The Abdus Salam
International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA

1960-5

ICTP Conference Graphene Week 2008

25 - 29 August 2008

Entangled spin-valley texture states in graphene in the quantum Hall regime

B. Doucot
CNRS, Paris, France

M. O. Goerbig
CNRS, Paris, France

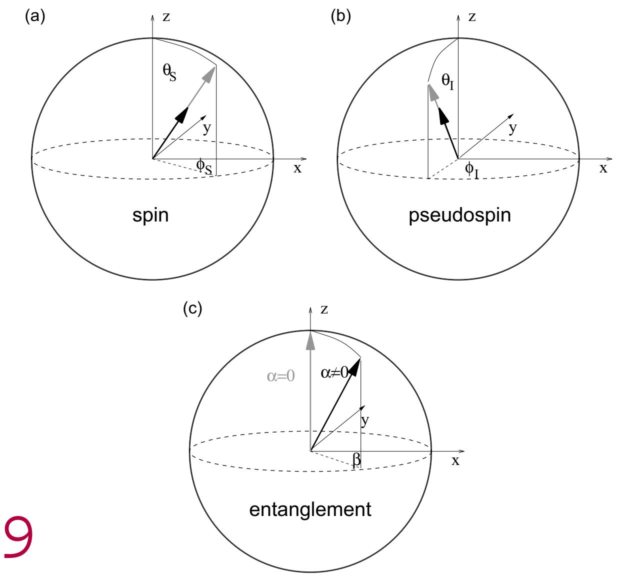
P. Lederer
CNRS, Paris, France

R. Moessner
CNRS, Paris, France

Entangled spin-valley texture states in graphene in the quantum Hall regime

Benoit Douçot
Mark O. Goerbig
Pascal Lederer
Roderich Moessner

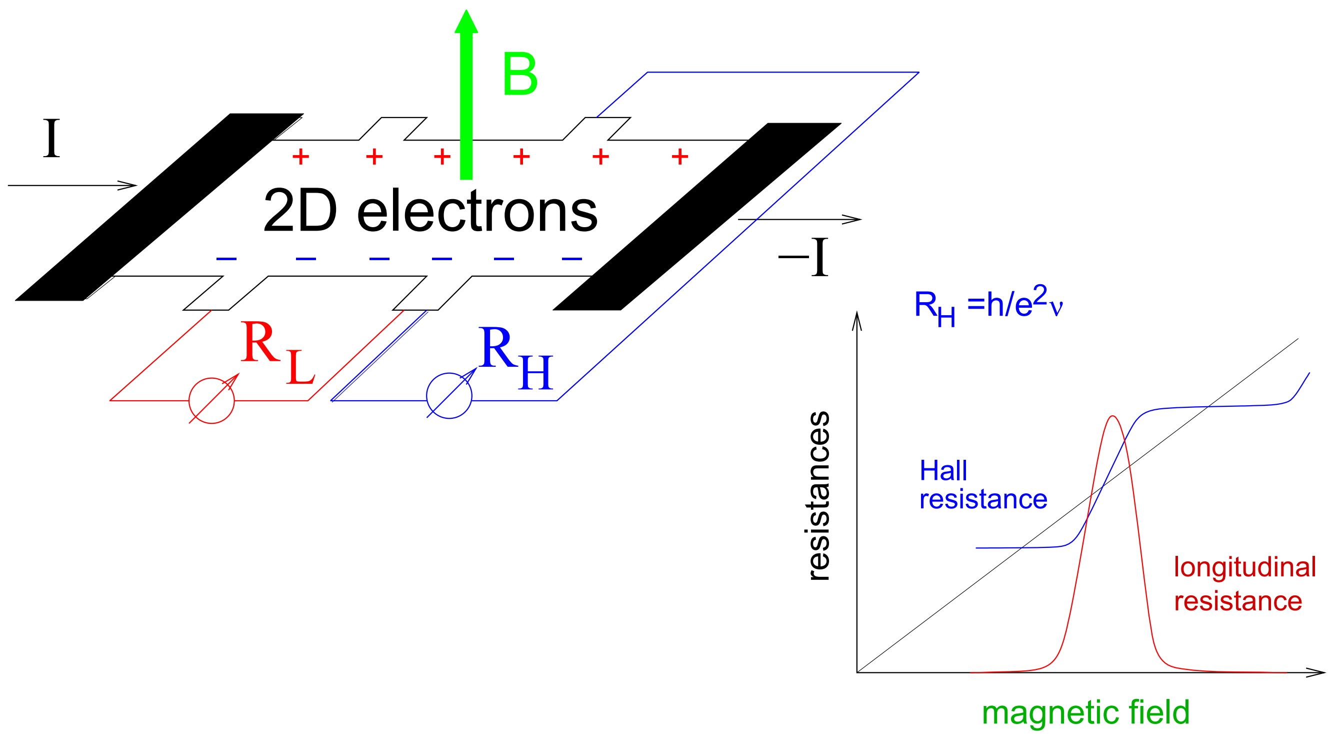
CNRS, Paris VI+XI, MPI-PKS



arXiv:0806.0229

Graphene Week 2008, ICTP Trieste

Quantum Hall Systems



- Quantum Hall system = 2D electrons in a perpendicular magnetic field at $T = 0$ (theoretician's limit)

Basic theoretical description

- Hamiltonian of 2D electrons (free or in a 2D crystal)

$$H(\mathbf{k}) \qquad\qquad \mathbf{k} \;:\; \text{wave vector}$$

