The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1957-4

Miniworkshop on Strong Correlations in Materials and Atom Traps

4 - 15 August 2008

Resonantly interacting degenerate atomic gases.

RADZIHOVSKY Leo

University of Colorado at Boulder
Department of Physics
Campus Box 390
CO 80309-0390 Boulder
U.S.A.

Resonant atomic gases

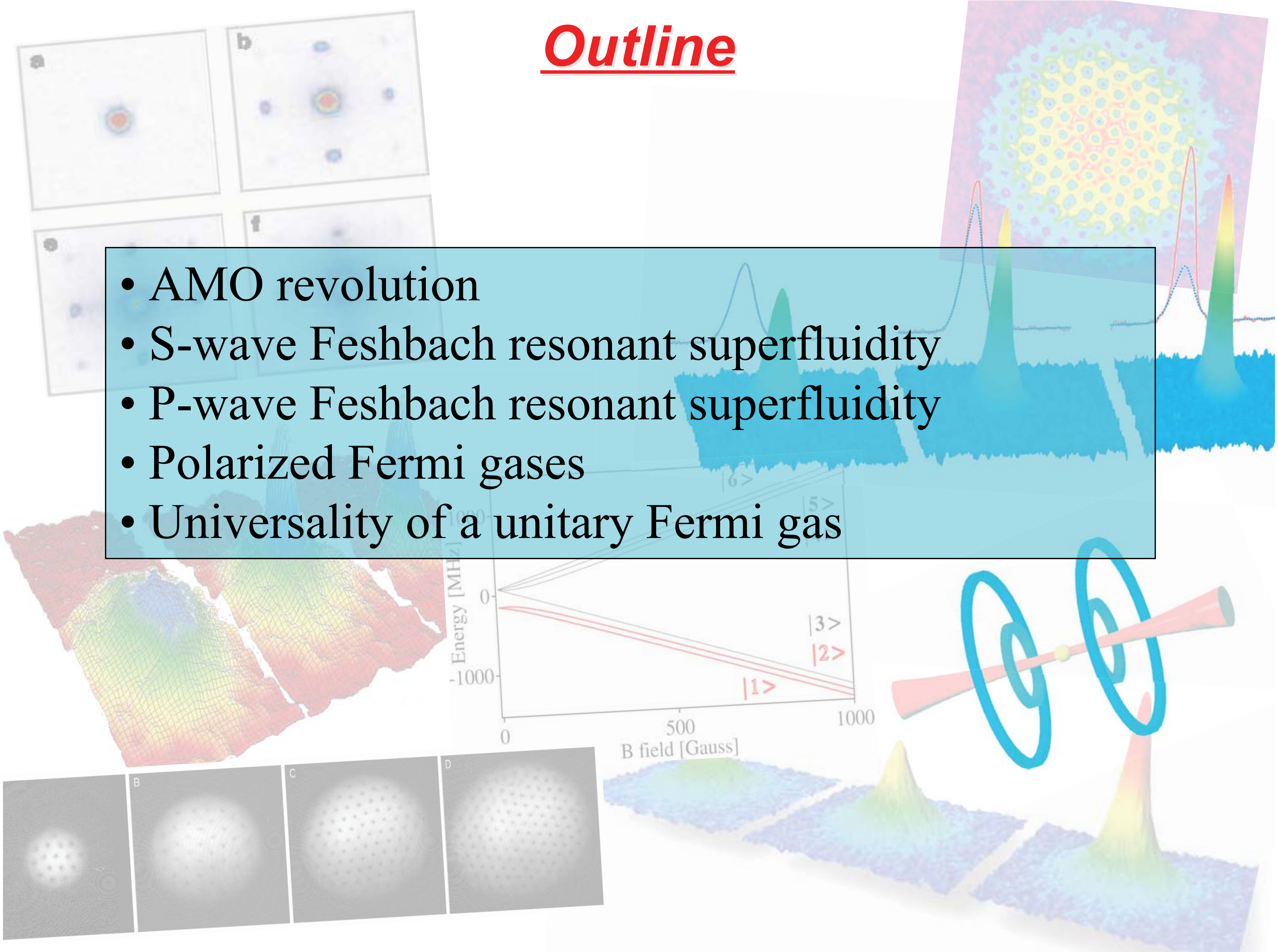
Leo Radzihovsky

for details see: *Gurarie, L.R., Annals of Physics '07*
Sheehy, L.R., Annals of Physics '07
L.R., Weichman, Park, Annals of Physics '08
Veillette, Sheehy, L.R., PRA, '07
Nikolic, Sachdev, PRA, '07

$: NSF, Packard

Outline

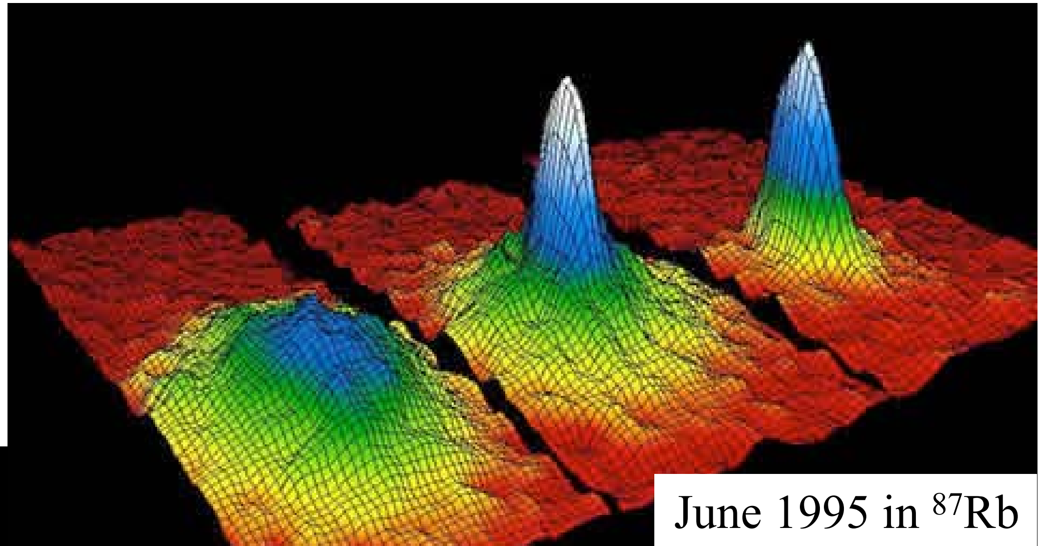
- AMO revolution
- S-wave Feshbach resonant superfluidity
- P-wave Feshbach resonant superfluidity
- Polarized Fermi gases
- Universality of a unitary Fermi gas



Revolution in AMO physics

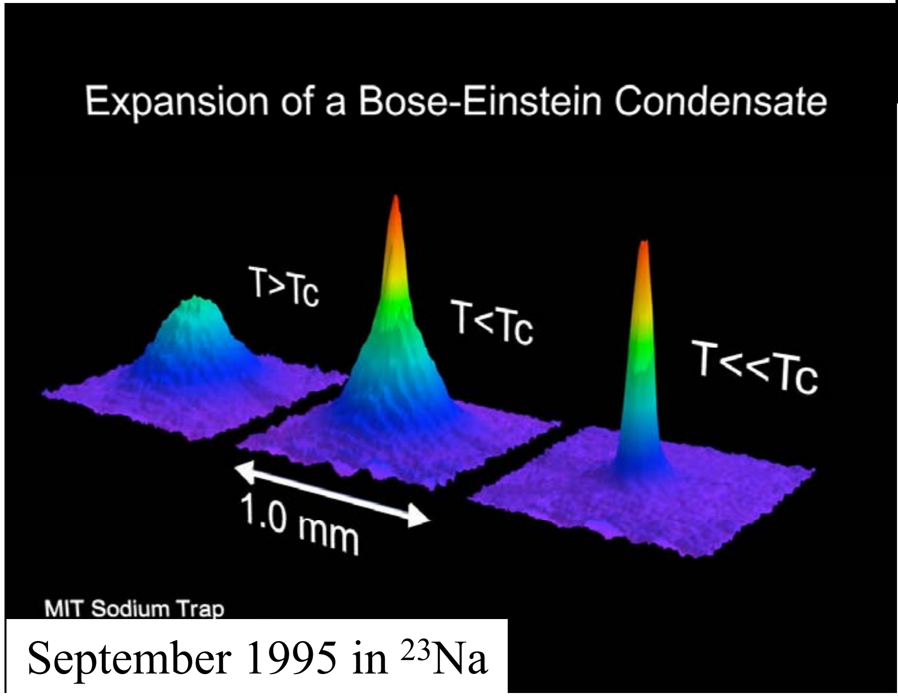
- degenerate Bose and Fermi atomic gases

Cornell and Wieman



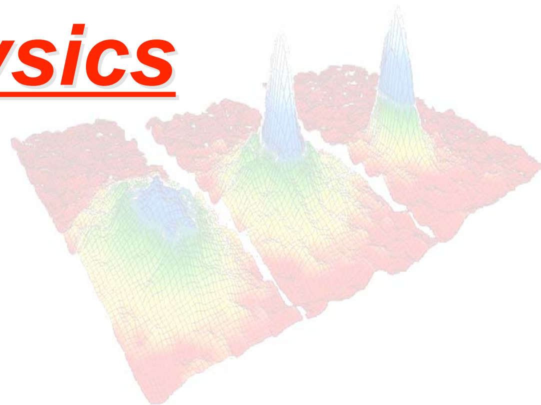
June 1995 in ^{87}Rb

Ketterle



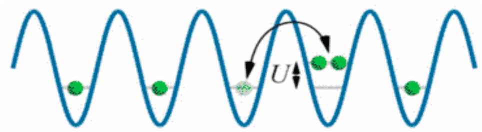
September 1995 in ^{23}Na

Revolution in AMO physics

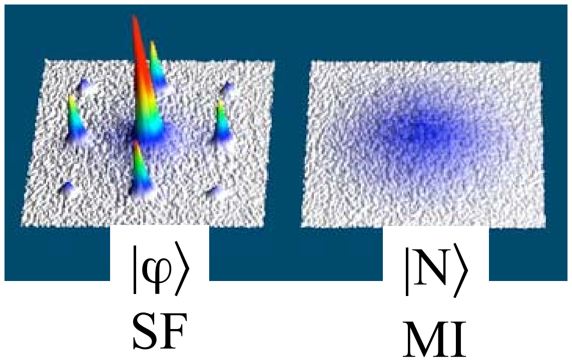


• degenerate Bose and Fermi atomic gases

• optical lattices



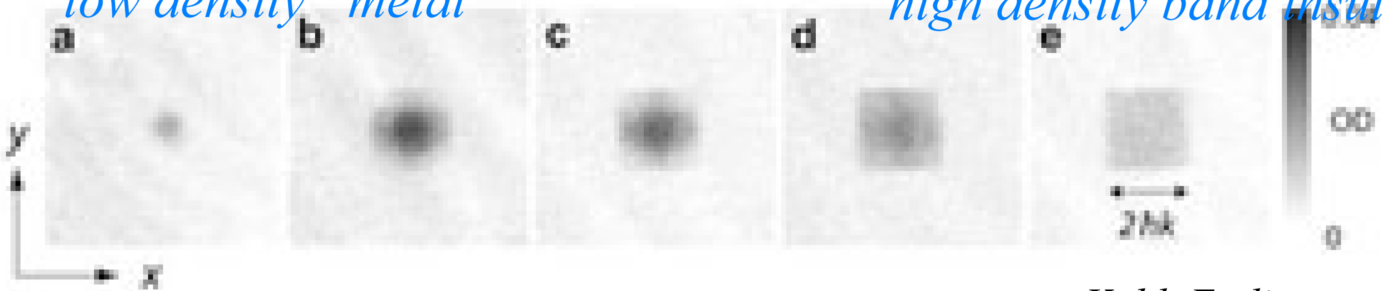
ac-Stark effect
(red-detuned, attractive)



Greiner, et al.

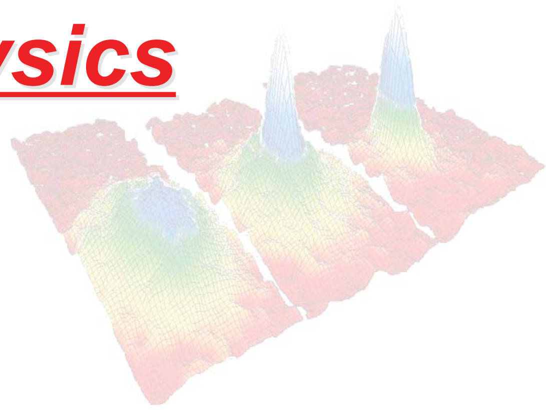
low density "metal"

high density band insulator



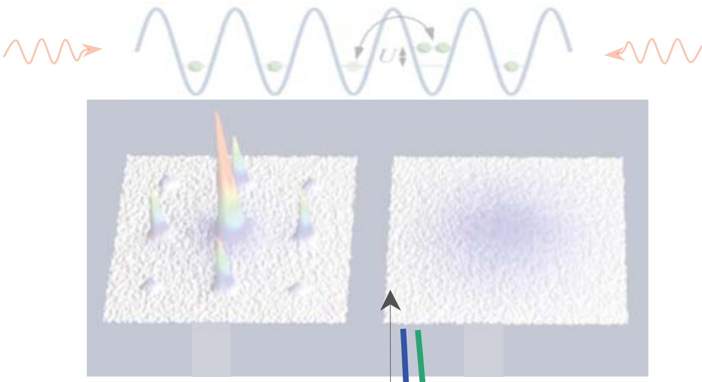
Kohl, Esslinger, et al. '05

Revolution in AMO physics

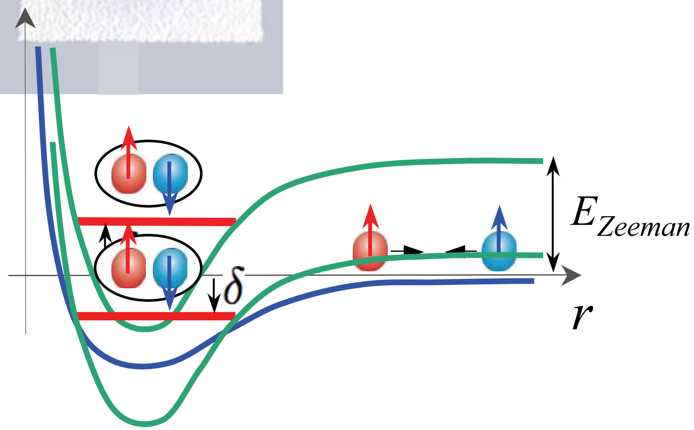


- degenerate Bose and Fermi atomic gases

- optical lattices

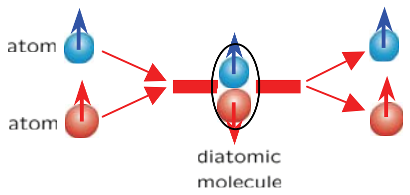


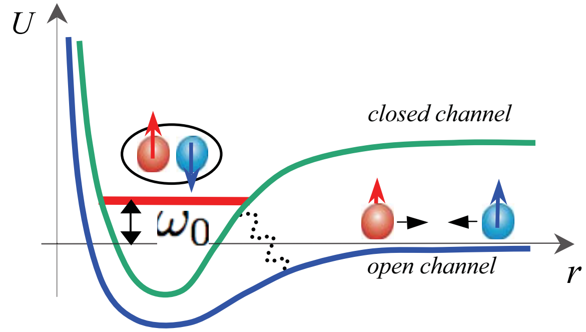
- Feshbach resonance



S-wave Feshbach resonant scattering

- tunability (strength and sign) of interactions (sudden and adiabatic)

