The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1965-19

9th Workshop on Three-Dimensional Modelling of Seismic Waves Generation, Propagation and their Inversion

22 September - 4 October, 2008

Tsunami Theory
Lecture 2: Extensions and Examples

Steven N. Ward

Institute of Geophysics
& Planetary Physics
University of California Santa Cruz
U.S.A.

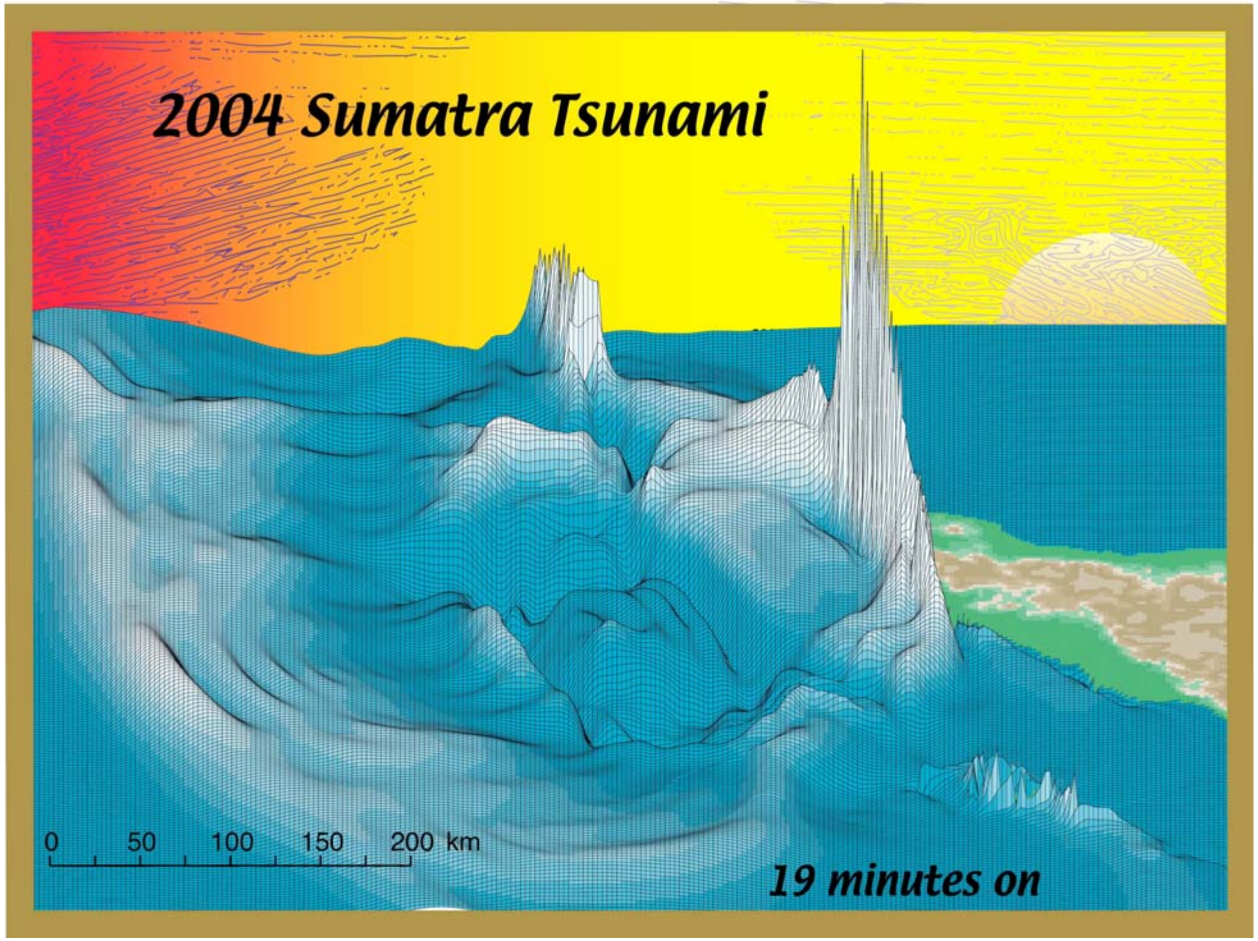
ward@pmc.ucsc.edu

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Tsunami Theory- Lecture 2: Extensions and Examples

ICTP, Trieste Italy 9/24/08

Steven N. Ward
Institute of Geophysics and Planetary Physics
University of California Santa Cruz



All we really need to do now is move to VARIABLE DEPTH OCEAN

At the ocean surface, (z=0) the frequency version of (6.1.3) for vertical tsunami motions from bottom uplift source starting at t_0 and lasting T_R seconds is

$$u_z(\mathbf{r},t)=\sum_{m=-\infty}^{m=\infty}\int_0^{\infty}\frac{k(\omega)\,d\omega}{2\pi u(\omega)\cosh(k(\omega)h)}J_m(\omega T(\omega,\mathbf{r}))e^{im\theta}\cos(\omega(t-t_0-\tau/2))$$
$$\times\frac{\sin(\omega\tau/2)}{-\omega T_R/2}\Big|_{\tau=0}^{\tau=\min(t-t_0,T_R)}\int_{Area}d\mathbf{r}_0U(\mathbf{r}_0)J_m(k(\omega)r_0)e^{-im\theta_0}$$

(6.2.1)

with travel time $T(\omega,\mathbf{r})=r/c(\omega)$. The $c(\omega)=\omega/k(\omega)$ and $u(\omega)$ are the tsunami phase and group velocities. In moving to a variable depth ocean, (3) becomes

$$u_z(\mathbf{r},t)=\sum_{m=-\infty}^{m=\infty}\int_0^{\infty}\frac{k_c(\omega)\,d\omega}{2\pi u_c(\omega)\cosh(k_c(\omega)h_c)}J_m(\omega T_p(\omega,\mathbf{r}))e^{im\theta}\cos(\omega(t-t_0-\tau/2))$$
$$\left[\frac{u_c(\omega)}{u(\omega)}\right]^{1/2}G(\mathbf{r})\times\frac{\sin(\omega\tau/2)}{-\omega T_R/2}\Big|_{\tau=0}^{\tau=\min(t-t_0,T_R)}\int_{Area}d\mathbf{r}_0U(\mathbf{r}_0)J_m(k_c(\omega)r_0)e^{-im\theta_0}$$

