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Workshop on Supersolid 2008

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Supersolidity or quantum metallurgy?

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Supersolidity or Quantum Metallurgy?

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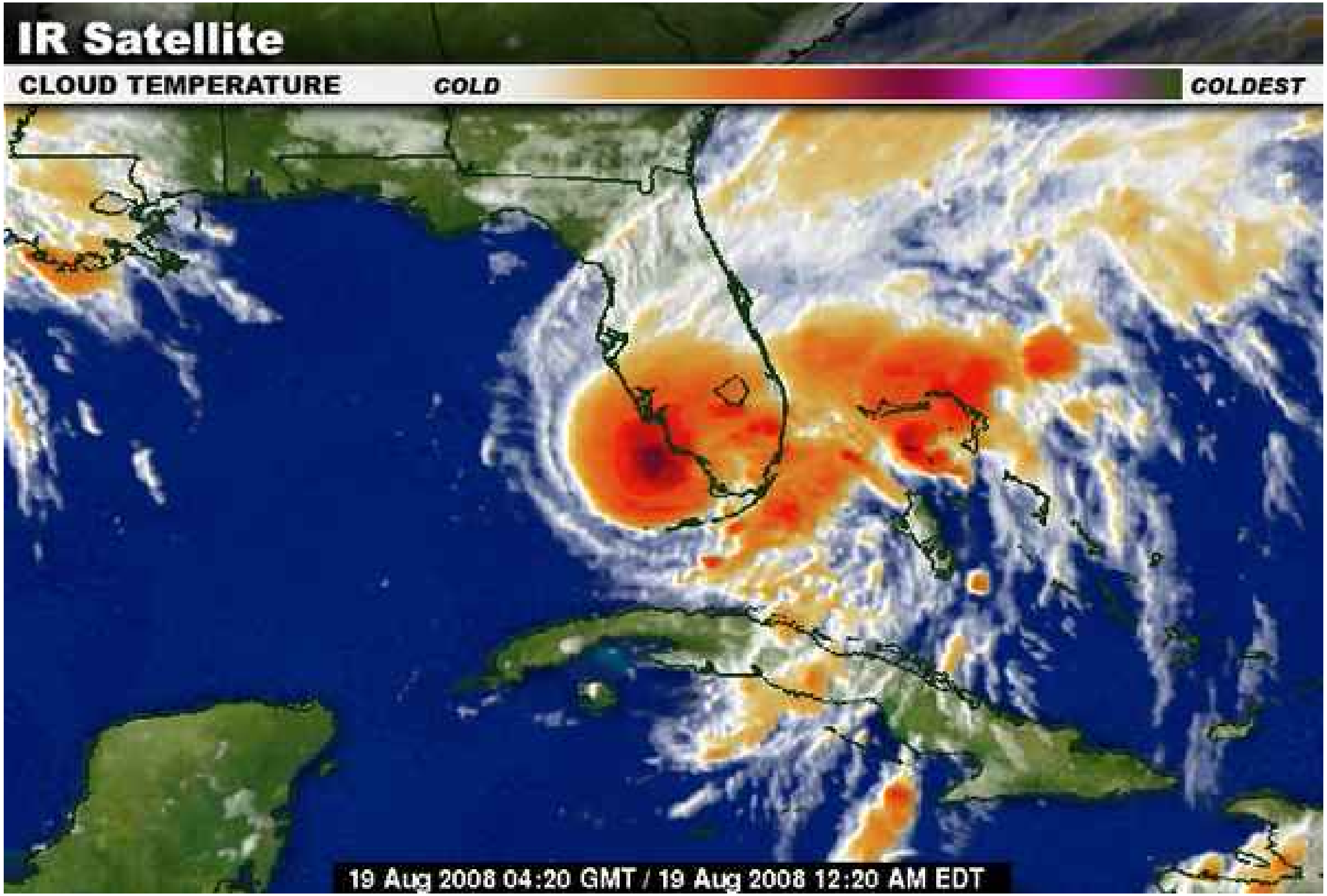
The Sunshine State?



