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On the scenarios for superfluid transition in the dislocation network in solid He-4

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On the scenarios for superfluid transition in the dislocation network

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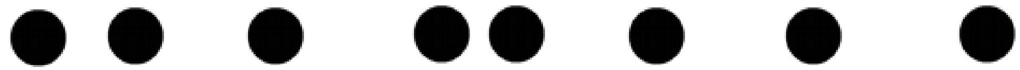
Threshold for the transition into the Andreev-Lifshits state (the Bethe-Peierls approximation)

Crystal state

Liquid state



Supersolid state

The model Hamiltonian

$$H = -t\sum_{\langle n,n'\rangle}\left[a_n^+ a_{n'} + h.c.\right] - g\sum_{\langle n,n'\rangle} a_n^+ a_n a_{n'}^+ a_{n'}$$

The first term describes the tunneling between the lattice sites in the presence of vacancies, the second term (g>0) describes the tendency toward the formation of the crystal

$a_n^+ ,\, a_n$ - the creation and annihilation operators of the hard core bosons.

Only one boson can be located at a given site: $a_n^2 = \left(a_n^+\right)^2 = 0$

Commutation relations for the operators $a_n ,\, a_n^+$

$$a_n a_n^+ + a_n^+ a_n = 1, \qquad \begin{array}{l} a_n a_m^+ - a_m^+ a_n = 0 \\ a_n a_m - a_m a_n = 0 \end{array} \quad m \neq n$$

Bethe-Peierls approximation

In this approximation the interaction of some particle at a given site with its nearest neighbors is taken into account exactly, and the interaction of its neighbors with the surrounding particles is taken into account in the self-consistent field approximation. The approximation is valid in case when the number of nearest neighbors z is much larger than one.

Wave function of the ground state in the Bethe-Peierls approximation

$$\left|\Phi_0\right\rangle = \prod_n \left(u + \mathrm{v} a_n\right)\left|1\right\rangle = \prod_n \left(u a_n^+ + \mathrm{v}\right)\left|0\right\rangle$$

$|u|^2 + |\mathrm{v}|^2 = 1,$ $\quad \left|0\right\rangle$ - wave function for the vacuum state

At $\mathrm{v} \neq 0$ the vacancies are present in the ground state and the wave function $\left|\Phi_0\right\rangle$ is a coherent superposition of a state in which the given site is filled and a state in which the given site is empty

The coefficients *u* and *v* are found from the condition of minimum of energy of the ground state (z is the number of nearest neighbors)

$$E_0 = \left\langle \Phi_0 \mid H \mid \Phi_0 \right\rangle = -\frac{zN}{2}\left[2t\mathrm{v}^2\left(1-\mathrm{v}^2\right) + g\left(1-\mathrm{v}^2\right)^2 \right]$$

The number of sites *N* is implied to be fixed.

Minimum of energy is reached at $\mathrm{v}^2 = \dfrac{t-g}{2t-g}$

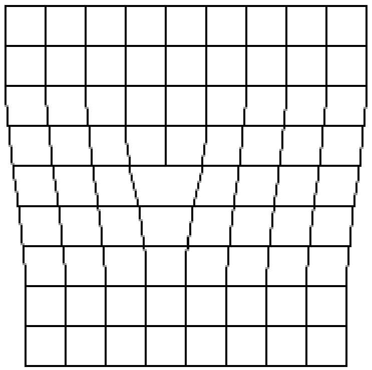
The quantity v^2 should satisfy the inequality $1 \geq \mathrm{v}^2 \geq 0$

