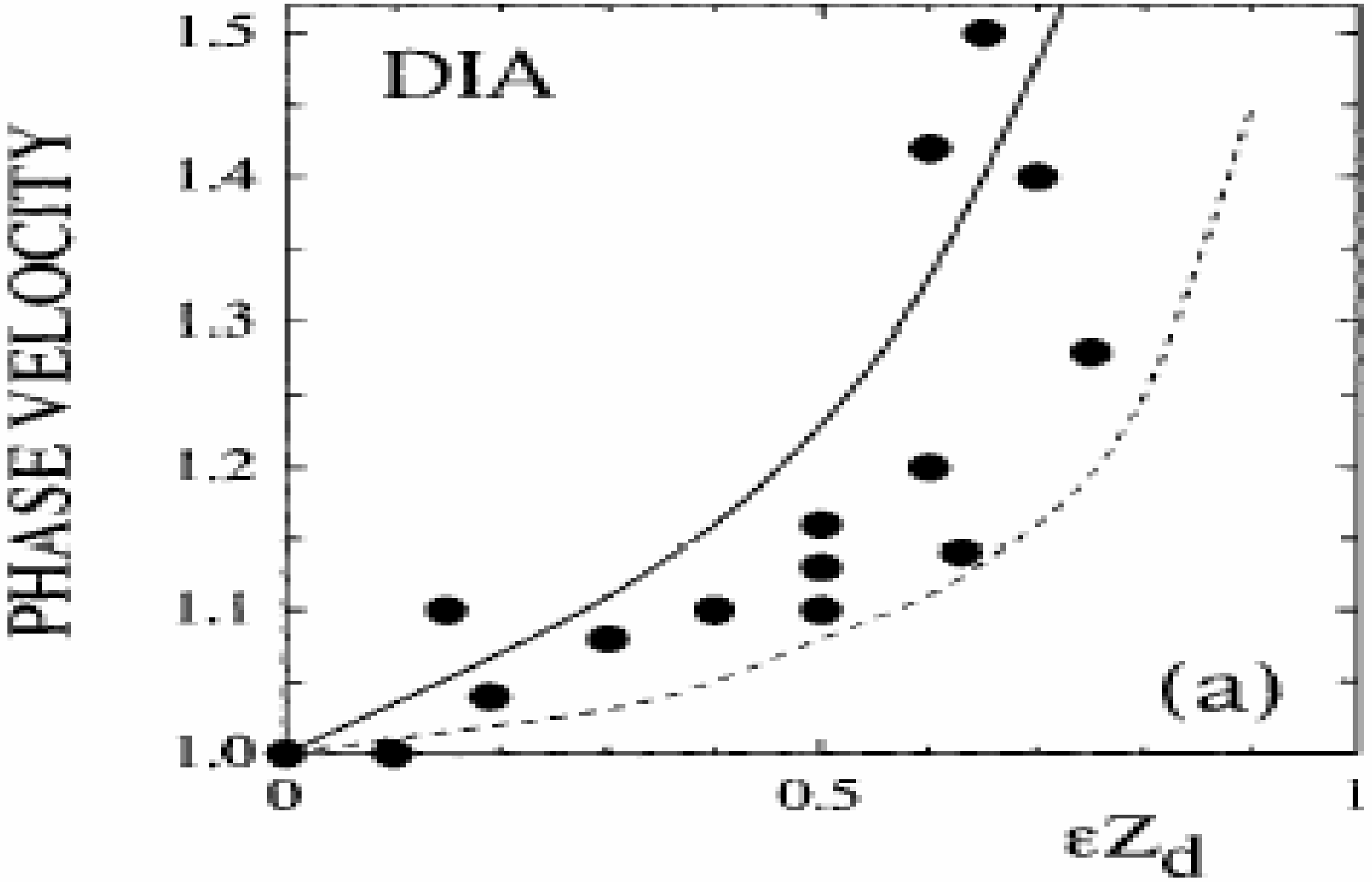
1. **DIA Waves**: The inequality $n_{i0}/n_{e0} >> 1$ (caused by the static negatively charged dust) introduces a new type of ion-acoustic waves (compression and rarefaction of ion density) known as **DIA** waves (**Shukla & Silin 1992**).

2. **DA Waves**: The mobile charged dust introduce a very low frequency and long wavelength waves (compression and rarefaction of dust density) known as **DA** waves (**Shukla 1989**).

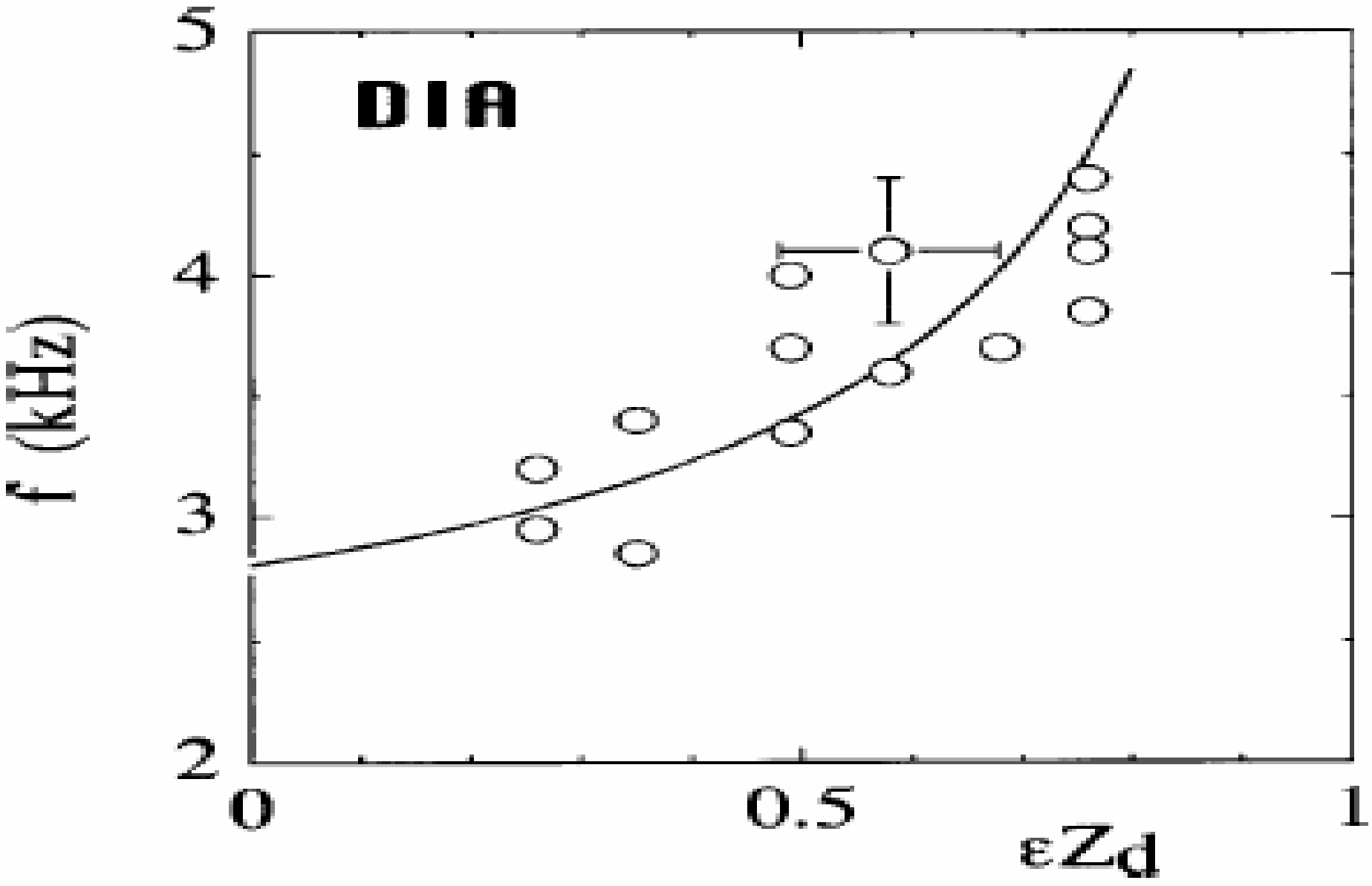
1. Static Dust: DIA Waves: $\frac{\omega}{k} = \sqrt{\left(\frac{n_{i0}}{n_{e0}}\right)\left(\frac{T_e}{m_i}\right)}$