(6.2.2)

where sub-c variables are calculated using the water depth h_c at the origin, e.g. k_c is found from $\omega^2=gk_c\tanh(k_ch_c)$. The origin is taken as a representative location near the source.

$$u_z(\mathbf{r},t) = \sum_{m=-\infty}^{m=\infty} \int_0^\infty \frac{k_c(\omega)\,d\omega}{2\pi u_c(\omega)\cosh(k_c(\omega)h_c)} J_m(\omega T_p(\omega,\mathbf{r}))e^{im\theta}\cos(\omega(t-t_0-\tau/2))$$

$$\left[\frac{u_c(\omega)}{u(\omega)}\right]^{1/2} G(\mathbf{r})\times\frac{\sin(\omega\tau/2)}{-\omega T_R/2}\Big|_{\tau=0}^{\tau=\min(t-t_0,T_R)} \int_{Area} d\mathbf{r}_0\, U(\mathbf{r}_0) J_m(k_c(\omega)r_0)e^{-im\theta_0}$$

Principal differences between (6.2.1) and (6.2.2) are:

1) Travel time $T_P(\omega,\mathbf{r})=P(r)/\bar{c}(\omega)$ is now calculated over a curved "ray path" of length P and mean phase speed $\bar{c}(\omega)$ over that path.

2) A new shoaling factor $[u_c(\omega)/u(\omega)]^{1/2}$ accounts for wave height changes due to water depth as the wave moves along. Generally you feather this factor in over a few minutes.

3) A new geometrical spreading factor $G(r)\leq 1$ takes into consideration the reduction of wave amplitudes into shadow zones.

4) Certain quantities $[k_c(\omega),u_c(\omega)$, etc] are evaluated at a representative location near the source.

Tsunami slow and grow as they near the coast.