The zero-point vacancies are present in the system if $t > g$

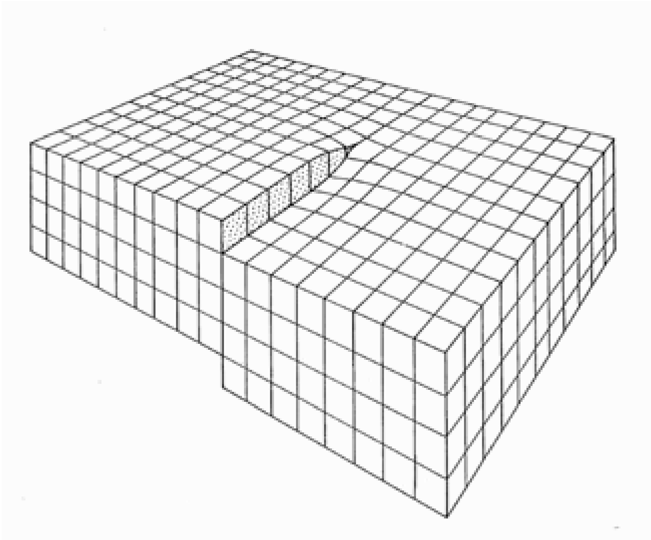
The appearance of zero-point vacancies is the threshold phenomenon

Microscopic model of the Andreev-Lifshits state in a core of screw dislocation

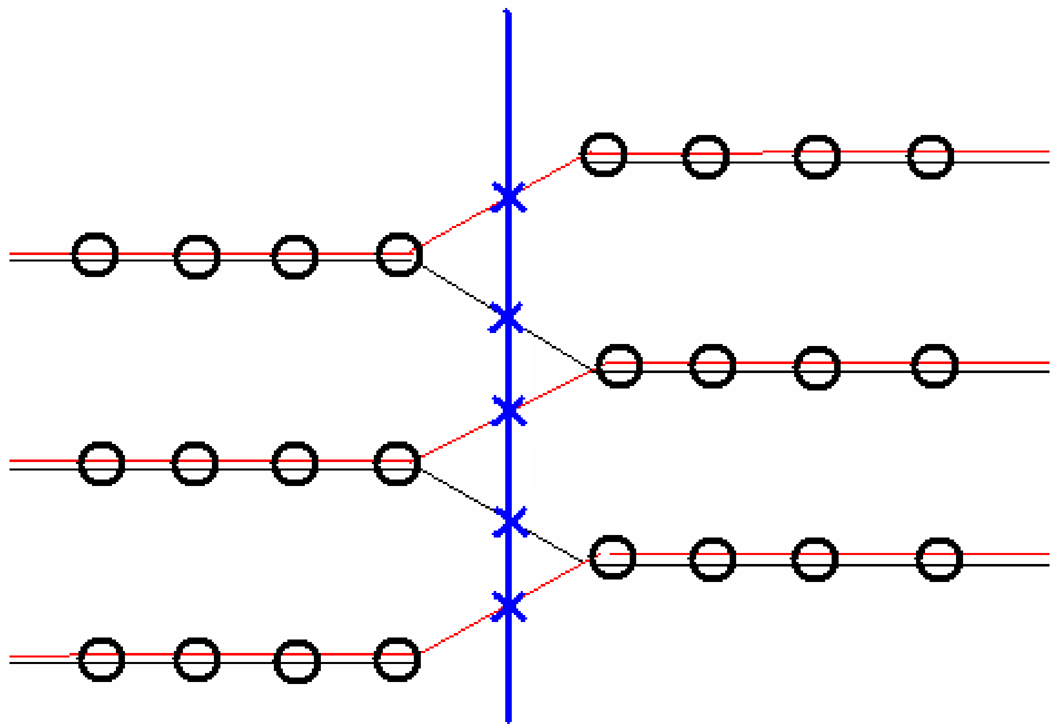
Edge dislocation



Screw dislocation



Screw dislocation in a two-dimensional crystal



Hamiltonian

$$H = -t\sum_{i=1}^{N}\left(a_i^+ a_{i+1} + a_{i+1}^+ a_i\right)$$

$$+U_1\sum_{i=1}^{N/2}\left(a_{2i-1}^+ a_{2i-1} a_{2i}^+ a_{2i} + a_{2i}^+ a_{2i} a_{2i+1}^+ a_{2i+1}\right)$$

$$-U_2\sum_{i=1}^{N/2}\left(a_{2i}^+ a_{2i} a_{2i+2}^+ a_{2i+2} + a_{2i-1}^+ a_{2i-1} a_{2i+1}^+ a_{2i+1}\right)$$

Ground state wave function in the Bethe-Peierls approximation

$$\left|\Phi\right\rangle = \prod_i \left(u_1 a_{2i-1}^{+} + \mathrm{v}_1\right)\left(u_2 a_{2i}^{+} + \mathrm{v}_2\right)\left|0\right\rangle$$

