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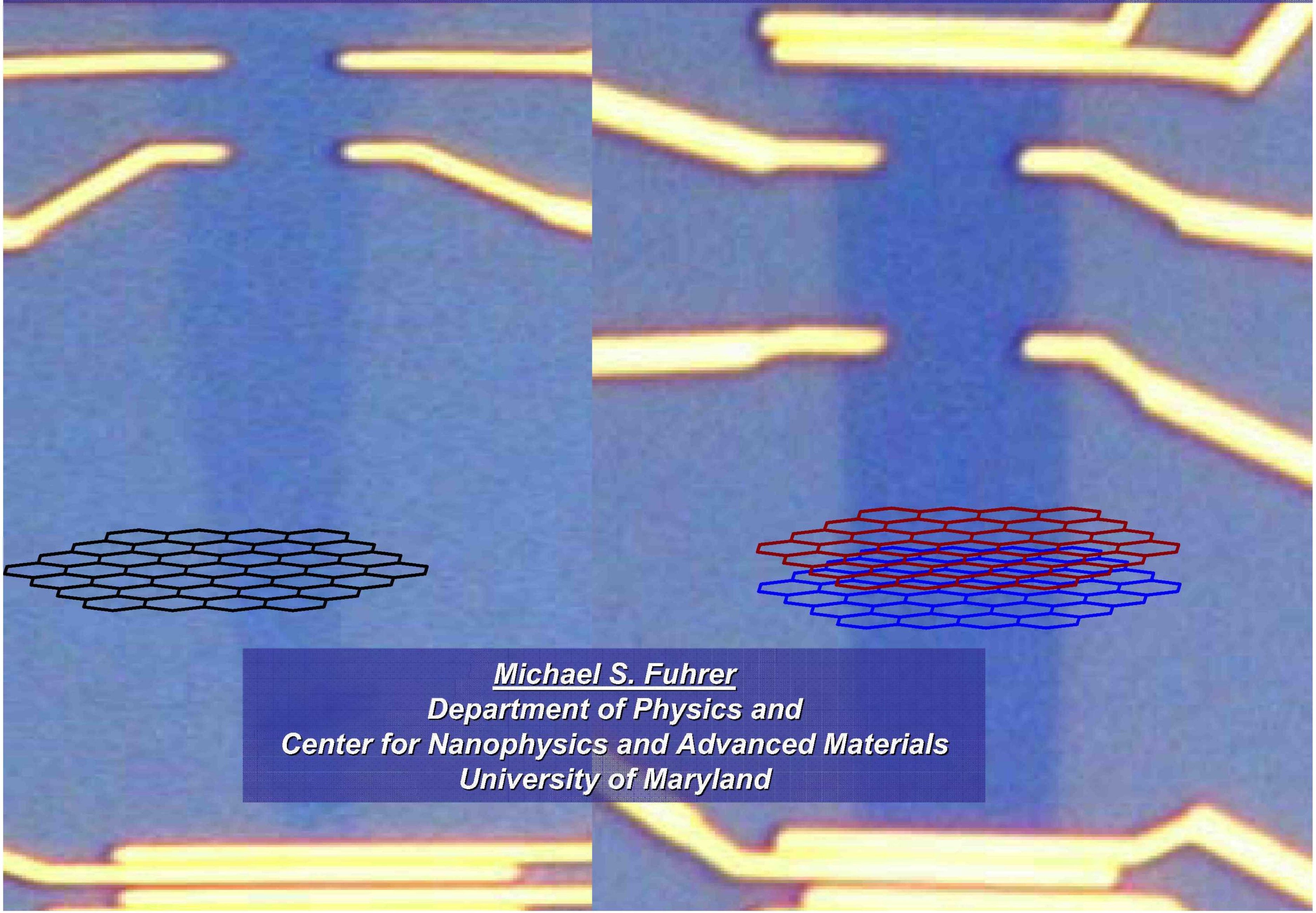
Probing diffusive and ballistic transport in graphene

M.S. Fuhrer

Department of Physics and
Center for Nanophysics and Advanced Materials
University of Maryland
U.S.A.

Probing Diffusive and Ballistic Transport in Graphene



Michael S. Fuhrer
Department of Physics and
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Outline

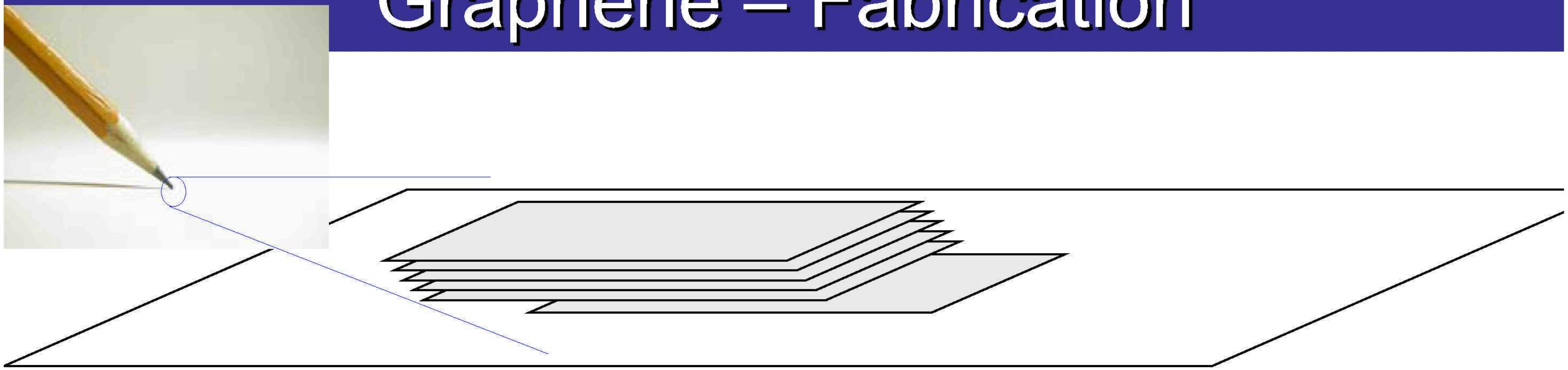
I. Introduction to Graphene
   "Massless" electrons
   Pseudospin and Berry's phase

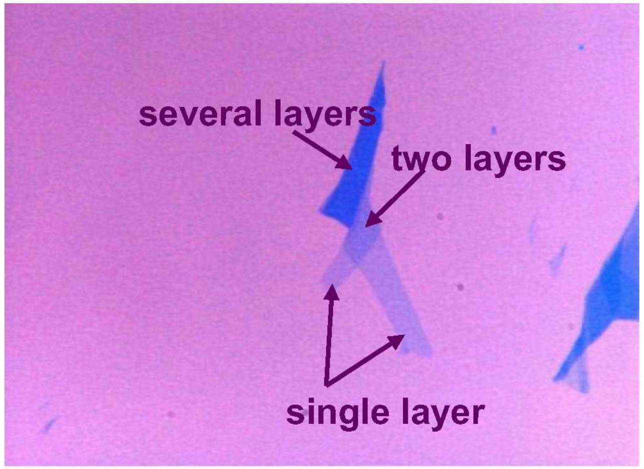
II. Fabrication and Characterization of Graphene on SiO_2
   Micro-Raman spectroscopy
   Scanning Tunneling Microscopy

III. Diffusive Transport in Graphene
   Boltzmann Transport
      Charged impurities
      Effect of Dielectric Environment
      Corrugations
      Phonons

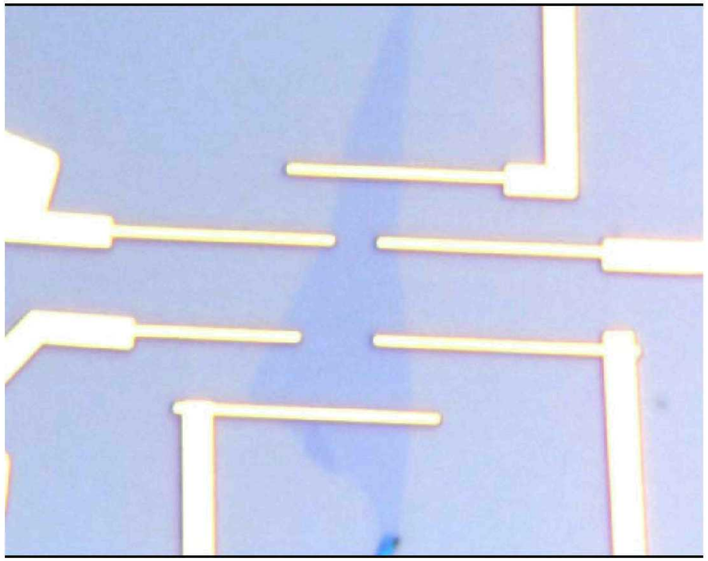
IV. Ballistic Transport in Short Graphene Devices
   Single and Bilayer Graphene
   Fabry-Perot Interference → Density of States

Graphene – Fabrication





Optical micrograph (layer thickness verified by AFM)



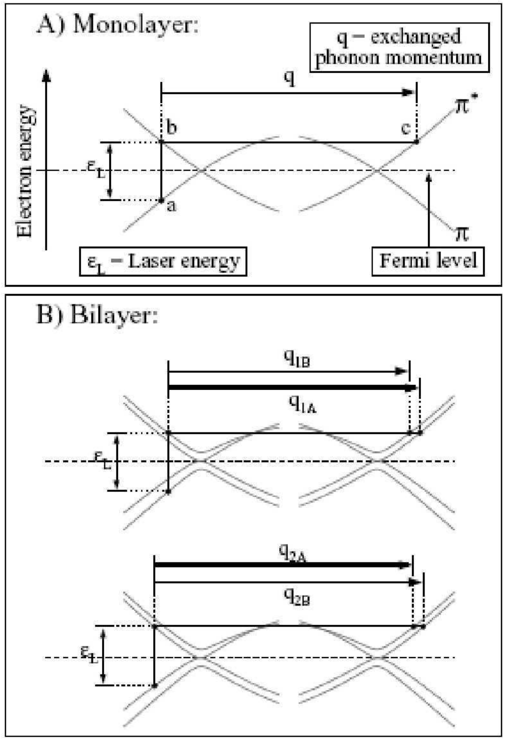
Single layer device after e-beam lithography

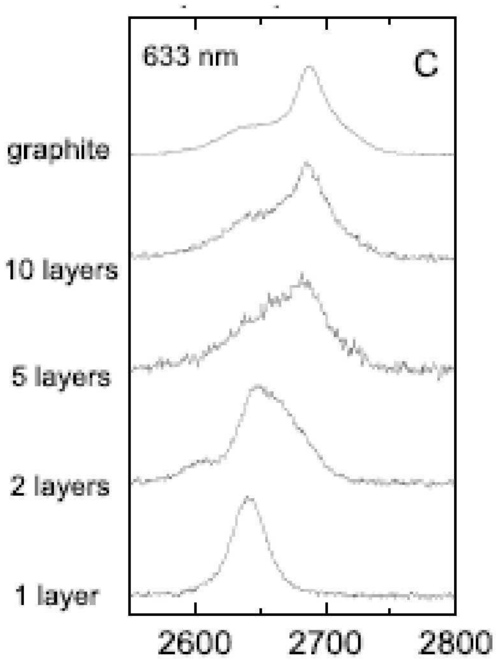
- Starting material is single-crystal Kish graphite
- Mechanically exfoliate on 300 nm SiO_2/Si chips

Method adapted from Novoselov, et al. *PNAS* **102** 10341 (2005)

Graphene fingerprint in Micro-Raman

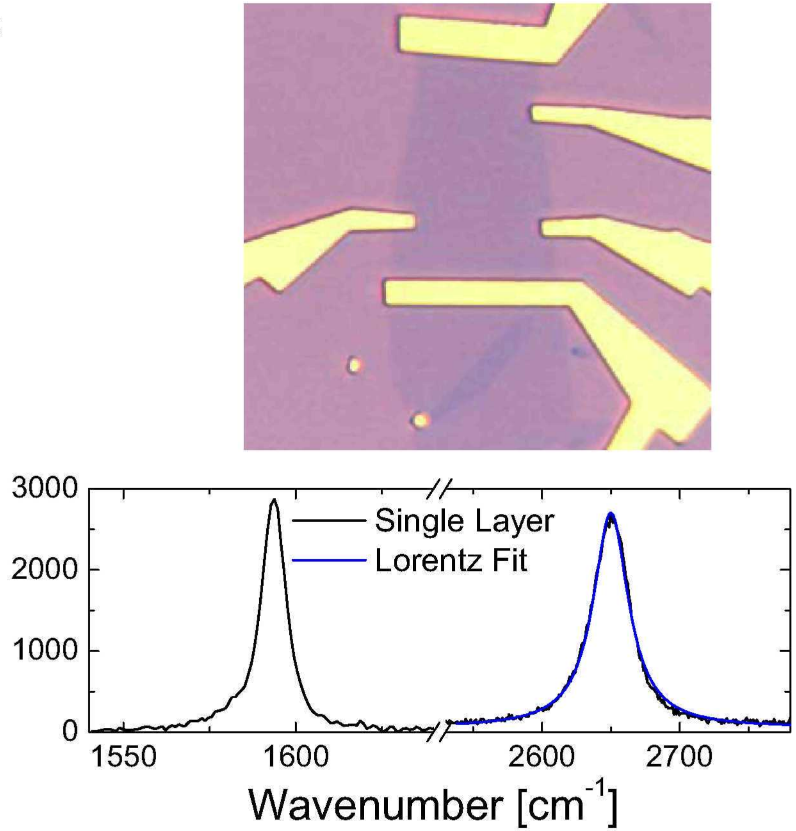
- Raman G' band is two-photon/two-phonon resonant excitation; sensitive to electronic structure of graphene





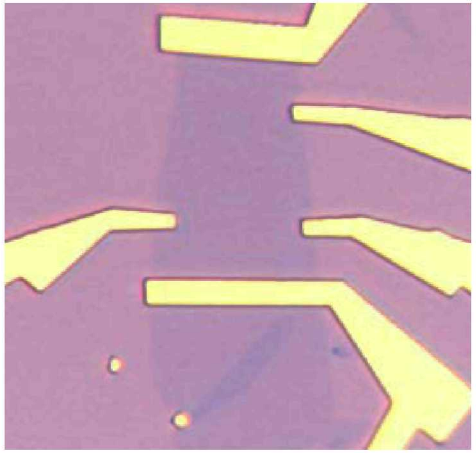
Ferrari, et al., *PRL* **97**, 187401 (2006)

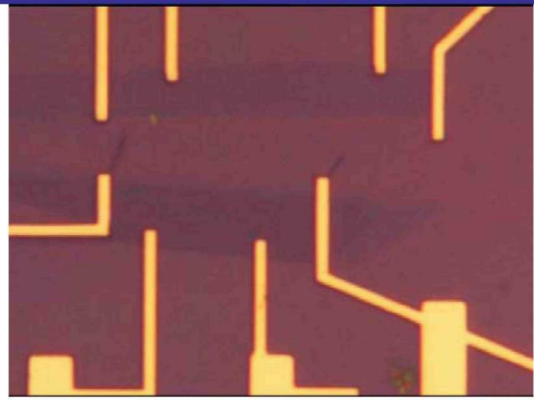
Fuhrer group sample

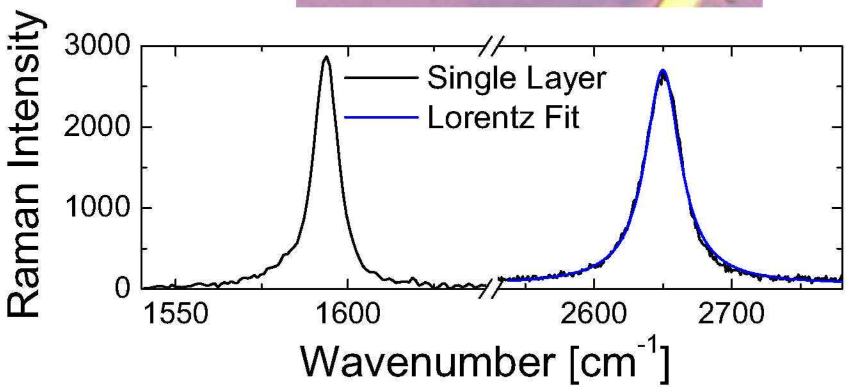


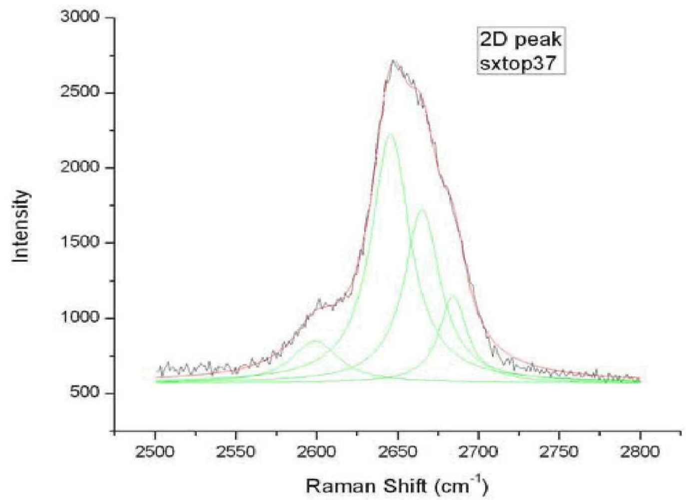
single Lorentzian G' peak indicates single-layer graphene

Single-layer vs. Bilayer in micro-Raman







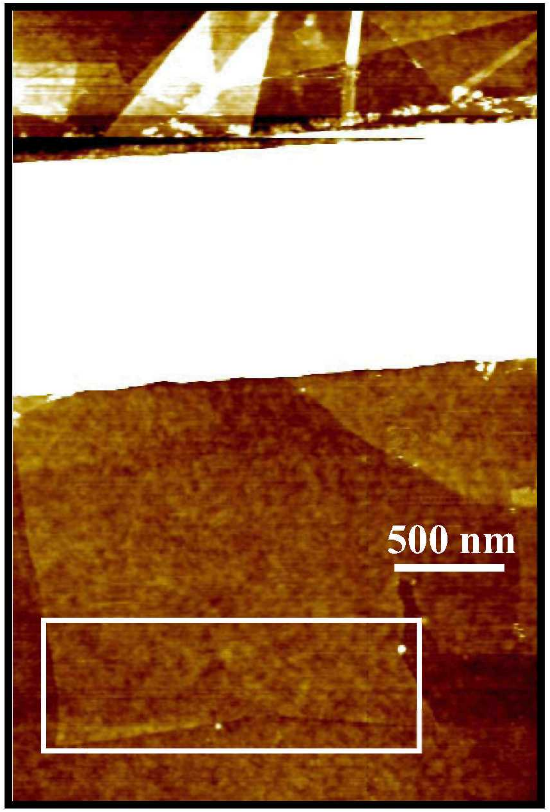


single Lorentzian G' peak indicates single-layer graphene

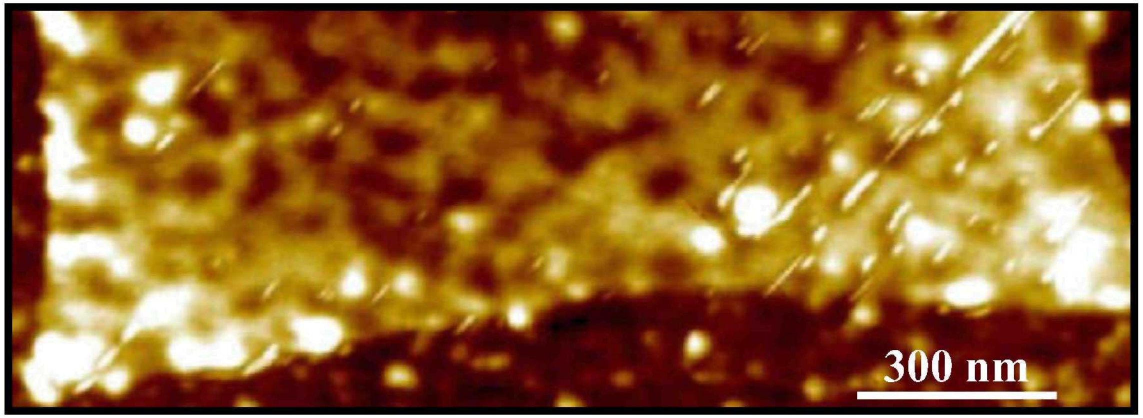
bilayer graphene shows characteristic four-fold splitting of G' peak

Removing Photoresist Residue from Graphene

Ishigami, et al., *Nano Letters* **7**, 1643 (2007)