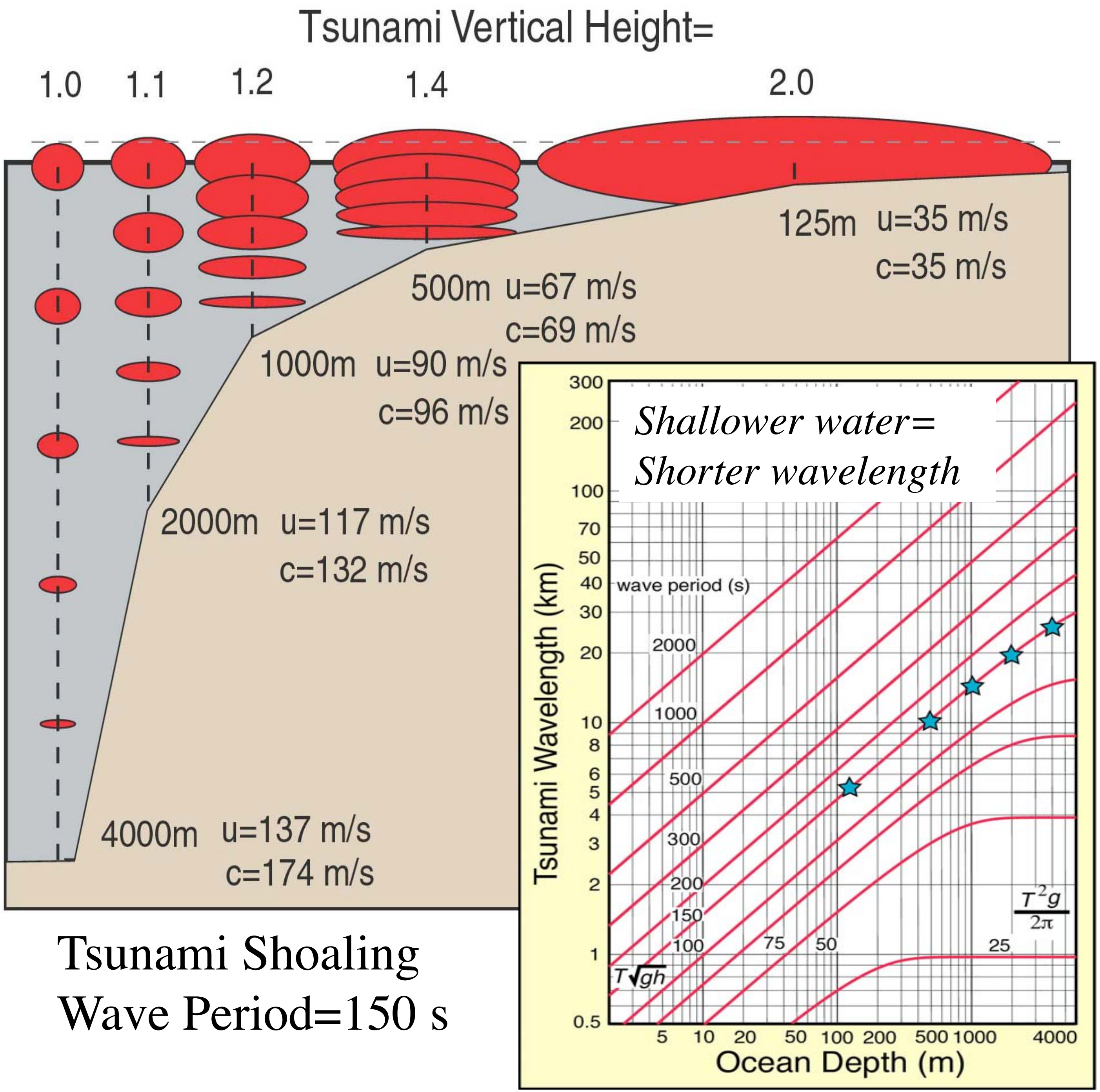
In deep water

V~500 mph

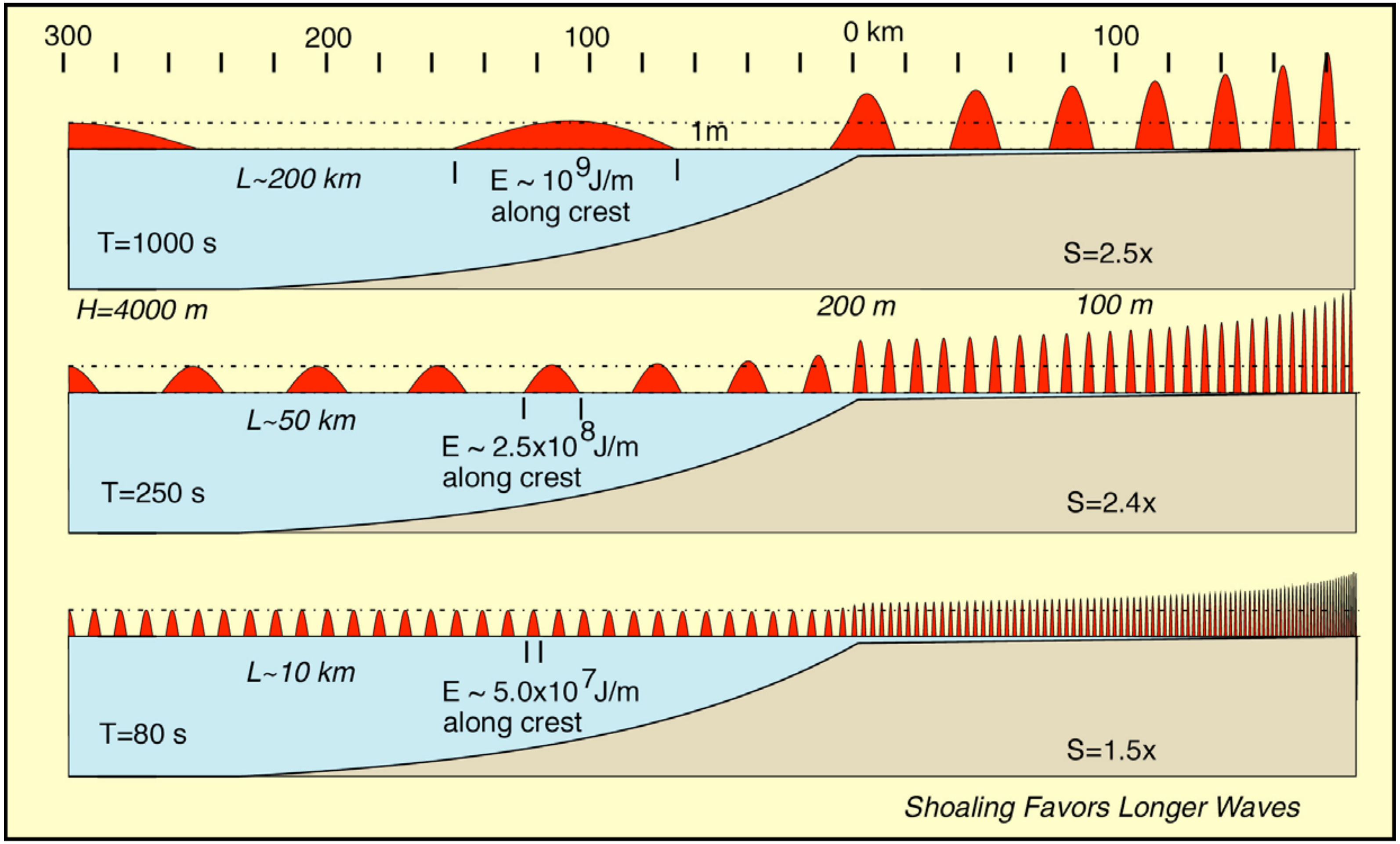
They come ashore about 20-30 mph.

Still -- Can't outrun one to high ground.

Grow by 3 -4x



Tsunami Shoaling
Wave Period=150 s

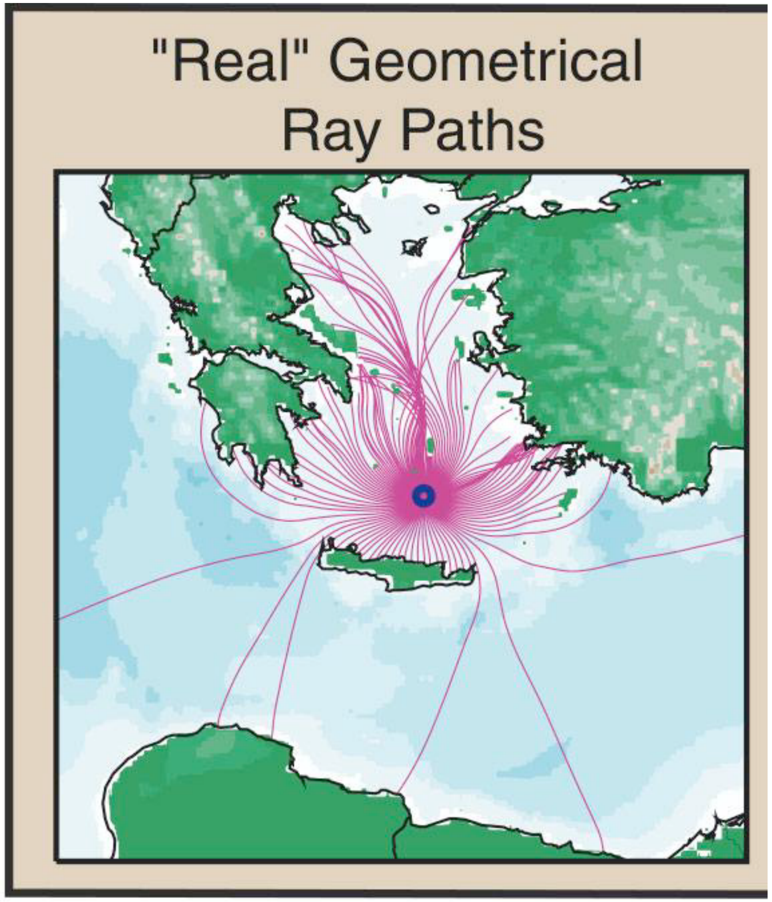


Tsunami steepen in shallow water, but generally do not become steep enough to break. Sorry, you can't surf a tsunami.

