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Miniworkshop on Strong Correlations in Materials and Atom Traps

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Cold Atoms in reduced dimensionality

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Strong correlations in 1D: ultra cold atoms in optical lattices

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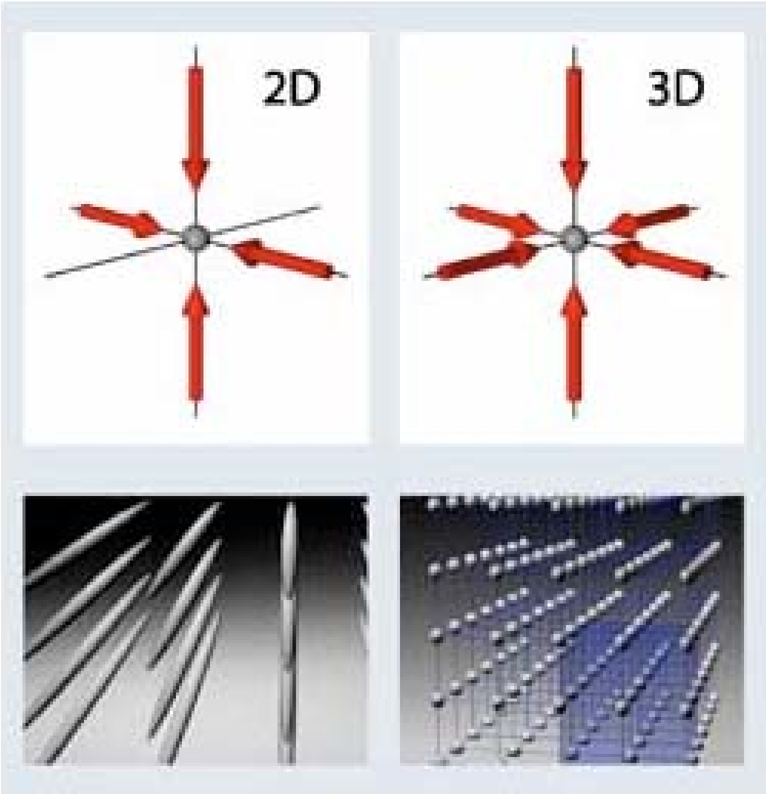
I. Background: ultra cold atoms trapped in optical lattices

- new environment for strong correlation physics
- select examples of strongly correlated states already created in cold atom systems
- advantages of cold atom lattices

New environment for quantum many body physics: ultracold atoms trapped in optical lattices



$$\lambda/2 \sim 400\text{nm}$$

Bloch (Mainz), Esslinger (ETH), Phillips (NIST), Weiss (U Penn), Ketterle (MIT),...

quantum degeneracy:

$$\mathrm{T} \sim 10 \text{ nK} \quad \sim 0.01 \mathrm{T}_{\mathrm{BEC}}$$
$$\sim 0.05 \ \mathrm{T}_{\mathrm{Fermi}}$$

$$\mathrm{N} \sim 10^5 \qquad \mathrm{n} \sim 10^{+15} \mathrm{cm}^{-3}$$

$$t_{\mathrm{hop}} \sim 1\mathrm{ms} \quad \frac{\text{pot.energy}}{\text{kin.energy}} \sim 2 - 100$$

atomic dipole trapping:

$$\Delta E_{\mathrm{latt}} = -\frac{1}{2}\alpha(\omega)\langle \mathcal{E}^2(r,t)\rangle_t$$

$$\alpha(\omega) = \mathrm{Re}\frac{|\langle e|\vec{p}\cdot\hat{\epsilon}|g\rangle|^2}{E_e - i\Gamma_e/2 - E_g - \omega}$$

$$\mathcal{E}_x = \mathcal{E}_x^0\ \hat{\epsilon}_x \left(e^{i2\pi x/\lambda} + e^{-i2\pi x/\lambda}\right) e^{i\omega t}$$