- The compression and rarefaction of ion density in presence of static negatively charged dust which cause $n_{i0}/n_{e0} >> 1$.
- The restoring force comes from the electron thermal pressure and the inertia is provided by the ion mass.
- The phase speed of the DIA waves can be 10 -- 100 times higher than the ion-acoustic speed $C_i=(T_e/m_i)^{1/2}$ because of the factor $n_{i0}/n_{e0} =1/(1- \varepsilon Z_d)$, which can be 10^2 --- 10^4 for many space and laboratory dusty plasma cases, where $\varepsilon=n_{d0}/n_{i0}$.
- DIA waves has been theoretically predicted by **Shukla & Silin (1992)** and experimentally observed by **D'Angelo et al (1996)**.

- **Theoretically and Experimentally measured DIA Wave Phase Speed (Merlino et al. 1998)**



- **Theoretically and Experimentally measured DIA Wave Frequency (Merlino et al. 1998)**

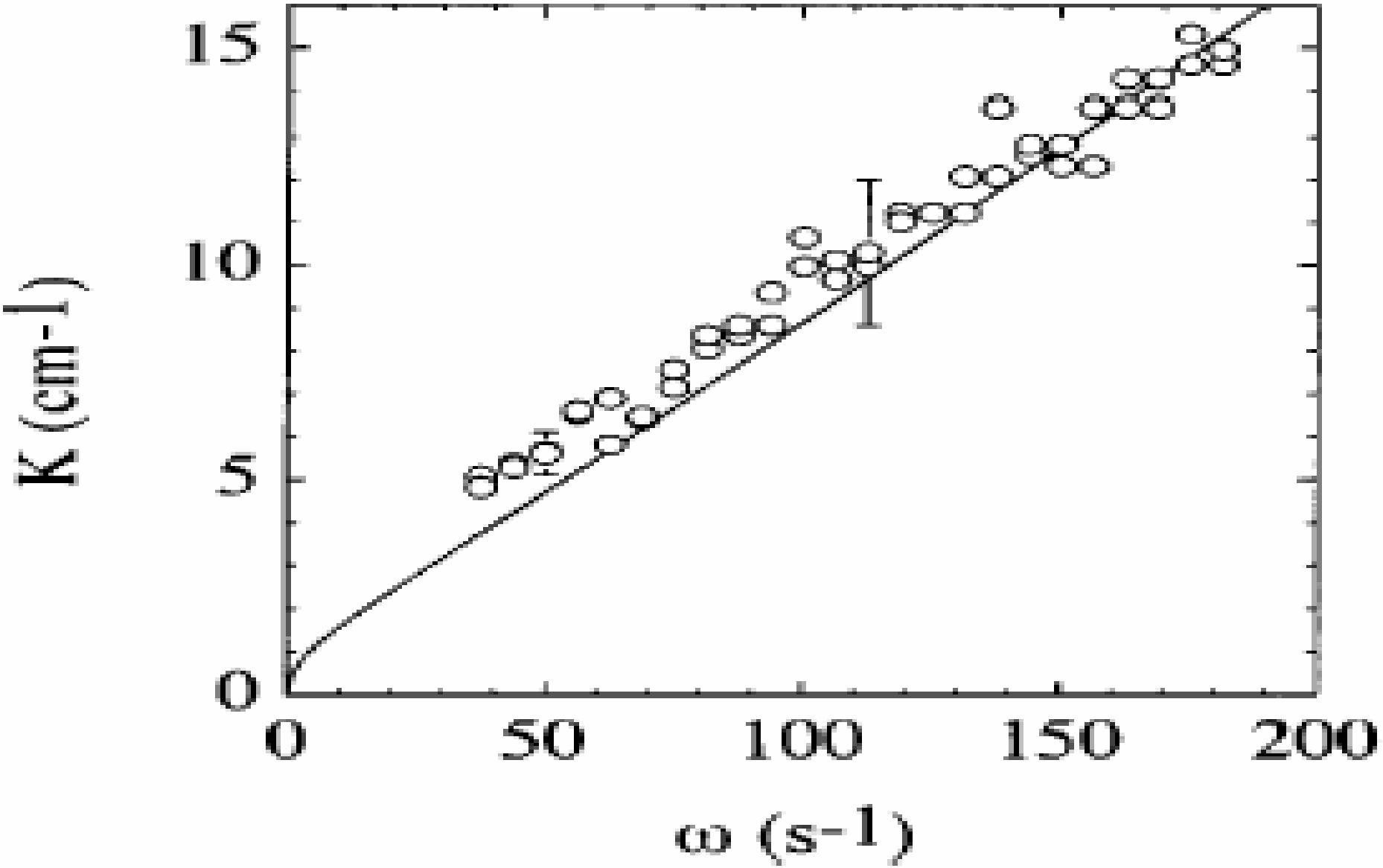


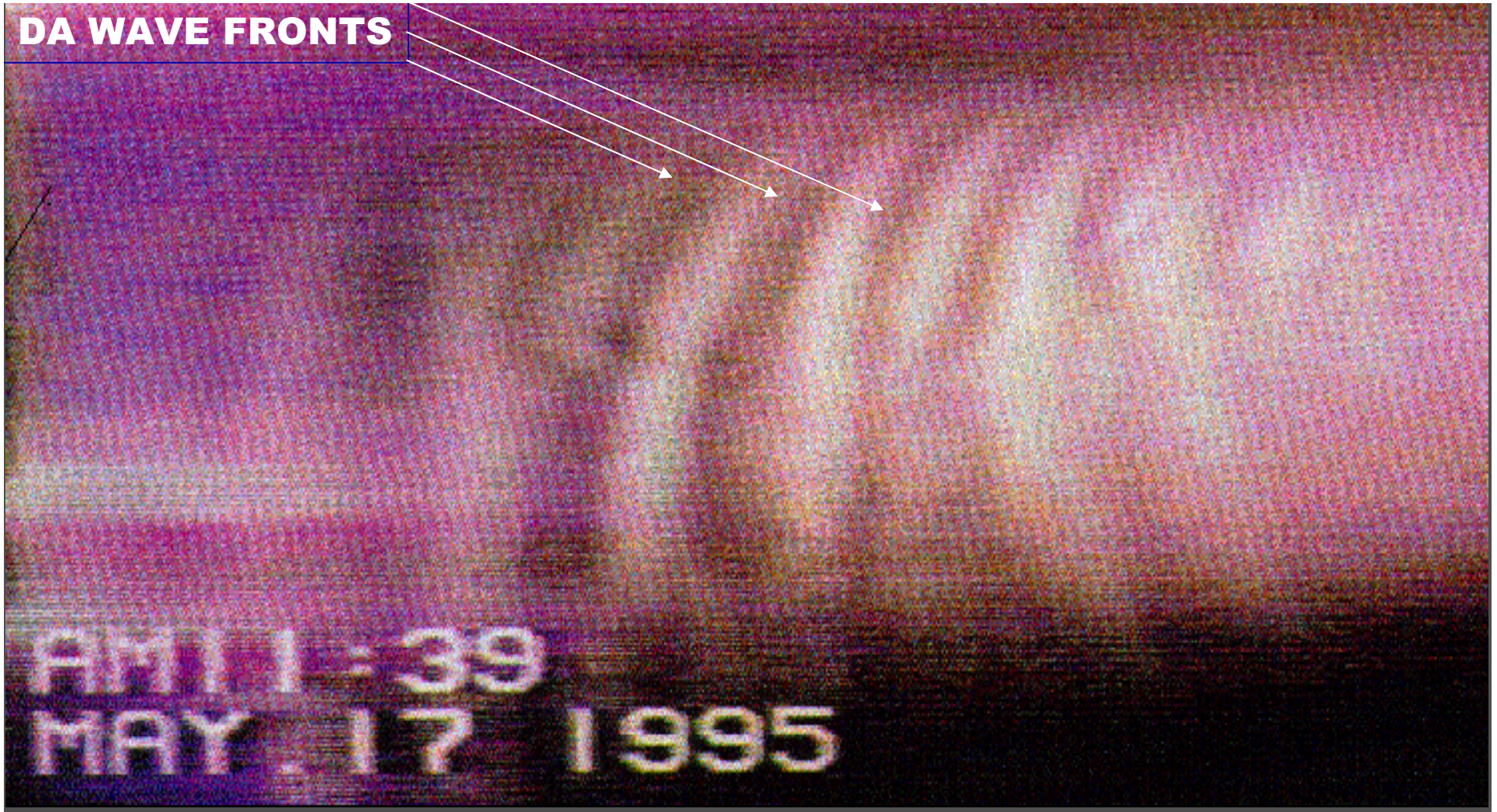
2. Mobile Dust: DA Waves:

$$\frac{\omega}{k} \approx \sqrt{\left(\frac{Z_d n_{d0}}{n_{i0}}\right)\left(\frac{Z_d T_i}{m_d}\right)}$$