Shoaling Coefficient $$S_L(\omega,\mathbf{r},\mathbf{r}_0)=\sqrt{\frac{u(\omega,h(\mathbf{r}_0))}{u(\omega,h(\mathbf{r}))}}\sim\sqrt[1/4]{\frac{h(\mathbf{r}_0)}{h(\mathbf{r})}}$$

I treat tsunami as mode-like vertically and ray-like laterally.

Geometrical Rays for Tsunami in Real Oceans are messy



Example of real tsunami rays

(1) huge lateral variations in velocity

From 250 m/s all the way to ZERO!

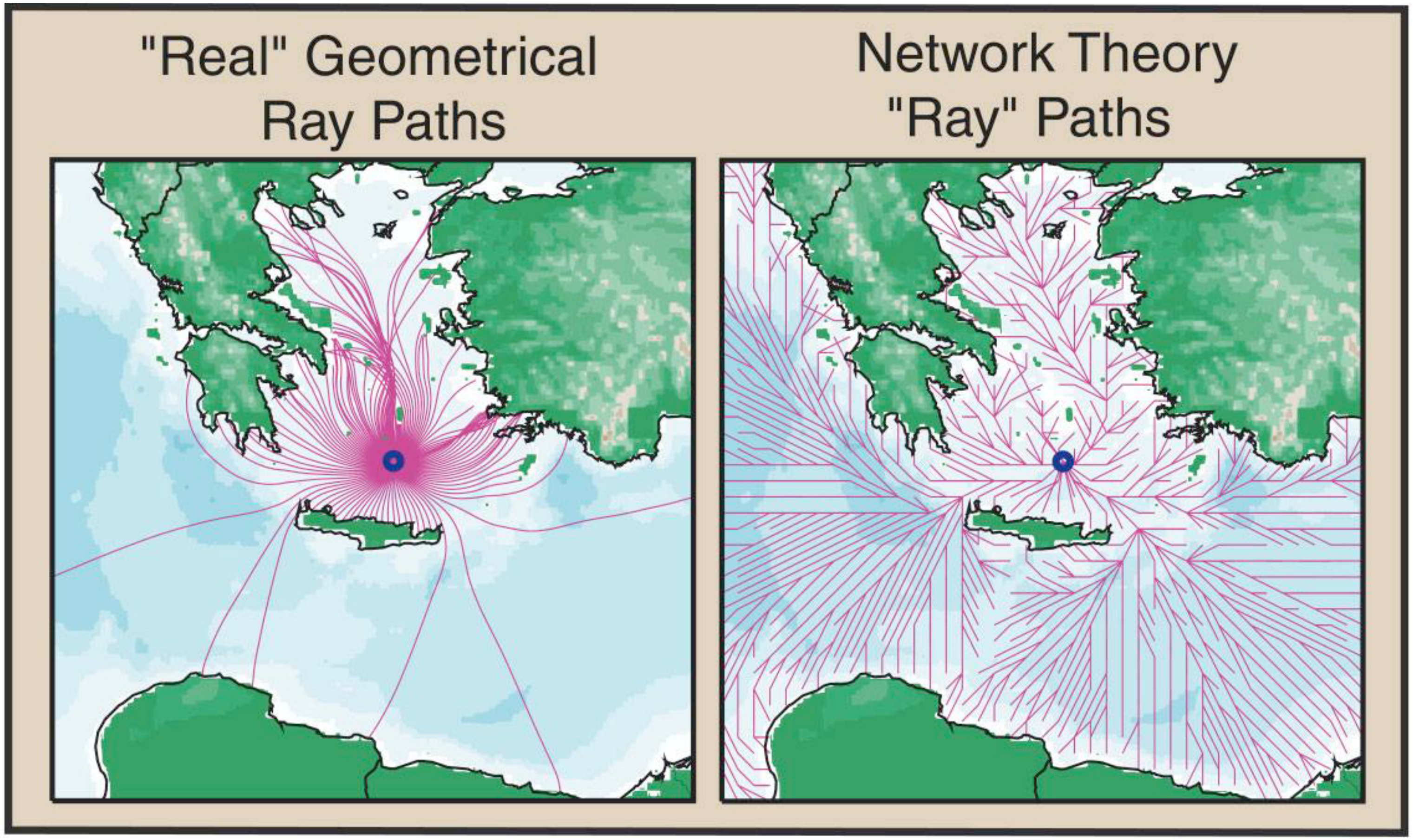
(2) highly dispersive (need rays for each frequency)

(3) caustics and shadows abound.

For these reasons, I have never found real tsunami rays very useful.

Instead of real rays, I employ **“Network rays”**

These are nothing more than an organized such for the Minimum Travel Time Paths. Because real rays are minimum time paths, Network rays bear some resemblance, but there is no physics involved. **Network rays fill all the space so tsunami will always reach any point with a water path to the source.**



No distinction, really, between refracted and diffracted waves.

Frequency dependent travel time $T_P(\omega,\mathbf{r})=P/\bar{c}(\omega)$ in a var iable depth ocean is to be evaluated along network rays with $\bar{c}(\omega)$ being the mean phase velocity along the path.

How exactly do I compute $T_P(\omega,\mathbf{r})$?

