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Gravitational Waves - Lecture 1

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Gravitational Waves

The dynamics of spacetime as the foundation for a new way to observe astronomical phenomena

Outline of these lectures

I. Basic concepts: Nature of gravitational waves; how they are generated, how they are measured.

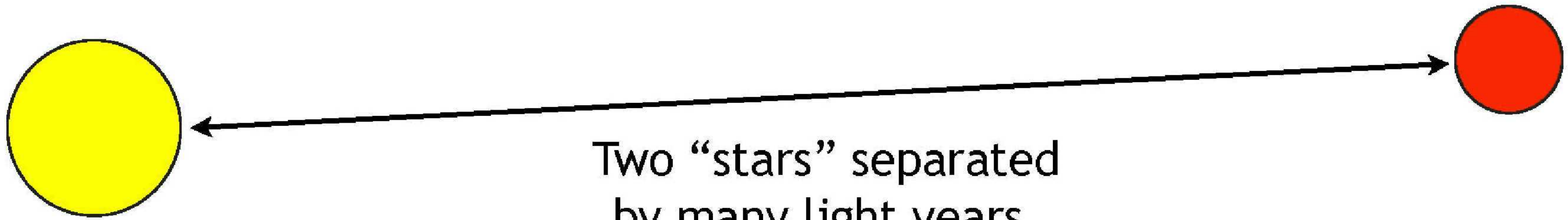
II. Details: Overview of general relativity; how gravitational waves arise in general relativity.

III. Measurement: How gravitational waves imprint themselves on matter; how we can exploit this to measure these waves.

IV. Astrophysics: What objects in nature are strong generators of gravitational waves. How to exploit these measurements to learn about the universe.

What are gravitational waves?

Newton's gravity has a ***huge*** shortcoming: Information about gravity is transmitted *instantaneously*.



Two "stars" separated by many light years.

What are gravitational waves?

Newton's gravity has a ***huge*** shortcoming: Information about gravity is transmitted *instantaneously*.

According to Newton, red star *immediately* "knows" about yellow star's new position!

Contradicts rule that information can travel *no faster* than the speed of light.

Review: Electromagnetic radiation

Start with Maxwell's equations — a complete *linear* classical, relativistic field theory.

Goal: Review *concepts* and *methods* used to describe radiation in classical field theory.
Many of these techniques carry over to gravity: Analogy to electromagnetic waves very useful!

CAUTION: Analogy *must* be justified rigorously! General relativity is *non-linear*; naive use of electromagnetic intuition can be misleading. Will explore more deeply in lecture #2.

Field equations

Start with the usual field equations:

Standard formulation in cgs units:

$$
\begin{aligned}
\vec{\nabla} \cdot \vec{E} &= 4\pi\rho \\
\vec{\nabla} \cdot \vec{B} &= 0 \\
\vec{\nabla} \times \vec{E} &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \\
\vec{\nabla} \times \vec{B} &= \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}
\end{aligned}
$$

Field equations

Useful to rewrite this in tensorial form — notation that carries directly over to gravity.

$$F_{\mu\nu} \doteq \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

Maxwell's equations become

$$\partial_\mu F^{\mu\nu} = 4\pi J^\mu$$

$$\partial_\lambda F_{\mu\nu} + \partial_\nu F_{\lambda\mu} + \partial_\mu F_{\nu\lambda} = 0$$

Notation introduced here ...

Einstein summation convention: Adjacent indices in "upstairs/downstairs" positions are summed.

$$\sum_{\beta=0}^{3} A^{\alpha\beta} B_{\beta\gamma} = A^{\alpha\beta} B_{\beta\gamma}$$

Raising and lowering of indices using the metric of special relativity:

$$F^{\alpha\beta} = \eta^{\alpha\mu}\eta^{\beta\nu}F_{\mu\nu} \qquad \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$$

$$\eta^{\alpha\gamma}\eta_{\gamma\beta} = \delta^{\alpha}{}_{\beta}$$ ($\delta^{\alpha}{}_{\beta}$ = identity)

$$\partial_\mu = \frac{\partial}{\partial x^\mu}$$ $c = 1$ (choice of units)

Change from fields to potentials

Since $F_{\mu\nu}$ is antisymmetric ($F_{\nu\mu} = -F_{\mu\nu}$), it can be constructed from a potential A_μ,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Suppose that we adjust potential using a *gauge transformation* with a scalar field Λ:

$$A_\mu \to A_\mu - \partial_\mu \Lambda$$

This leaves $F_{\mu\nu}$ unchanged: The fields are *invariant* to the gauge choice.