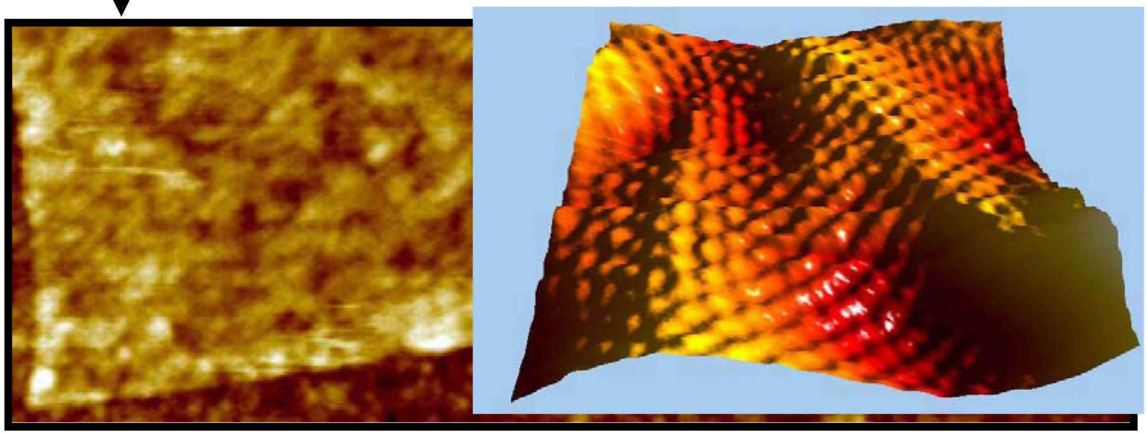


Residues from PMMA/MMA photoresist



Novel photoresist residue removal process

Anneal in flowing H_2 at 400°C



Complete removal of photoresist residues

Atomically clean STM images

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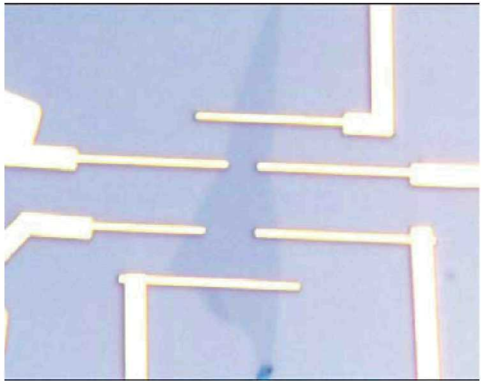
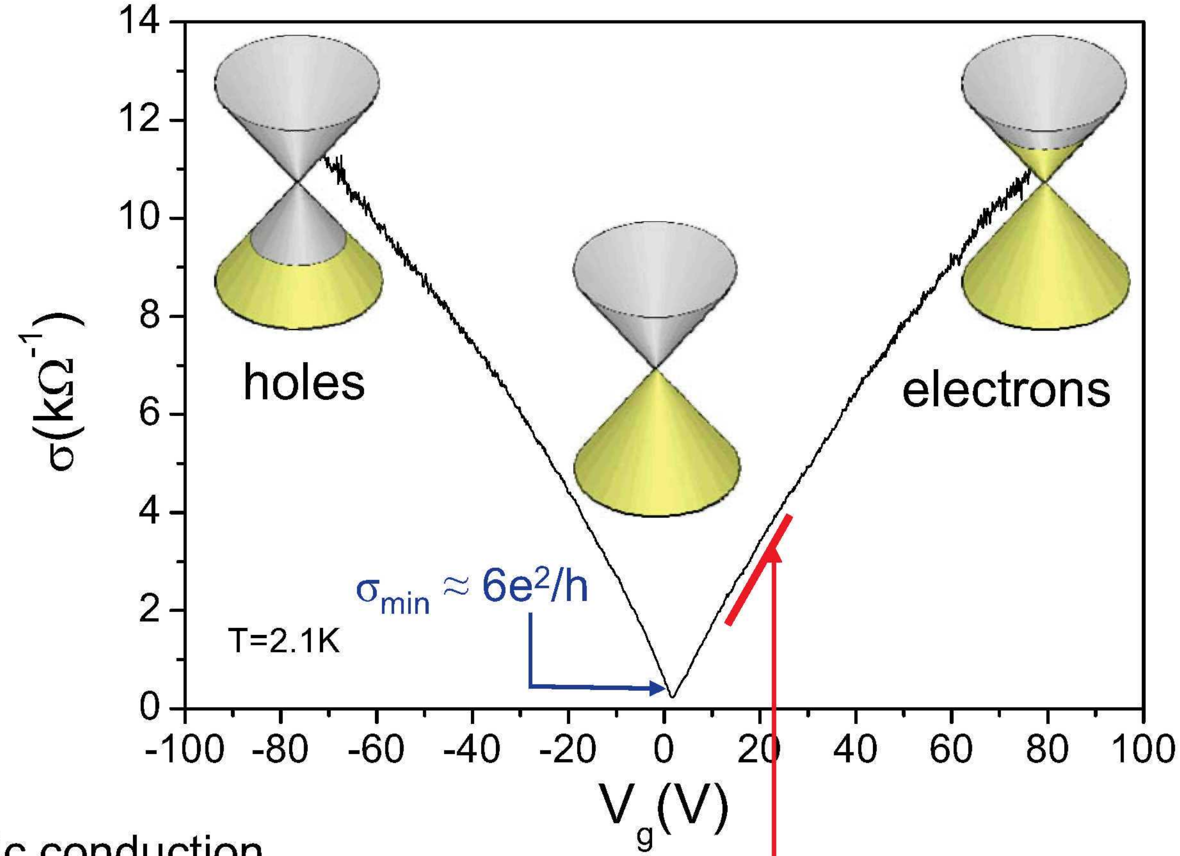
Electrical Characterization of Graphene





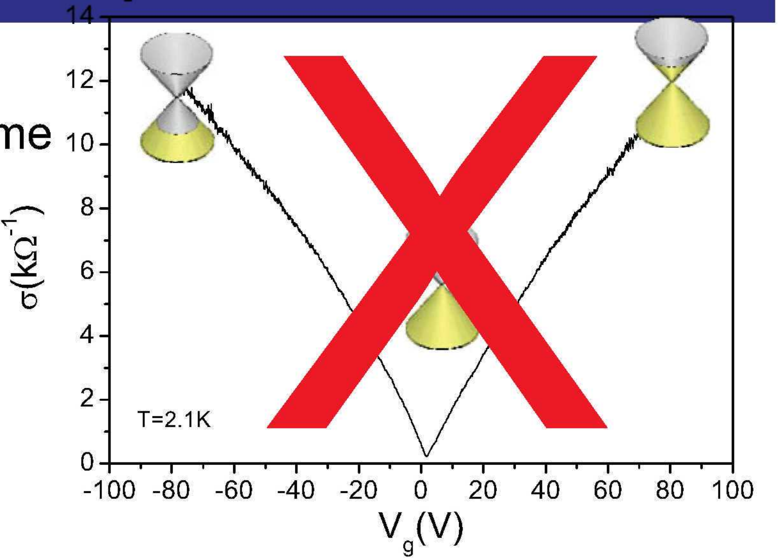
$$\mu_{FE} = \frac{1}{e}\frac{d\sigma}{dn} = \frac{1}{c_g}\frac{d\sigma}{dV_g}$$

- Ambipolar, symmetric conduction
- Finite minimum conductivity ~ [4-10]e^2/h
- Field-effect mobility up to 20,000 cm^2/Vs
- on/off ratio of 50-100:1
- Sheet resistance of ~100 Ω/square at high V_g

Boltzmann Transport

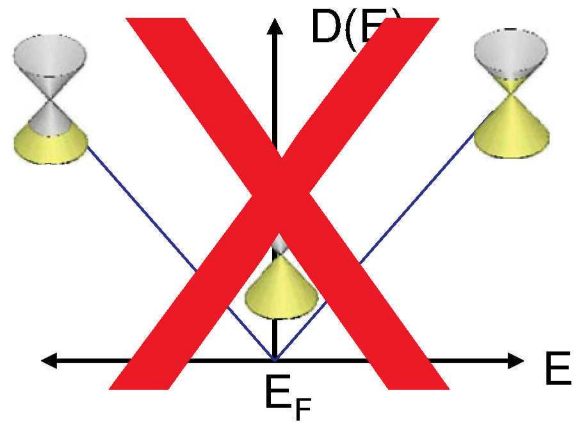
$$\sigma = \frac{e^2 v_F^2}{2} D(E)\tau$$

D(E) is density of states
τ is momentum relaxation time
v_F is Fermi velocity



Graphene: $D(E) = \frac{2E_F}{\pi\hbar^2 v_F^2}$

$$\sigma = \frac{e^2}{\pi\hbar^2} E_F \tau$$ or $$\sigma = \frac{2e^2}{h} k_F l$$



Fermi's Golden Rule:

$$\frac{1}{\tau} \propto \frac{2\pi}{\hbar} \left|\langle k|V|k'\rangle\right|^2 D(E)$$

$$\tau \propto D(E)^{-1} \propto E_F^{-1}$$

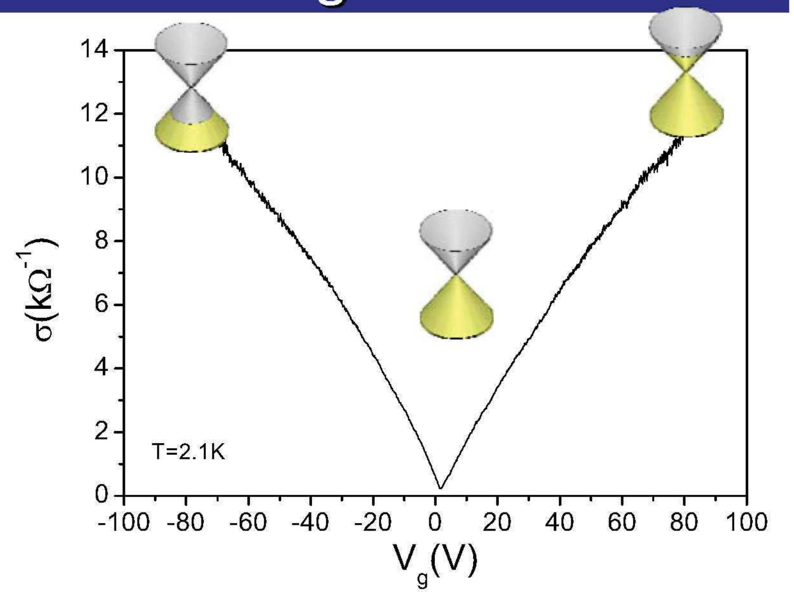
⇒ σ is independent of E_F!

True for point defects, phonons
see e.g. T. Ando (1996)

How to explain linear $\sigma(V_g)$?

$$\sigma = \frac{e^2}{\pi\hbar^2} E_F \tau$$

$$\frac{1}{\tau} \propto \frac{2\pi}{\hbar} \left| \langle k | V | k' \rangle \right|^2 D(E)$$



Interaction must be *q*-dependent

$$q = |\mathbf{k} - \mathbf{k}'| \sim k_F$$

Coulomb interaction: $V_{Coulomb} = \frac{2\pi e^2}{\kappa q}$

N.B. In graphene, *screened* Coulomb interaction remains ~1/k_F

$\Rightarrow$ $\sigma \sim E_F^{1/2} \sim n \sim V_g$

See:

Ando, *J. Phys. Soc. Jpn.* **75**, 074716 (2006)
Nomura & MacDonald *PRL* **98**, 076602 (2007)
Cheianov & Fal'ko *PRL* **97**, 226801 (2006)
Hwang, Adam, & Das Sarma, *PRL* **98**, 186806 (2007)

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Charged Impurity Scattering: Potassium Doping in UHV

J. H. Chen, et al. *Nature Physics* **4**, 377 (2008)

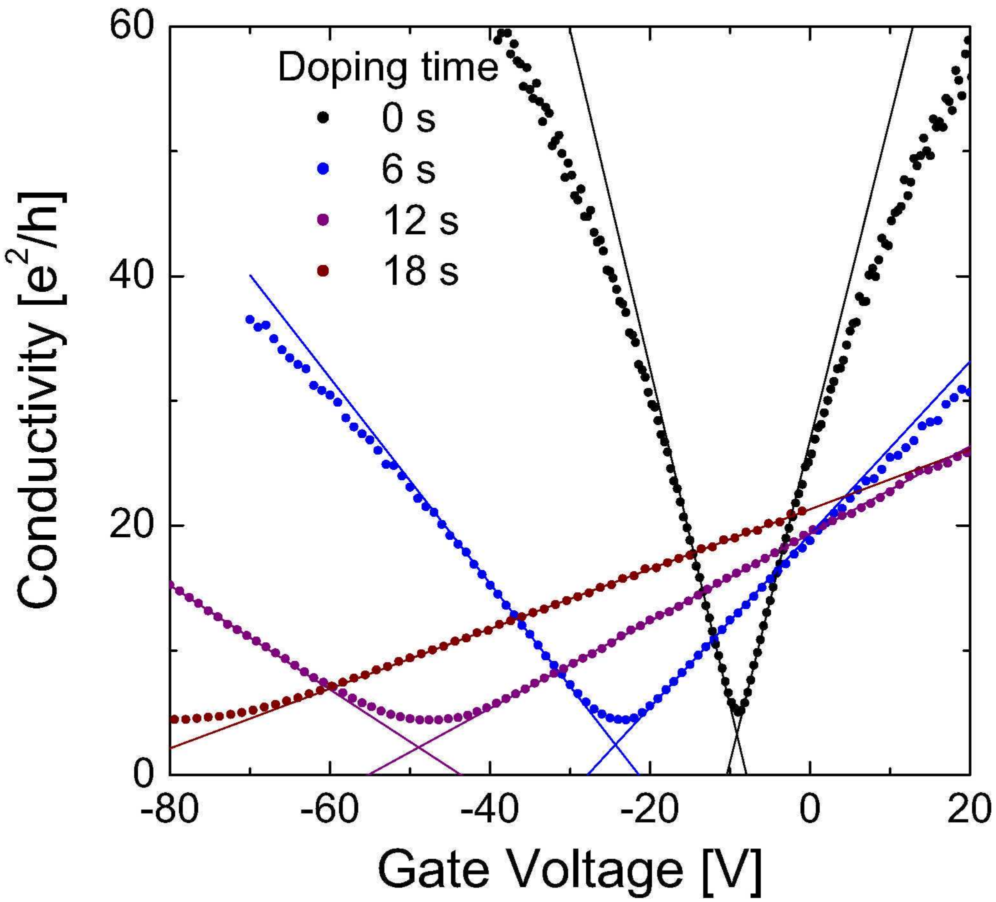
- Clean graphene in UHV at T = 20 K
- Potassium evaporated on graphene from getter

Upon doping with K:

1) mobility decreases
2) $\sigma(Vg)$ more linear
3) σ_{min} shifts to negative V_g
4) plateau around σ_{min} broadens
5) σ_{min} decreases (slightly)

All these feature predicted for Coulomb scattering in graphene

Adam, et al., PNAS 104, 18392 (2007)

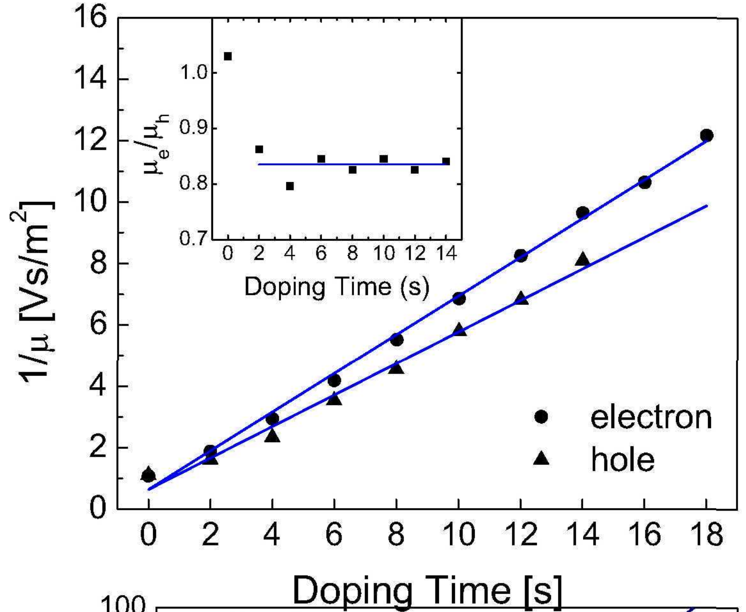


Charged Impurity Scattering: High Carrier Density
J. H. Chen, et al. *Nature Physics* **4**, 377 (2008)

- Theory:

[Adam, et al., *PNAS* **104**, 18392 (2007)]

$$\left[\mu(\mathrm{cm^2/Vs})\right] = \frac{C}{\left[n_{imp}(\mathrm{cm^{-2}})\right]} \qquad C = 5\mathrm{x}10^{11}\ \mathrm{Vs}$$

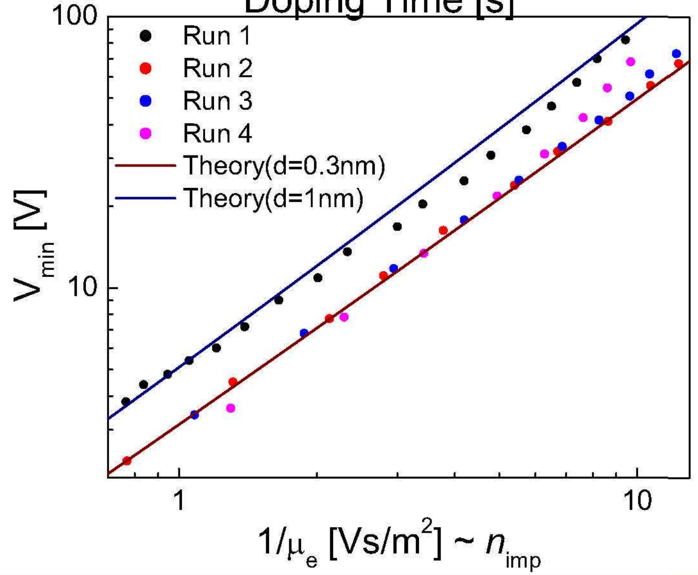


- 1/μ vs. doping time
- V_{min} vs. 1/μ

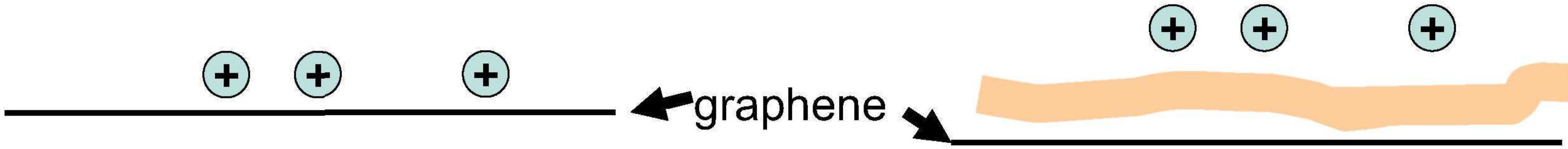
→ consistent with C = 5x10^{11}Vs

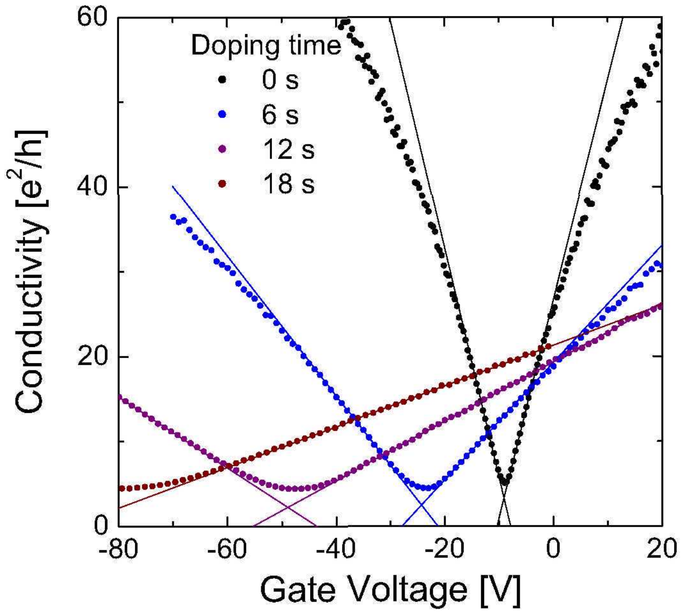
- *e-h* asymmetry predicted for Dirac fermions

[Novikov, *APL* **91**, 102102 (2007)]



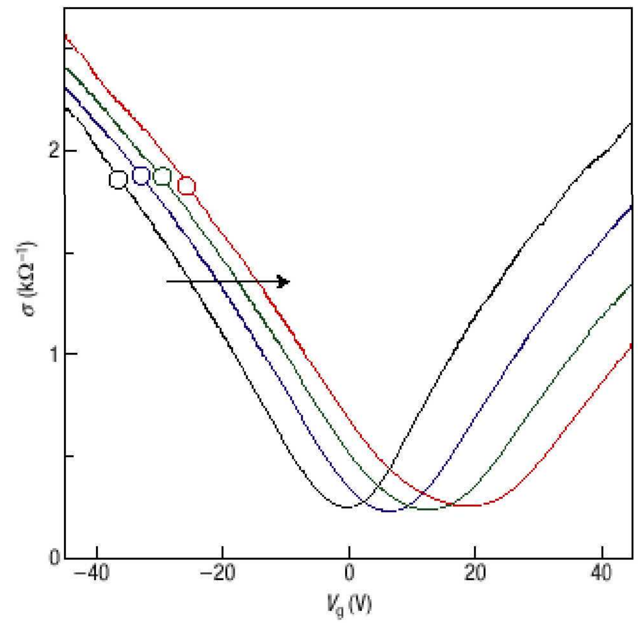
Why must graphene be atomically clean?





