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Summer School in Cosmology

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Cosmology & Particle Physics
Lecture 1 & 2

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Lecture 1.

DARK MATTER.

* THE simplest – and hence most appealing – SCENARIO: WIMPs (WEAKLY INTERACTING MASSIVE PARTICLES).

Assumptions

(1) THERE EXIST NEW STABLE, HEAVY, ELECTRICALLY NEUTRAL PARTICLES X

(2) X'S CAN BE CREATED AND DESTROYED IN PAIRS ONLY

$$X + X \leftrightarrow \text{STANDARD MODEL PARTICLES}$$

(ASSUMING X = ITS ANTIPARTICLE)

(3) TEMPERATURE IN THE UNIVERSE WAS HIGH, $T_{MAX} \gtrsim M_X$

NB: (2a) If $X \neq \bar{X}$, THEN PAIR CREATION AND ANNIHILATION

$$X + \bar{X} \leftrightarrow \text{SM PARTICES}$$

+ NO ASYMMETRY BUILT IN,

$$n_X = n_{\bar{X}}$$

PARAMETERS:

M_X

ANNIHILATION CROSS SECTION

$\sigma_a(v)$ AT NON-RELATIVISTIC

VELOCITY v.

$\Downarrow$

CALCULATE PRENT MASS DENSITY,

REQUIRE $\Omega_X = \Omega_{DM} \simeq 0.2$

Need TO *know*: HUBBLE EXPANSION RATE

AT TEMPERATURE T

- FRIEDMANN EQN.

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi}{3 M_{pl}^2}\,\rho$$

- ENERGY DENSITY AT TEMPERATURE T:

STEFAN - BOLTZMANN LAW

$$\rho = \frac{\pi^2}{30}\, g_* T^4$$

g_* = eFF. NUMBER OF RELATIVISTIC DEGREES OF FREEDOM

STANDARD MODEL AT $T \sim 100$ GeV $\Rightarrow g_* \simeq 100$

$\Downarrow$

EXPANSION RATE

$$H^2 = \frac{8\pi}{3 M_{Pl}^2} \cdot \frac{\pi^2}{30} g_* T^4$$

$\Downarrow$

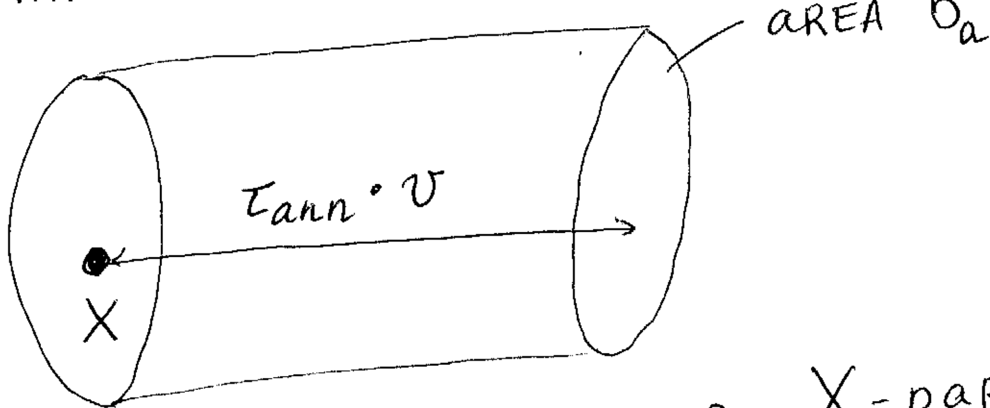
$$\boxed{H = \frac{T^2}{M_{Pl}^*}} \quad ; \quad M_{Pl}^* = \frac{M_{Pl}}{1.66 \sqrt{g_*}}$$

$\Downarrow$

$M_{Pl}^* \simeq 10^{18}$ GeV

at $T \simeq 100$ GeV

Compare this to <u>ANNIHILATION RATE</u> OF X-PARTICLES



ANNIHILATION: meet another X-particle in volume $\tau_{ann} \cdot v \cdot \sigma_a$

$\Downarrow$

$$\tau_{ann} \cdot v \cdot \sigma_a \cdot n_X \simeq 1$$

$$\boxed{\Gamma_{ann} = \frac{1}{\tau_{ann}} \simeq n_X \langle \sigma_a v \rangle}$$

THERMAL AVERAGE

FREEZE OUT:

- AT HIGH TEMPERATURES $\Gamma_{ann} \gg H \Leftrightarrow$ MANY CREATIONS/ANNIHILATIONS IN HUBBLE TIME
- CREATION/ANNIHILATION STOPS WHEN

$$\boxed{\Gamma_{ann} \simeq H} \Leftarrow \text{FREEZE-OUT CONDITION}$$

$\Downarrow$

EQUATION FOR FREEZE-OUT TEMPERATURE

$$n_X(T) \langle \sigma_a v \rangle_T \simeq H(T)$$

EQUILIBRIUM DENSITY OF X-PARTICLES.