Using gauge freedom

This gives us freedom to choose the gauge Λ in a manner that is as convenient as possible.

Our choice: adjust Λ so that A_μ is *divergence-free* ("Lorenz gauge").

$$\Box\Lambda = \partial^\mu A_\mu^{\rm old} \longrightarrow \partial^\mu A_\mu^{\rm new} = 0$$

Can now write Maxwell's equation for the potential in a particularly simple form:

$$\Box A_\mu = -4\pi J_\mu$$

where $\Box = \partial^\mu \partial_\mu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Solution via *Green's function*

Method of Green's functions: Begin with an equation of the form

$$\mathcal{D} f(\vec{x}, t) = s(\vec{x}, t)$$

Linear differential operator → $\mathcal{D}$

Field → $f(\vec{x}, t)$

A source → $s(\vec{x}, t)$

Replace the source with a Dirac delta function. Assert that some function *G* solves this:

$$\mathcal{D} G(\vec{x}, t; \vec{x}', t') = \delta(\vec{x} - \vec{x}')\delta(t - t')$$

Solution via *Green's function*

Notation: $(t, \vec{x})$ is the *field point*, event where we evaluate *f*.

$(t', \vec{x}')$ is the *source point*, event where we evaluate *s*.

Trivially, we have

$$s(\vec{x}, t) = \int dt' \int d^3x' s(\vec{x}', t')\delta(\vec{x} - \vec{x}')\delta(t - t')$$

By linearity, we therefore deduce

$$f(\vec{x}, t) = \int dt' \int d^3x' s(\vec{x}', t')G(\vec{x}', t'; \vec{x}, t)$$

Solution via *Green's function*

Green's function for wave operator well known (e.g., Jackson, *Classical Electrodynamics*):

$$\Box G(\vec{x}', t'; \vec{x}, t) = \delta(\vec{x} - \vec{x}')\delta(t - t') \longrightarrow$$

$$G(\vec{x}', t'; \vec{x}, t) = -\frac{1}{4\pi} \frac{\delta[t' - (t - |\vec{x} - \vec{x}'|)]}{|\vec{x} - \vec{x}'|}$$

Simple exact solution for potential:

$$A_\mu(\vec{x}, t) = \int d^3x' \, \frac{J_\mu(\vec{x}', t - |\vec{x} - \vec{x}'|)}{|\vec{x} - \vec{x}'|}$$

Expand in multipoles

Goal now: Find a form of A_μ appropriate in "radiation zone" far from the source.

Step 1: Expand A_μ and J_μ in Fourier modes.

$$A_\mu(\vec{x}, t) = A_\mu(\vec{x}) e^{-i\omega t}\,, \quad J_\mu(\vec{x}, t) = J_\mu(\vec{x}) e^{-i\omega t}$$

Solution becomes

$$A_\mu(\vec{x}) = \int d^3x' \, \frac{J_\mu(\vec{x}') e^{ik|\vec{x} - \vec{x}'|}}{|\vec{x} - \vec{x}'|}$$

where $k = \omega$ is the wavenumber.

Expand in multipoles

Goal now: Find a form of A_μ appropriate in "radiation zone" far from the source.

Step 2: Consider "far zone", $r >> 1/k$. Exponential rapidly oscillates; we can put

$$|\vec{x} - \vec{x}'| \simeq r - \hat{n} \cdot \vec{x}'$$

Leading solution, to $O(1/r)$:

$$A_\mu(\vec{x}) = \frac{e^{ikr}}{r} \int d^3x' J_\mu(\vec{x}')e^{-ik\hat{n}\cdot\vec{x}'}$$

$$= \frac{e^{ikr}}{r} \sum_b \frac{(-ik)^b}{b!} \int d^3x' J_\mu(\vec{x}')(\hat{n} \cdot \vec{x}')^b$$

Expand in multipoles

Look at b = 0; consider time and space components of the potential separately.