Outline

- Can the existing experiments on solid ^{4}He be interpreted using "metallurgical" concepts? Which experiments *require* a "supersolid" interpretation?
- Modeling torsional oscillator (TO) experiments: what does a TO actually measure?
  - TO response function for a viscoelastic solid
  - Period shifts and dissipation
- Modeling specific heat experiments
  - Binding of ^{3}He to edge dislocations
  - Schottky anomaly due to ^{3}He desorption from dislocations

Torsional oscillator: rigid body

- Equation of motion for a rigid solid:

$$\left[(I_{\mathrm{cell}} + I_{\mathrm{He}})\frac{d^2}{dt^2} + \gamma\frac{d}{dt} + \alpha\right]\theta(t) = \tau_{\mathrm{ext}}(t)$$

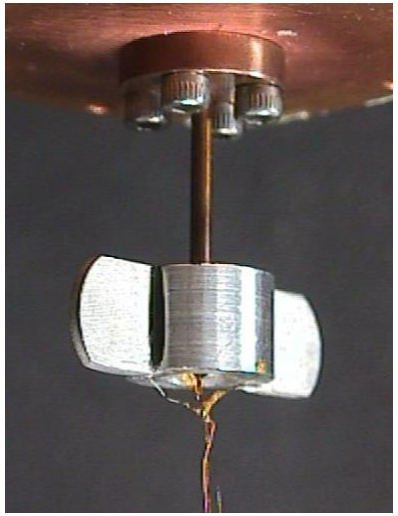
- Resonant period:

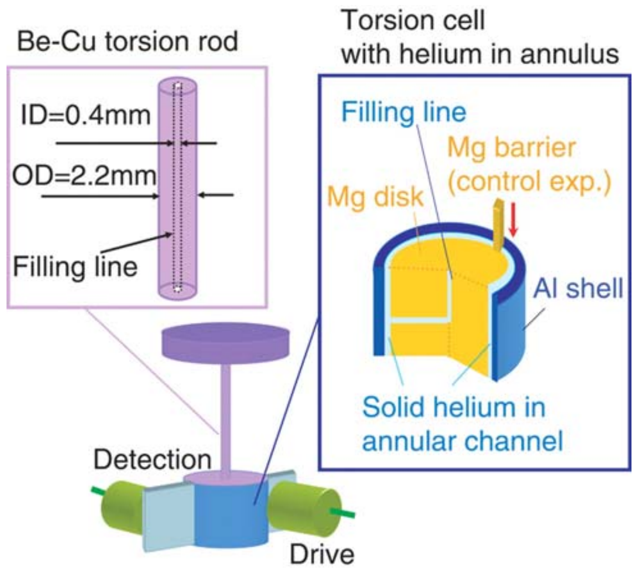
$$P = P_0\left(1 + \frac{1}{8Q^2} + \ldots\right), \quad Q = \frac{\sqrt{\alpha I_{\mathrm{total}}}}{\gamma} = \mathcal{O}(10^6)$$

$$P_0 = 2\pi\sqrt{\frac{I_{\mathrm{cell}} + I_{\mathrm{He}}}{\alpha}}$$

$$\frac{\Delta P}{P_0} = \frac{1}{2}\frac{\Delta I_{\mathrm{tot}}}{I_{\mathrm{tot}}} = \mathcal{O}(10^{-5})$$

- What happens if the solid ^{4}He is not rigid?





Torsional oscillator: elastic solid

- Equation of motion for a TO containing an elastic solid [Nussinov et al. (2007)]:

$$\underbrace{\left(I_{\rm cell}\frac{d^2}{dt^2}+\gamma\frac{d}{dt}+\alpha\right)\theta(t)=\tau_{\rm ext}(t)}_{\text{equation of motion for unloaded cell}} \quad + \quad \underbrace{M(t)}_{\text{back reaction from elastic solid}}$$

- Back action: moment that the solid ^{4}He exerts on the walls of the cell (linear response):

$$M(t)=\int dt' g(t-t')\theta(t'), \quad M(\omega)=g(\omega)\theta(\omega)$$

- Oscillator response function: $\chi(\omega)=\theta(\omega)/\tau_{\rm ext}(\omega)$

$$\chi^{-1}(\omega)=-I_{\rm cell}\omega^2-i\gamma\omega+\alpha-g(\omega)$$

- The complex poles of the response function determine the resonant frequency and dissipation of the system.

Elastic response of the solid

- *All* of the information about the solid ^{4}He is contained in $g(\omega)$. It has the following properties:
  - analytic in upper half frequency plane;
  - real and imaginary parts obey Kramers-Kronig relations;
  - low frequency behavior must be a rigid solid:
    $g(\omega) = I_{\mathrm{He}}\omega^2 + \mathcal{O}(\omega^3)$
- To calculate $g(\omega)$ we need to solve the equation of motion for an elastic solid:
  $\rho\partial_t^2 u_i = \partial_j \sigma_{ij}, \quad u_{ik} = (\partial_k u_i + \partial_i u_k)/2$
- Hooke's Law (nonlocal in time):
  $$\sigma_{ij}(t) = \int dt'\, K_{ijkl}(t-t')u_{kl}(t'), \quad \sigma_{ij}(\omega) = K_{ijkl}(\omega)u_{kl}(\omega)$$

Viscoelasticity

- Isotropic elasticity:

$$K_{ijkl}(\omega) = \lambda(\omega)\delta_{ij}\delta_{kl} + \mu(\omega)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

