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Theory of induced superconductivity in graphene

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Theory of induced superconductivity in graphene

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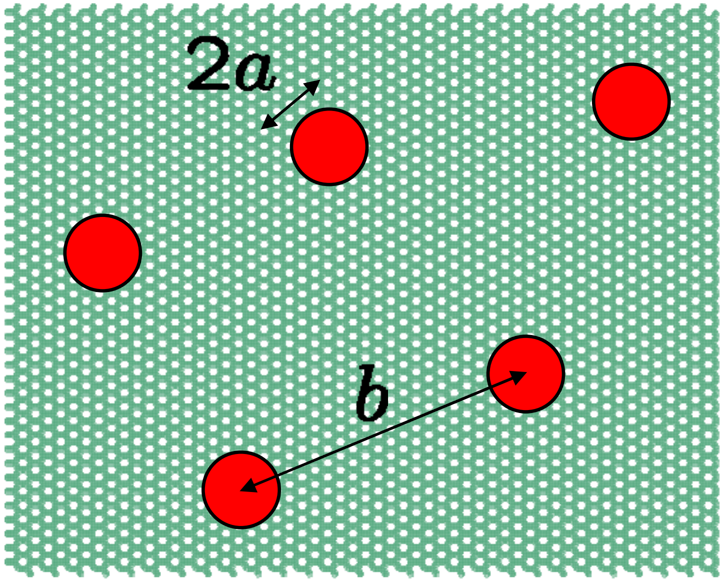
Graphene + superconducting islands



Graphene can be made superconductive
if small superconductive islands
are placed on top of it

$$T_c \sim \frac{\hbar D}{b^2}$$

can be Kelvins

disordered graphene

scales: $l \leq a \ll b$

Superconductive proximity effect in graphene

Experiments on SGS junctions:

Proximity effect

- Orsay, 2007
- Riverside, 2007

Josephson current

- Delft, 2007
- Rutgers, 2007

Conclusion (e): phase coherent transport

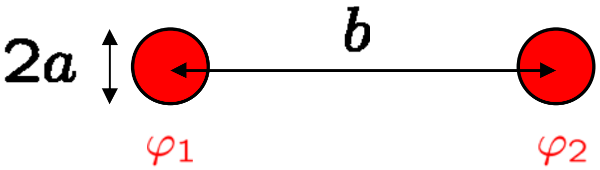
graphene is essentially disordered with $l \sim 30\,\mathrm{nm}$

transport is diffusive rather than ballistic

Conclusion (t): all relativistic beauty of graphene has gone

proximity effect is described by the Usadel equation

Josephson coupling via disordered graphene

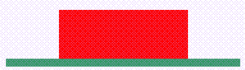


$$\mathcal{H} = -E_J(b,T)\cos(\varphi_1 - \varphi_2)$$

- Usadel equation (at $l < r$)
- linearization ($a \ll b$): $F_E(\mathbf{r}) = F_E^{(\varphi_1)}(\mathbf{r}-\mathbf{r}_1) + F_E^{(\varphi_2)}(\mathbf{r}-\mathbf{r}_2)$

$$E_J(b,T) = 16\pi^3 gT \sum_{\omega_n>0} \frac{P(\sqrt{\omega_n r^2/2D})}{\ln^2(\omega_n a^2/D)}, \qquad P(z) = z\int_0^\infty K_0(z\cosh t)K_1(z\cosh t)\,dt$$

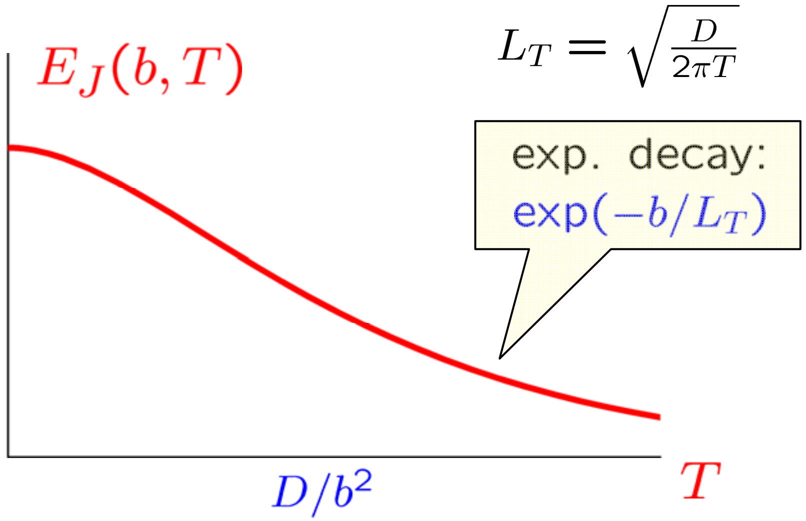
good contact



$$E_J(b,0) = \frac{\pi^3}{4}\frac{g}{\ln^2(b/a)}\frac{D}{b^2}$$

tunnel barrier

$$E_J(b,0) = \frac{G_T^2}{8\pi g}\frac{D}{b^2}$$



Thermal phase transition

JJ array:

$$\mathcal{H} = -\sum_{ij} E_J(b,T)\cos(\varphi_i - \varphi_j)$$

XY model:

$$\mathcal{H} = \frac{\Upsilon(T)}{2}\int d\mathbf{r}\,(\nabla\varphi)^2$$

Superfluid stiffness: $\Upsilon(T) = \frac{c}{2b^2}\sum_j |\mathbf{r}_i - \mathbf{r}_j|^2 E_J(\mathbf{r}_i - \mathbf{r}_j, T)$

The systems exhibits the **BKT transition** at $\Upsilon(T_c) = (2/\pi)T_c$

Josephson coupling is short-ranged at $T \sim T_c$

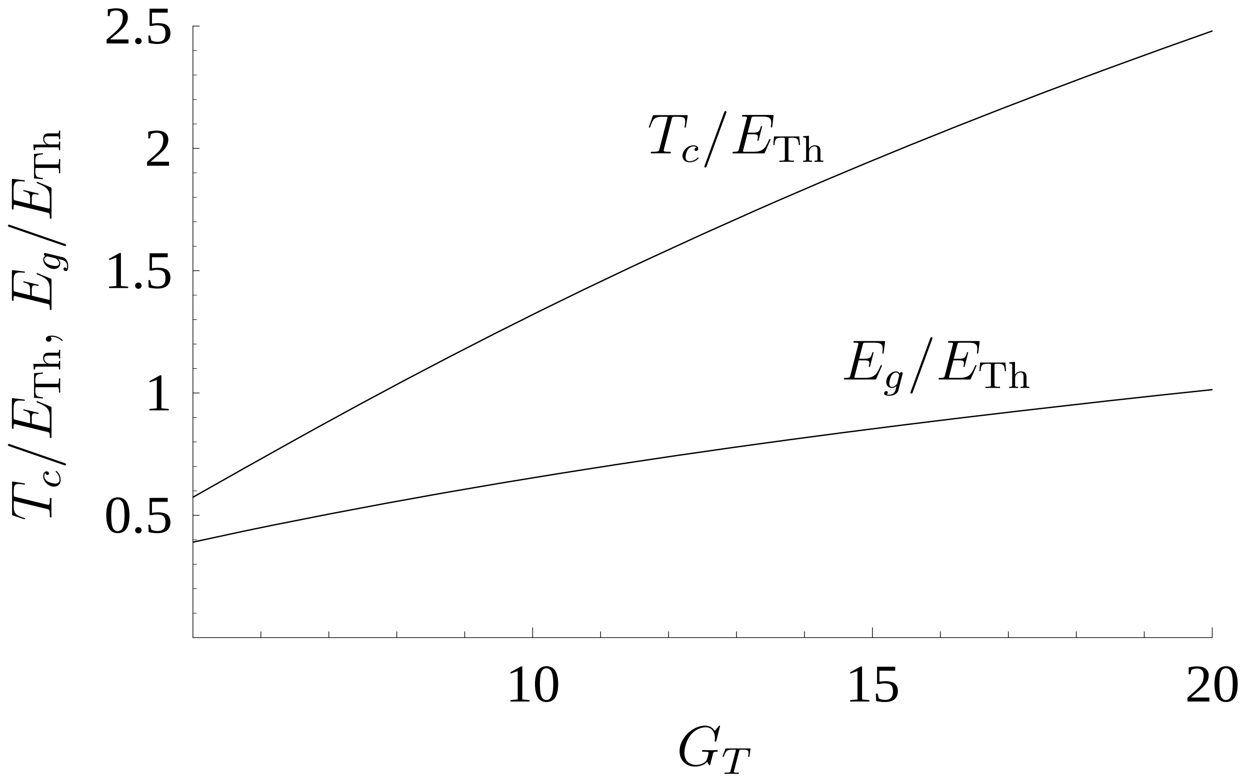
$$T_c = \gamma\, E_J(T_c)$$

$$E_{\text{Th}} = \frac{D}{b^2}$$

for triangular lattice $\gamma = 2.94$ (Butera & Comi, 1994)