Time component, go back to the original exact solution and expand from it:

$$\begin{aligned} A_0(\vec{x}, t) &= \int d^3x' \frac{J_0(\vec{x}', t - |\vec{x} - \vec{x}'|)}{|\vec{x} - \vec{x}'|} \\ &\simeq \frac{1}{r} \int d^3x' J_0(\vec{x}', t - r) \\ &= -\frac{1}{r} \int d^3x' \rho(\vec{x}', t - r) = -\frac{q(t - r)}{r} \end{aligned}$$

Just Coulomb solution!

Charge is conserved: $q(t - r) = q$, a **constant.**

The timelike component is just the constant, non-radiative 1/r piece of the potential.

Now examine spatial components: for $b = 0$,

$$\vec{A}(\vec{x},t) = \frac{e^{i(kr-\omega t)}}{r}\int d^3x'\,\vec{J}(\vec{x}')$$

Theorem: $$\int \vec{x}'(\vec{\nabla}\cdot\vec{J})d^3x' = -\int d^3x'\,\vec{J}(\vec{x}')$$

(Easily proved: Integrate by parts and use the fact that *J* is confined to a bound region.)

Continuing ...

$$\vec{A}(\vec{x},t) = -\frac{e^{i(kr-\omega t)}}{r}\int d^3x'\,\vec{x}'(\vec{\nabla}\cdot\vec{J})$$

Simplify this using continuity: $\partial\rho/\partial t + \vec{\nabla}\cdot\vec{J} = 0$

Apply Fourier modes: $\vec{\nabla}\cdot\vec{J} = i\omega\rho$

$$\begin{aligned}\vec{A}(\vec{x},t) &= -\frac{i\omega e^{i(kr-\omega t)}}{r}\int d^3x'\,\vec{x}'\,\rho(\vec{x}) \\ &\equiv -\frac{i\omega e^{i(kr-\omega t)}}{r}\vec{p}\end{aligned}$$

(Defined "electric dipole moment" ***p*** here.)

Simplifies a bit further

Because of Fourier decomposition, we have

$$e^{i(kr-\omega t)}\vec{p} \equiv e^{-i\omega(t-r)}\vec{p} = \vec{p}(t-r)$$

Fold this in, we get a very compact (and hopefully familiar!) result:

$$\vec{A} = -\frac{i\omega\vec{p}(t-r)}{r} = \frac{1}{r}\left(\frac{d\vec{p}}{dt}\right)_{t-r}$$

*The **leading** radiation potential falls off as 1/r and is proportional to the variation in the source's electric dipole moment.*

Go to higher order ...

Examine the contribution of higher terms in *b*:

$$A_\mu(\vec{x}) = \frac{e^{ikr}}{r} \sum_b \frac{(-ik)^b}{b!} \int d^3x' \, J_\mu(\vec{x}') (\hat{n} \cdot \vec{x}')^b$$

Each term defines a higher multipole contribution.
Terms divide into electric and magnetic moments:

$$A \simeq \frac{1}{r} \sum_l \frac{d^l}{dt^l} (E_l + M_l)$$

where

$$E_l \sim \int (x')^l \rho(x') \, d^3x' \qquad M_l \sim \int (x')^l v(x') \rho(x') \, d^3x'$$

Order of magnitude of terms

Consider bound source with total charge *Q*, internal speeds *v*, and size *L*. Take source to be varying in a more-or-less periodic manner. Then,

$$E_l \sim QL^l \longrightarrow \frac{d^l E_l}{dt^l} \sim Qv^l \sim Q\left(\frac{v}{c}\right)^l$$

$$M_l \sim QvL^l \longrightarrow \frac{d^l M_l}{dt^l} \sim Qv^{l+1} \sim Q\left(\frac{v}{c}\right)^{l+1}$$

Each higher moment is suppressed by (v/c) ... At given multipole order, magnetic moments contribute (v/c) less than the electric moment.