Energy of the ground state

$$E = -\frac{N}{2}\left[4t u_1 u_2 \mathrm{v}_1 \mathrm{v}_2 + 2U_1 u_1^2 u_2^2 - U_2\left(u_1^4 + u_2^4\right)\right]$$

The coefficients u_1 and u_2 are found from the equations

$$\partial E/\partial u_1 = 0, \quad \partial E/\partial u_2 = 0 \qquad \left(\mathrm{v}_i = \sqrt{1-u_i^2}\right)$$

Uniform solution $$u_1 = u_2 = u = \frac{2t^2}{(2t + U_1 - U_2)^2}$$

with the energy (per particle) $$E_{\text{supersolid}} = -\frac{4t^3}{(2t + U_1 - U_2)^2}$$

corresponds to the supersolid state in the core of screw dislocation

Non-uniform solution $u_1 = 1, \quad u_2 = 0$ with the energy $E_{\text{solid}} = -U_2$

corresponds to the crystal state

Conditions for supersolidity

$$0 < u < 1, \quad E_{\text{supersolid}} < E_{\text{solid}}$$

The condition on tunneling amplitude under which zero-point vacancies appear at the dislocation line

$$1. \quad U_1 << U_2, \qquad t > t_c \approx 1.4 U_2$$

$$2. \quad U_1 >> U_2, \qquad t > t_c \approx \frac{1}{2}\left(U_1^2 U_2\right)^{1/3}$$

The transition in the state with zero-point vacancies is a threshold one.

One can expect that at the dislocation line t is large than in the bulk and the threshold is reached.

Critical temperature for the superfluid transition in a dislocation network

Zero-point vacancies emerge at the dislocation line.

But the dislocation line is a one-dimensional object.

Is it possible a genuine superfluidity along it?

The answer on this question is the following.

In real physical systems one deals not with an isolated dislocation line of an infinite length, but with a system of dislocations that crossed and form a spatial network.

The transition into superfluid state may occur as a phase transition, and the temperature of the transition does not depend on the size of the system.

It depends only on the length of the segment of the network.

Superfluid density

The behavior of a system of spinless one-dimensional bosons is described by the partition function written in the form of the functional integral.

$$Z = \int D\Psi^*(x,\tau) D\Psi(x,\tau) \exp\left[-S(\beta)\right]$$

The action is

$$S(\beta) = \int_0^\beta d\tau \int_0^l dx \left[\Psi^*(x,\tau) \frac{\partial \Psi(x,\tau)}{\partial \tau} + H(\tau) \right]$$

with the Hamiltonian

$$H(\tau) = -\frac{\hbar^2}{2M} \Psi^* \frac{\partial^2}{\partial x^2} \Psi - \gamma n \Psi^* \Psi + \frac{\gamma}{2} \left(\Psi^* \Psi \right)^2$$

At low temperatures the main contribution to the partition function comes from the function Ψ_0 that correspond to the extremum of S and from the functions Ψ close to Ψ_0.

The condition of vanishing of the first variation of the action $\delta S = 0$ yields the equation for Ψ_0.

The solution

$$\Psi_0 = \sqrt{n}$$

n is the average one-dimensional density of bosons

The functions close to Ψ_0 can be written in the form

$$\Psi(x,\tau) = e^{i\varphi(x,\tau)}\sqrt{\rho(x,\tau)} \approx e^{i\varphi(x,\tau)}\sqrt{n}\left(1 + \frac{\delta\rho(x,\tau)}{2n}\right)$$

$$|\Psi|^2 \equiv \rho \approx n + \delta\rho, \quad \delta\rho << n$$

at low temperatures

The Fourier transformation for the ρ and φ variables does not diagonalize the action (due to the presence of the term $\Psi^*(x,\tau)\partial\Psi(x,\tau)/\partial\tau$ in the action)