1) Evaluate a zero frequency travel time along network ray

$$T_P(\omega = 0,\mathbf{r}) = \int_{ray\ path} dp/\sqrt{gH(\mathbf{p})}$$

2) Find a "mean" ocean depth $\overline{H}_P$ associated with the network ray path of length P,

$$\overline{H}_P = P^2/gT_P^{\ 2}(0,\mathbf{r})$$

3) Determine a "mean" wavenumber $\bar{k}(\omega)$ associated with the network ray by solving

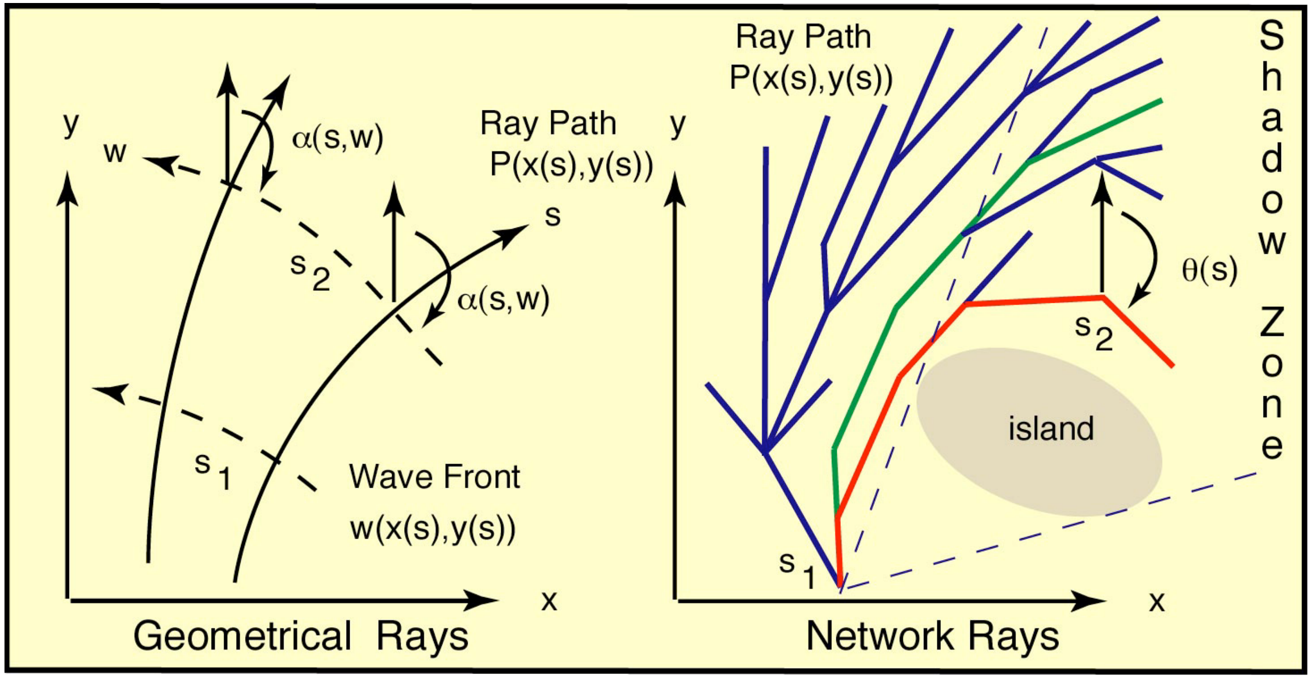
$$\omega^2 = g\bar{k}(\omega)\tanh[\bar{k}(\omega)\overline{H}_P]$$

Then $\bar{c}(\omega)$, the mean phase velocity along the path is $\bar{c}(\omega) = \omega/\bar{k}(\omega)$ and $T_P(\omega,\mathbf{r})=P/\bar{c}(\omega)$

Network rays give travel time, but what about amplitude?

"Ray" Geometrical Spreading. Because network rays do not have spreading coefficients in the usual sense, I take hint from *Geometrical Theory of Diffraction* and replace curvature of the wavefront with "curvature" of the ray.

For tsunami spreading the distinction between refraction and diffraction blurs



$$G(s_1, s_2) =$$

$$\exp\left[-\frac{1}{2}\int_{s_1}^{s_2}\frac{\partial\alpha(s)}{dw}ds\right] \Rightarrow \exp\left[-\frac{1}{2}\int_{s_1}^{s_2}\left[\frac{1}{s}+\frac{\partial\theta(s)}{ds}\right]ds\right] = \frac{1}{\sqrt{s}}\exp\left[-\frac{\Theta}{2}\right]$$

Other Thoughts:

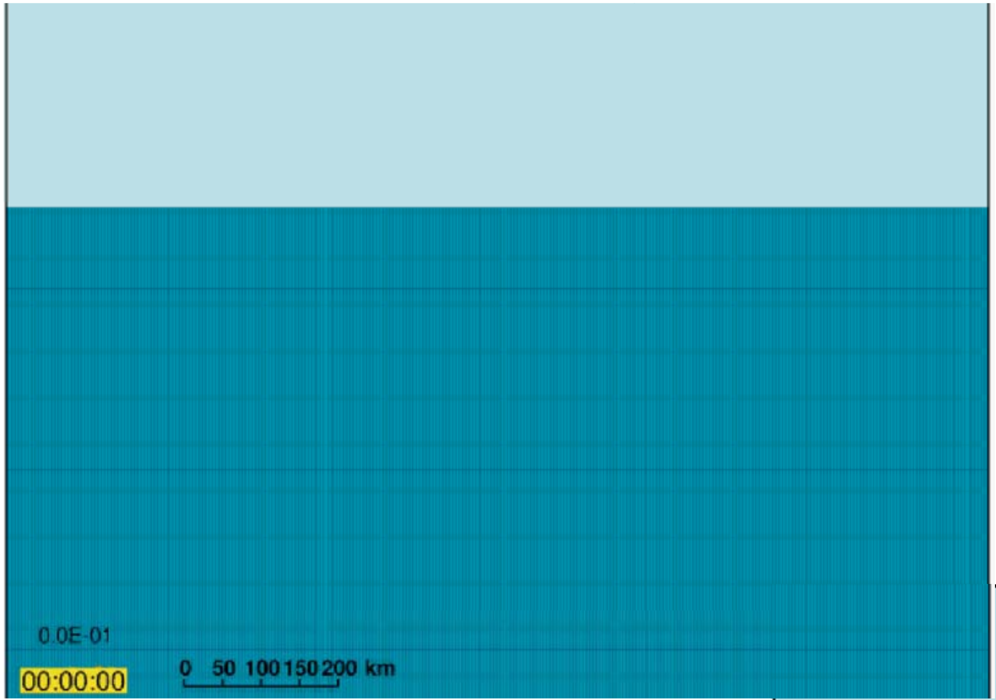
Generally I am interested in the wave train that waves following the fastest arriving path. As such I ignore ---

later direct multi-paths and reflections.