- at $T > T_c$, phases of different islands are uncorrelated
- at $T < T_c$, all phases phases look in the same direction

Transition temperature normalized to Thouless energy $E_{th} = D/b^2$ as function of interface conductance G_{int}



g = 6   b/a = 10

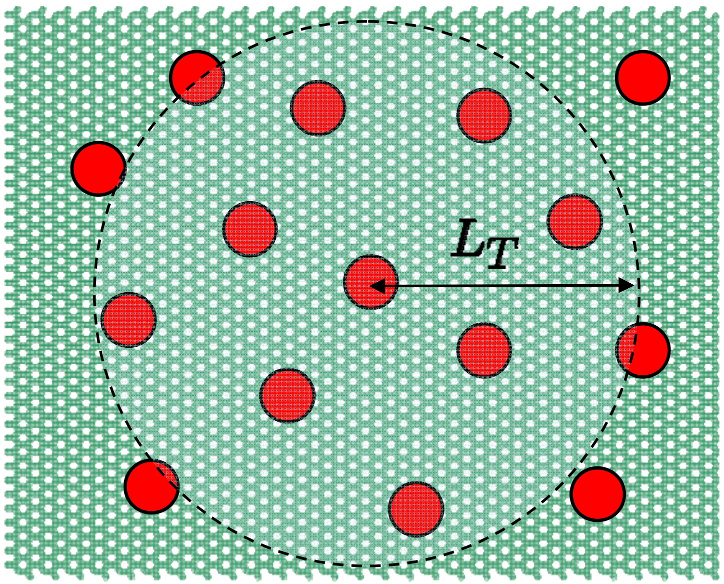
Superfluid density below T_c

At $T \ll T_c$, the Josephson coupling is long-ranged.

Many islands contribute to the superfluid stiffness $\Upsilon(T)$:

$$\Upsilon(T) = \frac{1}{2b^2}\int_0^\infty r^2 E_J(r,T)\, 2\pi r\, dr = \frac{\pi^5}{4}\frac{g}{\ln^2(b/a)}\frac{E_{\mathrm{Th}}^2}{T}$$

L_T^2 $\quad$ L_T^{-2} $\quad$ L_T^2



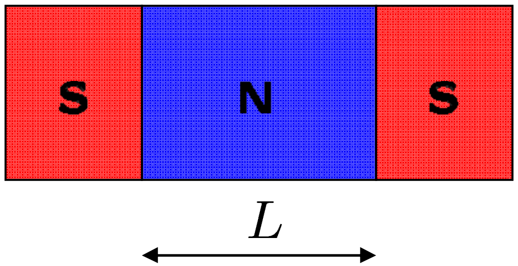
Strong divergence of $\Upsilon(T \to 0)$

Did we forget something?

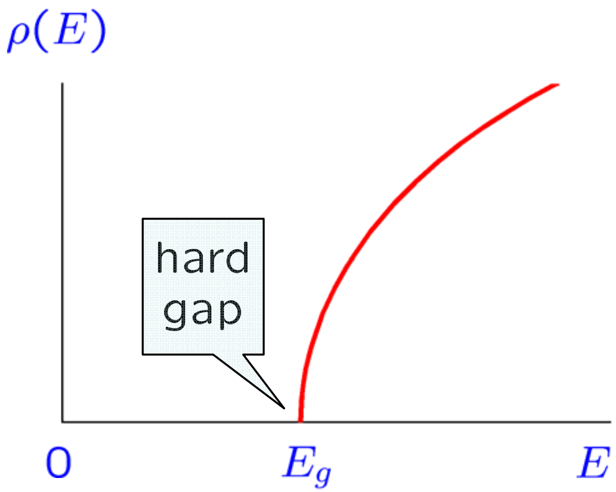
?