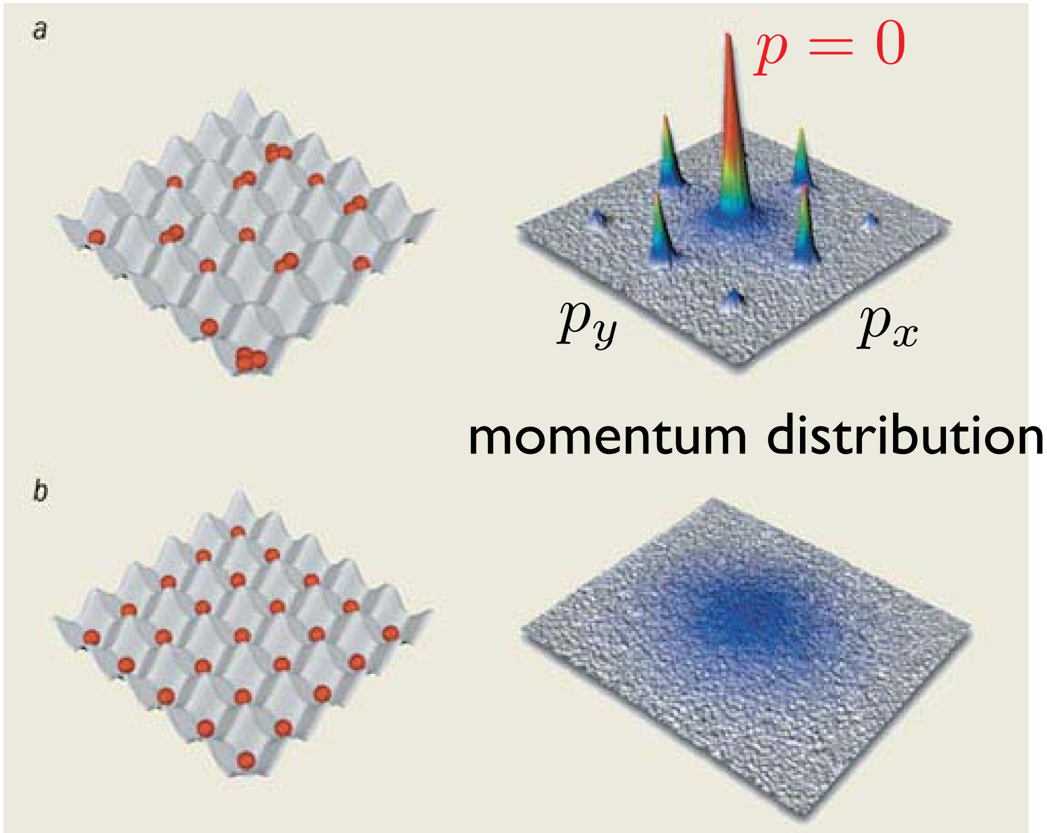
ultra cold atoms in optical lattices

many-body interplay: N_i vs ϕ_i

i lattice site label

weak lattice potential:
fixed ϕ_i : superfluid

strong lattice potential:
fixed N_i :
crystal: Mott Insulator



Greiner et al. Nature '02

Tonks gas: 1D hard core bosons $K \to 1^+$

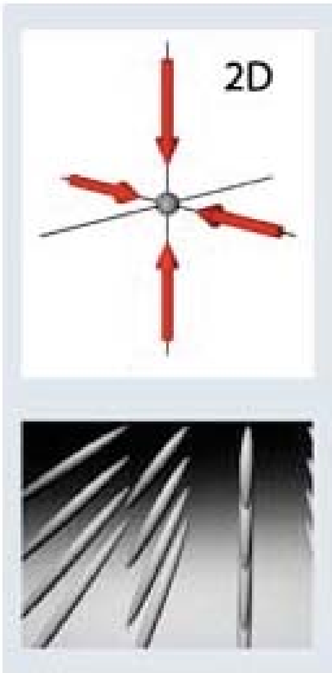
1D regime: $\omega_\perp \gg \mu \sim V_0 \bar{n} > T$

expt: $\omega_\perp \sim 10\mathrm{kHz} \sim 1\mu\mathrm{K}$

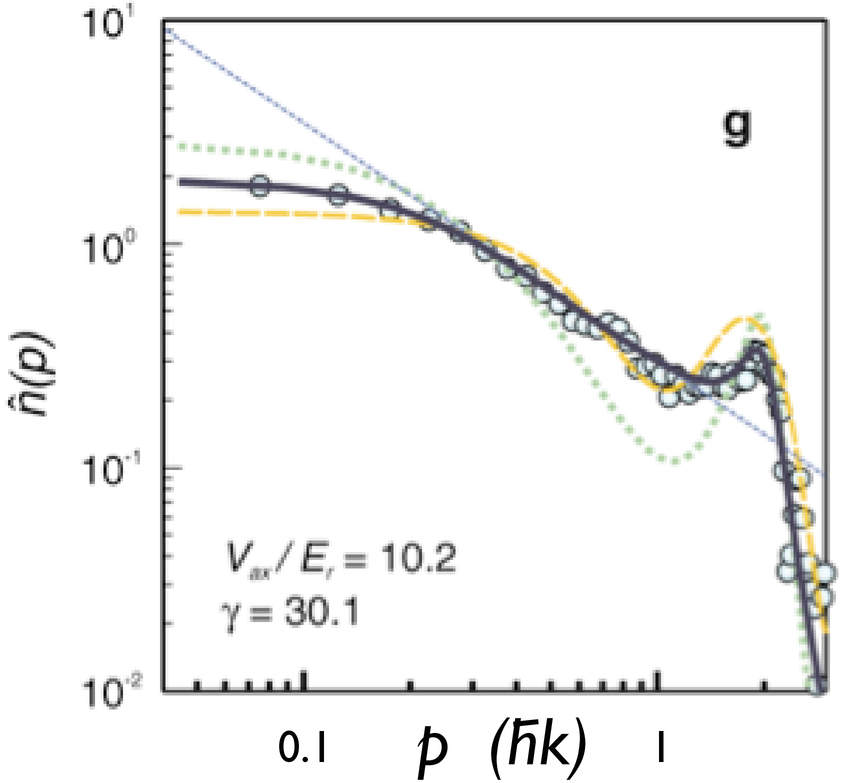
Luttinger liquid theory:

$$n(p \to 0) \sim p^{-1+1/2K}$$

also: Kinoshita et. al., Science 2004, Stoerferle et. al., PRL 2004. ....



momentum distribution



Paredes et. al. Nature 2004

cold atom lattices advantages

- know Hamiltonian precisely, clean
- systems parameters continuously tunable:
  - ✳ hopping, interaction: laser amplitude, Feshbach resonance
  - ✳ mixtures of atoms (F and/or B), molecules
  - ✳ lattice geometry, dimensionality
- long time, length scales: follow time evolution

extremely versatile laboratory for strong correlation physics

However: relatively few probes to characterise states

II. 1D & quasi-1D cold atom systems

- 1D quantum systems: introduction to Luttinger liquid and bosonization
- dimensional crossover for bosons: 1D to 3D
- binary mixtures in 1D (B+B, B+F, F_1+ F_2)
- extremely strong interactions and multi-band systems

1D: enhanced quantum fluctuations

- 1D interacting bosons: strong phase fluctuations no BEC, but still Bogoliubov phonons (Lieb, Liniger '63)
- 1D interacting fermions: low energy excitations not fermionic, but are collective bosonic density waves (charge, spin). (Lieb, Wu '66,...)

particle-
hole
continuum

$2\pi\rho_0$

momentum

Castro-Neto et al PRB '94

Interacting bosons or fermions:
Luttinger liquid   (Haldane '81)

- low energy excitations: gapless collective modes
- no true long range order, power law correlations
- spin-charge separation, "fractionalization"

Methods: no BEC: not Gross-Pitaevskii/mean field, but
1) weak coupling renormalisation group
2) bosonization/CFT,   3) strong coupling expansion
4) exact results (XXZ, sineGordon)