Summary of E&M radiation

Our final solution has "electric" and "magnetic" multipole contributions of the form

$$A_l^E \sim \frac{1}{r}\frac{d^l}{dt^l}E_l \qquad A_l^M \sim \frac{1}{r}\frac{d^l}{dt^l}M_l$$

where

$$E_{l+1}/E_l \sim v = v/c$$

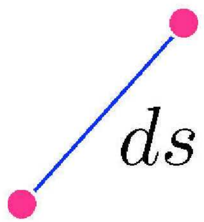
$$M_l/E_l \sim v = v/c$$

The *leading* radiation comes from the electric *l* = 1 moment; *l* = 0 cannot radiate due to charge conservation.

Equivalent gravitational quantities

In general relativity, role of "field" played by *curvature*. It gives a precise description of gravitational *tidal* forces.

Role of "potential" played by the *spacetime metric*. Tells us the distance between two events in spacetime:



$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

Note that $g_{\alpha\beta}$ is dimensionless if coordinates have dimensions of length.

Equivalent gravitational quantities

When gravity is weak, spacetime metric is nearly that which describes special relativity:

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

Perturbation $h_{\alpha\beta}$ encodes all the properties of gravity in this limit ... including radiation.

Particularly nice: The potential $h_{\alpha\beta}$ is governed by an equation very similar that which governs A_μ — shares many of its properties.

Multipolar expansion of metric

From electromagnetic analysis, we expect to have "gravitoelectric" and "gravitomagnetic" contributions to the perturbation $h_{\alpha\beta}$:

$$h_l^M \sim \frac{1}{r}\frac{d^l}{dt^l}M_l \qquad h_l^S \sim \frac{1}{r}\frac{d^l}{dt^l}S_l$$

M_l and S_l are "mass" and "current" moments:

$$M_l \sim \int (x')^l \rho(x') d^3x'$$

$$S_l \sim \int (x')^l v(x') \rho(x') d^3x'$$

Wrong dimensions!

h must be dimensionless. Correction factors: Multiply all masses by G/c^2; multiply all powers of time by *c*; divide all velocities by *c*.

$$h_l^M \sim \frac{1}{r}\frac{G}{c^{l+2}}\frac{d^l}{dt^l}M_l \qquad h_l^S \sim \frac{1}{r}\frac{G}{c^{l+3}}\frac{d^l}{dt^l}S_l$$

Now use these to deduce the leading order behavior of radiation in general relativity.

Mass monopole

Consider the contribution from M_0:

$$h_0^M \sim \frac{G}{c^2}\frac{M_0}{r} \qquad M_0 = \int d^3x'\,\rho(x') \equiv M_{\rm system}$$

Newtonian potential! Conservation of mass/energy means it is constant: *not* radiation.

Perfectly valid solution ... but conservation principles means that it is not radiation.

Mass dipole

Next, consider M_1:

$$h_1^M \sim \frac{1}{r}\frac{G}{c^3}\frac{dM_1}{dt} \qquad M_1 = \int x'\rho(x')d^3x'$$

dM_1/dt has same dimension as momentum. Careful analysis shows this term is proportional to system's total linear momentum.

Term therefore *must* be ***purely gauge***: We can always set it to zero by boosting into a frame in which total momentum is zero. Cannot oscillate (conservation): **Not radiation**.

Current dipole

The next possible multipole is S_1:

$$h_1^S \sim \frac{1}{r}\frac{G}{c^4}\frac{dS_1}{dt} \qquad S_1 = \int x' v(x') \rho(x') d^3x'$$

S_1 has same dimension as angular momentum J. Careful analysis confirms this: $h^S{}_1 \propto dJ/dt$.

Global conservation of angular momentum means $dJ/dt = 0$: Conservation principle "protects" the spacetime from such terms.

No S_1 radiation!

