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ICTP Conference Graphene Week 2008

25 - 29 August 2008

Theory of Quantum Transport in Graphene and Nanotubes

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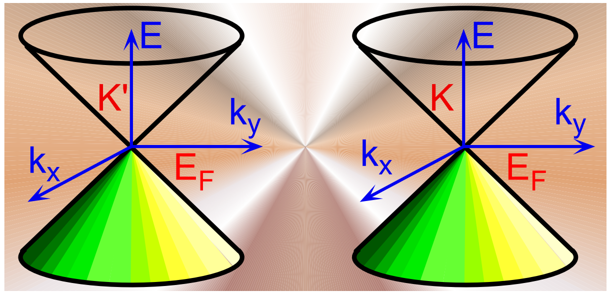
Theory of Quantum Transport in Graphene and Nanotubes

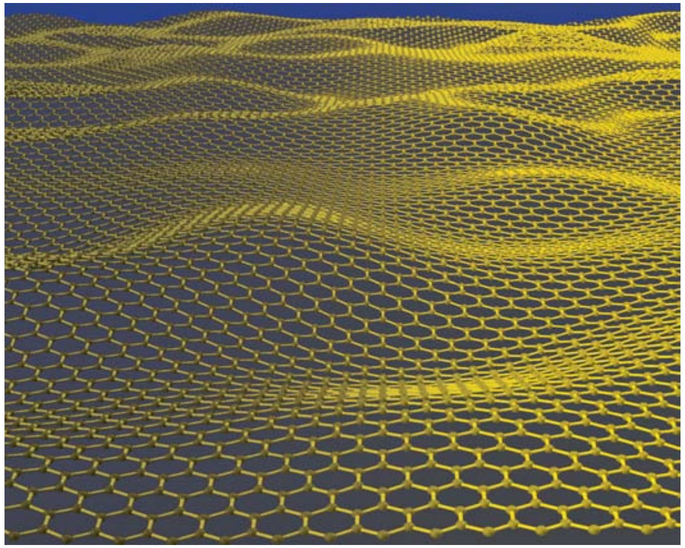
Tsuneya ANDO

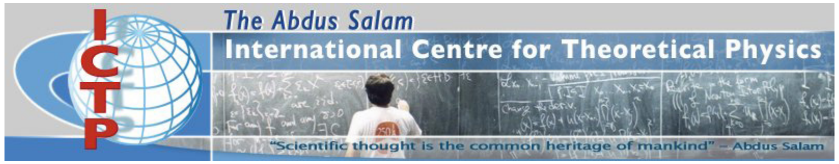
1. **Introduction**
   - Weyl's equation for neutrino
   - Berry's phase & topological anomaly
2. **Zero mode anomalies**
   - Density of states and conductivity
   - Dynamical conductivity
   - Self-consistent Born approximation
3. **Special time reversal symmetry**
   - Symmetry crossover
4. **Phonons and electron-phonon interaction**
   - Acoustic phonon
   - Optical phonon
   - Zone-boundary phonon
5. **Summary**

Trieste, Aug 29 (Fri) 2008

ICTP Conference Graphene Week 2008, Trieste, Italy
August 25–29, 2008 [14:30–15:20 (40+10)]

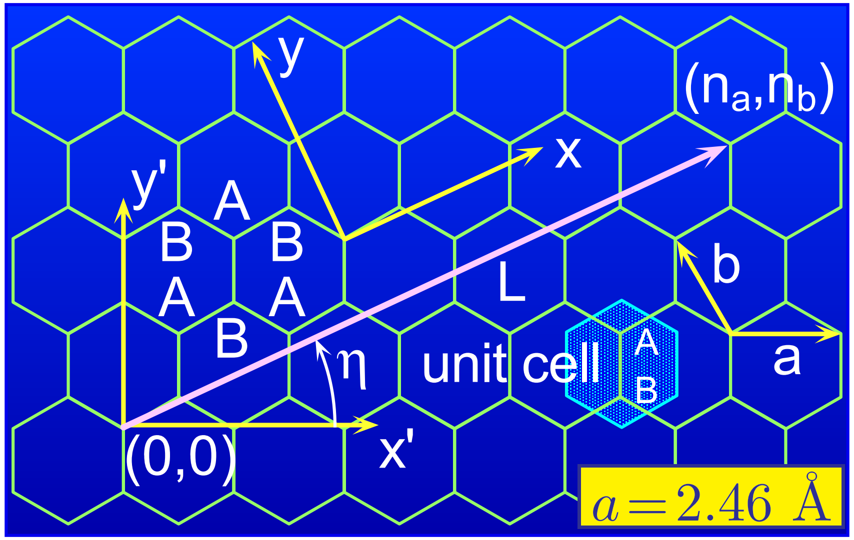






Effective-Mass Description: Neutrino or Massless Dirac Electron

Graphene (Triangular antidot lattice)

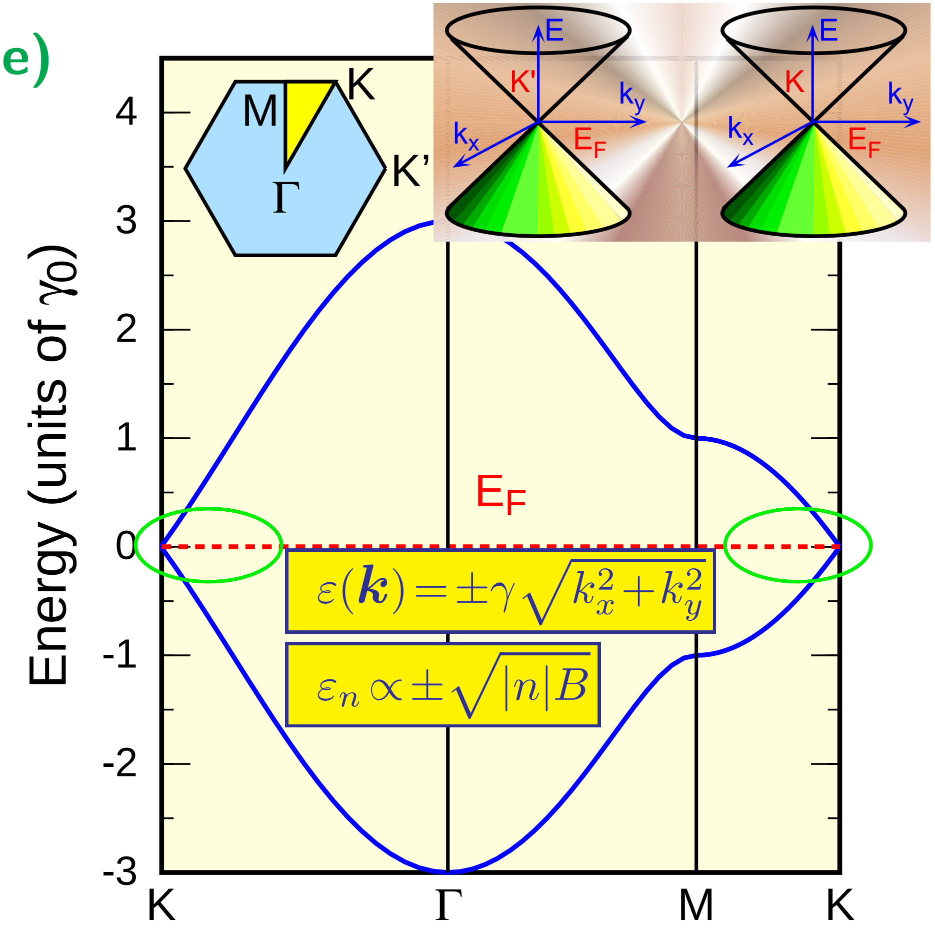


Weyl's equation for neutrino

$$\Leftrightarrow \ \gamma(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}})\boldsymbol{F}(\boldsymbol{r}) = \varepsilon\boldsymbol{F}(\boldsymbol{r})$$

$$\Leftrightarrow \ \gamma(\sigma_x\hat{k}_x + \sigma_y\hat{k}_y)\boldsymbol{F}(\boldsymbol{r}) = \varepsilon\boldsymbol{F}(\boldsymbol{r})$$

$$\begin{pmatrix} 0 & \gamma(\hat{k}_x - i\hat{k}_y) \\ \gamma(\hat{k}_x + i\hat{k}_y) & 0 \end{pmatrix}\begin{pmatrix} F^A(\boldsymbol{r}) \\ F^B(\boldsymbol{r}) \end{pmatrix} = \varepsilon\begin{pmatrix} F^A(\boldsymbol{r}) \\ F^B(\boldsymbol{r}) \end{pmatrix}$$



$$\varepsilon(\boldsymbol{k}) = \pm\gamma\sqrt{k_x^2 + k_y^2}$$

$$\varepsilon_n \propto \pm\sqrt{|n|B}$$

Wave Vector

$\hat{\boldsymbol{k}} = -i\vec{\nabla}$

Velocity: $v_F = \gamma/\hbar$

K': $\boldsymbol{\sigma} \to \boldsymbol{\sigma}^*$

Massless (Dirac)
Constant velocity (~light, cannot stop)
Topological anomaly

$v_F \sim c/300 \ (\gamma_0 \sim 3\,\mathrm{eV})$

$\gamma = \sqrt{3}\gamma_0 a/2$ (γ_0 : **Hopping integral**)

Topological Anomaly and Berry's Phase

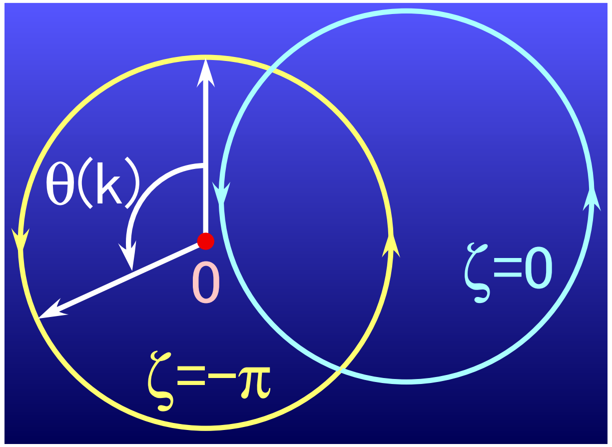
Weyl's equation : Neutrino ⇔ Helicity ($\boldsymbol{\sigma} \leftrightarrow \boldsymbol{k}$)

$$\gamma(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}})\,\boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r})=\varepsilon_s(\boldsymbol{k})\,\boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r})\qquad \boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r})=\frac{1}{\sqrt{L^2}}\exp(i\boldsymbol{k}\cdot\boldsymbol{r})\,R^{-1}[\theta(\boldsymbol{k})]\big|s\big)$$

$$R(\theta\pm2\pi)=-R(\theta)\qquad R(-\pi)=-R(+\pi)$$

$$\varepsilon_s(\boldsymbol{k})=s\,\gamma\,|\boldsymbol{k}|\qquad s=\pm1$$

$$\boxed{\begin{aligned}&R(\theta+2\pi)\\&\;=e^{-i\zeta}R(\theta)\end{aligned}}$$

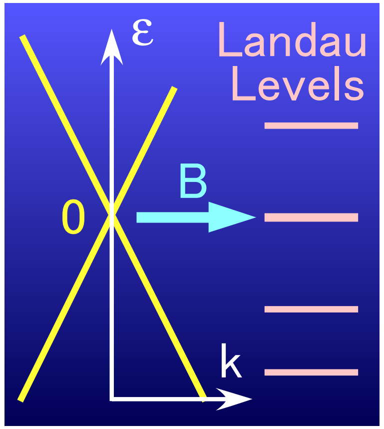


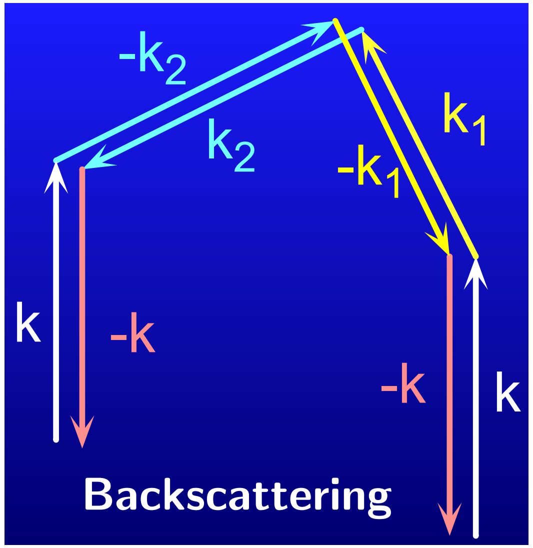
Pseudo spin ⇒ Berry's phase

$$\zeta=-i\int_0^T dt\Big\langle s\boldsymbol{k}(t)\Big|\frac{d}{dt}\Big|s\boldsymbol{k}(t)\Big\rangle=-\pi$$

Landau levels at $\varepsilon=0$ [*J.W. McClure, PR 104, 666 (1956)*]

$$\chi=-\frac{g_{\rm v}g_{\rm s}\gamma^2}{6\pi}\Big(\frac{e}{c\hbar}\Big)^2\int\Big(-\frac{\partial f(\varepsilon)}{\partial\varepsilon}\Big)\delta(\varepsilon)\,d\varepsilon$$





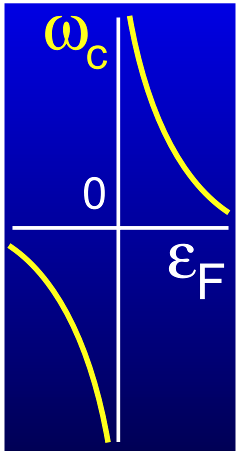
Absence of backscattering

Metallic CN with scatterers

⇒ Perfect conductor

T. Ando & T. Nakanishi, JPSJ 67, 1704 (1998)

Zero-Mode Anomaly: Boltzmann Conductivity



Equation of motion: $\hbar\frac{d\boldsymbol{k}}{dt}=-\frac{e}{c}\boldsymbol{v}\times\boldsymbol{B}\ \Rightarrow\ \omega_c=\frac{eBv^2}{c\,\varepsilon_F}$ $\boxed{m_c\propto\varepsilon_F\propto\sqrt{n_s}}$

Semiclassical: $\varepsilon_n=\pm\sqrt{|n|+\frac{1}{2}}\,\hbar\omega_B\ \Leftarrow\ \oint k_x dk_y=\pm\frac{2\pi}{l^2}\Big(|n|+\frac{1}{2}\Big)$

Full quantum: $\varepsilon_n=\pm\sqrt{|n|}\,\hbar\omega_B$ $\quad\hbar\omega_B=\sqrt{2}\,\frac{\gamma}{l}\quad l=\sqrt{\frac{c\hbar}{eB}}$

Density of states: $D(\varepsilon)=\frac{|\varepsilon|}{2\pi\gamma^2}$ ⇒ **Zero-gap semiconductor**

Boltzmann conductivity

$$\sigma(\varepsilon_F)=e^2D^*D(\varepsilon_F)=\frac{e^2}{\pi^2\hbar}\frac{1}{4W}$$