The following substitution diagonalizes the action

$$\rho(k,\tau) = \left(\frac{n\varepsilon(k)}{E(k)l}\right)^{1/2}\left[\eta(k,\tau) + \eta^*(-k,\tau)\right]$$

$$\varphi(k,\tau) = \frac{1}{2i}\left(\frac{E(k)}{\varepsilon(k)nl}\right)^{1/2}\left[\eta(k,\tau) - \eta^*(-k,\tau)\right]$$

Here $\varepsilon(k) = \dfrac{\hbar^2 k^2}{2m}$ is the kinetic energy, and

$E(k) = \left[\varepsilon^2(k) + 2\gamma n\varepsilon(k)\right]^{1/2}$ is the Bogolyubov spectrum

The Bogolyubov spectrum emerges from the condition that the action is diagonal in η and η^* variables

The result (the partition function Z) coincides with the partition function of an ideal Bose gas for which the particles with the momentum $\hbar k$ have the energy E(k).

The free energy of the system is equal to

$$F = T\sum_{k} \ln\left[1 - \exp\left(-\beta E\left(k\right)\right)\right]$$

The function F is an analytical function of the temperature. There is no phase transition in the system considered.

If the fluctuations of Ψ at the extremum point are small the system behaves like a superfluid one.

The current induced by wall motion

Hamiltonian $H = H_0 - j\mathrm{v}$, where

$$j = \frac{i\hbar}{2}\left(\Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x} \right)$$ is the operator for the current

v is the velocity of the walls

The average current induced by wall motion is $\langle j \rangle = \rho_n \mathrm{v}$, where

$$\rho_n = -\sum_k \hbar^2 k^2 \frac{\partial N_B(k)}{\partial E(k)} = \frac{\pi T^2}{3\hbar c}$$ is the normal density

N_B is the Bose distribution function, $c = (\gamma n)^{1/2}$ is the sound velocity.

We see that at low temperature $\rho_n < \rho$. According to the Landau argumentation it means that the liquid in a one-dimensional wire can be superfluid

It does not contradict to the known statement that there is no superfluidity in a one-dimensional system of infinite length.

In our scheme we neglected a possibility of vanishing of the superfluid density $\rho_s = \rho - \rho_n$ at some points, called **the phase slipping centers**.

Such a neglecting is not justified in a **one-dimensional system of infinite length.** But it can be done in description of thermodynamic properties of **two or three dimensional networks of crossed one-dimensional wires.**

The appearance of phase slipping centers in a network should be taken into account in the description of kinetics of the system above the temperature of transition to a superfluid state.

Kosterlitz-Thouless transition in a two-dimensional regular network of one-dimensional wires

In the case of continuous medium the critical temperature is given by the equation

$$T_c = \frac{\pi}{2} \frac{\hbar^2 \rho_{s2}(T_c)}{M}$$

where ρ_{s2} is a two-dimensional superfluid density

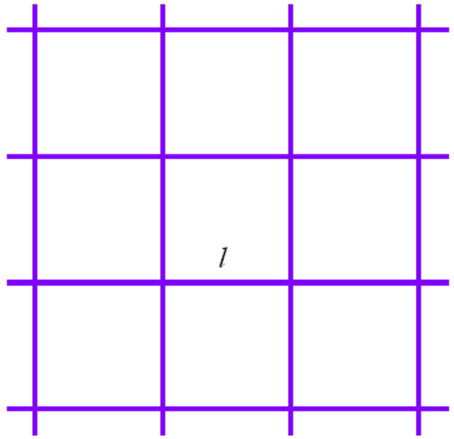
The Kosterlits-Thouless phase transition is connected with that the energy of the vortex pair diverges logarithmically at large distances between the vortices.

At such distances the difference between the continuous medium and the network is not important and the equation for T_c should contain the two-dimensional superfluid density for the network.

The equation for T_c should contain the two-dimensional superfluid density for the network. The latter quantity is equal to the number of superfluid bosons per a segment times the number of segments per unit area:

$$\rho_{s2} = \rho_{s1} l \cdot \frac{2}{l^2} = \frac{2\rho_{s1}}{l}$$

(for quadratic network)



ρ_{s1} is the one-dimensional density of bosons at the wire

l is the length of the segment

Critical temperature for the two-dimensional network

$$T_c = \frac{\pi \hbar^2 \rho_{s1}(T_c)}{M \ell}$$

High transparence of the vertexes of the network is assumed. At low transparence the answer may changed, see poster by Fil and Shevchenko on BEC in a 3D network

Rate for decay of quasi superfluid state in a two-dimensional network

(in collaboration with D.V.Fil)

We consider zero-point vacancies in the dislocation network as weakly non-ideal Bose gas.