1D low energy theory: Bosonization

Luther, Emery, Haldane, ...

density-phase representation:

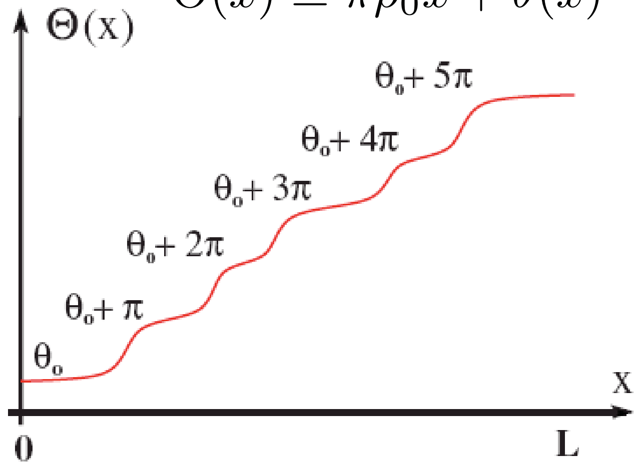
$$\psi_B(x) = \sqrt{\rho(x)}e^{i\phi(x)}$$

discreteness of particles:

$$\rho(x) = \sum_i \delta\left(x - x_i\right)$$

$$= \left[\rho_0 + \frac{1}{\pi}\partial_x\theta(x)\right] \sum_{m=-\infty}^{\infty} e^{i2m[\pi\rho_0 x+\theta(x)]}$$

$$\Theta(x) \equiv \pi\rho_0 x + \theta(x)$$



$$[\phi(x), \partial_y\theta] = i\delta(x - y)$$

Bosons:

$$[\psi_B(x), \psi_B^\dagger(x')] \sim \delta(x - x')$$

$$\psi_B(x) = A\sqrt{\rho_0 + \frac{1}{\pi}\partial_x\theta} \sum_{j=-\infty}^{\infty} e^{i2j[\theta(x)+\pi\rho_0 x]}\ e^{i\phi(x)}$$

Fermions:

$$\left\{\psi_F(x), \psi_F^\dagger(x')\right\} \sim \delta(x - x')$$

$$\psi_F(x) = A\sqrt{\rho_0 + \frac{1}{\pi}\partial_x\theta} \sum_{j=-\infty}^{\infty} e^{i(2j+1)[\theta(x)+k_F x]} e^{i\phi(x)}$$

$$k_F = \pi\rho_0$$

contrast (3D) quantum hydrodynamics/Bogoliubov:
only long wavelength density fluctuations

1D interacting bosons: Luttinger liquid description

continuum model (Lieb & Liniger '63):

$$H = \int dr \left[\frac{1}{2m} \nabla\psi^\dagger(r)\nabla\psi(r) - \mu\psi^\dagger(r)\psi(r) + \frac{V_0}{2}\psi^\dagger(r)\psi^\dagger(r)\psi(r)\psi(r) \right]$$

$$V_0 = 2\omega_\perp a_{3D}$$

lattice model:

$$H_{\mathrm{BoseHubbard}} = -t\sum_i b^\dagger_{i+1} b_i + \mathrm{H.c.} - \mu \sum_i b^\dagger_i b_i + \frac{U}{2}\sum_i n_i(n_i+1)$$

cold atoms: contact interaction accurate

both models bosonize to:

incommensurate filling:

$$H_{Lutt.liq} = \frac{v_s}{2\pi}\int dx \left[K(\partial_x\phi)^2 + \frac{1}{K}(\partial_x\theta)^2 \right]$$

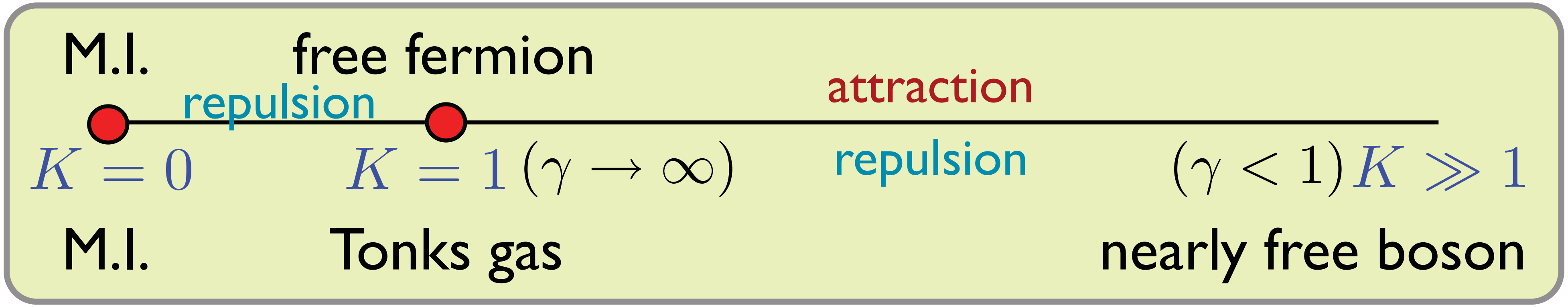
valid for: $x \gg \xi$

K interaction parameter, v_s sound velocity

$$K \propto \mathrm{compressibility} = \left[\rho_0^2\, \partial\mu/\partial\rho_0\right]^{-1}$$

Luttinger liquid: single species (B or F)

$$H_{Lutt.liq} = \frac{v_s}{2\pi} \int dx \left[K(\partial_x \phi)^2 + \frac{1}{K} (\partial_x \theta)^2 \right]$$