Mass quadrupole

Next in the sequence:

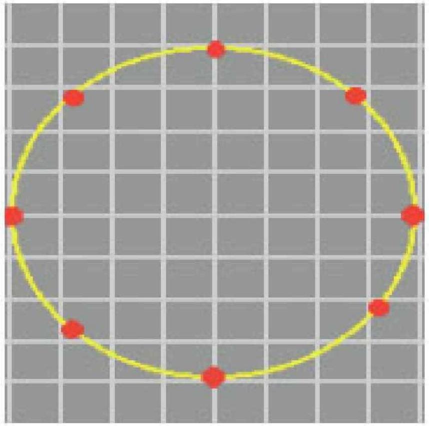
$$h_2^M \sim \frac{1}{r}\frac{G}{c^4}\frac{d^2 M_2}{dt^2} \qquad M_2 \sim \int x'_1 x'_2 \rho(x') d^3 x'$$

M_2 is not protected by conservation: This multipole is "allowed" to radiate!

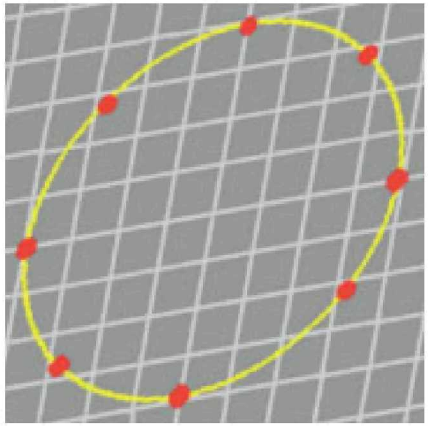
Leading gravitational radiation comes from the l = 2 mass quadrupole moment; l = 0 cannot radiate due to conservation of mass and energy, l = 1 cannot radiate due to conservation of momentum and angular momentum.

What *is* this radiation??

Tides are "fundamental" gravity fields in general relativity ... gravitational wave should therefore be oscillatory *tidal* field:



Oscillating tides: Alternating stretch and squeeze of "test masses" along orthogonal axes.



The metric amplitude *h* describes the tidal *strain* $\delta L/L$: The change in test mass separation divided by separation.

Magnitude of the effect

How big is this strain? Use formula we found with dimensional analysis to estimate effect:

$$h_{\rm rad} \sim \frac{1}{r}\frac{G}{c^4}\frac{d^2 M_2}{dt^2} \qquad \frac{d^2 M_2}{dt^2} \sim \frac{d^2}{dt^2}\int x'_1 x'_2 \rho(x') d^3x'$$

$v_{\rm int}$ = internal speeds $\sim M_{\rm tot} v_{\rm int}^2$

$KE_{\rm int}$ = internal kinetic energy $\sim KE_{\rm int}$

The metric amplitude *h* describes the tidal *strain δL/L*: The change in test mass separation divided by separation.

Magnitude of the effect

How big is this strain? Use formula we found with dimensional analysis to estimate effect:

$$\longrightarrow h_{\rm rad} \sim \frac{1}{r}\frac{G}{c^4} KE_{\rm int}$$

Typical values: $KE_{\rm int} \sim 1\,M_\odot c^2 \sim 1.8 \times 10^{54}\,{\rm erg}$

$$r \sim 100\,{\rm Mpc} \sim 3.1 \times 10^{26}\,{\rm cm}$$

Constants: $G/c^4 = 8.26 \times 10^{-50}\,{\rm cm/erg}$

Result: $h_{\rm rad} \sim 10^{-21} - 10^{-22}$!!!

Tiny effect

Smallness of strain reflects gravity's weakness: Weakest of the 4 fundamental forces. Makes direct detection extremely challenging.

Also opportunity: Weakness of GWs means that they propagate from source to earth with essentially no scatter or absorption.

The promise: Direct detection can open a window onto processes hidden from view using photons! Strong GW emission often associated with processes that are hidden from view or dark ... and often highly energetic as well.

Energy content

Despite small amplitude, *energy* carried by waves can be enormous.

Electrodynamics: Energy flux carried by waves determined by the Poynting vector:

$$\frac{dE}{dt} = \frac{c}{4\pi}\int(\vec{E}\times\vec{B})\cdot d\vec{a} \sim r^2|d\vec{A}/dt|^2$$

To properly define GW energy flux, must build an analogous quantity. ***Quite tricky***: requires analysis at second-order in perturbation theory!