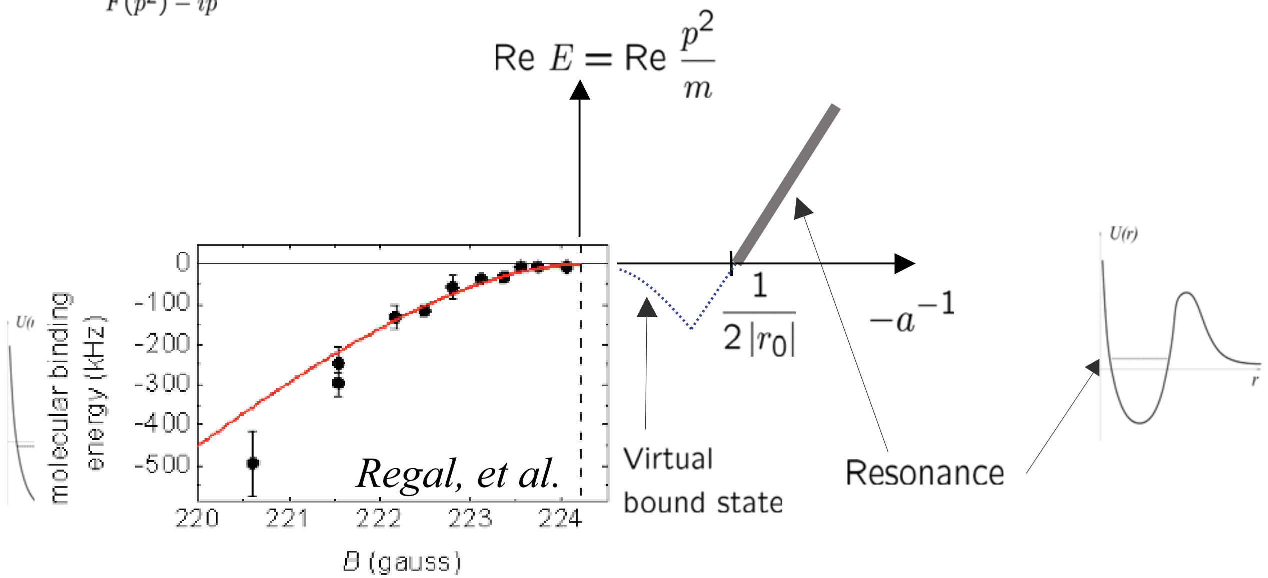


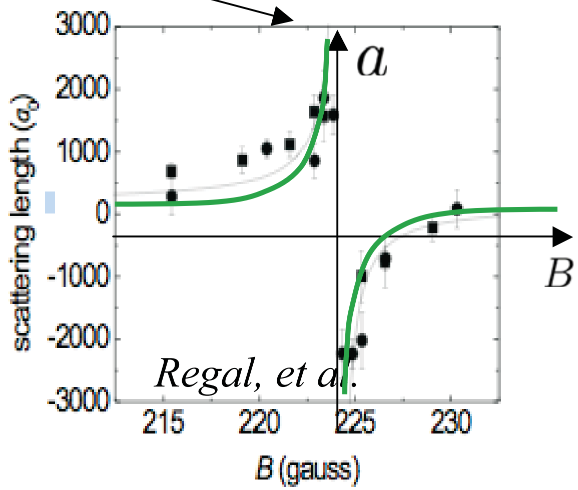


$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger \frac{\hat{p}^2}{2m}\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger$$

$$\longrightarrow \quad f_s(p) = \frac{1}{-a^{-1} + \frac{r_0}{2}p^2 - ip}, \quad \text{with } a \sim -\frac{g^2}{\omega_0} \sim a_{bg}\frac{\Delta B}{B_0 - B}, \quad r_0 \sim -\frac{1}{g^2}$$

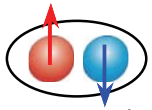
$$f_s(p) = \frac{1}{F(p^2) - ip}$$





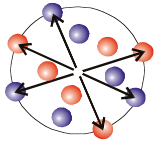
S-wave resonant fermionic superfluidity

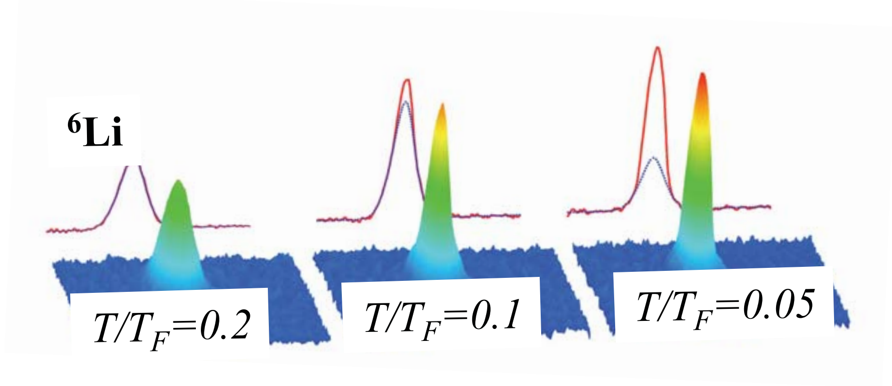
- molecular BEC *(Regal, Jin '03)*



K_2 , Li_2

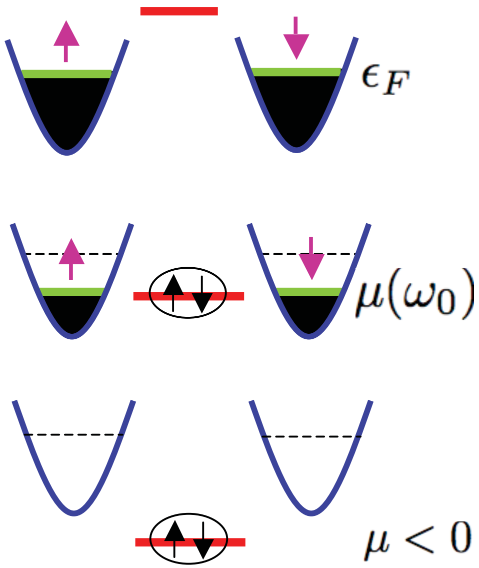
- BCS superfluid *(Regal, Jin 04*
  Zwierlein, Ketterle '04)

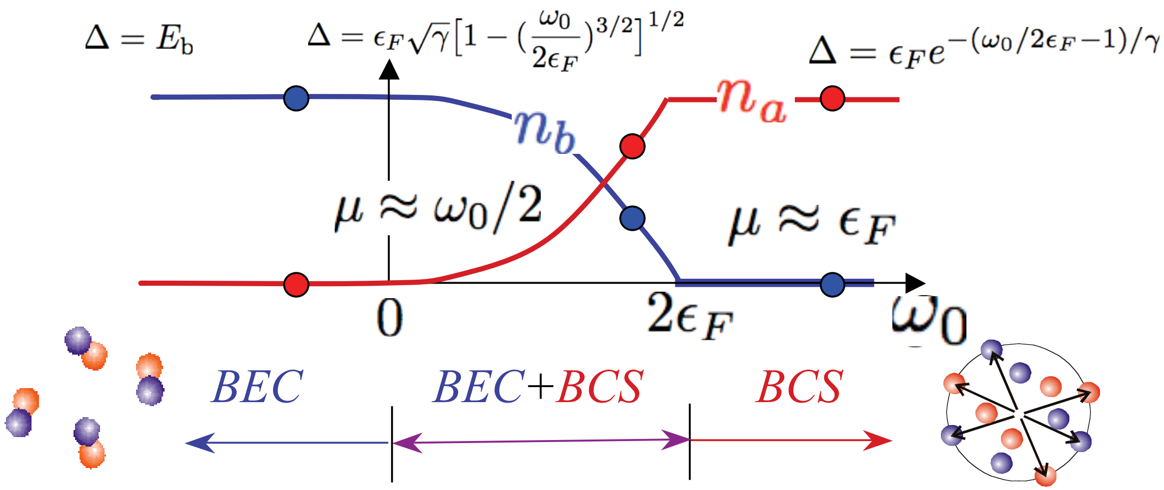




- BCS-BEC crossover:

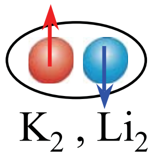
$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m} - \mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} - 2\mu + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger$$



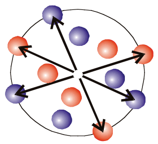


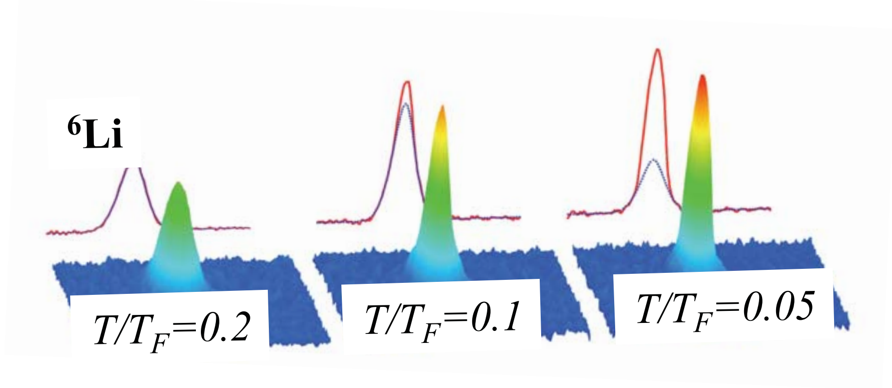
S-wave resonant fermionic superfluidity

- molecular BEC *(Regal, Jin '03)*



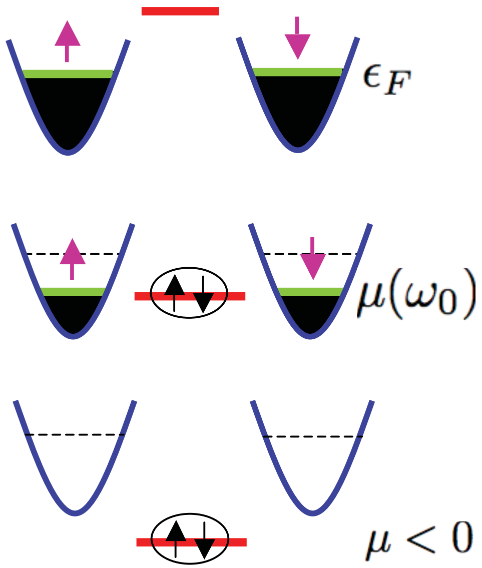
- BCS superfluid *(Regal, Jin 04*
  Zwierlein, Ketterle '04)

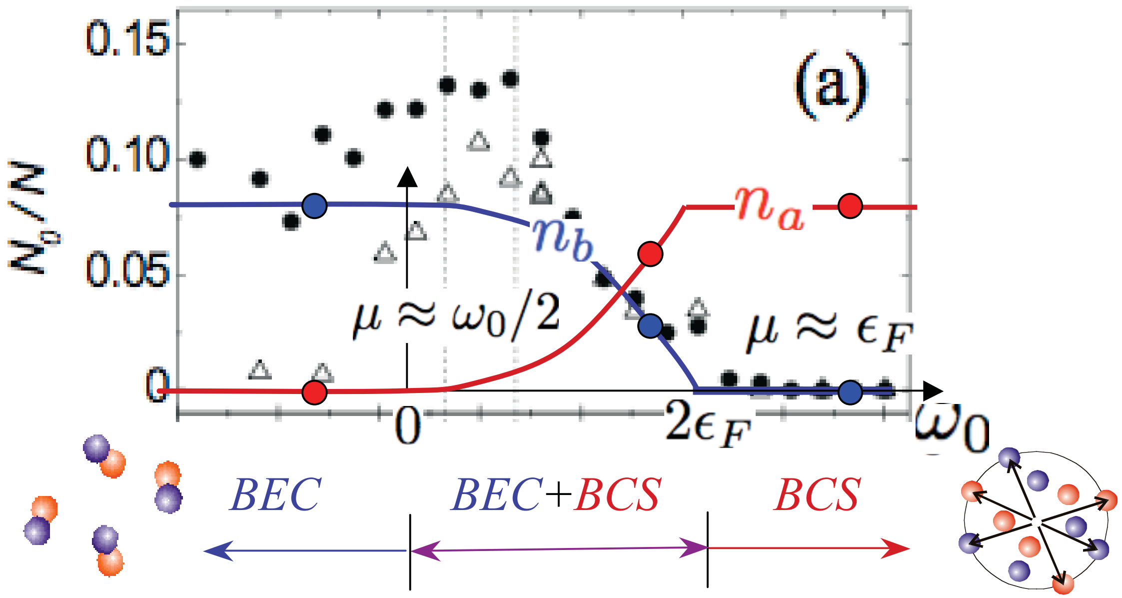




- BCS-BEC crossover:

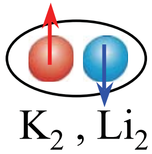
$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m} - \mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} - 2\mu + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger$$



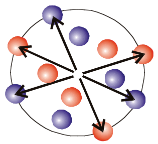


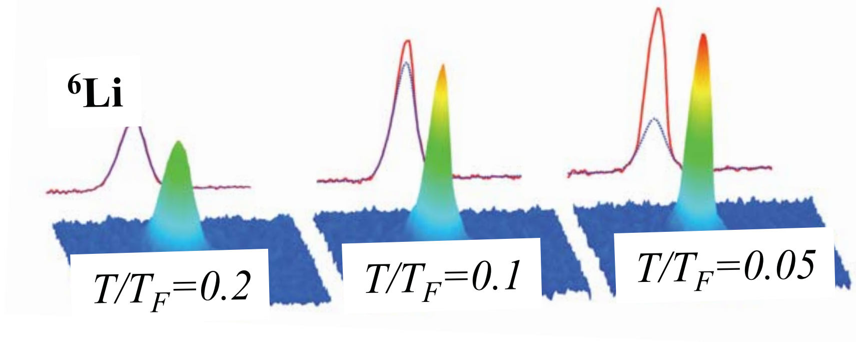
S-wave resonant fermionic superfluidity

- molecular BEC *(Regal, Jin '03)*



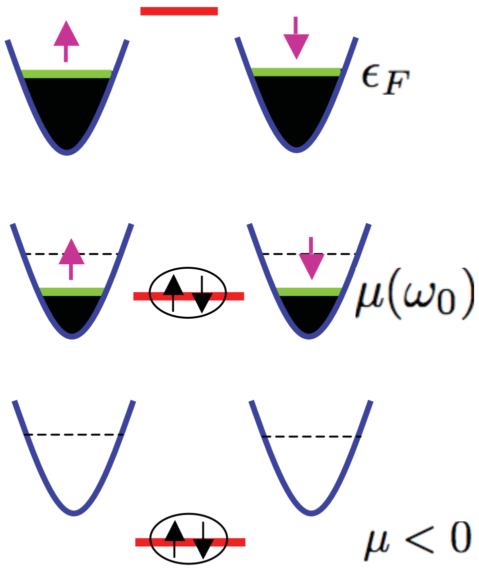
- BCS superfluid *(Regal, Jin 04*
  Zwierlein, Ketterle '04)

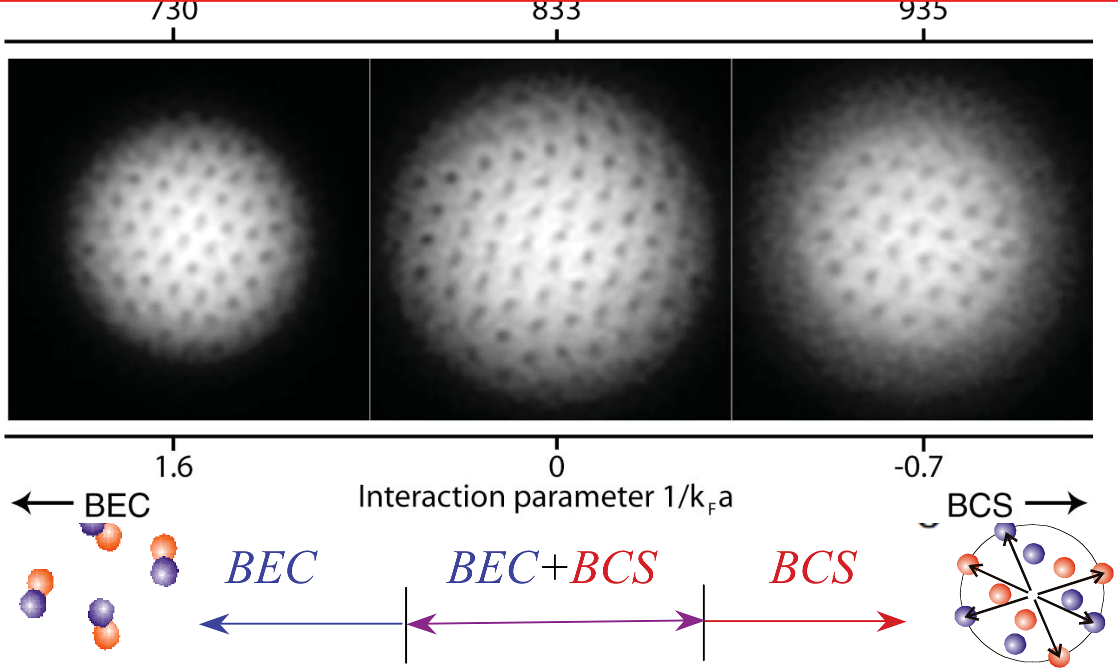




- BCS-BEC crossover:

$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m} - \mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} - 2\mu + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger$$





S-wave resonant superfluidity

$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m} - \mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} - 2\mu + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger$$

dimensionless coupling: $\gamma \sim \left(\frac{g\sqrt{n}}{\epsilon_F}\right)^2 \sim \frac{g^2}{\epsilon_F^{1/2}} \sim \frac{1}{r_0 n^{1/3}}$

$\gamma_{202G}^{^{40}K} \approx 5,\ \Delta B \sim 1G \sim 100\mu K$

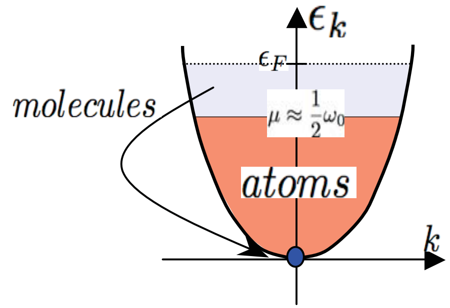
$\gamma_{544G}^{^{6}Li} \approx 0.1,\ \Delta B \sim 0.1G \sim 10\mu K$

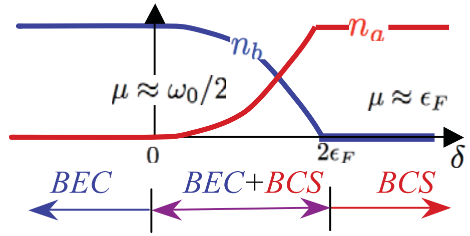
$\epsilon_F \sim 1\mu K$

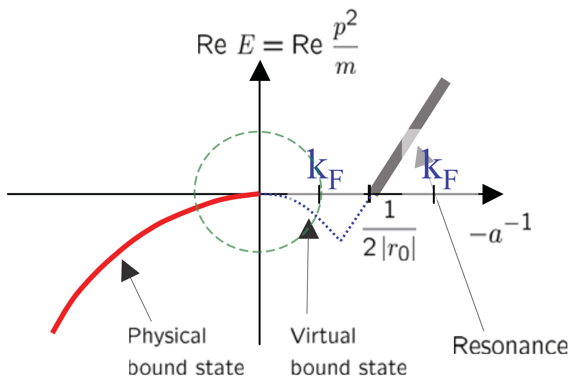
- **narrow** resonance $\gamma \ll 1 \ \rightarrow\ \mathrm{MFT}: \phi(x) = B$
- **broad** resonance $\gamma \gg 1$

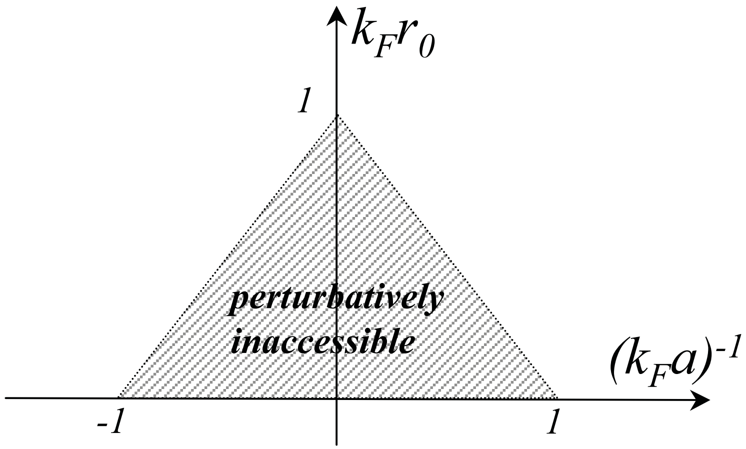
Strongly coupled ϕ and ψ

⇨ *MFT quantitatively uncontrolled*





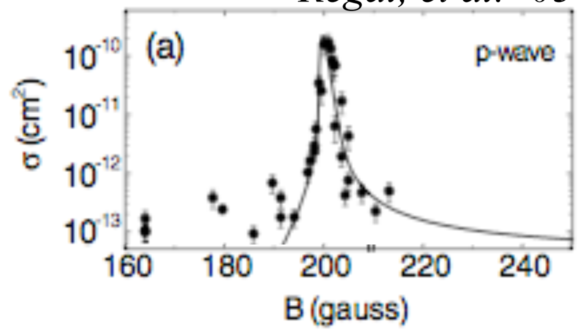




$$\gamma \approx |T_{k_F}| n/\epsilon_F \approx \frac{1}{(k_F a)^{-1} - k_F r_0 + 1}$$

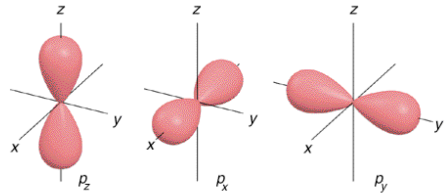
Finite angular momentum superfluidity

Regal, et al. '03

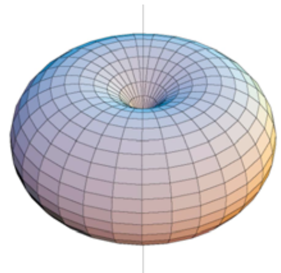


Motivation:

- *p-wave Feshbach resonances exist*
- *examples of ^{3}He and high-Tc superconductors*
- *multiple superfluids phases*

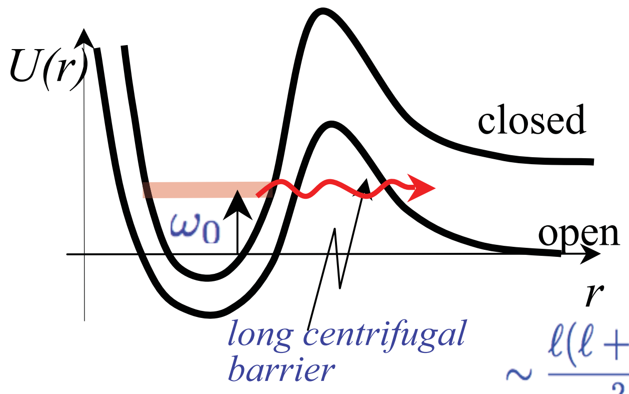


- *anisotropic gap with gapless excitations*



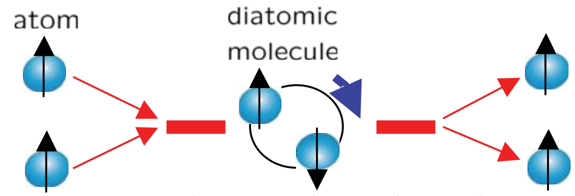
- *conventional (thermal and quantum) and topological phase transitions with detuning*
- *non-Abelian vortex excitations ⇒ topological QC?*

P-wave Feshbach resonant scattering



$\sim \frac{\ell(\ell+1)}{r^2}$ ➡ escape (molecular life) time $\tau \sim \Gamma^{-1} \sim E^{-\frac{3}{2}} \gg E^{-1}$, for $E \to 0$

naturally ***narrow!***



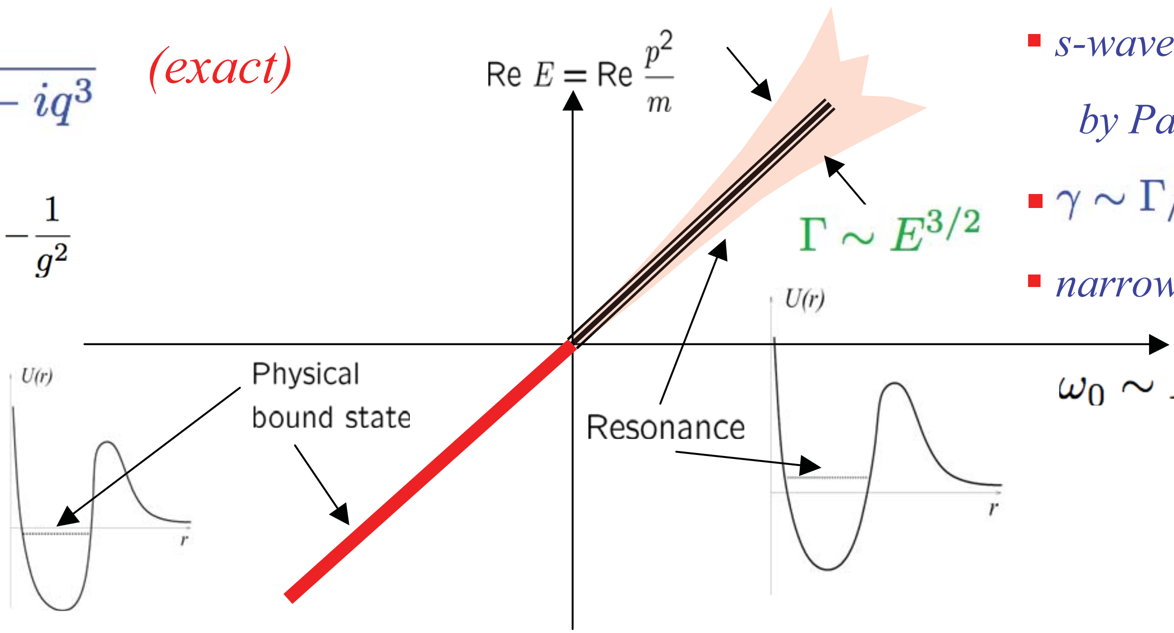
$\hat{b}^{\dagger}_{\mathbf{q},\alpha} = (\hat{b}^{\dagger}_{\mathbf{q,x}},\ \hat{b}^{\dagger}_{\mathbf{q,y}},\ \hat{b}^{\dagger}_{\mathbf{q,z}})$

$$\mathcal{H}_{2ch} = \psi^{\dagger}\frac{\hat{p}^2}{2m}\psi + \vec{\phi}^{\dagger}\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\vec{\phi} - ig\vec{\phi}\cdot\psi^{\dagger}\nabla\psi^{\dagger}$$

$$f_p = \frac{q^2}{-v^{-1} + \frac{q_0}{2}q^2 - iq^3}$$ *(exact)*

with $v^{-1} \sim -\frac{g^2}{\omega_0}$, $q_0 \sim -\frac{1}{g^2}$

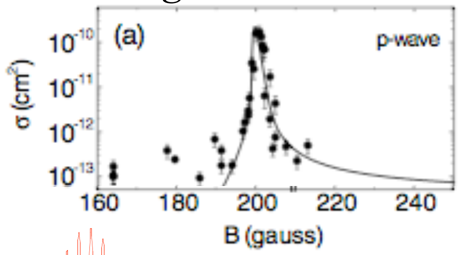
$$f_p(q) = \frac{q^2}{F(q^2) - iq^3}$$



- *s-wave suppressed by Pauli principle*
- $\gamma \sim \Gamma/E \sim E^{\frac{1}{2}} \ll 1$
- *narrows with* ϵ_F, n

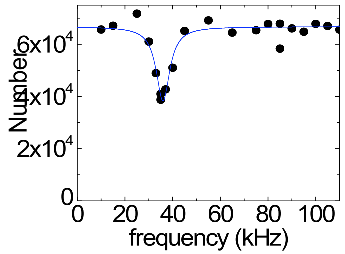
Experimental hopes for p-wave superfluidity

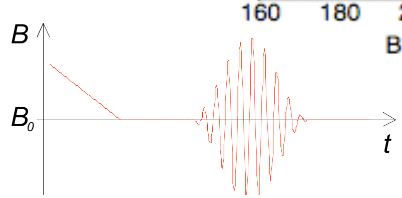
Regal, et al.'03



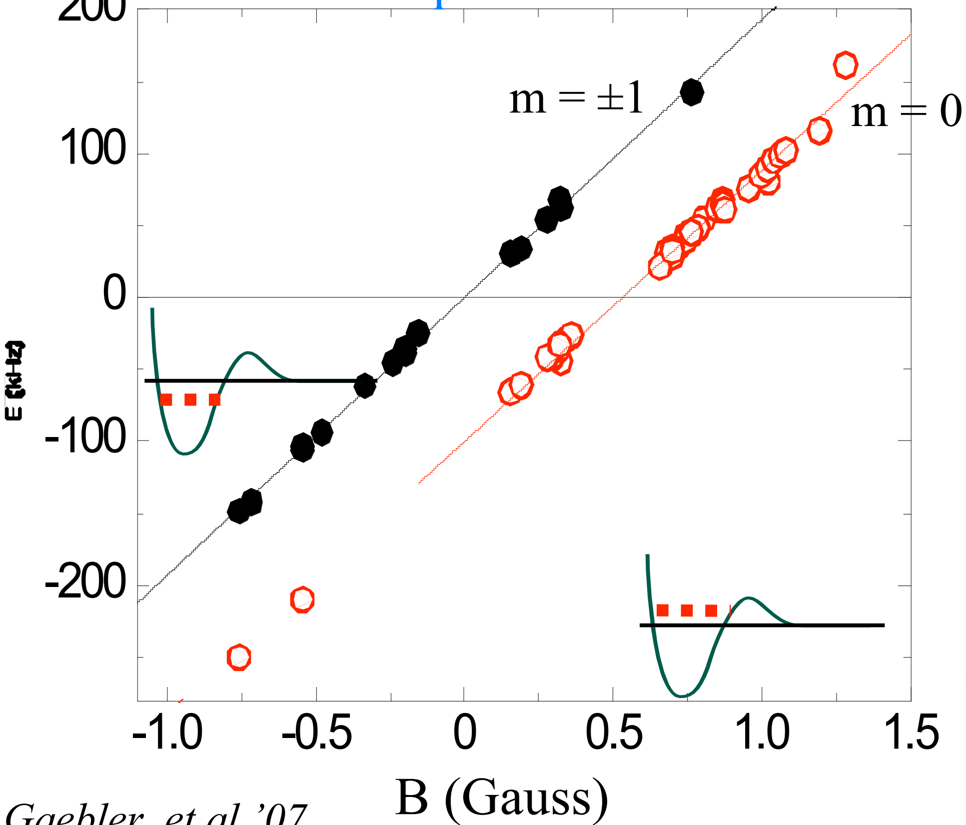
- p-wave Feshbach resonance in ^{40}K, ^{6}Li
- making p-wave molecules:

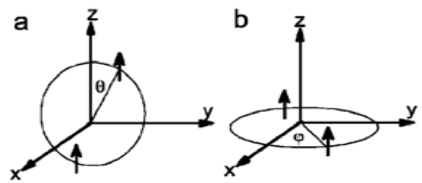
  resonant disappearance of atoms with oscillating B(t)





resonance position

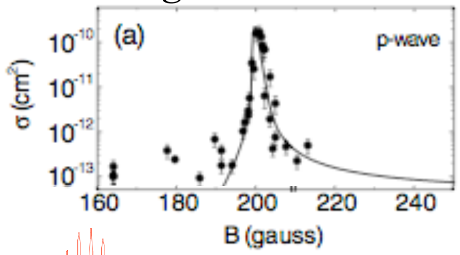




Gaebler, et al.'07

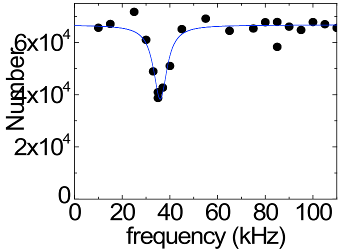
Experimental hopes for p-wave superfluidity

