





1954-12

Summer School in Cosmology

21 July - 1 August, 2008

Cosmology & Particle Physics Lecture 3 & 4

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Example: STANDARD MODEL (AND ITS EXTENSIONS).

Φ= HIGGS FIELD

 $V(\phi) = \lambda \left(\phi^{\dagger}\phi - \frac{v^2}{2}\right)^2$

=> 〈ゆ〉= で

IN VACUO

WHAT HAPPENS AT HIGH TEMPERATURE? LOW

CONDENSED MATTER:

SYMMETRY AT LOW T

HIGHI U

SYMMETRY AT HIGHT

MELTING CRYSTAL => RESTORATION OF TRANSLATIONAL SYMMETRY SUPERCONDUCTIVITY: CONDENSATE OF COOPER PAIRS AT LOW T

NO CONDENSATE AT HIGH T

"CONDENSATES EVAPORATE".

EXPECT THAT $\langle \phi \rangle_T = 0$ IN STANDARD MODEL

(SOMEWHAT SLOPPY, SEE BELOW).

SYMMETRY RESTORED AT HIGH T.

- AT FINITE T SYSTEM SITS

 AT MINIMUM OF FREE ENERGY,

 NOT ENERGY (NEGLECTING CHEMICAL POTENTIAL).
- · EFFECTIVE POTENTIAL:

 SPECIFY VALUE OF → ("BY HANDS"),

 TAKE SYSTEM IN THERMAL EQUILIBRIUM

 W.R.T. ALL OTHER PARAMETERS

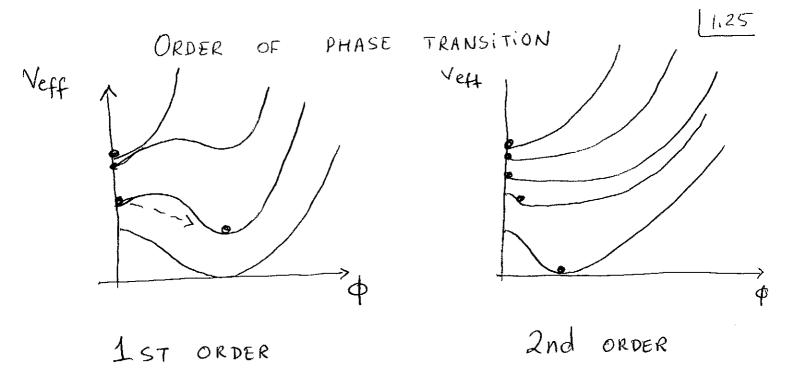
 (E.G. PARTICLE DISTRIBUTIONS ARE

 THERMAL)

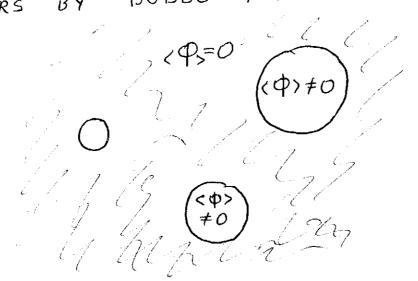
Veff $(\Phi, T) = FREE ENERGY DENSITY AT GIVEN <math>\Phi$

 $(\Phi)_T$: minimum of $V_{eff}(\bar{\Phi},T)$

NB: V(\$) AT T=0 IS EFF. POTENTIAL AT T=0: ENERY = FREE ENERGY AT ZERO T.



1ST ORDER: MOST INTERESTING FOR COSMOLOGY
OCCURS BY BUBBE NUCLEATION



MICROSCOPIC BURBLES OF NEW PHASE SPONTANEOUSLY CREATED AND THEN EXPAND.

EFFICIENT IF OF ORDER 1 BUBBLE CREATED IN HUBBLE TIME.

THEN BUBBLES EXPAND TO & SIZE ROUGHLY
OF ORDER OF HUBBLE SIZE

$$H^{-1}(T_{EW}) = \frac{M_{Pe}^{*}}{T_{EW}^{2}} \sim \frac{10^{18} \text{ GeV}}{10^{4} \text{ GeV}^{2}} \sim 10^{14} \text{ GeV}^{-1} \sim 1 \text{ cm}$$

Bubble Grows From R~ $T_{\rm EW} \sim (100\,{\rm GeV})^{-1} \sim 10^{-16}\,{\rm cm}$ To macroscopic Size of order 1 cm.