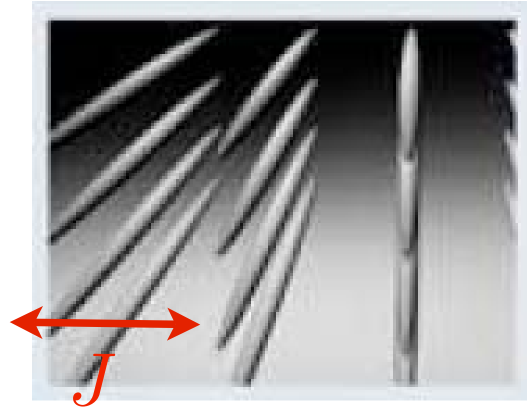
NB: no phase transitions
(except for Mott Insulator)

specific to 1D: $\gamma = Mg/\rho_0$

correlation functions: Luttinger liquid power law correlations

eg. boson momentum distribution $n(p \sim 0) \sim p^{-1+1/2K}$

a) 1D to 3D



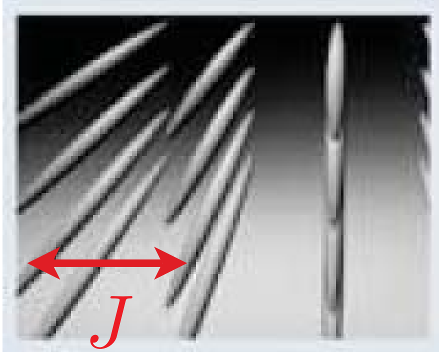
Why study 1D to 3D?

- experimental geometry (Esslinger, Bloch, Weiss, Phillips, ...) : continuous tuning of tunneling of atoms between tubes.
- theoretical: how does weird 1D physics get suppressed going to higher D? (cf. quasi-low D materials e.g. Bechgaard salts, spin chains...)

Methods: still use bosonization for 1D fluctuations, for 3D correlations: mean-field+RPA, variational, RG

Efetov & Larkin '75, Donohue & Giamarchi '01

1D to 3D: dimensional crossover



- Single species of bosons in a 1D tube:
  isolated tube = Luttinger liquid
- Allow atoms to hop between tubes:
  dimensional crossover to a 3D superfluid

$$T_c \approx \mu \sqrt{\frac{4J}{\mu}}(0.61 + 0.70K) \qquad (K > 2)$$

Ho, Cazalilla, Giamarchi
PRL 2004, NJP 2006

condensate fraction $\propto (J/\mu)^{1/(4K-1)}$

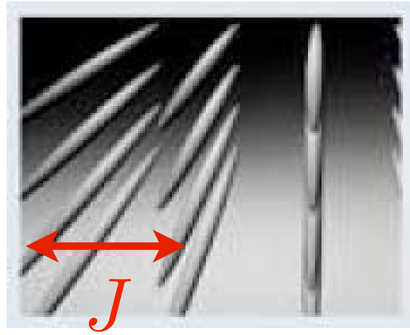
excitations in superfluid:

- amplitude mode gapped: $\Delta_+ \sim \tilde{f}(K)\, \mu (J/\mu)^{2K/(4K-1)}$
- phase mode: $\omega_-^2(q, \mathbf{Q}) = v_s^2 q^2 + 2\left(\frac{v_\perp}{b}\right)^2 \sum_{j=y,z} (1 - \cos Q_j b)$

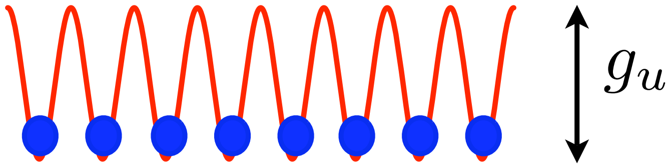
transverse velocity: $v_\perp(T=0) \sim \mu (J/\mu)^{2K/(4K-1)}$

1D to 3D: deconfinement transition

Ho, Cazalilla, Giamarchi PRL 2004, NJP 2006



single species
of bosons



- strong enough interaction + commensurate densities: isolated tube = 1D Mott Insulator

- coupling 1D MI together: deconfinement to 3D superfluid, when

$$J > J_c \propto g_u^{2/3}$$

condensate fraction:

$$\psi_0^2(T=0) \propto \rho_0 \left(1 - |J_c/J|^3\right)$$

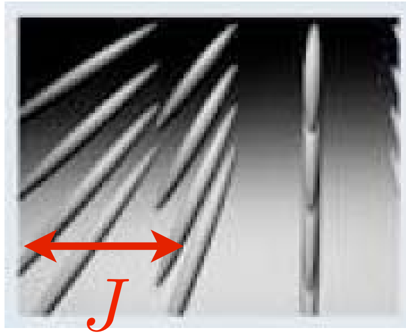
weak-coupling RG:

$$\frac{dJ}{d\ell} = (2 - \frac{1}{2K})J$$

$$\frac{dg_u}{d\ell} = (2 - K)g_u$$

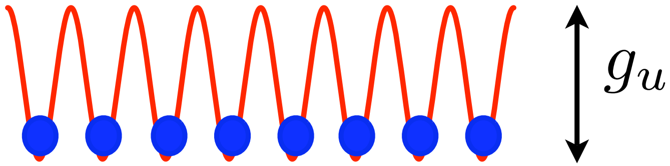
$$\frac{dK}{d\ell} = 4J^2 - g_u^2 K^2$$

1D to 3D: deconfinement transition



Ho, Cazalilla, Giamarchi   PRL 2004, NJP 2006

single species
of bosons



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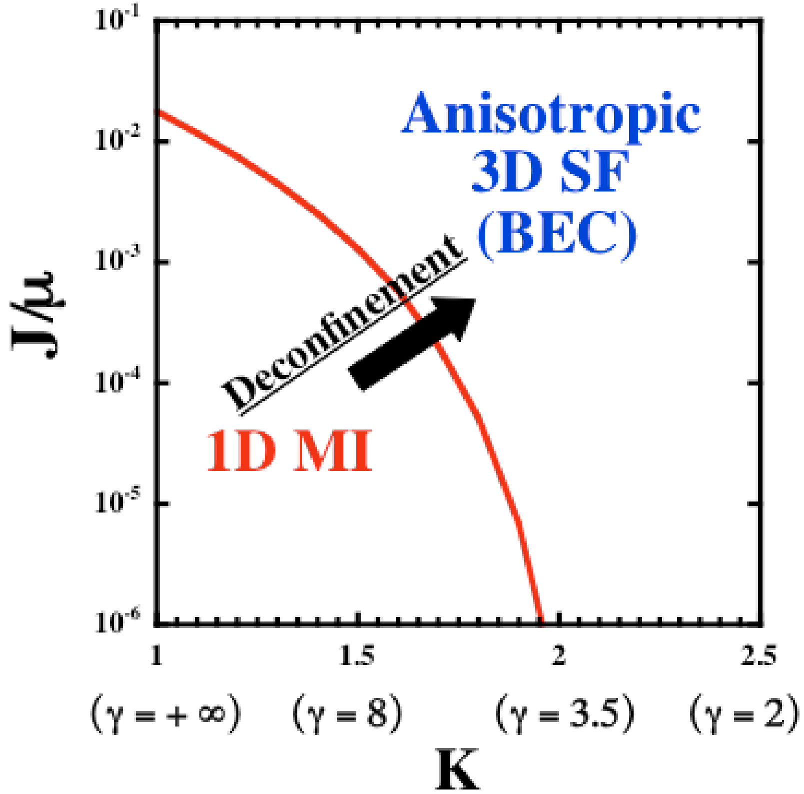


Mean field theory at SF-MI transition: Heisenberg spin-chain mapping at K=1/2

★ Nature of transition cannot be deduced from weak-coupling RG.

★ mean-field part as before: $\langle\psi(x)\rangle = \psi_0$

Luttinger Liquid $\quad H_{LL} \longrightarrow J_H \sum_n \mathbf{S}_{\mathbf{R}}(n)\mathbf{S}_{\mathbf{R}}(n+1)$

Mott potential $\quad g\cos 2\theta_{\mathbf{R}} \longrightarrow h^z(-1)^n S^z_{\mathbf{R}}(n)$

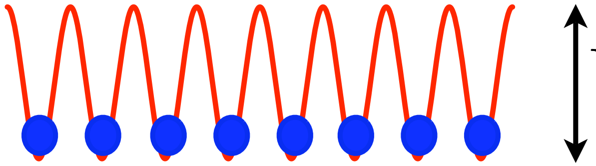
interchain hop $\quad J\psi_0 \cos\phi_{\mathbf{R}} \longrightarrow J\psi_0(-1)^n S^x_{\mathbf{R}}(n)$

★ trick: combine x-, z- staggered fields into one via a canonical spin rotation

★ continuous transition: $\quad J > J_c \propto g_u^{2/3}$

condensate fraction: $\quad \psi_0^2(T=0) \propto \rho_0\left(1 - |J_c/J|^3\right)$

excitation spectrum: single type of bosons in 1D



$V_{||}(t)$

$$V_{||}(t) = V_{||} + \delta V_{||} \cos \omega t$$

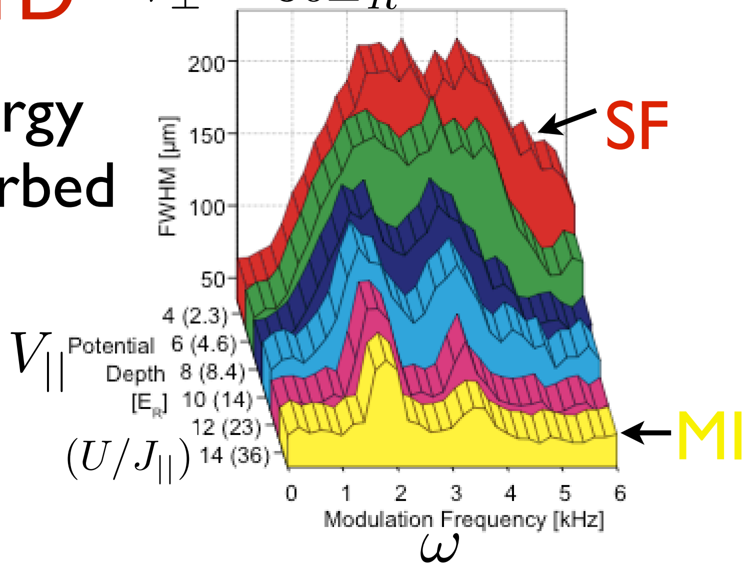
energy absoprtion rate:

$\sim \left(\frac{\omega}{\mu}\right)^{2K-1}$ SF

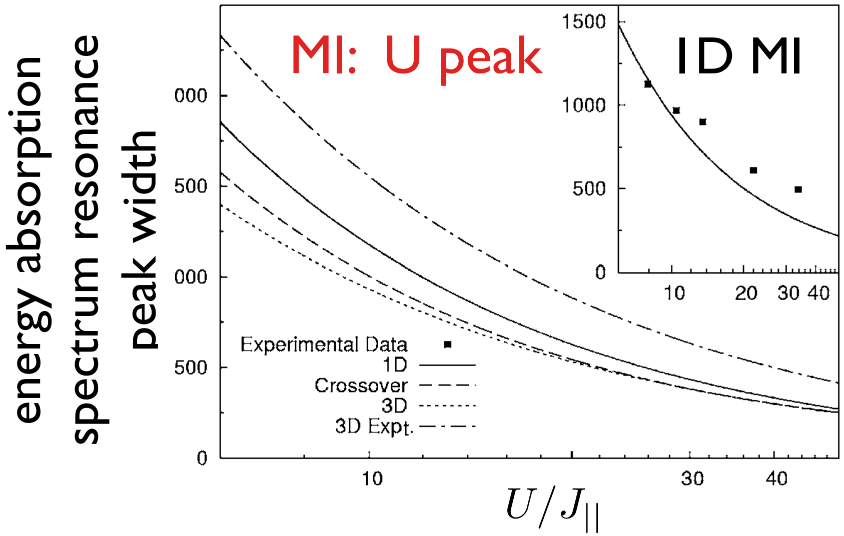
$\sim \omega \sqrt{1 - \left[\frac{\omega - U}{J_{||}(2n_0+1)}\right]}$ MI