Einstein relation

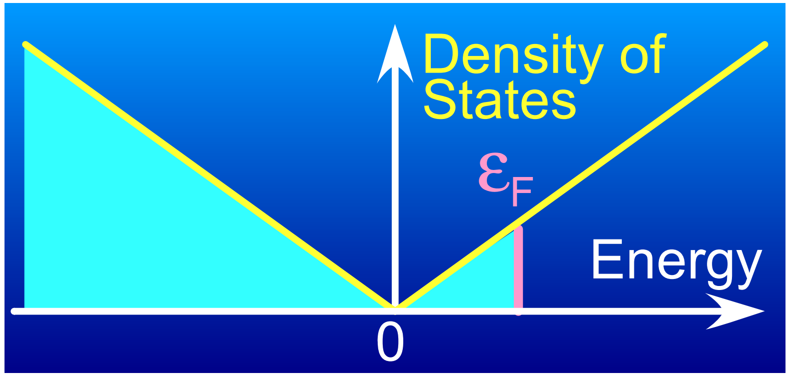
$$D^*=v_F^2\tau=\frac{\gamma^2}{\hbar^2}\tau\qquad W=\frac{n_{\rm i}u^2}{4\pi\gamma^2}$$

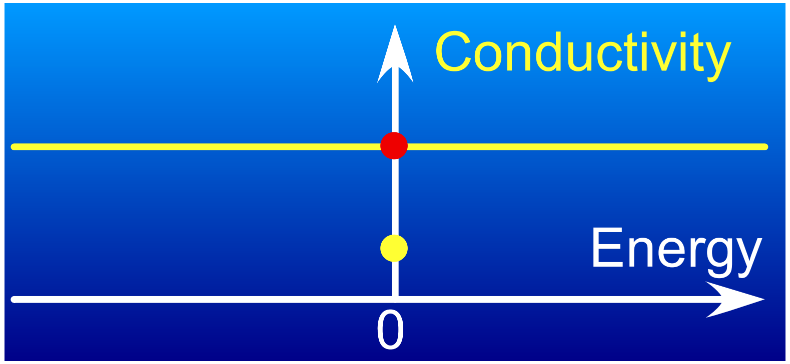
$$\frac{\hbar}{\tau}=2\pi n_{\rm i}u^2D(\varepsilon_F)\qquad\tau\propto D(\varepsilon_F)^{-1}$$

u **Impurity strength**
$n_{\rm i}$ **Impurity density**

⇒ **Independent of ε_F (Metal!)**

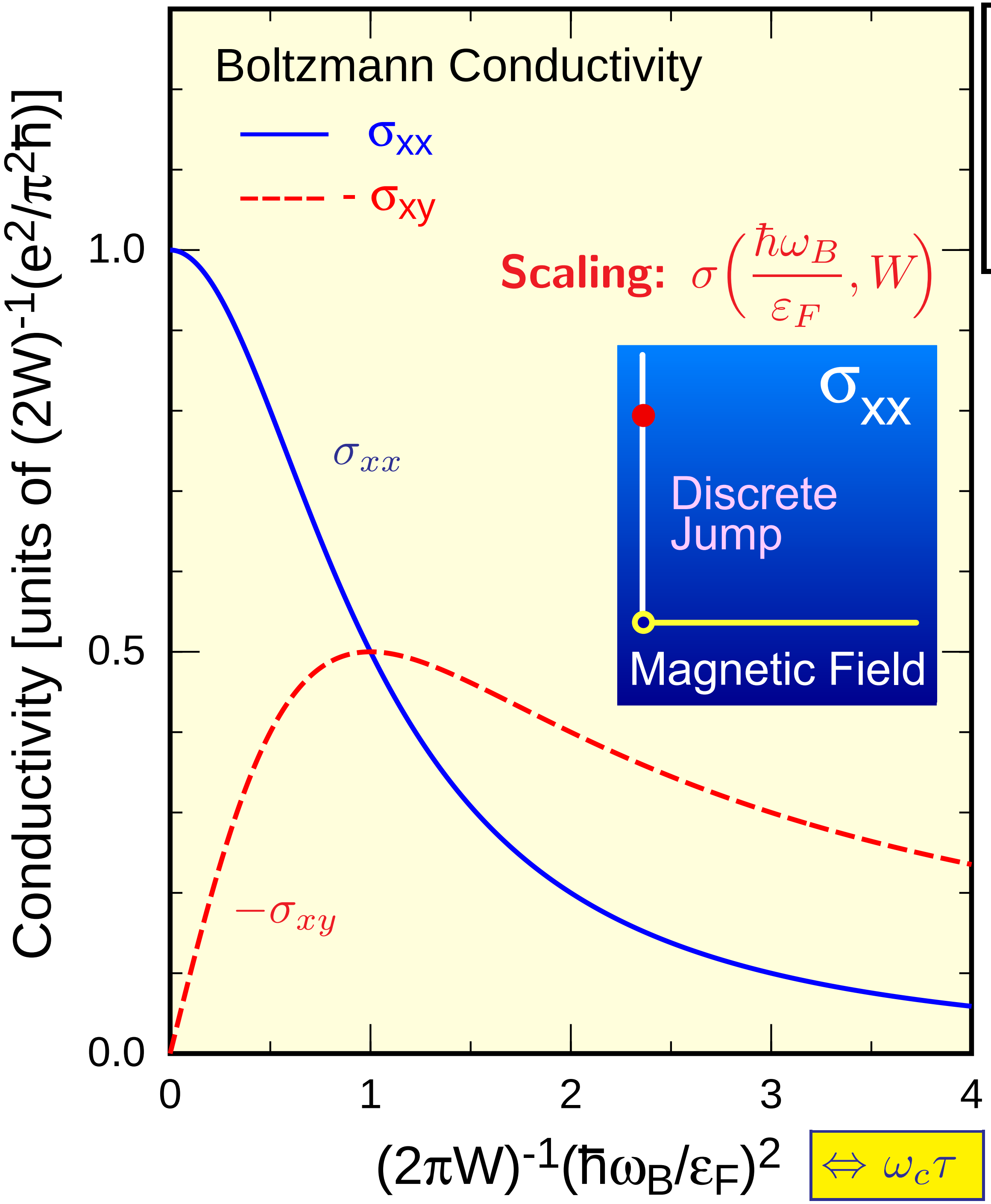
⇒ $\sigma(0)$ for $D(0)=0$?





Conductivity Tensor in Magnetic Fields

Y. Zheng & T. Ando, PRB 65, 245420 (2002)



Cyclotron frequency

$$\omega_c = \frac{eBv^2}{c\,\varepsilon_F}$$

$$\frac{\hbar}{\tau} = 2\pi|\varepsilon_F|W$$

$$\omega_c\tau \propto \left(\frac{\hbar\omega_B}{\varepsilon_F}\right)^2$$

$$\hbar\omega_B = \sqrt{2}\,\frac{\gamma}{l}$$

Magneto-conductivity

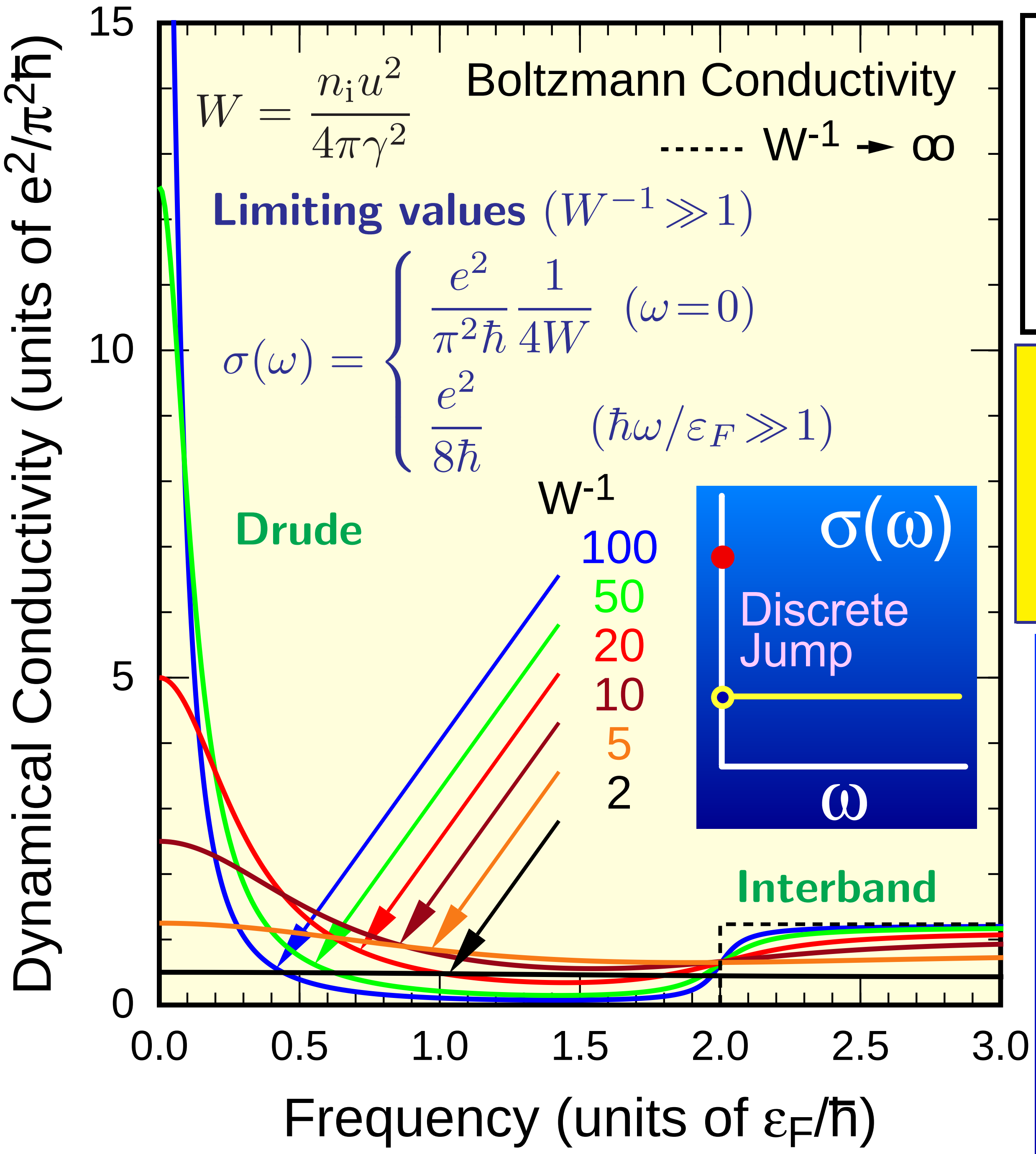
$$\sigma_{xx} = \frac{\sigma_0}{1+\omega_c^2\tau^2}$$

$$\sigma_{xy} = -\frac{\sigma_0\omega_c\tau}{1+\omega_c^2\tau^2}$$

$$\sigma_0(W) = \frac{e^2}{\pi^2\hbar}\,\frac{1}{2W}$$

Dynamical Conductivity $\sigma(\omega)$

T. Ando, Y. Zheng, H. Suzuura, JPSJ *<u>71</u>, 1318 (2002)*



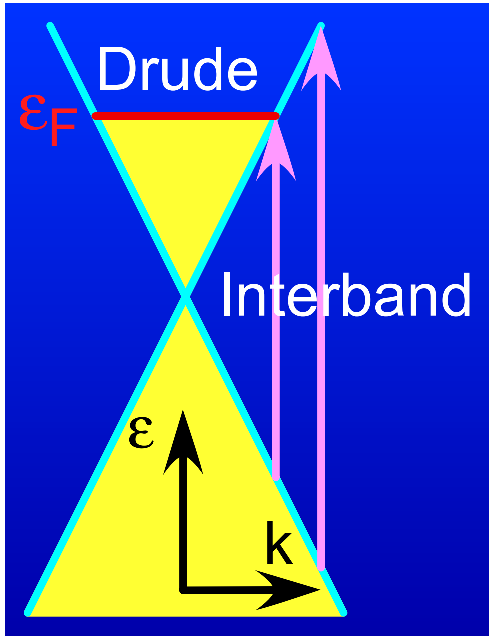
$$W = \frac{n_i u^2}{4\pi\gamma^2}$$

Limiting values $(W^{-1} \gg 1)$

$$\sigma(\omega) = \begin{cases} \dfrac{e^2}{\pi^2\hbar}\dfrac{1}{4W} & (\omega=0) \\ \dfrac{e^2}{8\hbar} & (\hbar\omega/\varepsilon_F \gg 1) \end{cases}$$

Scaling $\sigma(\omega)=\sigma\left(\dfrac{\hbar\omega}{\varepsilon_F}, W\right)$

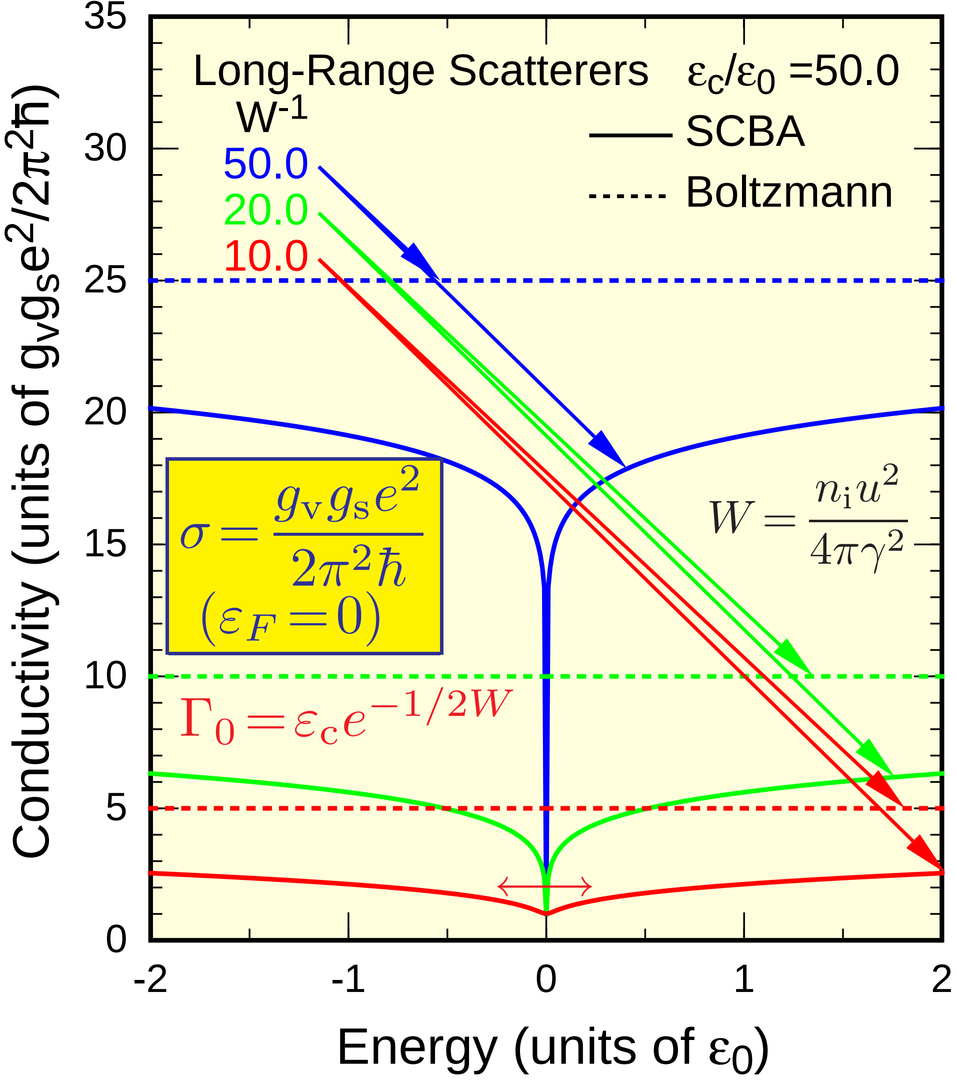
Discrete jump of $\sigma(\omega)$ **at** $\varepsilon_F=0$