UNIVERSE BOILS

STRONGLY OUT-OF-THERMAL-EQUILIBRIUM, HENCE INTERESTING.

DOES THIS REALLY OCCUR?

NOT IN STANDARD ModeL, BUT IN

SOME OF ITS EXTENSIONS.

WILL SOON KNOW FROM LHC ?

TO SEE THIS: CALCULATE EFFECTIVE POTENTIAL.

* THERMODYNAMICS: FREE ENERGY DENSITY

1ST LAW ZERO CHEMICAL POTENTIAL):

$$dE = -pdV + TdS \qquad E = gV$$

$$S = sV$$

V.dp + pdV = -pdV + TsdV + TVds

$$\frac{dg = TdS}{\left[TS = p + g\right]} \Longrightarrow \left[f = -p\right]$$

$$V_{eff}(\tau, \phi) = - P(\tau, \phi)$$

Need to calculate pressure

MINIMUM OF FREE ENERGY : HIGHER PRESSURE (NB: WiNS).

Veff
$$(T,\phi) = \lambda \left(\phi^2 - \frac{v^2}{2}\right)^2 + \text{TEMPERATURE}$$
CORRECTIONS

"TREE LEVEL" (NB: THERMAL LOOPS = TREE LEVEL FROM QFT VIEWPOINT PLASMA EFFECTS)

IDEAL GAS APPROXIMATION

ONE LOOP: NEGLECT INTERACTIONS OF PARTICLES IN PLASMA, CALCULATE CONTRIBUTIONS OF FREE PARTICLES INTO PRESSURE.

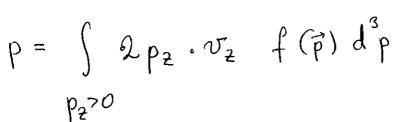
WHY DEPENDS ON P? BECAUSE MASSES

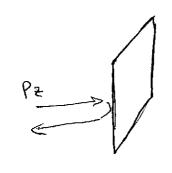
DEPEND ON +:

ON
$$\Phi$$
:
$$M_{W}(\phi) = \frac{9}{\sqrt{2}} \phi , \quad M_{t} = y_{t} \cdot \phi , \text{ etc.}$$

TEMPERATURE One Lopp: CORRECTIONS to Veff

to a particle apieces





PRESSURE ON A WALL:

2PZ: MOMENTUM TRANSFER TO WALL

Uz f(p): number of PARTICUES OF MOMENTUM P, HITTING WALL IN UNIT TIME PER UNIT AREA

dp.f(p): Number DENSITY OF PARTICLES OF MOMENTUM P

$$V_2 = \frac{P_2}{E}$$
; $\langle p^2 \rangle = \frac{1}{3} \not{p}^2 \not{p}$

$$P = \int \frac{1}{3} \frac{p^{2}}{E} f(p) d^{3}p = 4\pi \int_{0}^{\infty} \frac{p^{3}dp}{E} \frac{1}{(2\pi)^{3}} \frac{1}{e^{E/T} \mp 1}$$

$$\frac{1}{1}$$

$$\frac{1}{1}$$

$$\frac{1}{2}$$

$$\frac{$$

FERMI-DIRAC :

$$E = \sqrt{p^2 + m^2(\phi)}$$

\$ larger than 0 => E larger => denominator in p larger => pressure smaller => Veft larger

Veff
$$(T, \phi)$$
 = const - $\lambda \phi^2 v^2 + \lambda \phi^4$
+ $\frac{T^2}{24} \propto \phi^2 + higher orders in $\phi$$

X = COMBINATION OF COUPLINGS:

if Mi (Φ) = hi Φ, THEN

 $\lambda = \sum_{\text{BOSONS}} g_i h_i^2 + \sum_{\text{FERMIONS}} \frac{1}{2} g_i h_i^2$

gi = # OF SPIN STATES.

NB: LARGE CONTRIBUTION FROM t-quark in SM.

SM: $\alpha = \frac{2}{v^2} \left(6M_W^2 + 3M_Z^2 + 6m_t^2 \right)$

* Symmetry indeed restored at T>Tc

WHERE _2

$$\lambda v^2 = \frac{T_c^2}{24} \propto \frac{m_H^2}{2}$$

Numerically, SM: Tc = 120 GeV. (mH 100 GeV)

1.30

1 << T

ORDER OF PHASE TRANSITION

If Veff ANALYTIC IN \$2

$$\frac{1}{\sqrt{V_{eff}}} = \frac{\lambda}{24} \left(T^2 - T_c^2\right) \varphi^2 + \lambda eff \varphi' + \dots$$

and ORDER.