J. H. Chen, et al. *Nature Physics* **4**, 377 (2008)

$$\frac{1}{\mu} = \frac{n_{imp}}{[5x10^{11} Vs]}$$



F. Schedin, et al. *Nat. Mat.* **6**, 652 (2007)

$$\frac{1}{\mu} = \frac{n_{imp}}{[10^{13} Vs]}$$

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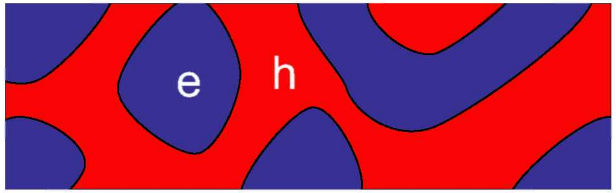
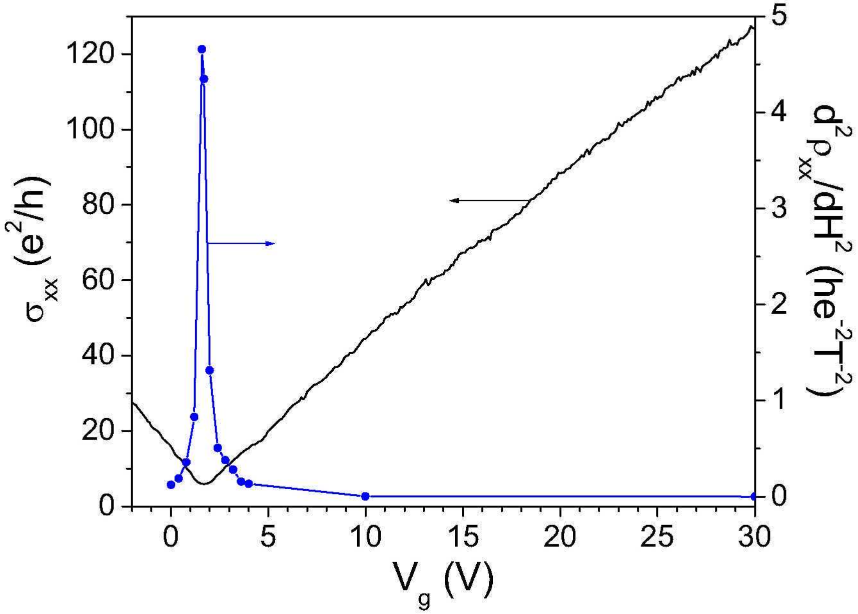
Magnetoresistance at Minimum Conductivity Point

S. Cho and M. S. Fuhrer, *PRB* **77**, 084102(R) (2008)

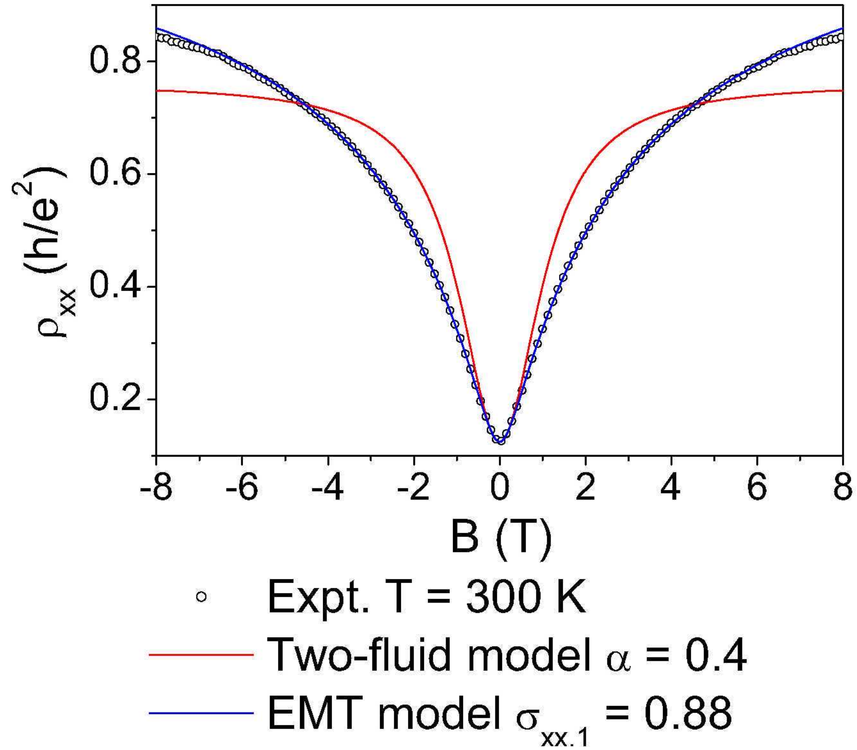
- At minimum conductivity point, graphene breaks into electron and hole "puddles"

Hwang, et al., *PRL* **98,** 186806 (2007); Adam, et al., *PNAS* **104**, 18392 (2007)





Large spike in magnetoresistance at Dirac point



Functional form of ρ(B):

effective medium theory for inhomogeneous e/h regions

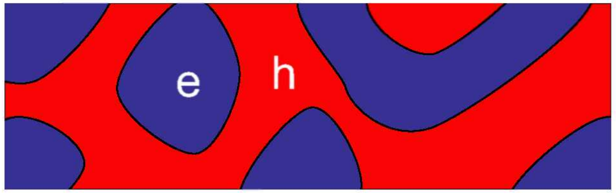
[Guttal and Stroud, *PRB* **71** 201304 (2005)]

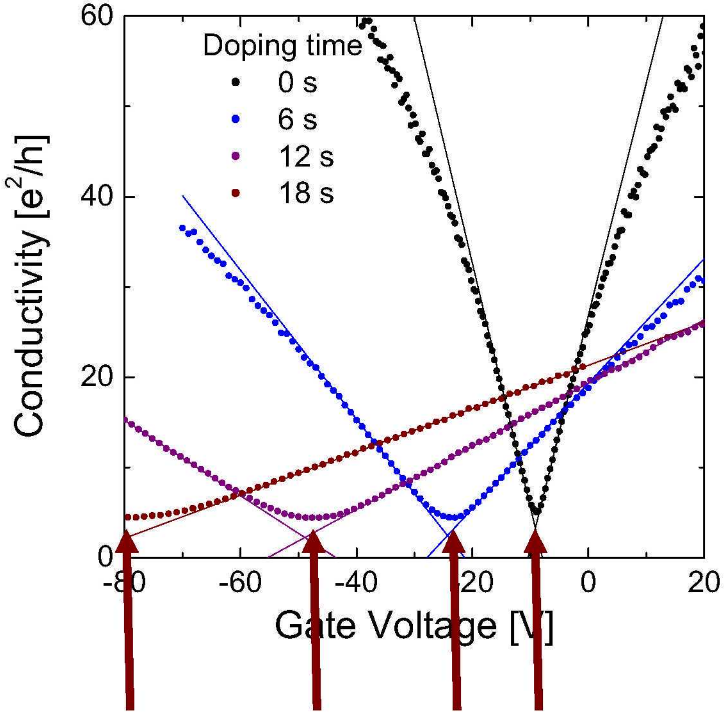
Charged Impurity Scattering: Minimum Conductivity

J. H. Chen, et al. *Nature Physics* **4**, 377 (2008)

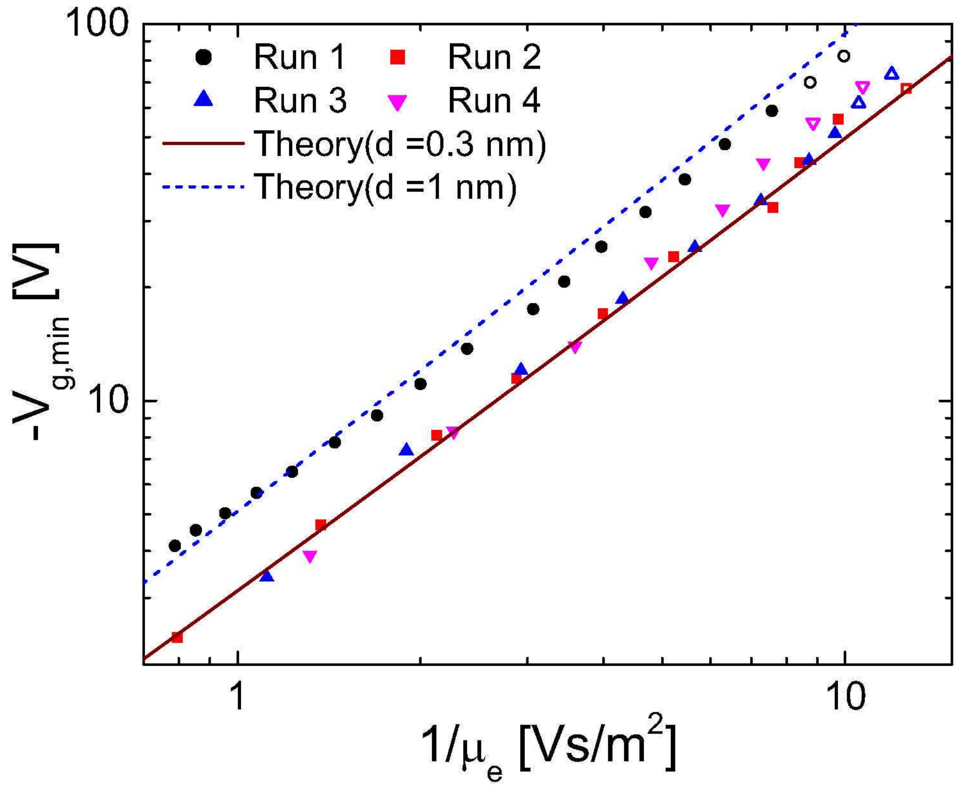
- At minimum conductivity point, graphene breaks into electron and hole “puddles”
- Minimum conductivity occurs when average *screened* potential is zero

Adam, et al., *PNAS* **104**, 18392 (2007)





minimum conductivity point ($V_{g,min}$)



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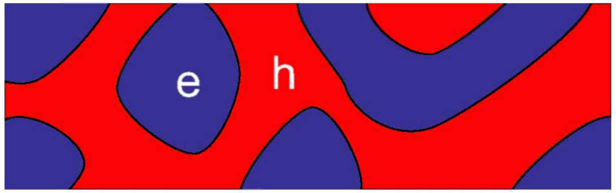

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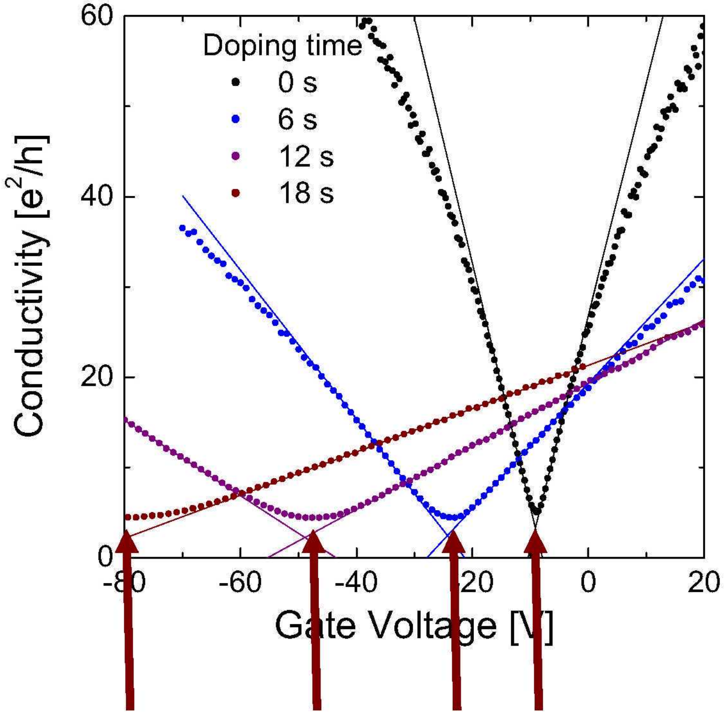

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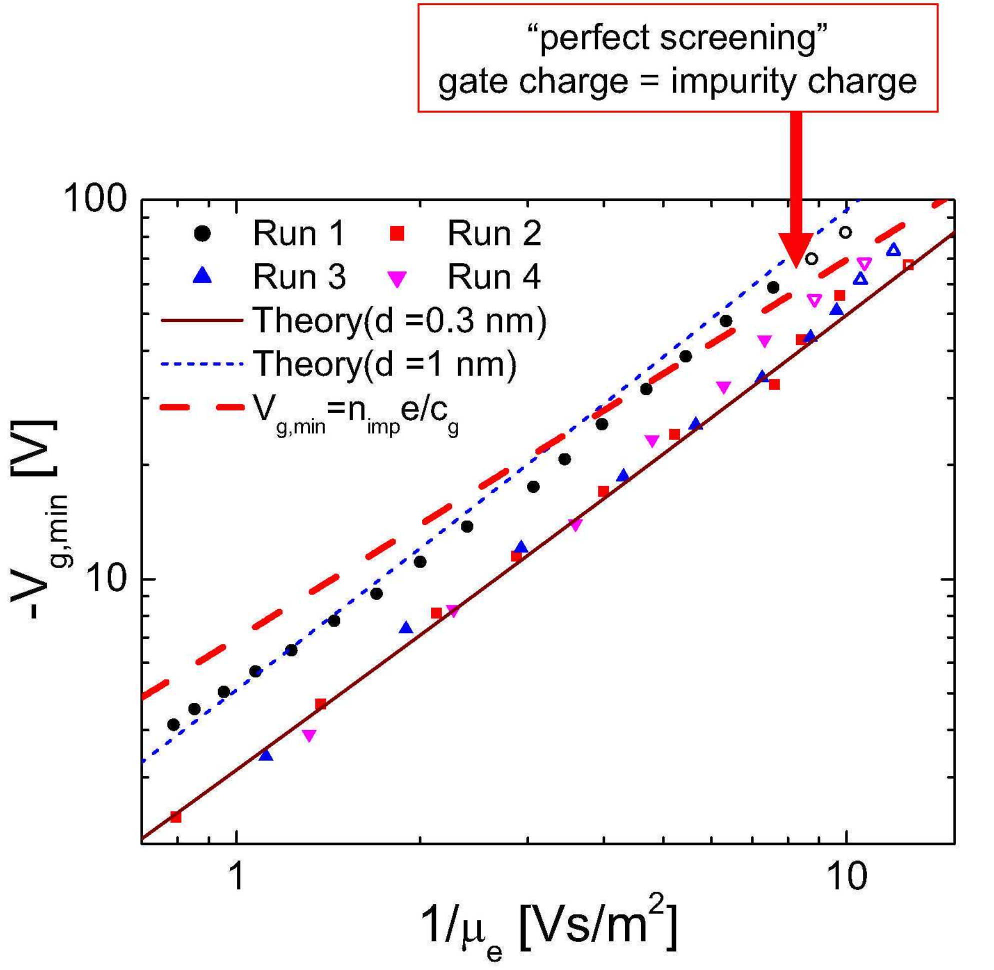

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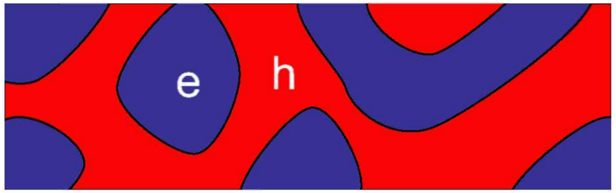

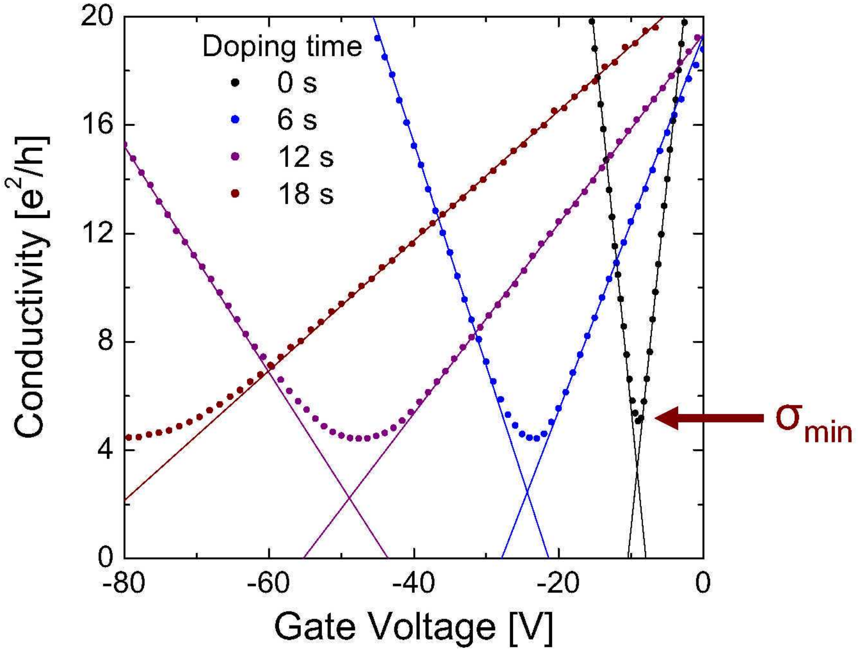

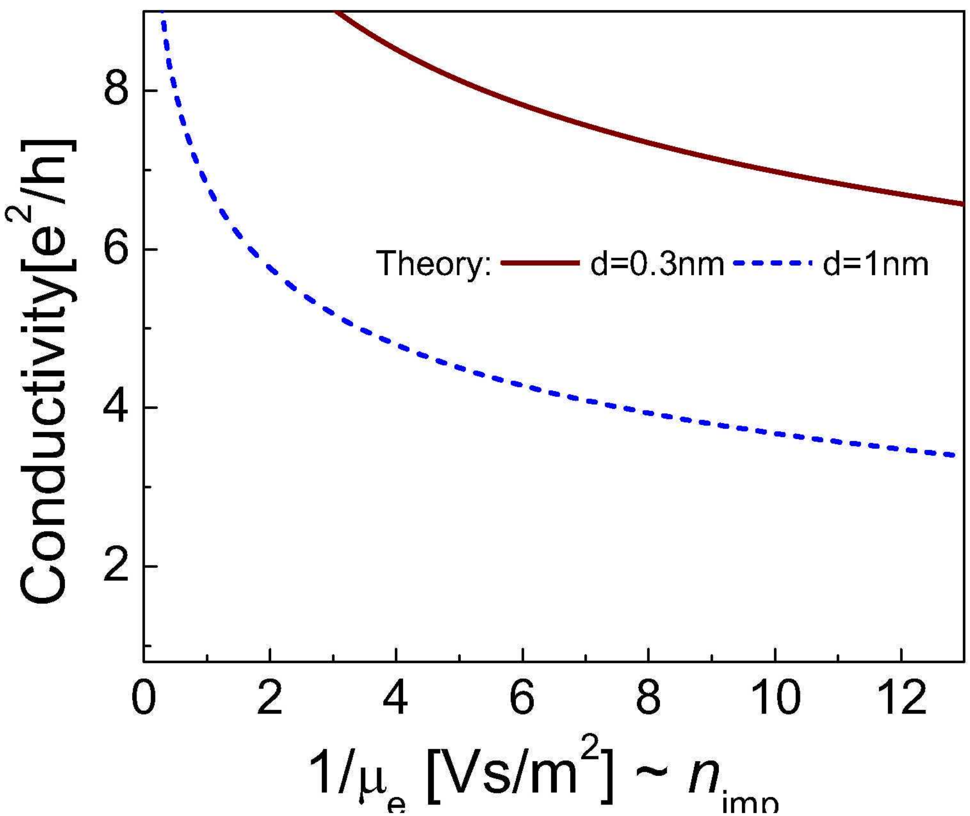

- Additional "residual conductivity" observed - extrapolation of linear $\sigma(V_g)$
- Not theoretically understood [but see Trushin, PRL 99, 216602 (2007) and Yan, ArXiv:0708.1569]

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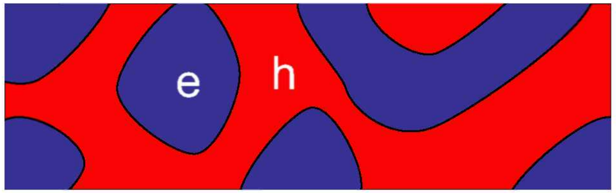

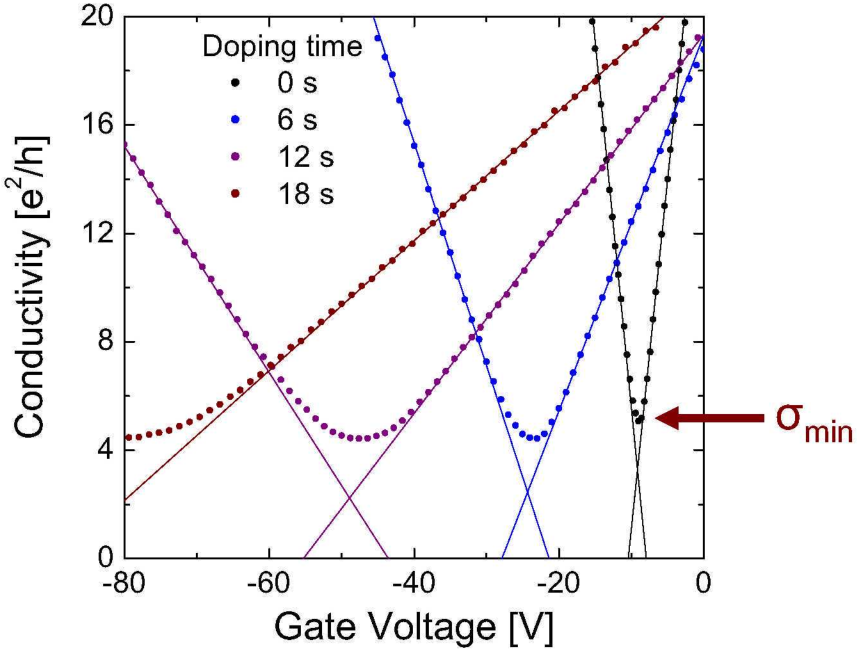

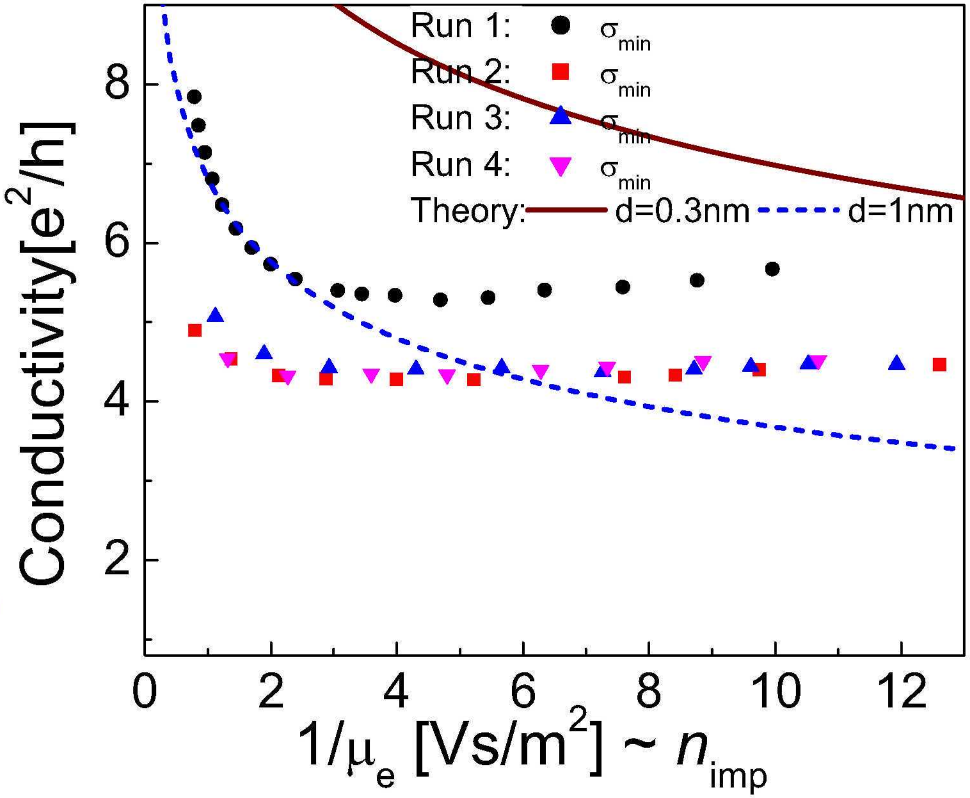

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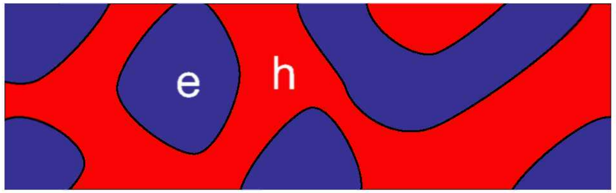