MAXWELL-BOLTZMANN LAW (ASSUMING $T < M_X$)

NUMBER OF SPIN D.O.F.

$$n_X = g_X \left(\frac{M_X T}{2\pi} \right)^{3/2} e^{-\frac{M_X}{T}}$$

[OBTAINED BY INTEGRATING MAXWELL-BOLTZMANN DISTRIBUTION

$$f(\vec{p}) = \frac{1}{(2\pi)^3} e^{-E(\vec{p})/T}$$

$$E(\vec{p}) = M_X + \frac{p^2}{2M_X}$$

NB: ZERO CHEMICAL POTENTIAL, SINCE PAIR-ANNIHILATION IN EQUILIBRIUM]

FREEZE-OUT EQN.

$$g_X \left(\frac{M_X T_f}{2\pi}\right)^{3/2} e^{-\frac{M_X}{T_f}} \langle \sigma_a \cdot v \rangle \simeq \frac{T_f^2}{M_{pl}^*}$$

$\Uparrow$ Γ_{ann} $\qquad$ $\Uparrow$ H $\Uparrow$ $\boxed{\text{SMALL}}$

MUST BE SMALL

M_X SOMEWHAT LARGER THAN T_f
(BUT NOT MUCH LARGER)

$\Downarrow$

$$\frac{M_X}{T_f} = \ln \frac{g_X M_X^{3/2} M_{pl}^* \langle \sigma_a \cdot v \rangle}{(2\pi)^{3/2} \, T_f^{1/2}}$$

$$= \ln \frac{g_X M_X M_{pl}^* \langle \sigma_a v \rangle}{(2\pi)^{3/2}} \equiv \text{"log"}$$

$$\simeq 30 \quad \text{FOR} \quad M_X \simeq 100 \text{ GeV}, \quad \langle \sigma_a v \rangle \simeq 10^{-8} \text{ GeV}^{-2}$$

ANNIHILATION/CREATION STOPS WHEN TEMPERATURE IS QUITE SMALLER THAN $M_X \Rightarrow$ X-PARTICLE DENSITY IS SMALL DUE TO $e^{-\frac{M_X}{T}}$.

NUMBER DENSITY AT FREEZE-OUT

$$n_X \langle \sigma_a \cdot v \rangle \simeq \frac{T_f^2}{M_{Pl}^*}$$

$$n_X(T_f) \simeq \frac{T_f^2}{M_{Pl}^* \langle \sigma_a v \rangle}$$

AFTER FREEZE-OUT, n_X decreases DUE TO EXPANSION OF THE UNIVERSE,

$$n_X \propto \frac{1}{a^3}$$

ROUGHLY SPEAKING, $n_X \propto T^3$

$$n_X(\text{TODAY}) \sim \underbrace{T_0}_{2.73K}{}^3 \; \frac{n_X(T_f)}{T_f^3}$$ (REFINE LATER)

$$n_X(\text{TODAY}) \sim T_0^3 \; \frac{1}{M_{Pl}^* \, T_f \langle \sigma_a v \rangle}$$

MASS DENSITY TODAY

$$\rho_X = M_X \cdot n_X \sim T_0^3 \frac{1}{M_{Pl}^* \langle \sigma_a v \rangle} \underbrace{\frac{M_X}{T_f}}_{\text{"log"}}$$

$$\boxed{\Omega_X = \frac{\rho_X}{\rho_c} \sim \frac{T_0^3}{M_{Pl}^* \rho_c} \; \frac{1}{\langle \sigma_a v \rangle} \cdot \text{"log"}}$$

