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International Centre for Theoretical Physics**



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ICTP Conference Graphene Week 2008

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Electric and Thermoelectric Transport in Graphene

P. Kim

Columbia University, New York, U.S.A.

Electric and Thermoelectric Transport in Graphene

Philip Kim

**Department of Physics
Columbia University**



Outlines

I. Electron transport in suspended graphene samples

K. I. Bolotin et. al., Solid State Comm. 146, 351-355 (2008);

K. I. Bolotin, et. al., arXiv:0805.1830.

II. Quantum oscillations in graphene pnp junctions

A. F. Young and P. Kim, arXiv:0808.0855;

will also be presented in the poster session by Andrea Young

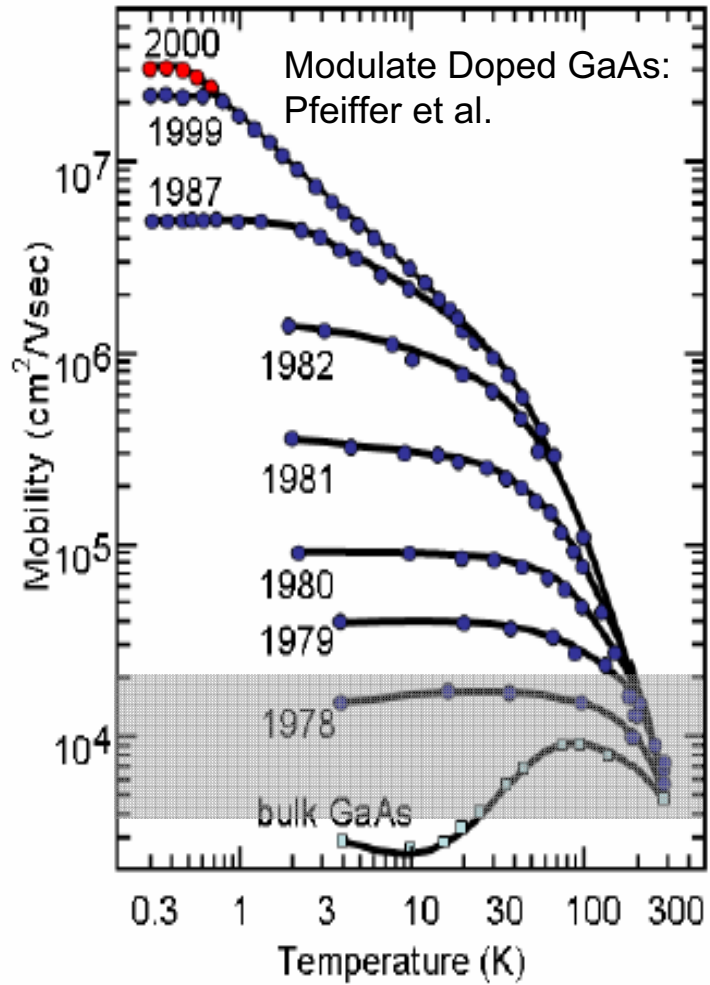
III. Thermoelectric power in graphene

Y. Zuev, W. Chang, and P. Kim, in preparation

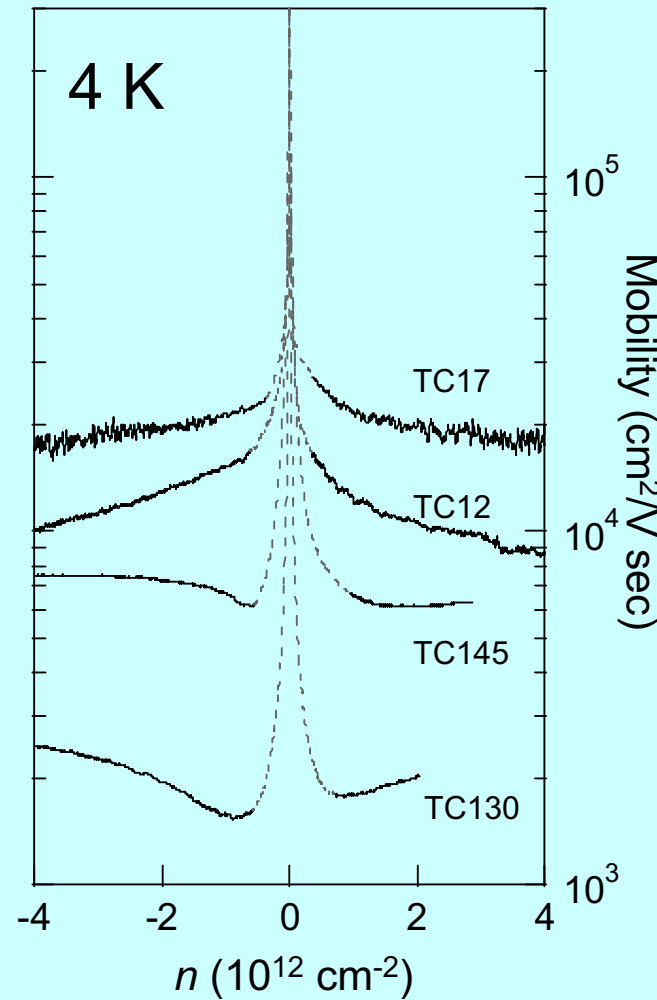
Graphene Mobility

$$\sigma = en\mu$$

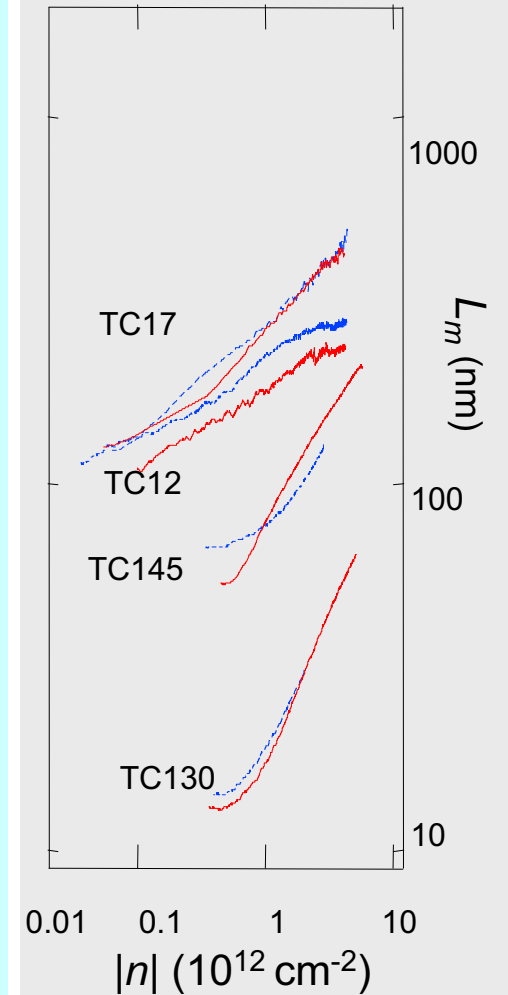
GaAs HEMT



Graphene Mobility



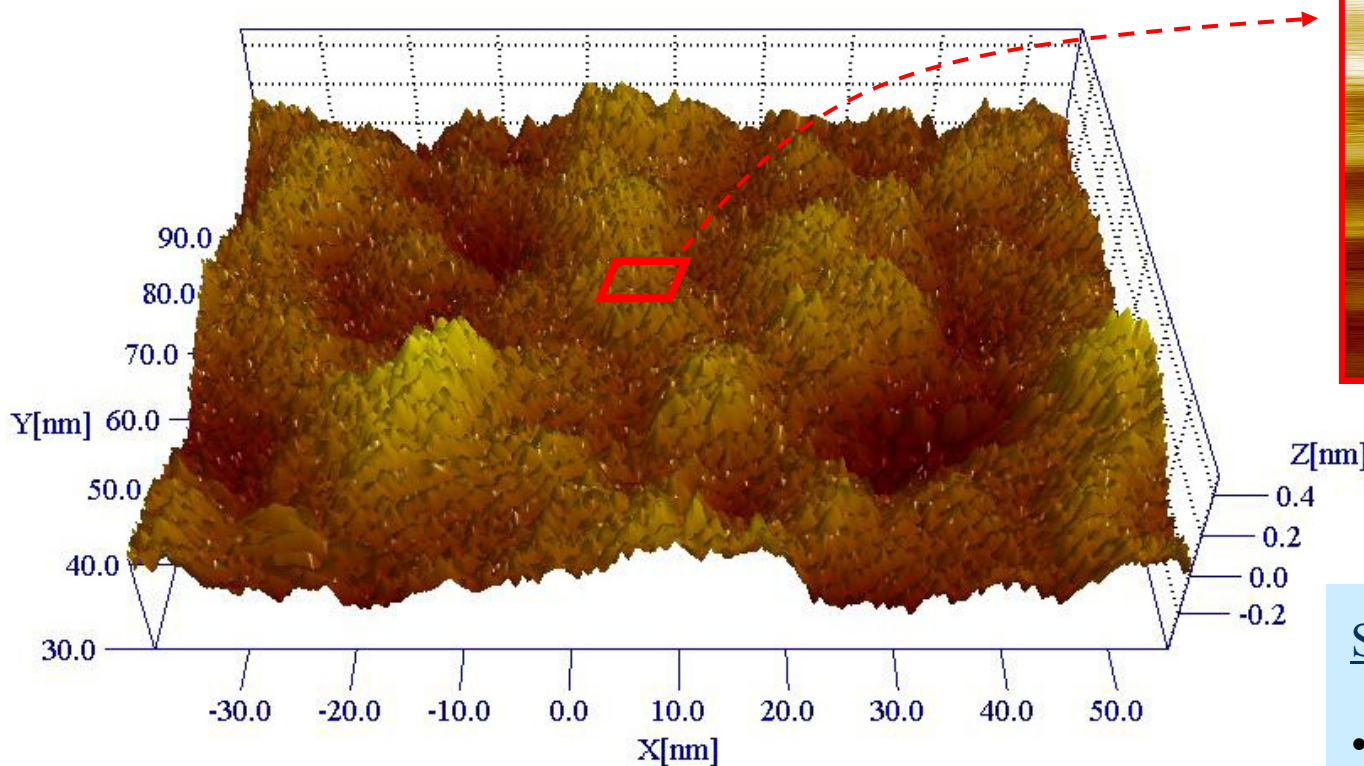
Mean free path



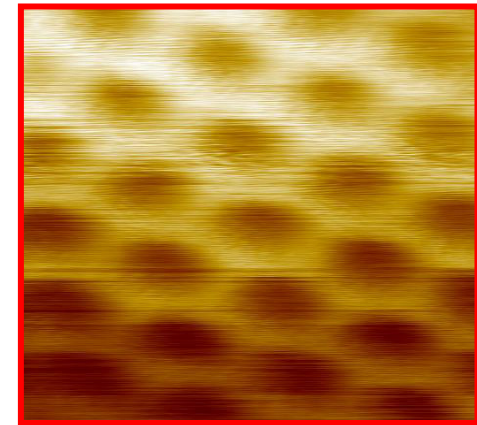
Tan et al. PLR (2007)

STM on Graphene

Ripples of graphene on a SiO₂ substrate



Atomic resolution



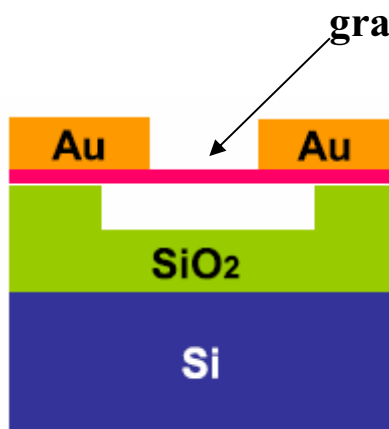
Scattering Mechanism?

- Ripples
- Substrate (charge trap)
- Absorption
- Structural defects

Elena Polyakova et al (Columbia Groups), PNAS (2007)

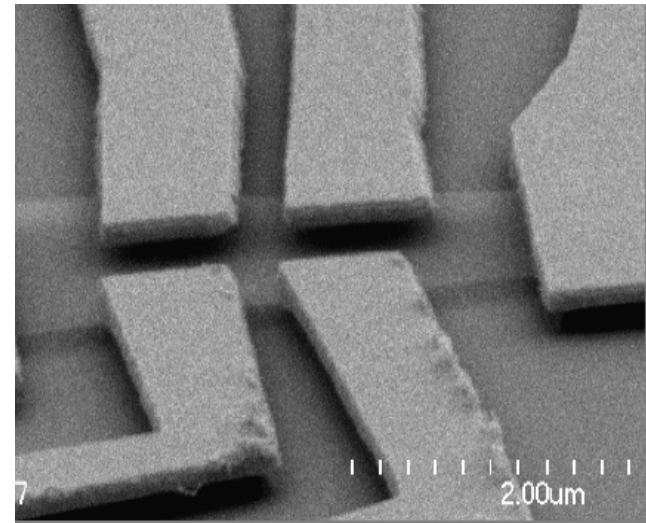
See also Meyer et al, Nature (2007) and Ishigami et al, Nano Letters (2007)

Toward High Mobility: Suspending Samples

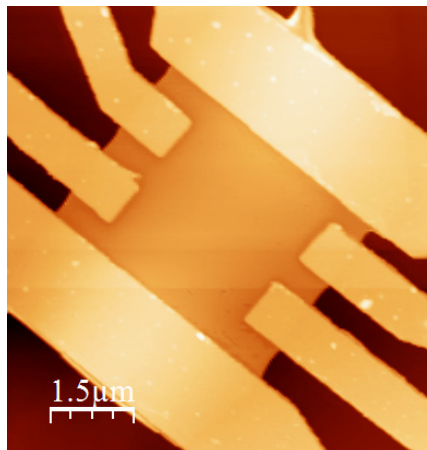


HF etching
-> critical pointing drying

SEM image of suspended graphene

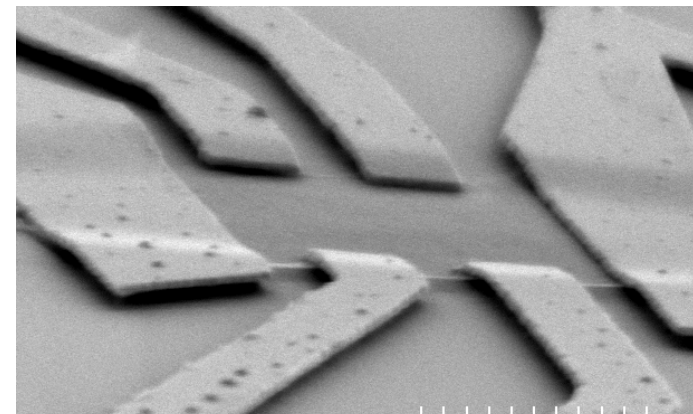
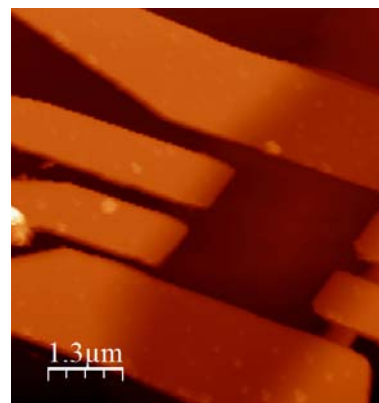


AFM image of suspended graphene



You should not apply to high gate voltage, otherwise...

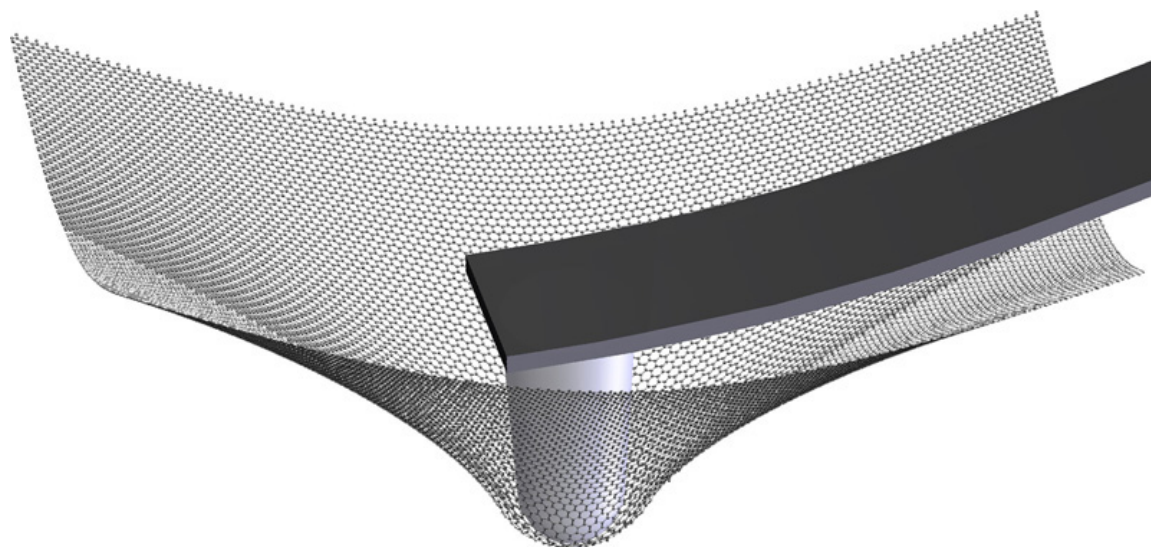
Collapsed graphene devices...