\+ Magnetic field via minimal coupling (Peierls substitution)

$$\hbar\mathbf{k} \rightarrow \mathbf{\Pi} = \mathbf{p} + e\mathbf{A} \qquad\qquad \mathbf{B} = \nabla \times \mathbf{A}$$

\+ Quantum mechanics: $[\Pi_x, \Pi_y] = -i\hbar^2/l_B^2$
with magnetic length $l_B = \sqrt{\hbar/eB}$

$\Rightarrow$ Harmonic oscillator ladder operators

$$a \propto \Pi_x - i\Pi_y \qquad a^\dagger \propto \Pi_x + i\Pi_y \qquad [a, a^\dagger] = 1$$

$\Rightarrow$ Energy (Landau) levels: $H(\mathbf{\Pi})\psi_n = H(a, a^\dagger)\psi_n = \epsilon_n\psi_n$

Degeneracy of energy levels

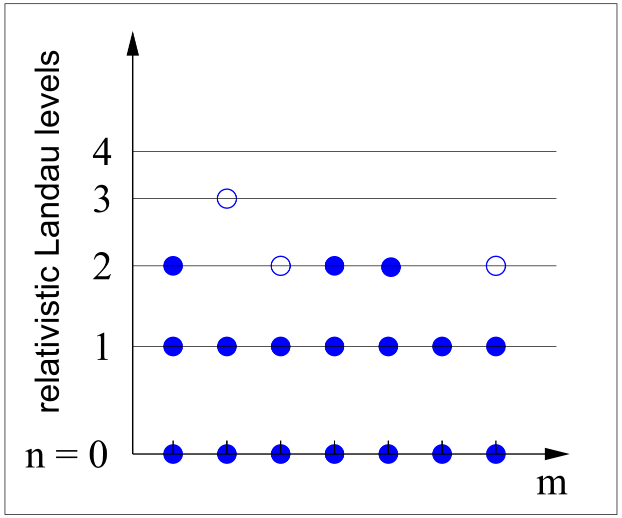
- Guiding centre operator (gauge dependent): $\tilde{\mathbf{\Pi}} = \mathbf{p} - e\mathbf{A}$
  – for symmetric gauge $\mathbf{A} = B(-y, x, 0)/2$
  $\longrightarrow$ constant of motion

degeneracy: $N_B = \mathcal{A}eB/h$

filling factor: $\nu = N_{el}/N_B$

in graphene:

$\epsilon_n = \pm\hbar\frac{v_F}{l_B}\sqrt{|n|} \propto \sqrt{B|n|}$



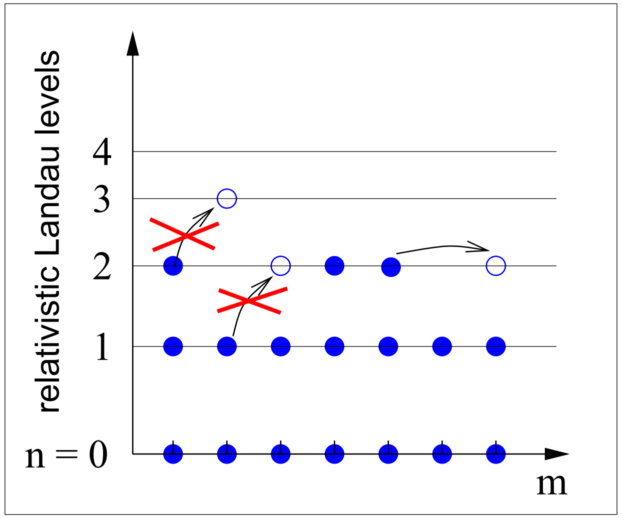
Degeneracy of energy levels

- Guiding centre operator (gauge dependent): $\tilde{\mathbf{\Pi}} = \mathbf{p} - e\mathbf{A}$
  – for symmetric gauge $\mathbf{A} = B(-y, x, 0)/2$
  $\longrightarrow$ constant of motion

degeneracy: $N_B = \mathcal{A}eB/h$

filling factor: $\nu = N_{el}/N_B$

in graphene:
$\epsilon_n = \pm\hbar\frac{v_F}{l_B}\sqrt{|n|} \propto \sqrt{B|n|}$



At $\nu \neq \pm 2(2n+1)$: “flat-band” limit of strong correlations

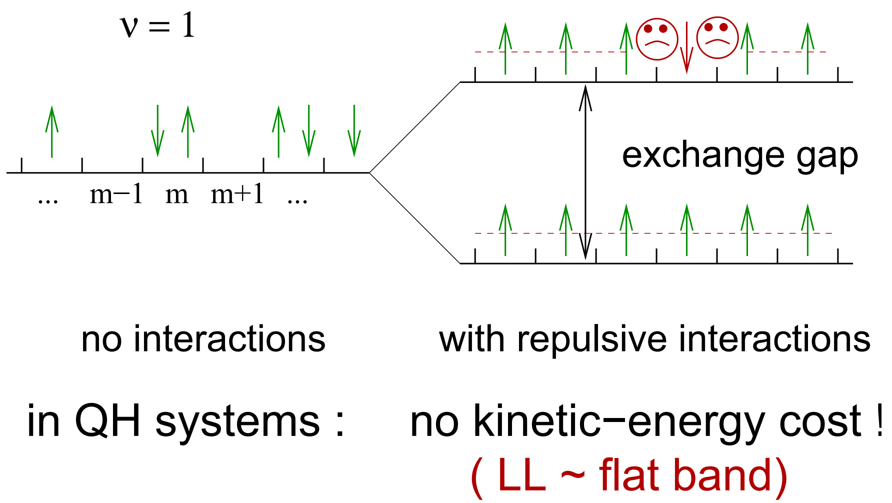
separation of energy scales, interactions lift degeneracy

Flat-band ferromagnetism at $\nu = 1$ (2DEG)

- Degenerate Landau levels have 'quenched' kinetic energy
- Splin-splitting due to Zeeman energy but in addition:

Coulomb repulsion favours anti-symmetric orbital wavefunction

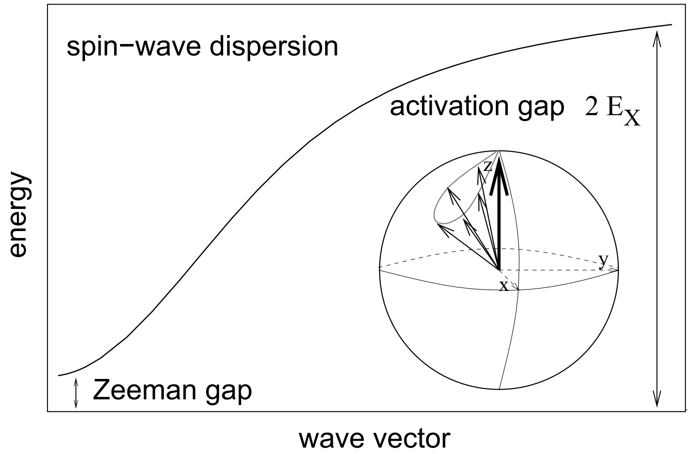
$\longrightarrow$ spin wavefunction: symmetric (ferromagnet)



- Collective effect ('exchange enhanced spin splitting')
- Zeeman splitting not necessary

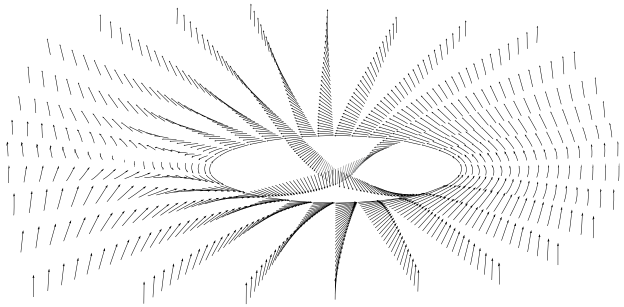
Topological spin excitations: Skyrmions

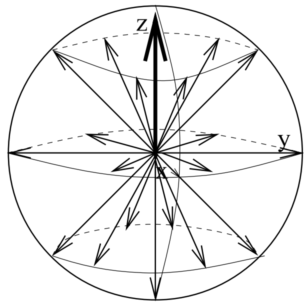
Spin waves:

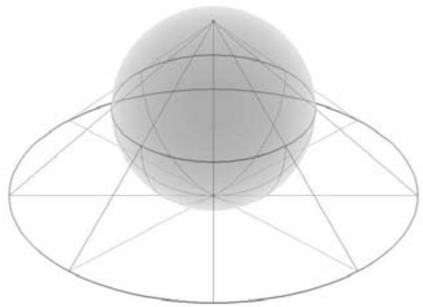


- non-topological
- charge-neutral

Skyrmions:





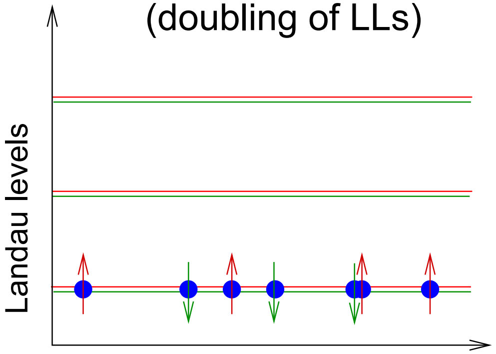


- topological
- quantised charge
- elementary excitation at $\nu = 1$ ($\longrightarrow$ energetics)

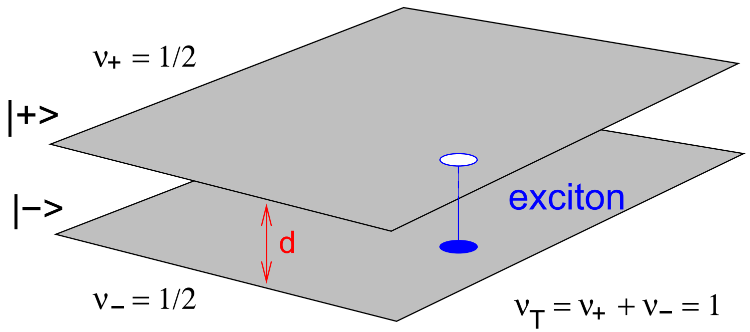
Multi-component systems (internal degrees of freedom)

(A) physical spin: SU(2)