Spectral gap

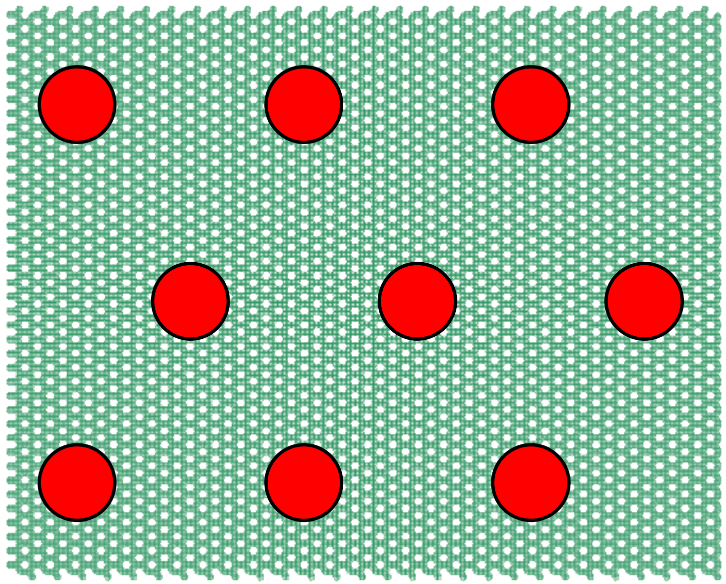
Minigap in an SNS junction



$$E_g \simeq 3.1 \frac{D}{L^2}$$



Regularly placed islands on graphene

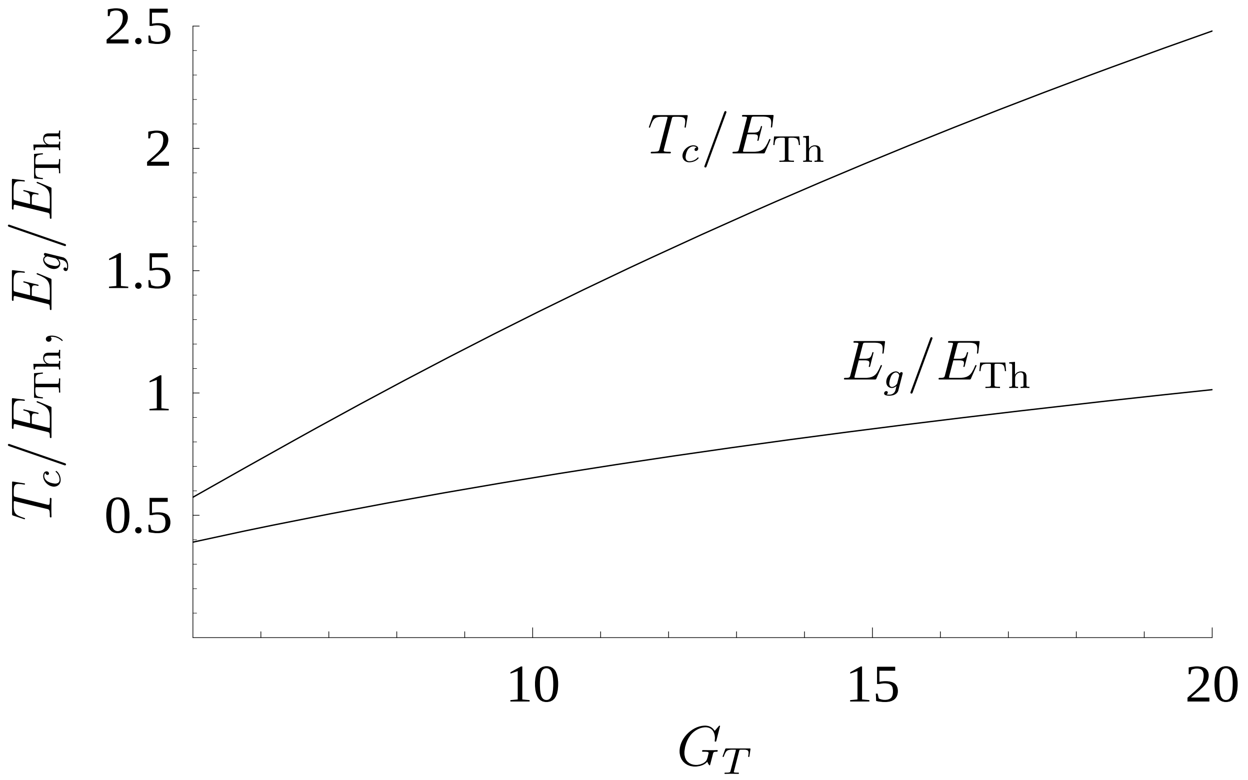


Numeric integration of the Usadel eq. gives

$$E_g \simeq \frac{2.6}{\ln(b/a)} \frac{D}{b^2}$$

Can be seen in STM experiments

Spectral gap normalized to Thouless energy $E_{th} = D/b^2$ as function of interface conductance G_{int}