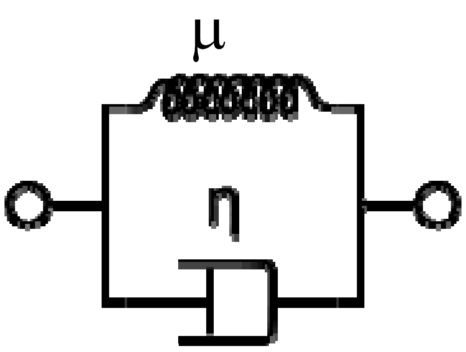
$$-\rho\omega^2\mathbf{u} = B(\omega)\nabla(\nabla\cdot\mathbf{u}) - \mu(\omega)\nabla\times\nabla\times\mathbf{u}, \qquad B = \lambda + 2\mu$$

- Shear motion of an elastic solid:

$$\rho\partial_t^2\mathbf{u} = \mu_0\nabla^2\mathbf{u}$$

- Navier-Stokes for a viscous fluid:

$$\rho\partial_t\mathbf{v} = \eta\nabla^2\mathbf{v} \quad \overset{\mathbf{v}=\partial_t\mathbf{u}}{\Longrightarrow} \quad \rho\partial_t^2\mathbf{u} = \eta\partial_t\nabla^2\mathbf{u}$$



- Combining (in "parallel"):

$$\rho\partial_t^2\mathbf{u} = (\mu_0 + \eta\partial_t)\,\nabla^2\mathbf{u}$$

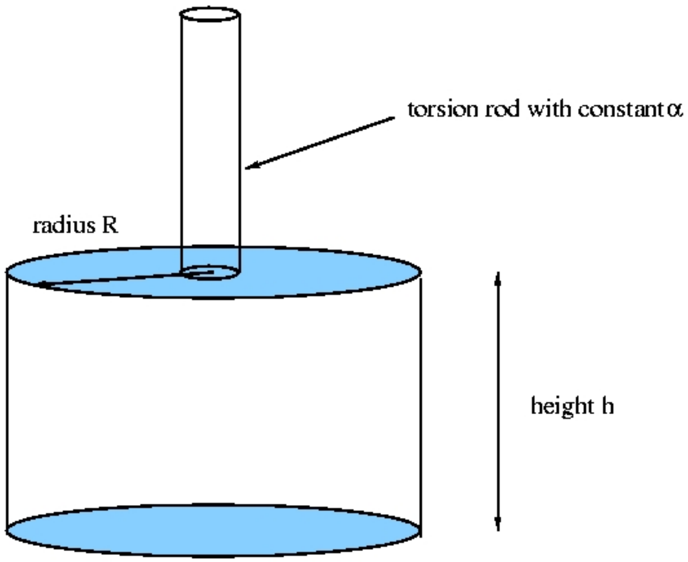
- Kelvin-Voigt model (internal friction):

$$\mu(\omega) = \mu_0 + i\eta\omega = \mu_0(1 + i\omega\tau), \quad \tau = \eta/\mu_0$$

Boundary value problem

- Cylindrical geometry, no slip boundary conditions (assume long cylinder):

$$\mathbf{u} = u_\theta(r) e^{i\omega t}\,\hat{\theta}, \quad u_\theta(r=R) = R\theta_0$$



- Equation of motion:

$$-\rho\omega^2 u_\theta = \mu(\omega)\left(\partial_r^2 + \frac{1}{r}\partial_r - \frac{1}{r^2}\right)u_\theta$$

- Solution:

$$u_\theta(r) = R\theta_0 \frac{J_1(kr)}{J_1(kR)}, \quad k^2 = \omega^2\rho/\mu(\omega)$$

- Shear stress exerted by the solid on the cell:

$$\sigma_{\theta r} = \mu(\omega)\left(\partial_r - \frac{1}{r}\right)u_\theta\Big|_{r=R} = -\theta_0 R^2 \rho\omega^2 \frac{J_2(kR)}{kRJ_1(kR)}$$

Putting it all together

- Integrate the shear stress over the area of the cell surfaces, multiply by the radius to find the moment that the solid 4He exerts on the cell:

$$M(t) = -\theta_0 \omega^2 I_{\rm He} \frac{4J_2(kR)}{kRJ_1(kR)} e^{i\omega t} \Longrightarrow g(\omega) = I_{\rm He}\omega^2 + I_{\rm He}\omega^2 \left[\frac{4J_2(kR)}{kRJ_1(kR)} - 1\right]$$