Experiments *[Z.Q. Li et al., Nat. Phys. <u>4</u>, 532 (2008)]*

Zero-Mode Anomalies (Self-Consistent Born Approximation)

Static conductivity



ε_0: **Arbitrary energy**
ε_c: **Cutoff energy (π-band width)**

Singularity at the Dirac point $(\varepsilon_F = 0)$

⇔ Fermi energy scaling

Magnetoconductivity

$$\sigma_{xx}(B) = \sigma_{xx}\left(\frac{\hbar\omega_B}{\varepsilon_F}\right), \text{ etc.}$$

Dynamical conductivity

$$\sigma(\omega) = \sigma\left(\frac{\hbar\omega}{\varepsilon_F}\right)$$

Diagonal conductivity σ_{xx}

N.H. Shon and T. Ando, JPSJ 67, 2421 (1998) $\dfrac{g_v g_s e^2}{2\pi^2\hbar}$

Quantum Hall effect σ_{xy}

Y. Zheng and T. Ando, PRB 65, 245420 (2002) $4\left(n+\dfrac{1}{2}\right)\dfrac{e^2}{h}$

Dynamical conductivity $\sigma(\omega)$

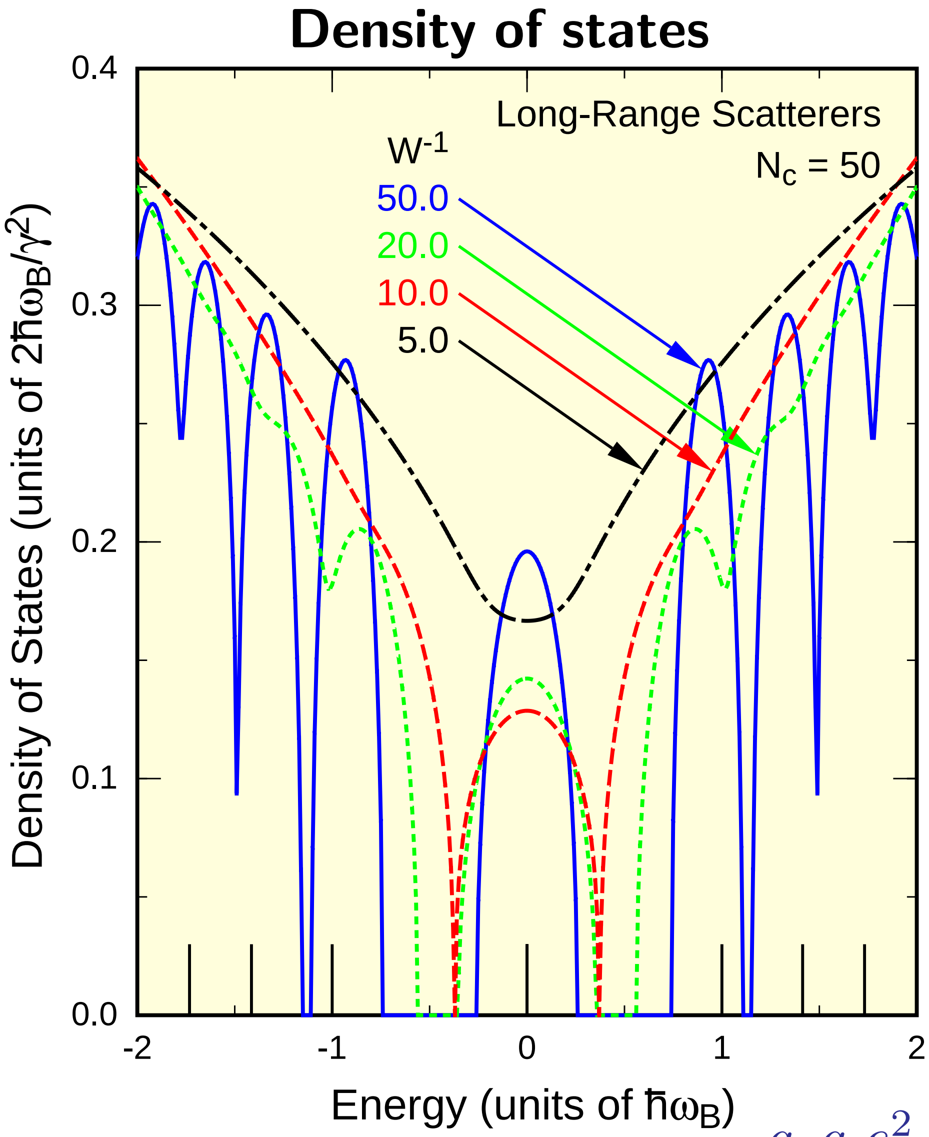
T. Ando, Y. Zheng, & H. Suzuura, JPSJ 71, 1318 (2002)

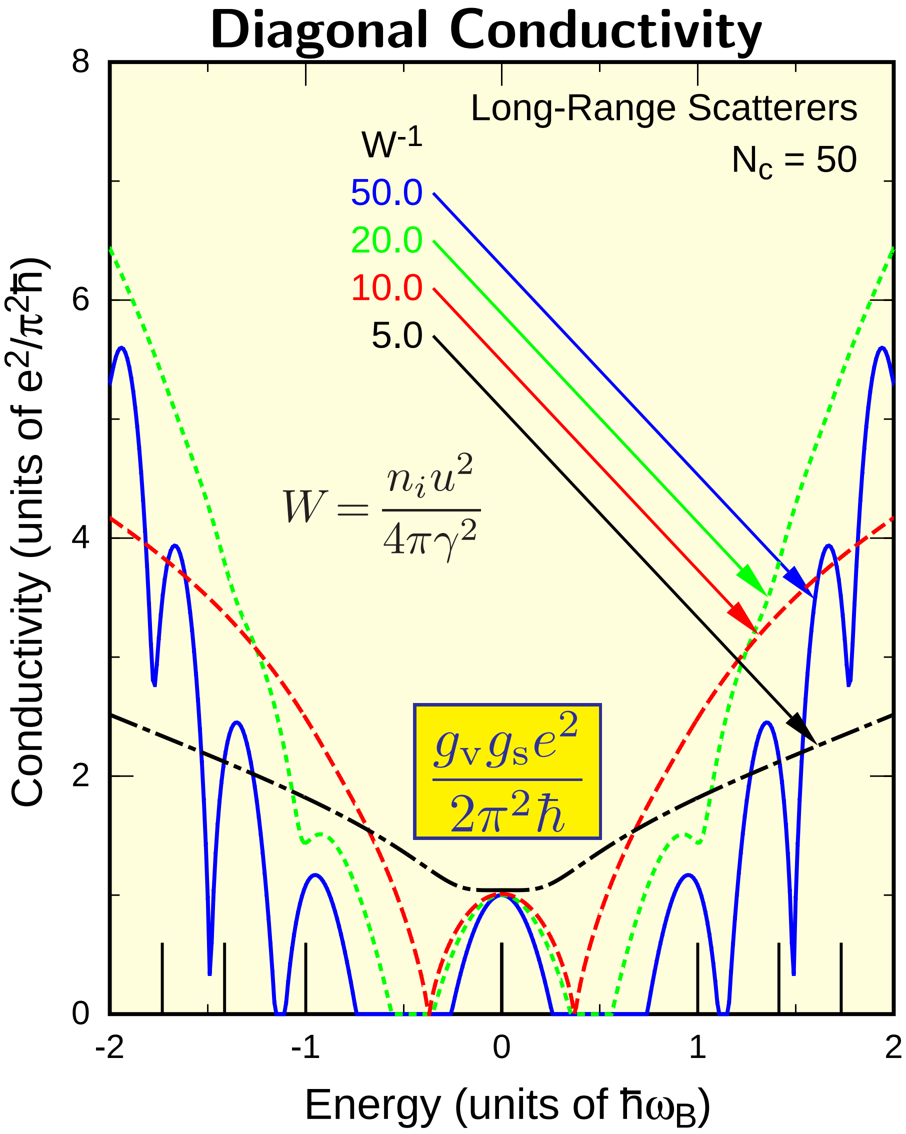
Diamagnetic susceptibility $\chi(\varepsilon_F)$

M. Koshino and T. Ando, PRB 75, 235333 (2007)

Self-Consistent Born Approximation: High-Magnetic Field

[*N.H. Shon and T. Ando, J. Phys. Soc. Jpn.* *67*, *2421 (1998)*]

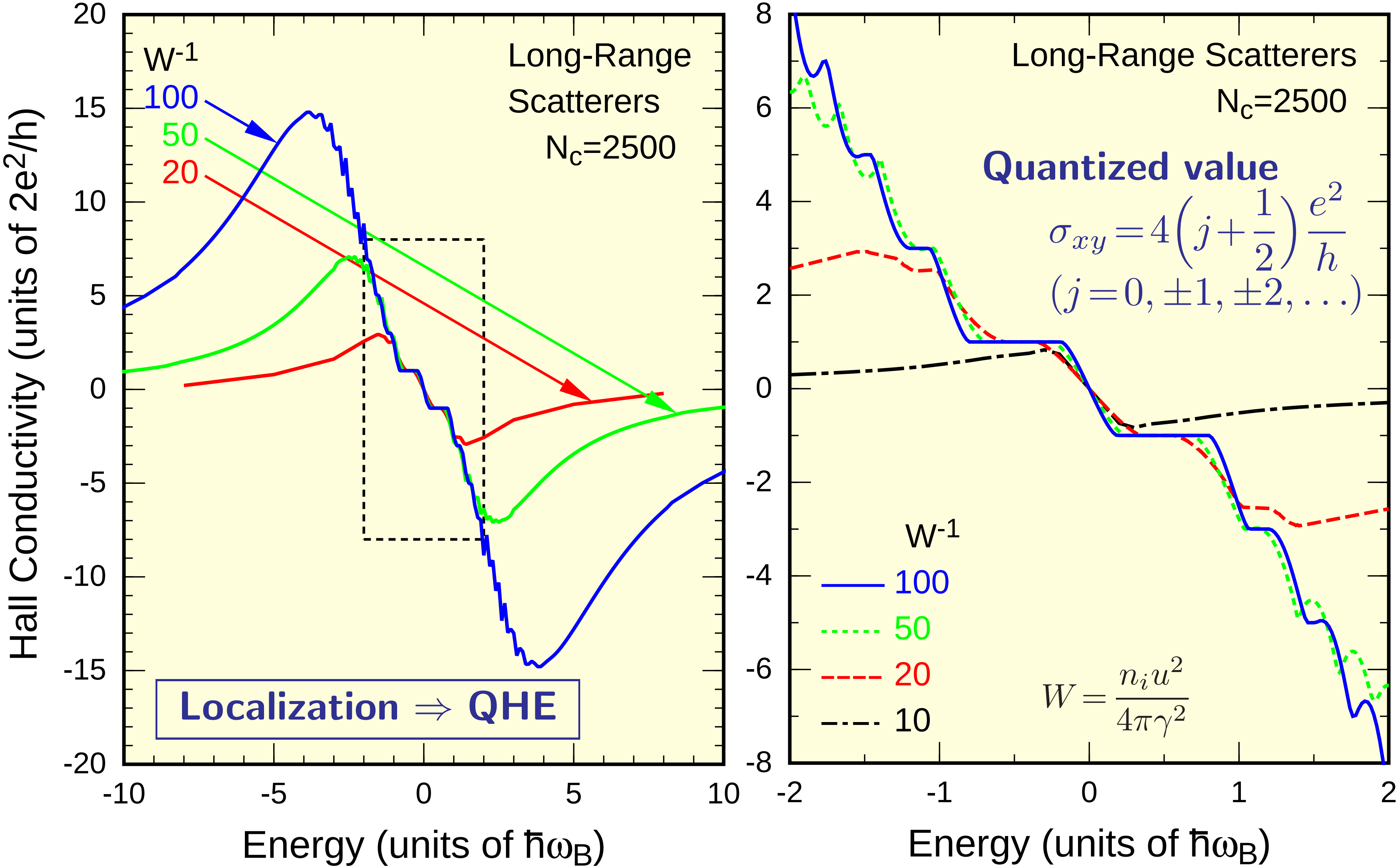




$\Gamma_N = \hbar\omega_B\sqrt{2W(1+\delta_{N0})}$ $\sigma_{xx}^{(N)} = \frac{g_\mathrm{v} g_\mathrm{s} e^2}{2\pi^2\hbar}(|N|+\delta_{N0})$ **Singular?** $(\hbar\omega_B \rightarrow 0,\ W \ll 1)$

Self-Consistent Born Approximation: Hall Conductivity

[*Y. Zheng and T. Ando, Phys. Rev. B* 65, *245420 (2002)*]



Localization ⇒ QHE

Quantized value

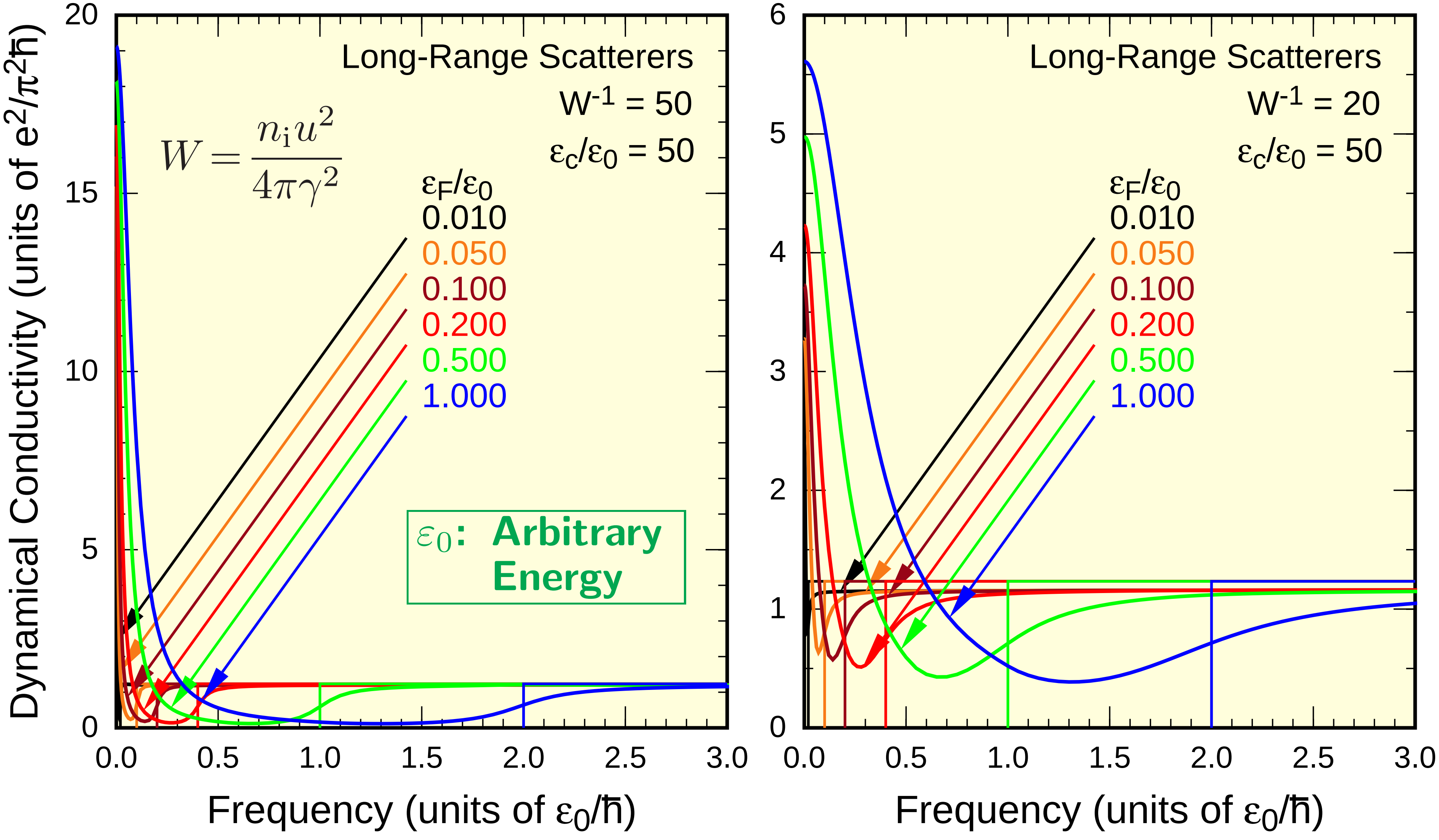
$$\sigma_{xy} = 4\Big(j + \frac{1}{2}\Big)\frac{e^2}{h}$$

$$(j = 0, \pm 1, \pm 2, \ldots)$$

$$W = \frac{n_i u^2}{4\pi\gamma^2}$$

Self-Consistent Born Approximation: Dynamical Conductivity

T. Ando, Y. Zheng, and H. Suzuura, JPSJ <u>71</u>, 1318 (2002)



- **Approximate $\hbar\omega/\varepsilon_F$ scaling except at $(\varepsilon_F, \omega) = (0, 0)$**
- **Experiments:** *Z.Q. Li et al., Nat. Phys. <u>4</u>, 532 (2008)*