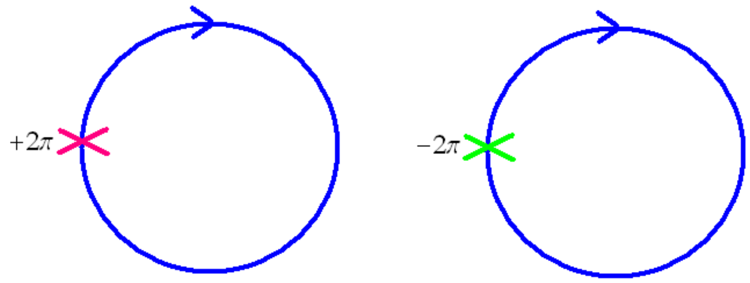
In the absence of viscosity macroscopic rarefactions or solitons in such a gas move along the wire conserving their momentum.

In reality, the momentum and the velocity of the soliton does not conserve due to non-zero second viscosity and the velocity can go to zero.

At the moment when the velocity of a soliton goes zero the shift of the phase of the order parameter along the soliton is changed on 2π . It means that the soliton is transformed to the phase slipping center.

Phase slipping centers in a ring

The 2π jump of the phase results in a increase or a decrease of the flow in a ring. The frequency of the former processes does not coincide with the frequency of the latter processes. The frequencies per unit length are given by the expression



$$\nu_{\pm} = \nu_0 \exp\left(\mp \frac{\pi \hbar n \mathrm{v}}{T} \right)$$

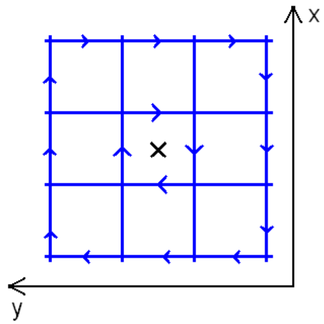
where

$$\nu_0 = \frac{4}{15\pi} \frac{\zeta_3 n}{\xi^2} \left(\frac{nT}{2\pi\hbar c} \right)^{1/2} \exp\left(-\frac{4}{3} \frac{\hbar n c}{T} \right)$$

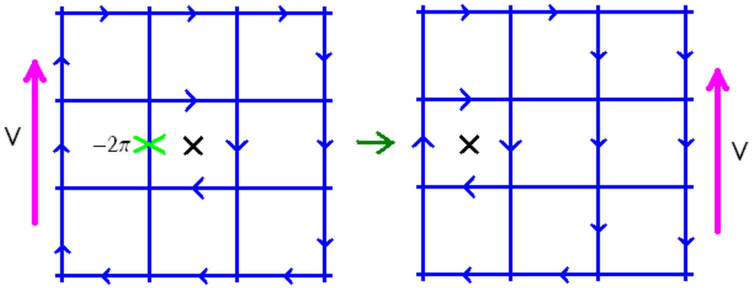
v is the superfluid velocity, ξ is the coherence length, and ζ_3 is the second viscosity

Motion of a vortex across the flow in the network

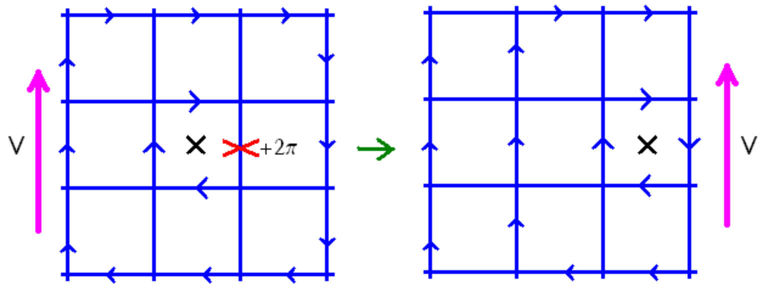
an isolated vortex



Phase slipping centers that result in vortex motion across the flow



vortex moves left



vortex moves right

In the absence of the flow the frequencies of the appearance of the phase-slip centers are the same for the right and the left segments of the vortex centered plaquette. An average the vortex does to move.