- Using $|k|R \sim 0.1$ expand the Bessel functions:

$$\chi^{-1}(\omega) \simeq -I_{\rm tot}\omega^2 - i\gamma_{\rm osc}\omega + \alpha - \frac{\rho R^2 \omega^4 I_{\rm He} F(R/h)}{24\mu(\omega)}$$

- Find approximate roots:

$$P \simeq \frac{2\pi}{\omega_0}\left[1 + \frac{\rho R^2 \omega_0^2 I_{\rm He} F(R/h)}{48\mu_0 I_{\rm tot}} \frac{1}{1+\tau^2\omega_0^2}\right] \qquad \Delta Q^{-1} \simeq \frac{\rho R^2 \omega_0^2 I_{\rm He} F(R/h)}{24\mu_0 I_{\rm tot}} \frac{\omega_0 \tau}{1+\tau^2\omega_0^2}$$

Properties of results

- TO is a probe of the shear modulus. The period shift and the dissipation are related!

$$\frac{\Delta P}{P} \propto \mathrm{Re}\left[\frac{1}{\mu(\omega)}\right], \quad \Delta Q^{-1} \propto \mathrm{Im}\left[\frac{1}{\mu(\omega)}\right]$$

- Corrections vanish for a rigid solid.
- The peak value of ΔQ^{-1} is independent of τ :

$$\Delta Q^{-1}\big|_{\max} = \underbrace{\frac{1}{48}}_{10^{-2}} \underbrace{\frac{I_{\mathrm{He}}}{I_{\mathrm{tot}}}}_{10^{-2}-10^{-3}} \underbrace{\left(\frac{\omega_0 R}{c_T}\right)^2}_{10^{-2}} = \mathcal{O}(10^{-6} - 10^{-7})$$

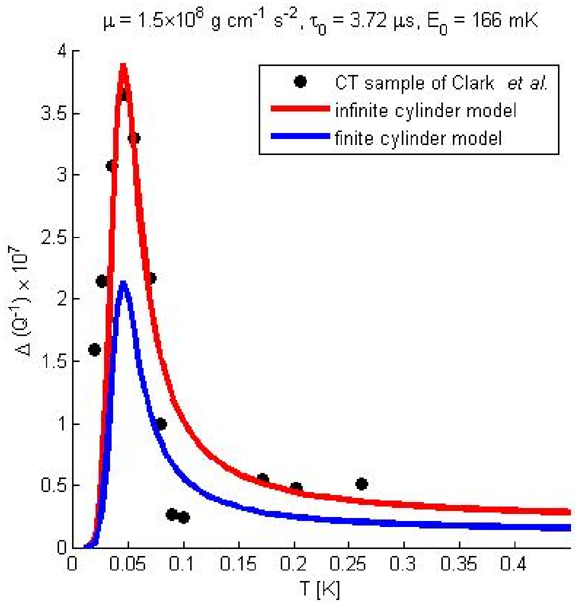
- At the peak,

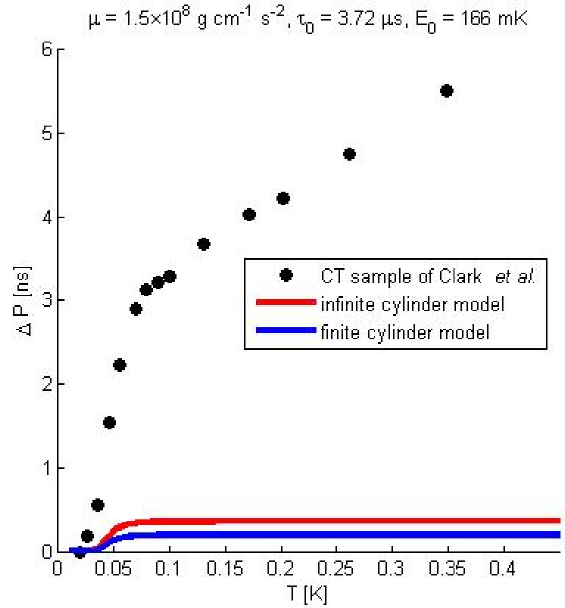
$$\Delta Q^{-1}\big|_{\max} = 2\Delta P/P\big|_{\max}$$

- For no dissipation, changing the shear modulus changes the period (inertial overshoot):

$$\frac{\Delta P}{P} = -\frac{1}{48}\left(\frac{\omega_0 R}{c_T}\right)^2 \frac{I_{\mathrm{He}}}{I_{\mathrm{tot}}} \frac{\Delta \mu}{\mu}$$

Fitting the TO experiments





$$\Delta Q^{-1} \simeq \frac{\rho R^2 \omega_0^2 I_{\rm He} F(R/h)}{24 \mu I_{\rm tot}} \frac{\omega_0 \tau}{1+\tau^2\omega_0^2}$$

$$P \simeq \frac{2\pi}{\omega_0}\left[1 + \frac{\rho R^2 \omega_0^2 I_{\rm He} F(R/h)}{48 \mu I_{\rm tot}} \frac{1}{1+\tau^2\omega_0^2}\right]$$

- Dissipation peak identifies long relaxation time on the order of 1ms $\tau = \tau_0 \exp(E_0/T)$. Dislocations?
- Model seems to only account for 10% of the period shift.