- The waves associated with the dynamics of dust, i.e., associated with the compression and rarefaction of dust density.
- The restoring force comes from electron and ion thermal pressures and the inertia is provided by dust mass. However, T_e and $k\lambda_D$ are not appeared in above dispersion relation, where $k\lambda_D \ll 1$ and $n_{e0}T_i/n_{i0}T_e \ll 1$ are assumed.
- A very low frequency and long wavelength wave mode, where f=10 - 100 Hz and λ=0.5 - 5.0cm.
- The DA waves are visible, even with naked eyes.

- **Dust-Acoustic Wave Dispersion Relation (Merlino *et al.* 1998)**





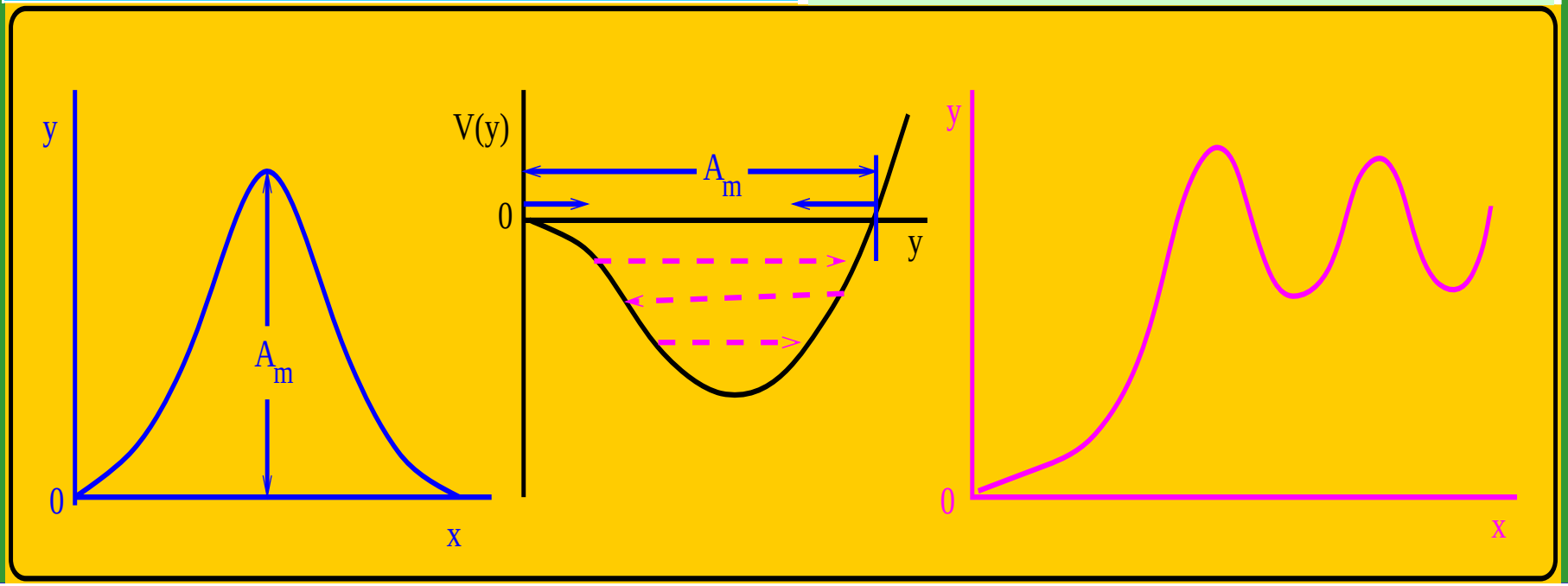
- The idea of DA waves was first presented by Shukla at Capri Meeting in 1989. Later, DA waves were theoretically predicted by Rao, Yu & Shukla (1990) and experimentally observed by Barkan, Merlino & D`Angelo (1995).

➢ What are solitary waves? How do they form? How do they convert to shock-like structures?

- **Solitary waves** are hump/dip shaped nonlinear waves of permanent profile due to balance between nonlinearity and dispersion (without dissipation). However, when dissipation is important, **shock waves** are formed.