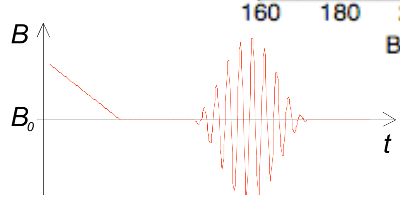
Regal, et al.'03



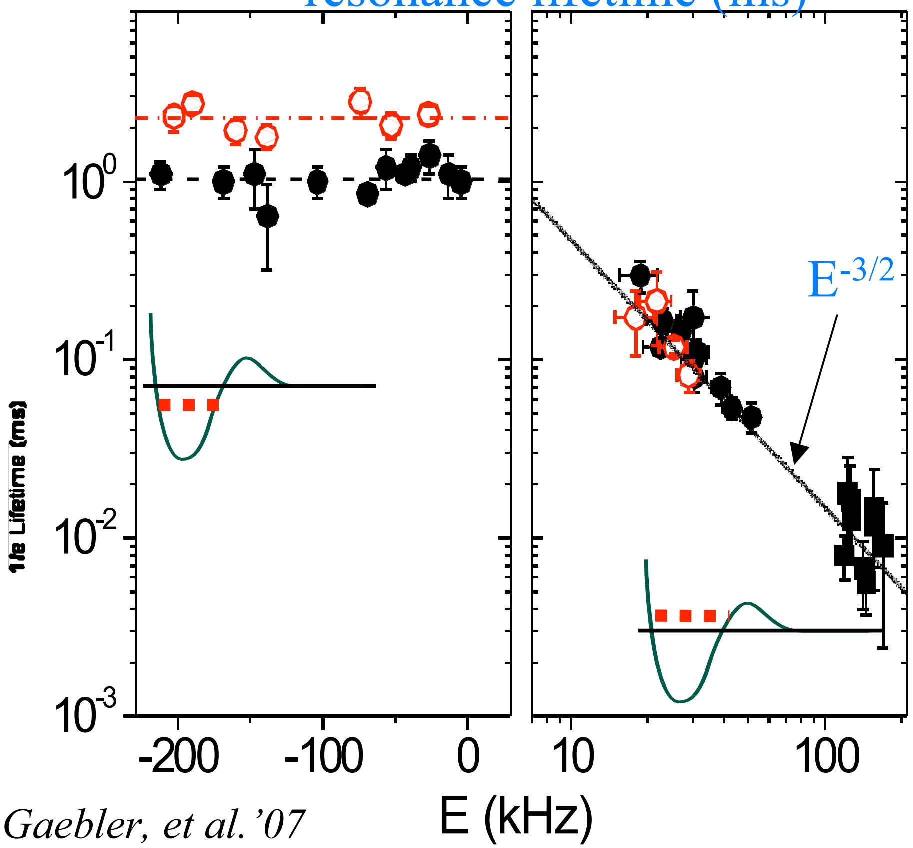
- p-wave Feshbach resonance in ^{40}K, ^{6}Li
- making p-wave molecules:

  resonant disappearance of atoms with oscillating B(t)

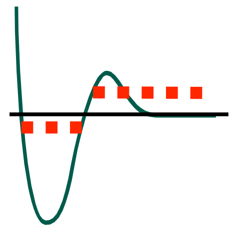




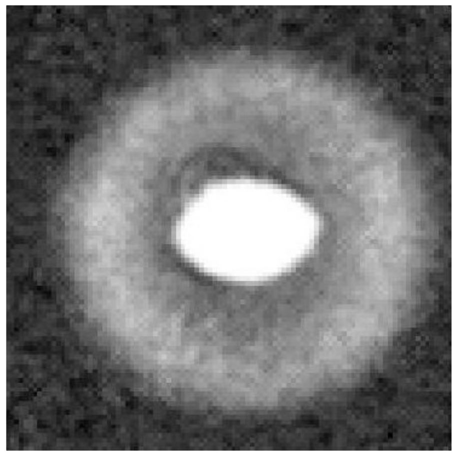
resonance lifetime (ms)



Gaebler, et al.'07



to see molecules:



look for energetic atoms

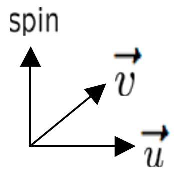
P-wave resonant superfluidity

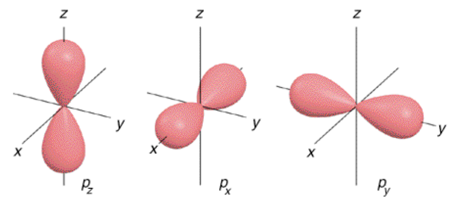
$$\mathcal{H}_{2ch} = \psi^\dagger \frac{\hat{p}^2}{2m}\psi + \vec{\phi}^\dagger \left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\vec{\phi} - ig\vec{\phi}\cdot\psi^\dagger\nabla\psi^\dagger$$

dimensionless coupling: $\gamma \sim \left(\frac{g\sqrt{n}k_F}{\epsilon_F}\right)^2 \sim g^2\epsilon_F^{1/2} \sim \frac{n^{1/3}}{q_0}$

- ***narrow*** resonance $\gamma \ll 1 \;\rightarrow\; \mathrm{MFT}: \vec{\phi}(x) = \vec{B}$

- ***complex vector*** order parameter:





$$\vec{B} = \vec{u} + i\,\vec{v} \;\Longleftrightarrow\; \psi_0 = B_z,\ \psi_\pm = \pm(B_x \pm iB_y)$$

$$\vec{B}\cdot\vec{k} = \sum_{m=0,\pm k} \psi_m Y_{1,m}(\hat{k})k$$

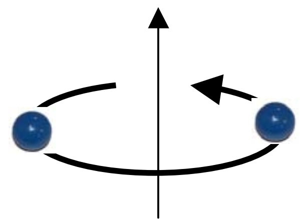
- sample states:

$v = 0 \;\Longleftrightarrow\; |m=0\rangle$ along $\vec{u}$

$(k_x \quad \beta - state\ in\ {}^3He)$

$u = v \;\Longleftrightarrow\; |m=1\rangle$ along $\vec{u}\times\vec{v}$

$(k_x + ik_y \quad "axial"\ Anderson - Morel\ state\ in\ {}^3He)$



$L_z = \pm 1, 0$

Mean-field theory $(\gamma \sim g^2 \epsilon_F^{1/2} \ll 1)$

$$H_p = \sum_{\mathbf{k}} \frac{k^2}{2m} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \sum_{\mathbf{q},\alpha} (\frac{q^2}{4m} + \epsilon_{0\alpha}) b^\dagger_{\mathbf{q},\alpha} b_{\mathbf{q},\alpha} + \sum_{\mathbf{k}} [\Delta_{\mathbf{k}}(\vec{B}) a^\dagger_{\mathbf{k}} a^\dagger_{-\mathbf{k}} + h.c.]$$

- *superfluid ground state:*

$$\text{molecular BEC } |\vec{B}\rangle \; + \; \text{Cooper pairing } |\text{BCS}_{\vec{\text{B}}}\rangle = \Pi_k (u_{\mathbf{k}} + v_{\mathbf{k}} a^\dagger_{-\mathbf{k}} a^\dagger_{\mathbf{k}})|0\rangle$$

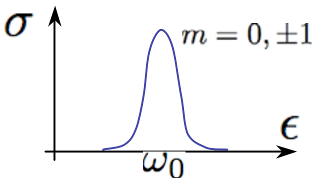
(closed) *(open)*

- *excitation spectrum:* $H_{ex} = \sum_{\mathbf{k}} E^{(a)}_{\mathbf{k}} \alpha^\dagger_{\mathbf{k}} \alpha_{\mathbf{k}} + \sum_{\mathbf{k},\alpha} E^{(m)}_{\mathbf{k},\alpha} \beta^\dagger_{\mathbf{k},\alpha} \beta_{\mathbf{k},\alpha}$

$$E^{(a)}_{\mathbf{k}} = \sqrt{\varepsilon^2_{\mathbf{k}} + \Delta^2_{\mathbf{k}}}, \qquad E^{(m)}_{\mathbf{k},\alpha} = \sqrt{\epsilon^2_{\mathbf{k}} + \mu_\alpha \epsilon_{\mathbf{k}}} \qquad \textit{with gap}: \;\; \Delta_k = 2g|\vec{B} \cdot \vec{k}|$$

- $\vec{B}, \;\; n_b, \;\; ,n_a, \;\; \mu$ *determined by :*

  - energy minimization (gap equation) $\rightarrow \dfrac{\partial E(\vec{B})}{\partial B_\alpha} = 0$
  - atom number equation $\rightarrow 2n_b + n_a = n$



Isotropic resonance at T=0 $(\omega_\alpha = \omega_0)$

$$E = (u^2+v^2)\left[\omega_0 - 2\mu + a_1 \ln\{a_0(u+v)\}\right] + a_1 \frac{u^3+v^3}{u+v} + a_2\left[\left(u^2+v^2\right)^2 + \frac{1}{2}\left(u^2-v^2\right)^2\right]$$

BCS

$$\text{BEC}: \quad (\vec{B}^* \cdot \vec{B})^2 + \frac{1}{2}|\vec{B}\cdot\vec{B}|^2$$

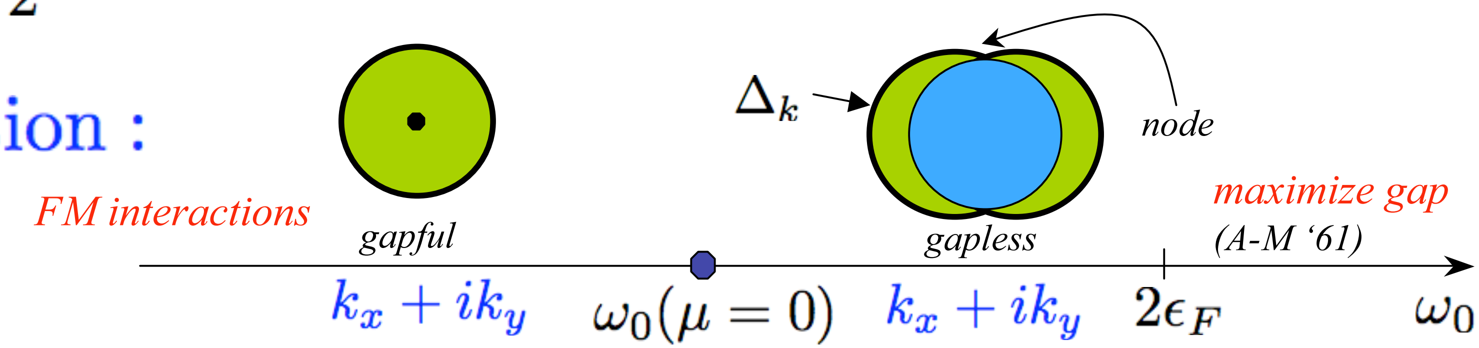
- BCS $(\omega_0 \gg 2\epsilon_F)$:
  - $\mu \approx \epsilon_F + \mathcal{O}(\gamma)$
  - $\dfrac{E_{k_x+ik_y}}{E_{k_x}} = \dfrac{1}{2}e > 1$

  ⟹ $(Anderson - Morel\ A_1\ phase)$

  $u = v \sim e^{-(\omega_0 - 2\epsilon_F)/\gamma\epsilon_F} \Longrightarrow k_x + ik_y\ (m=1)$

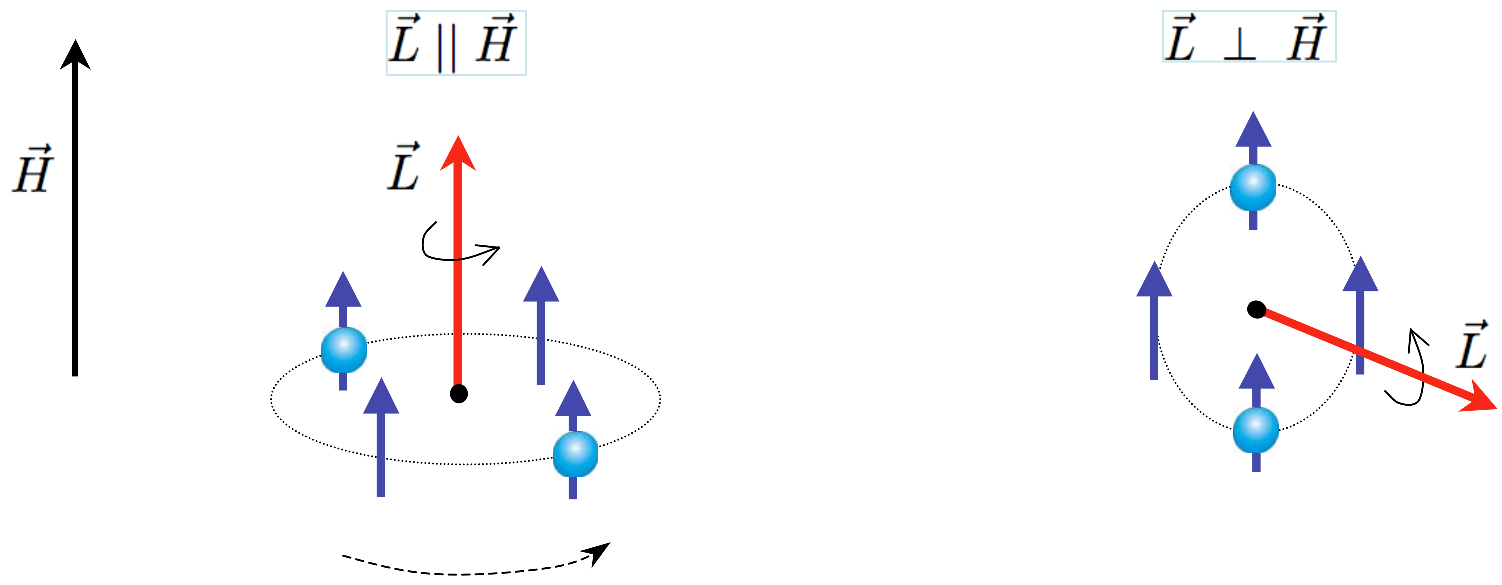
- BEC $(\omega \ll 2\epsilon_F)$:
  - $\mu \approx \dfrac{1}{2}\omega_0 + \mathcal{O}(\gamma)$ ⟹ $u = v \approx \sqrt{n - n(\omega_0/2)} \Longrightarrow k_x + ik_y\ (m=1)$

- transition :

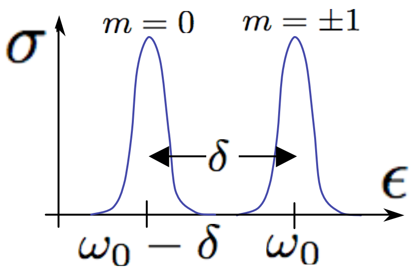


Dipolar-interaction FR splitting

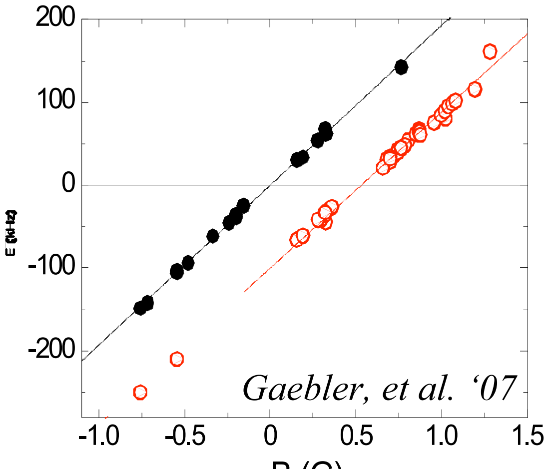
Ticknor, et al '03



$$H_{\text{molecule}} = \omega_{\parallel} b^{\dagger}_{\parallel} b_{\parallel} + \omega_{\perp} \vec{b}^{\dagger}_{\perp} \cdot \vec{b}_{\perp} \qquad \omega_{\parallel} < \omega_{\perp}$$



Ticknor, Regal, et al. '03

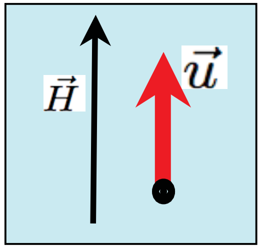


$$E = \sum_{\alpha} (u_{\alpha}^2 + v_{\alpha}^2)[\omega_{\alpha} - 2\mu + \ldots$$

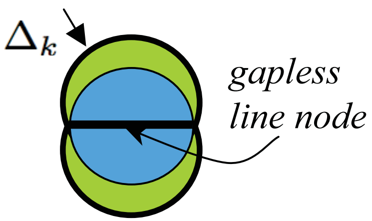
P-wave superfluid phases

$$E=\sum_{\alpha}(u_\alpha^2+v_\alpha^2)\left[\omega_{0\alpha}-2\mu+a_1\ln\{a_0(u+v)\}\right]+a_1\frac{u^3+v^3}{u+v}+a_2\left[(u^2+v^2)^2+\frac{1}{2}(u^2-v^2)^2\right]$$

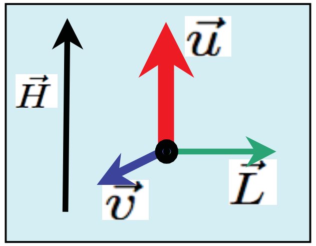
- k_x - *state:* *β-phase of* ^3He $(m_{||}=0)$