Measurement of the Elastic Properties and Intrinsic Strength of Monolayer Graphene

Changgu Lee,^{1,2} Xiaoding Wei,¹ Jeffrey W. Kysar,^{1,3} James Hone^{1,2,4*}

We measured the elastic properties and intrinsic breaking strength of free-standing monolayer graphene membranes by nanoindentation in an atomic force microscope. The force-displacement behavior is interpreted within a framework of nonlinear elastic stress-strain response, and yields second- and third-order elastic stiffnesses of 340 newtons per meter (N m^{-1}) and -690 N m^{-1} , respectively. The breaking strength is 42 N m^{-1} and represents the intrinsic strength of a defect-free sheet. These quantities correspond to a Young's modulus of $E = 1.0$ terapascals, third-order elastic stiffness of $D = -2.0$ terapascals, and intrinsic strength of $\sigma_{\text{int}} = 130$ gigapascals for bulk graphite. These experiments establish graphene as the strongest material ever measured, and show that atomically perfect nanoscale materials can be mechanically tested to deformations well beyond the linear regime.



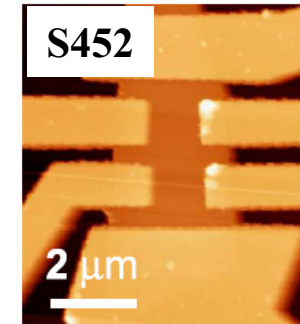
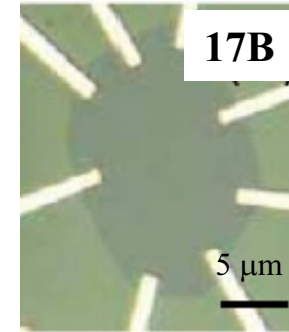
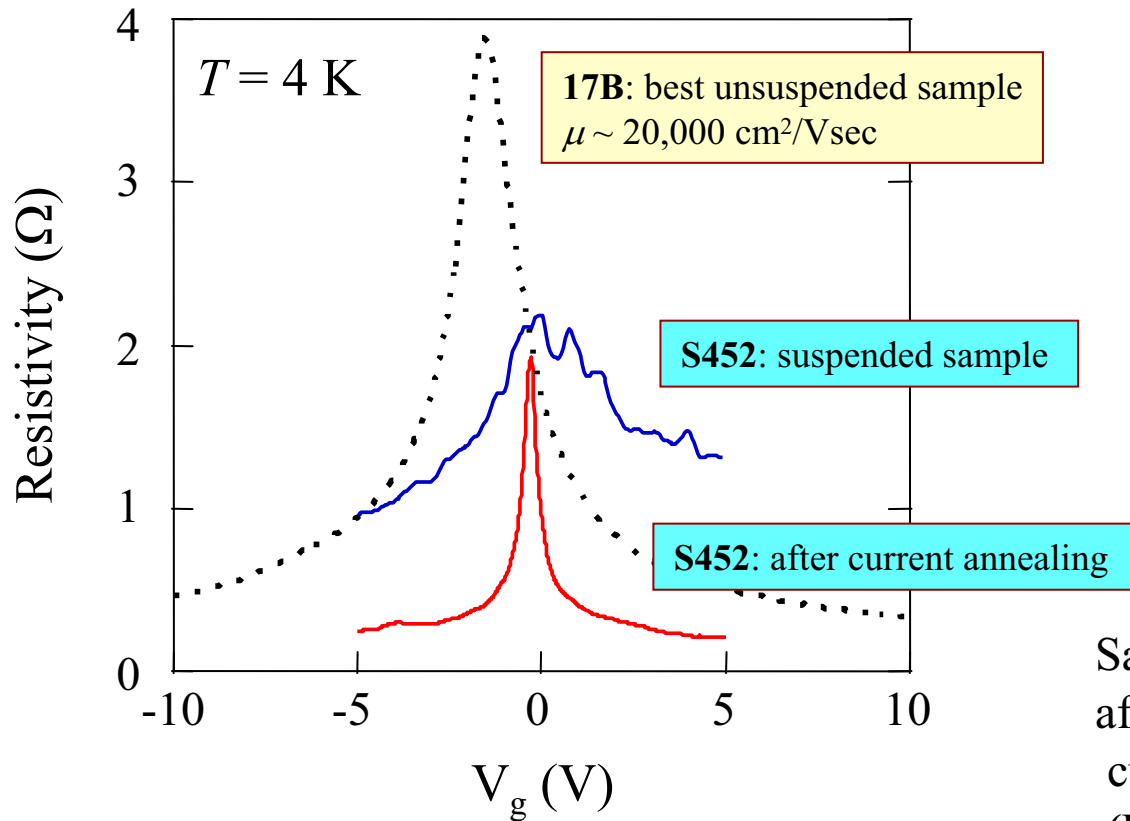
Young's modulus

~ 1 TPa

Critical Strain

~ 25 %

Cleaning Graphene Surfaces: Annealing



Sample quality can be improved after self-heating annealing:
current density $\sim 10^8$ A/cm²
(Developed by Bachtold et al, APL (2007))

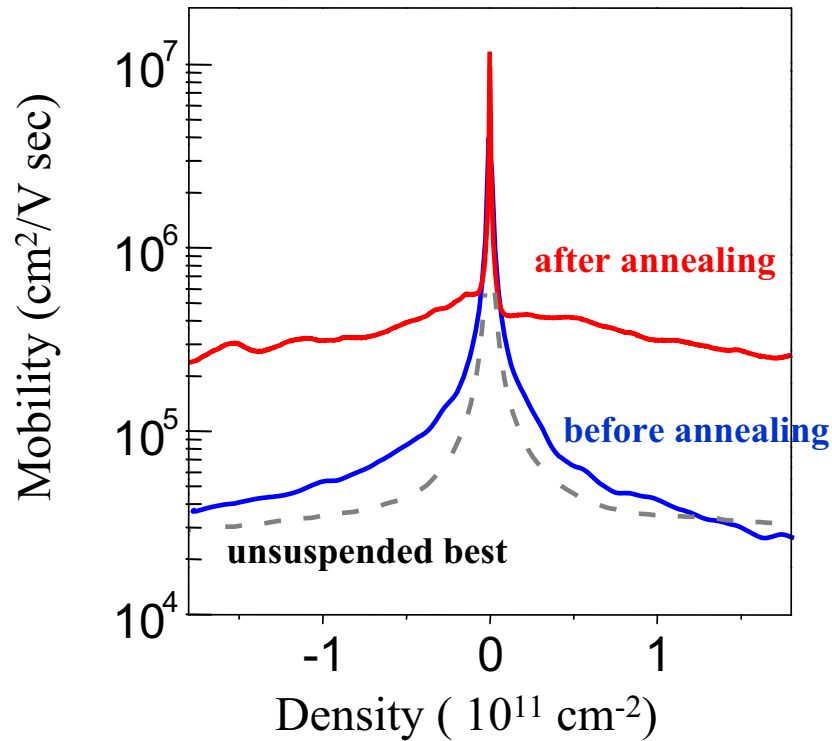
Unsuspended (300 nm SiO₂): $C_g = 115$ aF/ μm^2

Suspended (~ 150 nm SiO₂): $C_g \sim 50$ aF/ μm^2

Characteristics of Suspended Samples: Mobility

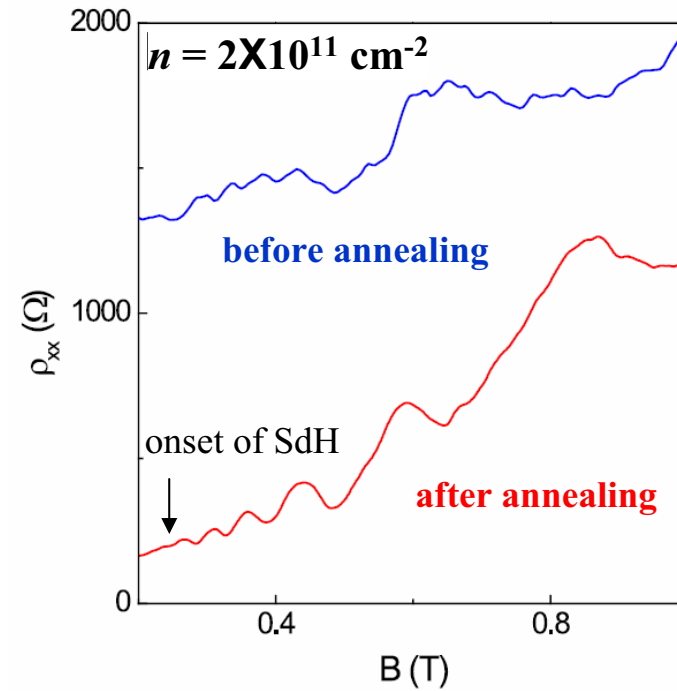
$$\mu = \sigma / en$$

Drude Mobility



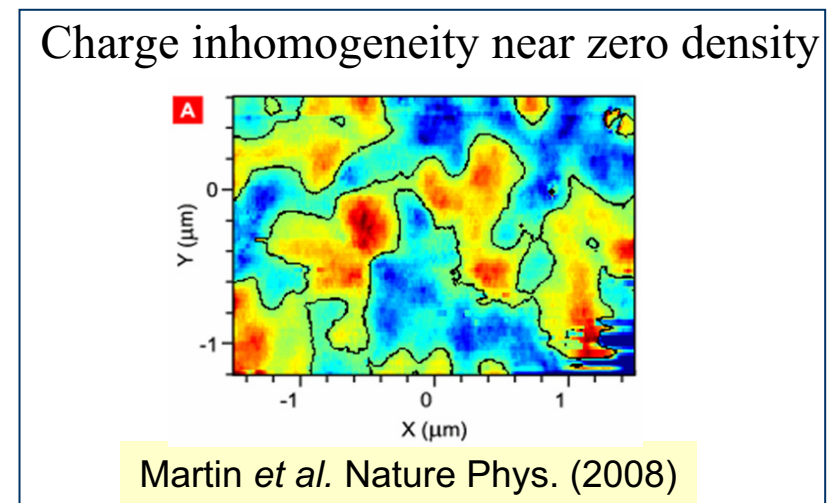
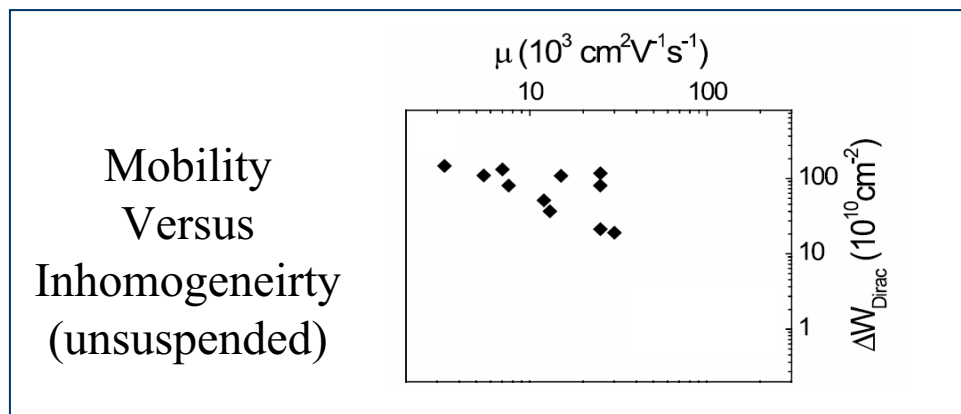
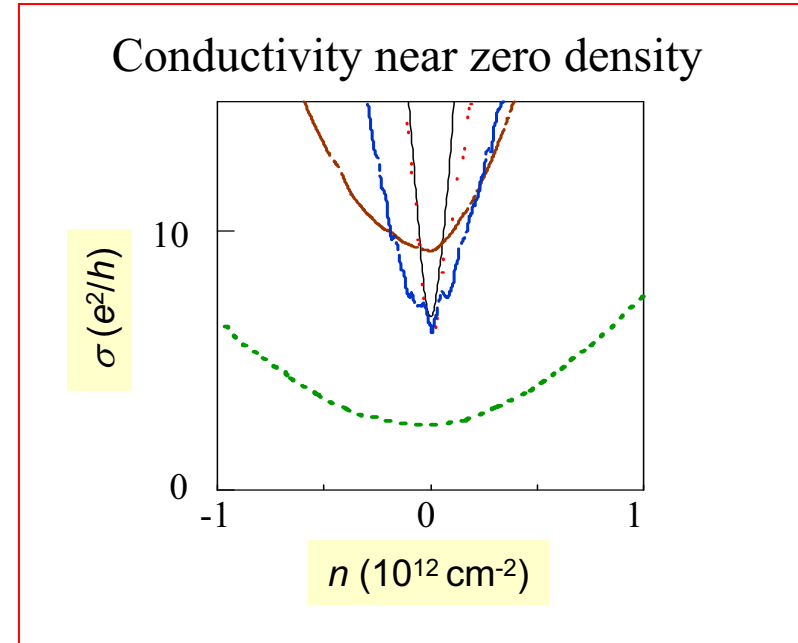
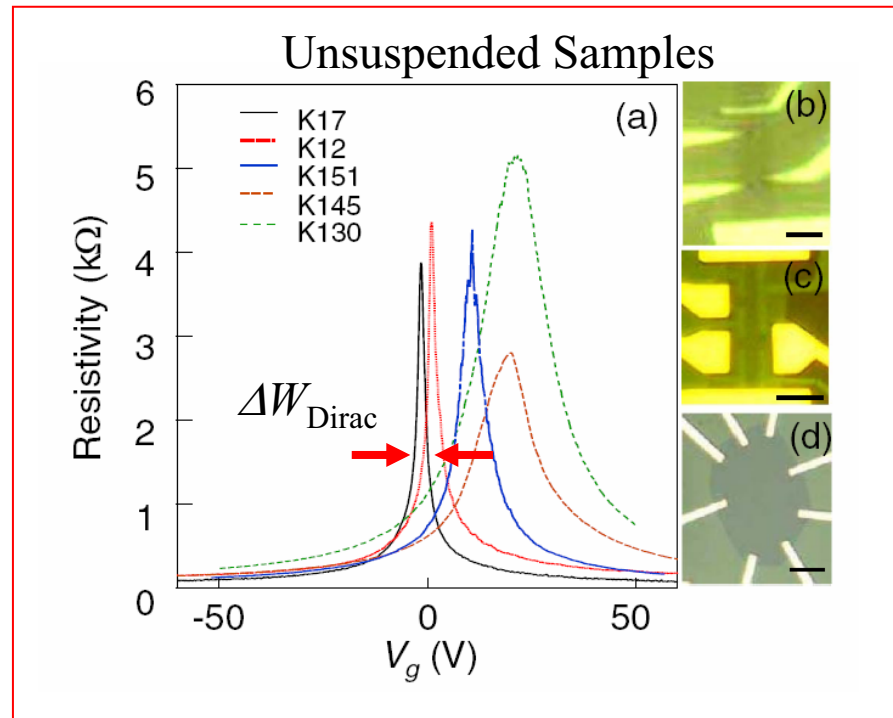
$$\mu \sim 250,000 \text{ cm}^2/\text{Vs} \\ @ 2.5 \times 10^{11} \text{ cm}^{-2}$$

SdH Oscillations

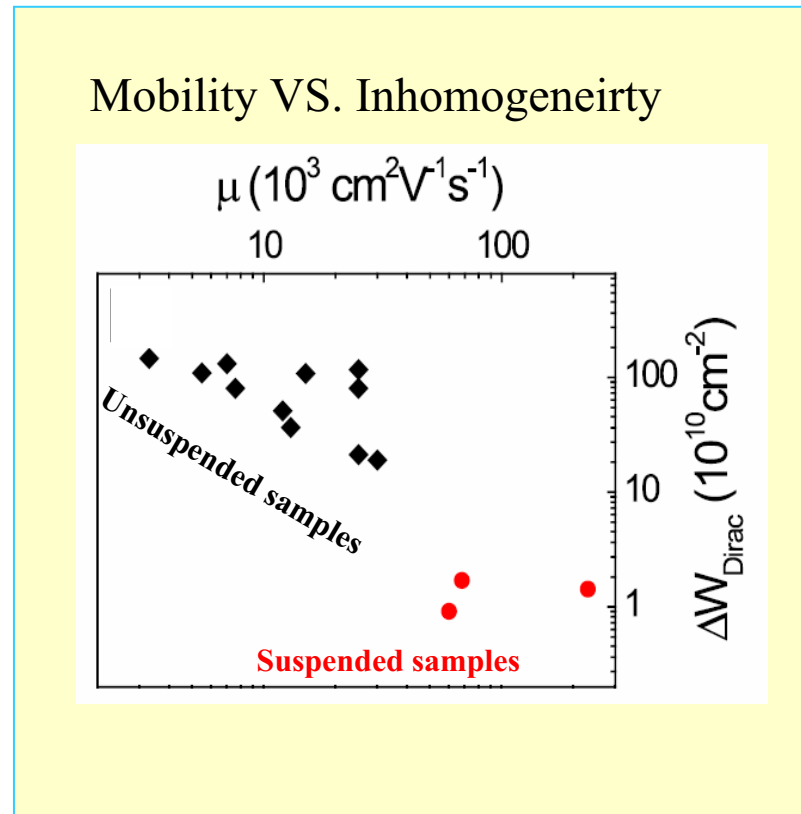
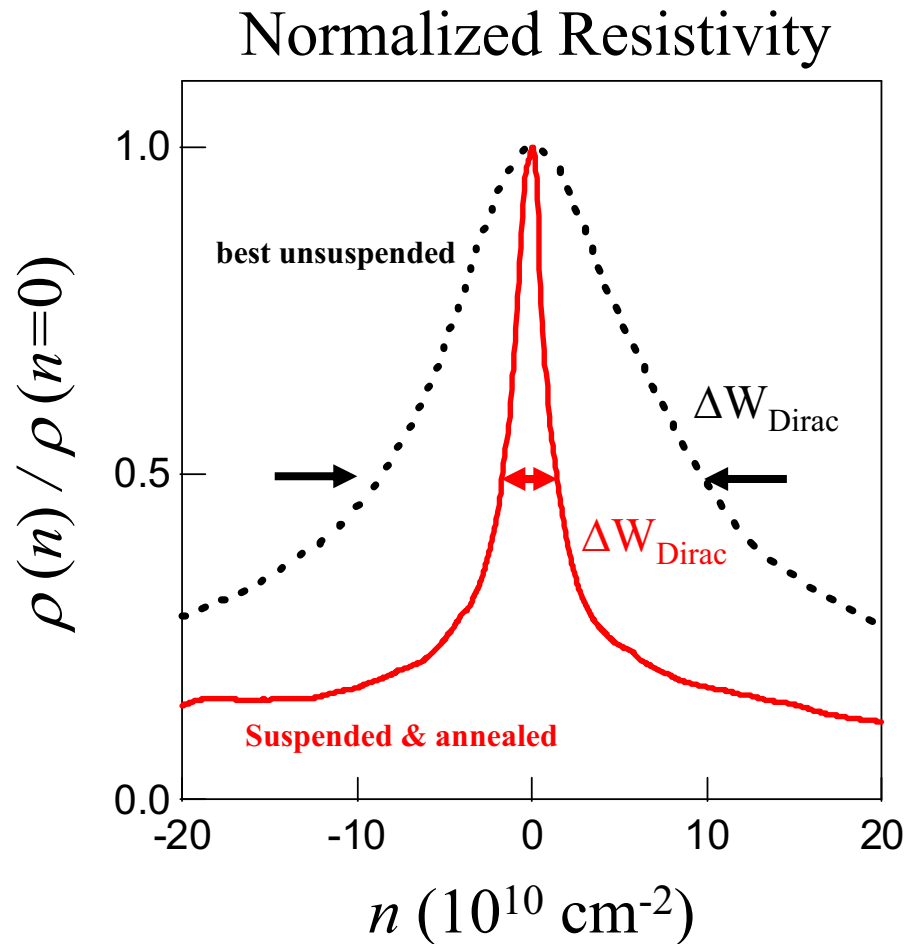