The formation of arbitrary amplitude stationary solitary and shock waves

$$\frac{1}{2}\left(\frac{dy}{dx}\right)^2 + V(y) = 0$$



- **Evolution equation for small amplitude solitary/shock waves: K-dV-Burgers equation (Karpman 1975):**

$$\frac{\partial y}{\partial t} + Ay\frac{\partial y}{\partial x} + B\frac{\partial^3 y}{\partial x^3} = C\frac{\partial^2 y}{\partial x^2}$$

The term containing B (C) is the dispersive (dissipative) term

When the dispersive term is much more important than the dissipative term: K-dV equation:

$$\frac{\partial y}{\partial t} + Ay\frac{\partial y}{\partial x} + B\frac{\partial^3 y}{\partial x^3} = 0$$

$$y = \left(\frac{3v_0}{A}\right)\ \mathrm{sech}^2\left[(x - v_0 t)\sqrt{\frac{v_0}{4B}}\right]$$

When the dissipative term is much more important than the dispersive term we have: Burgers equn:

$$\frac{\partial y}{\partial t} + Ay\frac{\partial y}{\partial x} = C\frac{\partial^2 y}{\partial x^2}$$

$$y = \left(\frac{v_0}{A}\right)\left\{1 - \tanh\left[(x - v_0 t)\frac{v_0}{2C}\right]\right\}$$

Basic Features of DIA and DA SWs

- The basic features of DIA and DA SWs have been seriously investigated by many authors during the last two decades. The basic features of DIA & DA SWs [Bharuthram & Shukla 1992; Mamun et al 1996] in comparison with IA SWs:

IA SWs ($V_{Te}>V_P>V_{Ti}$)	DIA SWs ($V_{Te}>V_P>V_{Ti}$)	DA SWs ($V_{Ti}>V_P>V_{Td}$)
$\phi>0$ (only)	$\phi>0$ when $\mu>1/3$ $\phi<0$ when $\mu<1/3$	$\phi<0$ (only)
$\Delta n_e>0$ $\Delta n_i>0$	$\Delta n_e>(<)0$ when $\mu>(<)1/3$ $\Delta n_i>(<)0$ when $\mu>(<)1/3$	$\Delta n_e<0$; $\Delta n_i>0$ $\Delta n_d<0$
$V>C_i$ $C_i=(T_e/m_i)^{1/2}$	$V>C_i\,\mu^{-1/2}$ $\mu=n_{e0}/n_{i0}$	$V>C_d$ $C_d=(Z_dT_i/m_d)^{1/2}$

• Aim

- These basic features are identified by assuming
  - isothermal electrons ($\gamma_e=1$, $p_e=n_e k_B T_e$ and $T_e=$const.), cold ions & static dust: DIA SWs.
  - isothermal electrons & ions ($\gamma_e=\gamma_i=1$, $p_{e,i}=n_{e,i} k_B T_{e,i}$ and $T_{e,i}=$const.) and mobile dust: DA SWs.

- The aim of our present talk is to consider a realistic dusty plasma model, to generalize al most all earlier works on DIA and DA SWs in an adiabatic dusty plasma, and to pinpoint the roles of the adiabaticity of electrons and ions on DIA and DA solitary waves.

❑ Static Dust: DIA Waves

- **Model:**

We consider an unmagnetized adiabatic DP with static dust [Shukla & Silin 1992]. The dynamics of 1D DIA waves:

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j u_j) = 0, \tag{1}$$

$$\frac{\partial p_j}{\partial t} + u_j \frac{\partial p_j}{\partial x} + \gamma_j p_j \frac{\partial u_j}{\partial x} = 0, \tag{2}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \Psi}{\partial x} - \frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x}, \tag{3}$$

$$0 = \frac{\partial \Psi}{\partial x} - \frac{1}{n_i}\frac{\partial p_e}{\partial x}, \tag{4}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = (1-\alpha)n_e - n_i + \alpha, \tag{5}$$

where n_j, u_j, p_j, ψ, t, x are normalized by n_{j0}, C_i, $n_{j0}k_BT_{j0}$, T_e/e, $1/\omega_{pi}$, C_i/ω_{pi}, respectively, $\sigma_j=T_{j0}/T_{e0}$ and $\alpha=Z_d n_d/n_{i0}$.

❖Small Amplitude:

- **RPM [Washimi & Taniuti 1966] reduces (1)- (4) to a K-dV Eq:**

$$\frac{\partial \Psi^{(1)}}{\partial \tau} + A_i \Psi^{(1)} \frac{\partial \Psi^{(1)}}{\partial \zeta} + B_i \frac{\partial^3 \Psi^{(1)}}{\partial \zeta^3} = 0,$$

where $\zeta = \varepsilon^{1/2}(z - v_0 t)$, $\tau = \varepsilon^{3/2} t$,

$$A_i == \frac{1}{2V_p}\left[1 + \gamma_e - 3\alpha + \sigma_i \frac{\gamma_i}{\gamma_e}(1-\alpha)(1+\gamma_i)\right],$$

$$B_i = \frac{\gamma_e^2}{2(1-\alpha)^2 V_p},$$

$$V_p^2 = \frac{\gamma_e}{1-\alpha} + \gamma_i \sigma_i.$$

- **The stationary SW solution of this K-dV Eq:**

$$\Psi^{(1)} = \Psi_m \, \mathrm{sech}^2(\xi/\Delta),$$

where $\psi_m=3u_0/A_i$ and $\Delta=(4B_i/u_0)^{1/2}$.

- **The basic features of DIA SWs for $\sigma_i=0$:**

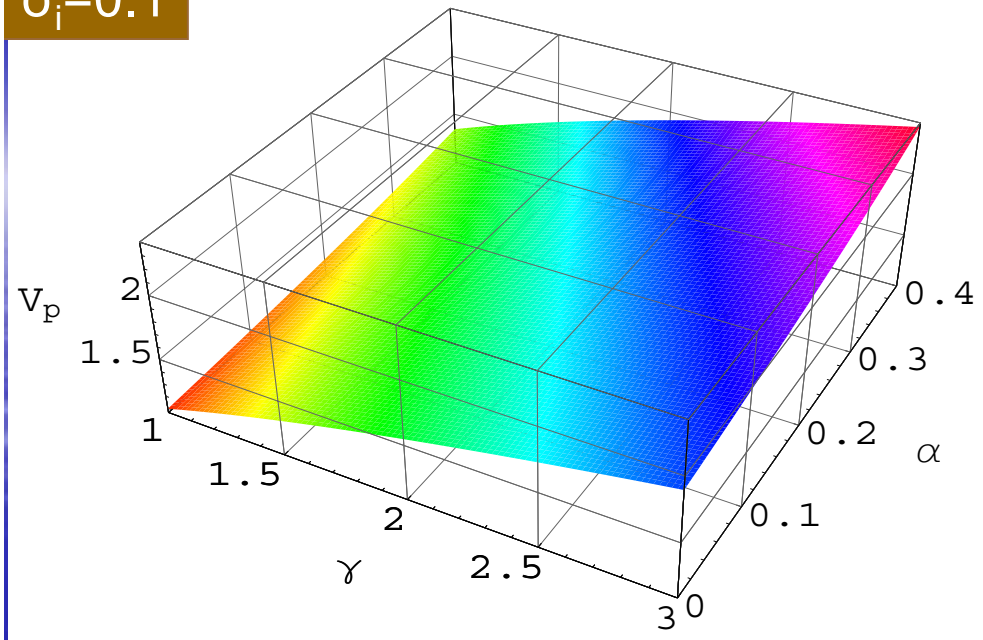
$$A_i = \frac{3}{2\sqrt{\gamma_e(1-\alpha)}}\left[\frac{1}{3}(1+\gamma_e)-\alpha\right]$$