Conclusions?

- Dissipation peak is accounted for naturally using a viscoelastic model. The derived timescale is much longer than microscopic timescales, suggesting a collective effect; dislocation depinning?
- A period shift accompanies the dissipation peak, but only accounts for 10% of the observed shift. Is the remainder NCRI?
- The period shift due to the dissipation is larger than the shift due to changes in the shear modulus.

Coupling superfluidity & elasticity

- Dorsey, Goldbart & Toner (2006): Landau model with coupling between superfluidity and elasticity (strain dependent T_c):

$$\mathcal{F}_{\rm ss} = \int_{\mathbf{x}} \left\{ \frac{1}{2} c_{ij}\, \partial_i \psi\, \partial_j \psi^* + \frac{1}{2} a^{(0)} |\psi|^2 + \frac{w}{4!} |\psi|^4 + \frac{1}{2} K_{ijkl}\, u_{ij}\, u_{kl} + \frac{1}{2} a_{ij}^{(1)}\, u_{ij} |\psi|^2 \right\}.$$

- Predictions
  - XY anomaly in specific heat (lambda transition)
  - Anomalies in elastic constants; shows up as a dip in the sound speed at the transition:

$$K_{ijkl} = -T \frac{\partial^2 F}{\partial u_{ij} \partial u_{kl}} = K_{ijkl}^{(0)} - \frac{1}{4T} a_{ij}^{(1)}\, a_{kl}^{(1)} \int_{\mathbf{x}} \int_{\mathbf{x}'} \langle |\psi(\mathbf{x})|^2 |\psi(\mathbf{x}')|^2 \rangle_0$$

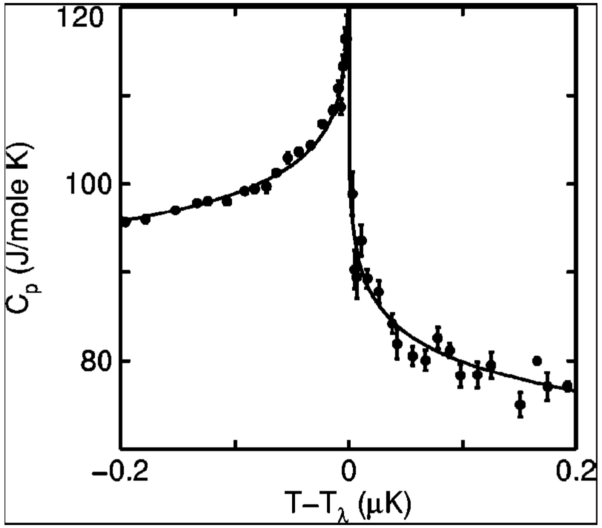
Specific heat near the λ transition

- The singular part of the specific heat is a correlation function:

$$S = -\partial F/\partial T \propto -\partial F/\partial a(T) = \int_{\mathbf{x}} \langle |\psi(\mathbf{x})|^2 \rangle$$

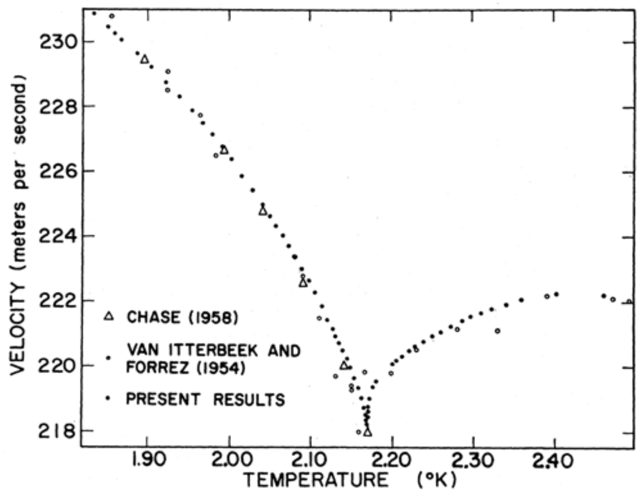
$$C = T(\partial S/\partial T) \propto \int_{\mathbf{x}} \int_{\mathbf{x}'} \langle |\psi(\mathbf{x})|^2 |\psi(\mathbf{x}')|^2 \rangle \sim A_{\pm}|t|^{-\alpha}$$

- For the λ transition, α= -0.0127.