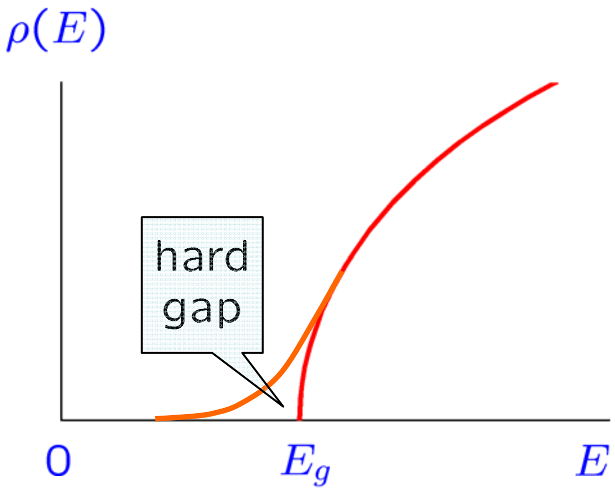


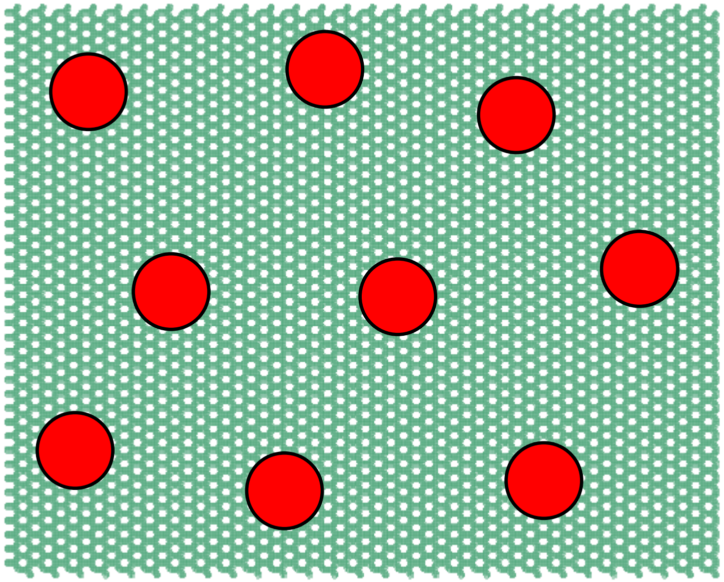
g = 6 b/a = 10

Spectral gap in an irregular system

Iregularly placed islands on graphene

The hard gap will be smeared





The relative smearing is expected to be small: $\dfrac{\delta E_g}{E_g} \sim \dfrac{1}{\ln(b/a)}$

Two-energy-scale superconductivity

bulk inhomogeneous superconductor

interacting SC islands with pair-wise interaction

normal

T

0

$E_g \simeq \frac{2.6\, E_{\mathrm{Th}}}{\ln(b/a)}$

$L_T \sim \xi$

$T_c \sim 4\, E_{th}$

$L_T \sim b$

determines the SC coherence length

$\xi \sim b\sqrt{\ln(b/a)}$

$\Upsilon(T) \sim g\frac{E_g^2}{T}$

$\Upsilon(T)$ saturates at $\Upsilon(E_g) \sim gE_g$

compare with a dirty SC: $\Upsilon(0) \sim g\Delta$

Magnetic field effect (low T, low H)

Small H: like in a 2D superconductor with $\Delta \sim E_g$

Critical field:

$$H_g \simeq \frac{cE_g}{eD} = \frac{2.6}{\pi \ln b/a} \frac{\Phi_0}{b^2}$$

$H_g \sim 100$ G
at $b \sim 300$ nm

$H < H_g$

- disordered vortex lattice
- core size $\xi \sim b \ln(b/a) \gg b$
- strong pinning

$H > H_g$

- spectral gap is totally suppressed
- gapples SC $\Leftrightarrow$ SC glass
- weakly frustrated

Magnetic field effect (low T, high H)