$$\mu > 1/B_{\text{SdH}} \sim 100,000 \text{ cm}^2/\text{Vs}$$

Charge Inhomogeneity near the Charge Neutrality Point: Unsususpended samples



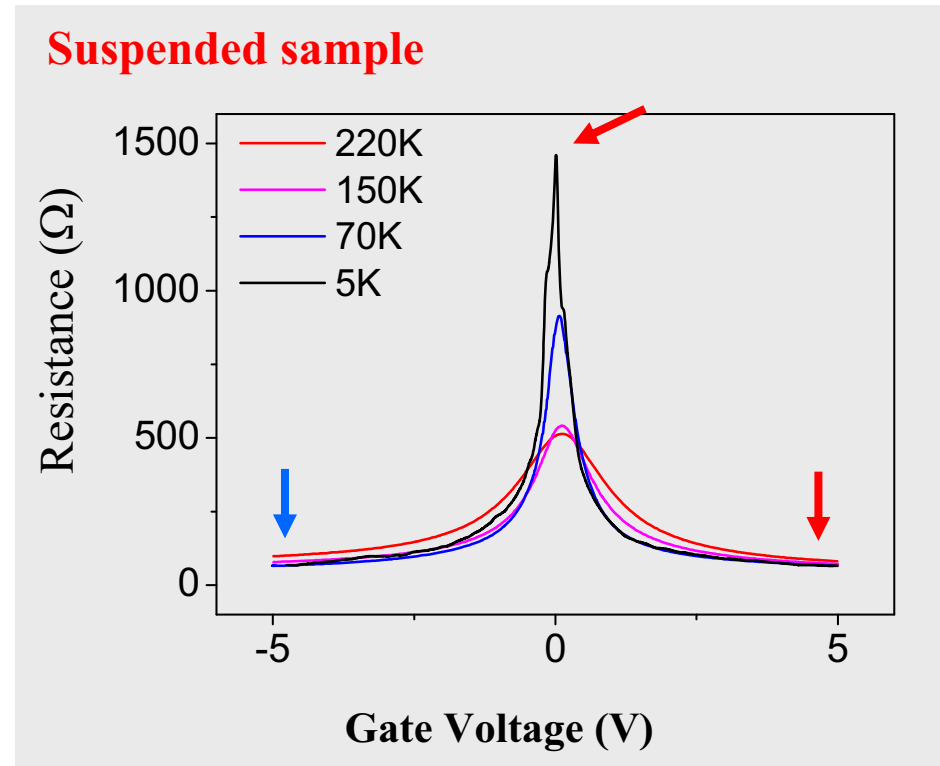
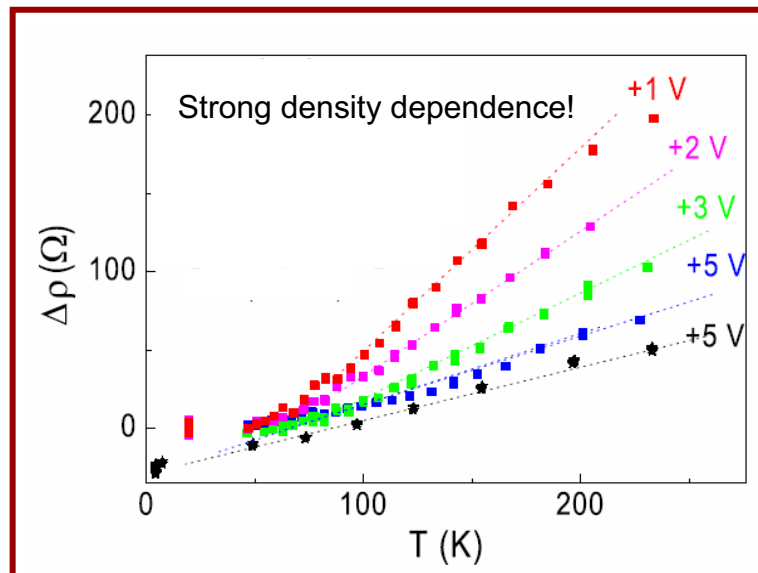
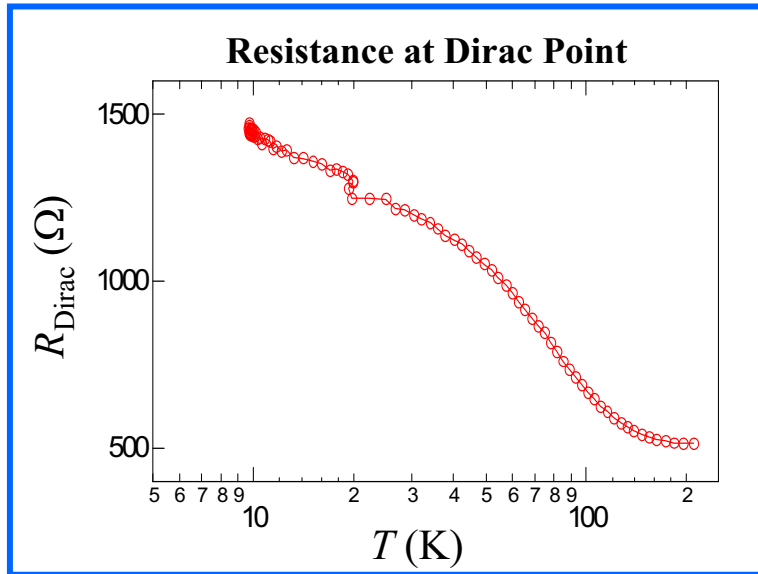
Toward High Mobility Samples



Density Inhomogeneity $< 10^{10} \text{ cm}^{-2}$ is possible!

Strong Temperature Dependence in Suspended Samples

K. Bolotin, K. Sikes, J. Hone, H. Stormer and P. Kim, PRL in press

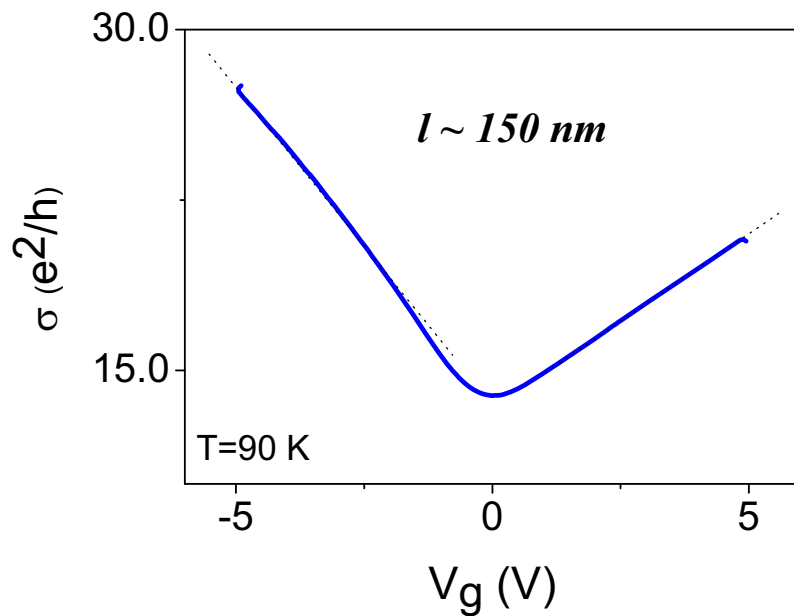


Theory estimates $dR/dT \sim 0.4 \Omega/K$, assuming acoustic phonons and electrons Vasko & Ryzhii PRB (2007)

Room Temperature mobility > 100,000 cm^2/Vsec

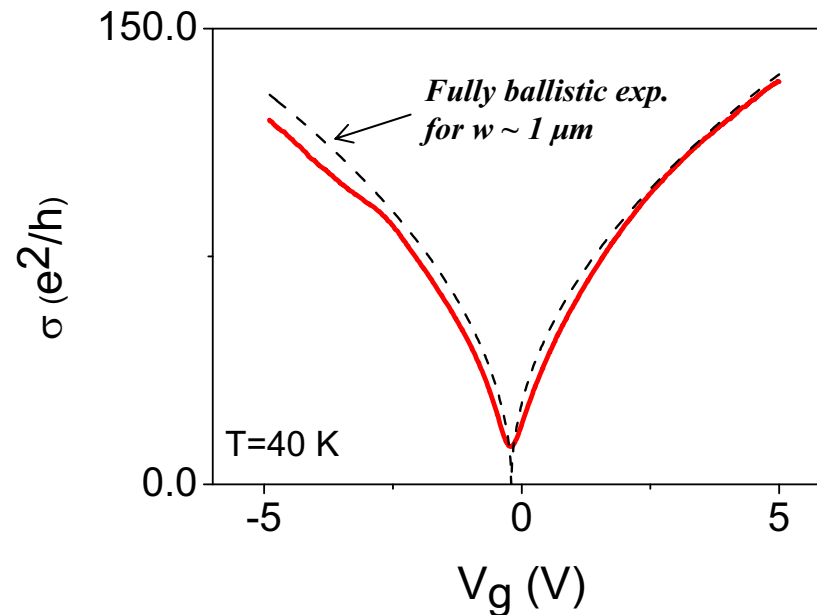
Ballistic Transport in Suspended Samples

before current annealing



- Mean free path $<$ sample size
- $\sigma(n) \sim n$, Coulomb scatterers dominated diffusive transport

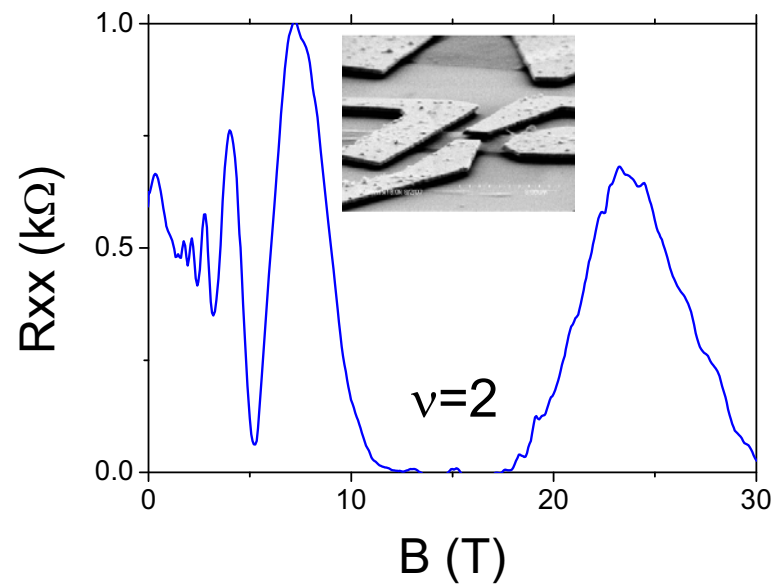
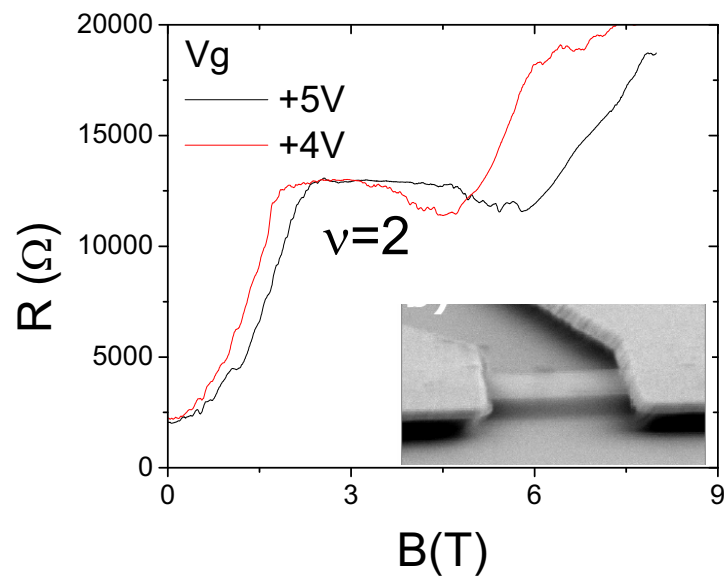
after current annealing



- Mean free path \sim sample size
- Conductance $\sim 4e^2/h \times (\text{number of modes})$
- $\sigma(n) \sim \sqrt{n}$, as expected for ballistic conductor
- Near ballistic transport at high density

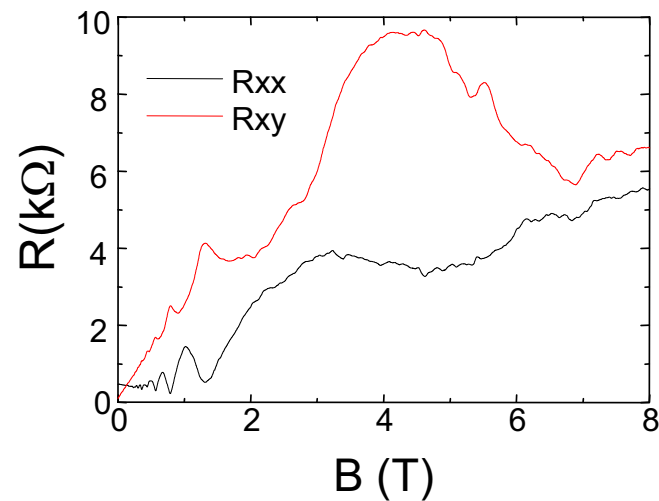
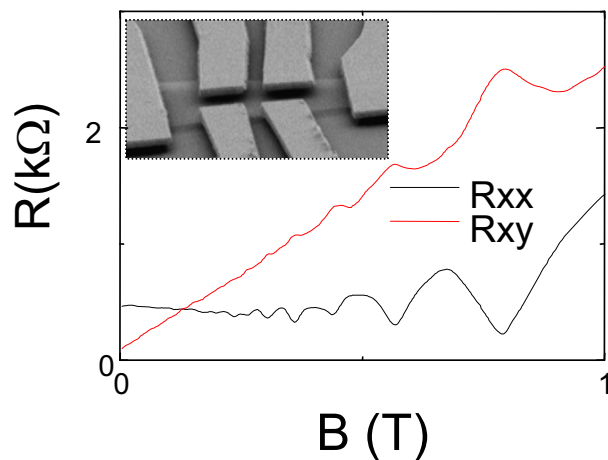
Similar results reported Du *et al.*, Nat. Nanotech (2008)

QHE in suspended sample



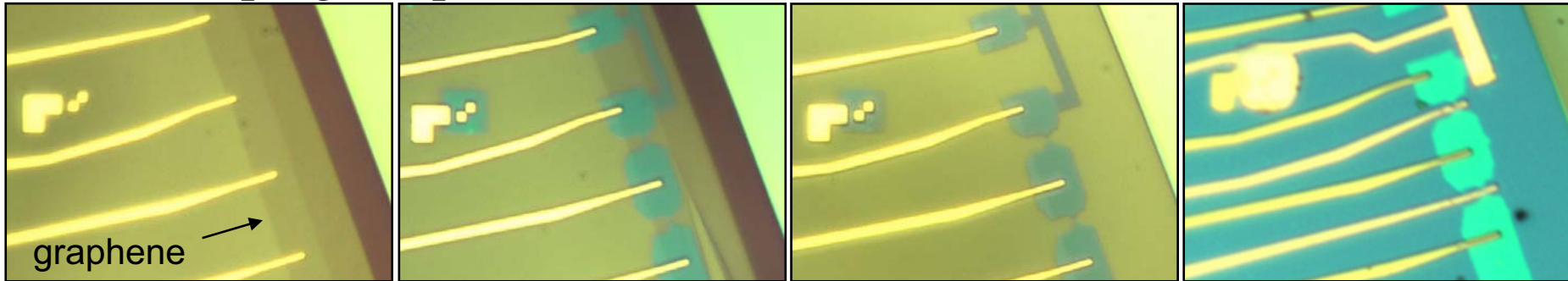
Typical samples:

Low Field



Graphene Device Fabrication

■ Developing Graphene Nanostructure Fabrication Process



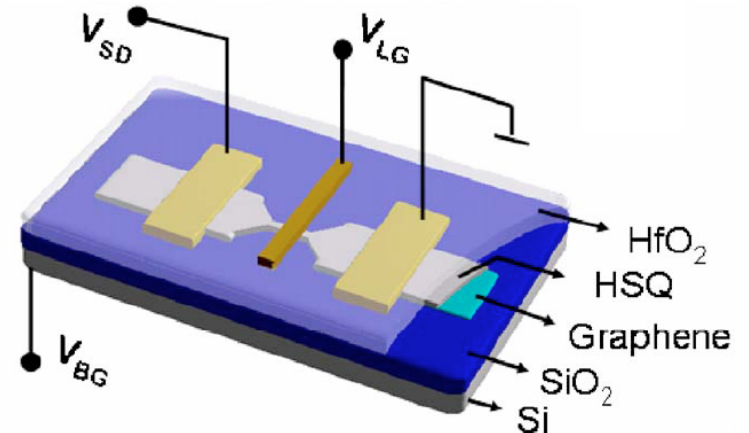
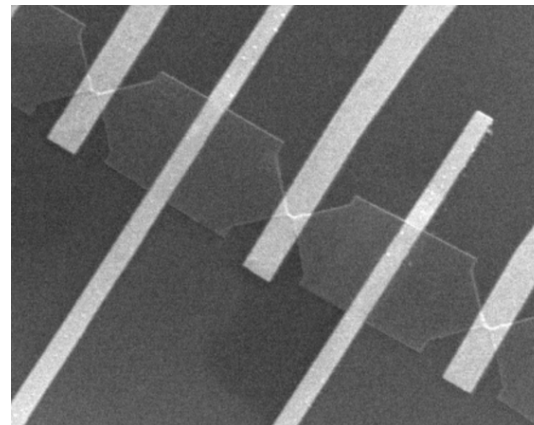
Contacts:
PMMA
EBL
Evaporation

Graphene patterning:
HSQ
EBL
Development

Graphene etching:
Oxygen plasma

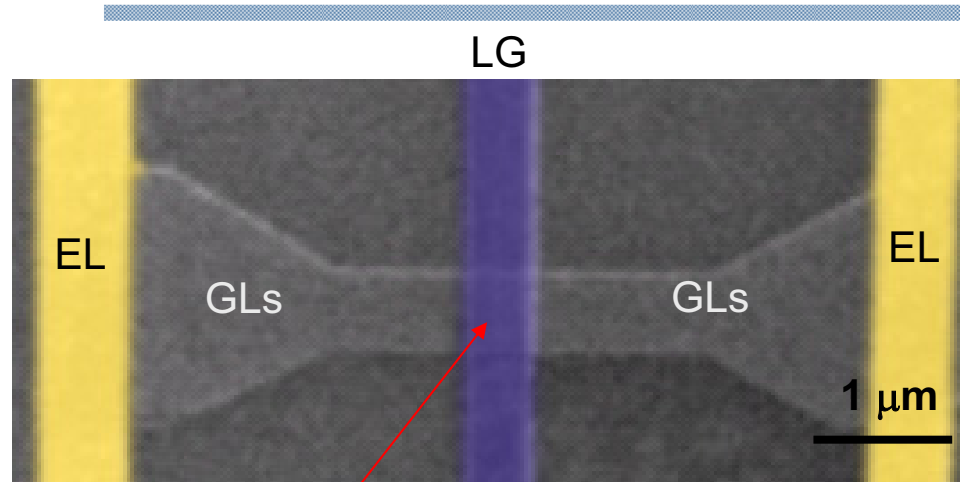
Local gates:
ALD HfO₂
EBL
Evaporation

■ Graphene device structure with local gate control

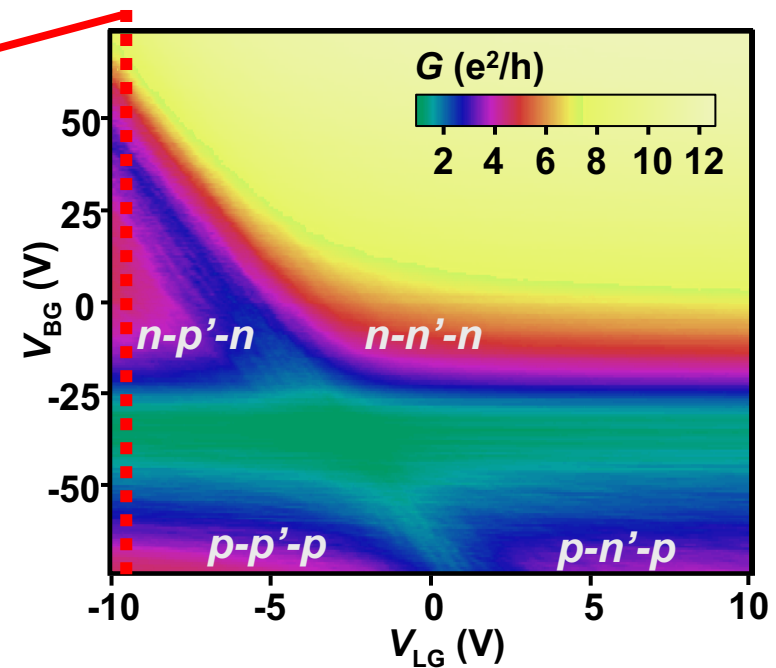
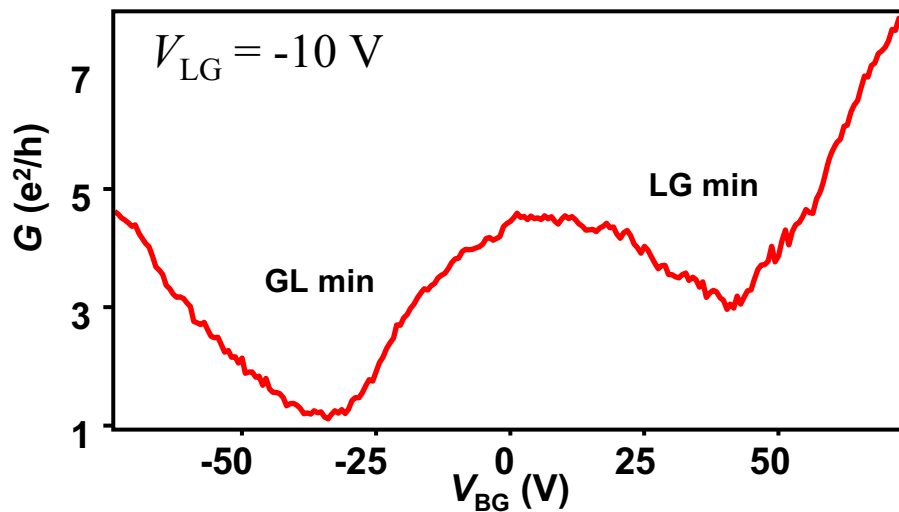
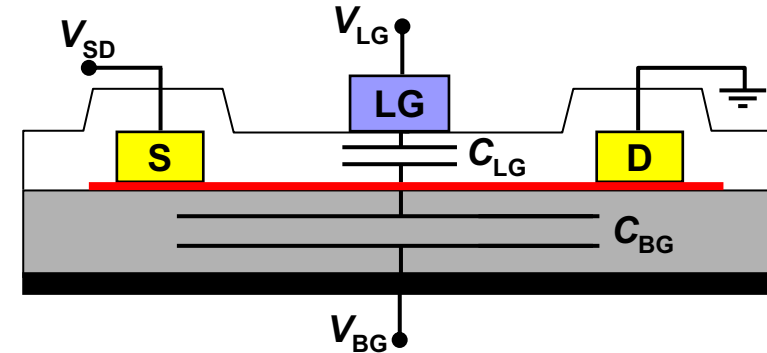


Oezylmaz, Jarrilo-Herrero and Kim APL (2007)

Graphene bipolar heterojunctions

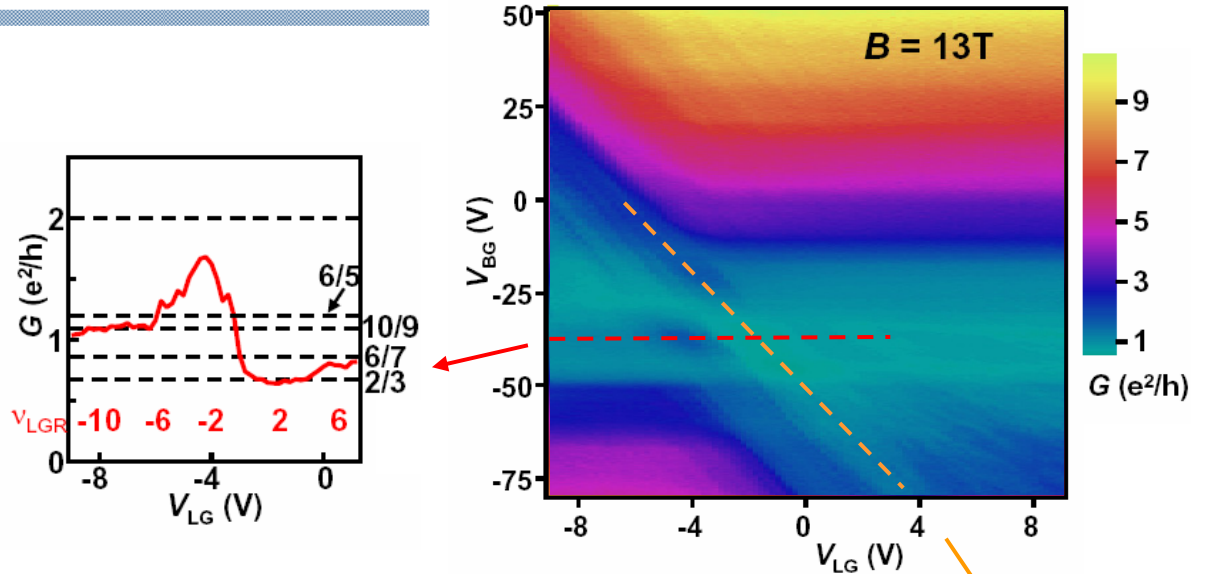
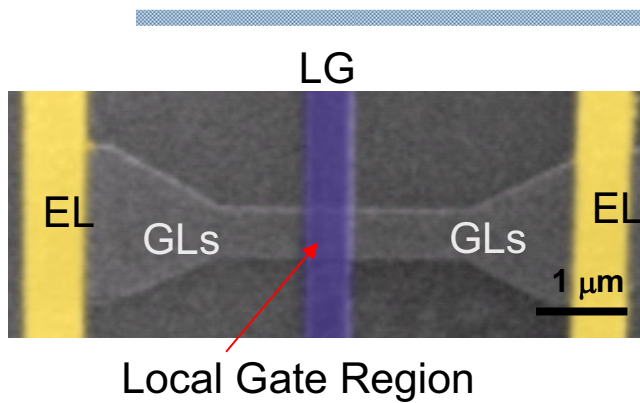


Local Gate Region

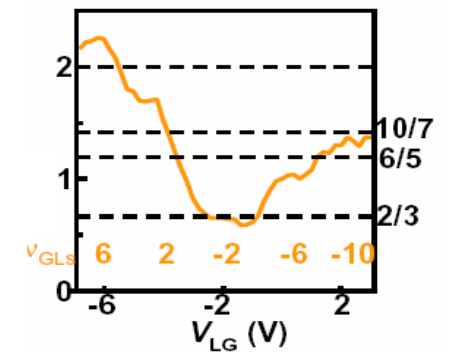
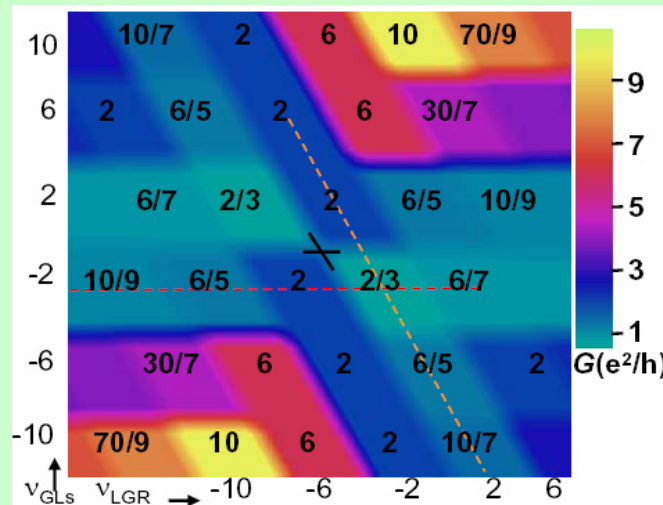
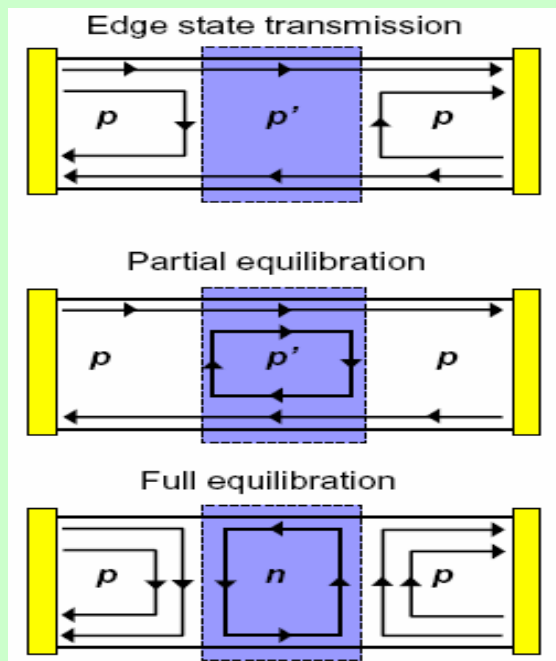


Oezylmaz, Jarrilo-Herrero and Kim PRL (2007)
 Related work by Huard *et al.* PRL (2007)

Graphene Quantum Hall Edge State Conduction



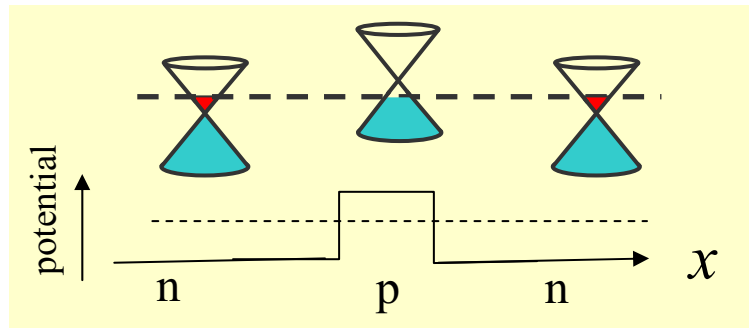
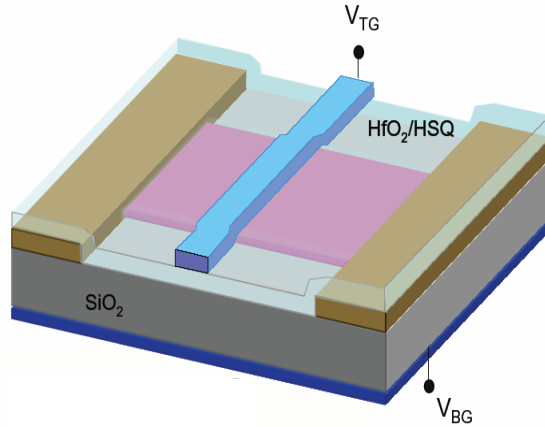
simple model (following Haug *et al*)



Oezylmaz, et al., PRL (2007) See also Related work by Williams *et al.* Science (2007)

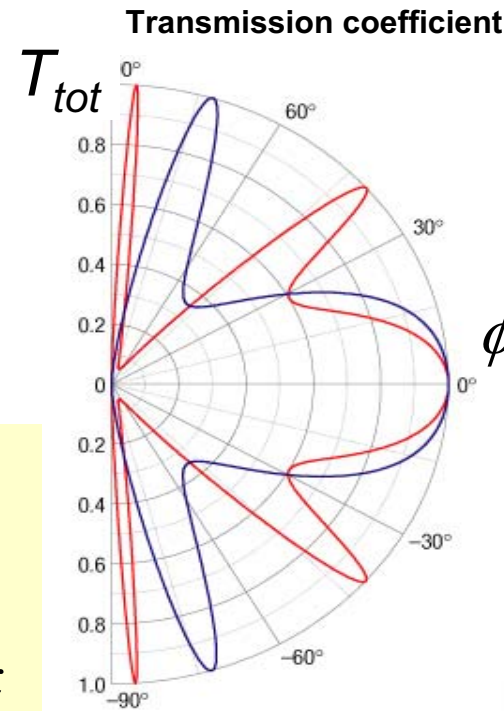
Ballistic Quantum Transport in Graphene Heterojunction

Graphene NPN junctions



Klein Tunneling

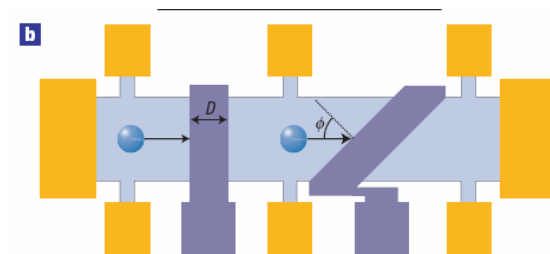
Novoselov et al, Nat. Phys (2006)