REFINED CALCULATION

T does NOT DECREASE EXACTLY LIKE a^{-1}

WHAT STAYS CONSTANT IS ENTROPY IN COMOVING VOLUME

$$s \cdot a^3 = \text{CONST}$$

$$s = \# \cdot g_* \cdot T^3 \quad ; \quad \frac{n_X}{s} = \text{const}$$

THIS GIVES

$$n_{X\,(\text{TODAY})} = \frac{g_{*,\text{eff}}\,(\text{TODAY})}{g_*(T_f)} \cdot \frac{T_0^3}{T_f^3} \cdot n_X(T_f)$$

EXTRA FACTOR IN CROSS SECTION:

$$\langle \sigma_a v \rangle = \frac{g_{*,\text{eff}}\,(\text{TODAY})}{g_*(T_f)} \; \frac{T_0^3}{M_{Pl}^* \rho_c \Omega_X} \cdot \text{"log"}$$

$g_{*,\text{eff}}\,(\text{TODAY}) = \frac{43}{11}$ (NEUTRINO INCLUDED AS IF MASSLESS)

$g_*(T_f) \simeq 100$

$$\Downarrow$$

$$\boxed{\langle \sigma v \rangle = 0.2 \cdot 10^{-8}\ \text{GeV}^{-2} = 1 \cdot 10^{-36}\ \text{cm}^2}$$

STILL WEAK SCALE.

IF THIS IS RIGHT MECHANISM,
WIMPs MUST BE CREATED BY LHC:

* PRODUCTION CROSS SECTION ~ ANNIHILATION σ_a
LARGE

* $M_X \lesssim$ TeV (OTHERWISE CROSS SECTION SMALL, $\sigma_a \lesssim \frac{1}{M_X^2}$)

SUPERSYMMETRY: NEUTRALINO

WARNING: σ_a OFTEN TOO LOW Fig.

NB: THIS MECHANISM, LIKE MANY OTHERS, MAY BE FALSIFIED BY COSMOLOGICAL OBSERVATIONS.

IT WORKS AT HOT STAGE

$\Downarrow$

$\frac{n_X}{s}$ IS THE SAME EVERYWHERE IN THE UNIVERSE

$\Downarrow$

NO ADMIXTURE OF CDM ISOCURVATORE MODE

EVEN SMALL ADMIXTURE OF CDM ISOCURVATURE MODE WOULD FALSIFY THIS MECHANISM

WEAKLY (LOG) DEPENDS ON M_X

<u>STRONGLY</u> DEPENDS ON ANNIHILATION CROSS SECTION

$$\langle \sigma_a \cdot v \rangle \sim \frac{T_0^3}{M_{Pl}^* \rho_c \Omega_X} \cdot \text{"log"}$$

NUMERICS: $T_0 = 2.73\,K = \frac{1}{0.08\,cm}$

$$M_{Pl}^* \simeq 10^{18}\ GeV$$

$$\rho_c = 5 \cdot 10^{-6}\ \frac{GeV}{cm^3}$$

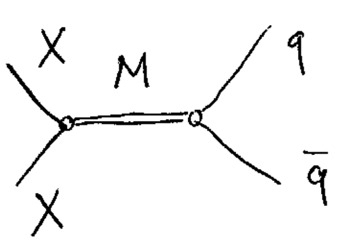
$$\Omega_X = 0.2$$

$$\text{"log"} \simeq 30$$

$$\Downarrow$$

$$\boxed{\begin{aligned}\langle \sigma_a \cdot v \rangle &\simeq 6 \cdot 10^{-8}\ GeV^{-2} \\ &\simeq 2 \cdot 10^{-35}\ cm^2\end{aligned}}$$

WEAK SCALE CROSS SECTION !



: $\sigma \sim \frac{\alpha^2}{M^2} \Rightarrow M \simeq 100\ GeV$ FOR $\alpha \simeq \frac{1}{30}$

LIKE IN SM

* Axions as Cold Dark Matter.

An alternative to WIMPs.

- Symmetry breaking and Nambu-Goldstone bosons

Consider scalar theory,
$\varphi(x)$: complex scalar field

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\varphi^*\partial_\nu\varphi - V(\varphi^*\varphi)$$

Invariant under global symmetry

$$\varphi \to e^{i\alpha}\varphi, \quad \varphi^* \to e^{-i\alpha}\varphi^*$$

Suppose that potential $V(\varphi^*\varphi)$ is such that its minimum occurs at

$$\varphi^*\varphi = f^2 \neq 0$$

As an example

$$V = \frac{\lambda}{4}(\varphi^*\varphi - f^2)^2$$

Minimum energy state (vacuum) not unique:

$$\varphi_{VAC} = e^{i\alpha} f$$

α is independent of x.