$$B_i = \frac{1}{2}\left(\frac{\gamma_e}{1-\alpha}\right)^{3/2}$$

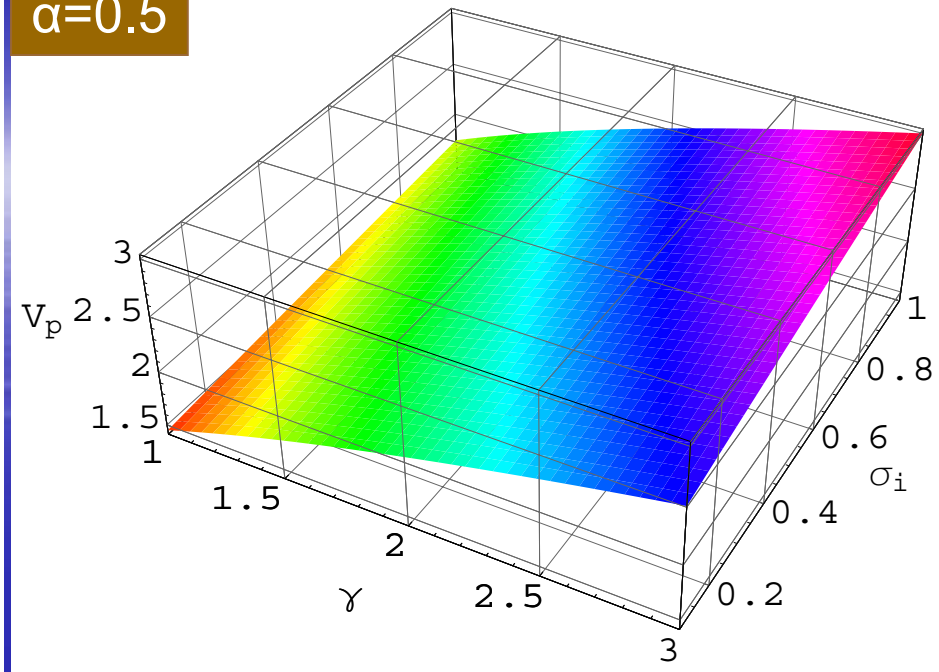
i.e. A_i or ψ_m is positive when $\alpha<(1+\gamma_e)/3$or negative when $\alpha>(1+\gamma_e)/3$, but $\alpha<1$. What does it mean?

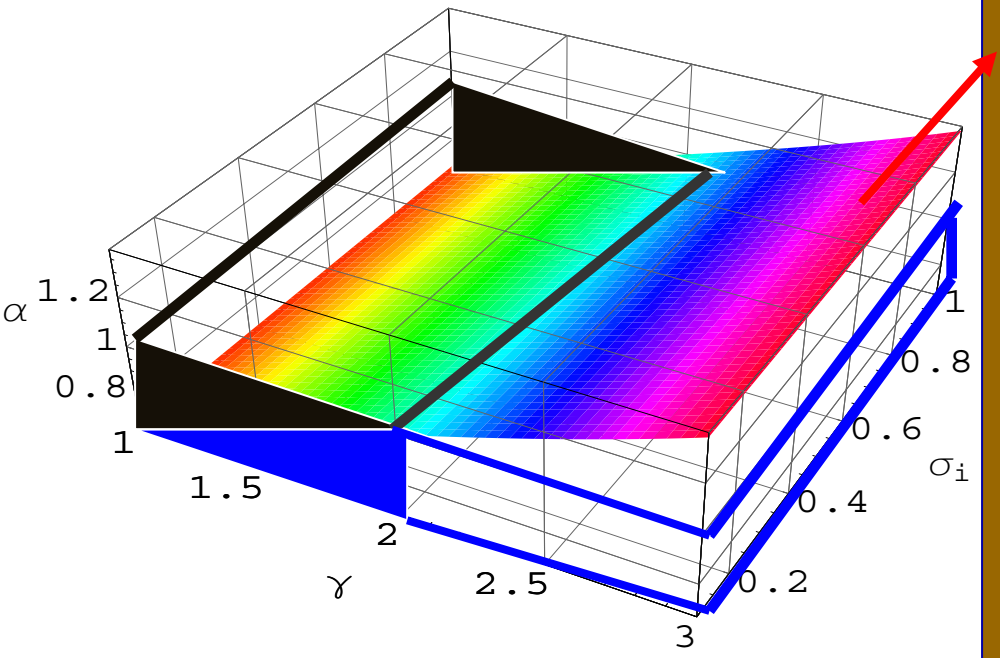
The variation of V_p, ψ_m & Δ with α, $\gamma=\gamma_e=\gamma_i$ and σ_i are shown as follows:

σ_i=0.1



α=0.5

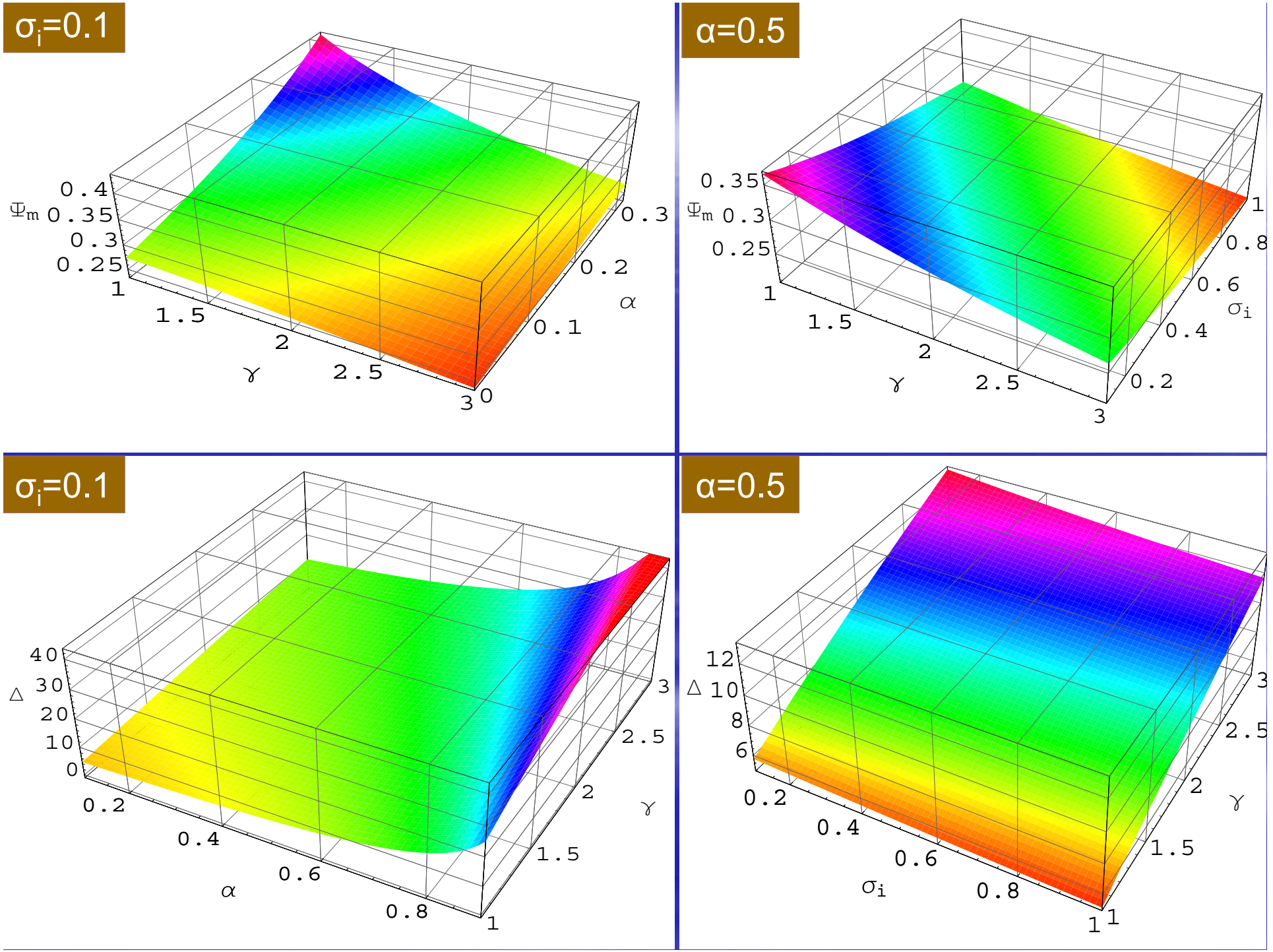




- **A (α,γ,σ_i)=0 surface**
- **possible values of α:0<α<1**
- **possible values of γ: 1≤γ≤3**
- **possible values of σ_i: 0<σ_i<1**