Energy content

Analogous quantity rigorously defined by Isaacson (1968): *stress-energy* tensor for grav. radiation.

General definition: $T^{\mu\nu}$ is the flux of momentum component p^μ in the x^ν direction. Hence, components T^{0i} describe energy flux:

$$\frac{dE}{dt} = \int T^{0i}\, da_i \sim \frac{c^3}{G} r^2 |dh/dt|^2$$

Consider a source measured with $h \sim 10^{-22}$, 100 Mpc away, periodic with frequency 100 Hz: $dE/dt \sim 10^{53}$ erg/sec $\sim 10^{20}$ solar luminosity!

Sources

Key thing for a source to be interesting: Needs to have a highly variable mass quadrupole moment — lots of mass moving very quickly.

Taken together, this means the source must be ***compact***: cannot get rapid variation unless spatial extent of the source is small.

Ideal sources:

Black holes **Stellar core collapse**

Neutron stars **Dynamics of the early universe**

Sources and measurement

Key characteristic: The frequency band in which your sources radiate.

4 bands usually discussed; ends of each band set by properties of sources and techniques used to make measurements in those bands.

High frequency: ~1 Hz < f < ~10^4 Hz.
Targets for ground-based GW antennae.

Low frequency: ~10^{-5} Hz < f < ~1 Hz.
Targets for space-based GW antennae.

Very low frequency: ~10^{-9} Hz < f < ~10^{-7} Hz.
Targets of pulsar timing GW measurements.

Ultra low frequency: ~10^{-5} H_0^{-1} < λ < ~H_0^{-1}.
Imprinted on cosmic microwave background.

High frequency band

Low end, roughly 1 Hz: Set by inability to isolate an instrument from terrestrial noise below this.

High end, roughly 10^4 Hz: Roughly 1/(shortest timescale likely to be associated with a source).

More carefully:

$$f_{\rm max} \sim \frac{1}{2\pi}\frac{v}{R}$$

$$v \le c$$

$$R \ge \frac{GM}{c^2}$$

$$f_{\rm max} \sim \frac{1}{2\pi}\frac{c^3}{GM}$$

$$\sim 10^4\,{\rm Hz}\left(\frac{M_\odot}{M}\right)$$

High frequency band

Low end, roughly 1 Hz: Set by inability to isolate an instrument from terrestrial noise below this.

High end, roughly 10^4 Hz: Roughly 1/(shortest timescale likely to be associated with a source).

Measurements made with ground-based gravitational wave antennae:

LIGO: Hanford, Washington & Livingston, Louisiana, United States

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Measurements made with ground-based gravitational wave antennae:

Virgo: Pisa, Italy (French-Italian collaboration)

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Measurements made with ground-based gravitational wave antennae:

GEO600, Hannover, Germany (German-British collaboration)

High frequency band

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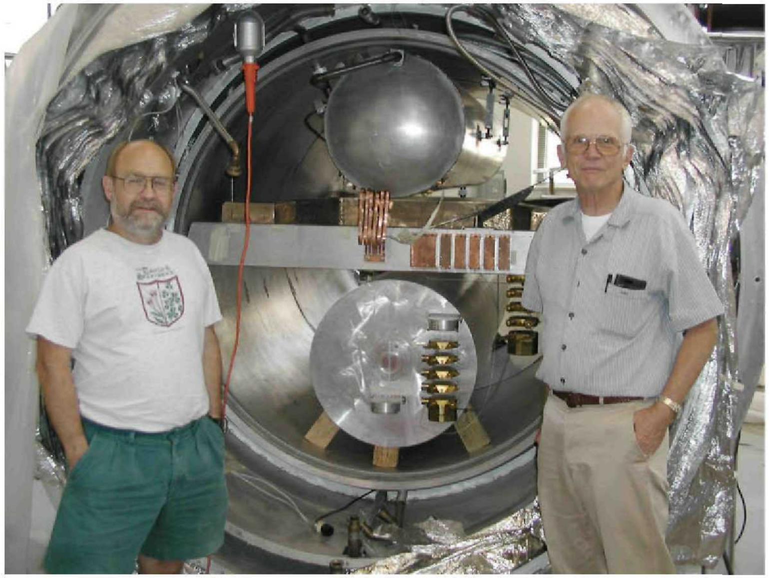
Measurements made with ground-based gravitational wave antennae:

Resonant mass detectors: Experiments in the United States, Italy, the Netherlands, Brazil (possibly others??)