Choose one vacuum and construct perturbation theory about it

CHOOSE

$$\varphi_{VAC} = f$$

<u>NB</u>: SPONTANEOUS SYMMETRY BREAKING: ACTION INVARIANT UNDER SYMMETRY, VACUUM IS NOT

PERTURBATIONS: PARAMETRIZE

$$\varphi = \rho(x)\, e^{i\theta(x)} \quad ; \quad \rho = f + h(x)$$

real

$\theta(x)$ SMALL

$$\mathcal{L} = \frac{1}{2} \partial_\mu \rho\, \partial^\mu \rho + \frac{1}{2} \rho^2\, \partial_\mu \theta \cdot \partial^\mu \theta - V(\rho)$$

Expand in $h(x)$ $\Longrightarrow$

$$\mathcal{L} = \frac{1}{2} \partial_\mu h\, \partial^\mu h + \frac{1}{2} f^2\, \partial_\mu \theta \cdot \partial^\mu \theta - \frac{1}{2} m_h^2 \cdot h^2 + \text{interaction}$$

$$m_h^2 = \left.\frac{\partial^2 V}{\partial \rho^2}\right|_{\rho = f}$$

$h(x)$: massive field

$\theta(x)$: <u>MASSLESS</u> field (Nambu - GOLDSTONE).

GOLDSTONE THEOREM

ONCE GLOBAL SYMMETRY IS BROKEN, THERE ARE ALWAYS MASSLESS FIELD(S)

EASY TO UNDERSTAND:

SYMMETRY OF ACTION

$$\varphi \to e^{i\alpha}\varphi \quad \Leftrightarrow \quad \rho e^{i\theta} \to \rho e^{i(\theta+\alpha)}$$

$$\theta \to \theta + \alpha$$

$$\Downarrow$$

NO TERMS WITHOUT DERIVATIVES IN ACTION

$$\Downarrow$$

NO mass FOR θ, $\frac{m^2}{2}\theta^2$ FORBIDDEN BY SYMMETRY

AXION = ALMOST NAMBU - GOLDSTONE BOSON

WHY NEED THE AXION?

CP - PROBLEM OF STRONG INTERACTIONS

QCD : - MASS TERM FOR QUARKS

$$\mathcal{L}_m = \sum_i \bar{q}_i m_i e^{i\alpha\gamma_5} q_i$$

$i = u, d, s, c, b, t$

α = common phase, ALLOWED BY ALL GAUGE SYMMETRIES.

Naively : PHASE α IRRELEVANT, UNDO BY FIELD REDEFINITION

$$q_i \to e^{-i\frac{\alpha}{2}\gamma^5} q_i$$

CANNOT DO THAT IN QUANTUM THEORY!

$\alpha \neq 0 \iff$ CP - VIOLATION.

- ANOTHER PARAMETER: Θ_{QCD}

$$\mathcal{L}_\theta = \frac{\alpha_s}{8\pi} \Theta_{QCD} \tilde{G}^a_{\mu\nu} G^{a\mu\nu}$$

$G^{a\mu\nu}$ ← GLUON FIELD STRENGTH

$\tilde{G}^a_{\mu\nu}$ ↑ $\frac{1}{2}\varepsilon_{\mu\nu\lambda\rho} G^{a\lambda\rho}$, DUAL TENSOR

Naively : Θ_{QCD} IRRELEVANT, SINCE $\tilde{G}^a_{\mu\nu} G^{a\mu\nu}$ = TOTAL DIVERGENCE

NOT TRUE IN QUANTUM THEORY!

Θ_{QCD} ALSO VIOLATES CP.

IN FACT, THE ONLY RELEVANT PARAMETER IS

$$\Theta_{eff} = \Theta_{QCD} + N_f \cdot \alpha$$

N_f ↑ NUMBER OF FLAVORS, $N_f = 6$.

EXPERIMENTAL CONSTRAINT FROM ELECTRIC DIPOLE MOMENT OF NEUTRON

$$|\Theta_{eff}| < 0.3 \cdot 10^{-9}$$

PROBLEM: WHY Θ_{eff} IS ESSENTIALLY ZERO?