- **A>0: blue bounded regoin: positive SWs**

- **A<0: black bounded region: negative SWs**


σi=0.1
Ψm
0.4
0.35
0.3
0.25
1
1.5
2
2.5
3
γ
0
0.1
0.2
0.3
α
α=0.5
Ψm
0.35
0.3
0.25
1
1.5
2
2.5
3
γ
0.2
0.4
0.6
0.8
1
σi
σi=0.1
Δ
40
30
20
10
0
0.2
0.4
0.6
0.8
1
α
1.5
2
2.5
3
γ
α=0.5
Δ
12
10
8
6
0.2
0.4
0.6
0.8
1
σi
1.5
2
2.5
3
γ

• Arbitrary Amplitude:

SPA [Sagdeev 1966] reduces (1) - (5) to an energy integral:

$$\frac{1}{2}\left(\frac{d\Psi}{d\xi}\right)^2 + V(\Psi) = 0,$$

where ξ=z - Mt and V(ψ) reads [Mamun 2008]:

$$V(\Psi) = C - \alpha\Psi - (1-\alpha)\left(1 + \frac{\gamma_e - 1}{\gamma_e}\Psi\right)^{\frac{\gamma_e}{\gamma_e - 1}} - \frac{M}{\sqrt{2}}\left(\Psi_1 + \Psi_2\right)^{\frac{1}{2}} - 2^{\frac{3}{2}}\sigma_i M^3\left(\Psi_1 + \Psi_2\right)^{-\frac{3}{2}},$$

$$C = 1 - \alpha + \sigma_i + M^2,$$

$$\Psi_1 = M^2 + 3\sigma_i - 2\Psi,$$

$$\Psi_2 = \sqrt{\Psi_1^2 - 12\sigma_i M^2}.$$

we numerically analyzed V(ψ), and obtained the same results that we mentioned in two previous slides with an exception that instead either +ve or -ve SWs, we have the coexistence of +ve and –ve SWs in the black shaded region (for details please see Mamun 2008).

❑ Mobile Dust: DA Waves

- **Model:**

We consider an unmagnetized DP with mobile dust The dynamics of 1D DA waves:

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0, \tag{1}$$

$$\frac{\partial p_s}{\partial t} + u_s \frac{\partial p_s}{\partial x} + \gamma p_s \frac{\partial u_s}{\partial x} = 0, \tag{2}$$

$$\frac{\partial p_e}{\partial x} = n_e \alpha \frac{\partial \Psi}{\partial x}, \tag{3}$$

$$\frac{\partial p_i}{\partial x} = -n_i \frac{\partial \Psi}{\partial x}, \tag{4}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{\partial \Psi}{\partial x} - \frac{\sigma}{n_d} \frac{\partial p_d}{\partial x}, \tag{5}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \mu_e n_e - \mu_i n_i + n_d, \tag{6}$$

where n_s, u_s, ψ, t, x are normalized by n_{s0}, C_d, T_{i0}/e, $1/\omega_{pd}$, C_d/ω_{pd}, respectively, $\mu_e=\mu/(1-\mu)$, $\mu_i=1/(1-\mu)$, $\mu=n_{e0}/n_{i0}$, $\alpha=T_{i0}/T_{e0}$,$\sigma=T_{d0}/Z_d T_{i0}$, and γ=adiabatic constant.

❖Small Amplitude:

- **RPM [Washimi & Taniuti 1966] reduces (1)- (4) to a K-dV Eq:**

$$\frac{\partial \Psi^{(1)}}{\partial \tau} + A_d \Psi^{(1)} \frac{\partial \Psi^{(1)}}{\partial \zeta} + B_d \frac{\partial^3 \Psi^{(1)}}{\partial \zeta^3} = 0,$$

where $\zeta = \varepsilon^{1/2}(z - v_0 t)$, $\tau = \varepsilon^{3/2} t$,

$$A_d = -B_d \left[\Gamma(\mu_e \alpha^2 - \mu_i) + \frac{1}{V_\mu^6}(3V_\mu^2 + \Gamma_\sigma) \right],$$

$$B_d = \frac{V_\mu^4}{2V_p}, \qquad V_\mu^2 = \frac{\gamma(1-\mu)}{1+\alpha\mu},$$

$$\Gamma = \frac{2-\gamma}{\gamma^2}, \qquad \Gamma_\sigma = \sigma\gamma(1+\gamma).$$

- **The stationary SW solution of this K-dV Eq:**

$$\Psi^{(1)} = \Psi_m \, \mathrm{sech}^2(\xi/\Delta),$$

where $\psi_m=3u_0/A_d$ and $\Delta=(4B_d/u_0)^{1/2}$.

- **The basic features of DA SWs for γ=1 and σ=0:**

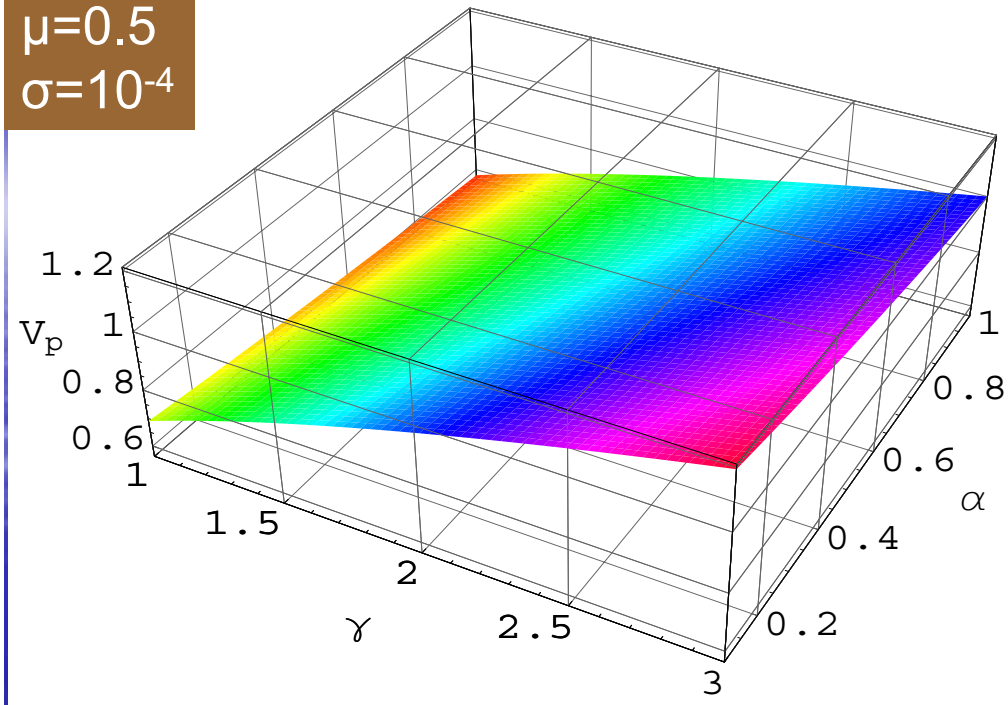
$$A_d = -\frac{1}{(1+\alpha\mu)^{1/2}(1-\mu)^{3/2}}[1+(3+\alpha\mu)\alpha\mu+\frac{1}{2}\mu(1+\alpha^2)]$$