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Sources typically stellar mass objects:

- "Compact" binaries (made of neutron stars or black holes)
- Vibrations of compact objects
- Stellar core collapse (inner engine of supernovae explosions)
- Cosmological phase transition artifacts

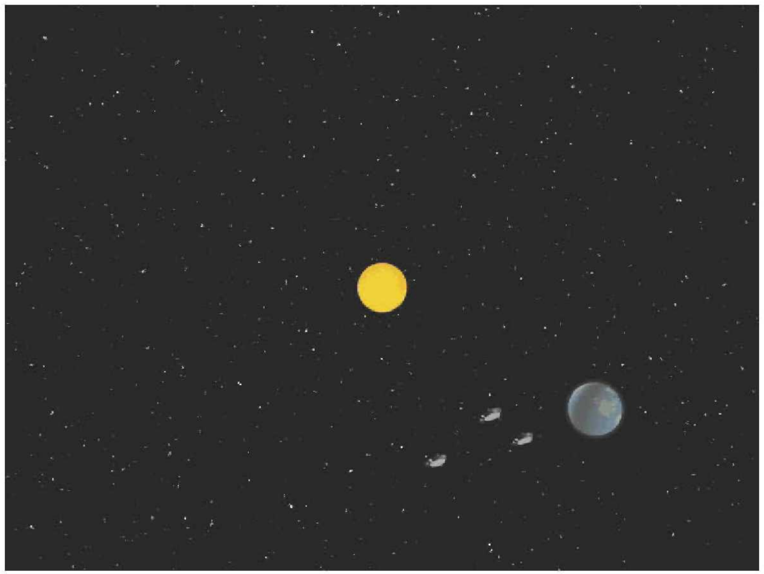
Low frequency band

Low end, roughly 10^{-5} Hz: roughly 1/(longest time for which we can control noise environment of spacecraft) ~ 1/(a few hours).

High end, roughly 1 Hz: in principle, try to join the high frequency band; in practice, set by photon counting statistics, limitations of data downlinks.

Measurements require space-based antennae:

"LISA," a planned mission under development by NASA (United States) and the European Space Agency



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Key sources massive black holes objects:

- Massive binaries formed by mergers of galaxies in hierarchical structure growth
- Binaries formed by capture of compact objects onto orbits of galactic center black holes
- Also stellar mass binaries which populate our galaxy

Very low frequency band

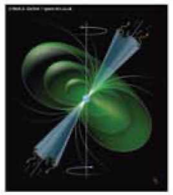
More interesting to describe range of this band in terms of period, rather than frequency:

Long end, roughly 30 years: Amount of data we have on pulsars!

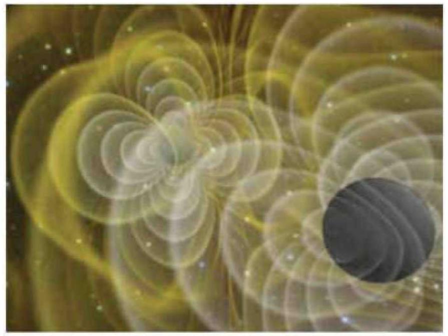
Short end, roughly 3 months: Need a few months of integration to beat down sources of noise.

Measurements based on using very stable millisecond pulsars as clocks:

fills spacetime with GWs ...