Clearly this is a huge simplification.

Certainly there may be certain situations where multi-paths and reflections are important. In those cases, other approaches will have to be taken.

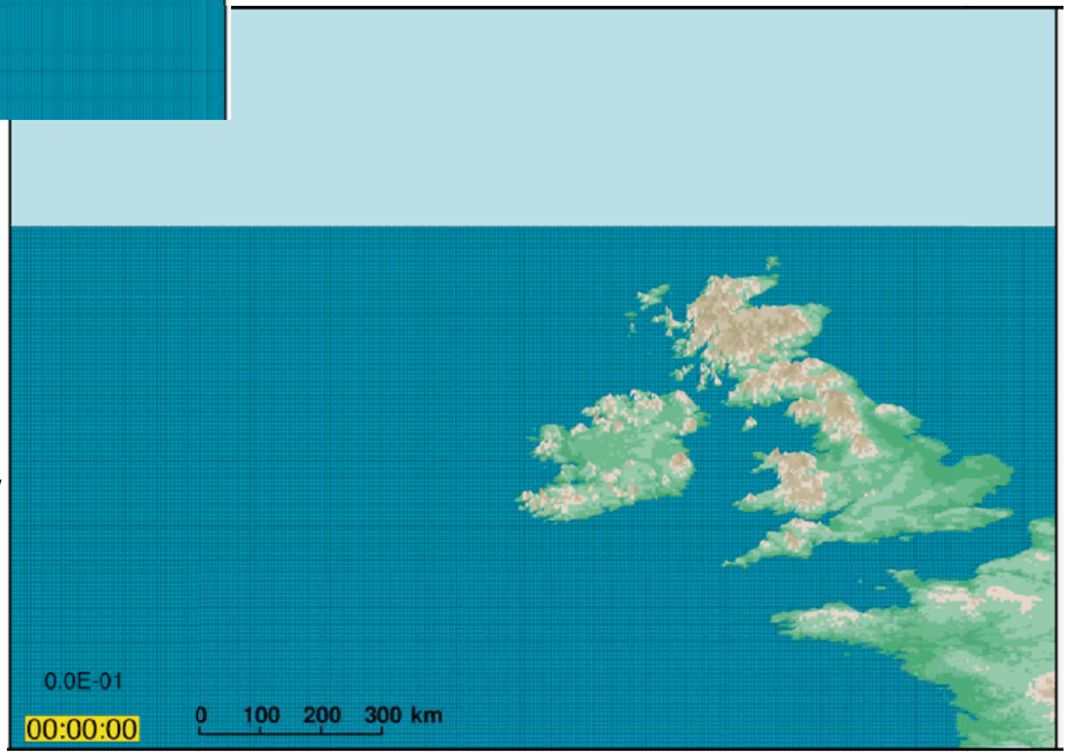
We will see how “run up” is handled later on.



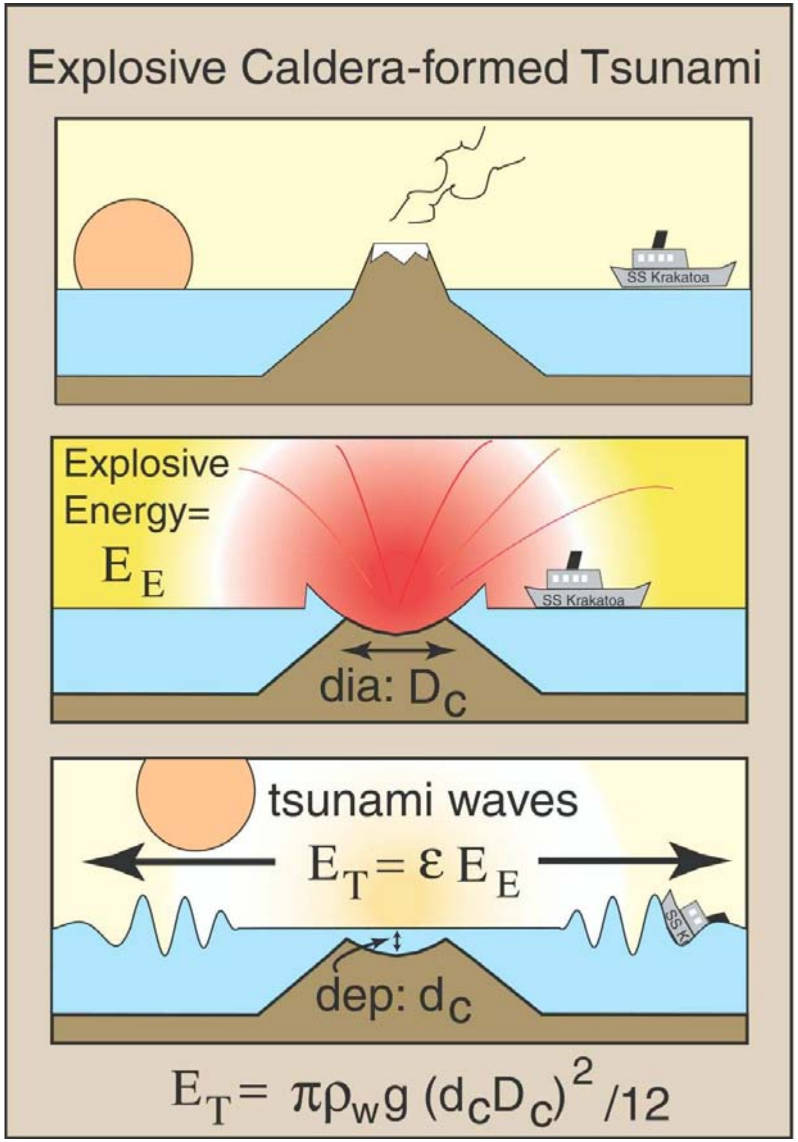
Impact tsunami in constant depth ocean. Nice circular rings.