Collimation:
(resonance)

$$k_{2x} = \pi n/L$$

Perfect transmission



$$n_p = 1 \times 10^{12} \text{ cm}^{-2}$$

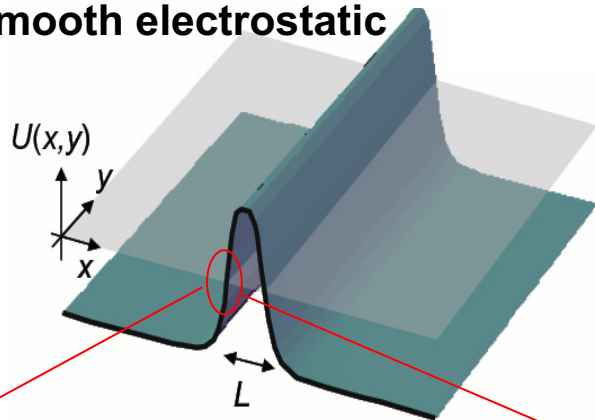
$$n_p = 3 \times 10^{12} \text{ cm}^{-2}$$

$$n_n = 0.5 \times 10^{12} \text{ cm}^{-2}$$

Realistic Graphene Heterojunction

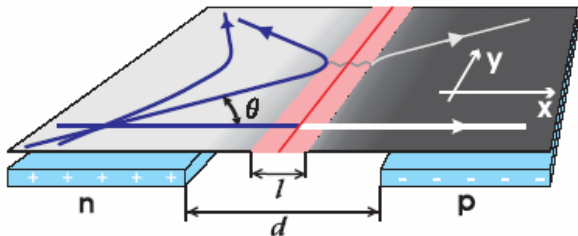
Mean free path < 100 nm

Smooth electrostatic



$n \sim 10^{12} \text{ cm}^{-2}$
 $\lambda_F \sim 30 \text{ nm}$
 $d_{\text{dielectric}} \sim 20 \text{ nm}$
 $L \sim 100 \text{ nm}$

Tunneling through smooth pn junction

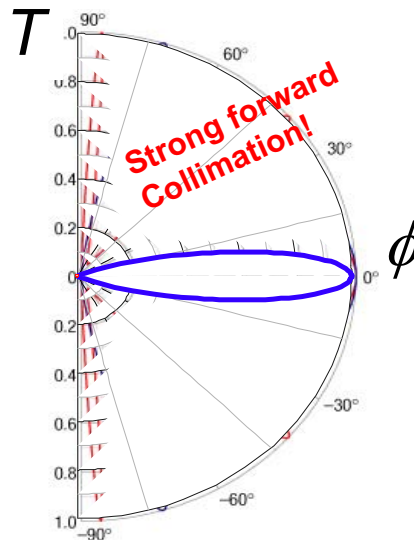


Cheianov and Fal'ko (2006)
 Zhang and Fogler (2008)

Tunneling through Classically Forbidden regime

$$T(\theta) = e^{-\pi(k_F d) \sin^2 \theta}$$

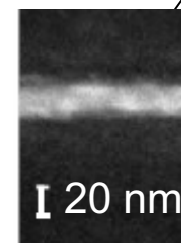
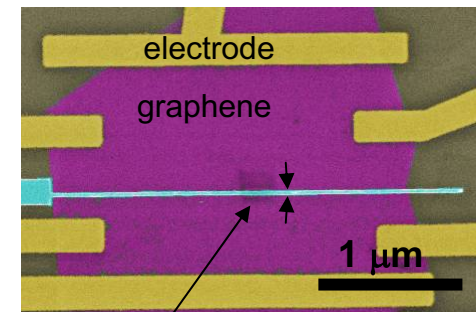
transmission



Requirements for Ballistic pn junctions

- Long Mean free path
 -> Ballistic conduction
- Large electric field ->
 Small d ->
 promote resonance

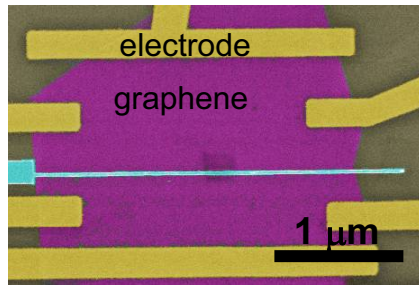
SEM image of device



Mean free path
 $\sim 50 \text{ nm}$

Transport Ballistic Graphene Heterojunction

Young and Kim (2008)



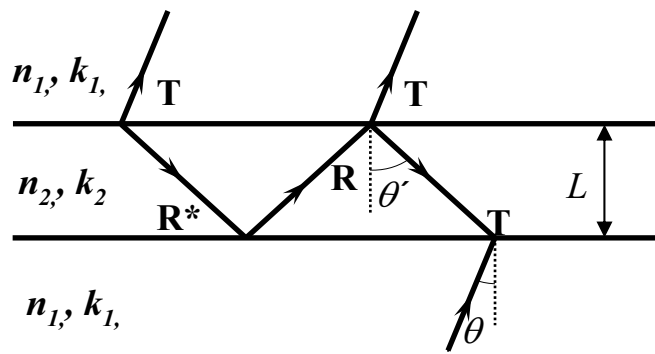
PN junction resistance

$$R = (\pi/2)(h/e^2)\sqrt{\hbar v / e|F_{pn}|}$$

Cheianov and Fal'ko (2006)

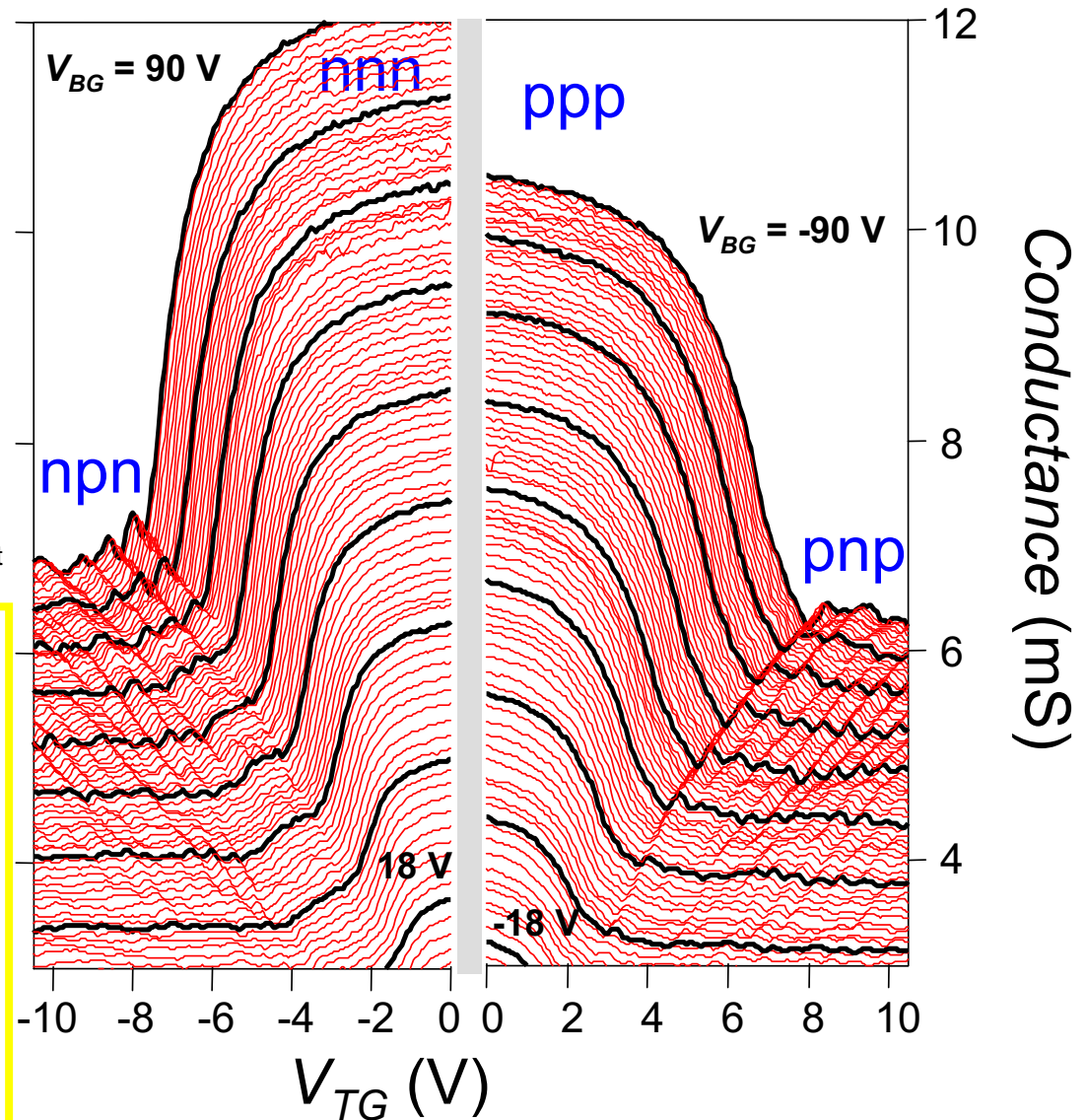
See also Shavchenko et al and Goldhaber-Gordon's recent preprint

Conductance Oscillation: Fabry-Perot



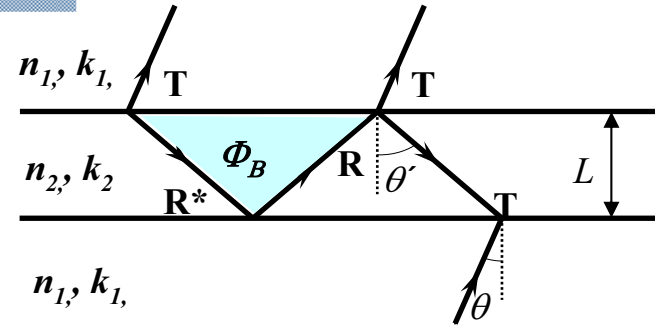
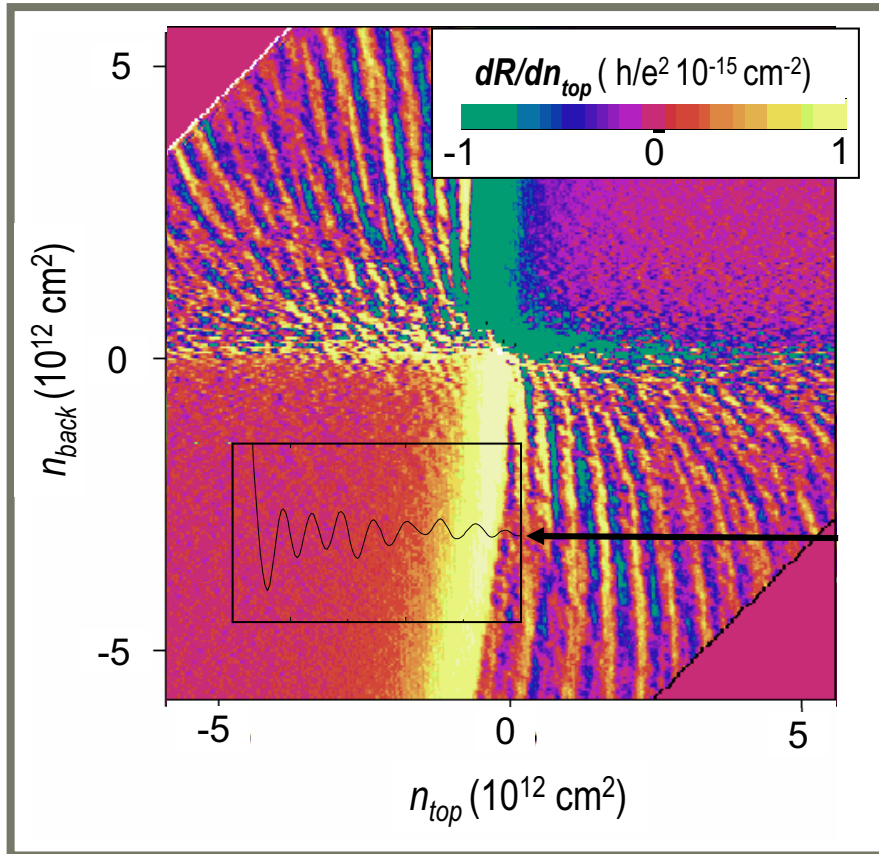
$$k_1/k_2 = \sin\theta' / \sin\theta$$

$$\Delta\phi = 2L / \cos\theta'$$



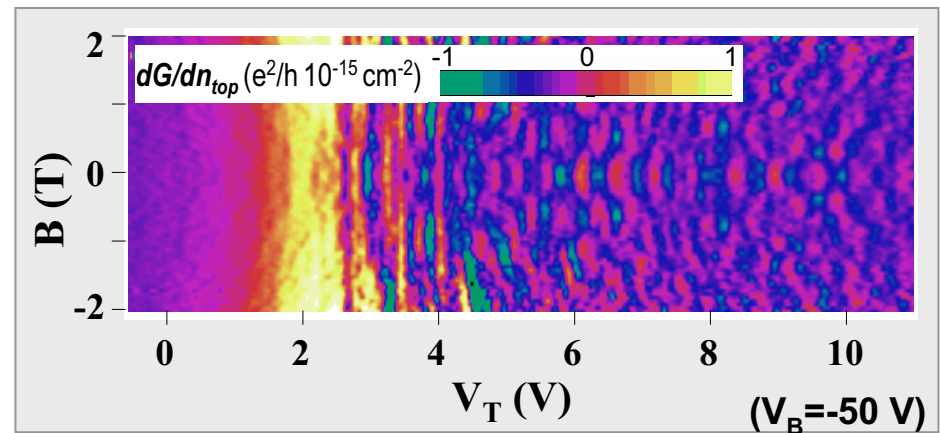
Quantum Oscillations in Ballistic Graphene Heterojunction

Resistance Oscillations



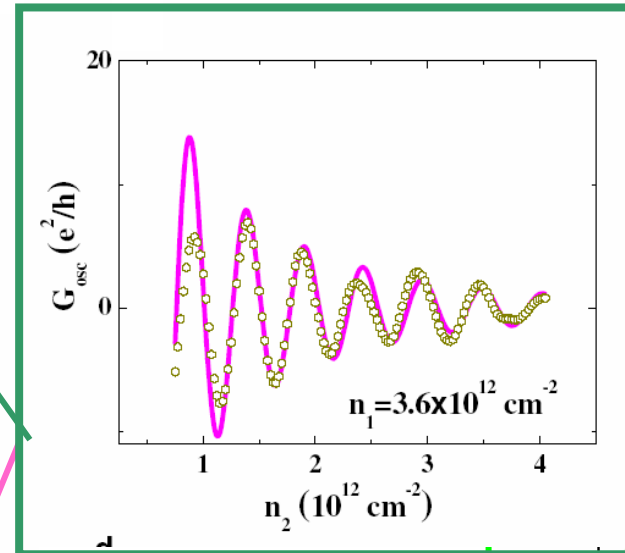
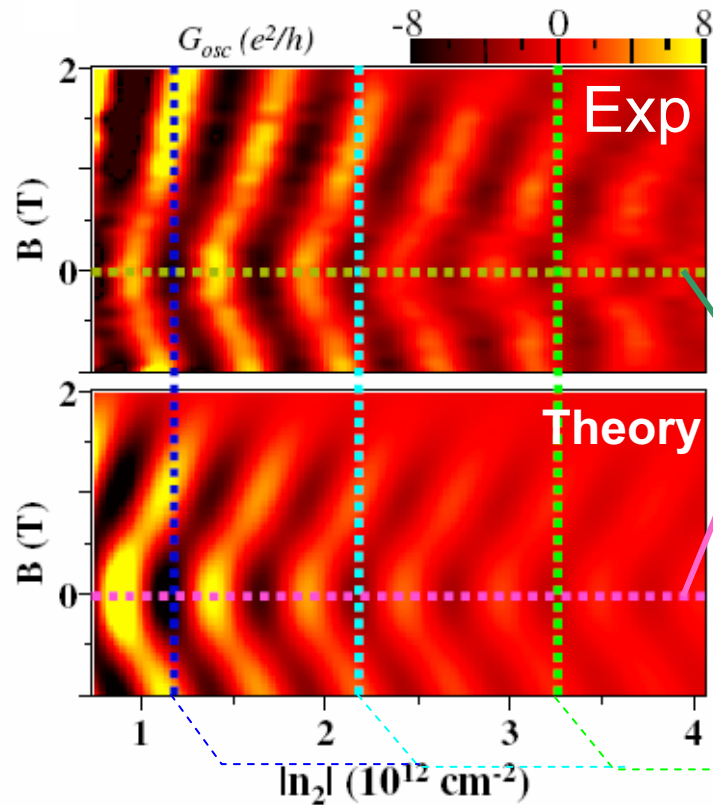
Magnetoresistance Oscillations ($B \neq 0$)

Aharonov-Bohm phase: $\Phi_B = B L^2 \sin \theta' / \cos \theta'$



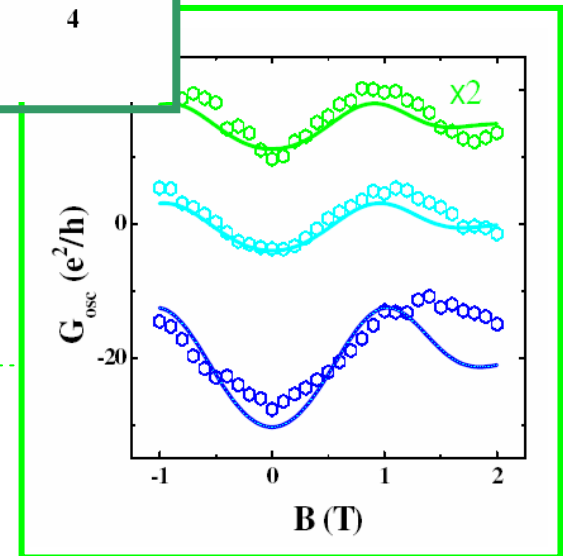
$$G_{osc} = e^{-2L/l_{LGR}} \frac{8e^2 W k_1}{h} \int \frac{d\theta}{2\pi} |T(\theta)|^2 |R(\theta)|^2 \cos \left(2Lk_2(1 + \theta^2/2) \right)$$