1D $V_{\perp} = 30 E_R$

energy absorbed



Stoeferle et.al. PRL 2004

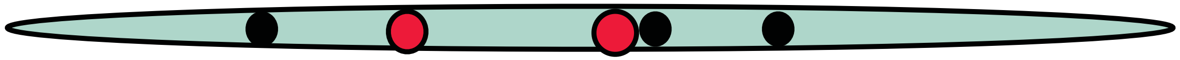


Iucci, Cazalilla, Ho, Giamarchi PRA(R) 2006

also, Kollath et al. PRA 2006

b) 1D binary mixtures

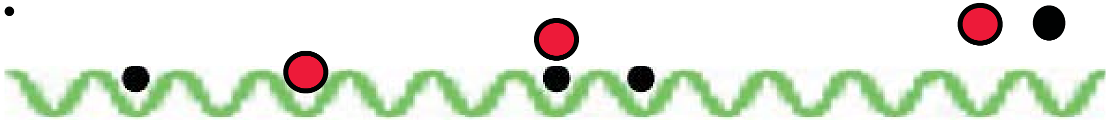
continuum model:



$$H = \int dx \sum_{\alpha} \left[\frac{\hbar^2}{2M_\alpha} \partial_x \Psi_\alpha^\dagger(x) \partial_x \Psi_\alpha(x) \right] + \frac{1}{2} \int dx \sum_{\alpha,\beta} g_{\alpha\beta}\, \rho_\alpha(x) \rho_\beta(x).$$

lattice model (Hubbard):

$\alpha, \beta = \uparrow, \downarrow$



$$H = -\sum_{\alpha,j} t_\alpha c_{\alpha j+1}^\dagger c_{\alpha j} + \mathrm{H.c.} + U \sum_j n_{\uparrow j} n_{\downarrow j}$$

NB: $N_\uparrow, N_\downarrow$ separately controlled, conserved

no spin flip processes

Binary mixtures (B+B or B+F or F+F) in 1D

similar densities (incommensurate)

Cazalilla & Ho
PRL '03

new coordinates: "spin" (-) and "charge" (+) modes

$$\theta_\pm = \frac{1}{\sqrt{2}}\left[\theta_1(x) \pm \theta_2(x)\right] \qquad \phi_\pm = \frac{1}{\sqrt{2}}\left[\phi_1(x) \pm \phi_2(x)\right]$$

spin - charge separation:

$$H_+ = \frac{v_+}{2\pi}\int dx \left[\frac{1}{K_+}(\partial_x\theta_+)^2 + K_+(\partial_x\phi_+)^2\right]$$

Lutt. Liq.
charge

$$H_- = \frac{v_-}{2\pi}\int dx \left[\frac{1}{K_-}(\partial_x\theta_-)^2 + K_-(\partial_x\phi_-)^2\right] + \frac{1}{2\pi}\int dx \left[g_b \cos\sqrt{8}\theta_-(x) - g_f(\partial_x\theta_-)^2\right]$$

sine-Gordon
spin

→ pairing or SDW

Binary mixtures in 1D tubes (B+B, B+F)

Cazalilla & Ho PRL (2003)

★ similar densities for each species

★ B: Tonks limit (strong interaction, low densities):

Kosterlitz-Thouless transition leads to:

"spin" gap ≠ 0 , "charge" gap =0 (incommensurate)

dominant instability (quasi-LRO):

$g_{12} < 0$:

B+F pair= finite k, fermionic!

Pairing

$g_{12} > 0$:

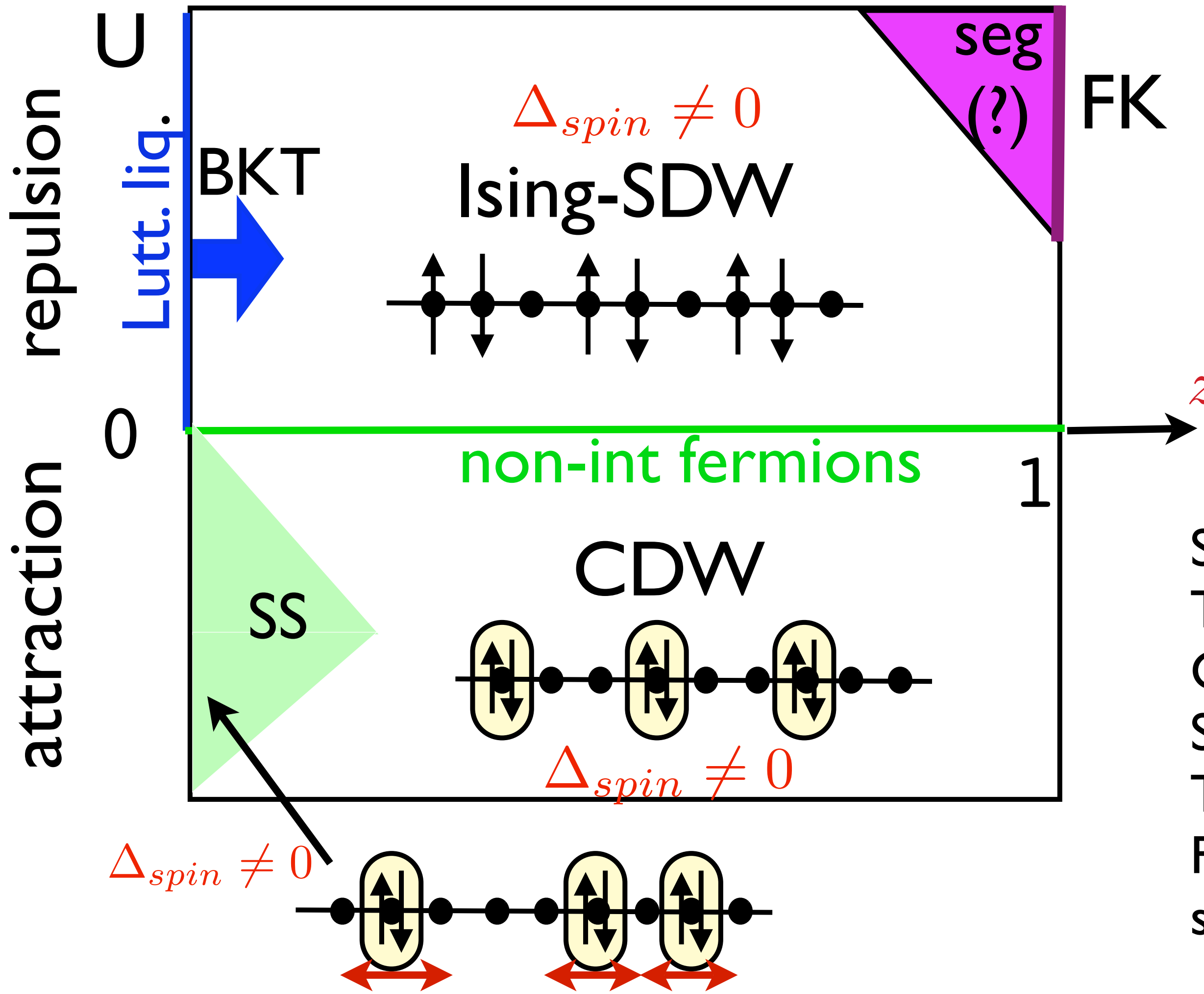
(different velocities)