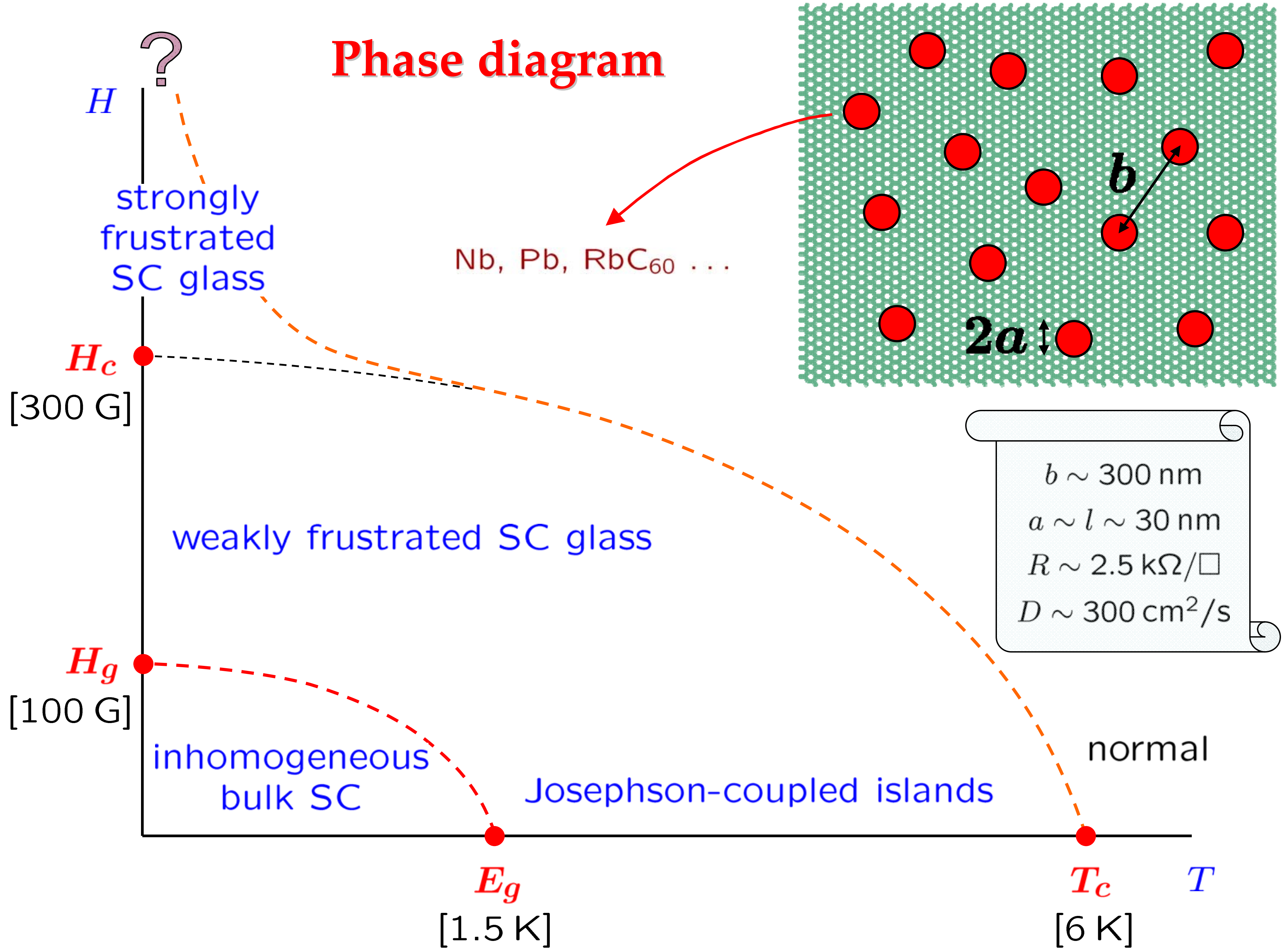
$$H_c \sim \frac{\Phi_0}{b^2} \gg H_g$$

At $H \sim H_c$ the Josephson coupling bewteen the nearest islands becomes totally frustated

average coupling: $\langle E_J(r_{ij}) \rangle \sim g \dfrac{D}{r_{ij}^2} \exp(-r_{ij}/L_H)$

mesoscopic fluctuations: $\sqrt{\langle E_J^2(r_{ij}) \rangle} \sim \dfrac{D}{r_{ij}^2}$

Phase diagram



Conclusions

1. Graphene can be made superconductive at Kelvins with very small part of area covered by superconductive islands

2. Spectral gap expected due to proximity effect should be measurable by low-temperature STM (and destroyed easily by magnetic field)

3. Transformation from continous disordered superconductor to weakly coupled junction's array is predicted with growth of either T or B

4. Approaching the neutrality point will lead to island's decoupling and T=0 quantum phase transition to metal