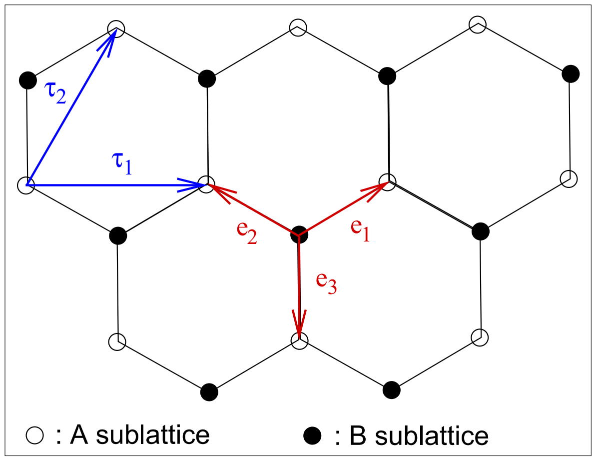
(doubling of LLs)



(B) bilayer: SU(2) isospin



(C) graphene (2D graphite)



two−fold valley degeneracy
$\longrightarrow$ SU(2) isospin

spin + isospin : SU(4)

Skyrmions in multicomponent systems with SU($N > 2$)

- General description of SU(N)/SU(4) Skyrmions
  Arovas, Karlhede, Lilliehöök, PRB **59**, 13147 (1999); Ezawa, PRL 82, 3512 (1999)

- SU(4) Skyrmion lattice in bilayer quantum Hall systems
  Bourassa et al., PRB **74**, 195320 (2006)

- SU(4) Skyrmions in graphene
  Yang, Das Sarma, MacDonald, PRB **74**, 075423 (2006)

Our approach:

- SU(4) description, but keeping track of the two SU(2) spin-isospin copies – spin-isospin entanglement
  Doucot, MOG, Lederer, Moessner, arXiv:0806.0229

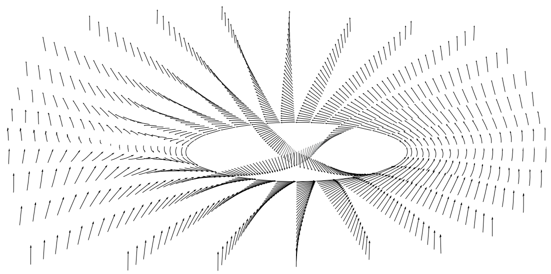
SU(2) Skyrmion parametrisation

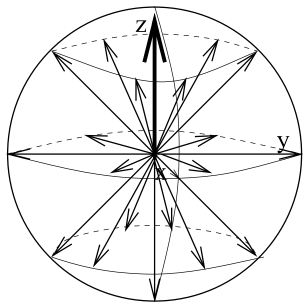
Slowly spatially varying spinor for Skyrmion of 'size' λ:

$$|\psi\rangle = \cos\frac{\theta(\mathbf{r})}{2}|\uparrow\rangle + \sin\frac{\theta(\mathbf{r})}{2}e^{i\phi(\mathbf{r})}|\downarrow\rangle = \begin{pmatrix} \cos\frac{\theta(\mathbf{r})}{2} \\ \sin\frac{\theta(\mathbf{r})}{2}\exp[i\phi(\mathbf{r})] \end{pmatrix}$$

with 'stereographic projection'

$$\tan\frac{\theta(\mathbf{r})}{2}\exp[i\phi(\mathbf{r})] = \frac{x+iy}{\lambda} \equiv \frac{z}{\lambda}$$





Skyrmions in graphene with SU(4) symmetry

Four-component spinors

$$\begin{aligned} w_1 &\rightarrow \uparrow, K \\ w_2 &\rightarrow \uparrow, K' \\ w_3 &\rightarrow \downarrow, K \\ w_3 &\rightarrow \downarrow, K' \end{aligned}$$

For an isotropic system, w_1,...,w_4 are analytic functions of spatial coordinate z.