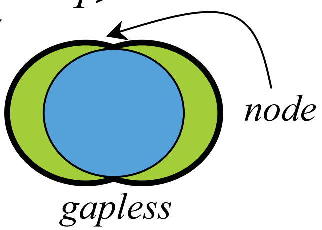
  - $u \approx u_0 e^{\delta}$, $v = 0$
  - equatorial node line for $\mu > 0 \Rightarrow C \sim T^{\alpha}$, ...
  - fully gapped for $\mu < 0 \Rightarrow C \sim e^{-|B-B_0|^2/T}$, ...
  - spontaneously broken symmetries: *U(1)*



- $k_x + i\,\sigma\, k_y$- *state:* *"deformed" A_1 -phase of* ^3He $(m_\perp=1)$



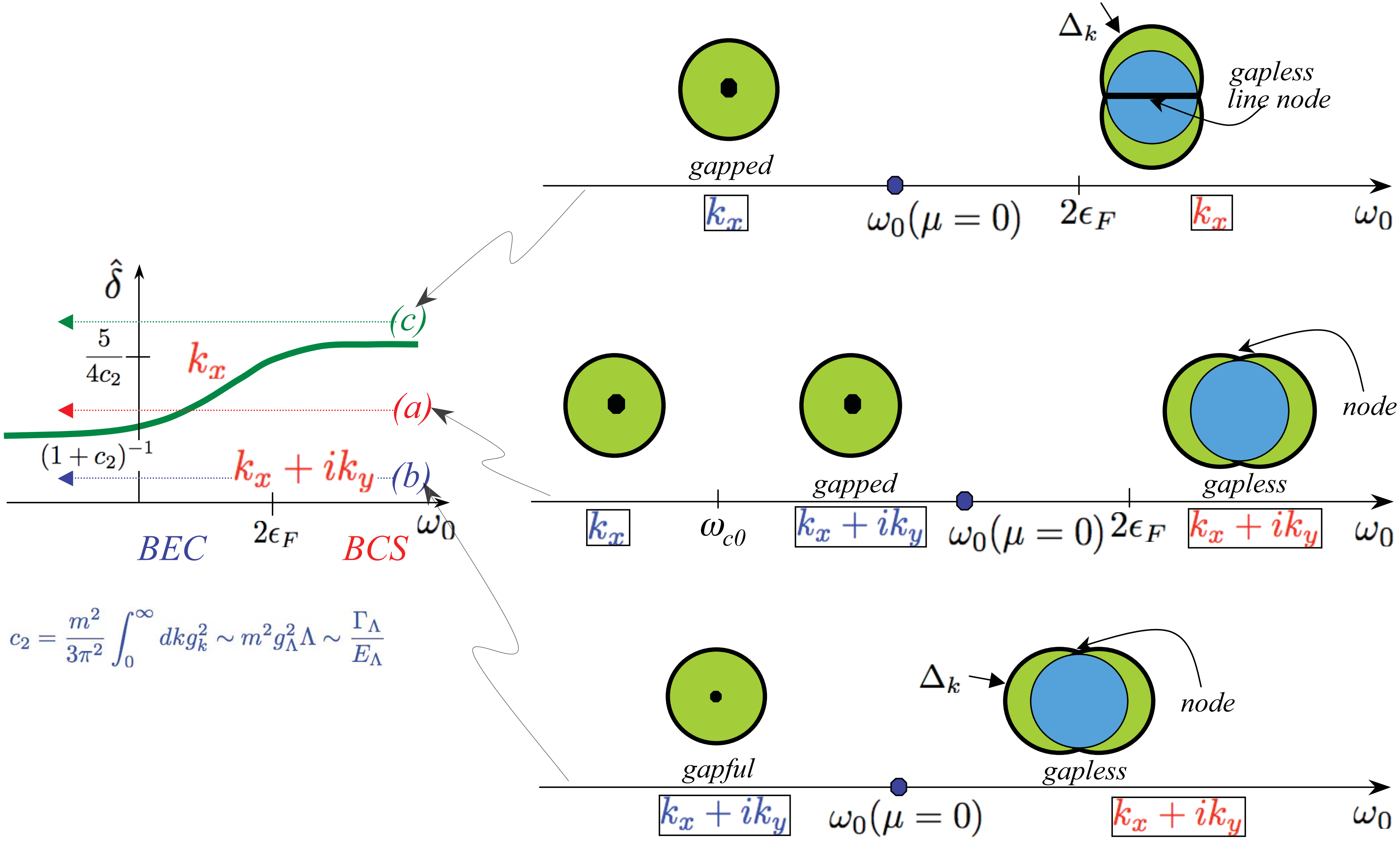
  - $u \approx u_0(1+\delta)\,e^{\delta/2}$, $v \approx u_0(1-\delta)\,e^{\delta/2} \Rightarrow |m_\perp=1\rangle + \delta\,|m_\perp=-1\rangle$
  - polar point nodes for $\mu > 0 \Rightarrow C \sim T^{\alpha}$, ...
  - fully gapped for $\mu < 0 \Rightarrow C \sim e^{-|B-B_0|^2/T}$, ...
  - spontaneously broken symmetries: *U(1), O(2), T*



T=0 phase diagrams

also see Cheng, Yip '05

$$E = \sum_\alpha (u_\alpha^2 + v_\alpha^2)\left[\omega_{0\alpha} - 2\mu + a_1 \ln\{a_0(u+v)\}\right] + a_1 \frac{u^3+v^3}{u+v} + a_2\left[(u^2+v^2)^2 + \frac{1}{2}(u^2-v^2)^2\right]$$

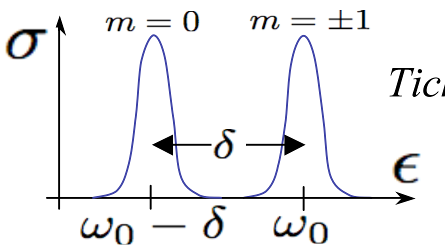


$$c_2 = \frac{m^2}{3\pi^2}\int_0^\infty dk g_k^2 \sim m^2 g_\Lambda^2 \Lambda \sim \frac{\Gamma_\Lambda}{E_\Lambda}$$

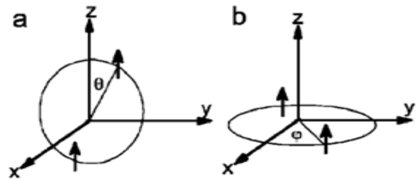
Anisotropic p-wave superfluidity

Gurarie, L.R., Andreev '05
Cheng and Yip '05

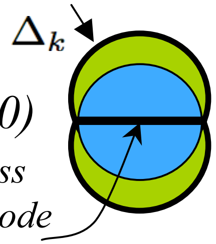
- *resonance splits:*



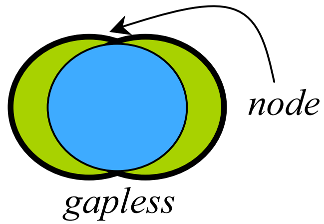
Ticknor, Regal, et al.

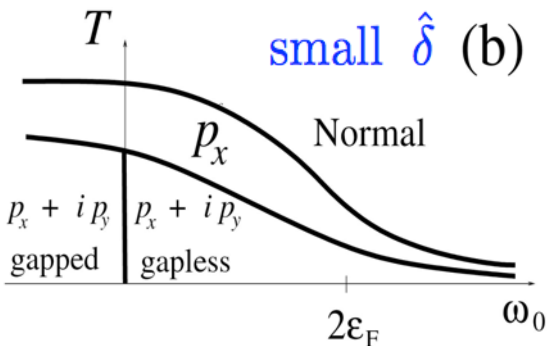


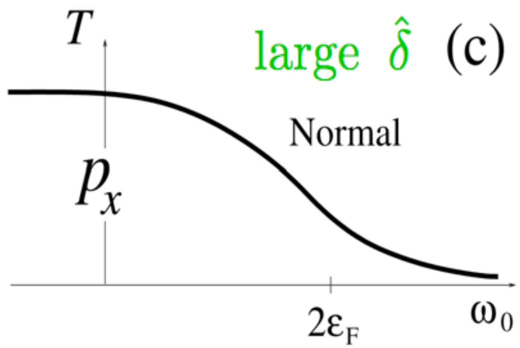
- *two competing states:* p_x *(m_z=0)* *gapless line node*

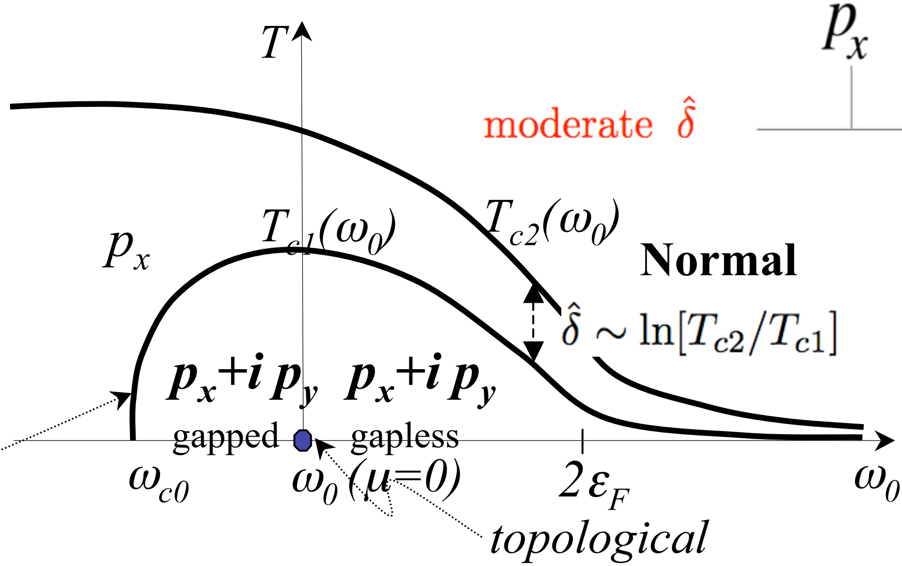


and $p_x+i\,p_y$ *(m_z=±1)*





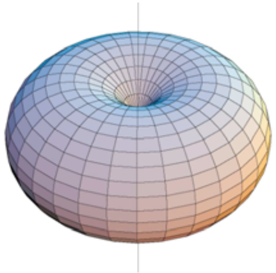


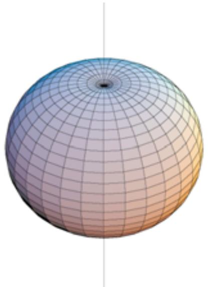


$T_{c1} \sim |\omega_0 - \omega_{0c}|^{1/2}$

Gapless $\longrightarrow$ gapped superfluid transitions

$p_x + i\, p_y$ $\xrightarrow{\mu \text{ changes sign}}$ $p_x + i\, p_y$





$$E_p = \sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + g^2\,(p_x^2 + p_y^2)\,|B|^2}$$

$$E_p = \sqrt{\left(\frac{p^2}{2m} + |\mu|\right)^2 + g^2\,(p_x^2 + p_y^2)\,|B|^2}$$

G. E. Volovik, JETP Lett. 80, 343 (2004)

p_x $\xrightarrow{\mu \text{ changes sign}}$ p_x

$$E_p = \sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + g^2\,p_x^2\,|B|^2}$$

$$E_p = \sqrt{\left(\frac{p^2}{2m} + |\mu|\right)^2 + g^2\,p_x^2\,|B|^2}$$

$p_x + i\,p_y$ superfluid in 2D

- Pfaffian (Moore-Read) state from FQH $\quad |p_x + ip_y{}_{BCS}\rangle = \prod_p \left[u_p + v_p a^\dagger_{-p} a^\dagger_p\right] |0\rangle$

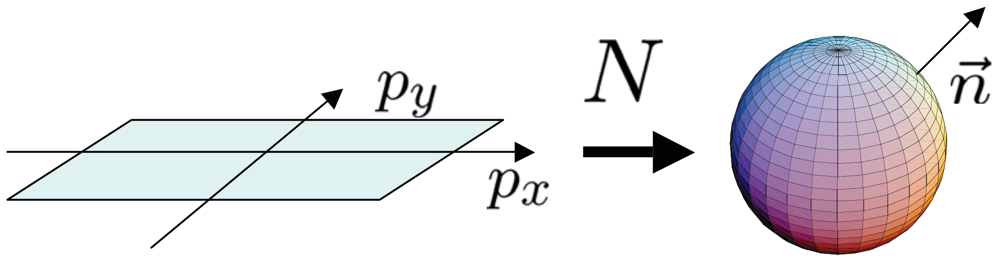
$$\Psi(z_1, z_2, \ldots, z_{2N}) = \sum_P (-1)^P \frac{1}{z_{P_1} - z_{P_2}} \frac{1}{z_{P_3} - z_{P_4}} \cdots \frac{1}{z_{P_{N-1}} - z_{P_N}}$$

- topological classification in terms of u_p and v_p

Anderson's pseudospin

$$\begin{cases} n_x + i n_y = 2v^* u \\ n_z = |v|^2 - |u|^2 \end{cases}$$

$$\vec{n} = \frac{1}{\sqrt{\left(\frac{p^2}{2m_a} - \mu\right)^2 + 4B^2\left(p_x^2 + p_y^2\right)}} \begin{pmatrix} 2gBp_x \\ -2gBp_y \\ \frac{p^2}{2m_a} - \mu \end{pmatrix}$$



Explicit calculations show that

N=0 if $\mu < 0$

N=1 if $\mu > 0$

$$N = \frac{1}{8\pi} \int d^2p \left[\vec{n} \cdot \partial_\alpha \vec{n} \times \partial_\beta \vec{n}\ \epsilon_{\alpha\beta}\right]$$

topological invariant

$p_x + i\,p_y$ superfluid in 2D

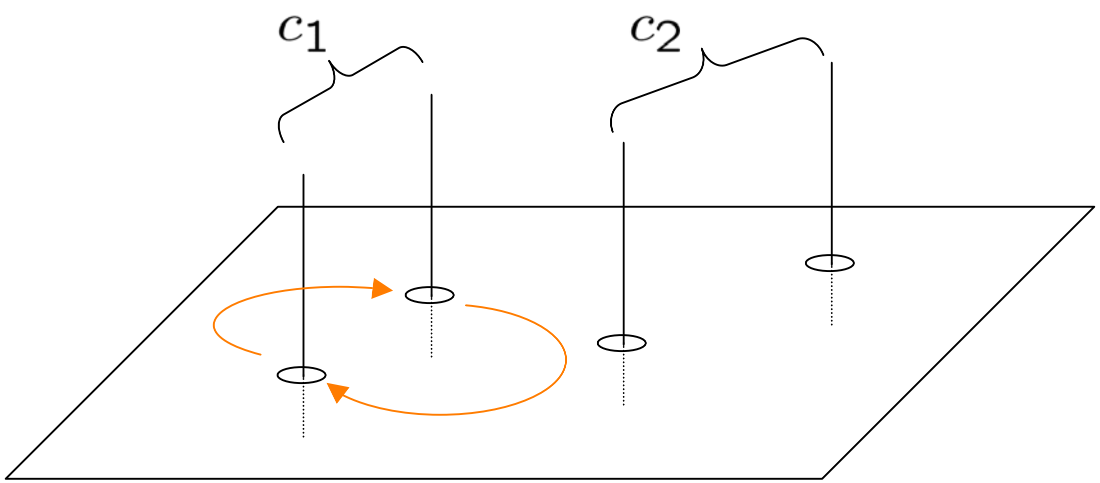
- Pfaffian (Moore-Read) state from FQH $|p_x + ip_{y\,BCS}\rangle = \prod_p \left[u_p + v_p a^\dagger_{-p} a^\dagger_p\right]|0\rangle$

$$\Psi(z_1, z_2, \ldots, z_{2N}) = \sum_P (-1)^P \frac{1}{z_{P_1} - z_{P_2}} \frac{1}{z_{P_3} - z_{P_4}} \cdots \frac{1}{z_{P_{N-1}} - z_{P_N}}$$

- topological classification in terms of u_p and v_p
- gapped (N=1, BCS) ⇒ gapped (N=0, BEC) superfluid transition at $\mu=0$

Read and Green, PRB 61, 10267 (2000)