Lipa et al., Phys. Rev. B (2003).

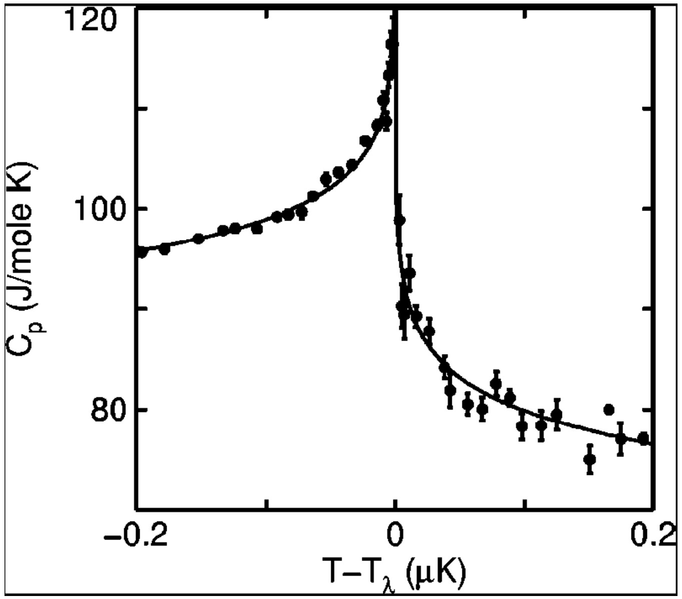
FIG. 7. Temperature dependence of the sound velocity in an extended region about the λ point.



Barmatz & Rudnick, Phys. Rev. (1968)

Specific heat I

High resolution specific heat measurements of the lambda transition in zero gravity.



J.A. Lipa et al.,
Phys. Rev. B **68**, 174518 (2003).

Specific heat near the putative supersolid transition in solid ^{4}He.

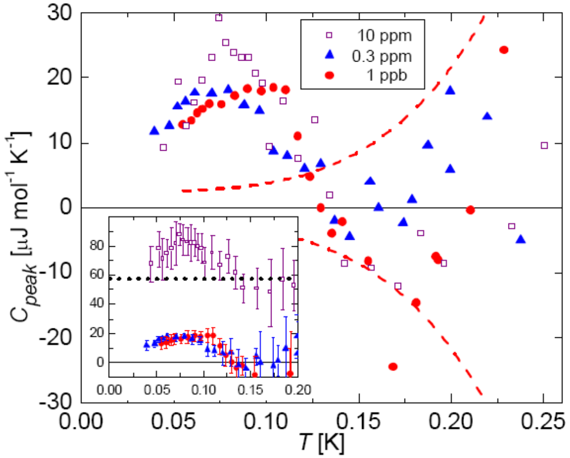
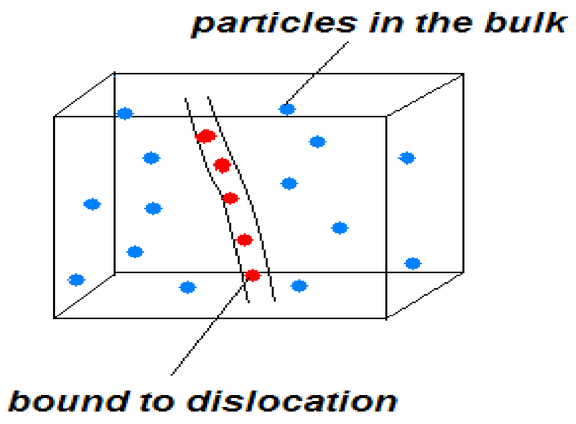


Figure 5 Specific heat peak of the 1 ppb, 0.3 ppm, and 10 ppm samples. The x_3-independent peak centred around 75 mK is revealed when the phonon contribution is subtracted. The red dashed lines indicate the uncertainty in the 1 ppb data. The uncertainty for x_3 = 0.3 ppm is comparable. For x_3 = 10 ppm, it is similar above 200 mK but decreases more gradually with decreasing temperature. For T < 100 mK the uncertainty is four times larger than that of the 1 ppb sample. The inset compares the specific heat of the three samples without the subtraction of the impuriton term of the 10 ppm sample (dotted line, 59 μJ mol^{-1} K^{-1}).