Topological density

- Berry connection

$$\begin{aligned} \mathcal{A} &= \frac{1}{i}\langle\Psi|\nabla\Psi\rangle \\ \oint \mathcal{A}\cdot d\mathbf{r} &= 2\pi Q_{\mathrm{top}} \end{aligned}$$

- $Q_{\mathrm{top}} = \pm 1$ for a skyrmion
- ‘restores’ commensurability in presence of hole (due to electric charge)

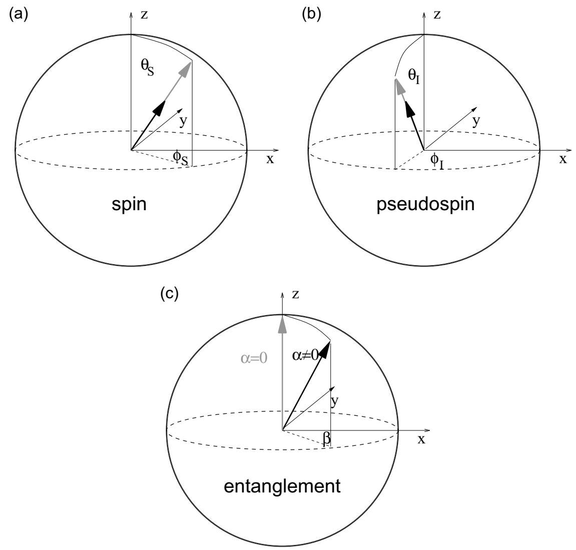
SU(2)×SU(2) parametrisation by Schmidt decomp.

Two SU(2) copies ⇒ four-spinor w (slowly varying)

- define local spinors $|\psi_{S,I}\rangle \perp |\chi_{S,I}\rangle$ for spin S and isospin I
- Schmidt decomposition for full SU(4) wavefunction:

$$\Psi(z) = \cos\left(\frac{\alpha}{2}\right)|\psi_S\rangle \otimes |\psi_I\rangle + \sin\left(\frac{\alpha}{2}\right)\exp(i\beta)|\chi_S\rangle \otimes |\chi_I\rangle$$

- S, I manifestly on equal footing
- third Bloch sphere appears
- "magnetisations" $\mathbf{m}_{S,I} = \cos\alpha\,\mathbf{n}_{S,I}$



Entanglement of spin and pseudospin

For $\sin\alpha \neq 0$, S and I are entangled:

$$\begin{aligned} \rho_S &= Tr_I\left(|\psi\rangle\langle\psi|\right) = \cos^2\frac{\alpha}{2}|\psi_S\rangle\langle\psi_S| + \sin^2\frac{\alpha}{2}|\chi_S\rangle\langle\chi_S| \\ m_S^a &= Tr\left(\rho_S S^a\right) = \cos\alpha\langle\psi_S|S^a|\psi_S\rangle = \cos\alpha\; n^a\left(\theta_S, \phi_S\right) \end{aligned}$$

Measure of entanglement:

$$\Xi = 1 - \sum_i \langle m^i_{S,I}\rangle^2 = \sin^2\alpha = 4|w_1 w_4 - w_2 w_3|^2$$

Factorisable state $\Leftrightarrow$ $\Xi = 0$

Topological density

Berry connection $\mathcal{A} = \frac{1}{i}\langle\Psi|\nabla\Psi\rangle$ and top. density $\mathcal{B} = \nabla \times \mathcal{A}$

$$\mathcal{A}(\mathbf{r}) = \sin^2\frac{\alpha}{2}\nabla\beta + \cos\alpha\left(\sin^2\frac{\theta_S}{2}\nabla\phi_S + \sin^2\frac{\theta_I}{2}\nabla\phi_I\right)$$

$$\begin{aligned}
\mathcal{B}(\mathbf{r}) \quad = \quad & \cos\alpha\{\rho_{\text{top}}\left[\mathbf{n}\left(\theta_S,\phi_S\right)\right] + \rho_{\text{top}}\left[\mathbf{n}\left(\theta_I,\phi_I\right)\right]\} \\
& + \rho_{\text{top}}\left[\mathbf{n}(\alpha,\beta)\right] \\
& + \sin^2\frac{\theta_S}{2}\rho_{\text{top}}\left[\mathbf{n}(\alpha,\phi_S)\right] + \sin^2\frac{\theta_I}{2}\rho_{\text{top}}\left[\mathbf{n}(\alpha,\phi_I)\right]
\end{aligned}$$

where $\rho_{top} = \frac{\epsilon^{ij}}{8\pi}\mathbf{n}(\theta,\phi)\cdot\left[\partial_i\mathbf{n}(\theta,\phi)\times\partial_j\mathbf{n}(\theta,\phi)\right]$