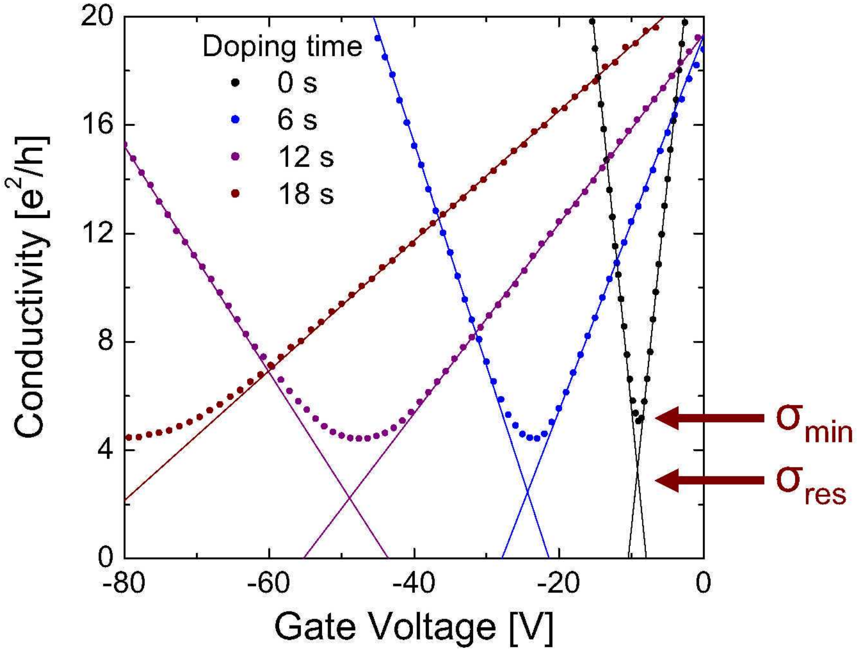

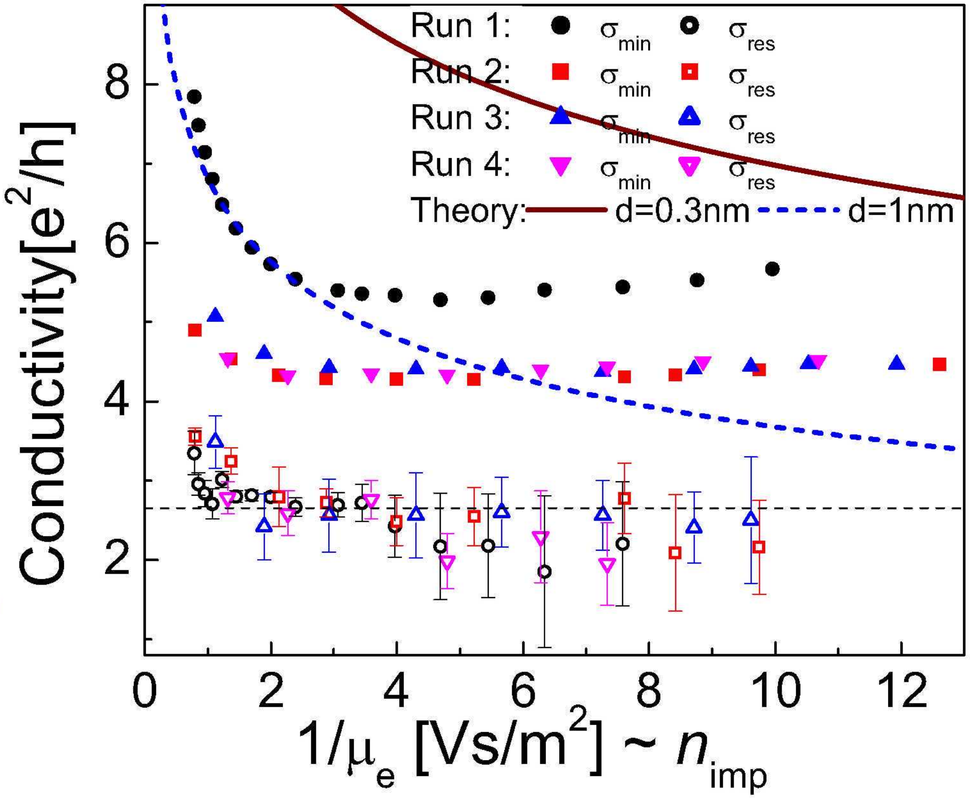

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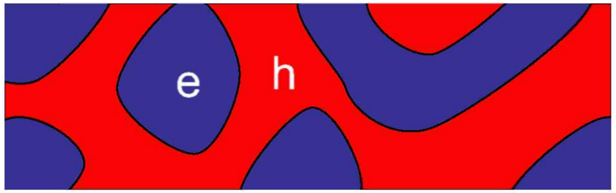
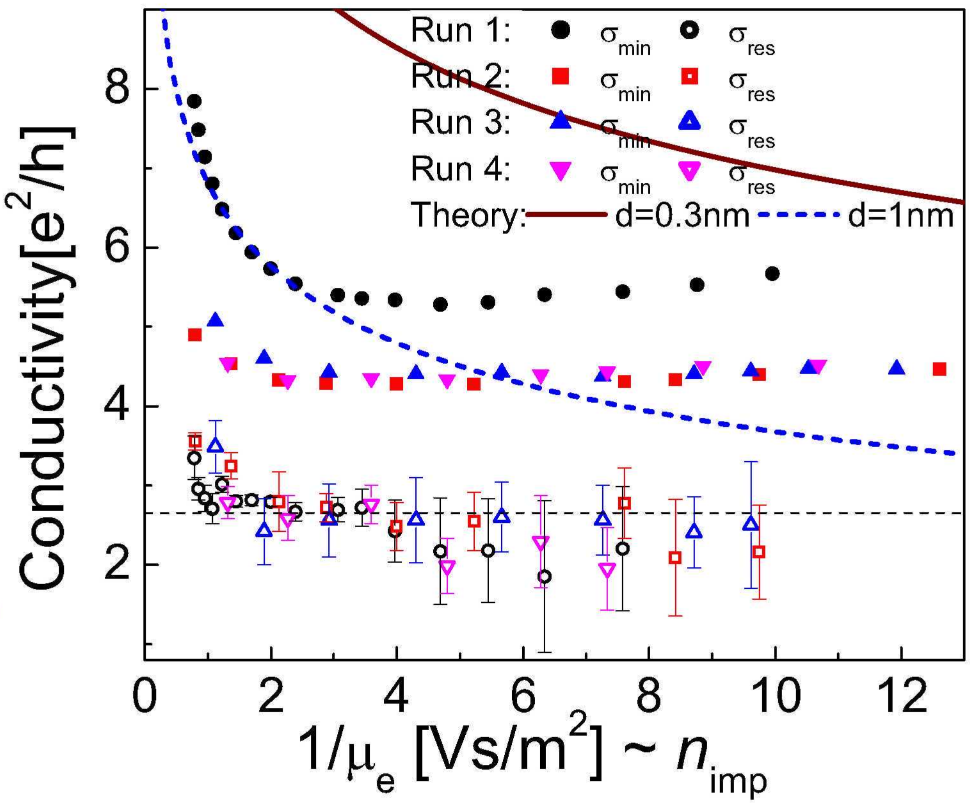


N.B.
σ_{min} *decreases* by factor of 1.5-2 when disorder is *increased*

NOT effect of 4-probe geometry error (sample is same!)

NOT effect of macroscopic inhomogeneity (this would *increase* conductivity!)



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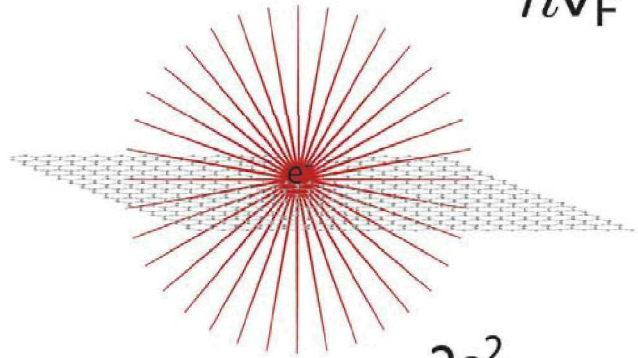
Tuning the "Fine Structure Constant"

C. Jang, et al. ArXiv:0805.3780

Dimensionless interaction strength

- $r_s \equiv PE/KE$

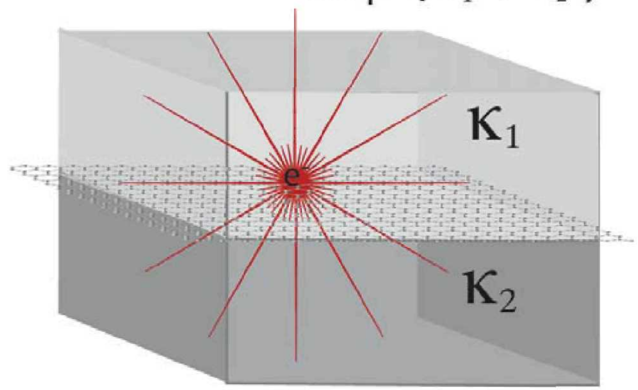
$$\alpha = \frac{e^2}{\hbar v_F}$$



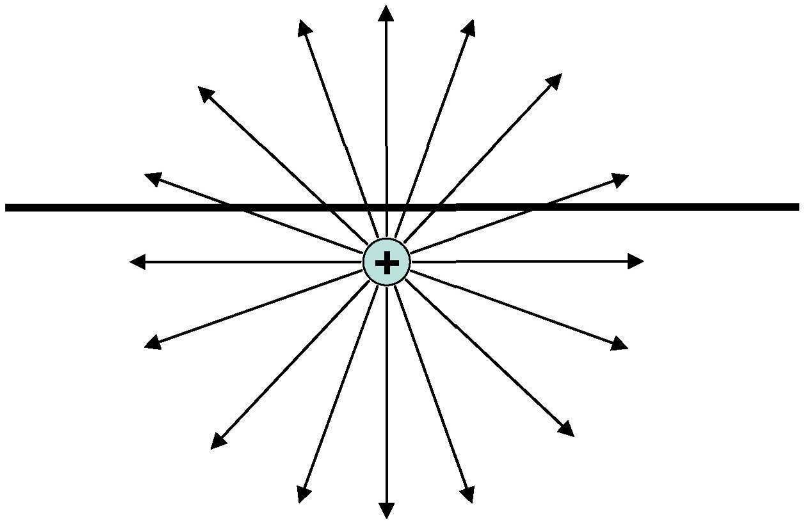
Graphene:

- $r_s = e^2/\hbar v_F = \alpha$ ($c \rightarrow v_F$)
- α is independent of carrier density in graphene
- Tunable through κ (dielectric constant of surroundings

$$\alpha = \frac{2e^2}{\hbar v_F\ (\kappa_1 + \kappa_2)}$$



How is an impurity screened?



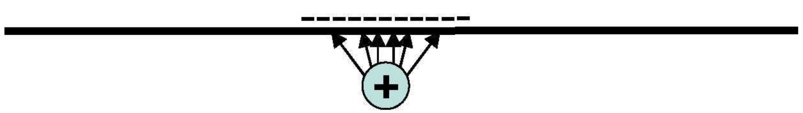
Potential in 2d plane

No screening:

$$V(q) = 2\pi e^2 \frac{e^{-qd}}{q}$$

How is an impurity screened?

large screening (metal):

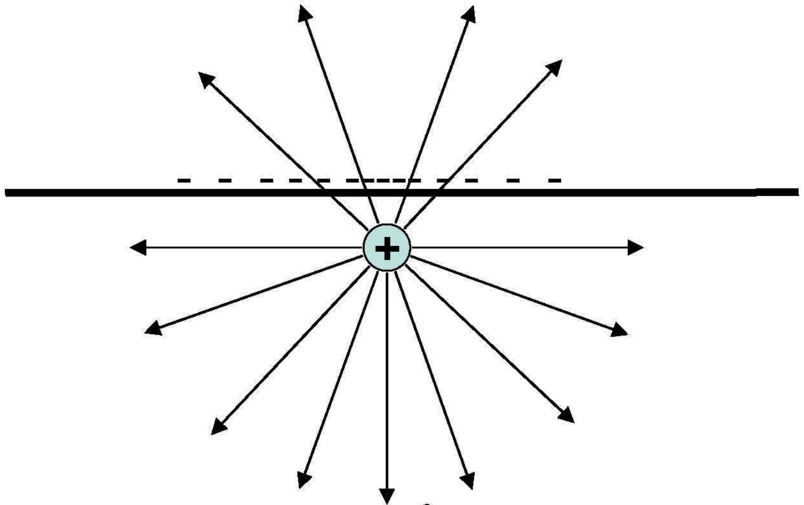


$$V(q) = 2\pi e^2 \frac{e^{-qd}}{q+q_s}$$

$$q_s >> q$$

q_s finite but large (~1/*a*):
potential is short-range
Confined to *r* ~ *a*

Graphene
k_F -dependent screening:



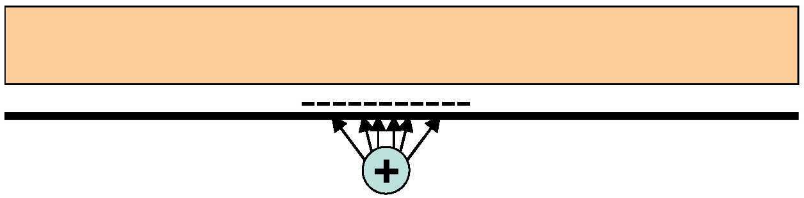
$$q_s = 4k_F r_s = \frac{4k_F e^2}{\kappa\hbar v_F}; \qquad q \sim 2k_F$$

$$V(q) = 2\pi e^2 \frac{e^{-qd}}{k_F\left(1+\frac{2e^2}{\kappa\hbar v_F}\right)}$$

Long-range character remains for *screened* potential

Adding a dielectric:

large screening (metal):

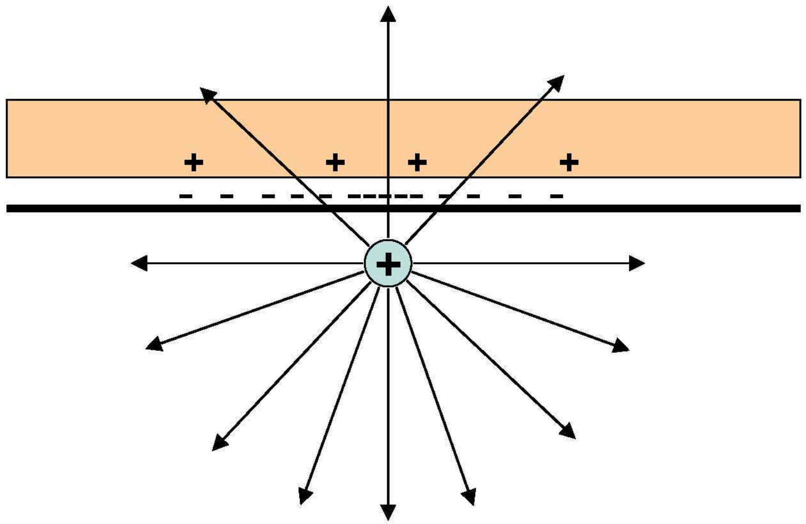


Dielectric cannot screen short-range (atomic scale) potential

$$V(q) \approx 2\pi e^2 \frac{e^{-qd}}{q_s}$$

May change q_s

Graphene k_F -dependent screening:



$$V(q) = \frac{2\pi e^2}{\kappa} \frac{e^{-qd}}{k_F\left(1 + 2\frac{e^2}{\kappa \hbar v_F}\right)}$$

Two effects:

1) Dielectric reduces field of screened impurity → μ *increases*

2) Dielectric reduces screening → μ *decreases*

Two Effects of Dielectric Screening

C. Jang, et al. ArXiv:0805.3780

Reducing α:

- Reduces interaction of carriers with charged impurities
  - Dominant effect for charged-impurity scattering
- Reduces screening by carriers
  - Dominant effect for short-range scattering

Within RPA:

$$\sigma_L = \frac{2e^2}{h}\frac{n}{n_{imp}}\frac{1}{F_L(\alpha)}; \qquad F_L(\alpha) = \pi\alpha^2 + \ldots$$

Coulomb scattering reduced:
Mobility μ_L *increases*

$$\sigma_S = \frac{\sigma_0}{F_S(\alpha)}; \qquad F_S(\alpha) = \frac{\pi}{2} - \frac{32\alpha}{3} + \ldots$$

Short-range scattering increased:
Conductivity σ_S *decreases*

$$\sigma_{\min} = n^* e\mu_L; \quad n^* \sim \alpha^2; \quad \mu_L \sim \frac{1}{\alpha^2}$$

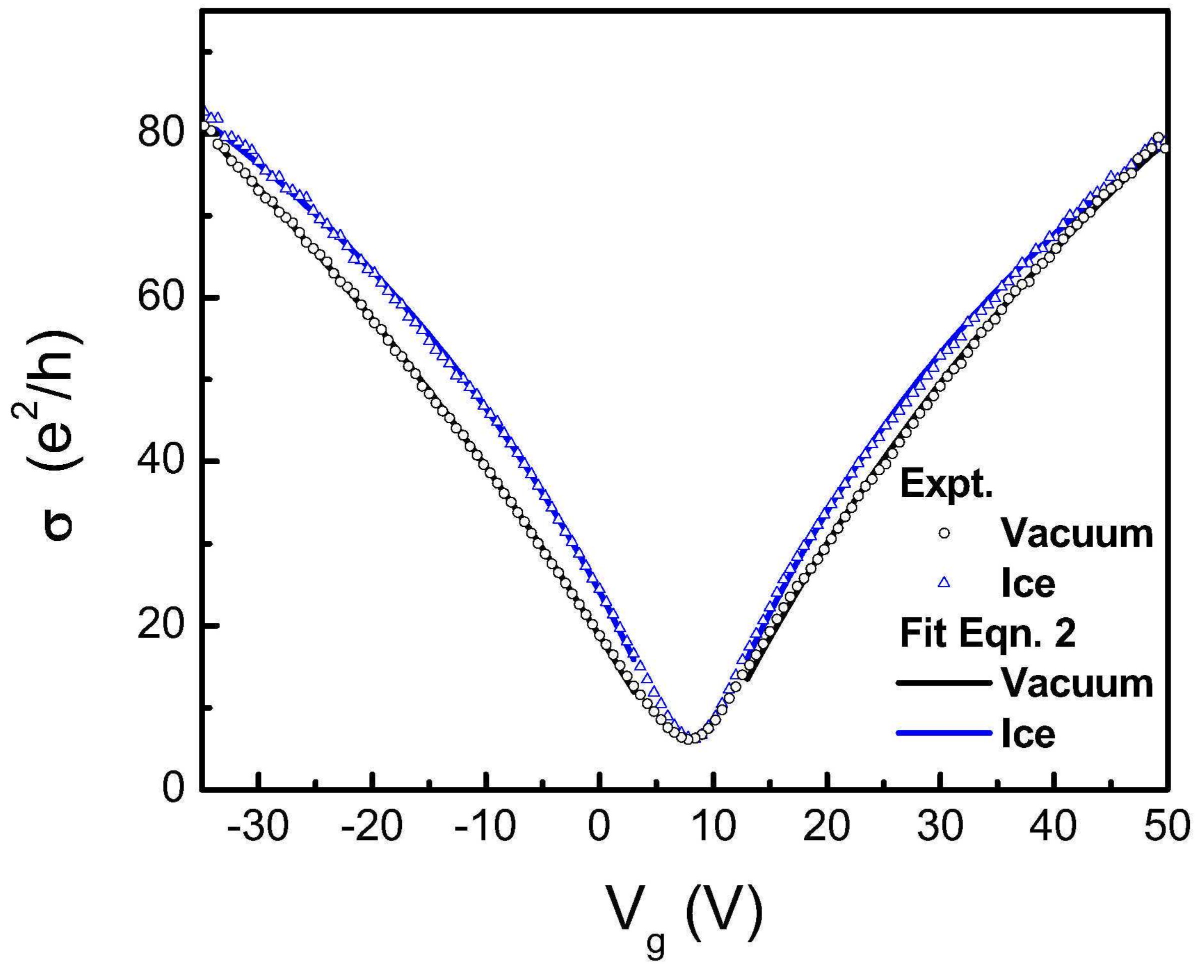
e-h puddle density decreased,
mobility increased:
Min. conductivity $\sigma_{\min}$ *constant*

Effects of Dielectric Screening

C. Jang, et al. ArXiv:0805.3780

Add ice to clean graphene in UHV:

α (SiO_2/vacuum) = 0.81 α (SiO_2/ice) = 0.56



Fit:

$$\sigma^{-1} = (ne\mu_L)^{-1} + \sigma_S^{-1}$$

Coulomb (long-range) scattering

Short-range scattering

(slight asymmetry in Coulomb scattering; take symmetric component of each)

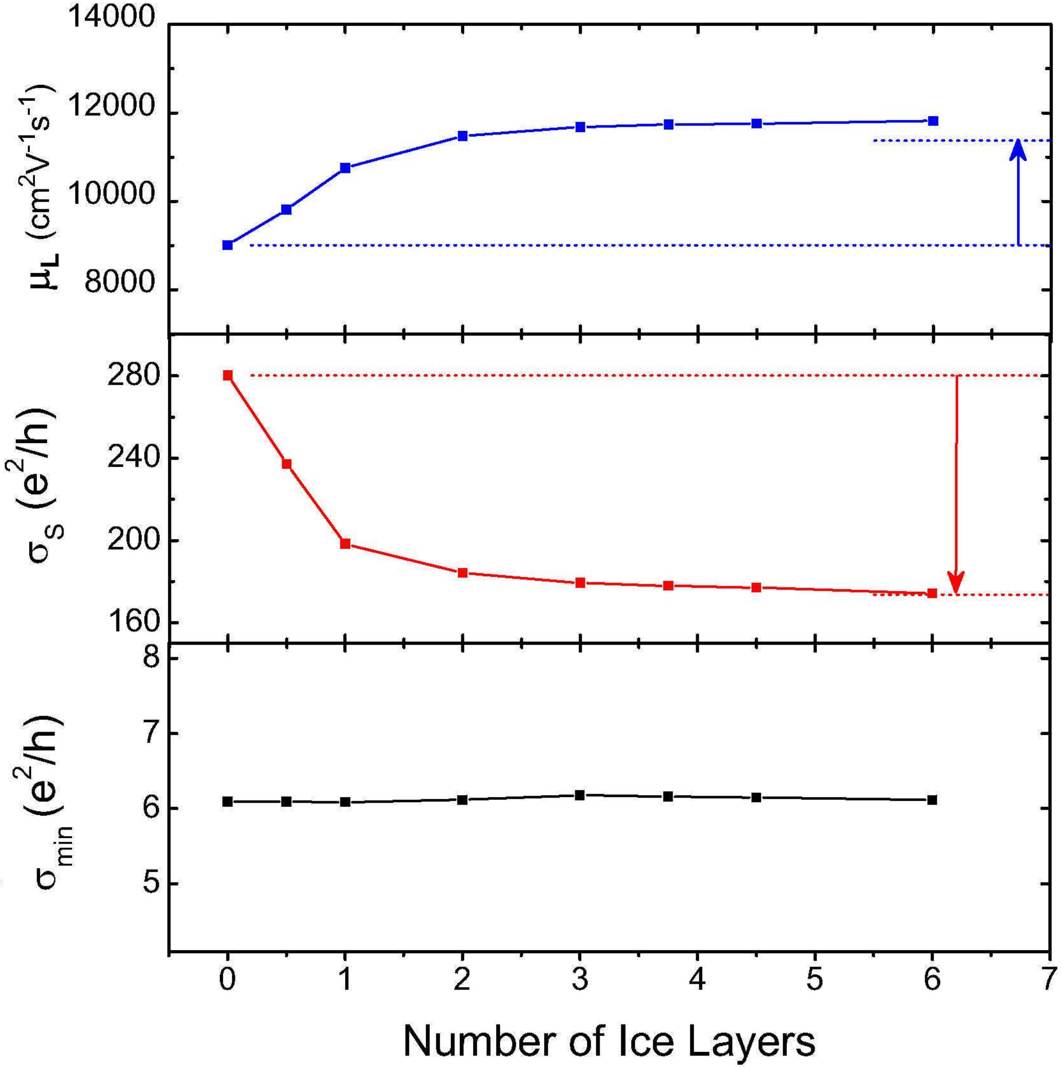
Dielectric Screening: Theory and Expt.