- vortex excitations with non-Abelian statistics *Ivanov, PRL (2001)*



one fermion (2 states –either empty or occupied fermion) per two vortices

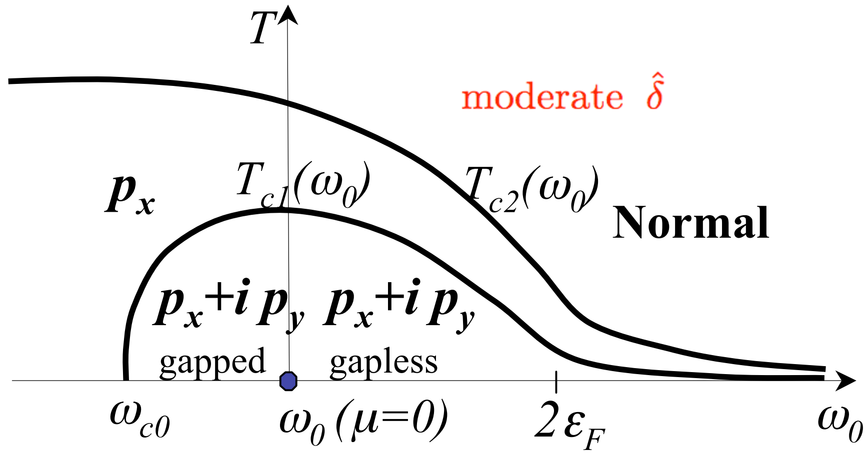
$2^{\frac{n}{2}}$ states per n vortices

- suggested to be used as qubits for quantum computers

Kitaev, Ann. Phys. 303, 2 (2003)

Summary of p-wave superfluidity

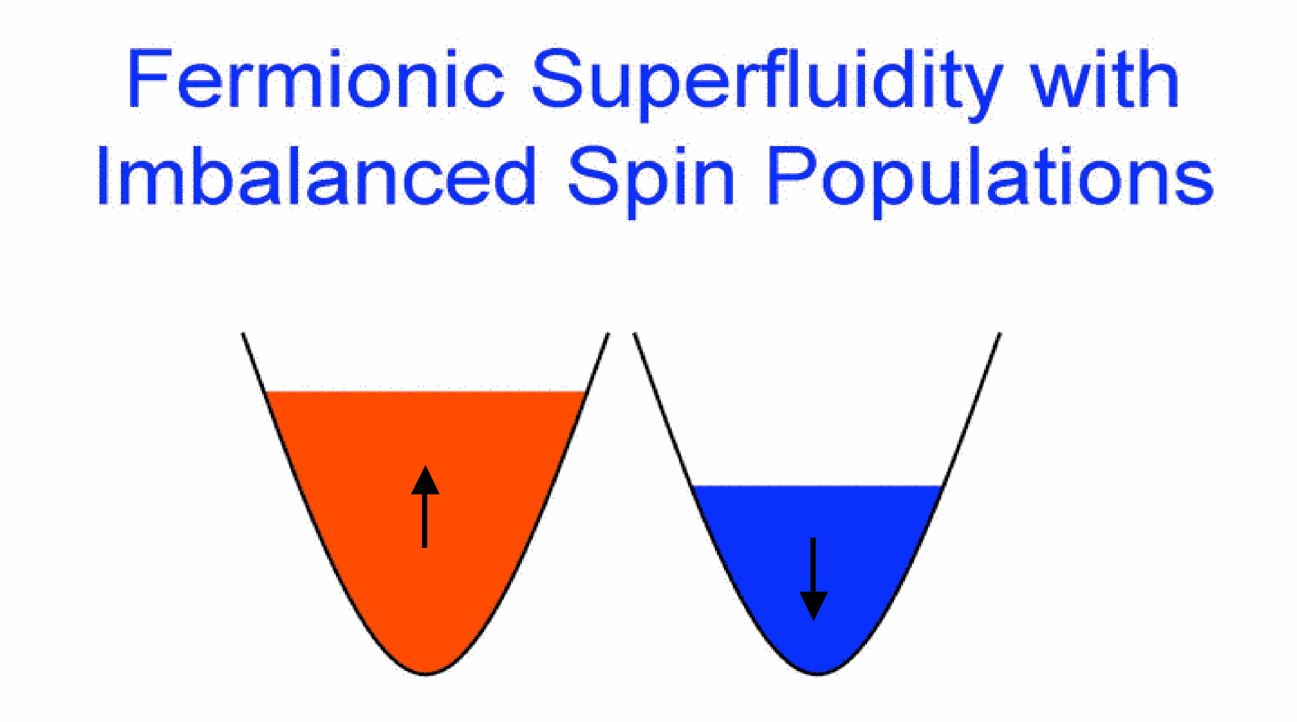
- mapped out T, $\omega_0 \propto B$, δ phase diagram for p-wave Feshbach resonant Fermi gas
  - *p_x and $p_x + i\, p_y$ superfluids*
  - *thermal, quantum and topological SF => SF transitions*



- quantitatively accurate description for small $\gamma=\Gamma/\varepsilon_F$ (low n)
- realization of topological states, majorana zero modes, and non-Abelian statistics of Pfaffian (Moore-Read) state
- p-wave Feshbach molecules observed in K^{40}
- …BUT
  - ❖ *short (msec) molecular lifetime (see Levinson, et al, PRL 2007)*
  - ❖ *what about Li^6*
  - ❖ *need better quantitative understanding of stability*

Imbalanced ("magnetized") BEC-BCS

- motivation: *superconductivity in B field, quarks-gluon plasma,…*
- natural realization in cold atoms: $H_h = H - h(N_\uparrow - N_\downarrow)$



$$\mathcal{H} = \psi_\sigma^\dagger\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_\sigma + \lambda\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$$

$$n = \psi_\uparrow^\dagger\psi_\uparrow + \psi_\downarrow^\dagger\psi_\downarrow, \quad \Delta n = \psi_\uparrow^\dagger\psi_\uparrow - \psi_\downarrow^\dagger\psi_\downarrow$$

$b_q = B_Q \, \delta_{q,Q}$ **Mean-field theory** *(valid for* $\gamma \sim g^2/\epsilon_F^{1/2} \ll 1$ *)*

$$H_{\mu,h} = H - \mu N - h\Delta N$$

$N = N_{a\uparrow} + N_{a\downarrow} + 2\,N_b$

$\Delta N = N_{a\uparrow} - N_{a\downarrow}$

- ***ground state:*** $|gs\rangle = \Pi'_{\mathbf{k}}\big(u_{\mathbf{k},\mathbf{Q}} + v_{\mathbf{k},\mathbf{Q}} a^\dagger_{-\mathbf{k}+\mathbf{Q}/2,\downarrow} a^\dagger_{\mathbf{k}+\mathbf{Q}/2,\uparrow}\big)|0\rangle$

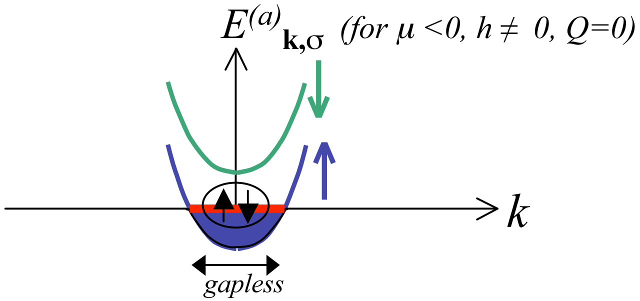
- ***ground state energy:*** $E_{gs} = (\frac{Q^2}{4m} + \delta - 2\mu)B_Q^2 - \sum_{\mathbf{k}}(E_k - \varepsilon_k) + \sum_{\mathbf{k}}[E_{\mathbf{k},\uparrow}\theta(-E_{\mathbf{k},\uparrow}) + E_{\mathbf{k},\downarrow}\theta(-E_{\mathbf{k},\downarrow})]$

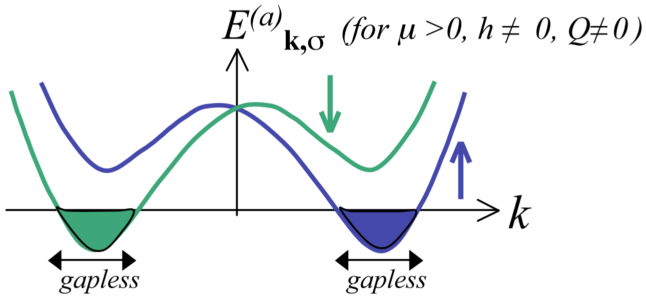
$$E_k = (\varepsilon_k^2 + g^2 B_Q^2)^{1/2}, \quad \varepsilon_k = \frac{k^2}{2m} - \mu + \frac{Q^2}{8m}$$

- ***excitation spectrum:*** $H_{ex} = \sum'_{\mathbf{k},\sigma} E^{(a)}_{\mathbf{k},\sigma}\alpha^\dagger_{\mathbf{k},\sigma}\alpha_{\mathbf{k},\sigma} + \sum'_{\mathbf{k},\sigma} E^{(b)}_{\mathbf{k},\sigma}\beta^\dagger_{\mathbf{k}}\beta_{\mathbf{k}}$

$$E^{(a)}_{\mathbf{k},\sigma} = E_k \mp (h + \mathbf{k}\cdot\mathbf{Q}/2m), \qquad E^{(b)}_k = \sqrt{\epsilon_k^2 + V_0\epsilon_k} \quad (\text{for } Q = 0)$$

(gapped and gapless k's) *(gapless k's collective; also phonons* $Q \neq 0$*))*





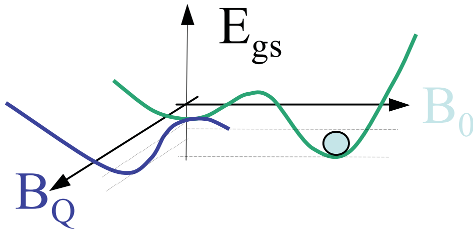
- ***determine*** $\boldsymbol{B_Q, N_b, N_{a\uparrow}, N_{a\downarrow}\ (\Delta N_a), Q}$ ***by:***

energy minimization ⇨ $\frac{\partial E_{gs}}{\partial B_Q} = 0$ *(gap equation),* $\frac{\partial E_{gs}}{\partial \mathbf{Q}} = 0$ *(*$P_{total} = 0$*)*

BSC and crossover regimes ($\delta > 0$)

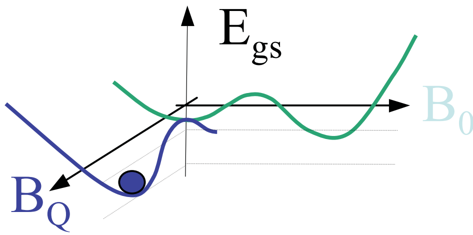
μ, h fixed

- ***BCS SF*** $B_0 \neq 0$, $B_Q = 0$, $\Delta N = 0$: $0 < h < h_c = \frac{1}{\sqrt{2}}\, g\, B_0(\mu,\delta)$

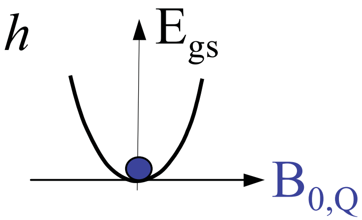


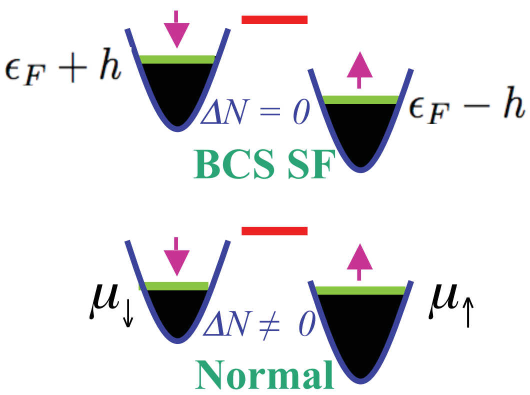
- ***FFLO*** $B_0 = 0$, $B_Q \neq 0$, $\Delta N \neq 0$: $h_c < h < h_{FFLO}(\delta) \xrightarrow{\delta \gg 2\epsilon_F} 1.1\, h_c$ *(Fulde-Ferrell)*
  $\xrightarrow{\delta \approx 2\epsilon_F} h_c$

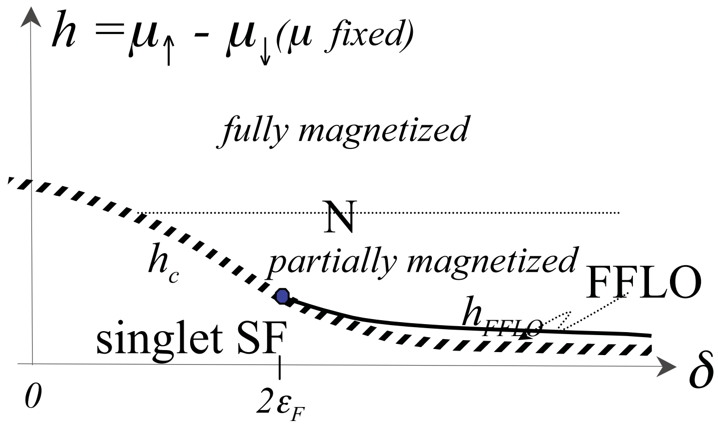
supersolid: *broken rotational and translational symmetry*



- ***Normal*** $B_0 = 0$, $B_Q = 0$, $\Delta N \neq 0$ *(Pauli "paramagnet")*: $h_{FFLO}(\delta) < h$

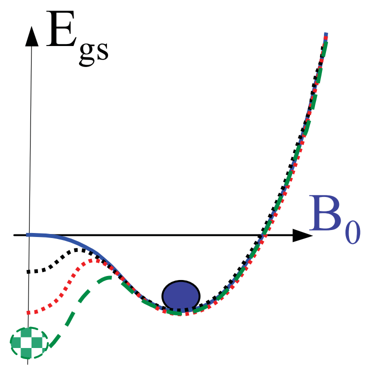






Phase separation $(\delta > 0)$

N, h fixed

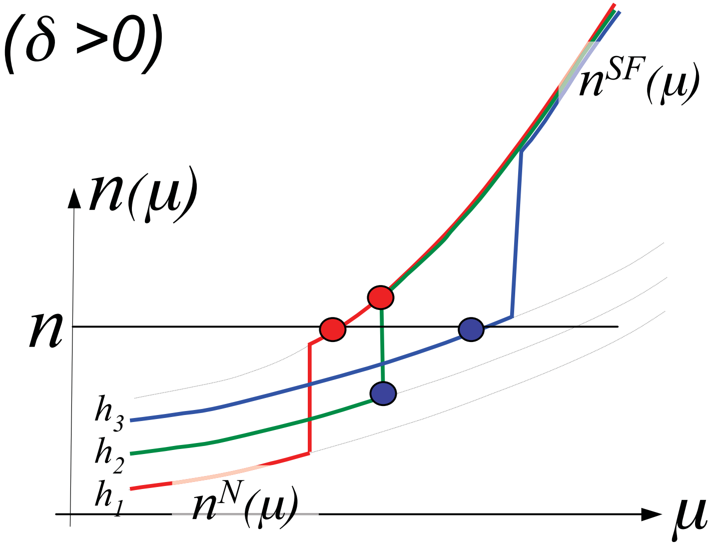


$$h_c$$

$$h_1 < h_{c1}(\delta) < h_2 < h_{c2}(\delta) < h_3$$

SF N

$$n = (1-x)n^{SF} + x\, n^N$$



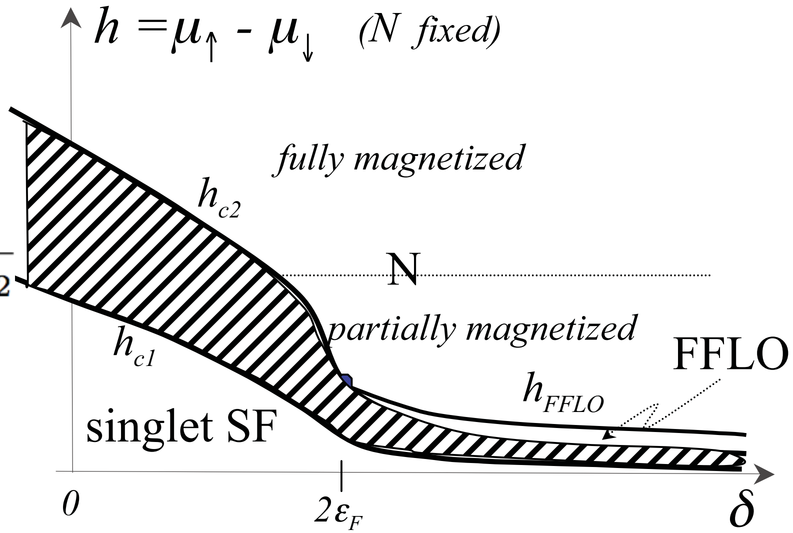
phase separation $x(h, \delta) = (n^{SF} - n)/(n^{SF} - n^N)$

$$h_{c2}(\delta) = h_c(\mu^{(FFLO)}(N, \delta), \delta) \qquad h_{c1}(\delta) = h_c(\mu^{(SF)}(N, \delta), \delta)$$