- one term depends on α, β *only*: $\rho_{top}(\alpha,\beta)$

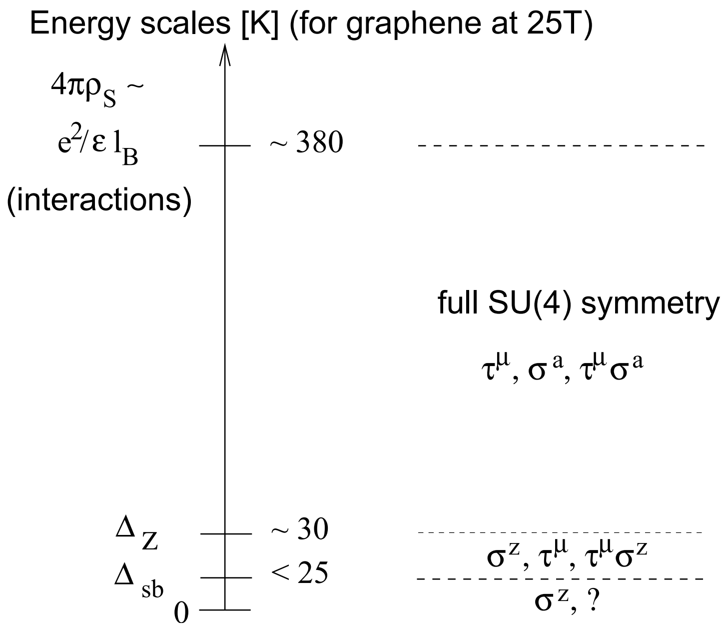
$\Rightarrow$ corresponding Skyrmion is entangled!

Realistic anisotropies in graphene

Hamiltonian can approximately have high $SU(4)$ symmetry

- Zeeman anisotropy: $SU(2) \to U(1)$
- Graphene: valley weakly split, $O(a/l_B \simeq 0.005\sqrt{B[\mathrm{T}]})$

  MOG, Moessner, Doucot, PRB 74, 161407 (2006)



Degenerate texture families in anisotropic system

Still have U(1) generators: σ^z , τ^z and $\sigma^z \otimes \tau^z$ ($\sigma^z \otimes \tau^\mu$)

$\Rightarrow$ define entanglement operator $\mathcal{T}(\gamma) = \exp\left(i\gamma\sigma^z \otimes \tau^z\right)$

$\Rightarrow$ generates families of degenerate Skyrmions

- family members differ in entanglement $\Xi(\gamma)$

$$\Xi_{\substack{\min \\ \max}} = 4\left(|w_1 w_4| \mp |w_2 w_3|\right)^2$$

$$\begin{aligned} \Xi_{\max} &= 1 - \left[\left(|w_1| + |w_4|\right)^2 + \left(|w_2| + |w_3|\right)^2\right] \\ &\quad \times \left[\left(|w_1| - |w_4|\right)^2 + \left(|w_2| - |w_3|\right)^2\right] \end{aligned}$$

- state dependent NMR rate (spin magnetisation)!

$$\langle S^+ S^- \rangle_\gamma = \cos^2(2\gamma)\langle S^+ S^- \rangle_{\gamma=0} + \sin^2(2\gamma)\langle (S^+\tau^z)(S^-\tau^z)\rangle_{\gamma=0}$$