$$B_d = \frac{1}{2}\left(\frac{1-\mu}{1+\alpha\mu}\right)^{3/2}$$

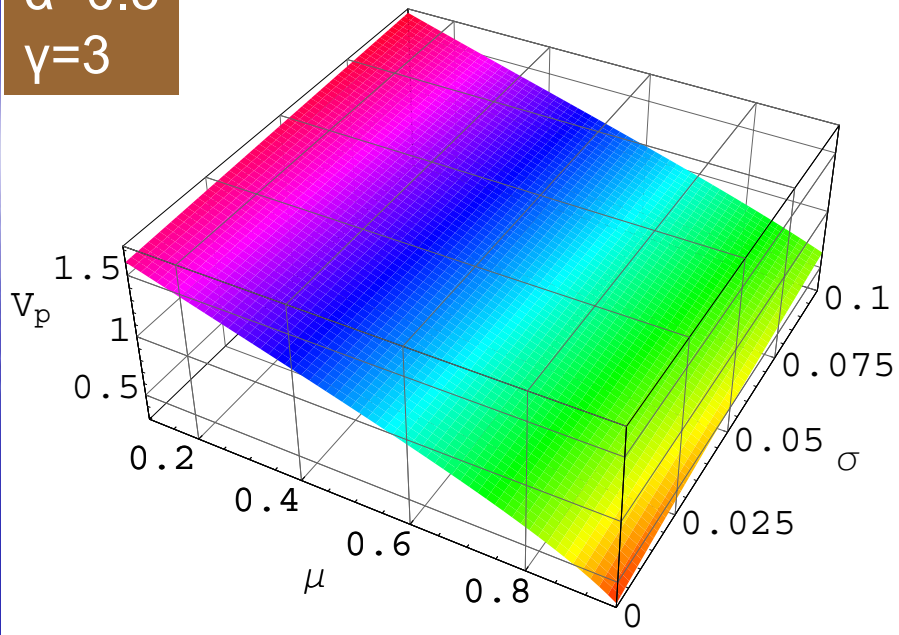
i.e. A_d or ψ_m is always negative for any possible values of α and μ. To include the effects of α and γ, we have also numerically analyzed A_d, and find that A_d is always negative for any possible values of α, μ, α and γ, [i.e. $0<\alpha\leq1$, $1\leq\gamma\leq3$, $0\leq\mu<1$, $0\leq\sigma<0.001$].

The variation of V_p, ψ_m & Δ with α, $\gamma=\gamma_e=\gamma_i$ and σ_i are shown as follows:

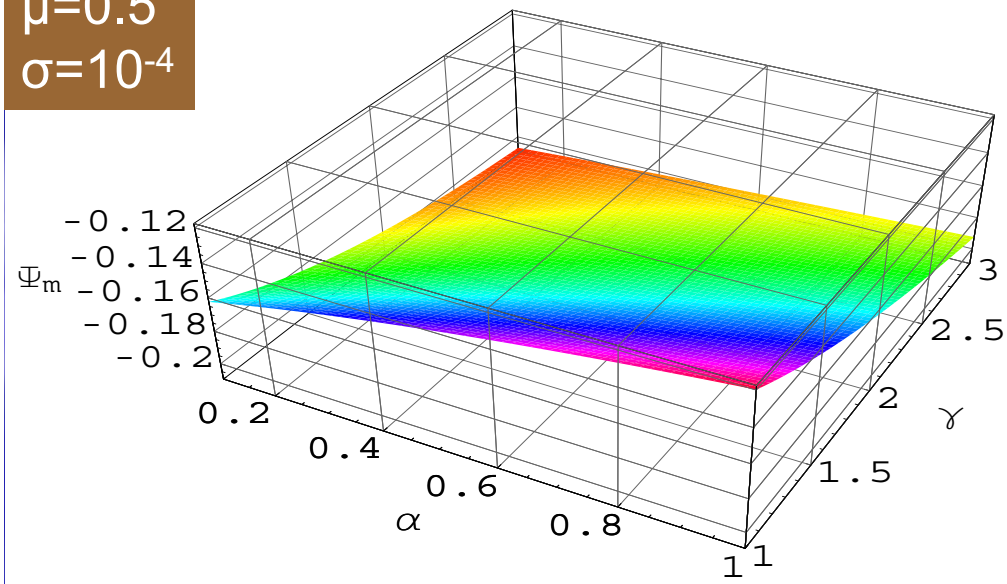
μ=0.5
σ=10^{-4}



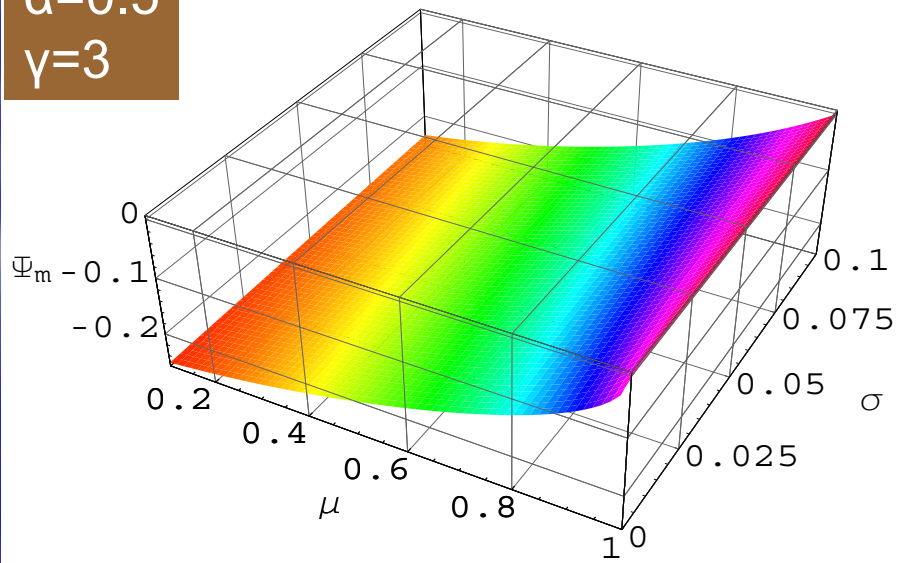
α=0.5
γ=3

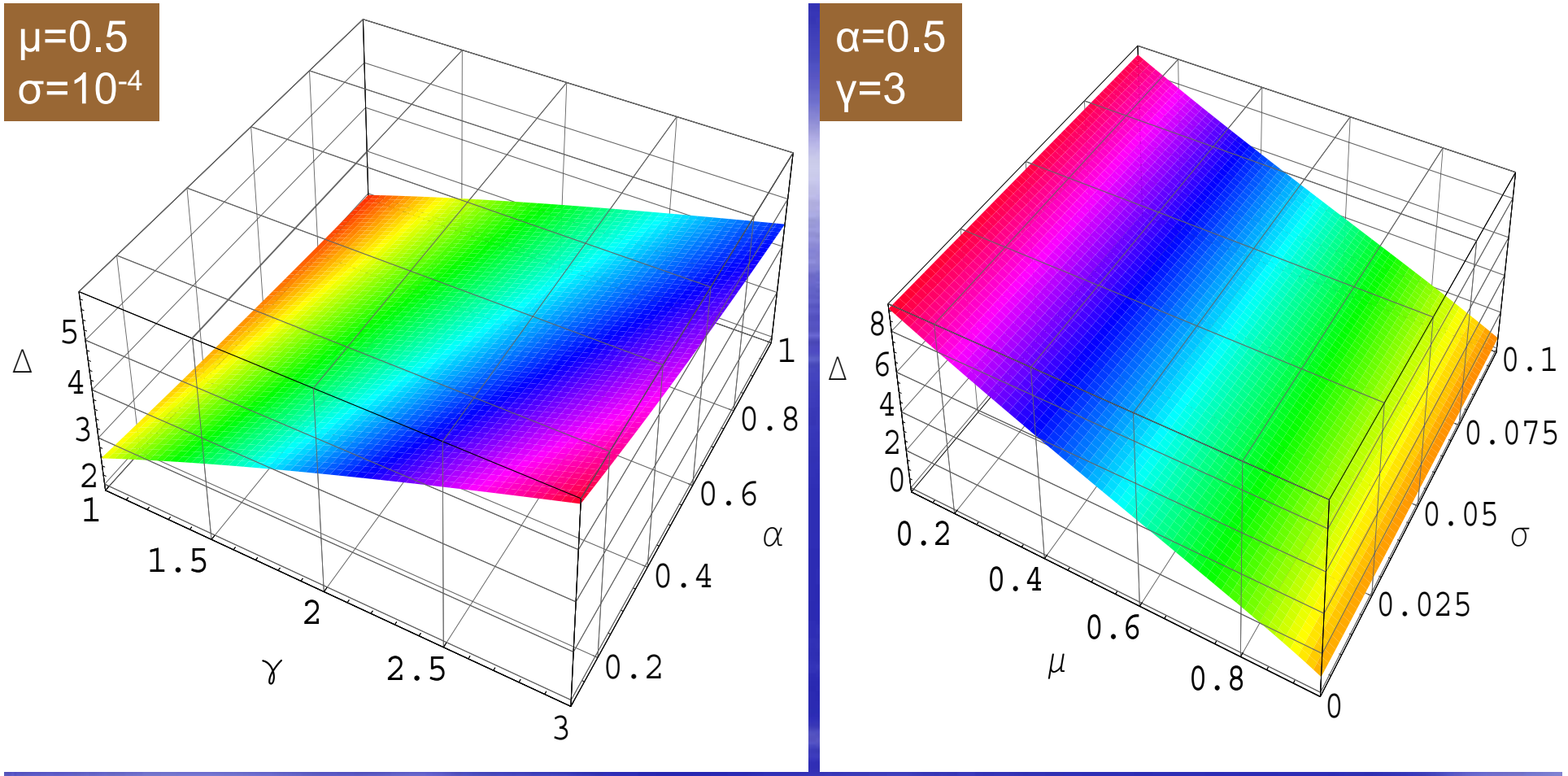


μ=0.5
σ=10^{-4}



α=0.5
γ=3





What do these figure imply?

- DA SW potential is always negative.
- γ has no role to change the polarity of DA SW.
- γ significantly modifies the basic properties of DA SW.
- α, μ, σ significantly modify the basic properties of DA SW.

- **Arbitrary Amplitude:**

SPA [Sagdeev 1966] reduces (1) - (6) to an energy integral:

$$\frac{1}{2}\left(\frac{d\Psi}{d\xi}\right)^2 + V(\Psi) = 0,$$

where ξ=z - Mt and V(ψ) reads [Mamun 2008]:

$$V(\Psi) = \frac{\alpha+\mu}{\alpha(1-\mu)} + M^2 + \sigma - \frac{\mu}{\alpha(1-\mu)}\left(1+\frac{2}{3}\alpha\Psi\right)^{\frac{3}{2}} - \frac{1}{1-\mu}\left(1-\frac{2}{3}\Psi\right)^{\frac{3}{2}}$$
$$-\frac{M}{\sqrt{2}}M_\sigma\left(\Psi_1+\sqrt{\Psi_1^2-12\sigma\frac{M^2}{M_\sigma^4}}\right)^{\frac{1}{2}} - 2^{\frac{3}{2}}\sigma\frac{M^3}{M_\sigma^3}\left(\Psi_1+\sqrt{\Psi_1^2-12\sigma\frac{M^2}{M_\sigma^4}}\right)^{-\frac{3}{2}},$$

$$\Psi_1 = 1 + 2\Psi/M_\sigma^2, \qquad M_\sigma = (M^2+3\sigma)^{1/2}.$$

we numerically analyzed V(ψ), and obtained the same results that we mentioned in two previous slides.

❑ Summary

- The charged dust do not only modify the existing plasma waves, but also introduce a number of new eigen modes. The most important ones, which are experimentally observed, are DIA and DA waves. The basic features of the DIA and DA solitary structures that may form in an adiabatic dusty plasma, are the focussing points of this talk.

- When $\alpha<(1+\gamma_e)/3$, only small amplitude +ve DIA SWs exist. On the other hand, when $\alpha>(1+\gamma_e)/3$ with $\alpha<1$, small amplitude -ve as well as large amplitude +ve DIA SWs exist,. This means that for isothermal electrons ($\gamma_e=1$) we can have -ve DIA SWs for $\alpha>2/3$, but for adiabatic electrons ($\gamma_e\neq1$) we can have -ve DIA SWs only for $\gamma_e<2$ since we cannot take $\alpha \geq 1$. We note that for $\gamma_e=3$ -ve DIA SWs cannot exist.

- **The adiabaticity of electrons does not only play a sifgnificant role to change the polarity of the DIA solitary waves, but also modify the other features (amplitude and with) of the DIA solitary waves.**

- **The DA SWs exist only for negative potential, but do not exist for positive potential.**

- **The adiabaticity of electrons and ions has no any role to change the polarity of the DA SWs. However, it significantly modifies the basic features of the DA SW. As γ is increased, the width increase, but the amplitude decreases.**

- **The other physical parameters, viz. α and μ, do also significantly modify the basic properties of the DA SWs. The amplitude increases with the increase of α and μ, but the width decreases with the increases of α and μ.**

- Because of time limit, I confined my talk to an unmagnetized DP. However, a number of investigations [Mamun 1998; Kotsarenko et al 1998a] on DA SWs in a magnetized DP have been made: (i) external magnetic field makes ES-SWs more spiky and (ii) ES-SWs becomes unstable: multi-dimensional instability [Mamun 1998b, 1998c].

- Very recently we have also investigated the effects of dust of opposite polarity on DA SWs: The dynamics of positive dust (in addition to negative ones) may allow negative and positive SWs to coexist [Mamun & Shukla 2002, Mamun 2008a, 2008b].

- The physics of nonlinear waves that we have discussed must play a significant role in understanding the properties of localized ES structures in space & laboratory dusty plasmas.

THANK YOU ALL