$0 < \delta \ll 2\epsilon_F$

$$h_{c2}(\delta) \approx 2^{3/2}\epsilon_F - \delta/2$$

$$h_{c1}(\delta) \approx \epsilon_F \sqrt{\frac{\gamma}{3}} \sqrt{1 - (\delta/2\epsilon_F)^{3/2}}$$



$2\epsilon_F \ll \delta$

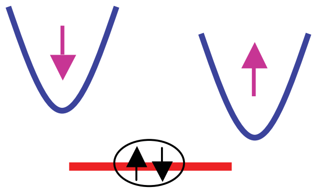
$$h_{FFLO}(\delta) \to 1.1 h_c$$

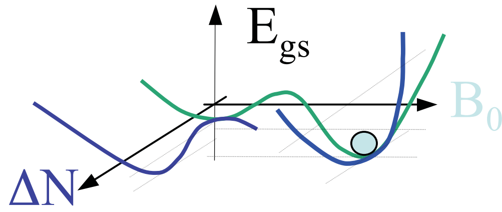
$$\left.\begin{array}{l} h_{c2}(\delta) \\ h_{c1}(\delta) \end{array}\right\} \to \frac{1}{\sqrt{2}}\Delta_{BCS}$$

N, h fixed

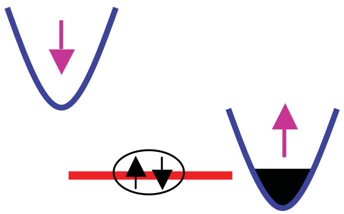
BEC regime $(\delta < 0)$

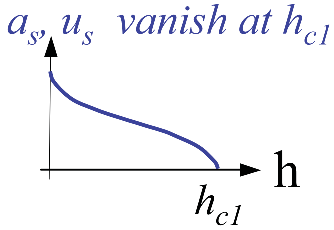
- ***BEC SF*** $B_0 \neq 0$, $B_Q = 0$, $\Delta N = 0$: $0 < h < h_m(\delta) \approx -\delta/2$

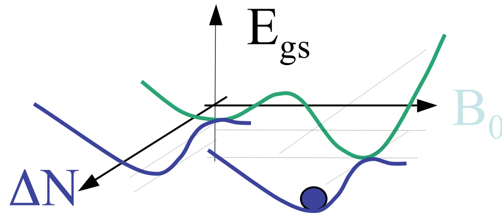




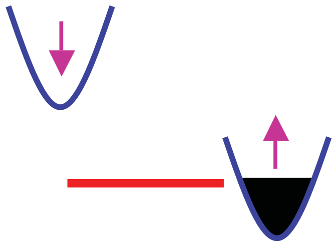
- ***SF$_M$*** $B_0 \neq 0$, $B_Q = 0$, $\Delta N \neq 0$: $h_m < h < h_{c1}(\delta) \approx -0.65\delta$

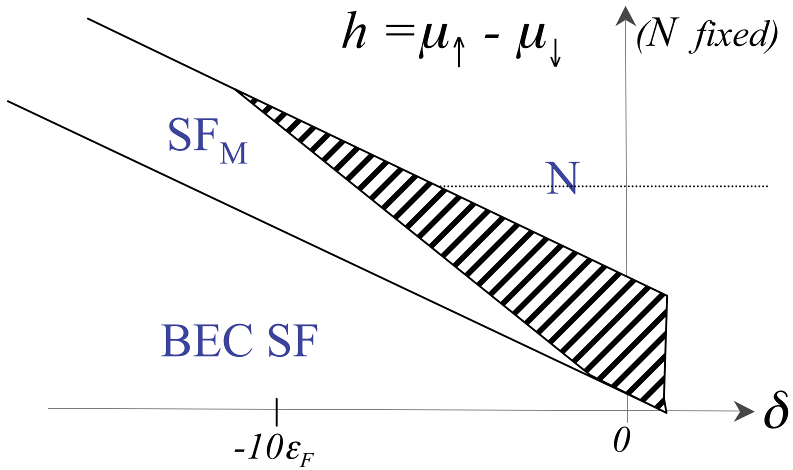


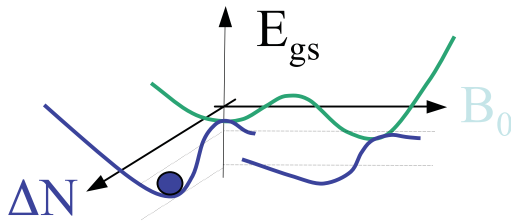




- ***Normal*** $B_0 = 0$, $B_Q = 0$, $\Delta N \neq 0$ *(Pauli "paramagnet")*: $h > h_{c2}(\delta) \approx 2^{3/2}\epsilon_F - \delta/2$

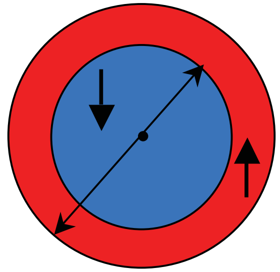




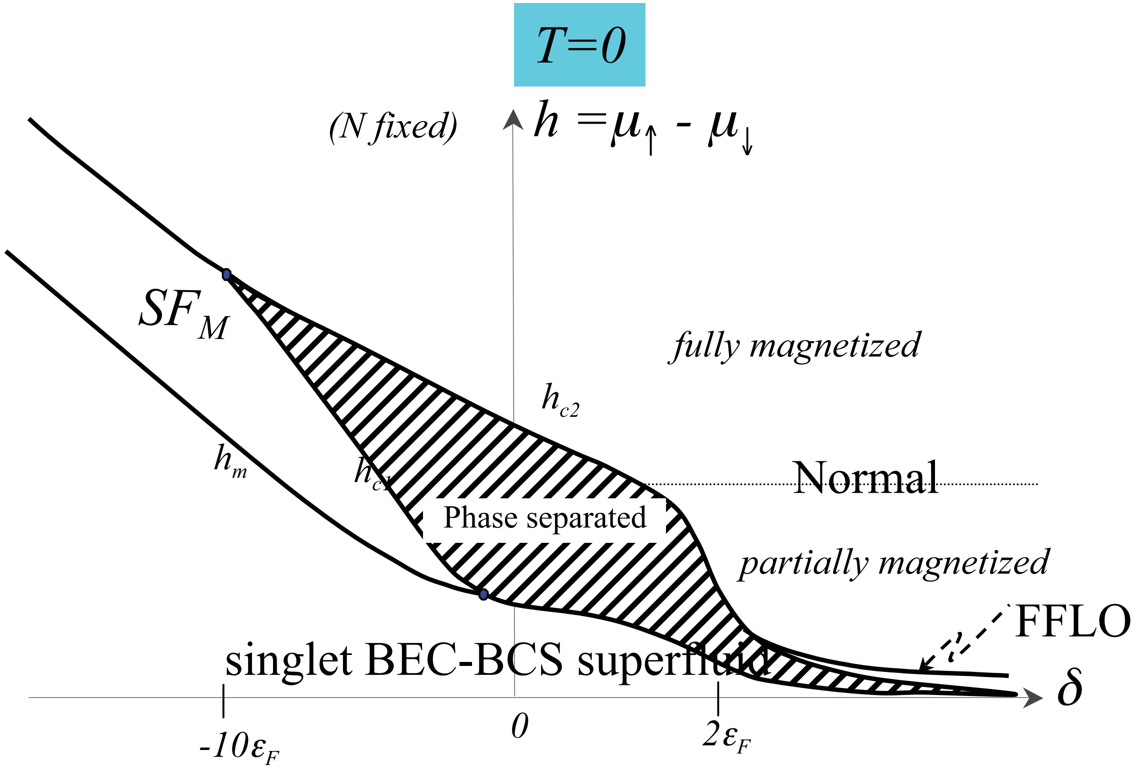


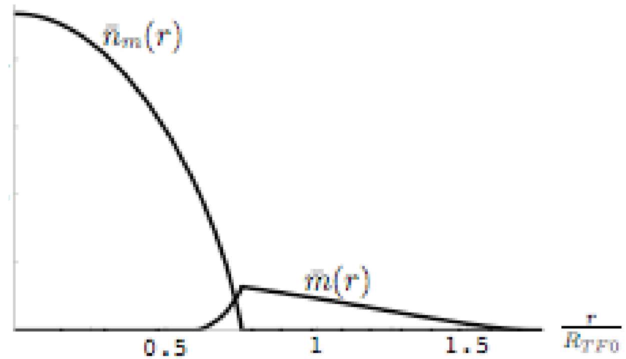
Imbalanced BEC-BCS

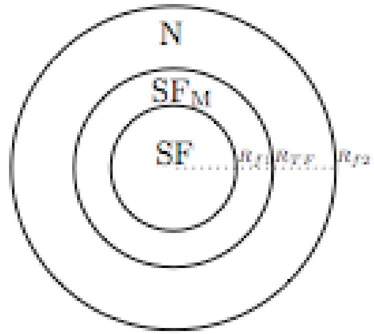
Sheehy, L.R. '05

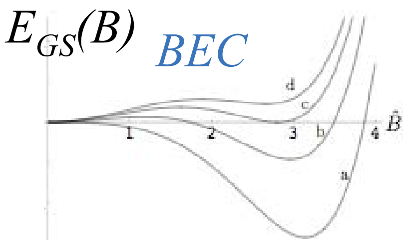


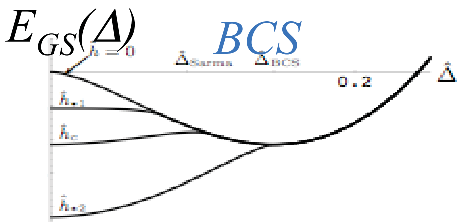
- 1st order transitions and phase separation





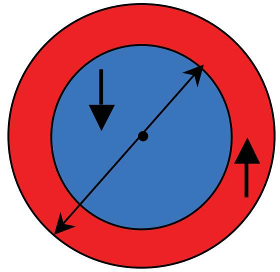






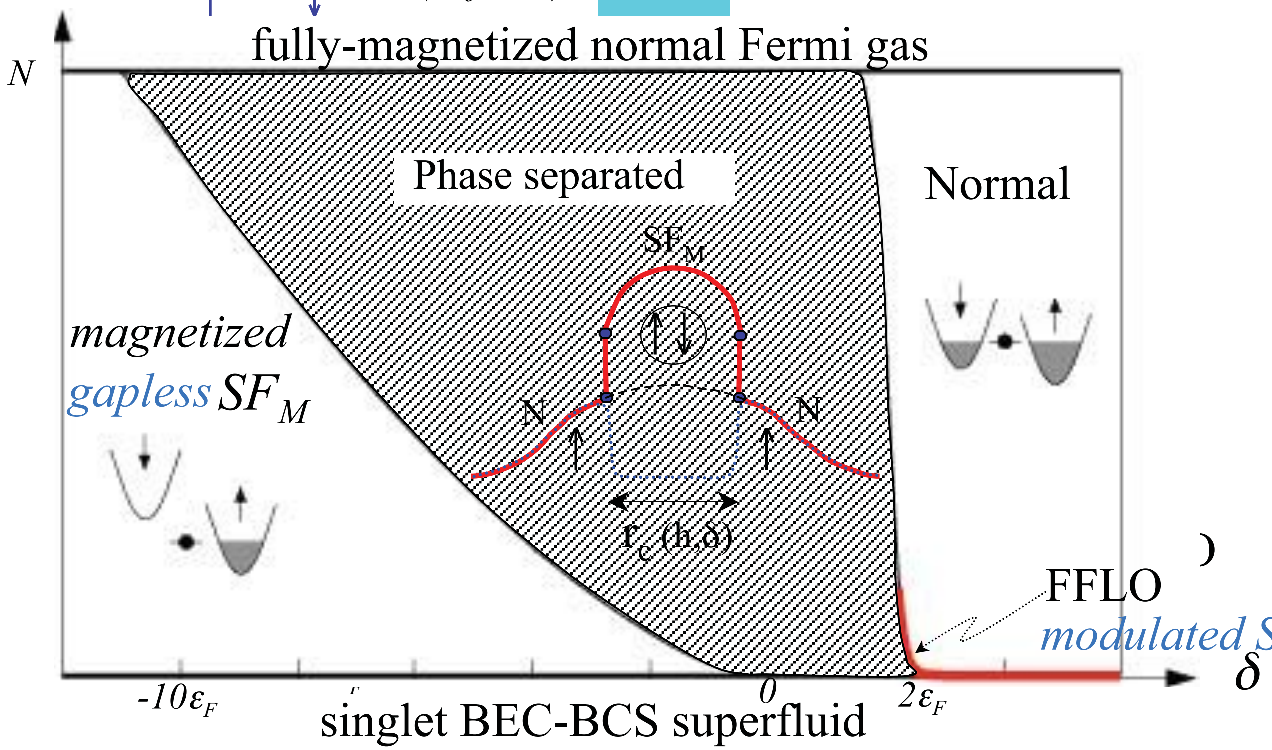
Imbalanced BEC-BCS

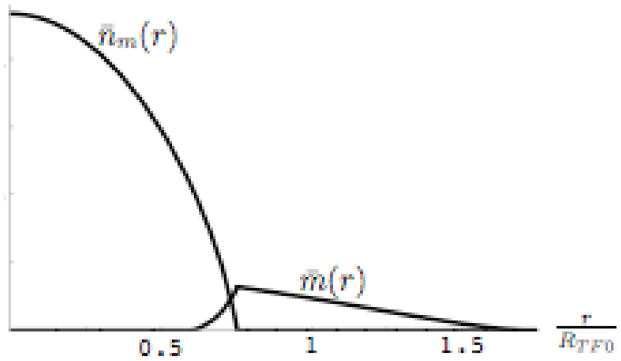
Sheehy, L.R. '05

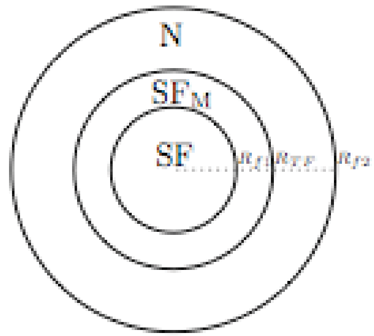


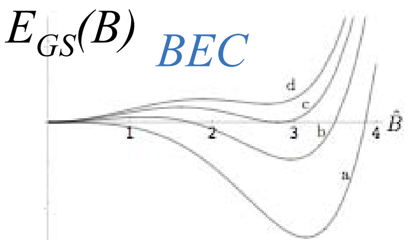
- 1st order transitions and phase separation

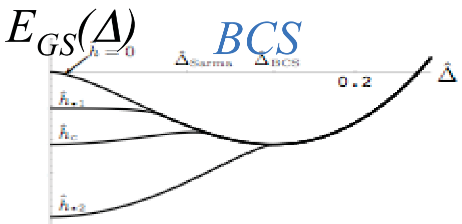
$\Delta N = N_\uparrow - N_\downarrow$ *(N fixed)* *T=0*