Diamagnetic Susceptibility: Disorder Effects

Singular diamagnetism

J.W. McClure,
Phys. Rev. 104, 666 (1956)
S.A. Safran & F.J. DiSalvo,
PRB 20, 4889 (1979)

$$\chi = -\frac{g_{\mathrm{v}} g_{\mathrm{s}} \gamma^2}{6\pi}\left(\frac{e}{c\hbar}\right)^2 \delta(\varepsilon_F)$$

Constant broadening Γ

H. Fukuyama, JPSJ 76, 043711 (2007)

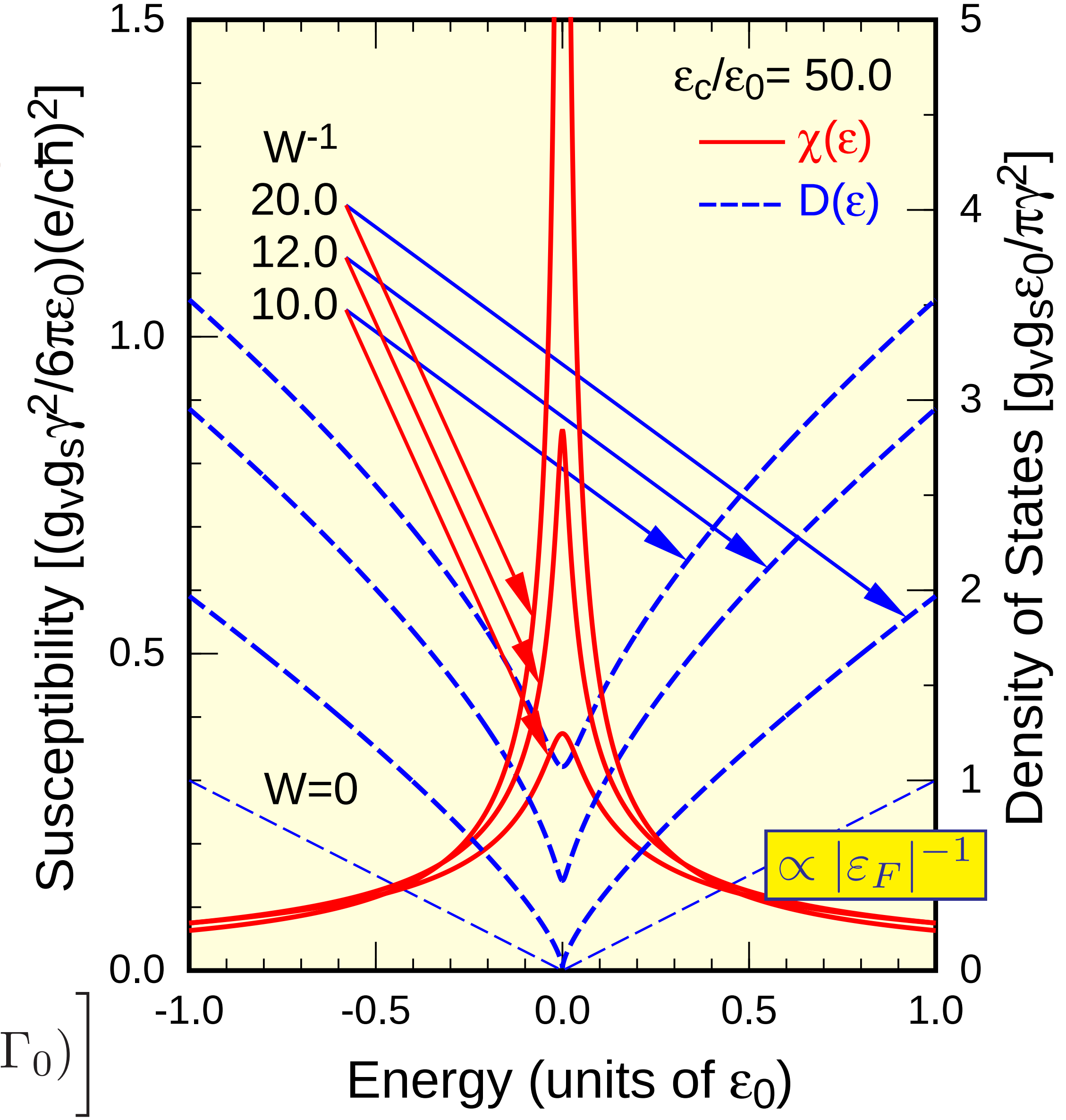
$$\delta(\varepsilon_F) \to \frac{\Gamma}{\pi(\varepsilon_F^2+\Gamma^2)}$$

Self-consistent Born approximation

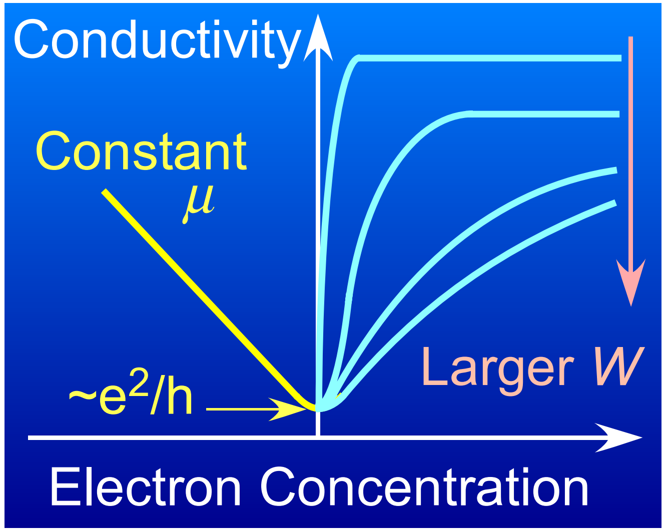
M. Koshino and T. Ando,
PRB 75, 235333 (2007)

$$\delta(\varepsilon_F) \to \frac{W}{2|\varepsilon_F|} \left[\frac{2W}{\pi\Gamma_0} \ (|\varepsilon_F|<\Gamma_0)\right]$$

Cutoff energy: $\Gamma_0 = \varepsilon_c \, e^{-1/2W}$



Sharp peak and long tail



Conductivity vs Concentration

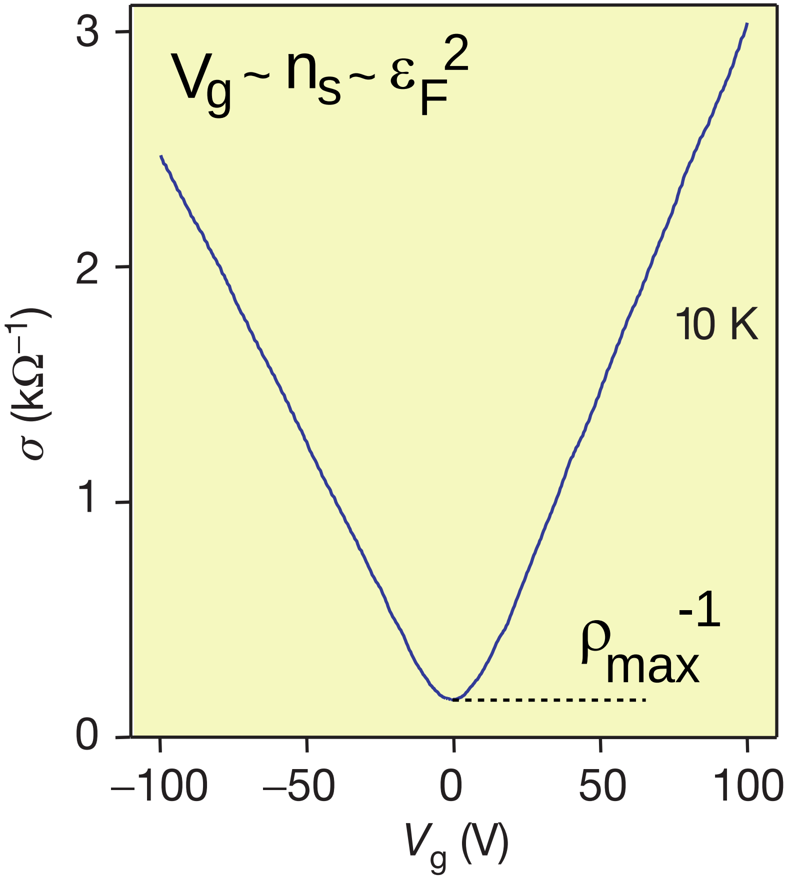
K.S. Novoselov et al., Nature 438, 197 (2005)

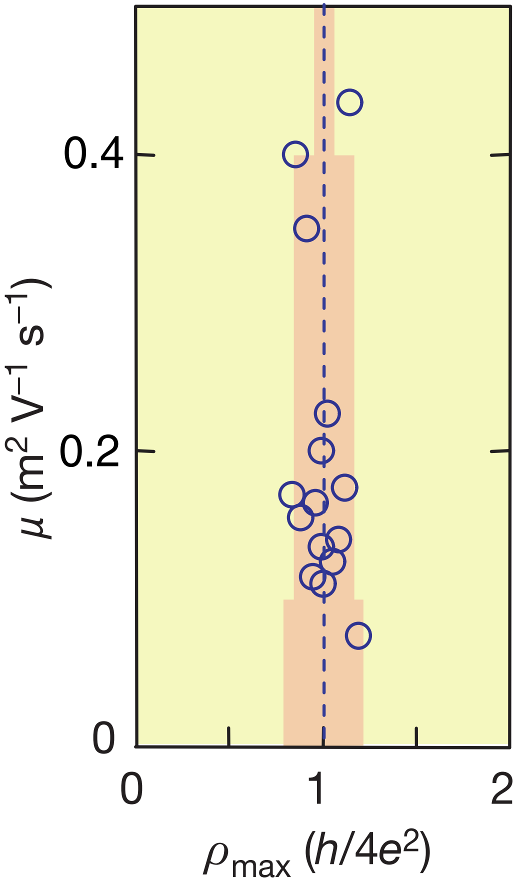
Scattering mechanisms

Boltzmann conductivity $\sigma(\varepsilon_{\mathrm{F}}) = \dfrac{e^2}{\pi^2\hbar}\dfrac{1}{4W}$

Constant mobility $\Leftrightarrow W \propto n_{\mathrm{s}}^{-1}$

Charged impurity with screening?





- *T. Ando, JPSJ 75, 074716 (2006)*
- *K. Nomura & A.H. MacDonald, PRL 96, 256602 (2006)*

Minimum conductivity?

- *A.K. Geim & K.S. Novoselov, Nat. Phys. 6, 183, (2007)*
- *Y.-W. Tan et al., PRL 99, 264803 (2007)*
- *K.I. Bolotin et al., SSC 146, 2351 (2008)*

Special Time Reversal Symmetry and Universality Class

Real time reversal (K↔K'): T $\quad \boldsymbol{F}_K^T = \sigma_z \boldsymbol{F}_{K'}^* \quad \boldsymbol{F}_{K'}^T = \sigma_z \boldsymbol{F}_K^* \quad T^2 = 1$

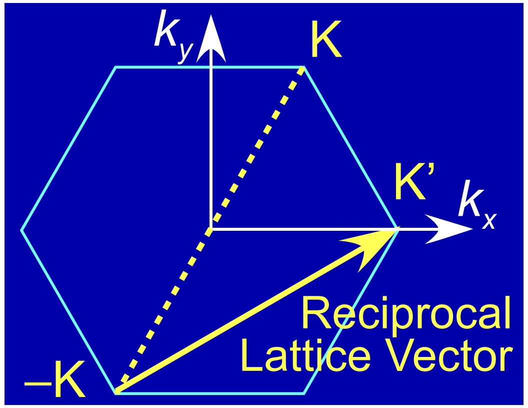
Special time reversal (within K and K'): S

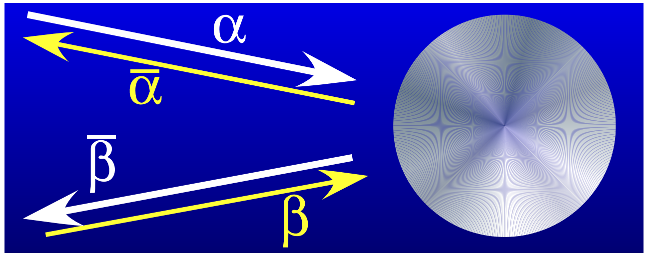
$$\boldsymbol{F}^S = K\boldsymbol{F}^* \quad K = -i\sigma_y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad K^2 = -1$$

$$\Rightarrow S^2 = -1$$

Time reversal of P

$$P^S = K\,{}^t\!PK^{-1} \Rightarrow (\boldsymbol{F}_\alpha^S, P^S\boldsymbol{F}_\beta^S) = (\boldsymbol{F}_\beta, P\boldsymbol{F}_\alpha)$$





Time reversal		Symmetry	Matrix
Real	$T^2 = +1$	Orthogonal	Real
Special	$S^2 = -1$	Symplectic	Quaternion
None		Unitary	Complex

Reflection coefficient: $r_{\bar{\beta}\alpha} = (\boldsymbol{F}_{\bar{\beta}}, T\boldsymbol{F}_\alpha) = (\boldsymbol{F}_\beta^S, T\boldsymbol{F}_\alpha) \Leftrightarrow r_{\bar{\alpha}\beta}$

T matrix: $T = V + V\dfrac{1}{E - \mathcal{H}_0 + i0}V + V\dfrac{1}{E - \mathcal{H}_0 + i0}V\dfrac{1}{E - \mathcal{H}_0 + i0}V + \cdots$

Real : $r_{\bar{\alpha}\beta} = (\boldsymbol{F}_\alpha^T, T\boldsymbol{F}_\beta) = (\boldsymbol{F}_\beta^T, T(\boldsymbol{F}_\alpha^T)^T) = +(\boldsymbol{F}_\beta^T, T\boldsymbol{F}_\alpha) = +r_{\bar{\beta}\alpha}$

Special: $r_{\bar{\alpha}\beta} = (\boldsymbol{F}_\alpha^S, T\boldsymbol{F}_\beta) = (\boldsymbol{F}_\beta^S, T(\boldsymbol{F}_\alpha^S)^S) = -(\boldsymbol{F}_\beta^S, T\boldsymbol{F}_\alpha) = -r_{\bar{\beta}\alpha}$