POSSIBLE SOLUTION: PROMOTE PHASE α (OR Θ_{QCD}) TO A FIELD. ARRANGE THAT ITS VACUUM EXPECTATION VALUE IS AT CP-CONSERVING POINT.

$$\Downarrow$$

QUARK MASS TERM

Θ_{eff} IN FACT

$$\sum_i \bar{q}_i m_i e^{i\Theta(x)\gamma^5} \cdot e^{i\alpha\gamma^5} q_i$$

IF NOT FOR QCD EFFECTS, $\Theta(x)$ SHOULD BE ARBITRARY CONSTANT IN VACUO

$$\Downarrow$$

SYMMETRY $\quad \Theta(x) \to \Theta(x) + \beta$

$\Theta(x)$ = Nambu - GOLDSTONE FIELD OF A $U(1)_{PQ}$ -SYMMETRY (PECCEI-QUINN)

IN QCD-VACUUM

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \neq 0$$

(negative)

Related to m_π and f_π

CHIRAL SYMMETRY BROKEN

[NB: ALMOST NAMBU-GOLDSTON BOSONS ARE $\pi^\pm, \pi^0$]

$$\Downarrow$$

POTENTIAL FOR AXION FIELD

$$V_\theta = + m_{u,d} \langle \bar{q}q \rangle \cos\left[\theta(x) + \theta_{eff}\right]$$

$$= -\frac{1}{4} m_\pi^2 f_\pi^2 \cos \bar{\theta}(x)$$

$$\bar{\theta}(x) = \theta(x) + \theta_{eff}$$

$m_\pi = 135$ MeV

$f_\pi = 93$ MeV

MINIMUM AT $\bar{\theta} = 0 \Rightarrow \bar{q} m e^{i\bar{\theta}\gamma^5} q = \bar{q} m q$

NO CP-VIOLATION

AXION MASS

$$\mathcal{L}_{\bar{\theta}} = \frac{1}{2} f_{PQ}^2 (\partial_\mu \bar{\theta})^2 - \frac{1}{8} m_\pi^2 f_\pi^2 \bar{\theta}^2$$

INTRODUCE $a(x) = f_{PQ} \cdot \bar{\theta}(x)$

[f_{PQ} = VEV OF PECCEI-QUINN FIELD $\varphi(x) = \rho(x) e^{i\theta(x)}$]

$$\Downarrow$$

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2} m_a^2 \cdot a^2$$

$$\boxed{m_a^2 = \frac{1}{4} \frac{m_\pi^2 f_\pi^2}{f_{PQ}^2}}$$

f_{PQ}: THE ONLY FREE PARAMETER, PQ-SYMMETRY BREAKING SCALE.

WE'LL SEE THAT GOOD FOR DARK MATTER

$$\boxed{f_{PQ} \sim (10^{12} - 10^{13})\ \text{GeV}}$$

$$\Downarrow$$

$$m_a \sim \frac{f_\pi m_\pi}{f_{PQ}} \sim \frac{0.1 \cdot 0.1}{10^{12-13}}\ \text{GeV} \sim 10^{-14} - 10^{-15}\ \text{GeV}$$

$$\boxed{m_a \sim (10^{-5} - 10^{-6})\ \text{eV}}$$

HOW CAN THIS BE <u>COLD</u> DARK MATTER?

$m_a < T_0$!?

PRODUCTION MECHANISM: (THERE ARE OTHER POSSIBILITIES)

MISALIGNMENT

HIGH TEMPERATURES: QCD WEAKLY INTERACTING ($\alpha_s(T)$ small, ASYMPTOTIC FREEDOM)

$$\Downarrow$$

CHIRAL SYMMETRY RESTORED

$$\Downarrow$$

$$V(\bar{\theta}) = 0$$

INITIAL VALUE OF $\bar{\theta}$ IN OUR UNIVERSE CAN BE ANYWHERE BETWEEN 0 AND 2π

(REMEMBER, $\bar{\theta}$ is PHASE)

$$a(x) = f_{PQ} \cdot \bar{\theta}(x)$$

$$\Downarrow$$

INITIAL VALUE

$a_i = f_{PQ} \cdot \bar{\theta}_i$ ANYWHERE FROM 0 TO $2\pi f_{PQ}$

AS $V(\bar{\theta})$ BUILDS UP (AT $T \sim T_{QCD} \sim 200\,\text{MeV}$)
AXION FIELD STARTS TO OSCILLATE AROUND 0.