"spin" density wave

schematic 1D "phase" diagram: fermions $N_\uparrow = N_\downarrow$

densities incommensurate to lattice: $\Delta_{charge} = 0$

Cazalilla, Ho, Giamarchi
PRL 2005



velocity difference parameter

$$z = \frac{t_\uparrow - t_\downarrow}{t_\uparrow + t_\downarrow}$$

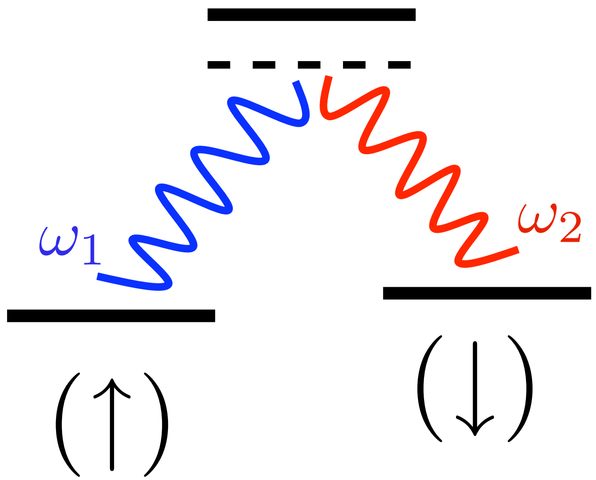
SS: singlet superfluid
TS: triplet superfluid
CDW: charge density wave
SDW: spin density wave
TLL: Tomonaga-Luttinger liq.
FK: Falicov-Kimball model
seg: segregation

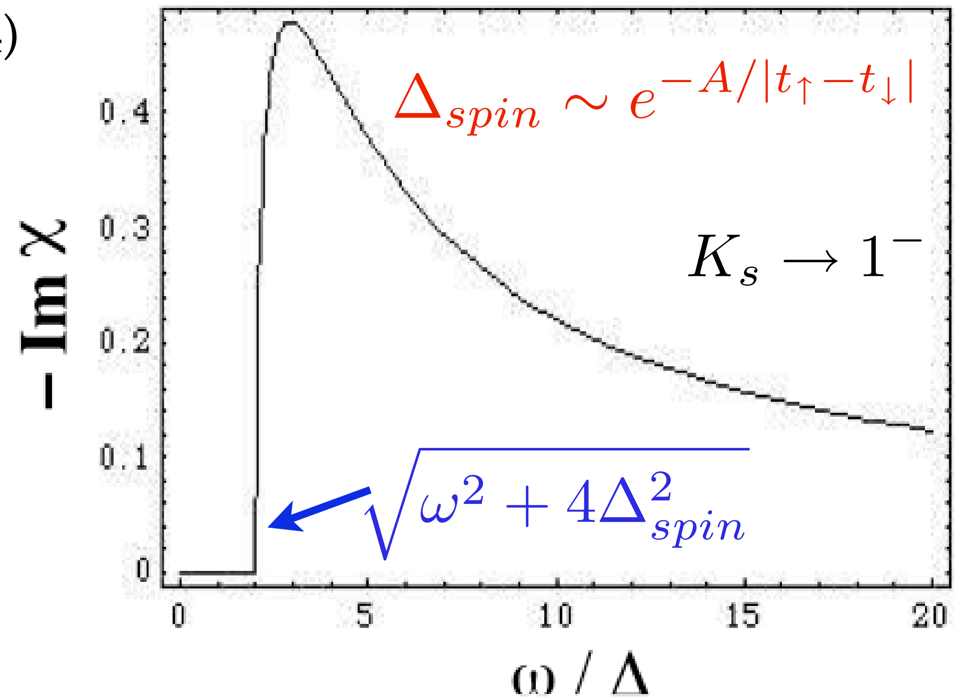
Binary mixtures of fermions in 1D tubes

spin-charge separation: spin gap measurement:

Cazalilla, Ho, Giamarchi PRL 2005

Buchler, Zoller & Zwerger, PRL (2004)





$$S^{+-}(t) = \langle S_T^+(t) S_T^-(0) \rangle$$

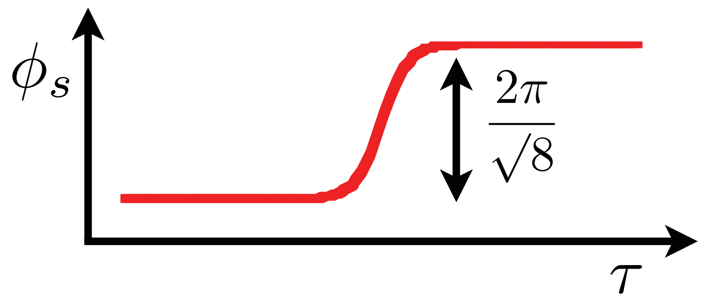
$$\omega = \omega_1 - \omega_2$$

$$\chi(q=0,\omega) = \mathrm{FourierTransf.}\, S^{+-}(t)$$

method: use exact form factors of integrable quantum sine-Gordon model (Lukyanov & Zamolodchikov)

quantum sine-Gordon model and form factors

$$\mathcal{A}_s = \frac{1}{2\pi K_s}\int dx d\tau \left[\frac{1}{v_s}(\partial_\tau \phi_s)^2 + v_s(\partial_x\phi_s)^2\right] - \frac{g}{2\pi}\int dx d\tau \cos\sqrt{8}\phi_s(x,\tau)$$



- elementary excitations: solitons, antisolitons, breathers
- spin flip connects ground state to a state with 2 solitons. (More solitons cost more spin gap energies.)
- Lukyanov Zamolodchikov 2001:

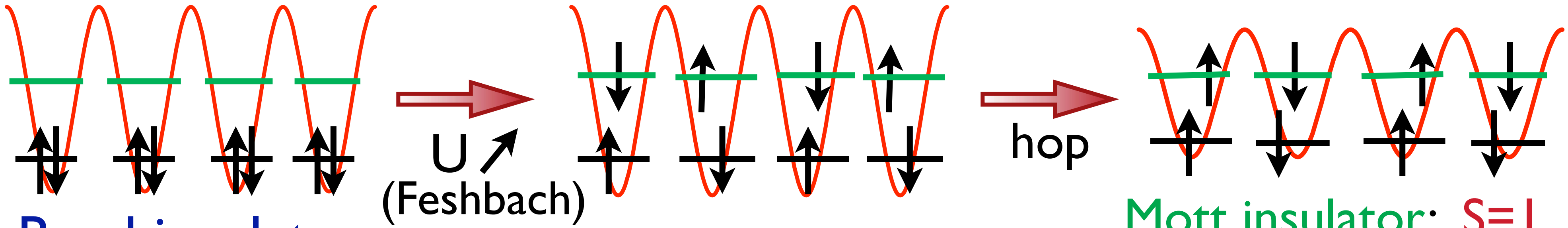
$$S^+(x,\tau) \propto e^{-i\sqrt{2}\theta_s(x,\tau)} \sin\sqrt{2}\phi_s(x,\tau)$$

$$\langle 0|S^+(0,0)|\theta_1,\theta_2\rangle \propto \sqrt{Z_2(K_s)}\, G(\theta_1-\theta_2)\ \cosh\left(\frac{\theta_1+\theta_2}{2}\right)$$

total momentum $P_2 = \frac{\Delta_s}{v_s}(\sinh\theta_1 + \sinh\theta_2)$ total energy $E_2 = \Delta_s(\cosh\theta_1 + \cosh\theta_2)$

c) Fermions with extreme strong coupling

multi-band models of optical lattices:



Band insulator

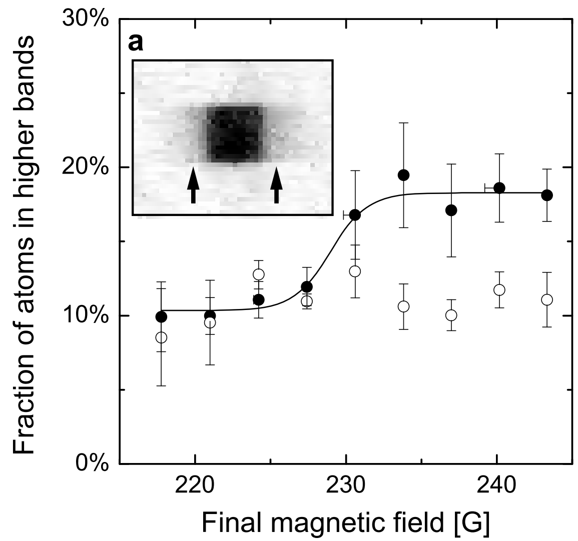
filling: 2 fermions/site

Mott insulator: S=1 antiferromagnet

2-band model: A.F. Ho PRA '06

$$H = \sum_{\alpha,\beta} \sum_{\sigma,i} -t_{\alpha\beta}\, c^\dagger_{i+1\sigma\alpha} c_{i\sigma\beta} + H.c.$$
$$- \sum_{\alpha i \sigma} \mu_\alpha\, n_{i\sigma\alpha} + \sum_{\alpha i} U_{\alpha\alpha}\, n_{i\uparrow\alpha} n_{i\downarrow\alpha}$$
$$+ \sum_{\alpha \neq \beta, i} U_{\alpha\beta} \left[n_{i\uparrow\alpha} n_{i\downarrow\beta} - S^+_{i\alpha} S^-_{i\beta} + \Delta^\dagger_{i\alpha} \Delta_{i\beta} \right]$$

Hund's



Kohl et al. PRL 2005

links with multi-orbital/band solids etc.

III. Outlook

What can we learn from cold atom lattices ?

- quantum simulator: analogue simulation of strong correlation models (Hubbard, Heisenberg, 1D systems incl. integrable models, ... See cond-mat!)
- quantum engineering: create new states not readily realisable in solids (supersolid?, topological order?-- eg. Pfaffian states, ... See cond-mat!)
- new and/or complementary experimental probes: need modelling or introduce new probes
- quantum dynamics, non-equilibrium
  in 1D: expt: Weiss, W. Phillips, Schmiedmayer,
  theory: Cardy, Rigol, Cazalilla, Gritsev & Demler ...