C. Jang, et al. ArXiv:0805.3780

Coulomb scattering reduced:
Mobility μ_L *increases*

Short-range scattering increased:
Conductivity σ_S *decreases*

e-h puddle density decreased,
mobility increased:
Min. conductivity σ_{min} *constant*



Dielectric Screening: Theory and Expt.

C. Jang, et al. ArXiv:0805.3780

N.B. There are NO fitting parameters here!

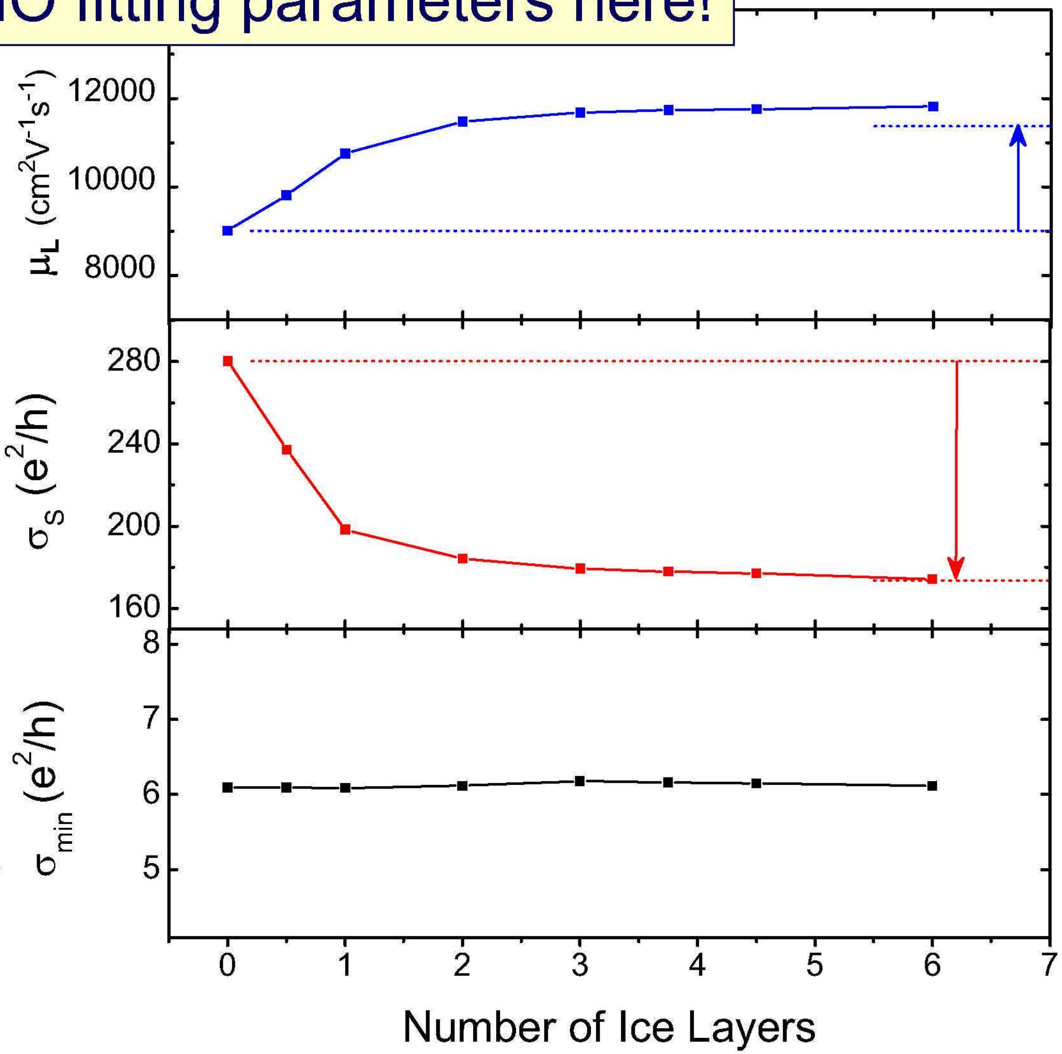
Coulomb scattering reduced:
Mobility μ_L *increases*

N.B. Mobility increases 30% for κ_{avg}: 2.45 → 3.55

Short-range scattering increased:
Conductivity σ_S *decreases*

N.B. σ_s decreases 40% for κ_{avg}: 2.45 → 3.55

e-h puddle density decreased,
mobility increased:
Min. conductivity σ_{min} *constant*



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Graphene Corrugation - Scattering

$\sigma = \frac{2e^2}{h} E_F \tau$ $\frac{1}{\tau} \propto \frac{2\pi}{\hbar} \left| \langle k | V | k' \rangle \right|^2 D(E)$

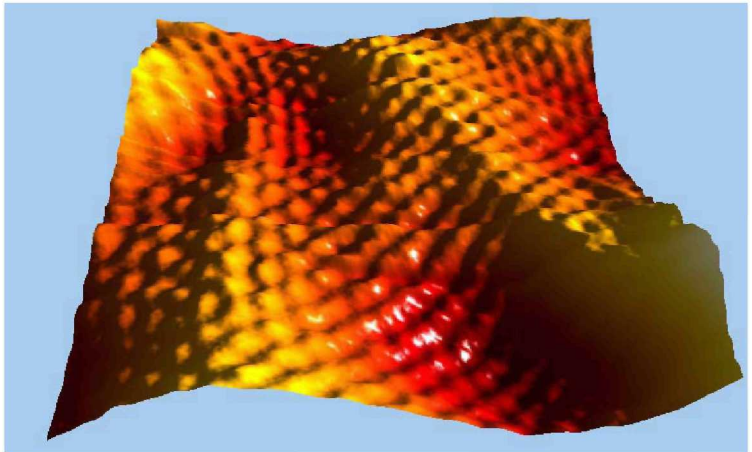
q-dependent interaction
→ carrier-density dependent σ(n)

1) Coulomb interaction: $q = |\boldsymbol{k} - \boldsymbol{k}'| \sim k_{\mathrm{F}} \Longrightarrow \sigma \sim n$

2) Corrugated graphene†: $\left\langle [h(r) - h(0)]^2 \right\rangle \propto r^{2H} \Longrightarrow \sigma \sim n^{2H\text{-}1}$

height-height correlation function

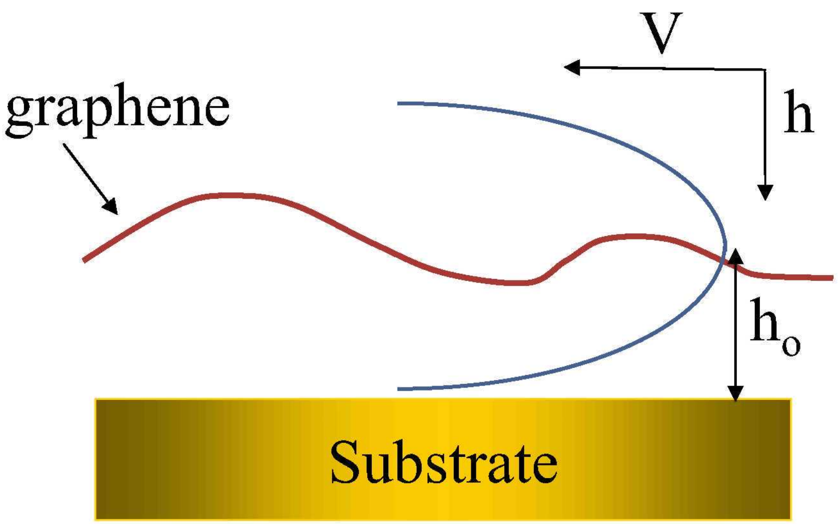
†Katsnelson & Geim, *Phil. Trans. R. Soc. A* **366**, 195-204 (2008)



What is exponent 2H?

Graphene Corrugation

Model 1:
Intrinsic graphene bending constrained via interface confining potential

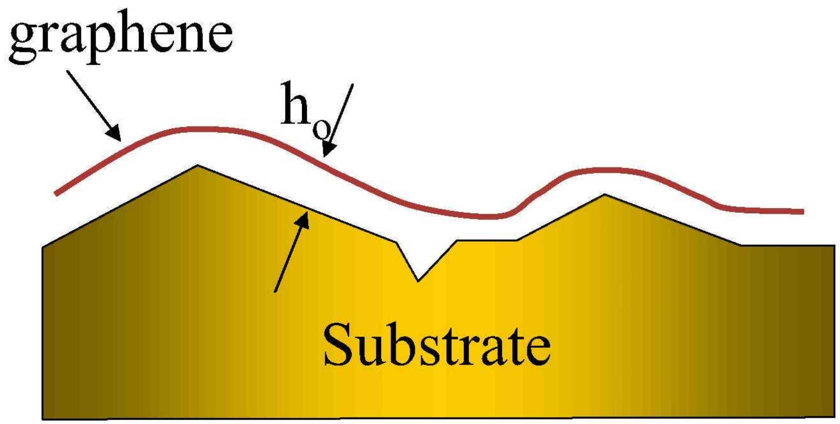


$$F = \frac{1}{2}\kappa\left[\nabla^2 h(x,y)\right]^2 + \frac{1}{2}Vh^2(x,y)$$

$$\left\langle \left(h(r) - h(0)\right)^2 \right\rangle \sim r^2$$

σ(*n*) ~ *n*
(mimics Coulomb scatting)

Model 2:
Corrugations determined by strong direct interaction governed by height variations of the substrate



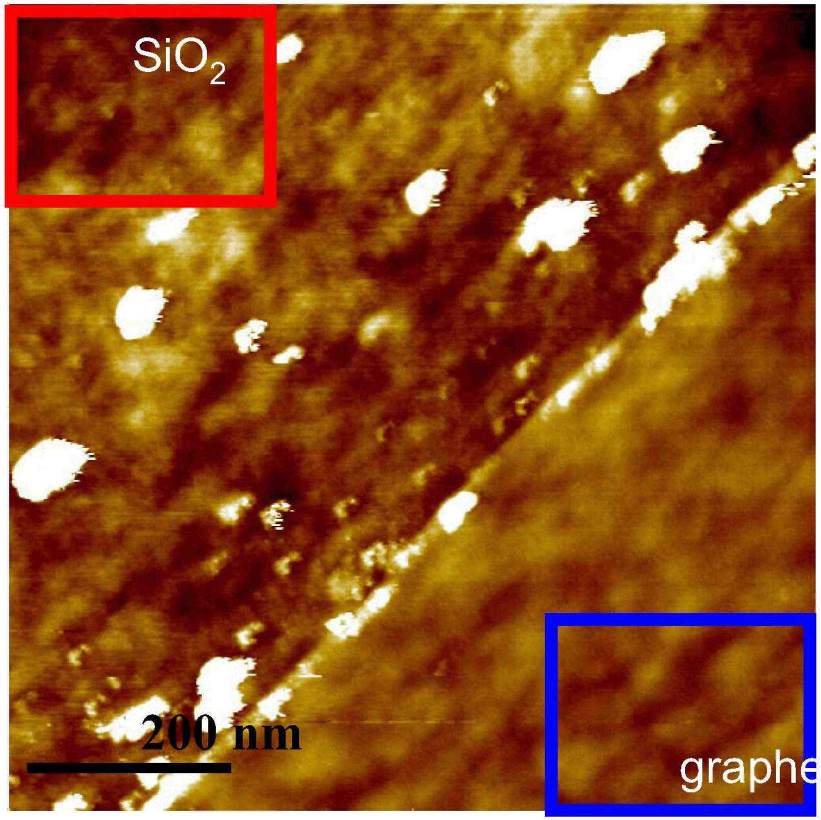
Height-height correlations will match those of the substrate.

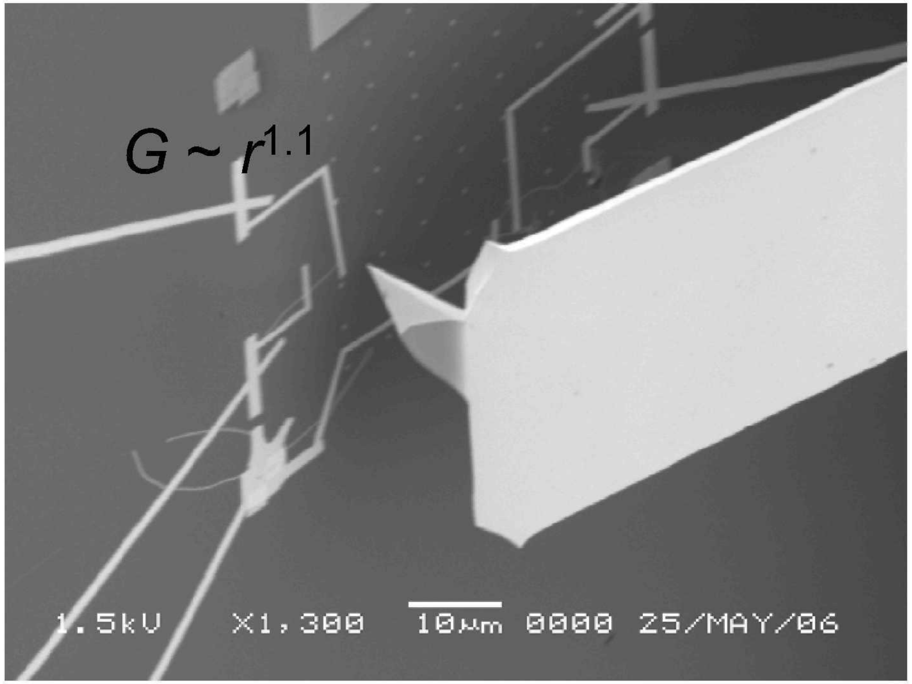
Typical non-equilibrium surfaces show: $\left\langle \left(h(r) - h(0)\right)^2 \right\rangle \sim r^{2H}$
with 2H ≈ 1.

σ(*n*) ~ *constant*
(mimics short range scattering)

Graphene Corrugations on SiO_2

Non-contact AFM image in UHV



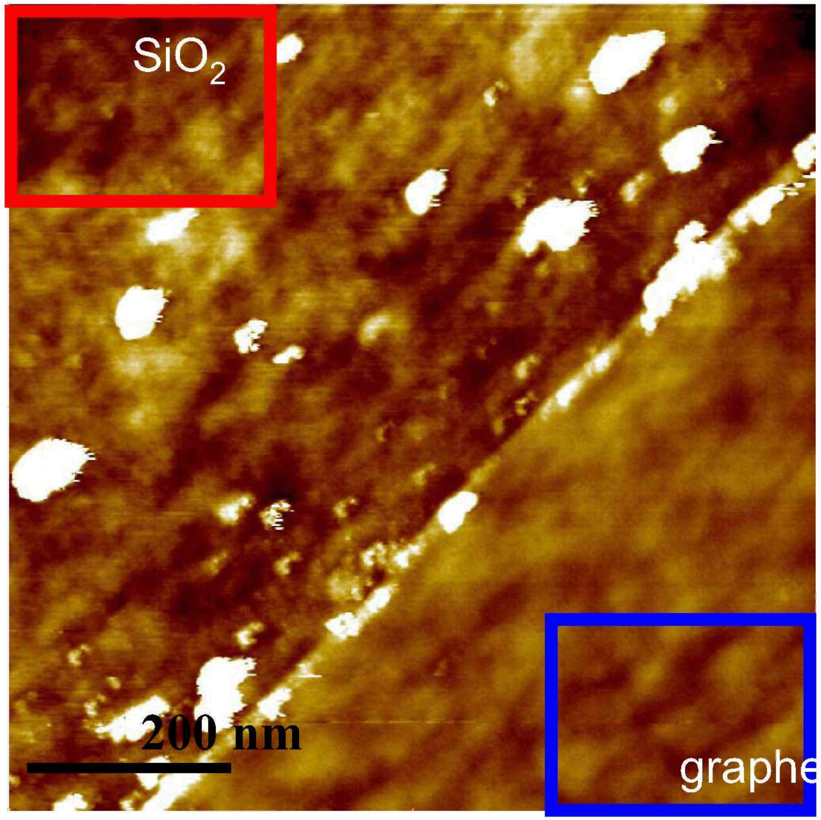


Height [nm]

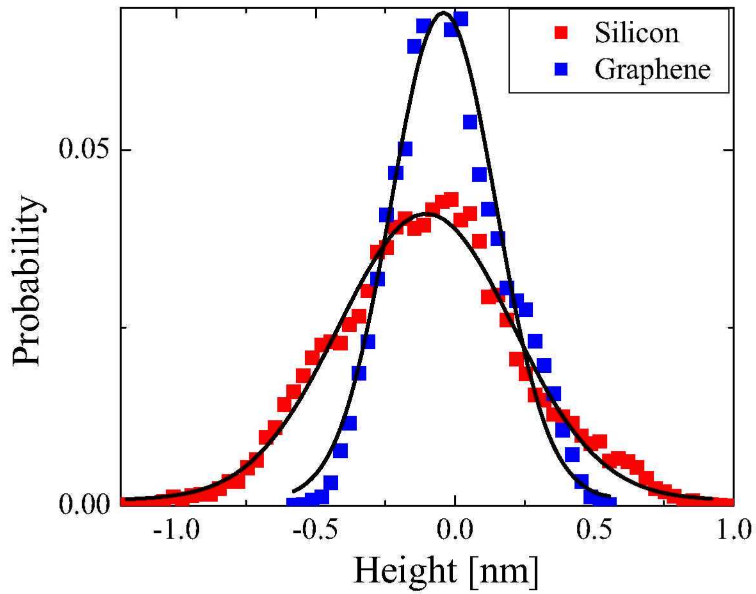
M. Ishigami et al., *Nano Letters* 7, 1643 (2007)

Graphene Corrugations on SiO_2

Non-contact AFM image in UHV



Oxide-graphene boundary

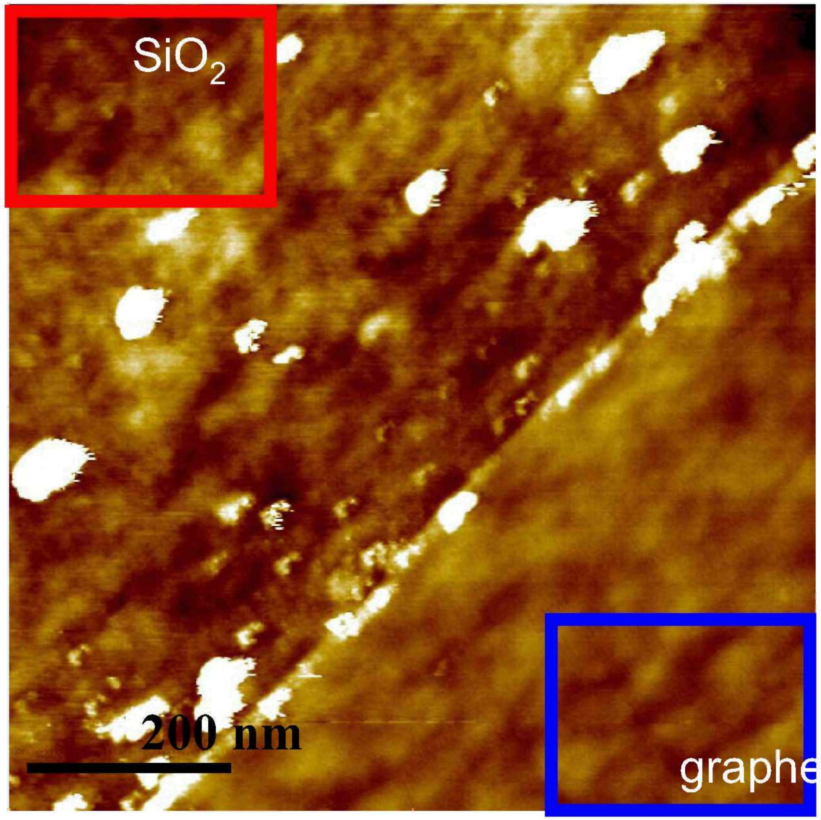


- σ_{oxide} = 3.1 Å and $\sigma_{graphene}$ = 1.9 Å
- Graphene 60% smoother than SiO_2