BUT Veff is NOT ANALYTIC IN P

BOSONS

$$f = -p = -\frac{1}{3 \cdot 2 + t^2} \int_{0}^{\infty} \frac{p' dp}{E} \frac{1}{e^{+E/T} - 1}$$

Infrared part:
$$E \ll T$$

$$\int_{0}^{\infty} \frac{1}{p^2 + m^2(\phi)}$$

TRY TO EXPAND $\simeq -\frac{1}{3.2\pi^2} \left[const - T \left(m^2(\phi) dp + T \right) m'(\phi) \frac{dp}{p^2} \right]$ in $m^2(\phi) \simeq \frac{1}{3.2\pi^2} \left[const - T \left(m^2(\phi) dp + T \right) m'(\phi) \frac{dp}{p^2} \right]$

Linear infrared "divergence" => PAGE (1.30a) NEXT CONTRIBUTION IS $-m^3(\Phi) \cdot T$

$$\gamma = \frac{1}{12\pi} \sum_{8050NS} g_i |h_i|$$

DIVIDE PRESSURE INTERRAL INTO UV PORT, p> 1 AND IR PART, P< 1, WHERE 1 «T $\Lambda \gg m(\Phi)$

UV-part: ANALYTIC IN \$2

 $-\frac{Tm(\Phi)}{6\pi^{2}}\int_{0}^{1}\frac{d\rho}{\rho^{2}+m_{(\Phi)}^{2}}=-\frac{Tm^{2}(\Phi)}{12\pi}$ $\frac{1}{2} \frac{1}{m(\Phi)} AT \Lambda >> m(\Phi)$

Cubic TERM - $\frac{T}{12\pi}$ m³(ϕ) = - $\frac{T}{12\pi}$ h³ ϕ ³

ON THE FACE OF IT:

FIRST ORDER PHASE TRANSITION

BUT: LIFE IS MORE COMPLICATED ...

ONTHE FACE OF IT: AT T=TC

Veft =
$$- \gamma T_c \varphi^3 + \lambda_{eff} \varphi^4$$
 $\lambda_{eff} \approx \lambda$

1

MINIMUM AT

AT
$$\Phi \simeq \frac{\gamma}{\lambda} T_c$$
 RECALL $m_H \sim \sqrt{\lambda} V$

HEAVY HIGGS => LARGE > => SMALL \$\overline{\Psi}\$

BUT AT SMALL & PERTURBATION THEORY DOES NOT WORK

• INFRARED PROBLEM OF BOSONS AT HIGH T χ : BOSONIC FIELD, $m_{\chi} = h \, \phi$

ITS DISTRIBUTION FUNCTION LE AT ECT

$$f(\vec{p}) = \langle A^{\dagger}(\vec{p}) A(\vec{p}) \rangle = \frac{1}{e^{\xi f}-1} \simeq \frac{T}{E}$$

OCCUPATION # LARGE => CLASSICAL FIELD

NB: Important LESSON: QUANTUM
BOSONIC FIELD BECOMES
CLASSICAL FOR IPI, M & T

ENERGY =
$$\int d^3p \sqrt{p^2 + m^2} \langle A_p^{\dagger} A_p \rangle$$

= $\frac{1}{2} \int d^3x \left[(\vec{\nabla} \mathcal{A})^2 + \dot{\chi}^2 + m^2 \chi^2 \right] = \int d^3p (p^2 + m^2) \chi_p^2$
 $\chi_p^2 \sim \frac{\langle A_p^{\dagger} A_p \rangle}{E(p)} \sim \frac{T}{E^2}$

FIELD FLUCTUATION

$$\langle \chi^2(\tilde{x}) \rangle \sim \int d^3 p \chi_p^2 \sim \int \frac{dp}{p} p^3 \chi_p^2$$

FIELD FLUCTUATION AT MOMENTUM SCALE P $\langle \chi^2(\bar{\mathbf{x}}) \rangle_p \sim p^3 \chi_p^2 \sim \frac{p^3 T}{E_p^2}$

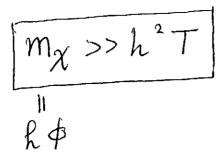
$$\langle \chi^2(\bar{x}) \rangle_p \sim p \chi_p \sim \frac{1}{E_p^2}$$