Under the flow these frequencies differ from each other. An average the vortex moves across the flow.

The average frequency of a jump of a given vortex center n in the direction of its motion is

$$\nu_{\text{jump}} = \left(\nu_{-} - \nu_{+}\right) l$$

The frequency of the pass of this vortex across the network is

$$\nu_{\text{pass}} = \nu_{\text{jump}} \frac{l}{L_y}$$ (the time of pass is $t_{pass} = t_{jump} \frac{L_y}{l}$)

The frequency of the pass of all vortexes across the network is

$$\nu_{\text{tot}} = \nu_{\text{pass}} n_{\text{vort}} L_x L_y$$

where n_{vort} is the density of vortices

Decrease of the phase shift and the superfluid velocity caused by the one vortex crossed the flow is

$$\Delta\varphi = \Delta\varphi_0 - 2\pi \qquad \mathrm{v} = \mathrm{v}_0 - \frac{2\pi\hbar}{L_x M}$$

Equation for the superfluid velocity

$$\frac{d\mathrm{v}}{dt} = -\nu_{\mathrm{tot}} \frac{2\pi\hbar}{L_x M} = -\frac{2\pi\hbar}{M}(\nu_- - \nu_+) n_{\mathrm{vort}} l^2$$

remind that $\nu_\pm = \nu_0 \exp\left(\mp \frac{\pi\hbar n \mathrm{v}}{T} \right)$

Relaxation time

$$\frac{d\mathrm{v}}{dt} = -\frac{4\pi^2\hbar^2 n v_0}{MT} n_{\mathrm{vort}} l^2 \mathrm{v}$$

(at small velocities $\mathrm{v} << \frac{\mathrm{T}}{\pi\hbar \mathrm{n}}$)

$$\mathrm{v} = \mathrm{v}_0 \exp\left(-t/\tau\right)$$

$$\tau = \frac{15}{16\pi}\frac{\xi^2}{\zeta_3}\frac{M}{\hbar^2 n^2}\frac{1}{n_{\mathrm{vort}} l^2}\left(\frac{2\pi\hbar c T}{n}\right)^{1/2} \exp\left(\frac{4}{3}\frac{\hbar n c}{T}\right)$$

Crossover temperature

τ_{exper} is the time of the experiment

If $\tau > \tau_{\text{exper}}$ the relaxation of the flow does not occur

The equation for the crossover temperature

$$\tau(T_*) = \tau_{\text{exper}}$$

The solution

$$T_* = \frac{4}{3}\frac{\hbar nc}{\ln\left(\tau_{\text{exper}}/\tau_0\right) + \ln\left(n_{\text{vort}} l^2\right)}$$

$\tau_0 \sim \dfrac{c}{\zeta_3 n^2}\dfrac{\hbar}{Mc^2}$ (under condition $\tau_{\text{exper}} >> \dfrac{\tau_0}{n_{\text{vort}} l^2}$)

Conclusion

1. In the core of a dislocation, where the lattice sites are situated in "wrong positions" the intersite tunneling amplitude increases considerable. The latter results in tendency toward formation of a one-dimensional "supersolid" in the dislocation core.

2. At rather high density of dislocations they intersect and form a 2D or 3D network. In such a network zero-point vacancies transit into superfluid state. The temperature of the phase transition T_c is of order $\hbar^2 n / Ml$.

3. Above T_c in a wide interval of temperatures (up to the temperature of degeneracy of zero-point vacancies $\hbar^2 n^2 / M$) a specific quasi superfluid state emerges. While there is no the long range order in the system, one can say about the time dependent phase of the order parameter $\varphi(\mathbf{r},t)$ and the superfluid velocity $\mathrm{v}_s(\mathbf{r},t) = \hbar \nabla \varphi / M$. Relaxation of v_s is determined by the frequency of appearance of phase slipping centers. The relaxation time $\tau \propto l^{-2} \exp\left(4\hbar nc / 3T\right)$.