Absence of backward scattering: $r_{\bar{\alpha}\alpha} = 0$ **(⇐ Berry's phase)**

Presence of perfect channel (Odd channel numbers)

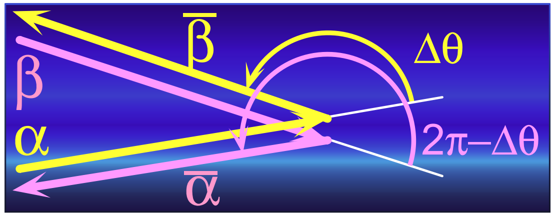
Metallic Nanotubes: Perfect Channel without Backscattering

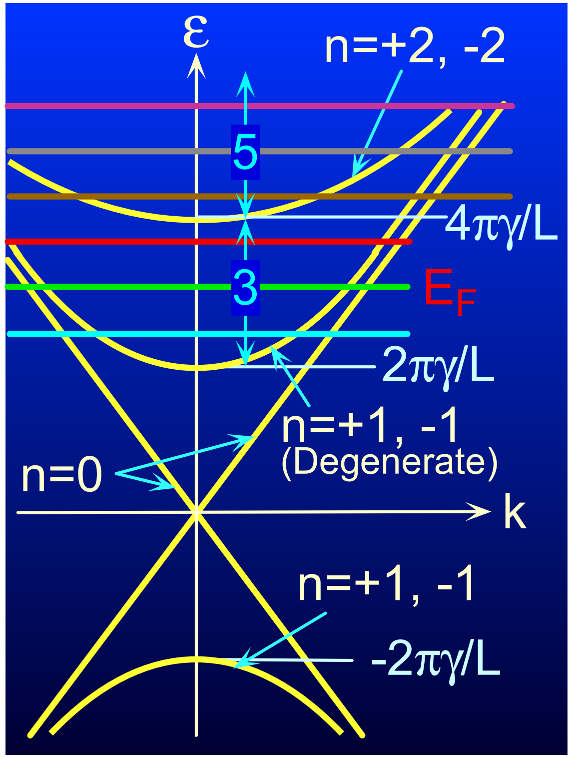
T. Ando and H. Suzuura, J. Phys. Soc. Jpn. 71, 2753 (2002)

Time reversal processes: $\boxed{\alpha \rightarrow \bar{\beta}}$ $\Leftrightarrow$ $\boxed{\beta \rightarrow \bar{\alpha}}$ $\Rightarrow$ $r_{\bar{\beta}\alpha} = -r_{\bar{\alpha}\beta}$

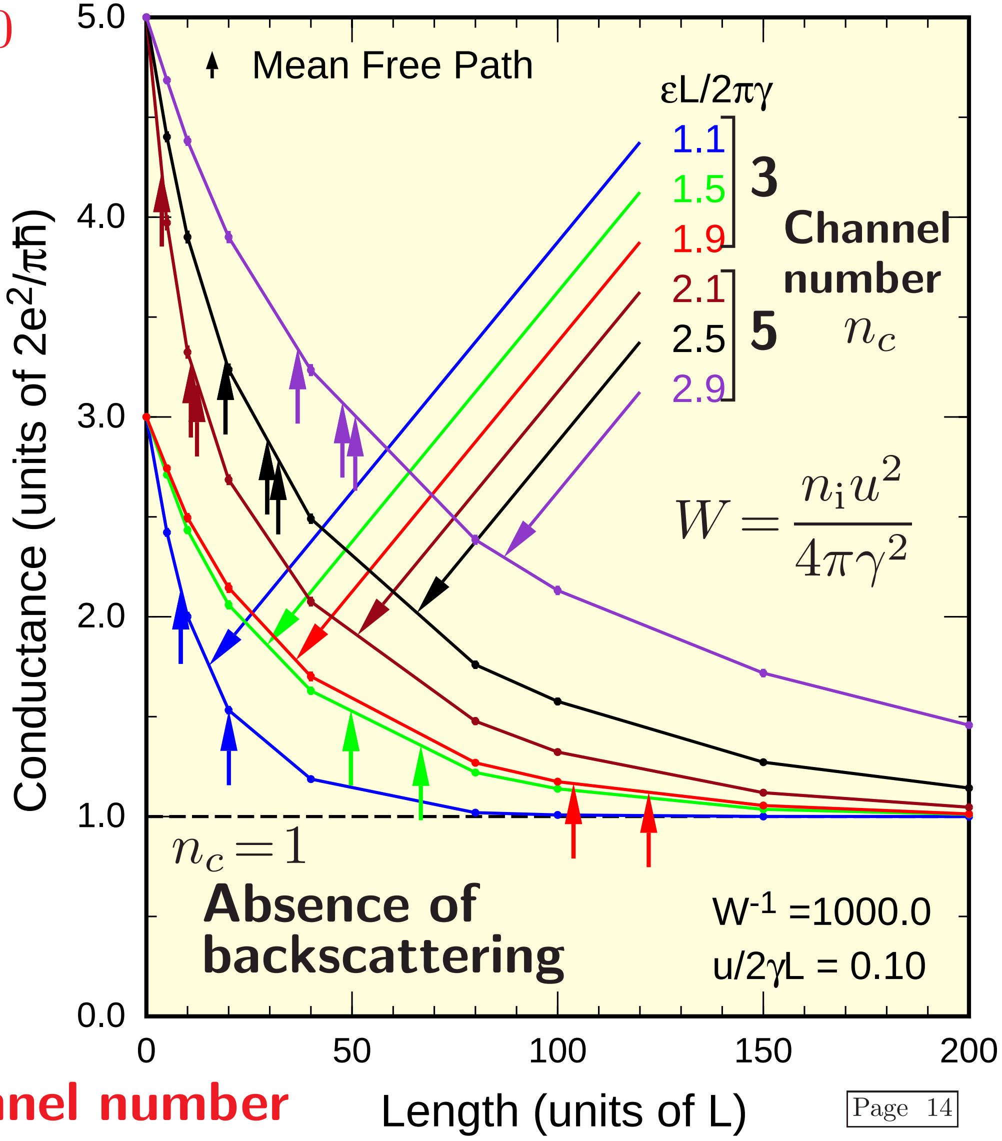
Reflection matrix ⇒ $\det(r) = 0$

⇒ **Perfect channel**





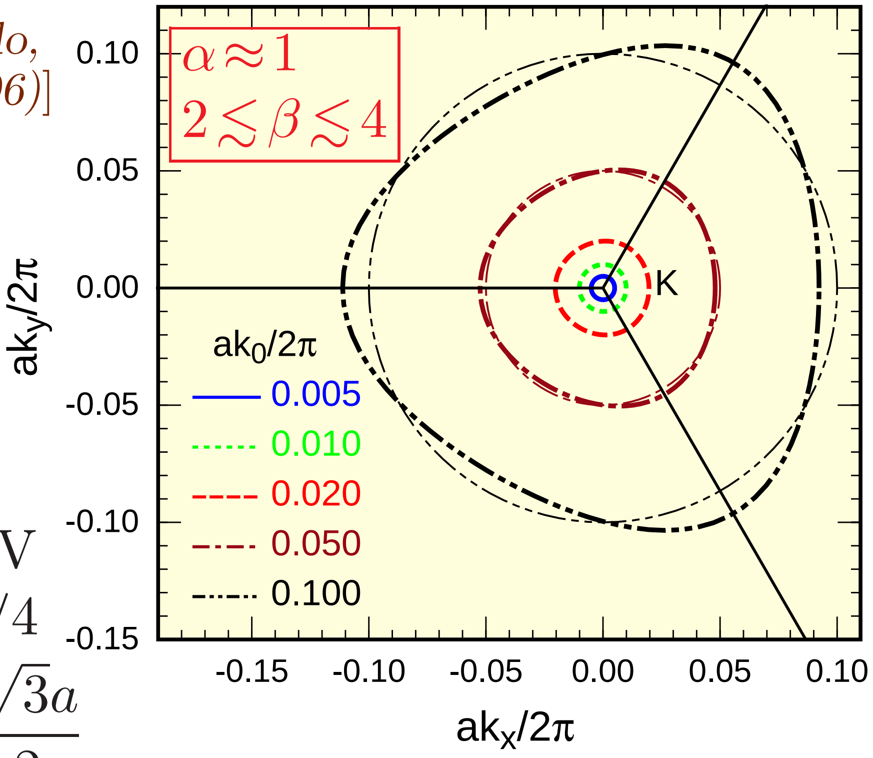
Odd channel number



Symmetry Breaking Effects: Symplectic ⇒ Unitary

Trigonal warping (S) *[H. Ajiki & T. Ando, JPSJ 65, 505 (1996)]*

$$\mathcal{H}' = \alpha \frac{\gamma a}{4\sqrt{3}} \begin{pmatrix} 0 & (\hat{k}_x + i\hat{k}_y)^2 \\ (\hat{k}_x - i\hat{k}_y)^2 & 0 \end{pmatrix}$$



Lattice distortion *[H. Suzuura & T. Ando, PRB 65, 235412 (2002)]*

$$\mathcal{H}' = g_1(u_{xx} + u_{yy}) + g_2[(u_{xx} - u_{yy})\sigma_x - 2u_{xy}\sigma_y]$$

Deformation potential : $g_1 \sim 16\ \mathrm{eV}$

Bond-length (b) **change:** $g_2 \approx \beta\gamma_0/4$

$$\beta = -\frac{d\ln\gamma_0}{d\ln b}, \quad \gamma = \frac{\sqrt{3}\gamma_0 a}{2}, \quad b = \frac{\sqrt{3}a}{2}$$

$$u_{xx} = \frac{\partial u_x}{\partial x} + \frac{u_z}{R} \quad u_{yy} = \frac{\partial u_y}{\partial y} \quad u_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

$$p = 1 - \frac{3}{8}\frac{\gamma'}{\gamma}$$

Curvature: $\mathcal{H}' = p\frac{\gamma a}{4\sqrt{3}}\left[\left(\frac{\partial^2 u_z}{\partial x^2} - \frac{\partial^2 u_z}{\partial y^2}\right)\sigma_x - 2\frac{\partial^2 u_z}{\partial x \partial y}\sigma_y\right]$

$$\gamma = -\frac{\sqrt{3}}{2}V_{pp}^{\pi}a$$

$$\gamma' = \frac{\sqrt{3}}{2}(V_{pp}^{\sigma} - V_{pp}^{\pi})a$$

Optical phonon: $\mathcal{H}' = -\frac{\beta\gamma}{b^2}\boldsymbol{\sigma} \times [\boldsymbol{u}_A - \boldsymbol{u}_B]$

[T. Ando, JPSJ 69, 1757 (2000)]

[K. Ishikawa & T. Ando, JPSJ 75, 084713 (2006)]

Symmetry Breaking Effects and Crossover

Intervalley (K↔K')

Symplectic ⇒ Orthogonal

- **Short-range scatterers** $(d/a<1)$
- **Zone-boundary phonon**

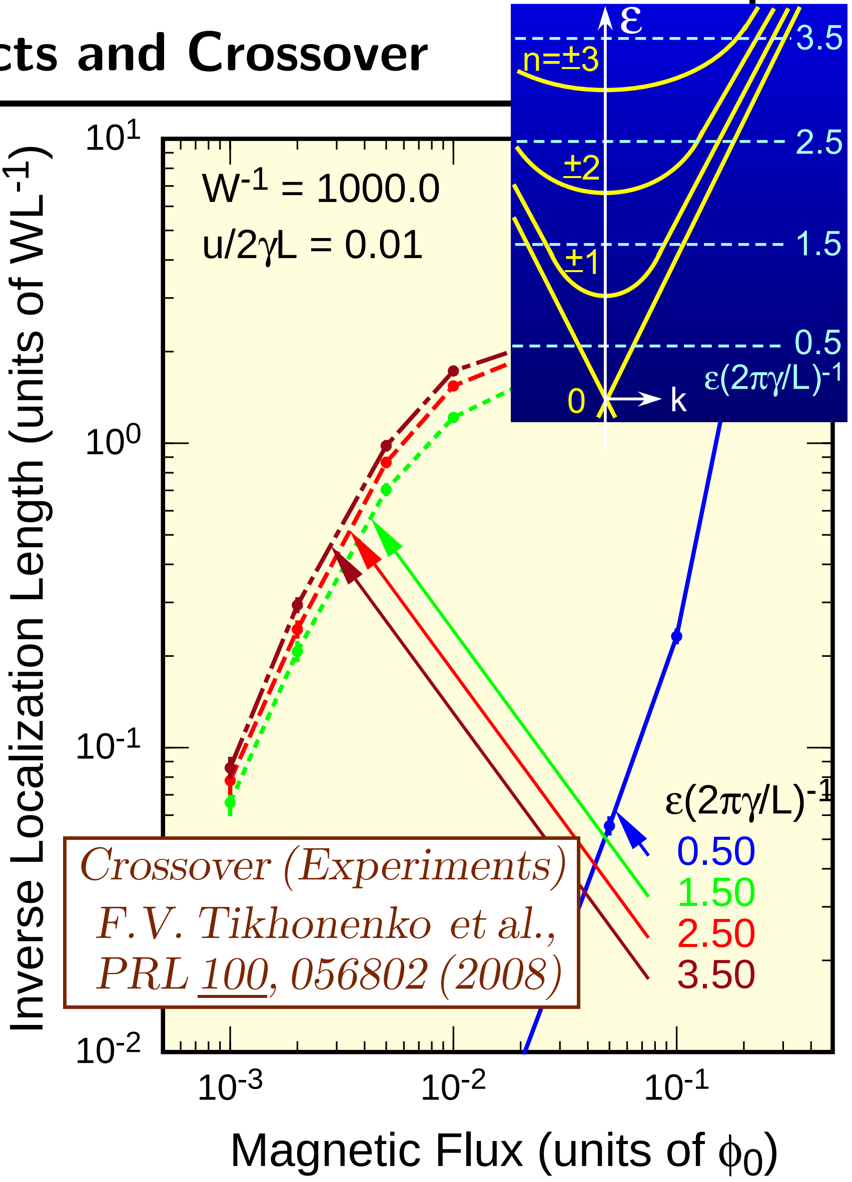
Metallic nanotubes *H. Suzuura & T. Ando*
JPSJ 77, No. 4 (2008)

Absence of backscattering: Robust
Perfect channel : Fragile

T. Ando, JPSJ 73, 1273 (2004)
TA & K. Akimoto, JPSJ 73, 2895 (2004)
K. Akimoto & TA, JPSJ 73, 2194 (2004)
T. Ando, JPSJ 75, 054701 (2006)