 $SELF-INTERACTION: h^2 \chi^4 \left[g^2 W.W.W.W; m_W \sim g \Phi \right]$

REQUIRE SELF-INTEACTION SMALL
COMPARED TO FREE TERMS AT MOMENTUM
SCALE P~MX

SCALE
$$\rho \sim m_{\chi}$$

$$1 >> \frac{h^2 < \chi^2 >^2}{m_{\chi}^2 < \chi^2 >} \sim \frac{h^2 < \chi^2 >}{m_{\chi}^2} \sim \frac{h^2 \rho^3 T}{m_{\chi}^2} \sim \frac{h^2 \Gamma^3}{m_{\chi}^2} \sim \frac{h^2 \Gamma^3}{m_{\chi}^2}$$



 ψ $\phi \gg kT$

RELATIVELY LARGE FIELDS ONLY SM: h → g, GAUGE COUPLING; X=W-field

Recall THAT AT CRITICAL TEMPERATURE $\phi \simeq \frac{\mathcal{X}}{\lambda} T$

 $\gamma \simeq \frac{1}{12\pi} \cdot \frac{9g^3}{2\sqrt{2}}$ (W, Z-CONTRIBUTIONS)

 $\psi \qquad \phi > 7 g T$ $\psi \qquad \chi \ll \frac{\chi}{g} \sim g^2 \Leftrightarrow m_H^2 \ll m_W^2$

TRUE FOR LIGHT HIGGS ONLY

CERTAINLY NOT TRUE FOR MH>114 GeV

PHASE TRANSITION" NON-PERTURBATIVE.

IN FACT, NO PHASE TRANSITION AT ALL,

SMOOTH CROSSOVER (cf. Liquid-VAPOR)

WHAT CAN MAKE PHASE TRANSITION STRONGLY FIRST ORDER?

New BOSONIC FIELD(S) STRONGLY ENOUGH coupled to Higgs FIELD. NEW FIELD; COUPLES TO HIGGS WITH hNEW, MNEW = hNEW . V RECALL

Ф(Tc) = ~ ~ Tc

 $\gamma = \frac{1}{12\pi} \sum_{\text{Bosons}} g_i |h_i|^3 = \frac{1}{12\pi} \sum_{\text{Bosons}} g_i \frac{m_i}{v^3}$ # OF DEGREES OF FREEDOM

PERTURBATION THEORY VALID IF

 $\frac{\chi}{2} T_c = \Phi(T_c) \gg h_i T_c \simeq \frac{m_i}{v} T_c$

WORKS IF

MH

TO THEW

NEED MANY DEGREES OF FREEDOM GNEW * LARGE COUPLING hNEW => LARGE MASS MNEW EXAMPLE: SCALAR TOP IN SUSY

* MAY HAVE "SOFT" (HIGGS-INDEPENDENT) MASS

Might must be small (Smaller THAN

100 GeV), OTHERWISE & DECOUPLES FROM

PLASMA AT T~Tc~ 100 GeV

* HIGGS - GIVEN MASS my = 172 GeV (= m+)

WITH NUMERICAL FACTORS ACCOUNTED FOR,
SUFFICIENT TO MAKE PHASE TRANSITION
STRONGLY FIRST ORDER.

THIS WILL DEFINITELY BE SEEN AT LHC.

OTHER OPTIONS: LESS CLEAR E.G. SINGLET SCALAR(S).



BARYON ASYMMETRY OF THE UNIVERSE.

QUANTITATIVE CHARACTERISTIC

$$\Delta_{B} = \frac{n_{B} - n_{\overline{B}}}{5}$$

STAYS CONSTANT IN TIME IF B IS CONSERVED
AND EVOLUTION CLOSE TO
ADIABATIC

RELATED TO MB=MB/NY VIA NUMERICAL FACTOR

$$\eta_{B} = 6.1 \cdot 10^{-10}$$
 => $\Delta_{B} = 0.9 \cdot 10^{-10}$

EARLY UNIVERSE: $n_B - n_{\overline{B}} = \frac{1}{3} (n_q - n_{\overline{q}})$ $S \simeq (n_q + n_{\overline{q}})$

$$\Delta B^{\gamma} \frac{n_q - n_{\overline{q}}}{n_q + n_{\overline{q}}}$$

ONE EXTRA QUARK PER 100 9-9 PAIRS.

WHERE DID IT COME FROM?

FROM 1980'S : INFLATION

ASYMMETRY = O AT THE REGINNING
GENERATED DURING EVOLUTION OF UNIVERSE.