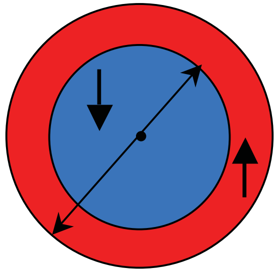




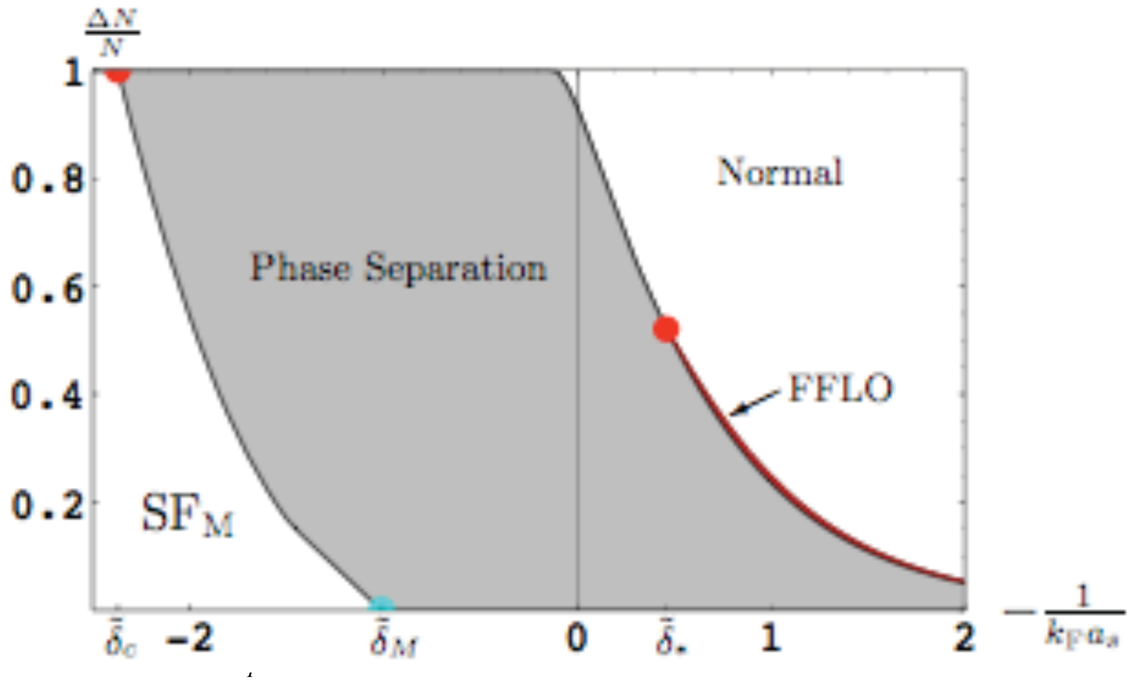
Imbalanced BEC-BCS

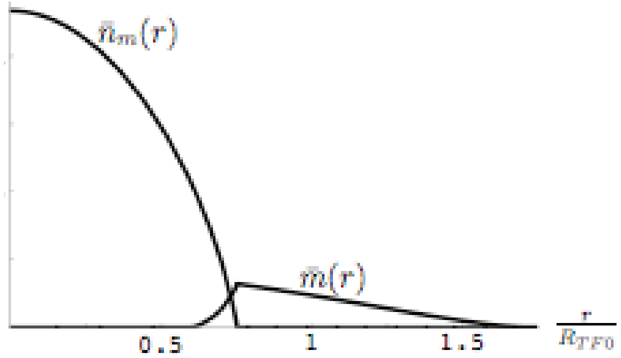
Sheehy, L.R. '05

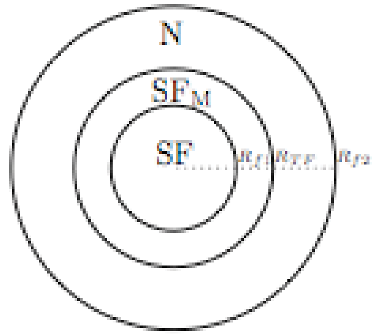
- 1st order transitions and phase separation



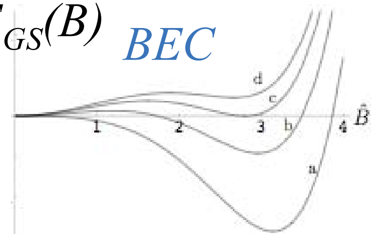
T=0



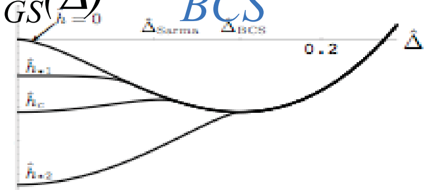




$E_{GS}(B)$ BEC

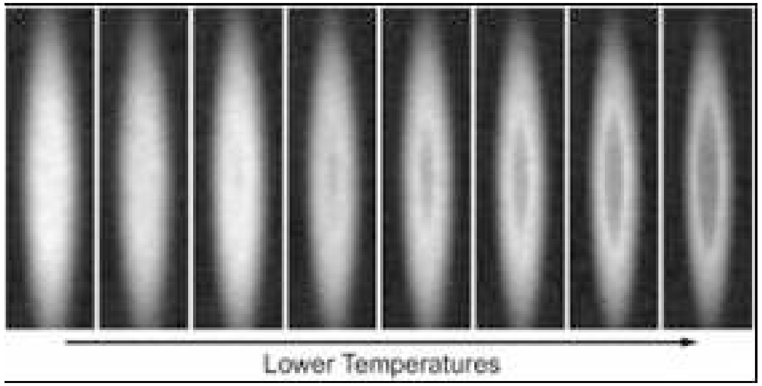


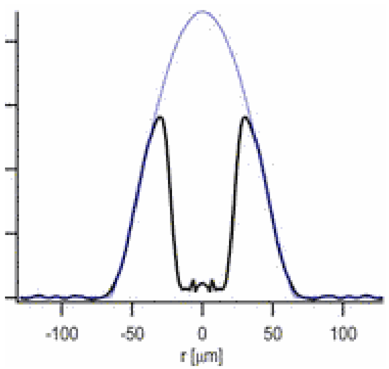
$E_{GS}(\Delta)$ BCS



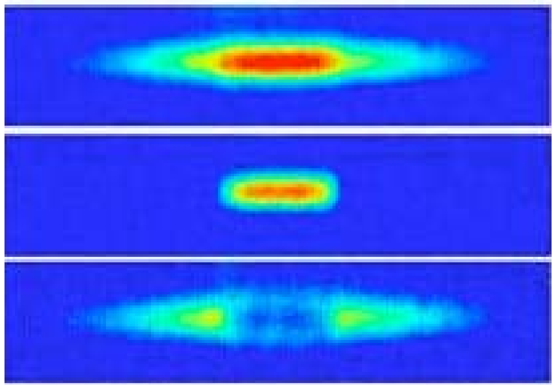
Imbalanced BEC-BCS experiments *(2006)*

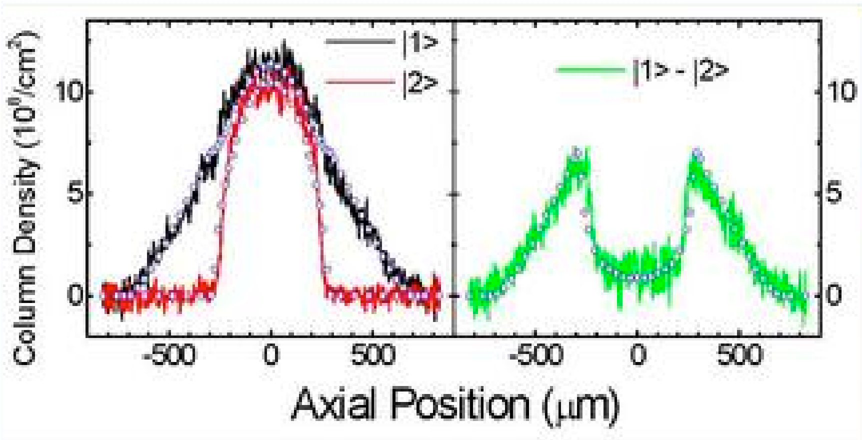
- Ketterle's experiments (vortices, phase separation)





- Hulet's experiments (phase separation, surface tension)



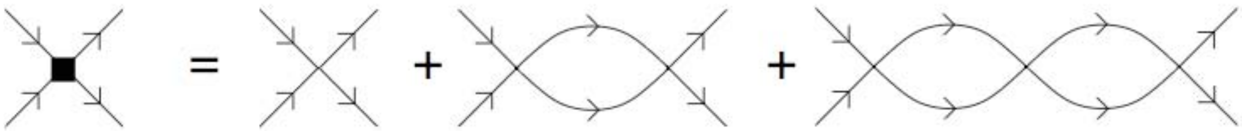


$\gamma >> 1$

Broad resonance scattering

$$\mathcal{H}_{2ch} \longrightarrow \mathcal{H}_{1ch} = \psi_\sigma^\dagger \left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_\sigma + \lambda \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

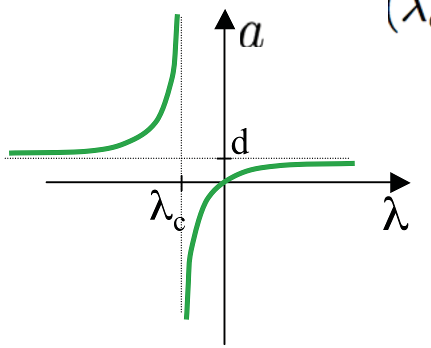
- scattering T-matrix relates λ to a:

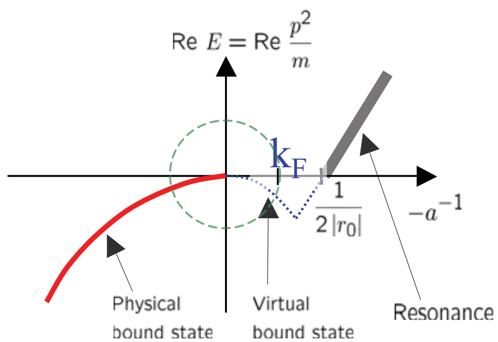
$T_{k\,k'} =$  $+ \ldots = \dfrac{\lambda}{1 - \lambda\Pi}$

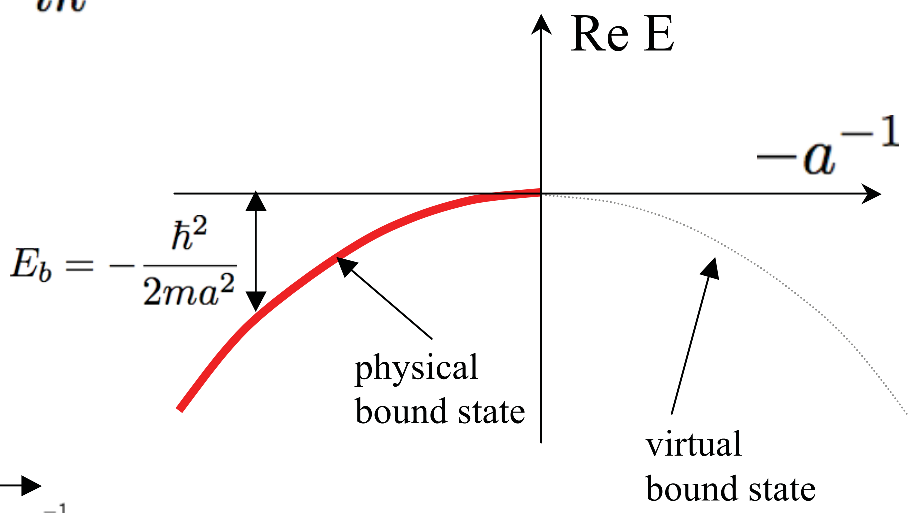
$$= -\frac{4\pi\hbar^2}{m} f_{kk'} \approx \frac{4\pi\hbar^2}{m} \frac{1}{a^{-1} + ik}$$

$$\longrightarrow \quad a = \frac{m}{4\pi\hbar^2} \frac{\lambda}{1 + \lambda/\lambda_c}$$

$$(\lambda_c = \pi d/m)$$







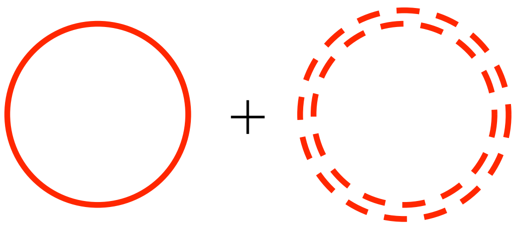
$\gamma >> 1$ Broad resonance superfluidity: Large N

- no small parameter for $k_F a \sim n^{1/3} a >> 1 \rightarrow$ *introduce 1/N*

$$\mathcal{H}_{1ch} \xrightarrow{Sp(2N)} \mathcal{H}_N = \psi^\dagger_{\sigma\alpha}\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_{\sigma\alpha} + \frac{\lambda}{N}\psi^\dagger_{\uparrow\alpha}\psi^\dagger_{\downarrow\alpha}\psi_{\downarrow\beta}\psi_{\uparrow\beta}$$

$$S[\phi] = -\frac{N}{\lambda}\int_0^\beta d\tau d^3r|\phi|^2 - N\mathrm{Tr}\log\left[-G_\phi^{-1}\right] \qquad G_\phi^{-1} = \begin{pmatrix} -\partial_\tau + \frac{\nabla^2}{2m} + \mu_\uparrow & \phi_x \\ \phi_x^* & -\partial_\tau - \frac{\nabla^2}{2m} - \mu_\downarrow \end{pmatrix}$$

$$\begin{aligned} f &= -\frac{1}{\beta V}\log\int D\phi e^{-S[\phi]}, \\ &= N f^{(0)} + f^{(1/N)} + \ldots \end{aligned}$$



MFT

$$f^{(0)} = -\frac{|\Delta|^2}{\lambda} - \int_k (E_k - \xi_k) - \sum_{\sigma=\pm}\int_k \log\left[1 + e^{-\beta(E_k + \sigma h)}\right]$$

Veillette, Sheehy, LR
Nikolic, Sachdev
also Nishida, Son
ε-expansion

$\gamma >> 1, k_F a \to \infty$

Universality at unitary point

T.L. Ho '04

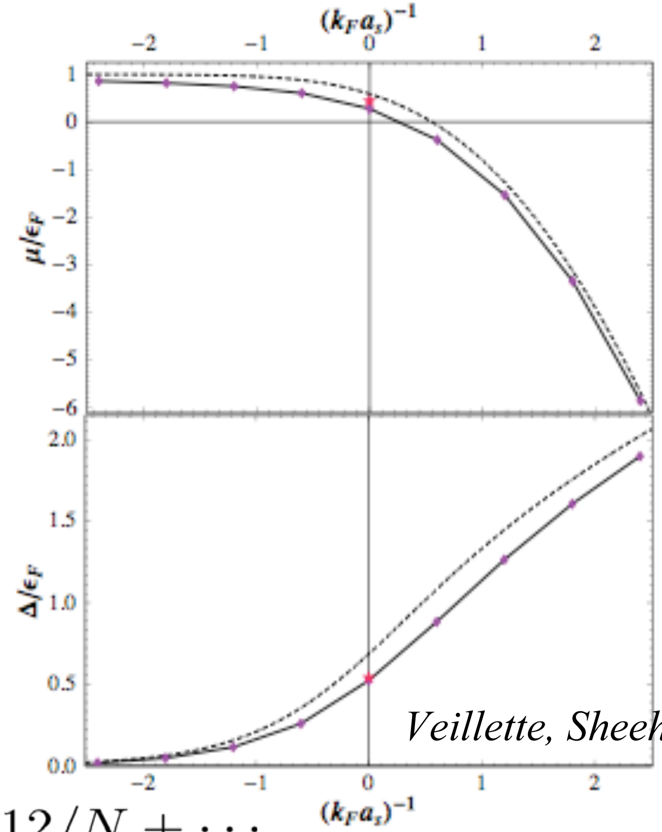
- $f_k = -1/(a^{-1} + i\,k) \to i/k$, k_F is the only scale

check in $N \to \infty$ (BCS) limit:

$$f(T,n) = n\epsilon_F \hat{f}(k_B T/\epsilon_F)$$

$$\frac{m}{2\pi\hbar^2 a} \to 0 = \int_k \left(\frac{1}{E_k} - \frac{1}{\epsilon_k}\right)$$

$$\begin{aligned} \epsilon &= \xi \frac{3}{5}\epsilon_F \\ \mu &= \xi\epsilon_F \\ \Delta &= \alpha\epsilon_F \\ \Delta_{exc} &= \alpha_{exc}\epsilon_F \\ k_B T_c &= \gamma\epsilon_F \\ B &= \xi\frac{2}{3}n\epsilon_F \end{aligned}$$



Veillette, Sheehy, LR

Result from 1/N $\xi = 0.5906 - 0.312/N + \cdots$

Exp with ^{40}K $\xi = 0.46^{+0.05}_{-0.12}$

Questions of current interest

- What are the big fundamental questions?

- Specific questions of current experimental interest:
  - *Unitary Fermi gas (universality)*
  - *Resonant Bose gas (beyond Beliaev)*
  - *Stability to 3-body collisions and other inelastic processes*
  - *Cooling and thermalization*
  - *Experimental probes (development and understanding)*
  - *Phases realizations (e.g., FFLO, p-wave SF, magnetism,…)*
  - *Nonequilibrium quantum dynamics*
  - …

On the horizon

- p-wave superfluidity?
- degenerate molecular gases?
- local many-body lattice models
- multi-site many-body lattice models ⇒ exotic models?
- quantum Hall regime?
- …

…but not before technical hurdles are overcome:

- cooling
- off-site interactions
- stability to inelastic processes near Feshbach resonances
- much larger clouds
- flat traps
- better and wider range of experimental probes
- …