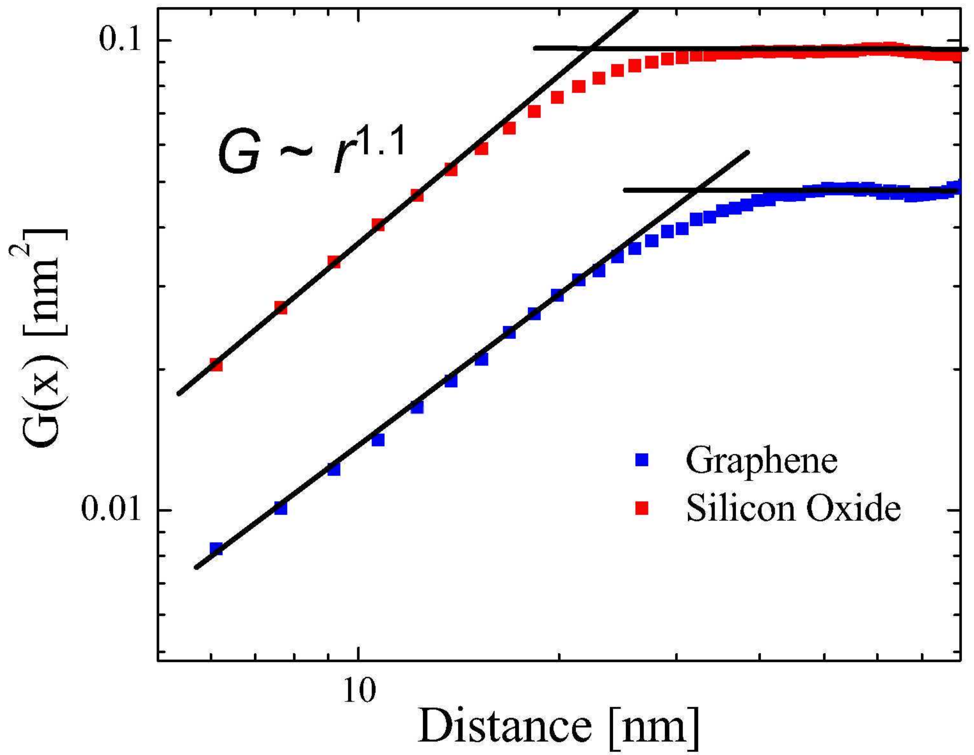
M. Ishigami et al., *Nano Letters* 7, 1643 (2007)

Graphene Corrugations on SiO_2

Non-contact AFM image in UHV



Oxide-graphene boundary



Height-height correlations function

$$\left\langle \left(h(r)-h(0)\right)^2 \right\rangle \sim r^{2H}$$

with 2H ≈ 1

$\sigma(n) \sim$ *constant*
(mimics short range scattering)

M. Ishigami et al., *Nano Letters* 7, 1643 (2007)

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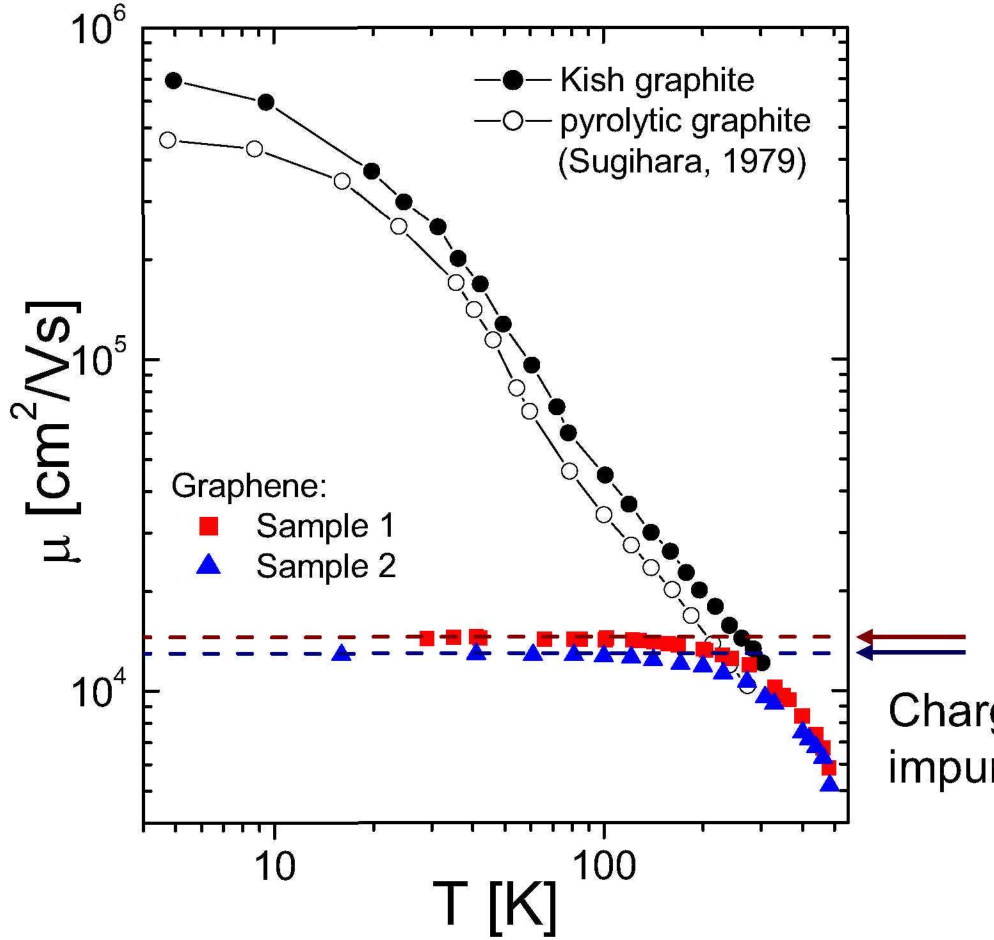
Limits of Mobility in Graphene

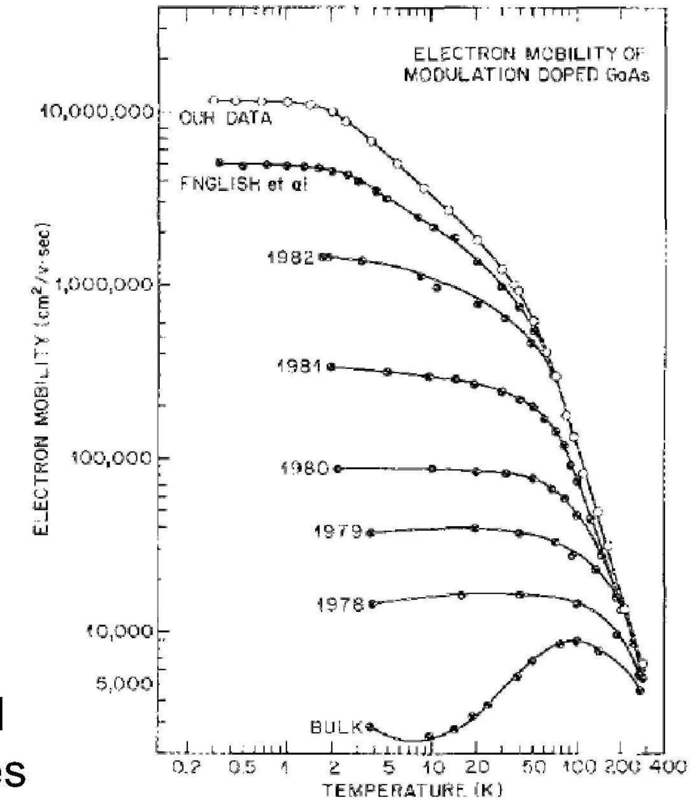
Bulk Graphite

and Graphene on SiO_2



Charged impurities

GaAs (Pfeiffer and West, 1989)



Electron-Phonon Scattering

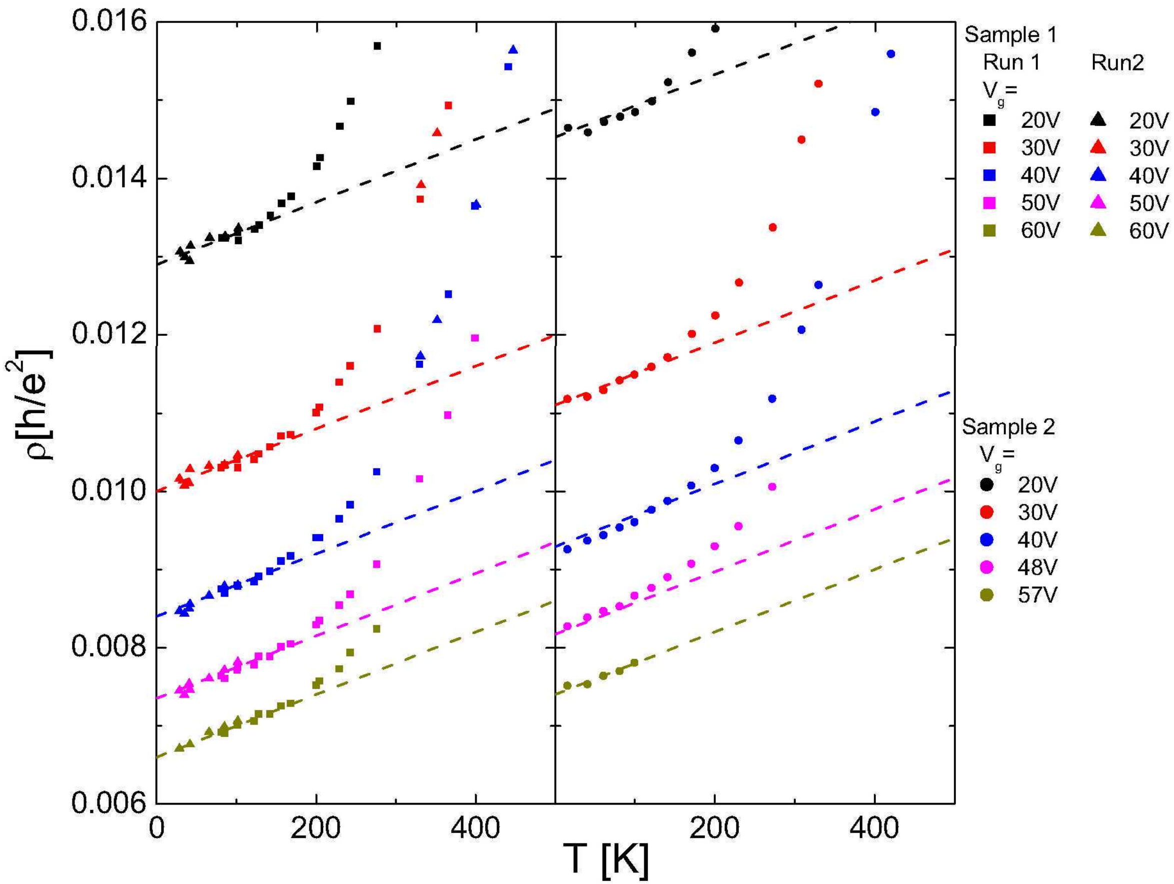
J. H. Chen, et al. *Nature Nanotechnology* **3**, 206 (2008)

Linear *T*-dependence at low *T*

Longitudinal acoustic phonons in graphene

$\rho = \rho_0 + AT$
A = 0.1 Ω/K

A is independent of charge carrier density, as predicted



$$\rho_A(T) = \left(\frac{h}{e^2}\right)\frac{D_A^2 k_B T}{16 e^2 \hbar^2 \rho_s v_s^2 v_F^{\ 2}}$$

→ D_A = 18 ± 1 eV

(good agreement w/CNT, graphite)

Electron-Phonon Scattering

J. H. Chen, et al. *Nature Nanotechnology* **3**, 206 (2008)

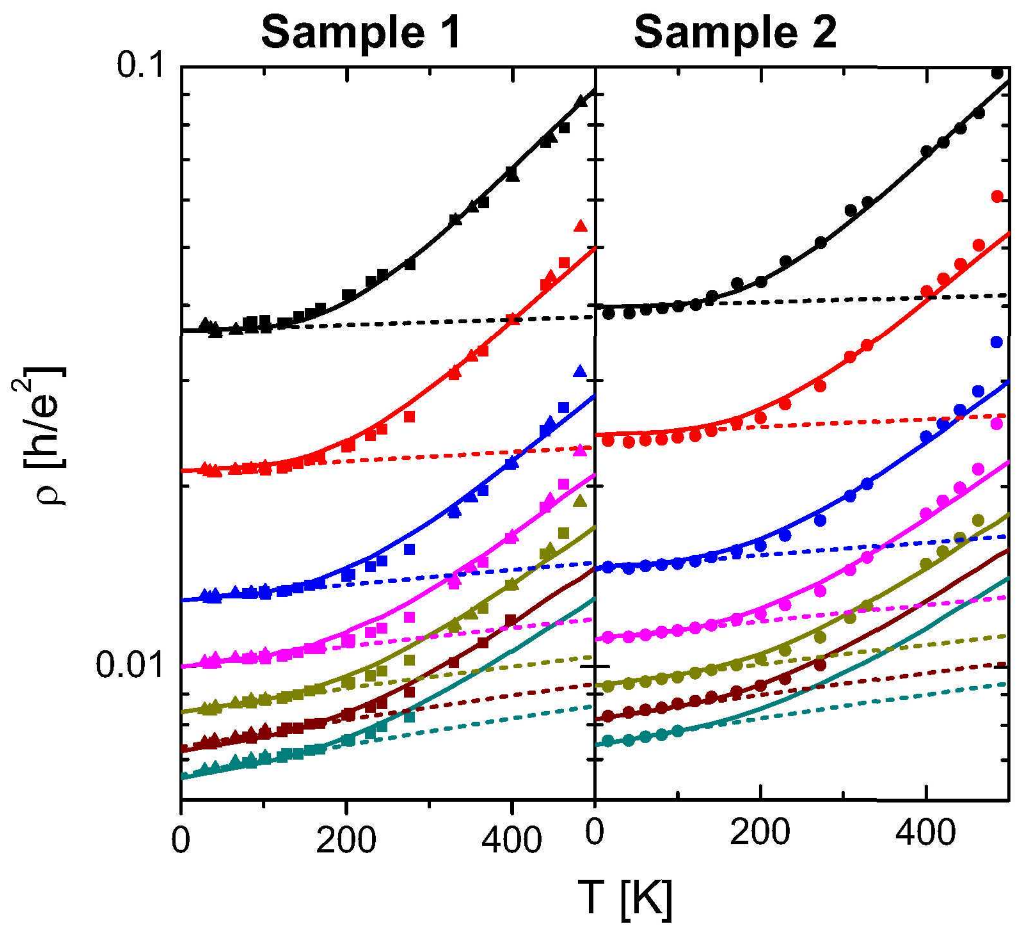
Activated *T*-dependence at high *T*

Consistent with:

LA phonons in graphene

$$\rho(n,T) = \rho_0 + AT + Bn^{-\alpha}\left(\frac{1}{e^{(59\,meV)/k_B T} - 1} + \frac{6.5}{e^{(155\,meV)/k_B T} - 1}\right)$$

polar optical surface phonons in SiO_2
(see Fratini and Guinea, ArXiv:0711.1303)



(3 global fit parameters for all curves)

Potential due to polar optical phonons is long-ranged; leads to density-dependent resistivity

Mobility Limits in Graphene

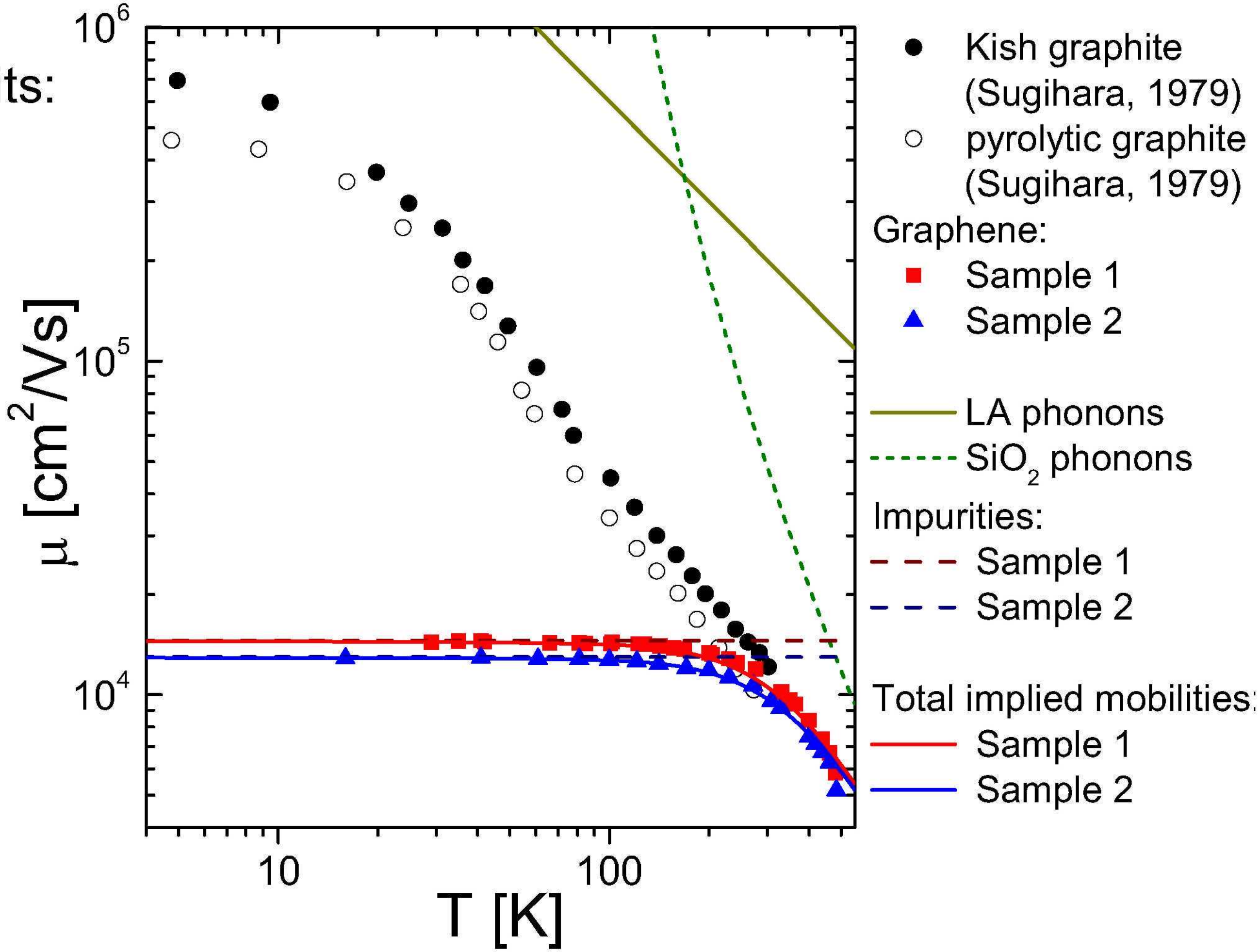
J. H. Chen, et al. *Nature Nanotechnology* **3**, 206 (2008)

Room Temperature Limits:

Currently:
$\mu_{RT} \sim 10^4\ cm^2/Vs$
(charged impurities)

Substrate-limited:
SiO_2 surface phonons:
$\mu_{RT} \sim 4 \times 10^4\ cm^2/Vs$

Intrinsic:
acoustic phonons:
$\mu_{RT} \sim 2 \times 10^5\ cm^2/Vs$
@ $n = 10^{12}\ cm^{-2}$



Room temperature mobility of **200,000 cm^2/Vs** possible!

Ballistic transport over >2 microns

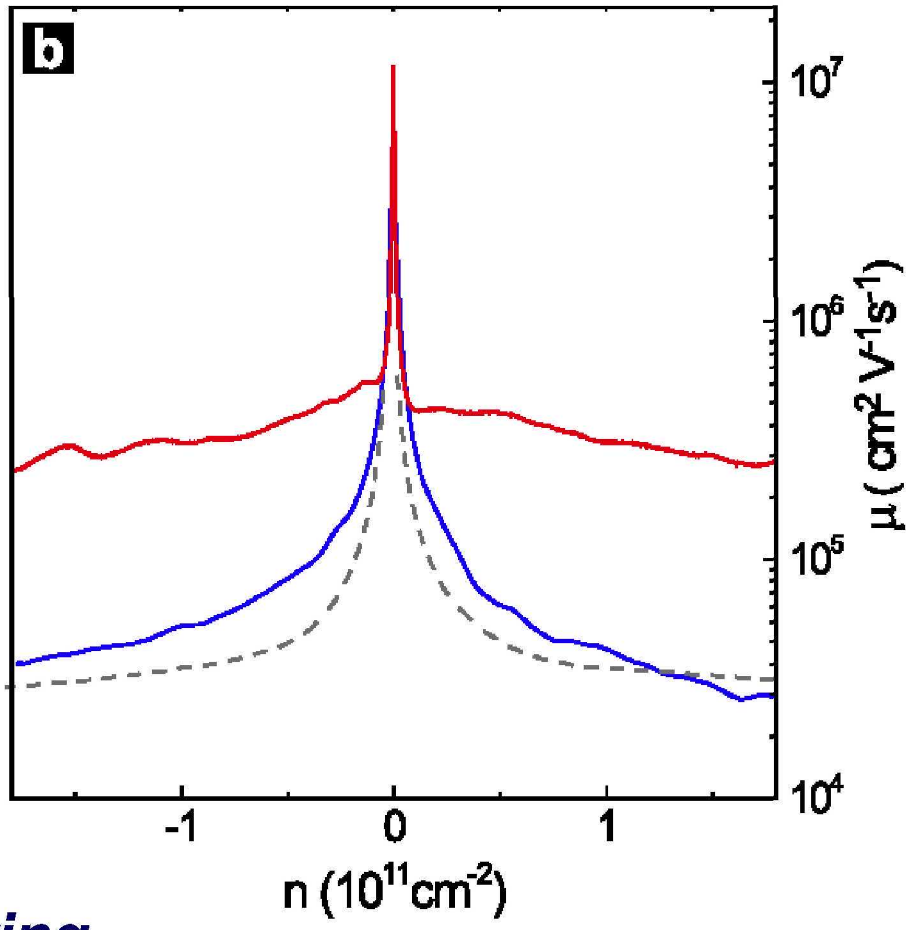
Suspended Graphene (Kim group, Columbia)

Kim, Stormer et al. (Columbia U.)