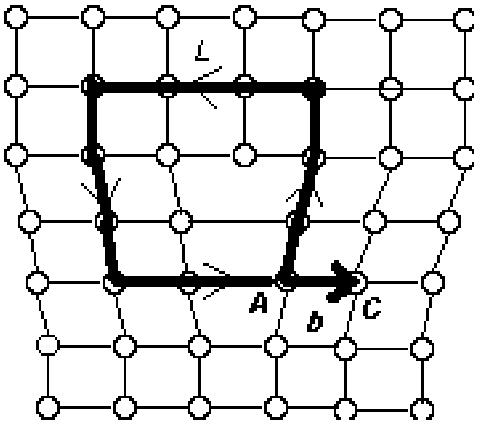
Lin, Clark, and Chan,
Nature (2007)

An alternative: lattice gas model

- Edge dislocations in ^{4}He provide an attractive potential for ^{3}He impurities.
- Bound ^{3}He impurities "evaporate" from the dislocations, increasing entropy and producing a bump in the specific heat.
- Divide the impurities into bound and free; two systems are in chemical equilibrium.
- Treat both systems classically.
- See T. N. Antsygina et al., Low Temp. Phys. 21, 453 (1995).



Binding of ^{3}He to dislocations



- Hydrostatic pressure due to an edge dislocation (continuum theory):

$$p = -\frac{1}{3}\sigma_{ii} = \frac{\mu b}{3\pi}\frac{1+\nu}{1-\nu}\frac{\sin\theta}{r}$$

- Effective potential due to a volume defect δV (Cottrell “atmosphere”):

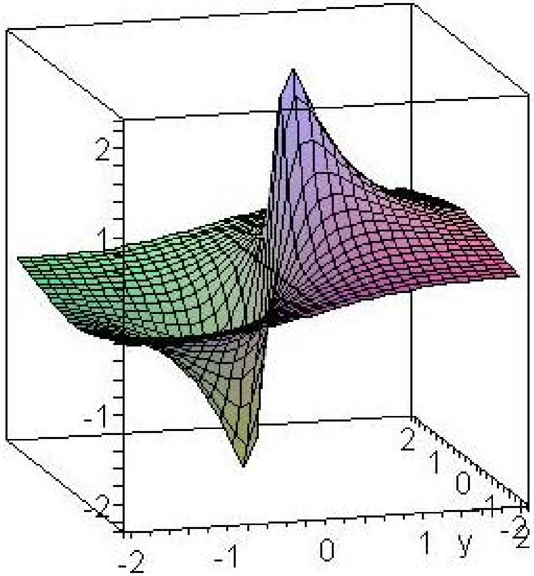
$$U(r,\theta) = p\delta V = U_0\frac{\sin\theta}{r}, \quad U_0 = \frac{\mu b}{3\pi}\frac{1+\nu}{1-\nu}\delta V$$

- Breaks down in the core due to diverging strains; need a cut off. The cut off will reduce the binding energy.

Details: Quantum dipole problem

Schrodinger equation:

$$-\frac{\hbar^2}{2m_3}\nabla^2\psi + U(r,\theta)\psi = E\psi$$



- Variational wavefunction:

$$\psi = \frac{A}{a}\sqrt{\frac{2}{\pi}}e^{-\frac{r}{a}} - \frac{\sqrt{1-A^2}}{a^2}\sqrt{\frac{8}{3\pi}}re^{-\frac{r}{a}}\sin\theta$$

- Variational estimate:

$$E_0 = -0.2397 m_3 U_0^2/\hbar^2 = -0.2397\frac{m_3}{\hbar^2}\left(\frac{\mu b}{3\pi}\frac{1+\nu}{1-\nu}\delta V\right)^2 \simeq -860\ \mathrm{mK}$$

- What about screw dislocations? Need nonlinear strains, $U(r) \sim 1/r^2$. See recent e-print by Corboz, Pollet, Prokof'ev and Troyer; binding energy $-E_0 = 0.8 \pm 0.1\ \mathrm{K}$

Specific heat II: some details

- N ^{3}He impurities, M defect sites that bind the impurities with energy ε .
- The defect sites have 0 or 1 ^{3}He impurities (two level system). Ignore correlations among sites and quantum statistics.
- Assume particles that have evaporated form a noninteracting gas.

$$N = \langle N_{\mathrm{gas}} \rangle + M \langle n_{\mathrm{site}} \rangle, \quad \langle n_{\mathrm{site}} \rangle = \left[1 + e^{-\beta(\epsilon + \mu)} \right]^{-1}$$

- Equate chemical potentials of the gas and the adsorbed particles:

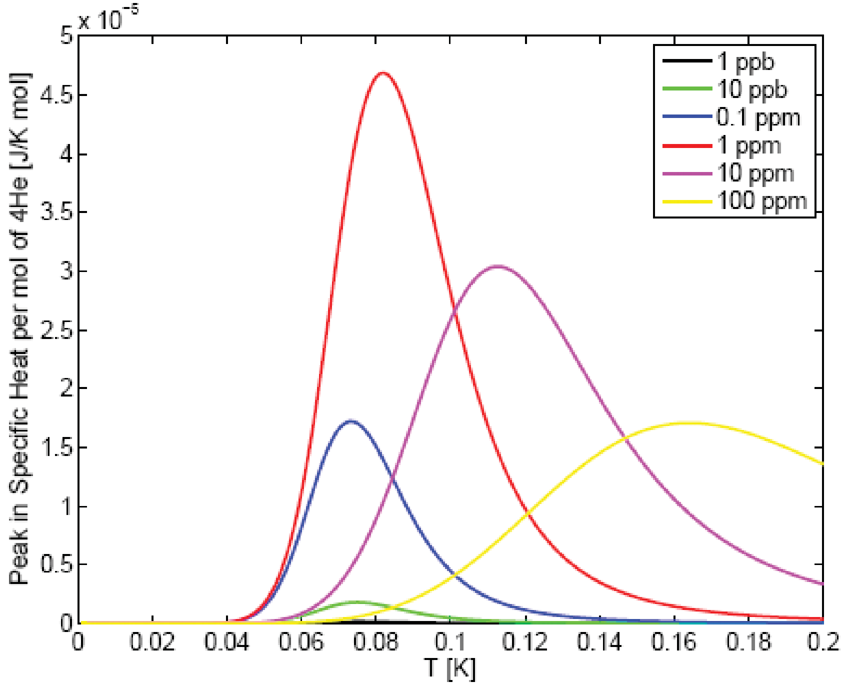
$$\langle N_{\mathrm{gas}} \rangle = \frac{1}{2} \left[N - M - p(T) + \sqrt{(N - M - p(T))^2 + 4Np(T)} \right]$$

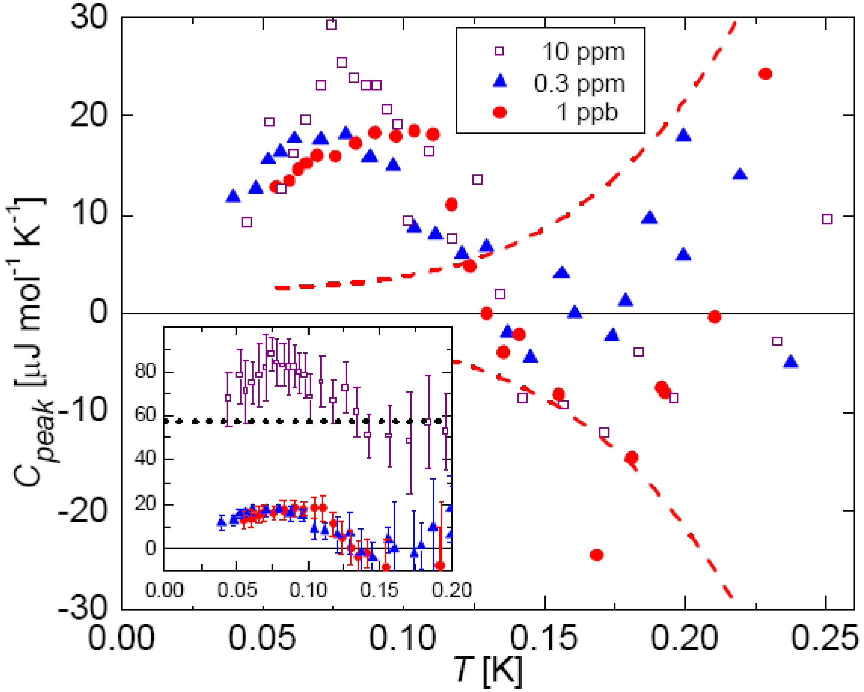
$$p(T) = V e^{-\epsilon/T} \left(\frac{2\pi m_3 T}{h^2} \right)^{3/2}$$

Properties

- Calculate the molar specific heat at constant N; complicated expression. Roughly, there is a background piece (from the gas particles) and a bump (Schottky anomaly) from the adsorbates.
- Two limits:
  - N>M (“saturated” case): size of the bump scales as M.
  - N<M (“unsaturated” case): size of bump scales with N.
- Peak appears at a temperature T^* such that $\epsilon + \mu(T^*) = T^*$. The peak position depends on the ^3He concentration (weakly).

Sample comparison with data





ε=0.6K ; defect site concentration= 0.3 ppm

Lin *et al*., Nature **449**, 1025 (2007)

Summary

- Dissipation peak in the TO response is well described by a simple viscoelastic model. A long time scale is identified, probably from dislocation physics.
- The viscoelastic model only accounts for about 10% period shift. Is the rest of the "spectral weight" at zero frequency? Is there a superfluid response?
- Specific heat feature appears to have a natural interpretation as a Schottky anomaly due to evaporation of ^{3}He impurities from dislocations. Is the binding energy related to the Arrhenius behavior of the viscoelastic model?