Quantum correction to conductivity

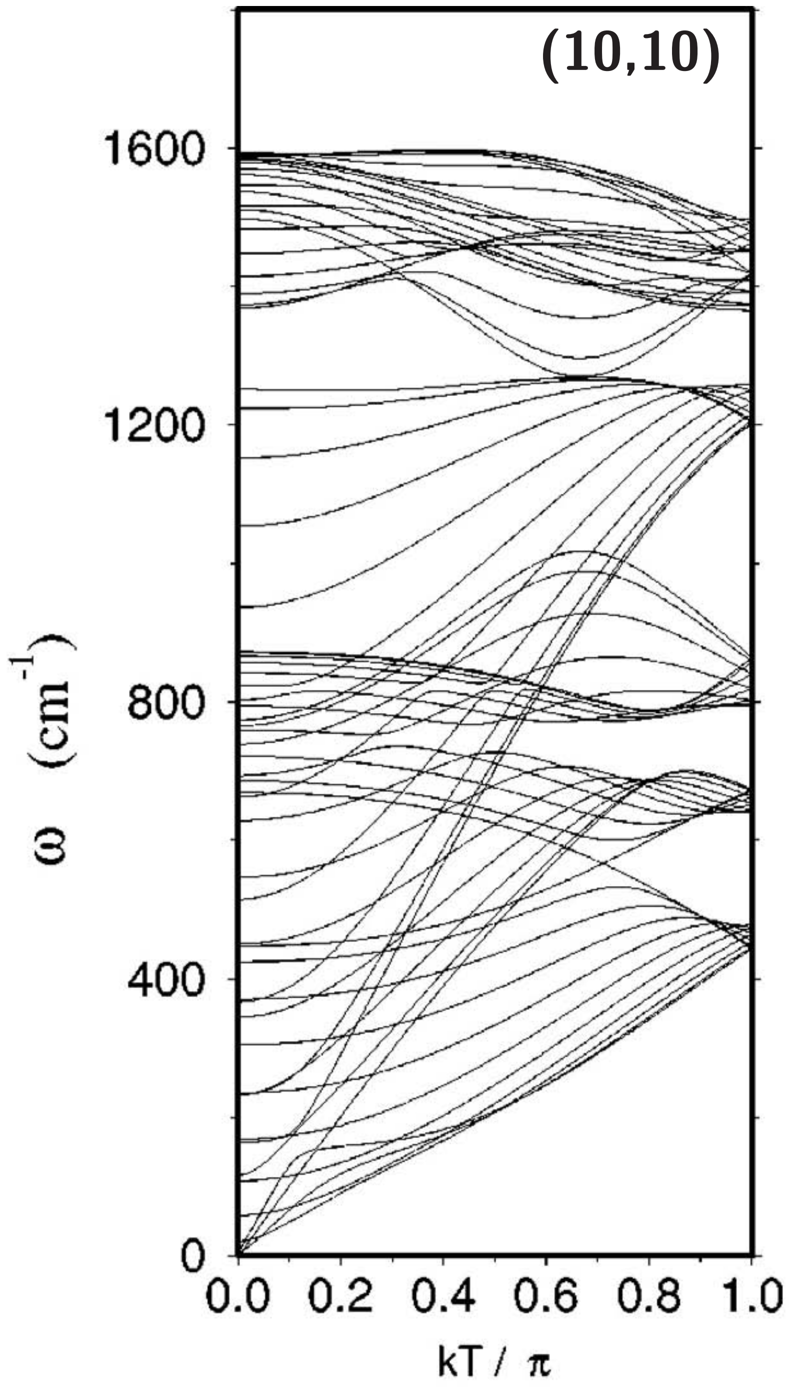
		Magnetoresistance
Orthogonal	$\Delta\sigma<0$	**Negative**
Symplectic	$\Delta\sigma>0$	**Positive**
Unitary	$\Delta\sigma=0$	**No**



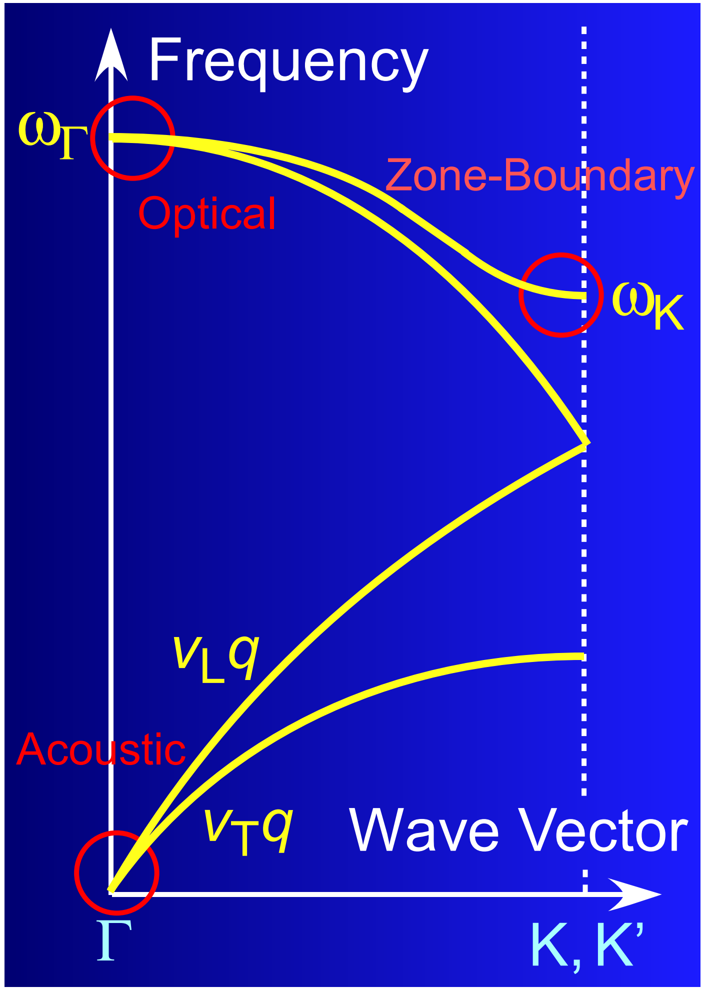
Crossover (Experiments)
F.V. Tikhonenko et al.,
PRL 100, 056802 (2008)

Crossover: *H. Suzuura and T. Ando, PRL 81, 266603 (2002)*
Experiments: *S.V. Morozov et al., PRL 97, 016801 (2006)* $(\Delta\sigma\approx0)$
Localization *X.-S. Wu et al., PRL 98, 136801 (2007)* $(\Delta\sigma>0)$
UCF **Theory:** *E. McCann et al., PRL 97, 146805 (2006)*

Phonons in Carbon Nanotubes



R. Saito et al., PRB 57, 4145 (1998)



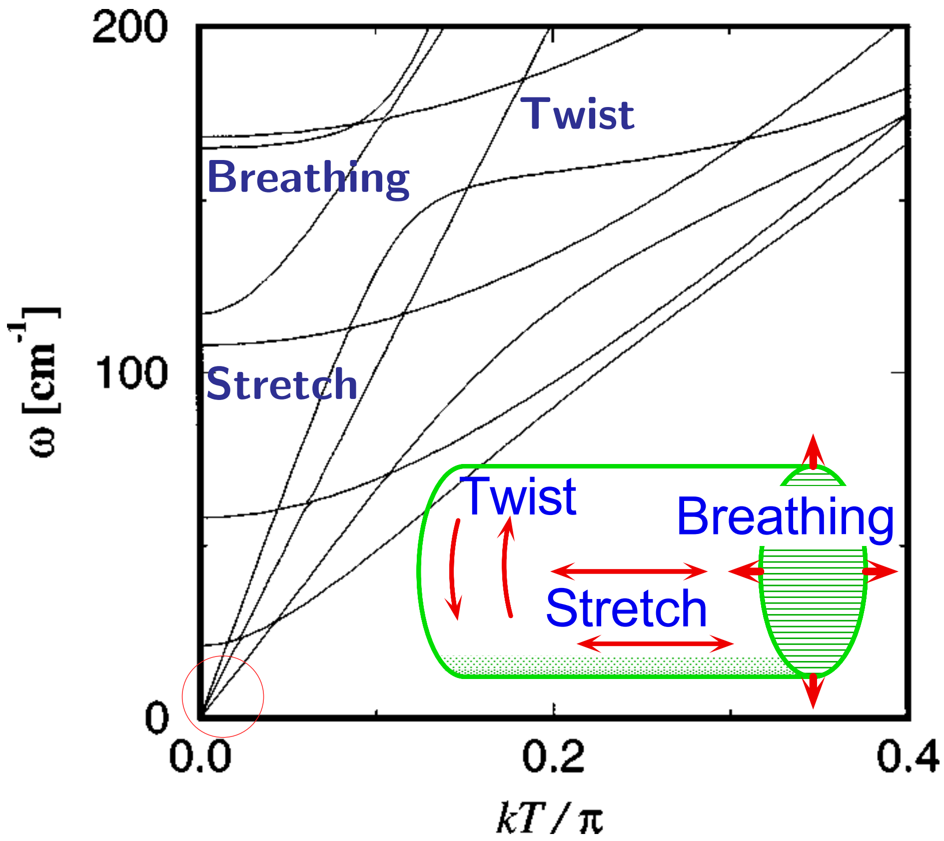
Continuum models in graphene

- **Acoustic phonons**
- **Optical phonons (zone-center)**
- **Zone-boundary phonon**

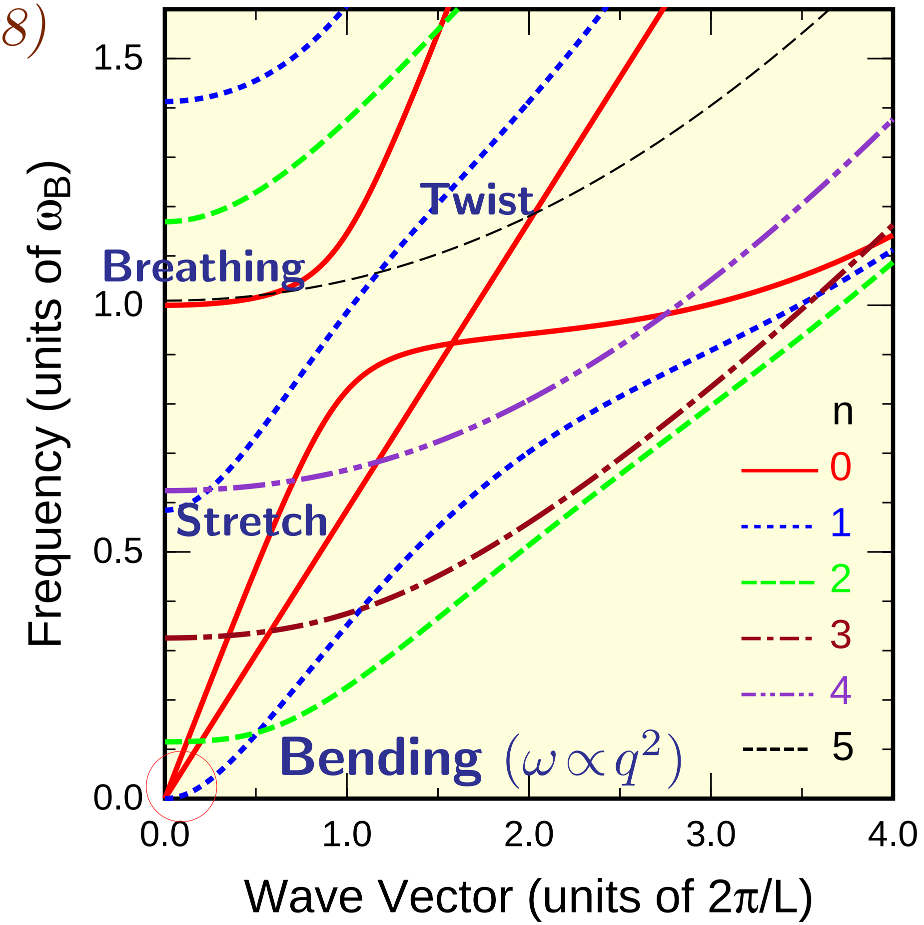
Long-Wavelength Phonons in Carbon Nanotubes

Force-constants model calculation

R. Saito et al., PRB 57, 4145 (1998)



Continuum model



Transverse modes
Twist
Longitudinal modes
Stretch + Breathing

H. Suzuura and T. Ando,
Phys. Rev. B 65, 235412 (2002)

$\omega_B \Leftarrow$ **Frequency of breathing mode** $(\propto 1/L)$

Continuum Phonon Model and Electron-Phonon Interaction

H. Suzuura and T. Ando, Phys. Rev. B 65, 235412 (2002)

Energy functional of lattice distortion $\boldsymbol{u}=(u_x,u_y,u_z)$

$$U_{\rm 2D}[\boldsymbol{u}] = \int dxdy\,\frac{1}{2}\Big(B(u_{xx}+u_{yy})^2+\mu\big[(u_{xx}-u_{yy})^2+4u_{xy}^2\big]\Big)$$

$$u_{xx}=\frac{\partial u_x}{\partial x}+\frac{u_z}{R},\quad u_{yy}=\frac{\partial u_y}{\partial y},\quad 2u_{xy}=\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}$$

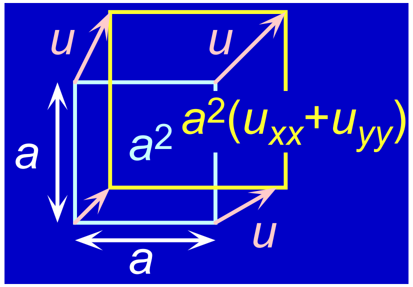
B **: Bulk modulus**
μ **: Shear modulus**

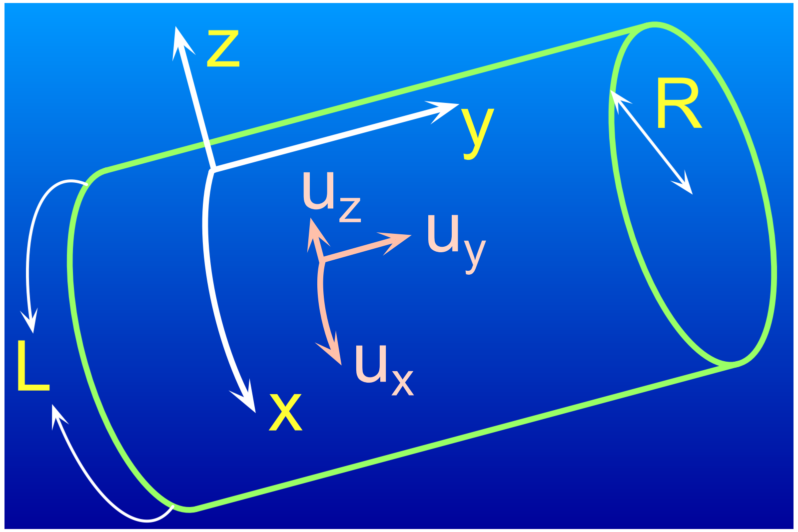
Electron-phonon interaction

Curvature

$$U_{\rm c}[\boldsymbol{u}]\propto\int\Big[\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{1}{R^2}\Big)u_z\Big]^2 d\boldsymbol{r}$$

$$V_{\text{el-ph}}=\begin{array}{c}\begin{array}{cc}KA & KB\end{array}\\ \begin{pmatrix}V_1 & V_2\\ V_2^+ & V_1\end{pmatrix}\end{array}$$





$$V_1=g_1\,(u_{xx}+u_{yy})$$

$$V_2=g_2\,e^{3i\eta}(u_{xx}-u_{yy}+2iu_{xy})$$

η: **Chiral angle**
g_1: **Deformation potential**
g_2: **Bond-length change** [cf. $(\mathrm{CH})_x$]
$(g_1\gg|g_2|)$

No contribution of g_1 in metallic CN

Phonon-Limited Resistivity

H. Suzuura and T. Ando, Physica E 6, 864 (2000)

Breathing mode

$$k_B T_B = \hbar\omega_B \;\Big(\sim \frac{2000}{R\,[\text{Å}]}\,\text{K}\Big)$$

Mean free path (Å)

$$\Lambda \sim 1000 \times R\,[\text{Å}] \times \frac{300}{T\,[\text{K}]}$$

$$\Lambda \sim 1\ \mu\text{m} \quad (R \sim 10\ \text{Å},\ T \sim 300\ \text{K})$$