OSCILLATING FIELD = COLLECTION OF SCALAR PARTICLES AT REST

EQN. FOR HOMOGENEOUS AXION FIELD $a(t)$
(APPROXIMATING $V(a) = \frac{1}{2} m_a^2 \cdot a^2$)

$$\ddot{a} + 3H\dot{a} + m_a^2 \cdot a = 0$$

OSCILLATIONS START AT TEMPERATURE T_{osc}
SUCH THAT

$$H(T_{osc}) \simeq m_a(T_{osc}).$$

ENERGY DENSITY AT BEGINNING OF OSCILLATIONS

$$\rho = \frac{1}{2} m_a^2(T_{osc}) \cdot a_i^2$$

$$\Downarrow$$

NUMBER DENSITY OF AXIONS AT BEGINNING OF OSCILLATIONS

$$n_a = \frac{\rho(T_{osc})}{m_a(T_{osc})} \sim m_a(T_{osc}) \cdot a_i^2 \sim H(T_{osc}) \cdot a_i^2$$

THEN NUMBER DENSITY DECREASES AS $\frac{1}{a^3} \approx T^3$ (AGAIN, BETTER USE ENTROPY CONSERVATION)

Today

$$n_a(\text{TODAY}) \simeq \frac{T_0^3}{T_{osc}^3} \, n_a(T_{osc}) \simeq \frac{T_0^3}{T_{osc}^3} \, H(T_{osc}) \cdot a_i^2$$

MASS DENSITY

$$\rho_a(\text{TODAY}) = m_a \cdot n_a \simeq \frac{T_0^3 \, m_a}{T_{osc}^3} \, H(T_{osc}) \, a_i^2$$

RECALL $\quad a_i = f_{PQ} \, \bar{\theta}_i \simeq \frac{f_\pi m_\pi}{2 m_a} \bar{\theta}_i$

$$H(T_{osc}) = \frac{T_{osc}^2}{M_{Pl}^*}$$

$$\Omega_a \simeq \frac{T_0^3}{T_{osc} \cdot M_{Pl}^* \cdot \rho_c} \cdot \frac{f_\pi^2 m_\pi^2}{4 m_a} \cdot \bar{\theta}_i^2 \quad \times \quad \underbrace{\frac{g_*(\text{TODAY})}{g_*(T_{osc})}}_{= \, 0.1}$$

TAKE $\quad T_{osc} \simeq T_{QCD} \simeq 200 \text{ MeV}$

$$T_0 = \frac{1}{0.1 \text{ cm}} \; ; \; M_{Pl}^* \simeq 10^{18} \text{ GeV}$$

$$\Omega_a \simeq 0.1 \, \frac{1}{10^{-3} \text{cm}^3} \cdot \frac{1}{0.2 \text{ GeV} \cdot 10^{18} \text{ GeV} \cdot 5 \cdot 10^{-6} \frac{\text{GeV}}{\text{cm}^3}} \cdot \frac{10^{-4} \text{ GeV}}{4 m_a} \cdot \bar{\theta}_i^2$$

$$\simeq \frac{2 \cdot 10^{-15} \text{ GeV}}{m_a} \, \bar{\theta}_i^2$$

$$\Omega_a \simeq \frac{2\cdot 10^{-6}\,\text{eV}}{m_a}\,\bar{\theta}_i^2$$

MORE ACCURATE ESTIMATE: FROM REALISTIC T-DEPENDEN AXION POTENTIAL

$$\Omega_a \simeq 0.2 \cdot \frac{4\cdot 10^{-6}\,\text{eV}}{m_a} \cdot \bar{\theta}_i^2$$

$$\bar{\theta}_i \in (0, \pi) \implies \text{NEED } m_a = 10^{-5} - 10^{-6}\ \text{eV}$$

NB: AXIONS AT REST $\Rightarrow$ COLD DARK MATTER, NO VELOCITY DISPERSION AT THE BEGINNING.

SEARCH: AXION DECAY

$$a \to \gamma\gamma$$

θ

$$\mathcal{L}_{a\gamma\gamma} = C\,\frac{\alpha}{8\pi}\,\frac{a}{f_{PQ}}\,F_{\mu\nu}\tilde{F}^{\mu\nu}$$

MODEL-DEPENDENT, OF ORDER 1 OR SOMEWHAT SMALLER

LIFETIME

$$\Gamma_a \sim C^2\left(\frac{\alpha}{8\pi}\right)^2 \frac{1}{f_{PQ}^2}\,\frac{m_a^3}{4\pi}$$

$$\tau_a \sim \frac{1}{C^2}\left(\frac{8\pi}{\alpha}\right)^2 \frac{f_\pi^2 m_\pi^2}{4 m_a^5}\cdot 4\pi \sim \frac{1}{C^2}\left(\frac{10^{-5}\,\text{eV}}{m_a}\right)^5 \cdot 10^{49}\ \text{s}$$