Solid State Comm. 146, 351-355 (2008), arXiv:0805.1830

Suspended graphene:

- >10x increase in mobility
- → 230,000 cm^2/Vs (T = 5 K)
- → 120,000 cm^2/Vs (T = 240 K)



Removing substrate:

Removes charged impurities

Removes RIP phonon scattering

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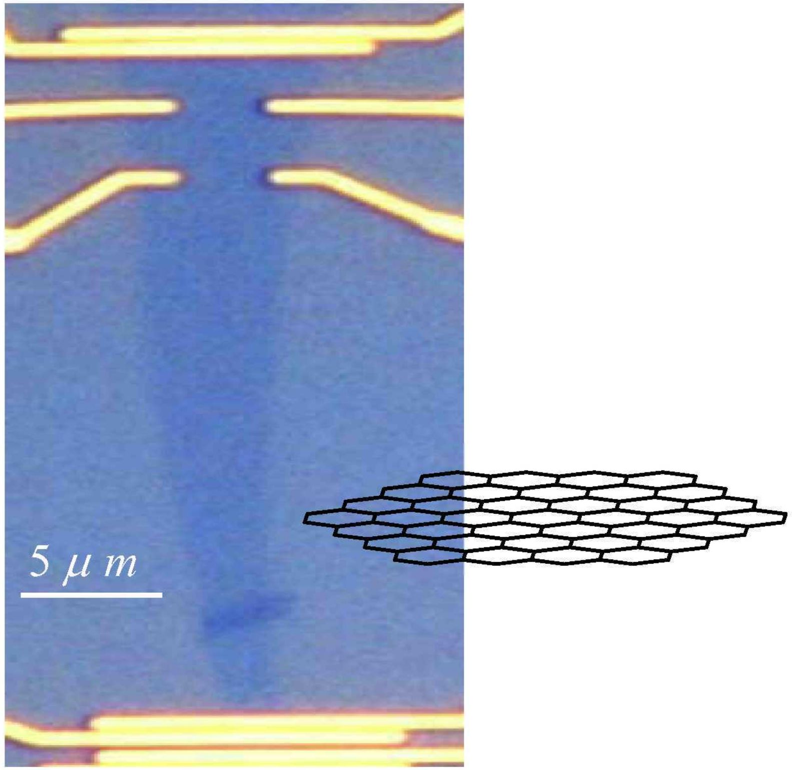
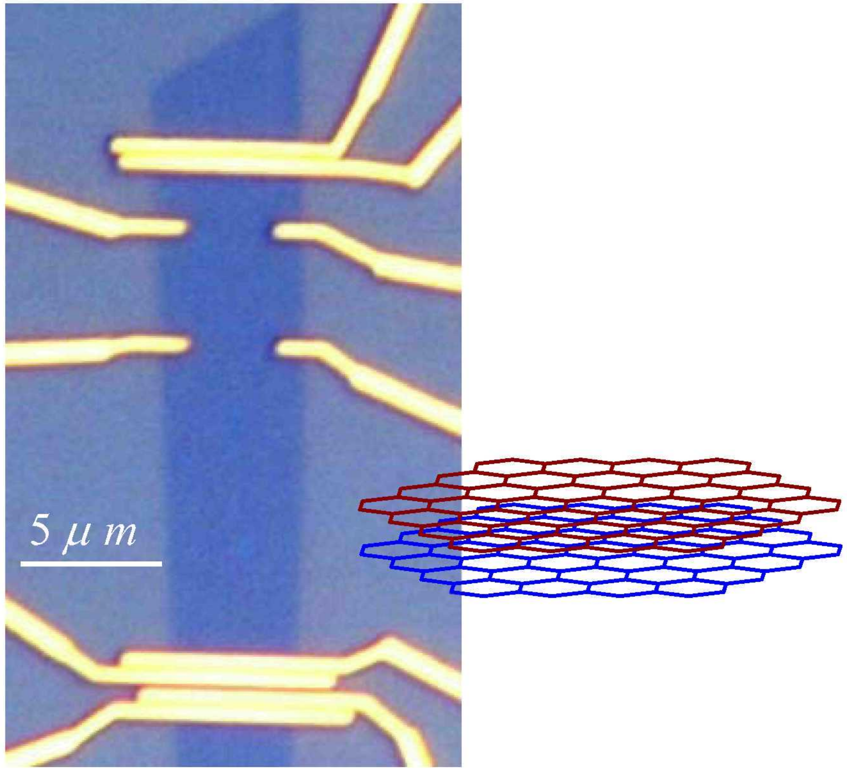
Graphene: Single layer vs. Bilayer



Single layer Graphene

$$w = 6.0\mu m$$

$$l = 2.6\mu m$$

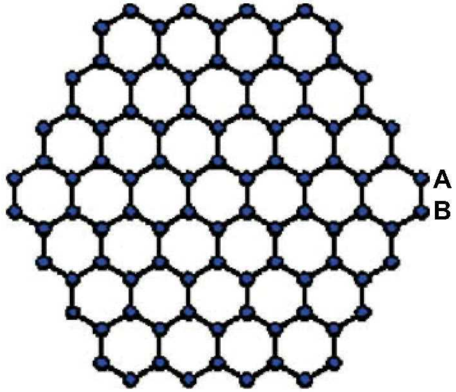


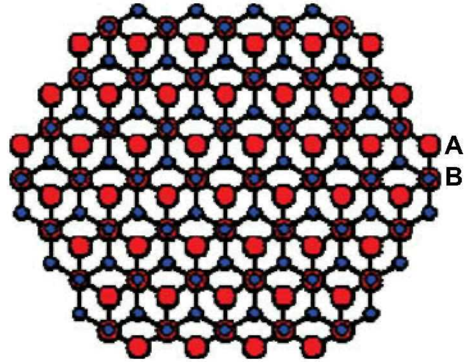
Bilayer Graphene

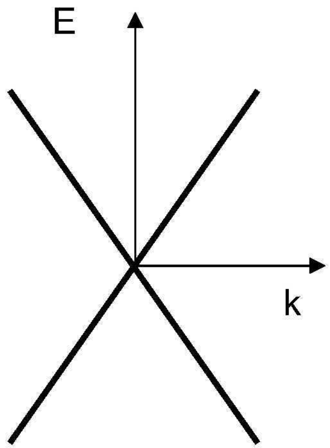
$$w = 5.2\mu m$$

$$l = 4.3\mu m$$

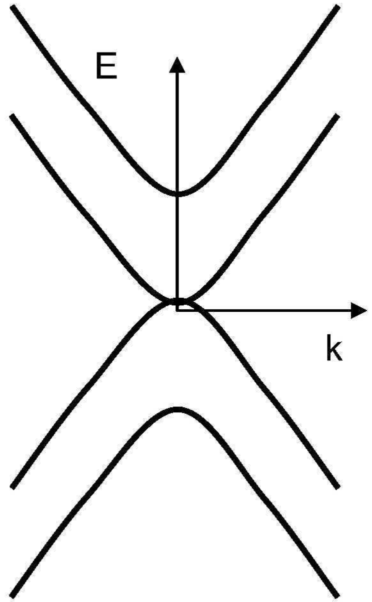
Bilayer Dispersion Relation







Break A-B degeneracy
→ bandgap

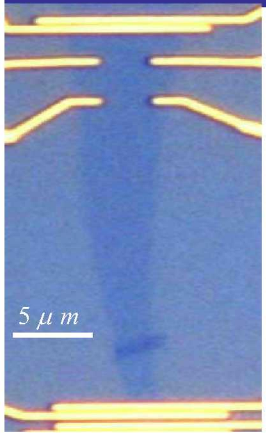


Bonding/anti-bonding states of two layers
→ split into 4 bands

$$E = \hbar v_F |\boldsymbol{k}|$$

$$E = \frac{\hbar^2 k^2}{2m^*}$$

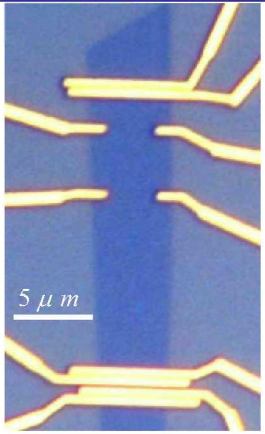
Quantum Hall Effect: Single Layer vs. Bilayer



Single layer:

$$\sigma_{xy} = \nu \frac{e^2}{h} \quad \nu = 4\left(n + \frac{1}{2}\right)$$

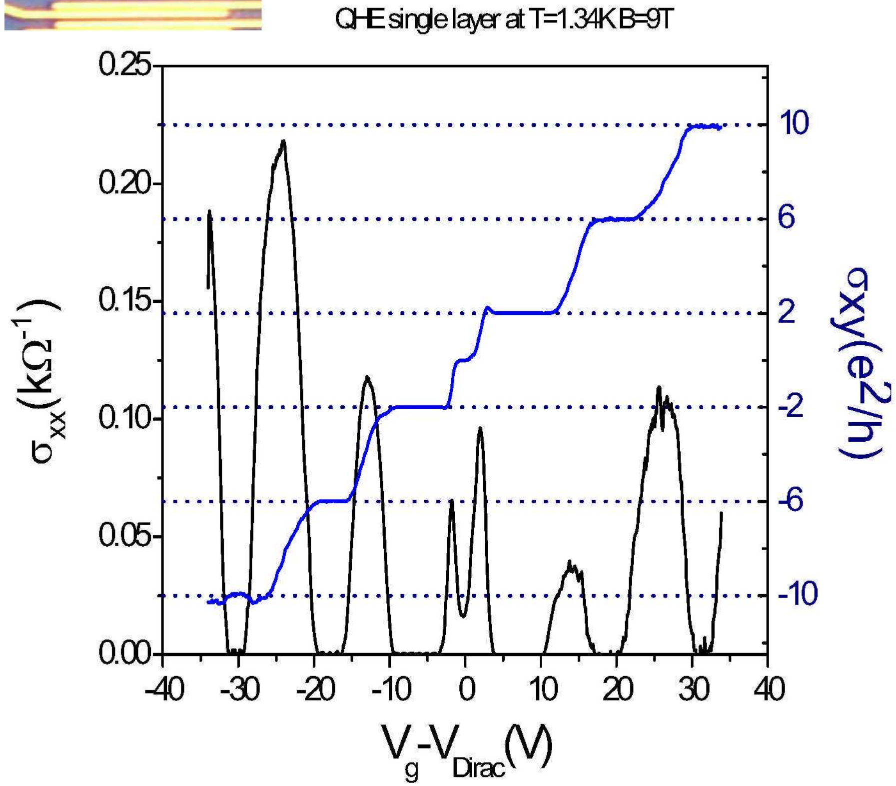
Berry's phase = π

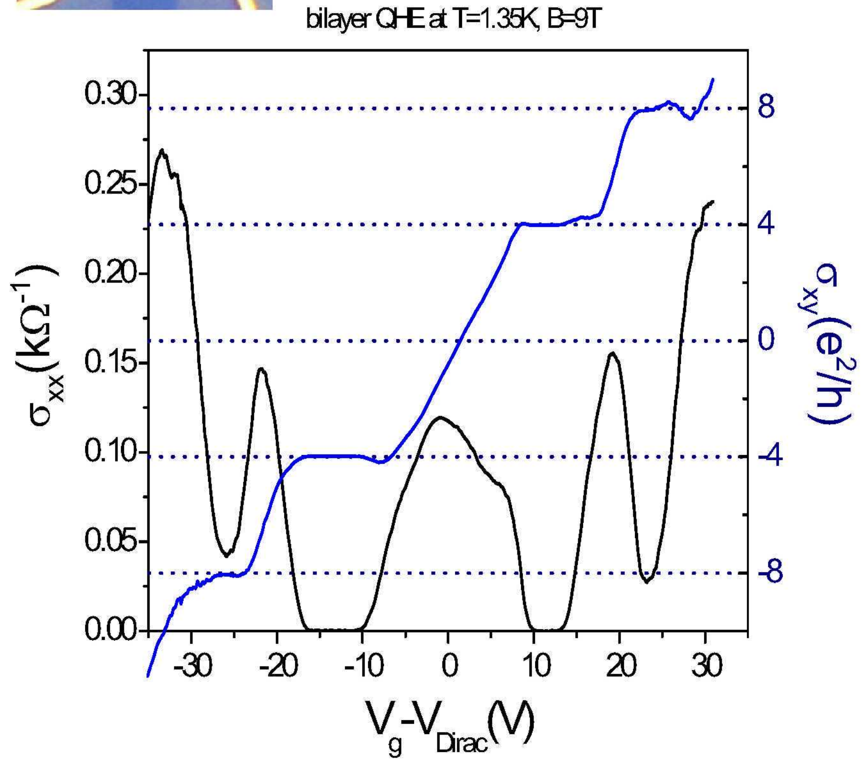


Bilayer:

$$\sigma_{xy} = \nu \frac{e^2}{h} \quad \nu = 4(n+1)$$

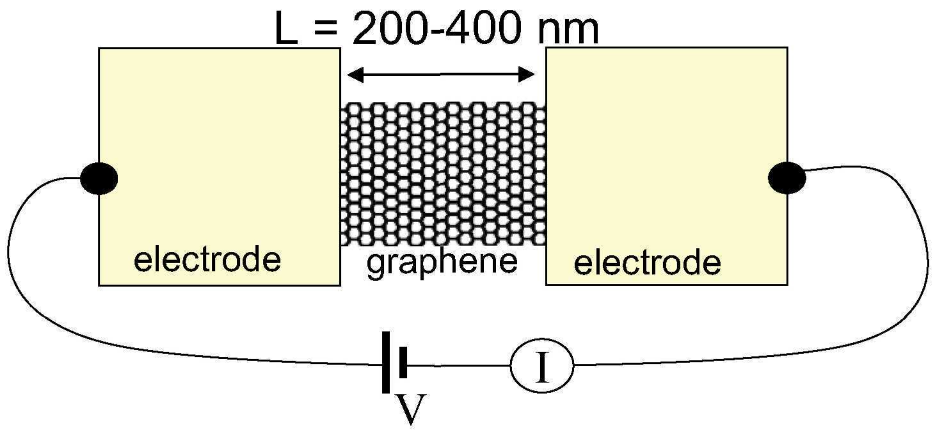
Berry's phase = 2π





See also: Zhang et al, 2005, Novoselov et al, 2005.

Fabry-Pérot Interference in CNT and Graphene

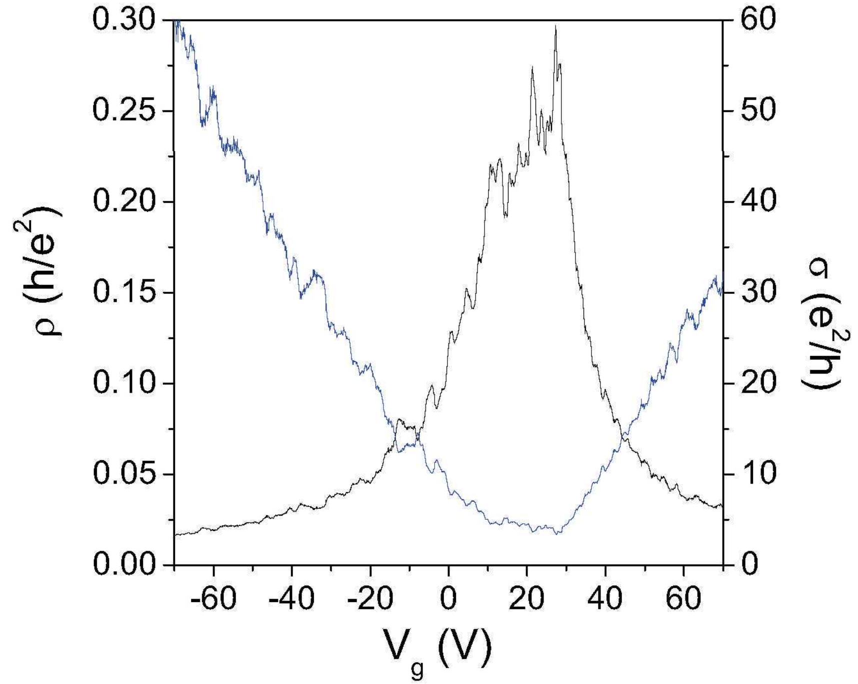


Short two-probe devices (L ~ mfp)

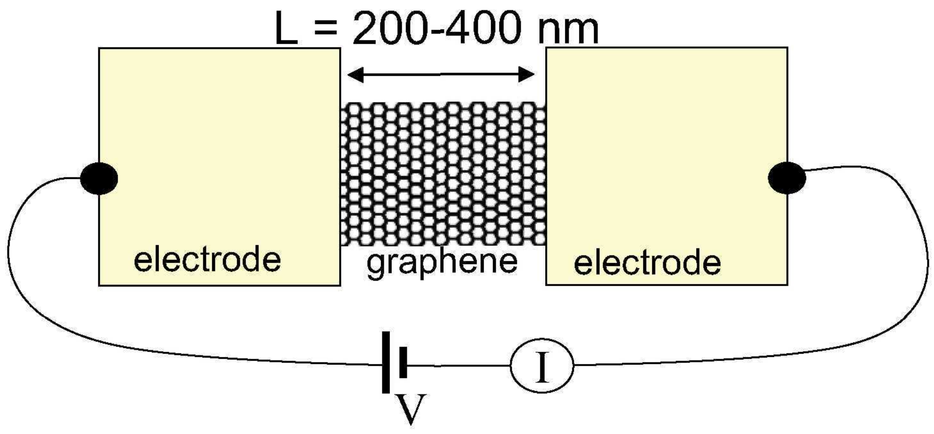
- Two-probe differential conductance shows oscillations as a function of gate and drain voltage

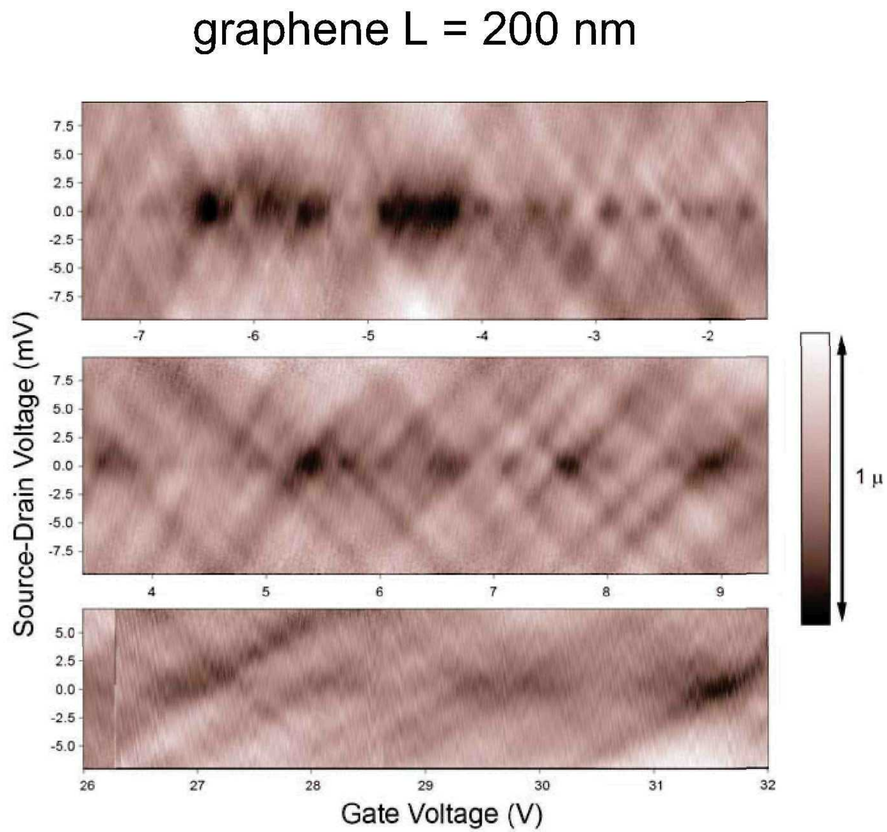
- NOT universal conductance fluctuations (UCF)

graphene L = 200 nm



Fabry-Pérot Interference in CNT and Graphene





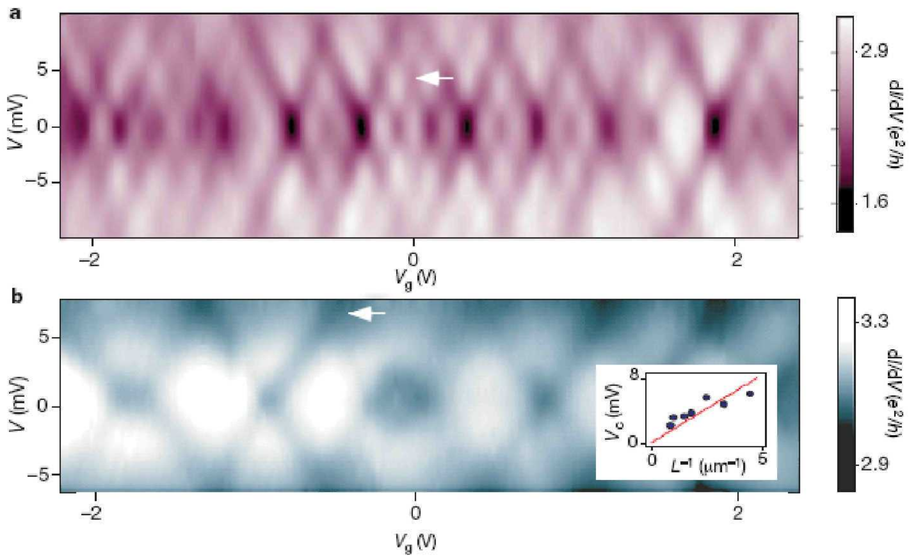
Short two-probe devices (L ~ mfp)

• Two-probe differential conductance shows oscillations as a function of gate and drain voltage

• NOT universal conductance fluctuations (UCF)

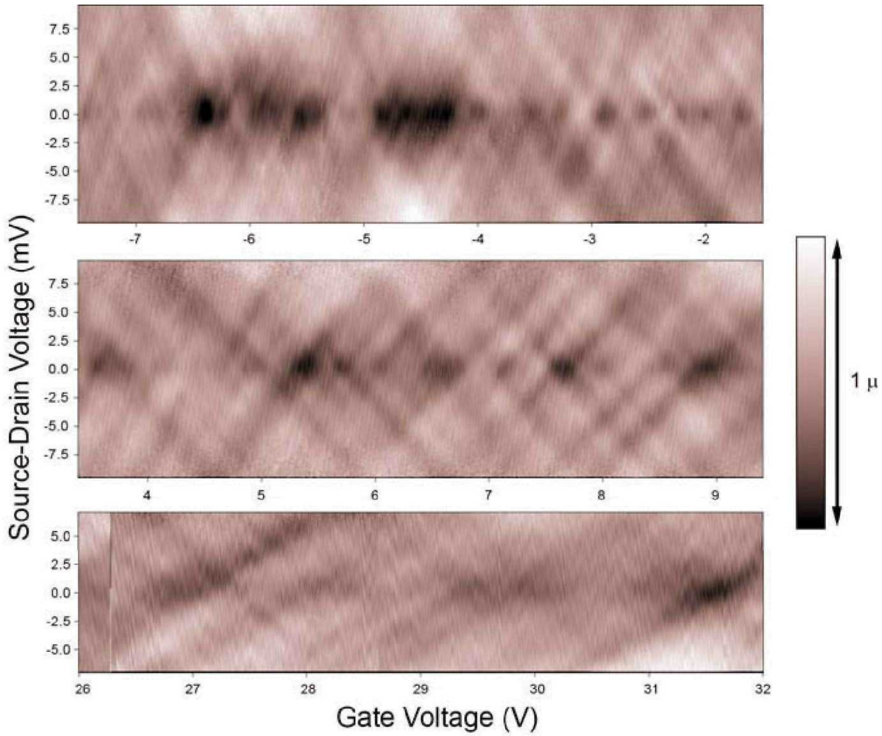
Fabry-Pérot Interference in CNT and Graphene

Similar phenomena observed in metallic CNTs



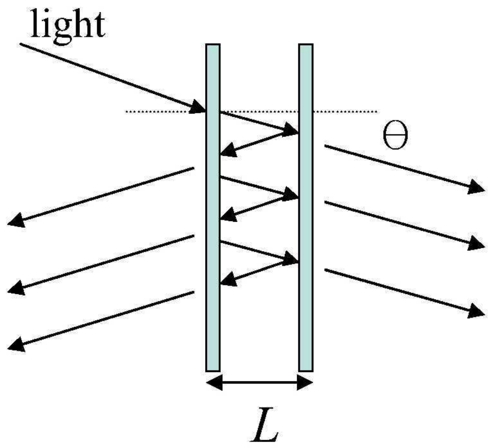
Metallic CNT:
Liang, et al. Nature **411**, 665 (2001)

graphene L = 200 nm

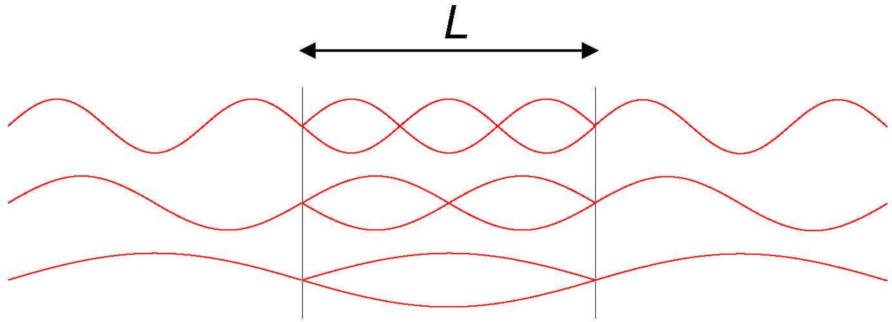


Graphene:
F. Miao, cond-mat/0703052 (2007)
S. Cho and M. S. Fuhrer, unpublished.