SAKHAROV CONDITIONS

- (1) BARYON NUMBER VIOLATION
- (2) C- AND CP-VIOLATION
- (3) THERMAL INEQUILIBRIUM
- (2): (2 AND CP DISTINGUISH PARTICLES FROM ANTIPARTICLES

IF CP OR C CONSERVED, PARTICLES AND ANTIPARTICLES BEHAVE IN THE SAME WAY

- (3): THERMAL EQUILIBRIUM: ALL ASYMMETRIES GET WASHED OUT, IF QUANTUM NUMBERS ARE NOT CONSERVED.
- (1) + (2) + (3) SHOULD WORK TOGETHER, AT THE SAME TIME
- QUALIFICATION: MAY FIRST GENERATE LEPTON
 NUMBER, IT WILL GET REPROCESSED INTO B.

Can baryon asymmetry be due to electroweak physics?

Baryon number is violated in electroweak interactions.

Non-perturbative effect

Hint: triangle anomaly in baryonic current B^{μ} :

$$\partial_{\mu}B^{\mu} = \left(\frac{1}{3}\right)_{B_q} \cdot 3_{colors} \cdot 3_{generations} \cdot \frac{g_W^2}{32\pi^2} \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$

 F_{uv}^a : $SU(2)_W$ field strength; g_W : $SU(2)_W$ coupling

Likewise, each leptonic current ($n = e, \mu, \tau$)

$$\partial_{\mu}L_{n}^{\mu}=rac{g_{W}^{2}}{32\pi^{2}}\cdotarepsilon^{\mu
u\lambda
ho}F_{\mu
u}^{a}F_{\lambda
ho}^{a}$$

Large field fluctuations, $F_{\mu\nu}^a \propto g_W^{-1}$ may have

$$Q \equiv \int d^3x dt \; \frac{g_W^2}{32\pi^2} \cdot \varepsilon^{\mu\nu\lambda\rho} F_{\mu\nu}^a F_{\lambda\rho}^a \neq 0$$

Then

$$B_{fin} - B_{in} = \int d^3x dt \ \partial_{\mu} B^{\mu} = 3Q$$

Likewise

$$L_{n, fin} - L_{n, in} = Q$$

B is violated, B-L is not.

How can baryon number be not conserved without explicit *B*-violating terms in Lagrangian?

Consider massless fermions in background gauge field $\vec{A}(\mathbf{x},t)$ (gauge $A_0=0$). Let $\vec{A}(\mathbf{x},t)$ start from vacuum value and end up in vacuum.

NB: This can be a fluctuation

Dirac equation

$$i\frac{\partial}{\partial t}\psi = i\gamma^0\vec{\gamma}(\vec{\partial} - ig\vec{A})\psi = H_{Dirac}(t)\psi$$

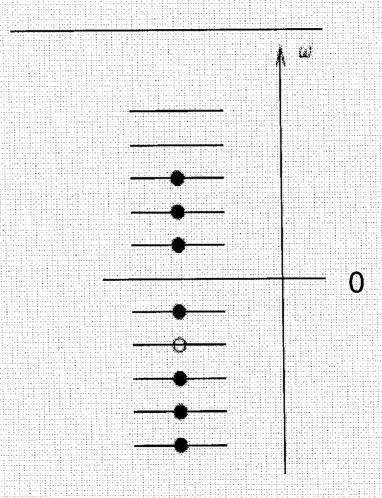
Suppose for the moment that \vec{A} slowly varies in time. Then fermions sit on levels of instantaneous Hamiltonian,

$$H_{Dirac}(t)\psi_n=\omega_n(t)\psi_n$$

How do eigenvalues behave in time?