Resonant Magneto-Oscillations in Graphene Heterojunctions



- Ballistic pn junction
- Collimation

See also Shytoy et al.,
[arXiv:0808.0488](https://arxiv.org/abs/0808.0488)

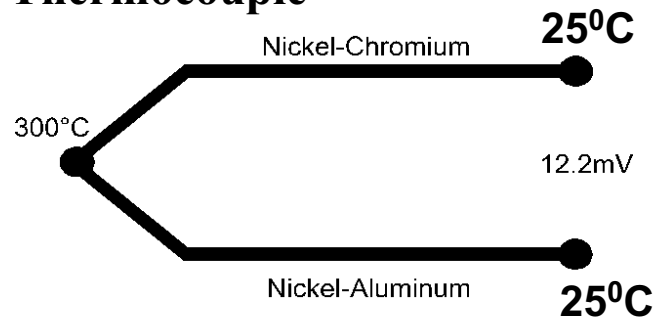


Two fitting parameters: $l_{LGR} = 27$ nm; $l_{GL} = 50$ nm

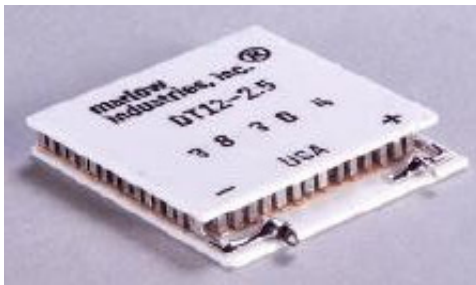
$$G_{osc} = e^{-2L/l_{LGR}} \frac{8e^2 W k_1}{h} \int \frac{d\theta}{2\pi} |T(\theta)|^2 |R(\theta)|^2 \cos \left(2Lk_2(1 + \theta^2/2) + \frac{eBL(L + l_{GL})}{\hbar} \theta \right)$$

Thermo Electricity

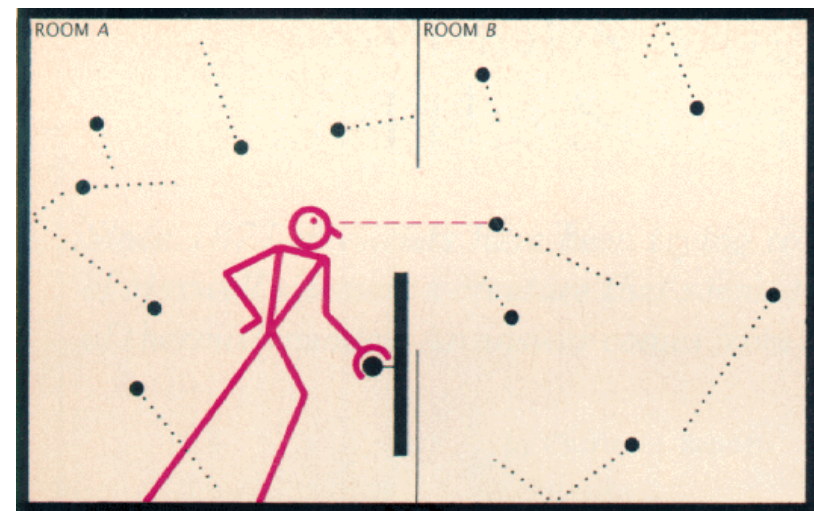
Thermocouple



- Thermoelectric (Peltier) cooler:



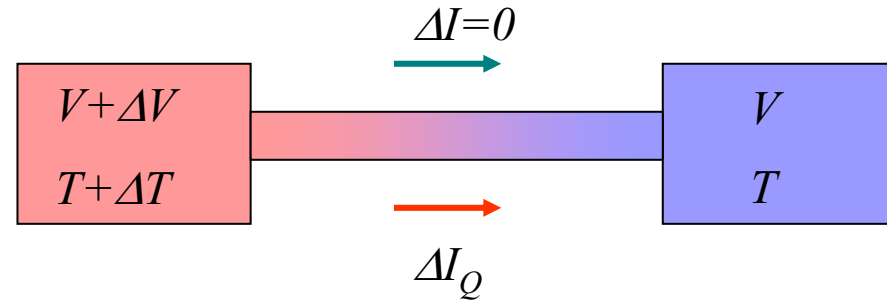
Entropy Flow with Charge



Maxwell's demon

Thermopower and Mott Formula

Thermoelectric Power (TEP)
 = Thermopower
 = Seebeck Coefficient



$$S = - \left(\frac{\Delta V}{\Delta T} \right)_{\Delta I = 0}$$

Microscopic theory (Boltzman transport theory, and etc.)

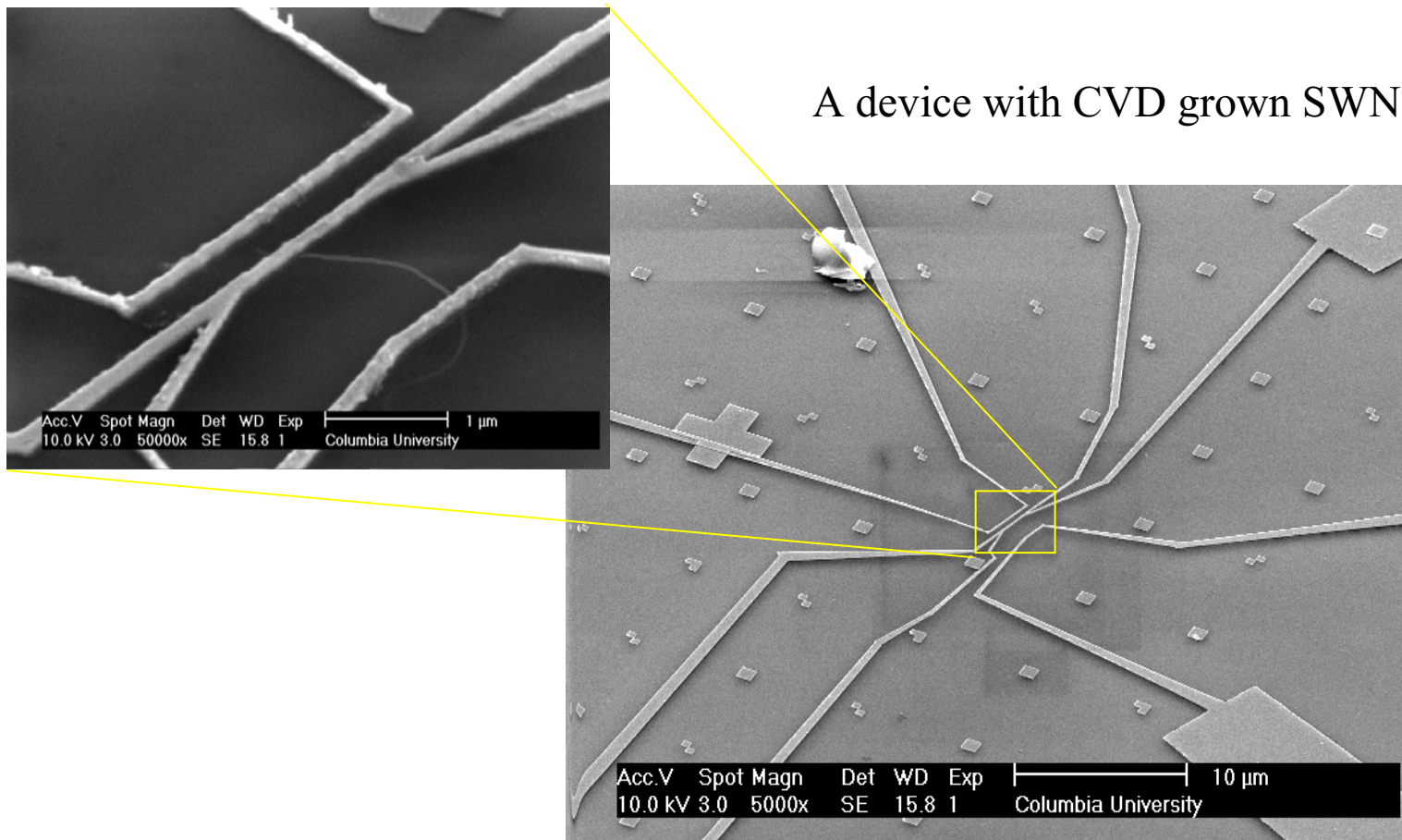
Mott's formula

$$S_d = \frac{-\pi^2 k_B^2 T}{3|e|} \frac{1}{\sigma} \frac{d\sigma}{dE} \Big|_{E_f}$$

σ : electric conductance

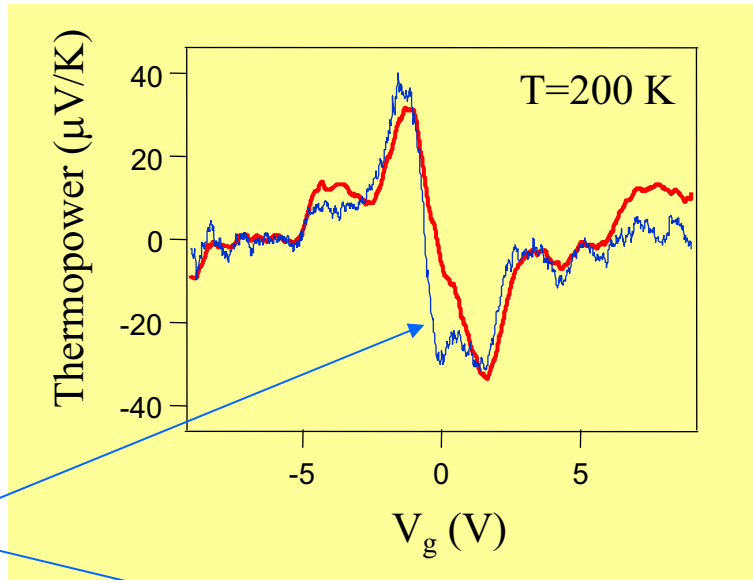
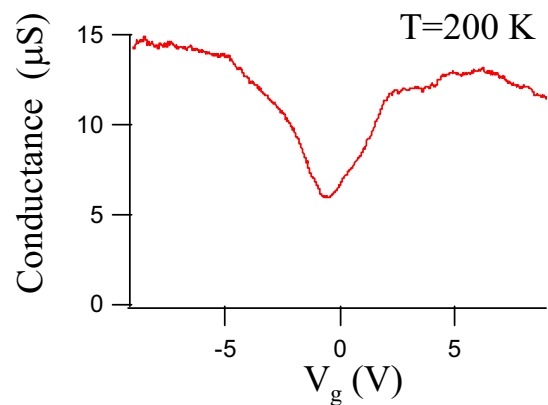
Mesososcopic Thermopower measurement of Nanotube

Microfabricated devices for electric and thermal transport

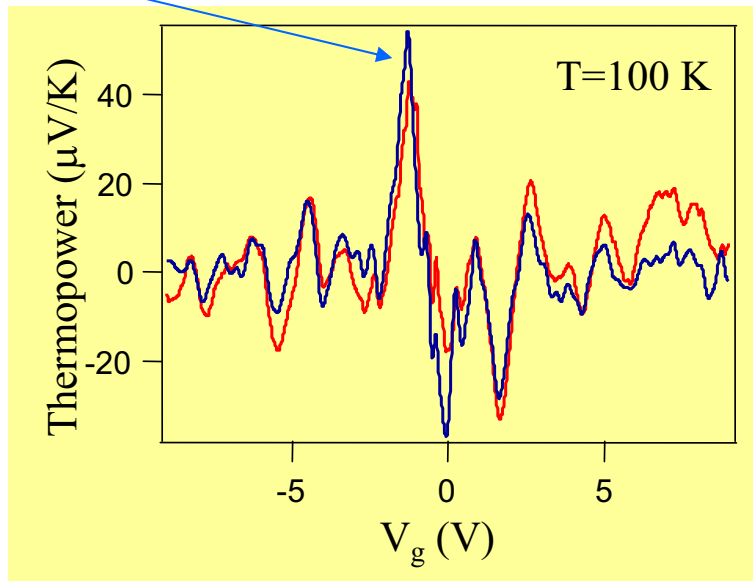
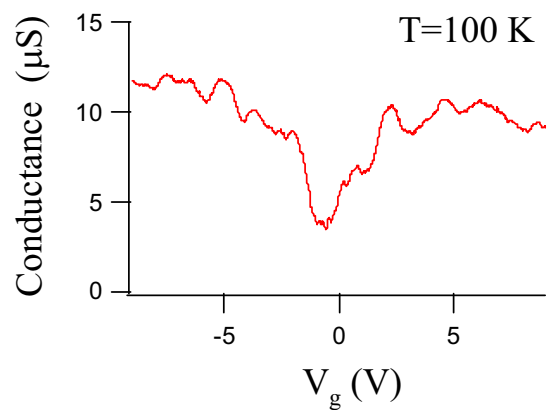


J. P. Small, K. Peretz, and P. Kim, PRL (2003)

Metallic Nanotube: Mott Formula

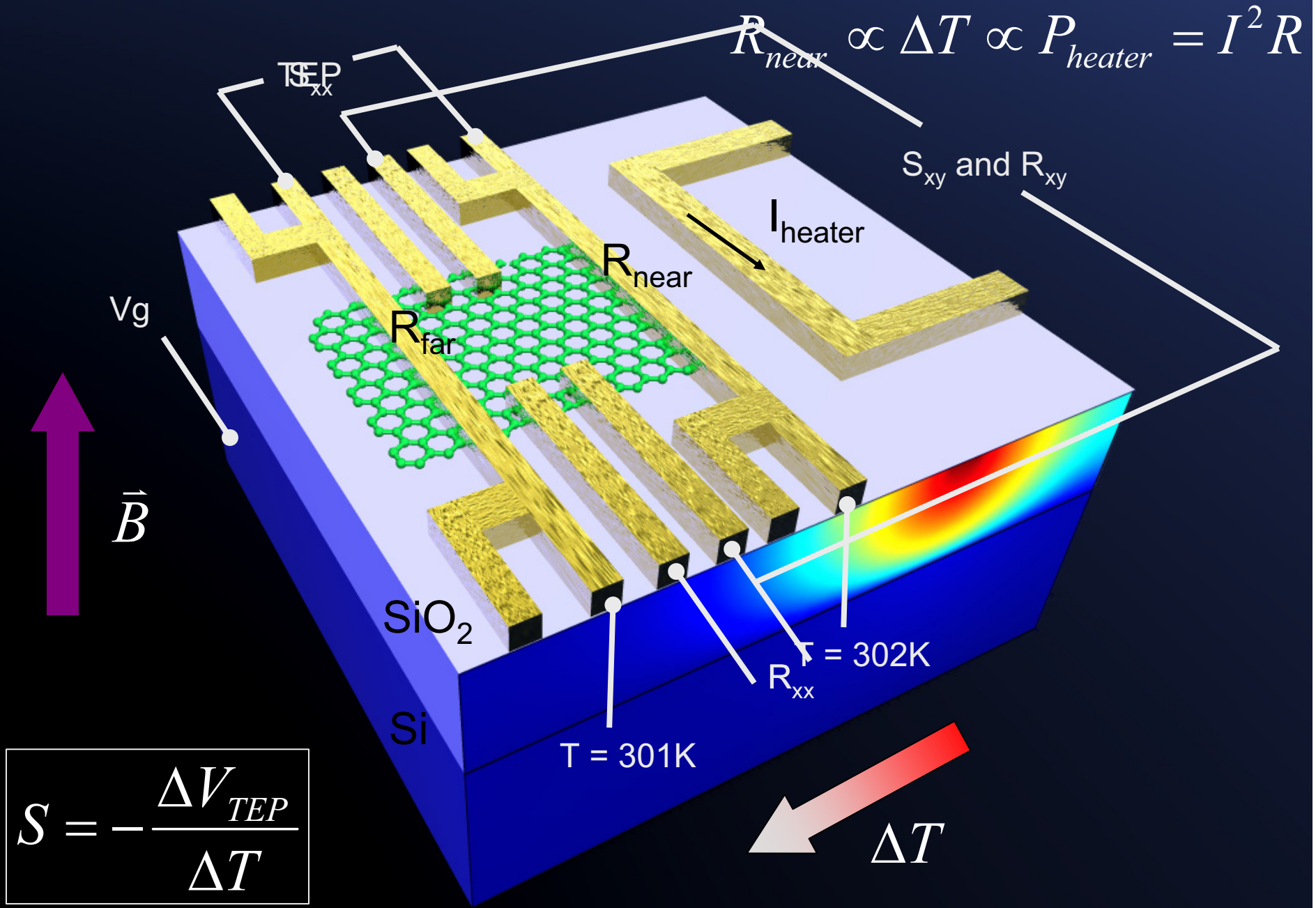


$$S_{Mott} = \frac{-\pi^2 k_B^2 T}{3|e|} \frac{1}{G} \left. \frac{dG}{dE} \right|_{E_f}$$



$$\Delta E_F = \frac{C_g}{DOS(E)} V_g$$

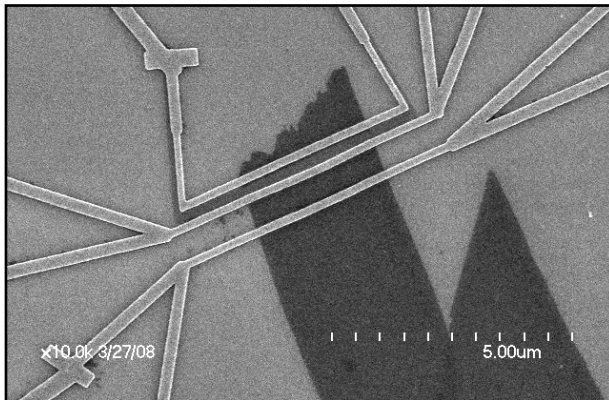
Device Diagram



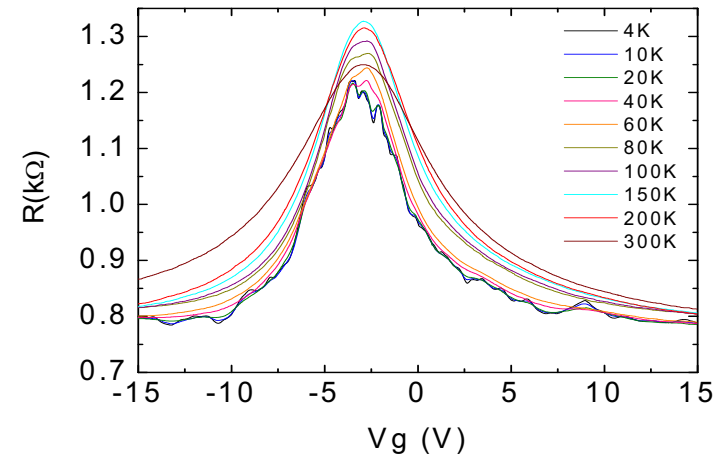
$$S = - \frac{\Delta V_{TEP}}{\Delta T}$$