Impact tsunami in variable depth ocean.

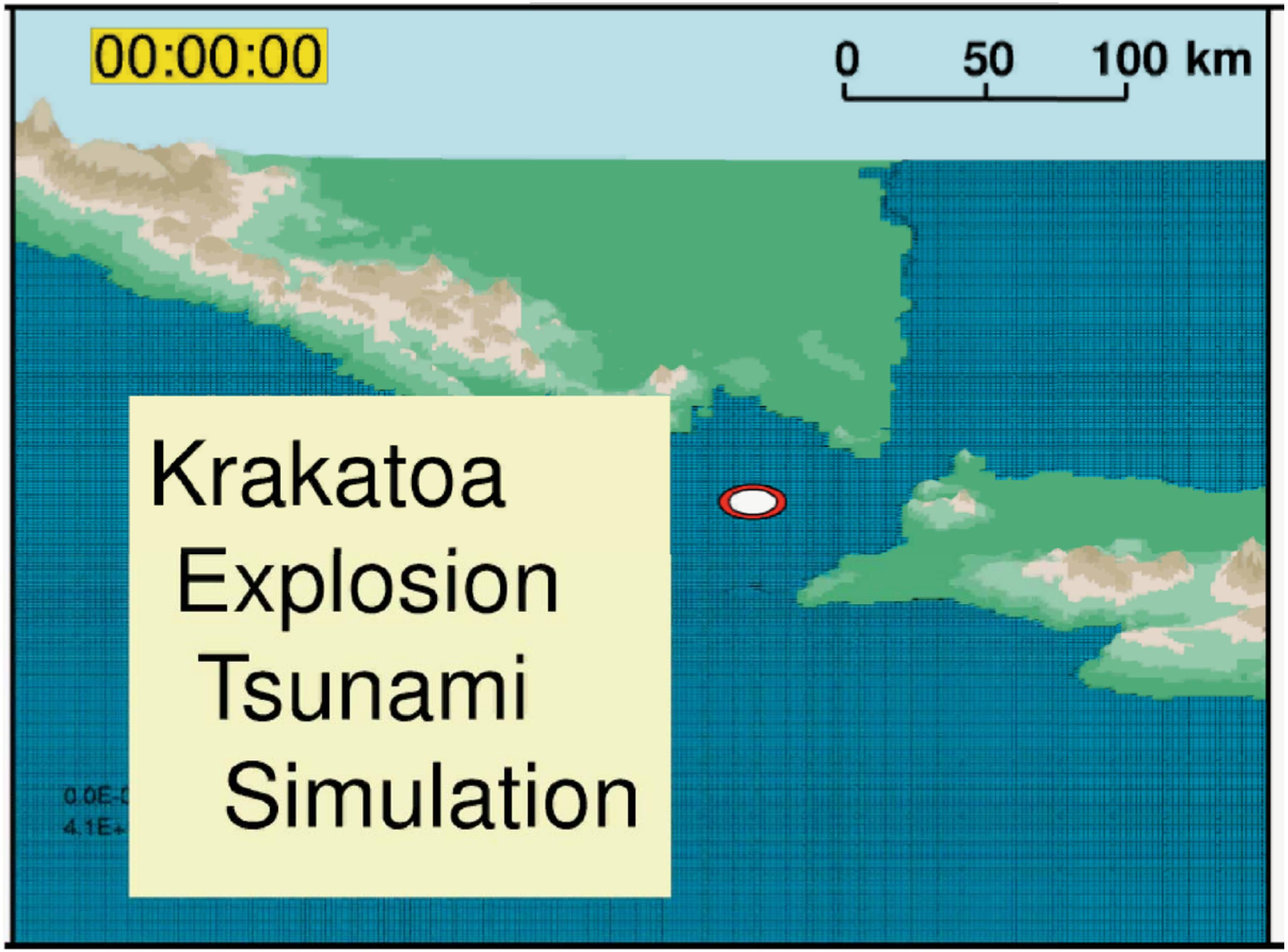
Tsunami waves crush together in the shallows and "bend around" obstacles.
Watch out Ireland!