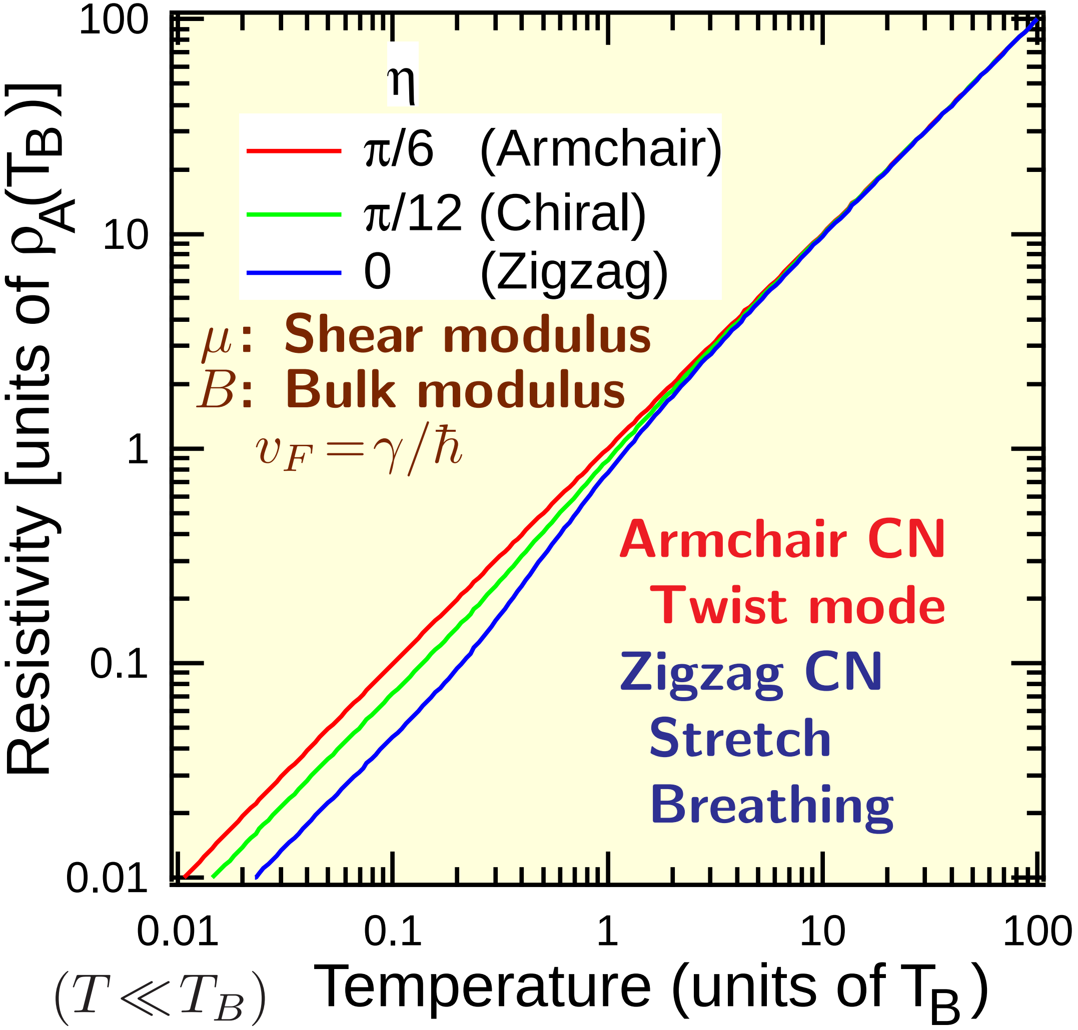
Resistivity

Armchair CN

$$\rho_A(T) = \frac{g_2^2 k_B T}{2e^2 \hbar v_F^2 R \mu}$$

Zigzag CN

$$\rho_Z(T) = \begin{cases} \dfrac{B}{B+\mu}\rho_A(T) & (T \ll T_B) \\ \rho_A(T) & (T \gg T_B) \end{cases}$$



Armchair ⇒ *C.L. Kane et al., Europhys. Lett. 41, 683 (1998)*

Photo-relaxation: *T. Hertel and G. Moos, PRL 84, 5002 (2000)*

Long-Wavelength Optical Phonon

[*K. Ishikawa and T. Ando, J. Phys. Soc. Jpn.* 75, *084713 (2006)*]

Optical phonon (lattice displacement $\boldsymbol{u}$): $\mathcal{H}_{\rm ph}=\sum_{\boldsymbol{q},\mu}\hbar\omega_0\Big(b^\dagger_{\boldsymbol{q}\mu}b_{\boldsymbol{q}\mu}+\frac{1}{2}\Big)$

$$\boldsymbol{u}(\boldsymbol{r})\equiv\frac{1}{\sqrt{2}}[\boldsymbol{u}_A(\boldsymbol{r})-\boldsymbol{u}_B(\boldsymbol{r})]=\sum_{\boldsymbol{q},\mu}\sqrt{\frac{\hbar}{2NM\omega_0}}(b_{\boldsymbol{q}\mu}+b^\dagger_{-\boldsymbol{q}\mu})\boldsymbol{e}_\mu(\boldsymbol{q})e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$$

Electron-phonon interaction $\beta_\Gamma=-\frac{d\ln\gamma_0}{d\ln b},\quad \gamma=\frac{\sqrt{3}\gamma_0 a}{2},\quad b=\frac{\sqrt{3}a}{2}$

Bond length

$$\mathcal{H}^{\rm K}_{\rm int}=-\sqrt{2}\frac{\beta_\Gamma\gamma}{b^2}\boldsymbol{\sigma}\times\boldsymbol{u}(\boldsymbol{r})\qquad \mathcal{H}^{\rm K'}_{\rm int}=+\sqrt{2}\frac{\beta_\Gamma\gamma}{b^2}\boldsymbol{\sigma}^*\times\boldsymbol{u}(\boldsymbol{r})$$

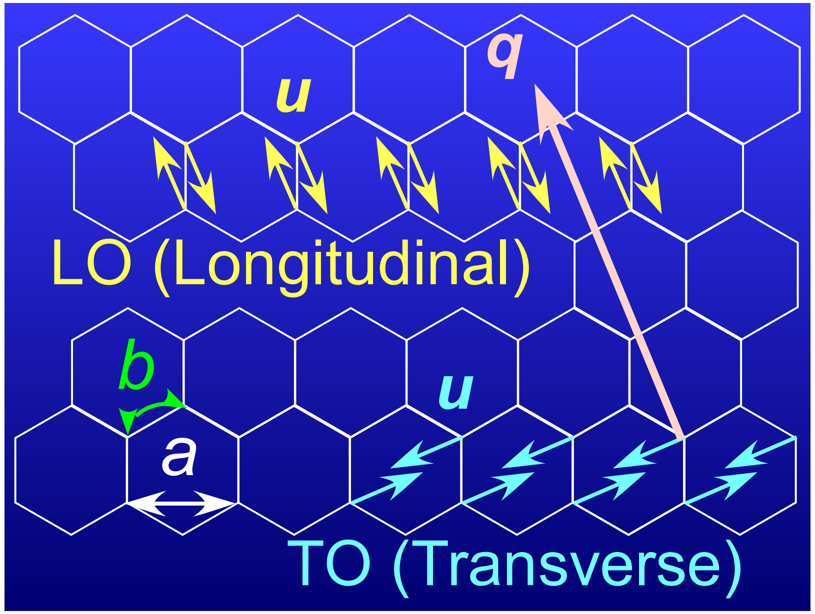
Dimensionless coupling constant

$$\lambda=\frac{g_{\rm v}g_{\rm s}}{4}\frac{36\sqrt{3}}{\pi}\frac{\hbar^2}{2Ma^2}\frac{1}{\hbar\omega_0}\Big(\frac{\beta_\Gamma}{2}\Big)^2$$

$$\approx 3\times10^{-3}\Big(\frac{\beta_\Gamma}{2}\Big)^2\qquad(4\gtrsim\beta_\Gamma\gtrsim 2)$$

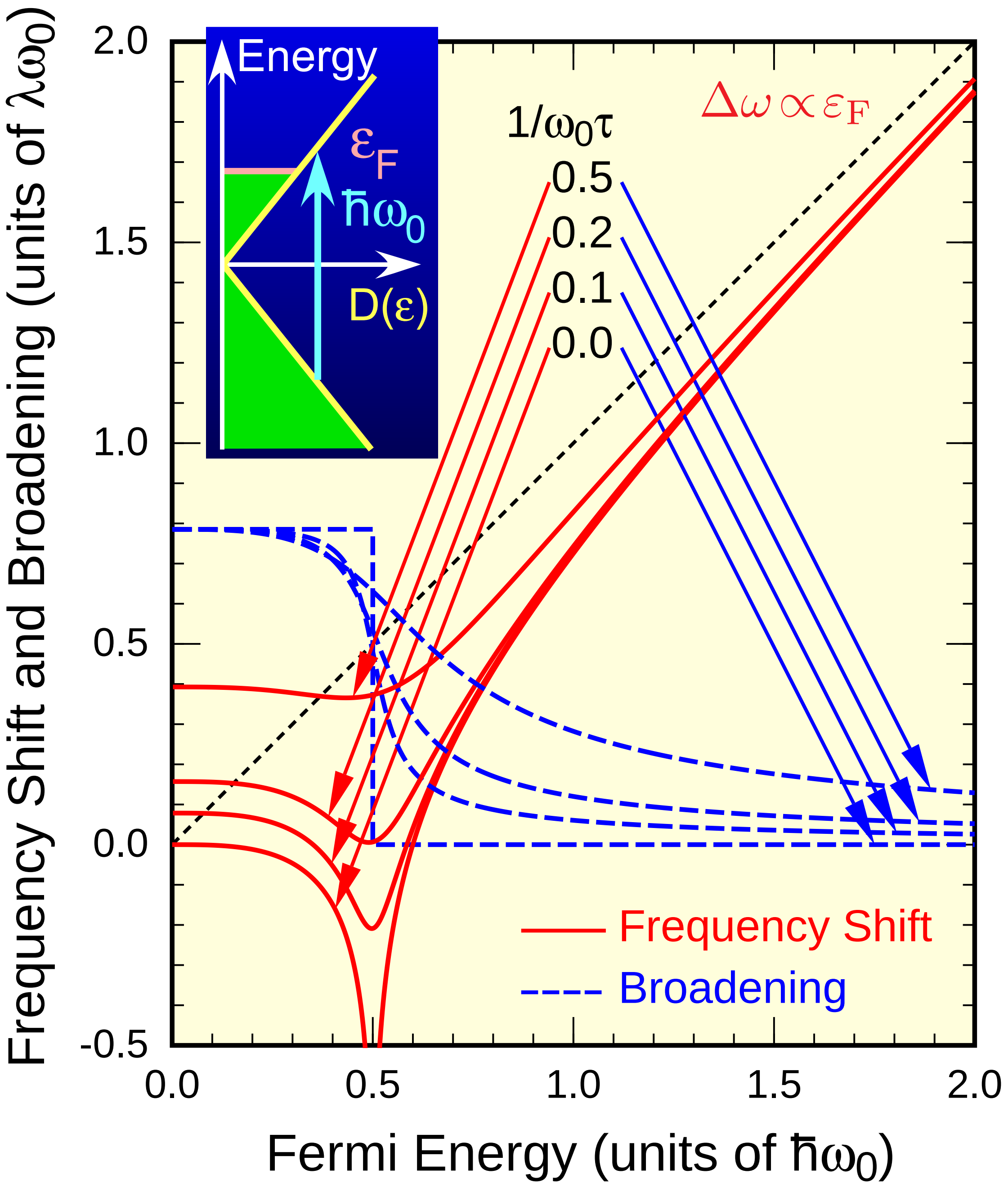
$$\hbar\omega_0=0.196\,\mathrm{eV}$$

$$M=1.993\times10^{-23}\,\mathrm{g}$$



Frequency Shift and Broadening

[T. Ando, J. Phys. Soc. Jpn. 75, 124701 (2006)]



Logarithmic singularity at $\varepsilon_F/\hbar\omega_0=1/2$

Phonon Green's function

$$D(\boldsymbol{q},\omega)=\frac{2\omega_0}{\omega^2-\omega_0^2-2\omega_0\Pi(\boldsymbol{q},\omega)}$$

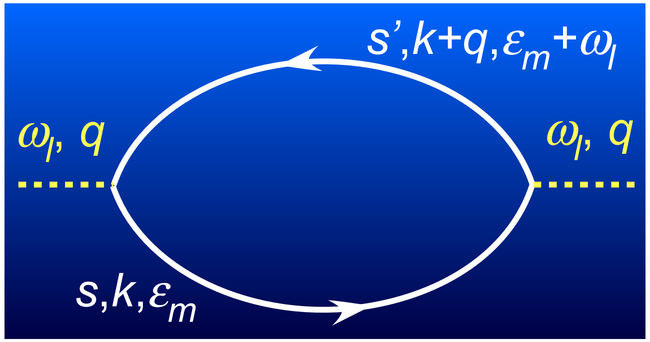
Energy shift: $\Delta\omega=\mathrm{Re}\tilde{\Pi}(\boldsymbol{q},\omega_0)$

Broadening : $\Gamma=-\mathrm{Im}\tilde{\Pi}(\boldsymbol{q},\omega_0)$

Self-energy

$$\tilde{\Pi}(\boldsymbol{q},\omega)=\Pi(\boldsymbol{q},\omega)-\Pi_{\varepsilon_F=0}(\boldsymbol{q},0)$$

(Avoid double counting)