Conductance and TEP (ThermoElectric power) in Graphene

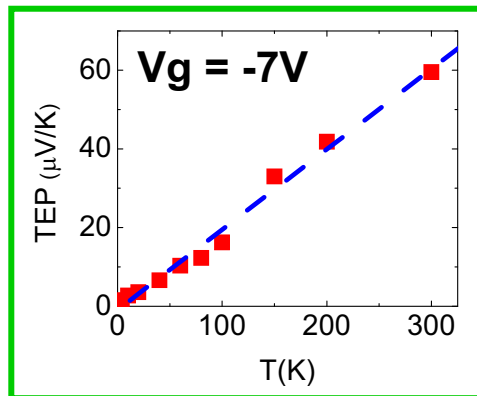
Graphene TEP device (SEM image)



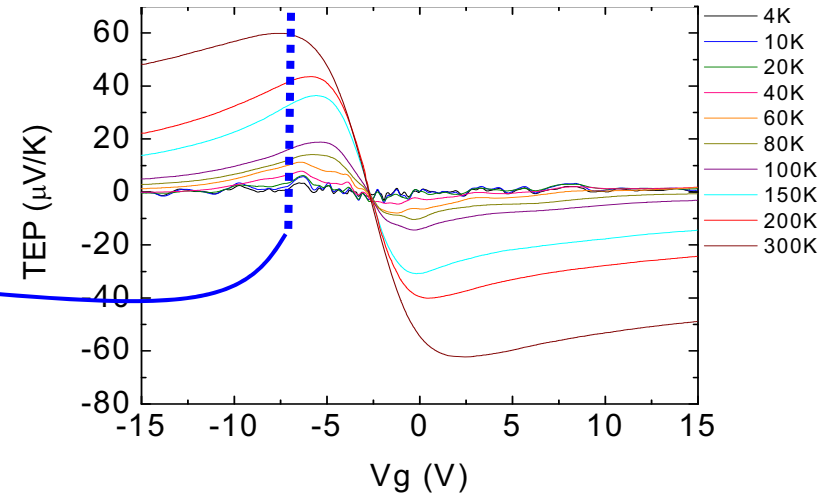
Temperature dependence of Resistance



Linear
T-dependence



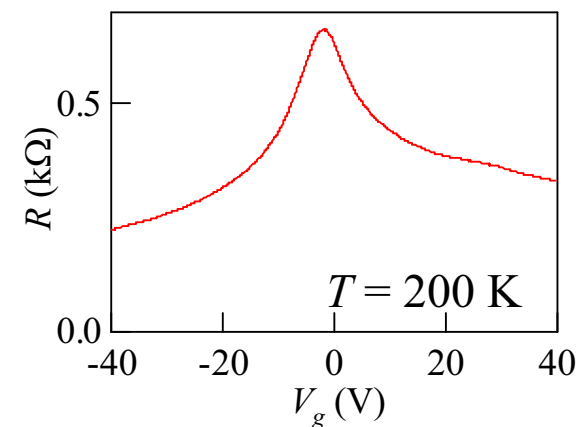
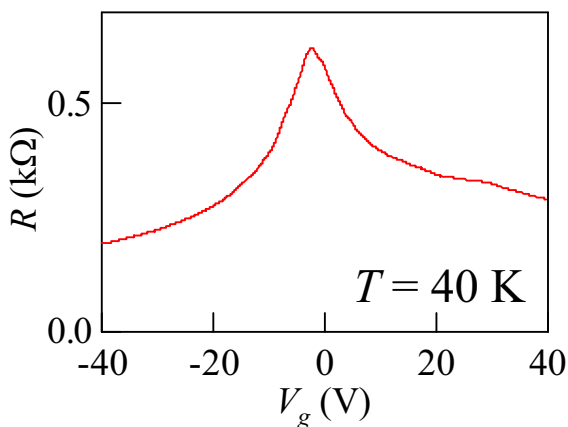
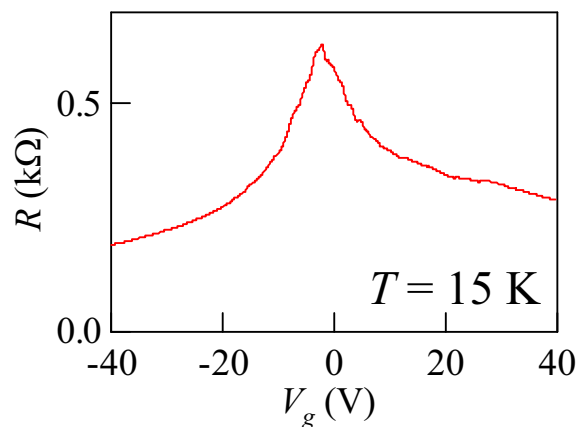
Temperature dependence of TEP



Mott relation :

$$S_{Mott} = \frac{\pi^2 k_B^2 T}{3|e|} \left. \frac{d \ln R}{dE} \right|_{E_f}$$

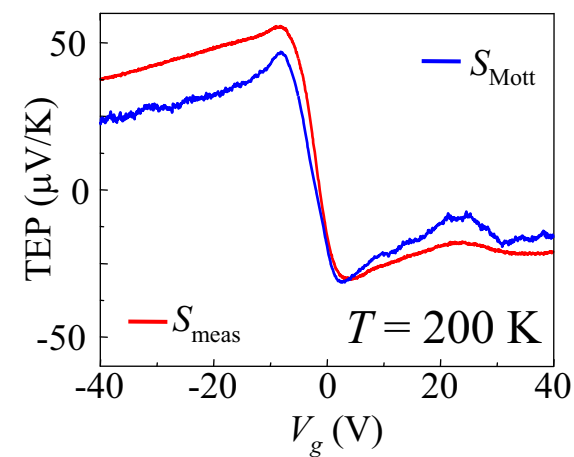
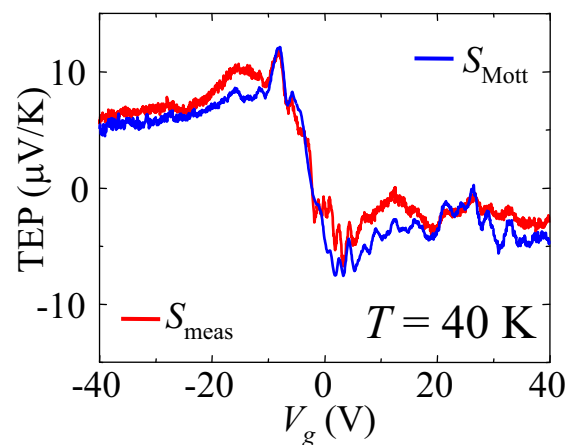
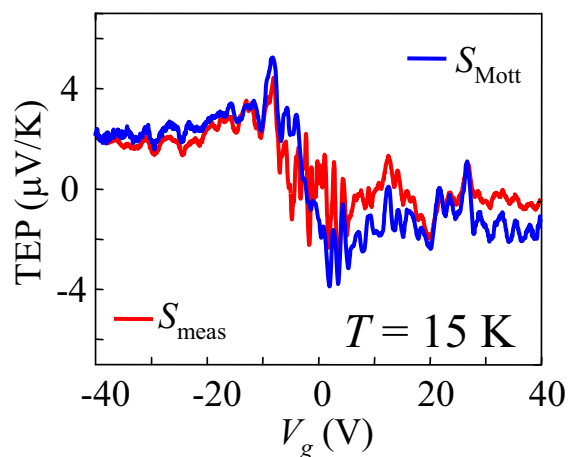
Quantitative Comparison with the Mott Formula



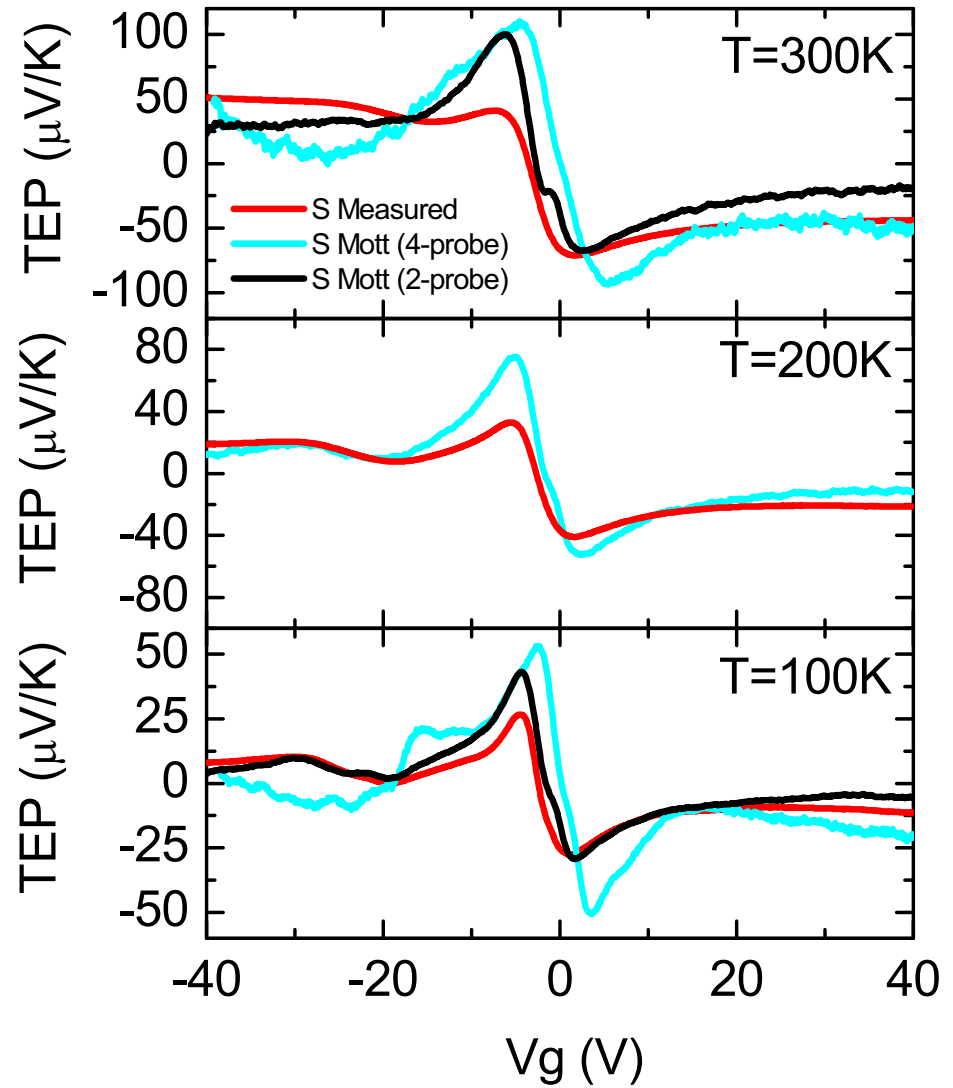
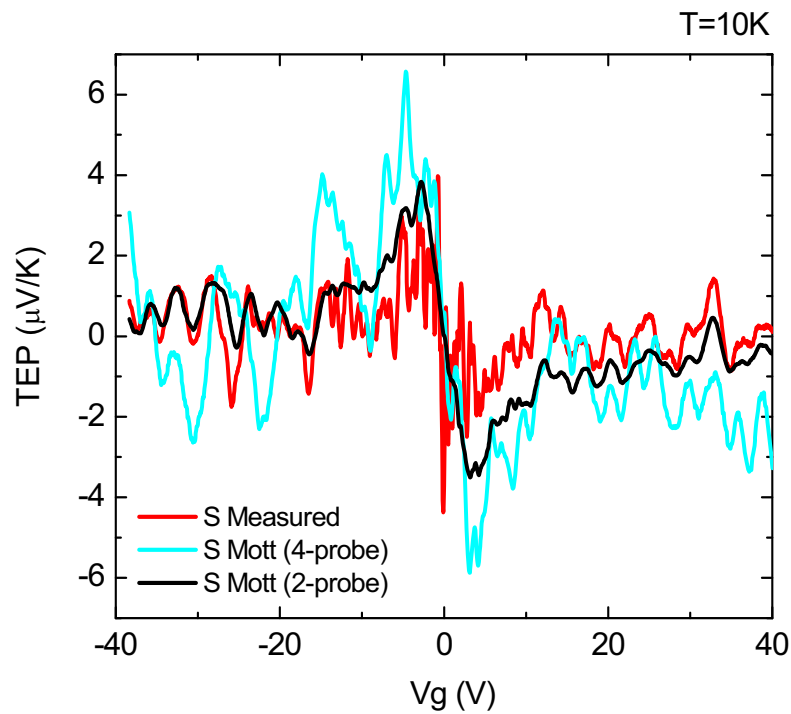
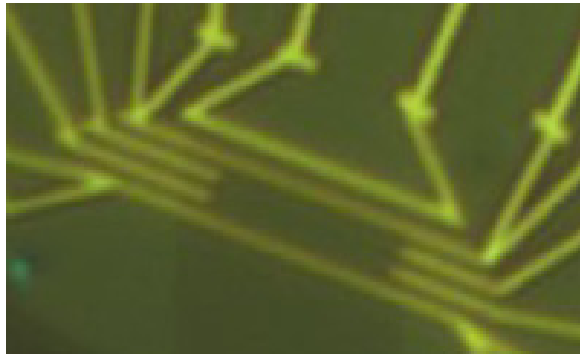
$$S_{Mott} = \frac{\pi^2 k_B^2 T}{3|e|} \left. \frac{d \ln R}{dE} \right|_{E_f}$$

$$E_F = \hbar v_F \sqrt{\pi C_g \Delta V_g}$$

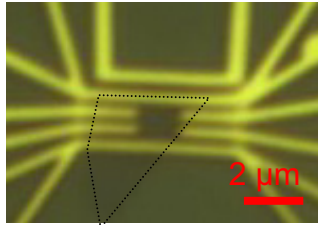
Mott relation holds!