Fabry-Pérot Interferometer



Light is transmitted when resonance occurs:



Resonance condition is: $L = n\frac{\lambda}{2} = n\frac{\pi}{k}$ ⇒ $k_n = \frac{n\pi}{L}$

Identical to particle-in-a-box states!

Light-like particles: $k = \frac{n\pi}{L}$

$$E_n = \hbar v_F k_n = n\frac{\hbar v_F \pi}{L}$$

Massive particles: $k = \frac{n\pi}{L}$

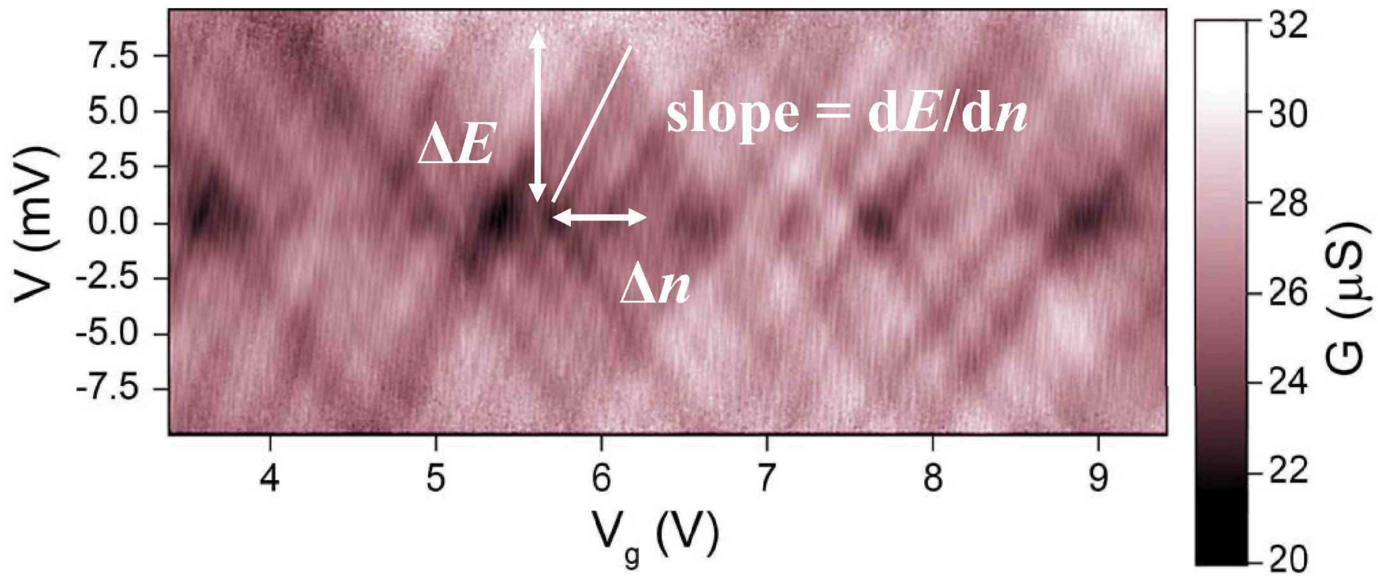
$$E_n = \frac{\hbar^2 k_n^2}{2m} = n^2 \frac{h^2}{8mL^2}$$

Fabry-Pérot Interference in CNT and Graphene

Simple picture:

V measures spacing of resonances in energy

V_g measures spacing of resonances in particle density



slope is proportional to dE/dn = 1/D(E)

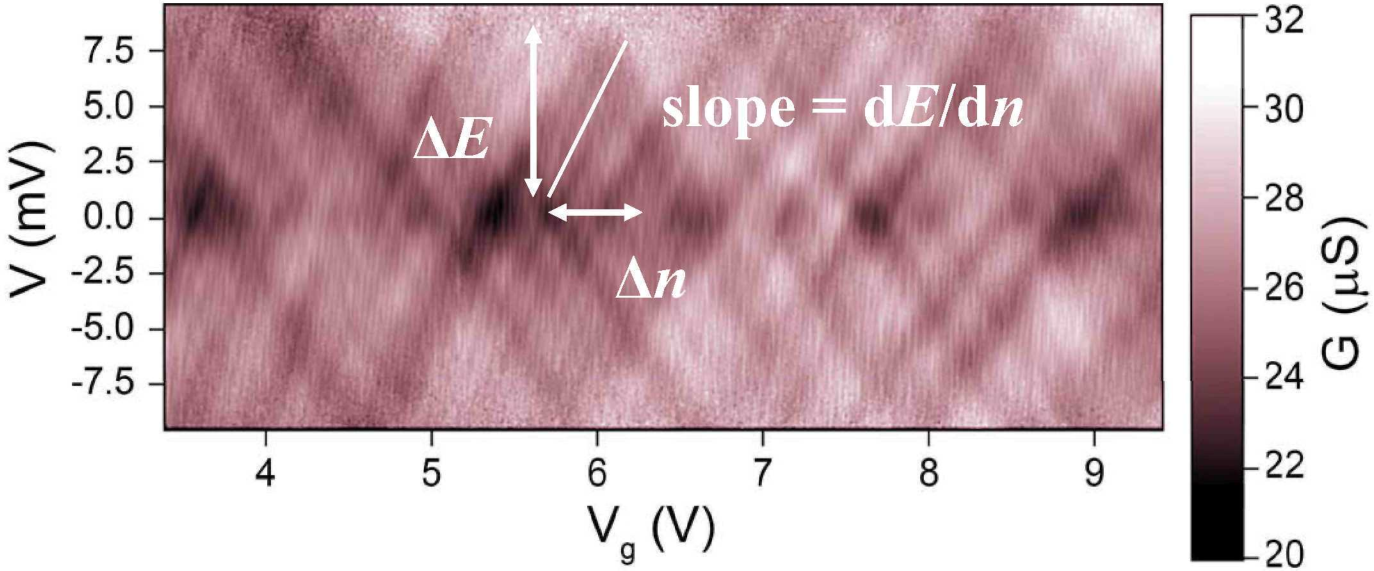
D(E) is density of states in energy

Fabry-Pérot Interference in CNT and Graphene

Simple picture:

V measures spacing of resonances in energy

V_g measures spacing of resonances in particle density



More careful treatment:

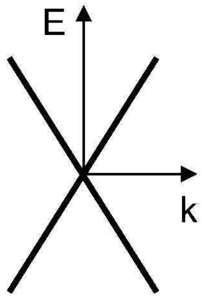
$$slope = \frac{\Delta V}{\Delta V_g} = \frac{\frac{hv_F}{eL}}{\frac{e}{c_g}\frac{hv_F}{2L}D(E)} = \frac{2c_g}{e^2}\frac{1}{D(E)}$$

Two-dimensional particle-in-a-box

Massless: Single-layer graphene

$$E = \hbar v_F |\boldsymbol{k}|$$

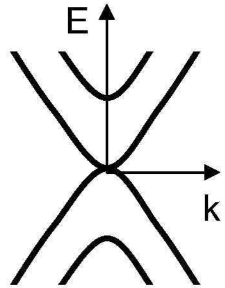
$v_F = 1\text{x}10^6$ m/s



Massive: Bilayer graphene

$$E = \frac{\hbar^2 k^2}{2m^*}$$

$m^* = 0.031\ m_e$



Particle in a box: $L = \frac{n\lambda}{2}$ $\boldsymbol{k} = (k_x, k_y) = \left(p\frac{\pi}{L}, q\frac{\pi}{L}\right)$ p, q = integer

$$E = hf\sqrt{p^2 + q^2}$$

$$E \propto k$$

$$E = \frac{h^2(p^2 + q^2)}{8m^* L^2}$$

$$E \propto k^2$$

Fermions: $N = \frac{g k_F^2 L^2}{4\pi}$ g is degeneracy

Fermi energy: $E_F = \hbar v_F \sqrt{\pi n}$

Density of states: $D = \frac{1}{\hbar v_F}\sqrt{\frac{gn}{\pi}} \propto n^{1/2}$

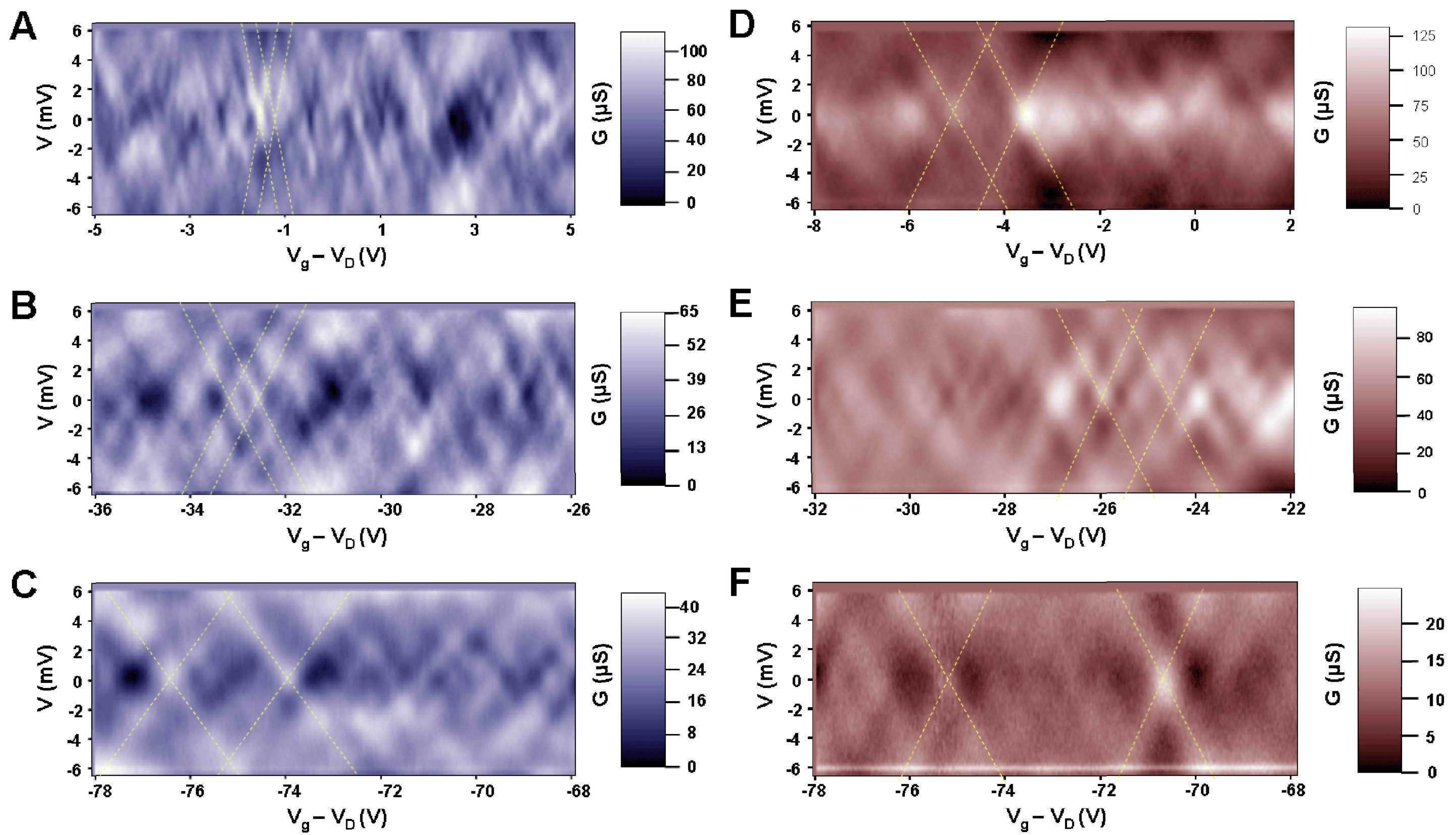
Fermi energy: $E_F = \frac{\pi \hbar^2 n}{2m^*}$

Density of states: $D = \frac{g m^*}{2\pi\hbar^2}$ indep. of n

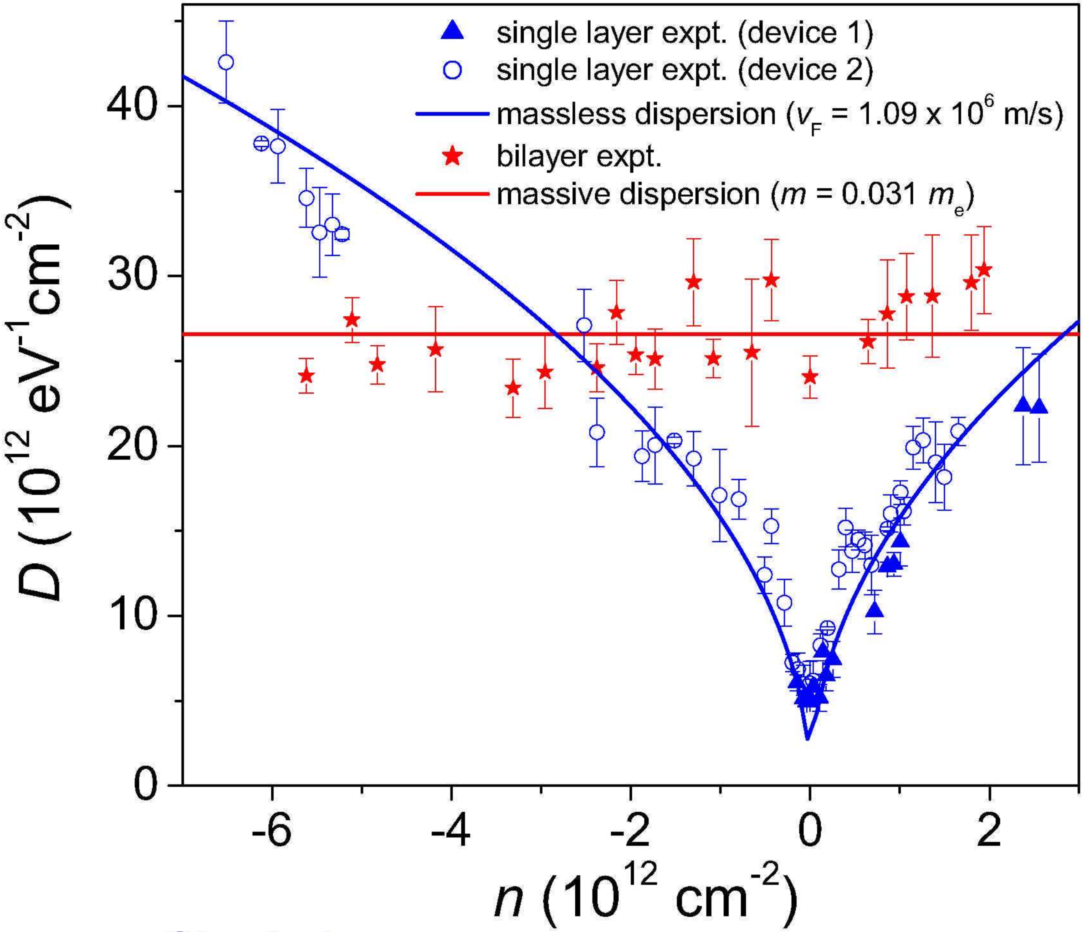
Gate voltage dependence of the slopes

Single Layer

Bilayer



Density of States: Single layer and Bilayer



$$slope = \frac{\Delta V}{\Delta V_g} = \frac{2c_g}{e^2}\frac{1}{D(E)}$$

Fits: Single layer

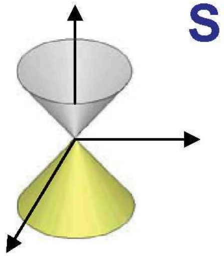
$$D(E) = \frac{1}{\hbar v_F}\sqrt{\frac{4n}{\pi}} \Rightarrow v_F = 1\text{x}10^6 \text{ m/s}$$

Bilayer

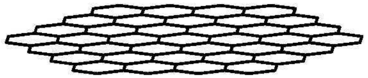
$$D(E) = \frac{2m^*}{\pi\hbar^2} \Rightarrow m^* = 0.031\ m_e$$

Excellent agreement with theoretical expectations

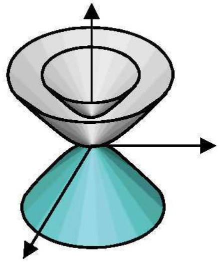
Outlook

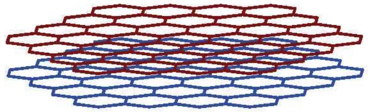


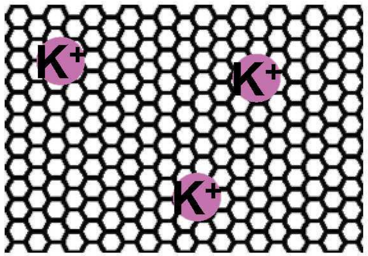
Single-layer graphene
Massless electrons



Bilayer graphene
Massive electrons

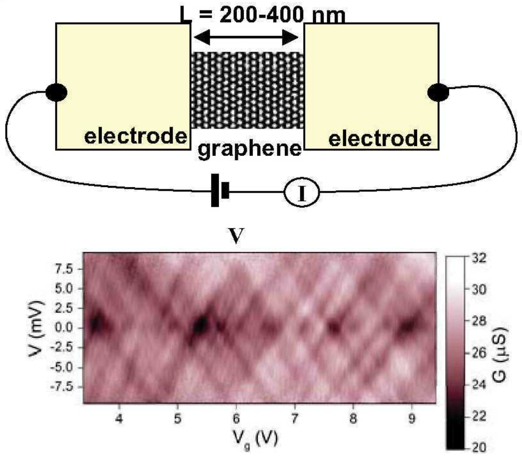






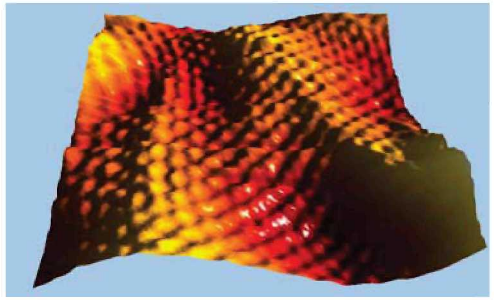
Surface science expts. can tune:

Dielectric constant
Charged impurities
Magnetic impurities
etc.



Mesoscopic expts.

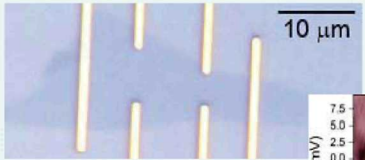
Ballistic transport of wave-like electrons



Scanned probe expts. can image:

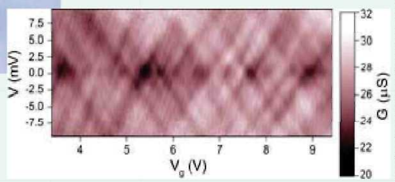
Atomic structure
Defects
Edges
Electron wavefunctions

Prof. Michael S. Fuhrer's Group
Sungjae Cho
Chaun Jang
Shudong Xiao
Alexandra Curtin

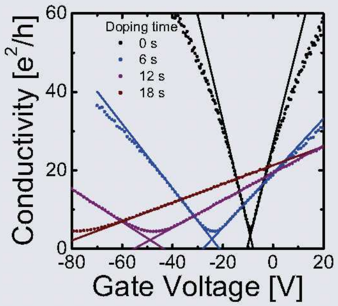


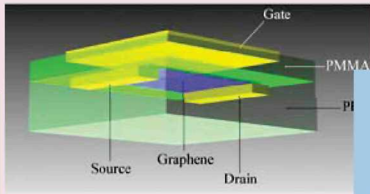
MR in graphene

graphene Fabry-Pérot



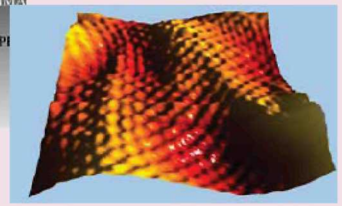
UHV doping, dielectric expts.





transfer-printing

STM



Prof. Ellen D. Williams' Group:
Dr. William Cullen
Prof. Masa Ishigami (now @ UCF)
Daniel Hines
Jianhao Chen

Prof. Sankar Das Sarma's Group
Dr. Shaffique Adam Dr. Euyheon Hwang
Dr. Enrico Rossi Wang-Kong Tse
Theory

More Info:
Fuhrer group: **www.physics.umd.edu/mfuhrer**
Williams group: **www.physics.umd.edu/spg**
Das Sarma group: **www.physics.umd.edu/cmtc**

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