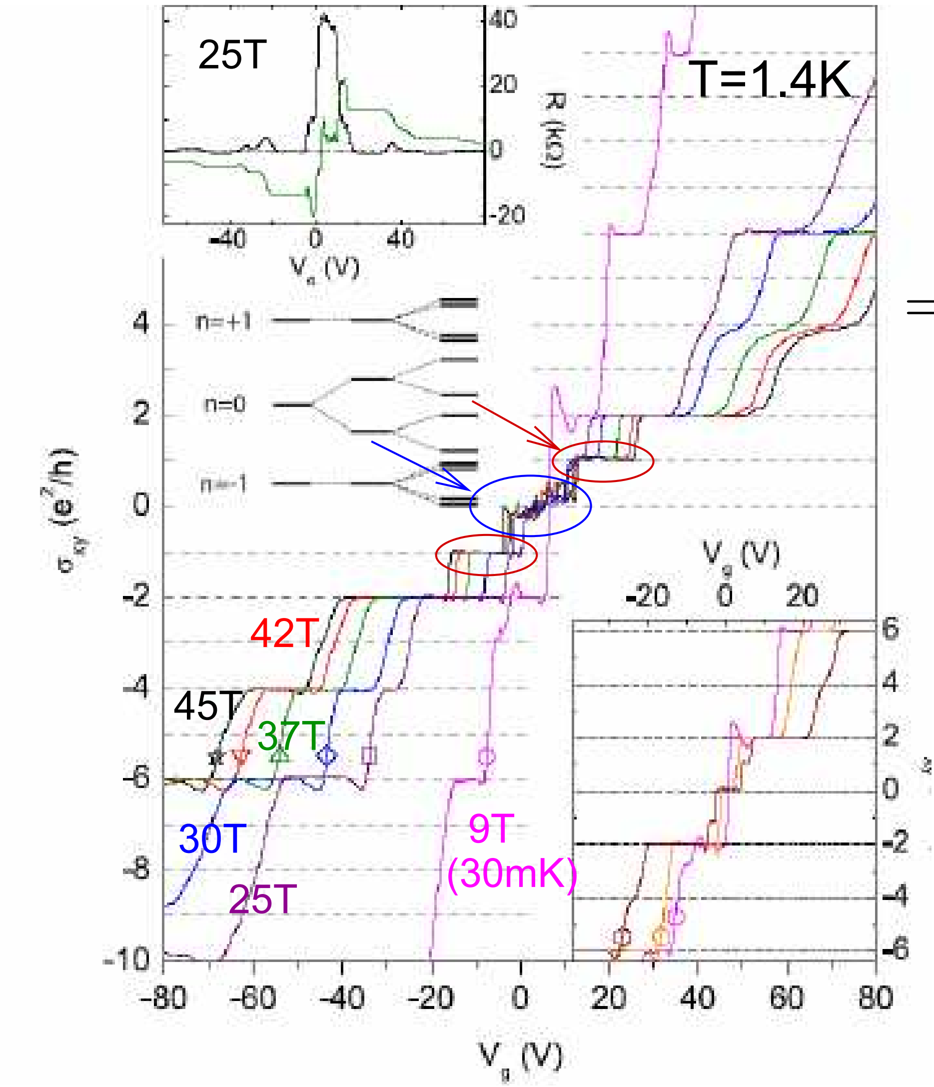
Conclusions and open questions

- Generic SU(4) textures in graphene exhibit entanglement
- Treatment of spin and isospin on equal footing
- Space-dependent entanglement as a source of topological charge
- Some symmetry operations generate entanglement
- Physical properties depend on the degree of entanglement

- Energetics of entangled textures ?
- Quantitative predictions for NMR (^{13}C in graphene) ?
- Other physical signatures of entanglement ?

$\longrightarrow$ How to probe physically the valley isospin ?

'SU(4) Skyrmions' in graphene?



- Plateaus at $\nu = 0, \pm 1, \pm 4$
  Zhang et al. PRL 06

$\Rightarrow$ some addt'l Landau levels individually resolved

- Simplest consideration: gaps vs. broadening Γ
- $E_{sk}^{n=0} \approx 4$meV$> \Gamma >$ $E_{sk}^{n=1} \approx 1.8$meV
- $\nu = 4$ plateau only at strong fields$\Rightarrow$ need E_Z
- other scenarios exist. . .