GW source



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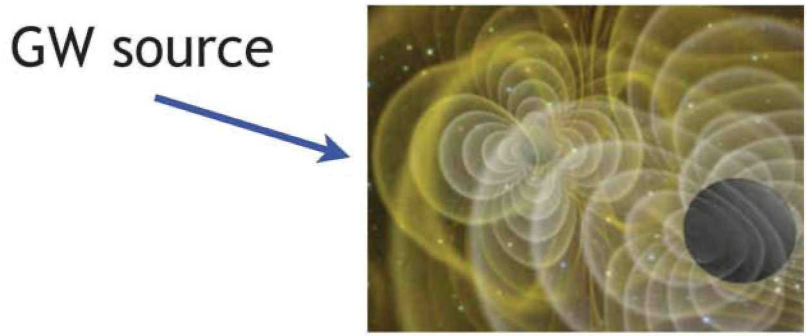
Long end, roughly 30 years: Amount of data we have on pulsars!

Short end, roughly 3 months: Need a few months of integration to beat down sources of noise.

Measurements based on using very stable millisecond pulsars as clocks:

... causing arrival time of pulses to vary periodically.



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Measurements based on using very stable millisecond pulsars as clocks:

If sources of noise are sufficiently well understood, and *if* there are enough pulsars used to make measurement, a sky of the "timing residuals" from pulsars will vary with a characteristic quadrupolar shape.

Very low frequency band

More interesting to describe range of this band in terms of period, rather than frequency:

Long end, roughly 30 years: Amount of data we have on pulsars!

Short end, roughly 3 months: Need a few months of integration to beat down sources of noise.

Measurements based on using very stable millisecond pulsars as clocks:

A background of *exactly* such waves is expected from massive binary black holes formed in the process of galaxy growth!

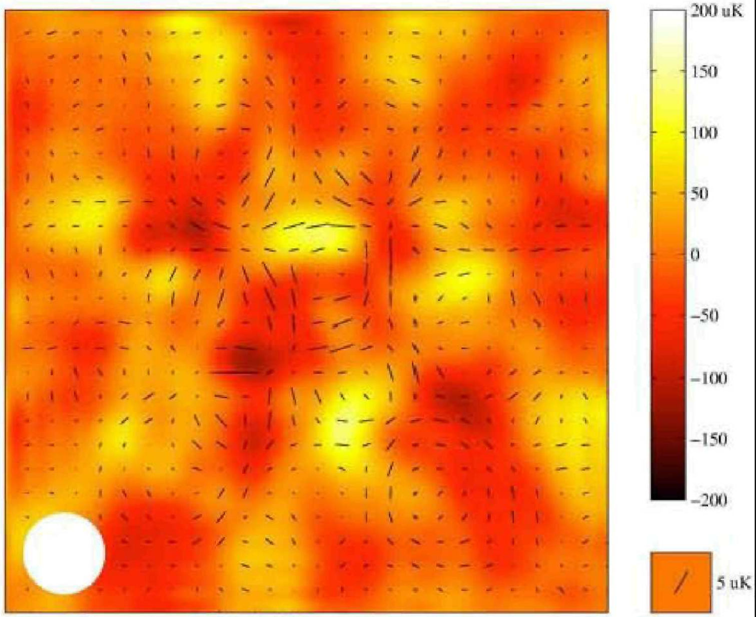
Ultra low frequency band

More interesting to describe range of this band in terms of wavelength, rather than frequency:

Long end, $\sim H_0^{-1}$: The size of the universe.

Short end, $\sim 10^{-4} H_0^{-1}$: Scales we can resolve in the cosmic microwave background.

Source here is primordial ground-state fluctuations in spacetime, amplified by inflation. Imprints on CMB with a unique polarization signature.



Map is 5 degrees square

Final wrap up

Key points to take away:

Gravitational radiation required by causality: Enforces rule that changes in the field can propagate no faster than the speed of light.

Leading order radiation *quadrupolar*: Monopole and dipole radiation prohibited by conservation laws.

Weak: Gravity weakest interaction. Makes GWs very hard to measure ... but means that radiation carries "pristine" information about dynamics of process that generates it since it barely scatters or absorbs.

Highly energetic: Enormous energy carried by waves. Connected to some of the most interesting and violent dynamical processes. Outstanding opportunity for a new tool to study our universe.