Two-Terminal Versus 4-Terminal Measurement

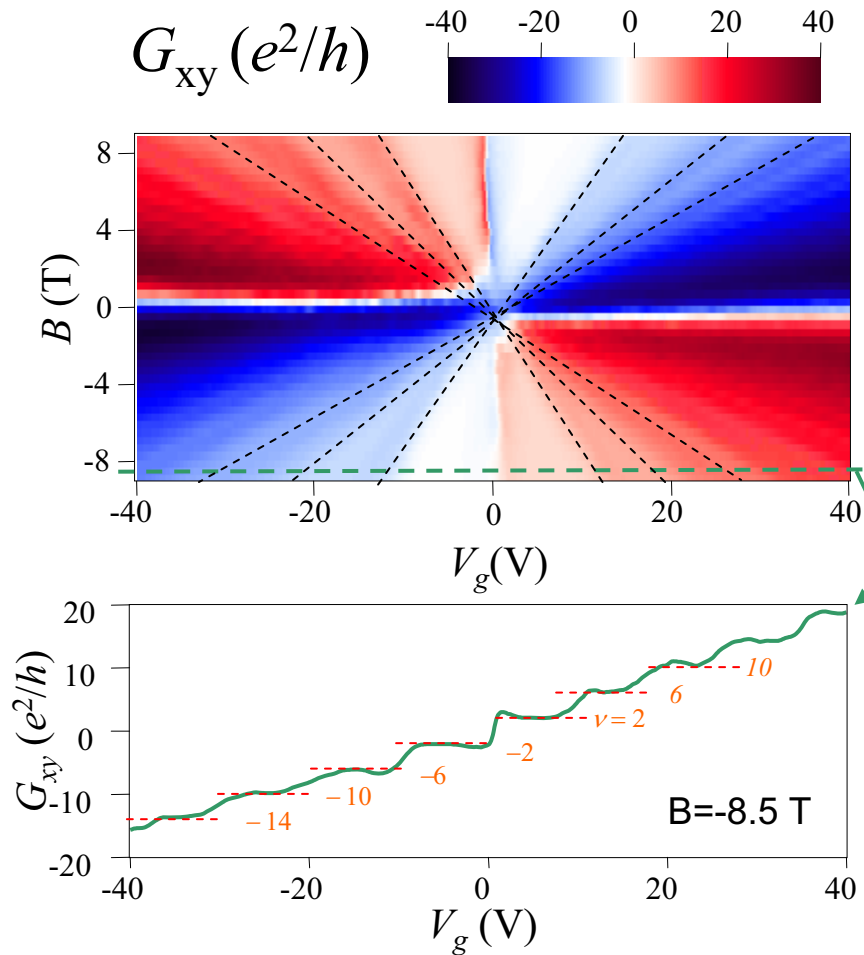


Magneto-Thermoelectric Measurement in Graphene



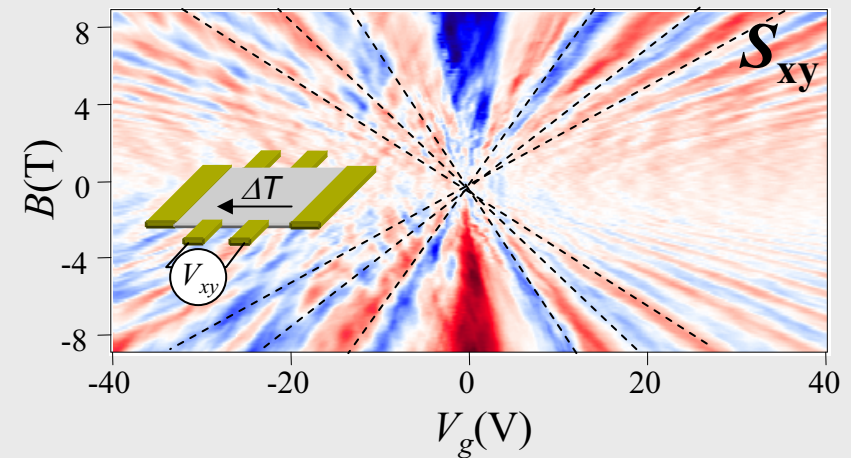
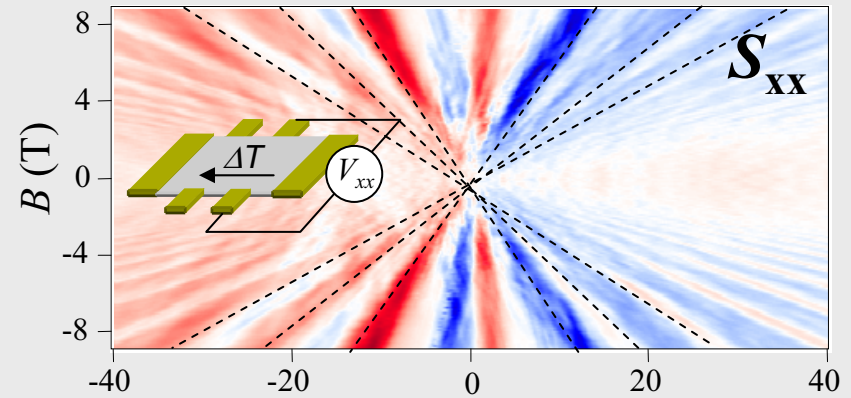
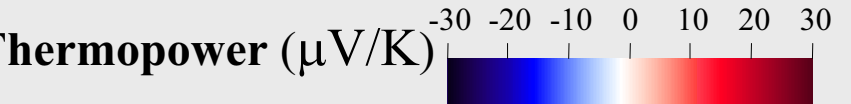
Multiterminal device
 Mobility $\sim 3000 \text{ cm}^2/\text{Vsec}$
 Measure R_{xx} and R_{xy}

magnetoconductance



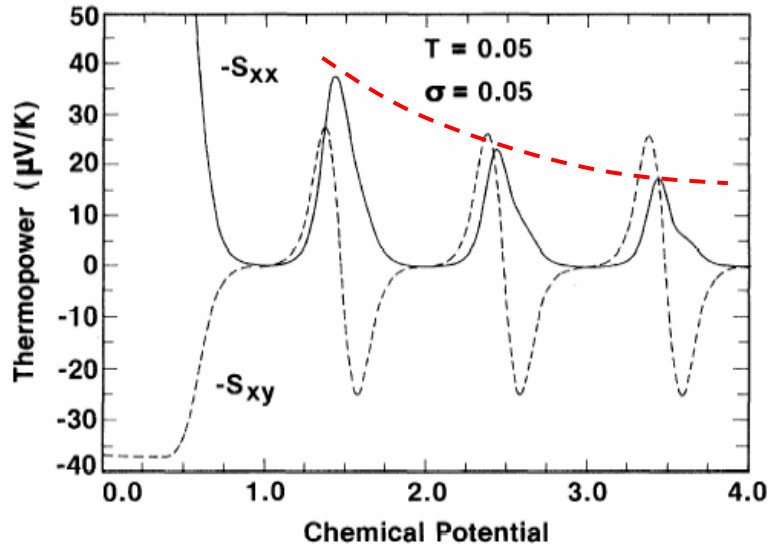
$T = 10 \text{ K}$

Thermopower ($\mu\text{V}/\text{K}$)



Thermopower in QH Regime in 2DEG

Jonhson and Girvin 1984:



$$S_{ij} = -\frac{\pi^2}{3} \frac{k_B^2 T}{e} \sum_k (\sigma^{-1})_{ik} \left(\frac{\partial \sigma}{\partial \mu} \right)_{kj}$$

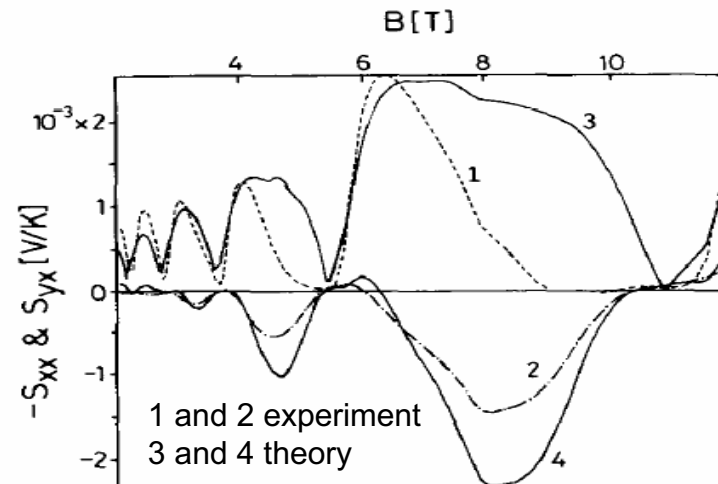
$$S_{xx}^{peak} = \frac{-\ln(2) k_B / e}{(N + 1/2)}$$

where N is number of 1D channels

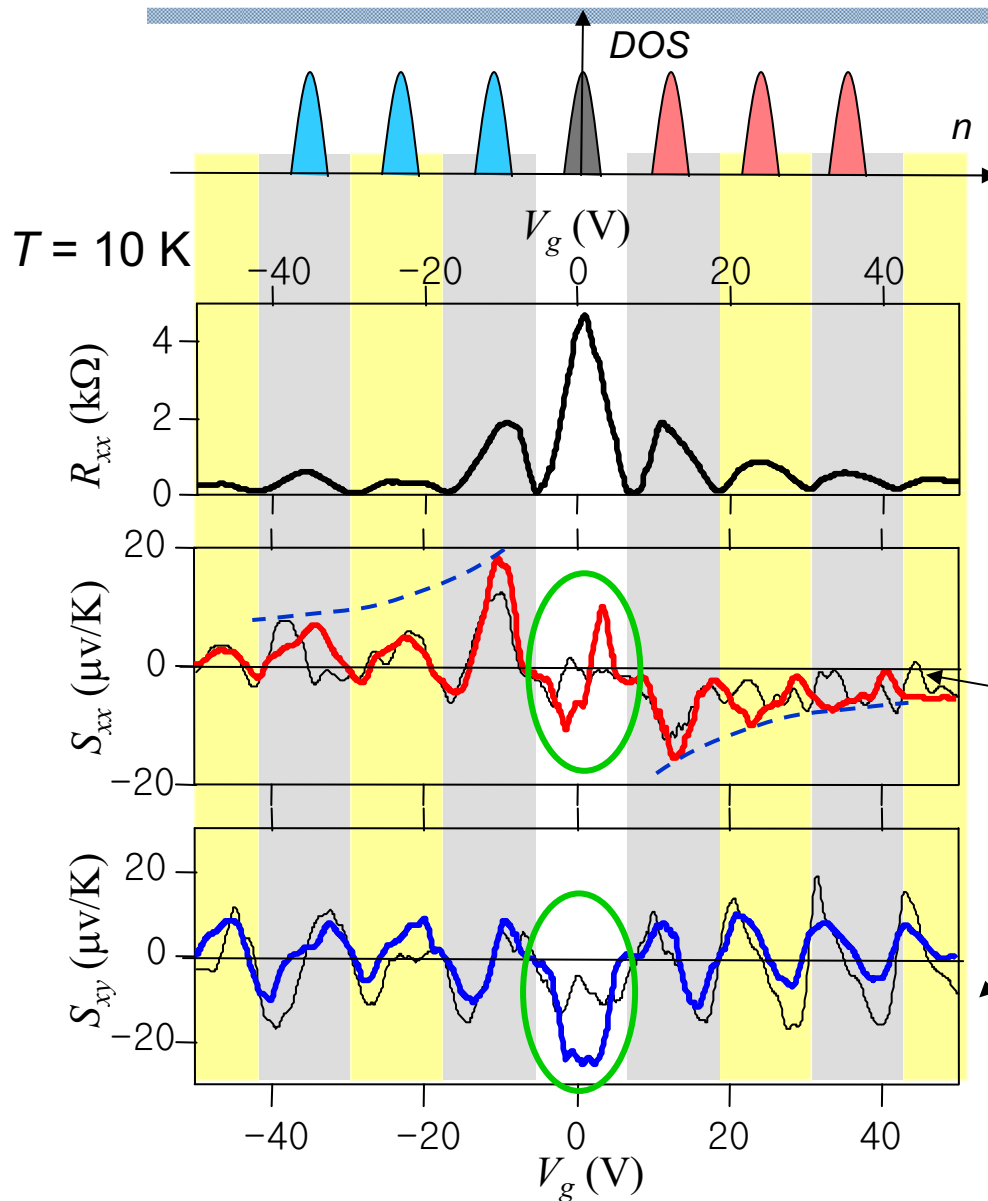
Obloh H. and Von Klizting, 1984 Surface Science

Fromhold TM et al., 1992 Surface Science

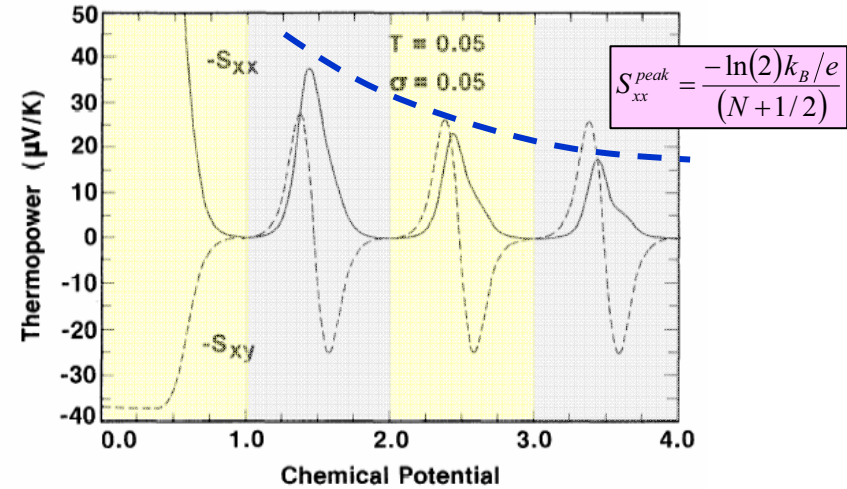
$$S = S_{diffusive} + S_{phonon drag}$$



QHE in magneto-TEP in graphene



Jonhson and Girvin PRB (1984):

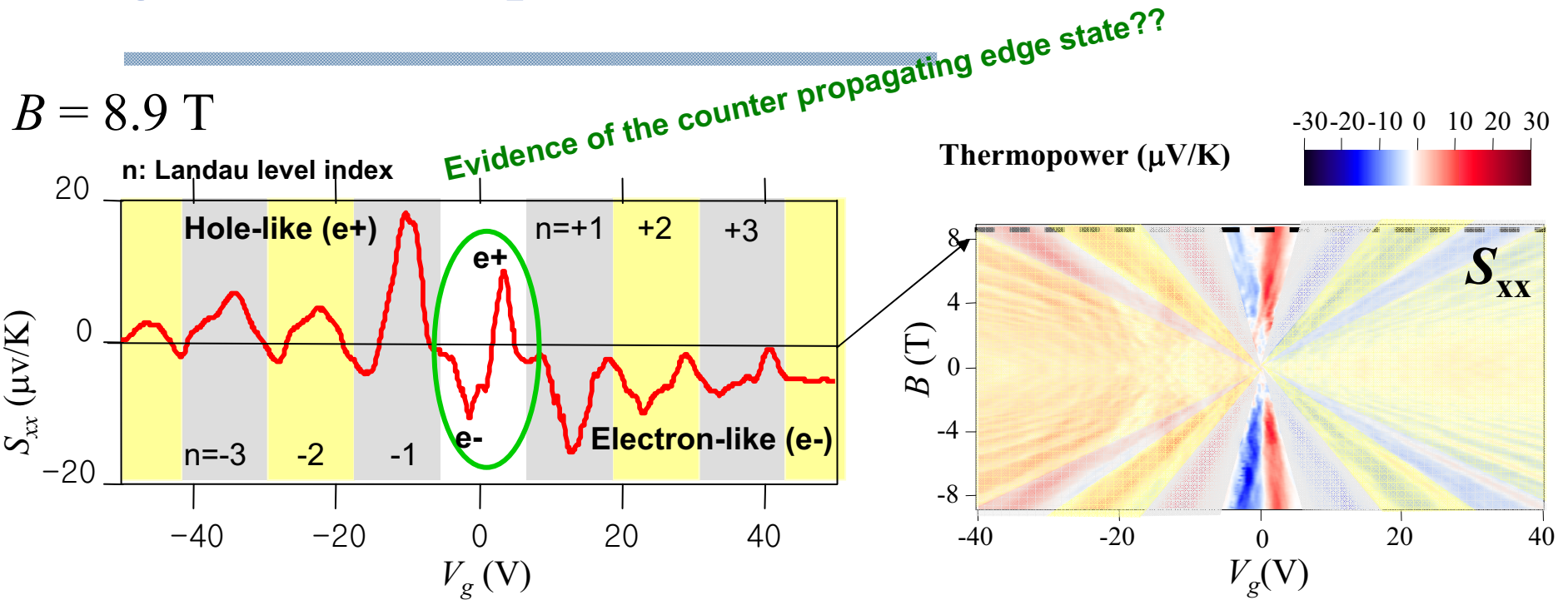


Generalized Mott Formula

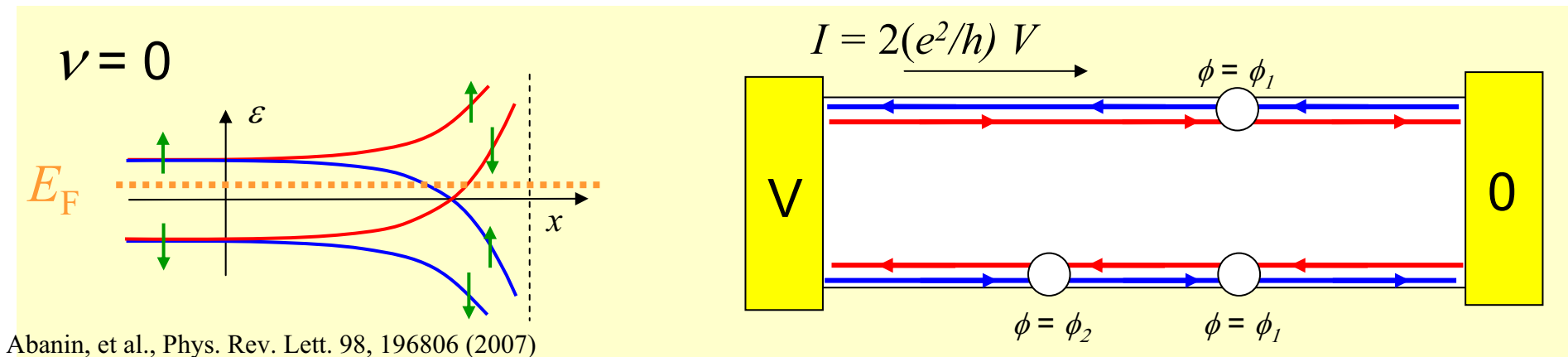
$$S_{ij} = -\frac{\pi^2}{3} \frac{k_B^2 T}{e} \sum_k (\sigma^{-1})_{ik} \left(\frac{\partial \sigma}{\partial \mu} \right)_{kj}$$

TEP at the Dirac point is unusual !!

Magneto-Thermopower at the Dirac Point



Counter propagating QH edge state at $\nu=0$



Abanin, et al., Phys. Rev. Lett. 98, 196806 (2007)

Conclusions

- Mobility of graphene can be greatly improved by suspending and subsequent annealing of samples.
- Quantum Oscillations in graphene heterojunction devices were observed and they indicate the collimated ballistic transport of the chiral particles across the p-n junctions.
- Thermoelectric power graphene agrees with generalized Mott Formula except near the Dirac point.

(Unspoken: variable range hopping in graphene nanoribbons, possible manybody effect in cyclotron resonance in graphene)

Challenges: Controlled Growth & Edge Control