10^{42} yrs

NUMBER DENSITY LOCALLY

$$n \simeq \frac{\rho_{DM}(\text{LOCAL})}{m_a} \simeq \frac{0.3\ \text{GeV}/\text{cm}^3}{10^{-5}\ \text{eV}} \simeq 10^{14}\ \frac{1}{\text{cm}^3} \simeq 10^{20}\ \frac{1}{\text{m}^3}$$

HOPELESS. NOT A SINGLE AXION DECAYS IN CUBIC METER IN LIFETIME OF UNIVERSE.

AXION - PHOTON CONVERSION IN STRONG BACKGROUND MAGNETIC FIELD.

INTERESTING POSSIBILITY:
AXION ISOCURVATURE PERTURBATIONS GENERATED AT INFLATION

RECALL

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)^2 - \frac{\lambda}{4}(\rho^2 - f_{PQ}^2)^2 + \frac{1}{2}\rho^2(\partial_\mu \bar{\theta})^2$$

ASSUME THAT AT THE END OF INFLATION (SOME 60 e-foldings BEFORE)

$$\Lambda_{QCD} \ll H \ll f_{PQ} \cdot \sqrt{\lambda}$$

THEN ρ = HEAVY FIELD, $m_\rho \sim \sqrt{\lambda} f_{PQ} \gg H$

$$\Downarrow$$

ρ sits AT minimum of potential, $\rho = f_{PQ}$

θ is light field (massless)

EVERY LIGHT FIELD BECOMES INHOMOGENEOUS DUE TO AMPLIFICATION OF VACUUM FLUCTUATIONS DUE TO INFLATION

CANONICALLY NORMALIZED AXION FIELD

$$a(x) = f_{PQ}\,\bar{\theta}(x)$$

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2$$

FLUCTUATIONS OF a HAVE AMPLITUDE

$$\langle \delta a^2 \rangle \equiv (\delta a)^2 = \frac{H^2}{(2\pi)^2}$$

Recall $\langle a \rangle \equiv a_i = f_{PQ}\,\bar{\theta}_i$

H: HUBBLE PARAMETER SOME 60 e-foldings BEFORE END OF INFLATION

STAYS TIME-INDEPENDENT UNTIL QCD EPOCH FOR ALL MODES OF INTEREST (THESE ARE SUPERHORIZON AT $T \sim \Lambda_{QCD}$)

$\Downarrow$

AXION OSCILLATIONS START FROM DIFFERENT INITIAL VALUES AT DIFFERENT PLACES IN THE UNIVERSE,

$$\frac{\delta a}{\langle a \rangle} \simeq \frac{H}{2\pi \cdot \bar{\theta}_i f_{PQ}}$$

AXION ENERGY DENSITY INHOMOGENEOUS

$\rho_a \simeq m^2 a^2$ $\qquad \dfrac{\delta\rho_a}{\rho_a} \simeq \dfrac{2\,\delta a}{a} \sim \dfrac{H}{\pi\,\bar{\theta}_i\, f_{PQ}}$

ALMOST FLAT SPECTRUM.

THESE ARE <u>ISOCURVATURE PERTURBATIONS</u>:

HOT PLASMA DOES NOT KNOW ABOUT THEM.

MUST BE SMALL: ISOCURVATURE MODES AT LEAST 10 TIMES SMALLER THAN ADIABATIC (WMAP)

$$\Downarrow$$

$$\frac{\delta\rho_a}{\rho_a} \lesssim 10^{-5} \Rightarrow H \lesssim \pi\,\bar{\theta}_i\, f_{PQ} \cdot 10^{-5}$$

Low INFLATION SCALE: $f_{PQ} \sim 10^{12}$ GeV

$$\Downarrow$$

$$H \lesssim 10^{8} \text{ GeV}$$

$$\Downarrow$$

Inflation energy scale

$$M_{infl} \equiv \rho_{infl}^{1/4} = \left(\frac{3}{8\pi} M_{Pl}^2 H^2\right)^{1/4} \lesssim 10^{13} \text{ GeV}$$

NO TENSOR PERTURBATIONS (GRAVITY WAVES).