Dirac picture at $\vec{A}=0$, $t\to\pm\infty$



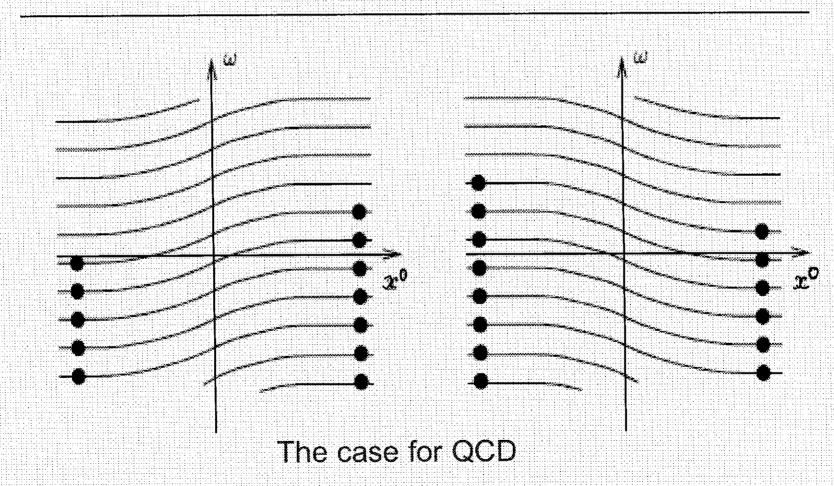


TIME EVOLUTION OF LEVELS

IN SPECIAL (TOPOLOGICAL) GAUGE FIELDS

Left-handed fermions

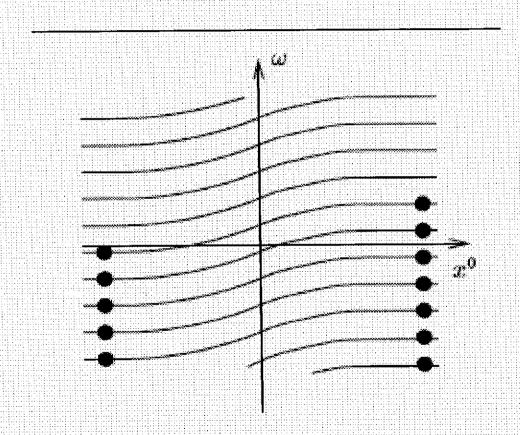
Right-handed



 $B = N_L + N_R$ is conserved, $Q^5 = N_L - N_R$ is not



If only left-handed fermions interact with gauge field, then number of fermions is not conserved



The case for $SU(2)_W$

Fermion number of every doublet changes in the same way



NB: Non-Abelian gauge fields only (in 4 dimensions)

QCD: Violation of Q^5 is a fact.

In chiral limit $m_u, m_d, m_s \to 0$, global symmetry is $SU(3)_L \times SU(3)_R \times U(1)_B$, not symmetry of Lagrangian $SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A$



Need large field fluctuations. At zero temprature their rate is suppressed by

$$e^{-rac{16\pi^2}{g_W^2}} \sim 10^{-165}$$

High temperatures: large thermal fluctuations ("sphalerons"). *B*-violation rapid as compared to cosmological expansion at

$$\langle \phi \rangle_T < T$$

 $\langle \phi \rangle_T$: Higgs expectation value at temperature T.

Possibility to generate baryon asymmetry at electroweak epoch, $T_{EW} \sim 100$ GeV ?

But Universe expands slowly. Expansion time

$$H^{-1} = rac{M_{Pl}^*}{T_{EW}^2} \sim 10^{14} \; ext{GeV}^{-1} \sim 10^{-10} \; ext{s}$$

Too large to have deviations from thermal equilibrium?



THE DULY WAY

FIRST ORDER EW PHASE TRANSITION

> AFTER PHASE TRANSITION BARYON ASYMMETRY SHOULD NOT BE WASHED OUT

AFTER PHASE TRANSITION (p>, > T

[STRONGER THAN SIMPLY EXISTENCE OF 1ST ORDER PHASE TRANSITION]

1

NEW BOSONS, STRONGLY INTERACT WITH HIGGS.

NEW SOURCE OF CP-VIOLATION

IF SO, THEN BARYON ASYMMETRY MAY BE GENERATED BY INTERACTIONS OF FERMIONS WITH BUBBLE WALLS.

Good CHANCE FOR LHC AND ILC.

IS THIS THE ONLY WAY TO GENERATE BARYON ASYMMETRY?

BY NO MEANS.

Competitor: LEPTOGENESIS.

NB: ANY MECHANISM OF GENERATION OF BARYON ASYMMETRY AT HOT STAGE WOULD BE INCONSISTENT WITH BARYON ISOCURVATURE PERTURBATIONS.

SAME STORY AS WITH CDM.

NB: PUZZLE: DDM ~ DB. WHY? LOOKS LIKE COINCIDENCE.

TO CONCLUDE:

BOTH COM AND BARYON ASYMMETRY MAY BE DUE TO PHYSICS AT 100 GEV SCALE. IF SO, WE WILL KNOW THAT IN OUR LIFETIME.

IF NOT, NEED A LOT OF LUCK.