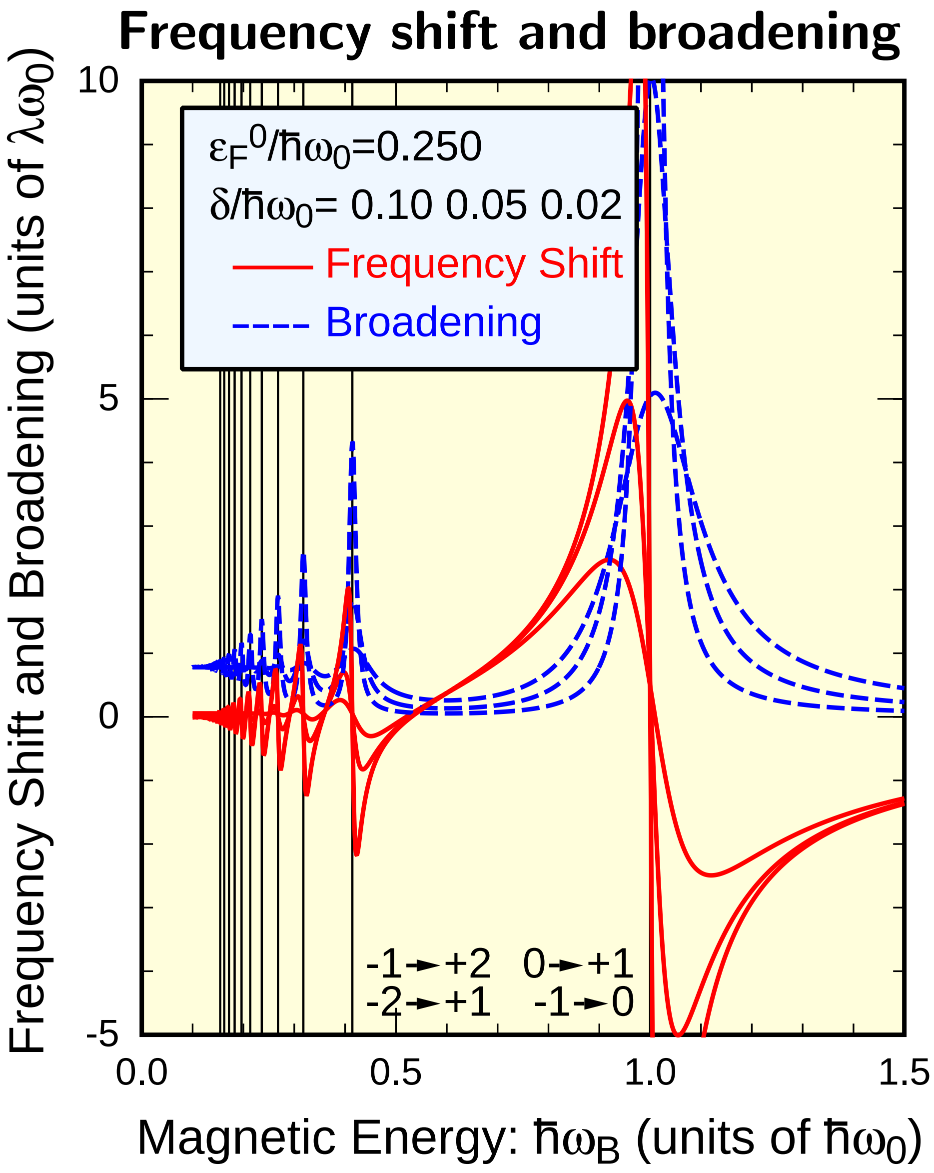


⇒ *M. Lazzeri & F. Mauri, PRL 97, 266407 (2006)*

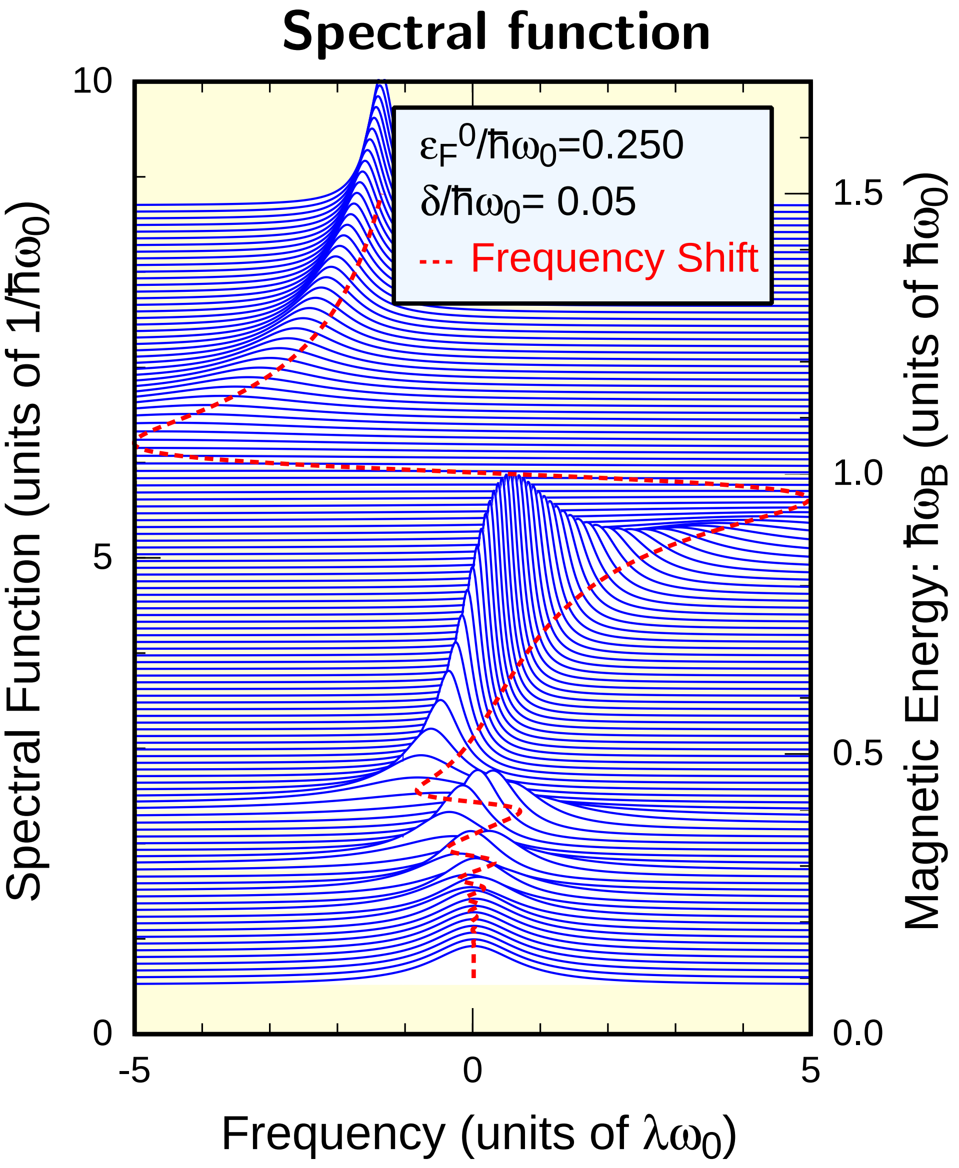
Magnetic Oscillation of Optical Phonon $(\varepsilon_{\mathrm{F}}^{0}/\hbar\omega_0 = 0.25)$

[*T. Ando, J. Phys. Soc. Jpn.* *76*, *024712 (2007)*]

Frequency shift and broadening



Spectral function



Zone-Boundary Phonon

[*H. Suzuura and T. Ando, JPSJ 77, 044703 (2008)*]

Highest frequency phonon: ω_K (Kekulé distortion)

Electron-phonon interaction $\beta_K = -\dfrac{d\ln\gamma_0}{d\ln b}$

$$\mathcal{H}_{\text{el-ph}} = 2\frac{\beta_K\gamma}{b^2}\begin{array}{c} \begin{array}{cc} K & K' \end{array} \\ \begin{pmatrix} 0 & \Delta(\boldsymbol{r})\sigma_y \\ \Delta(\boldsymbol{r})^\dagger\sigma_y & 0 \end{pmatrix} \end{array}$$

$$\Delta(\boldsymbol{r}) = \sum_{\boldsymbol{q}}\sqrt{\frac{\hbar}{2NM\omega_K}}(b_{K'\boldsymbol{q}}+b^\dagger_{K-\boldsymbol{q}})\,e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$$

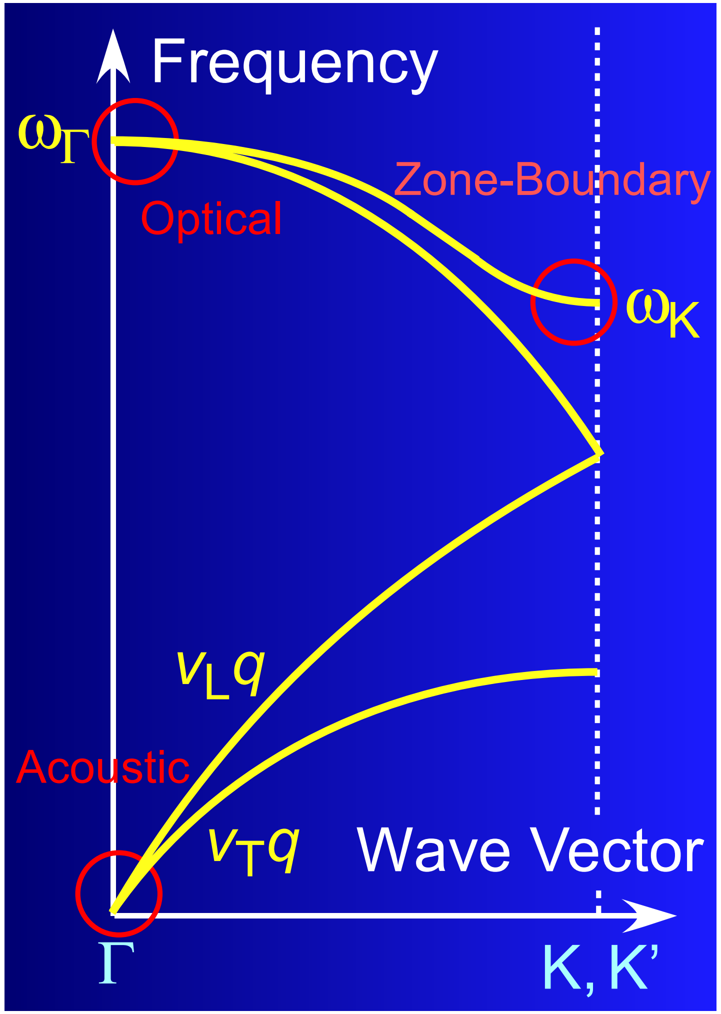
cf: N.A. Viet et al., JPSJ 63, 3036 (1994)

Dimensionless coupling constant

$$\lambda_K = \frac{36\sqrt{3}}{\pi}\frac{\hbar^2}{2Ma^2}\frac{1}{\hbar\omega_K}\Big(\frac{\beta_K}{2}\Big)^2$$

$$\approx 3.5\times10^{-3}\Big(\frac{\beta_K}{2}\Big)^2 \quad (\hbar\omega_K = 0.161\,\text{eV})$$

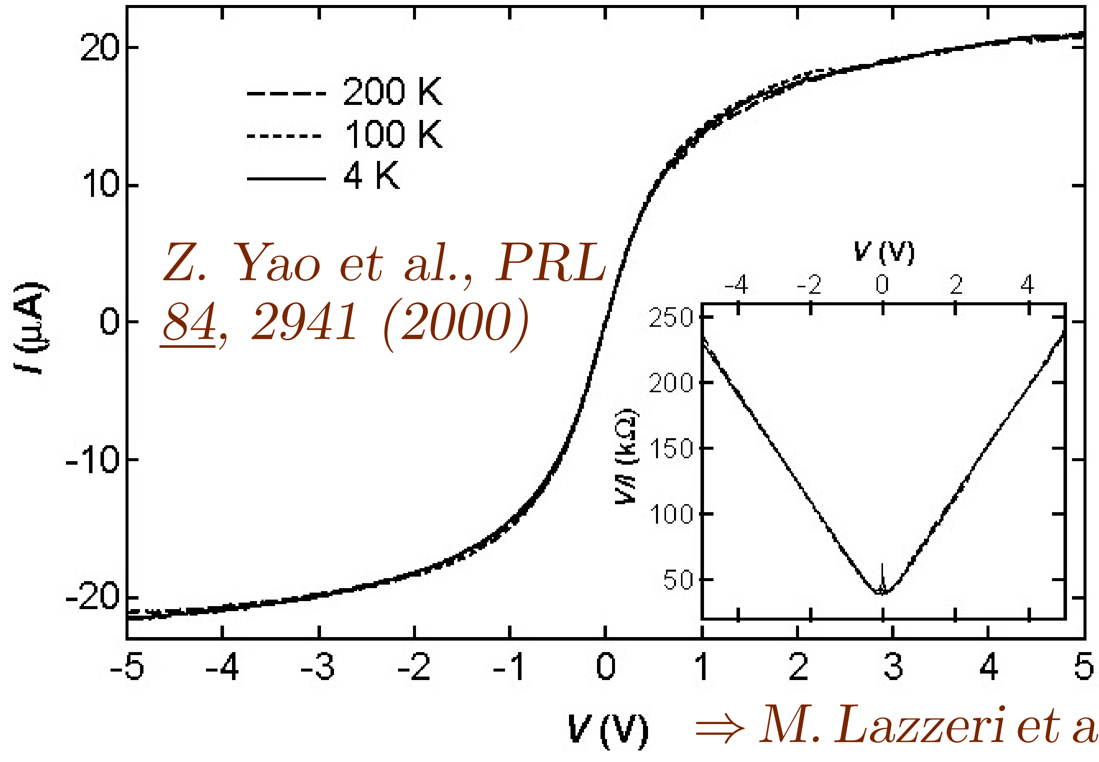
$$cf:\ \lambda_\Gamma \approx 2.9\times10^{-3}\Big(\frac{\beta_\Gamma}{2}\Big)^2 \quad (\hbar\omega_\Gamma = 0.196\,\text{eV})$$



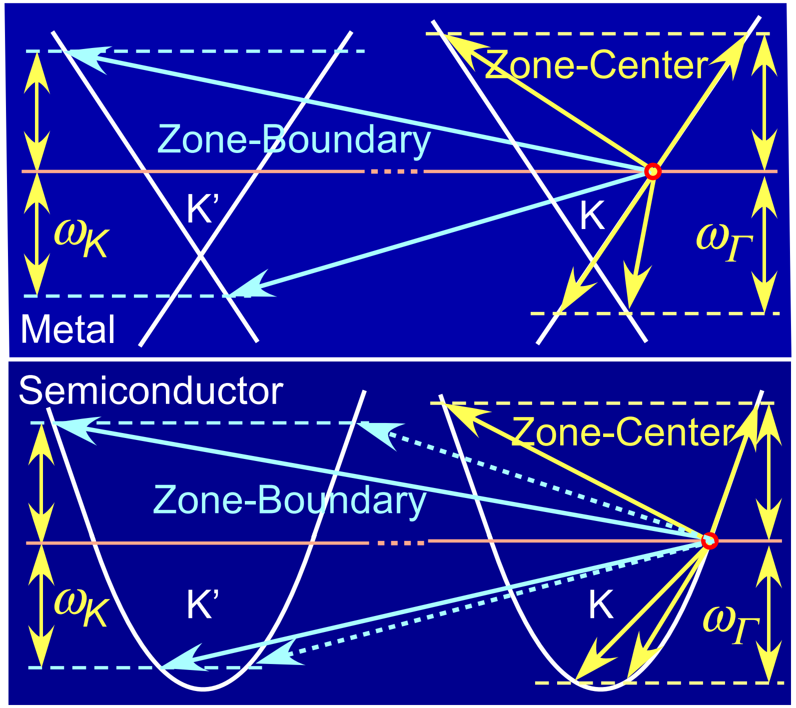
ω_K: Dominant in high-field transport

High-Field Transport: Zone-Center vs Zone-Boundary Phonons

Phonon	**Zone-center**	**Zone-boundary**
Scattering	**Isotropic intra-valley**	**Backward inter-valley**
Relaxation time	$\frac{\hbar}{\tau_\Gamma} \approx \frac{1}{2}\lambda_\Gamma \frac{2\pi\gamma}{L}$	$\frac{\hbar}{\tau_K} \approx \lambda_K \frac{2\pi\gamma}{L}$
Mean free path	$\Lambda_\Gamma \approx \frac{L}{\pi\lambda_\Gamma} \sim 110 \left(\frac{2}{\beta_\Gamma}\right)^2 L$	$\Lambda_K \approx \frac{L}{2\pi\lambda_K} \sim 45 \left(\frac{2}{\beta_K}\right)^2 L$



Z. Yao et al., PRL *84*, *2941 (2000)*



⇒ *M. Lazzeri et al., PRL* *95*, *236802 (2005)*

Summary: Quantum Transport in Graphene and Nanotubes

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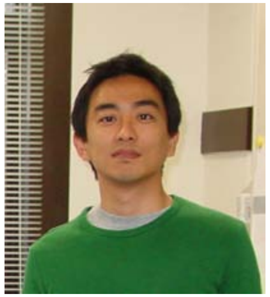

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1. Introduction
   - Weyl's equation for neutrino
   - Berry's phase & topological anomaly
2. Zero mode anomalies
   - Density of states and conductivity
   - Dynamical conductivity
   - Self-consistent Born approximation
3. Special time reversal symmetry
   - Symmetry crossover
4. Phonons and electron-phonon interaction
   - Acoustic phonon
   - Optical phonon
   - Zone-boundary phonon

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