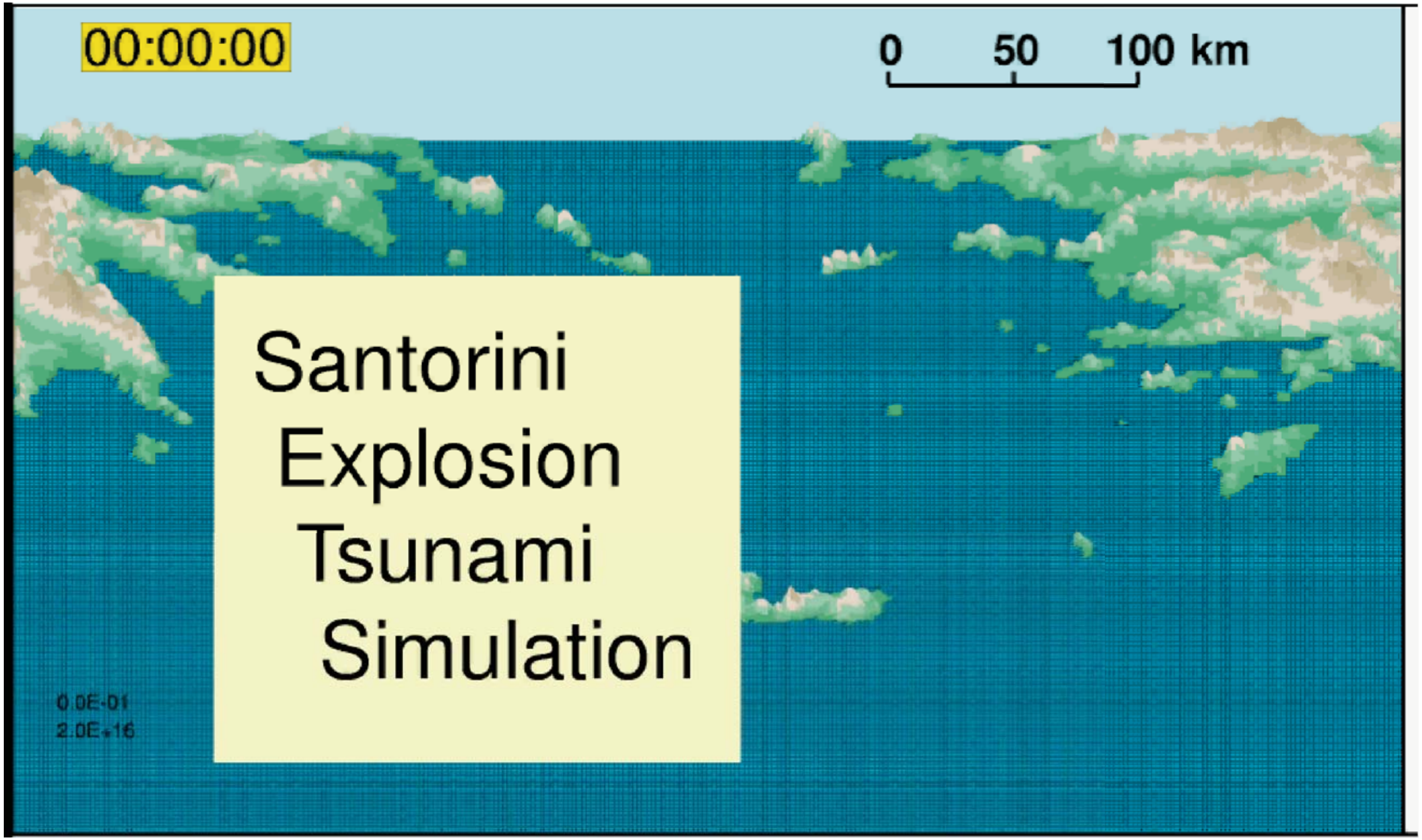


Certain explosive volcanoes can be considered as “impact-like”.

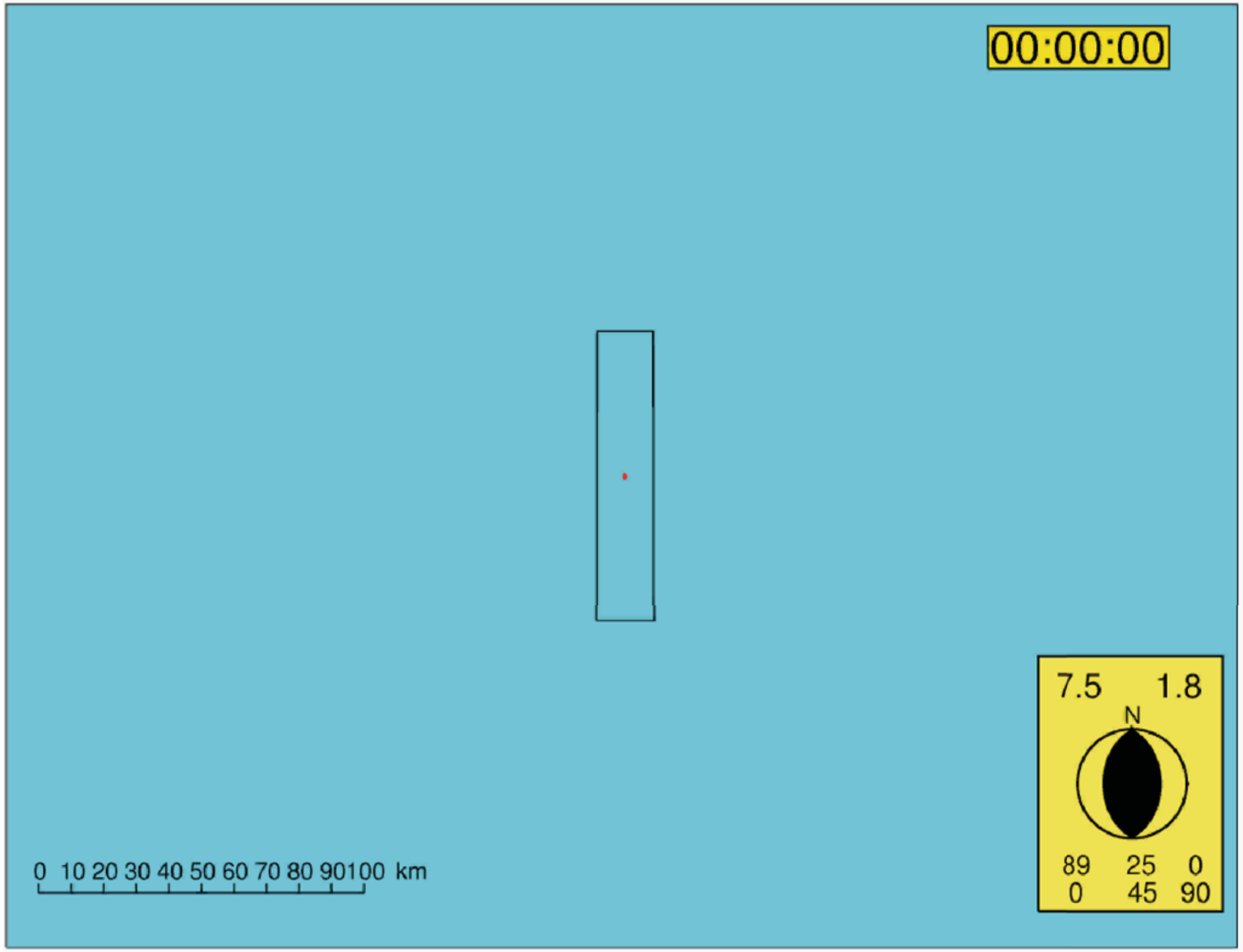


Explosive Sources=Uniform Radiation Pattern

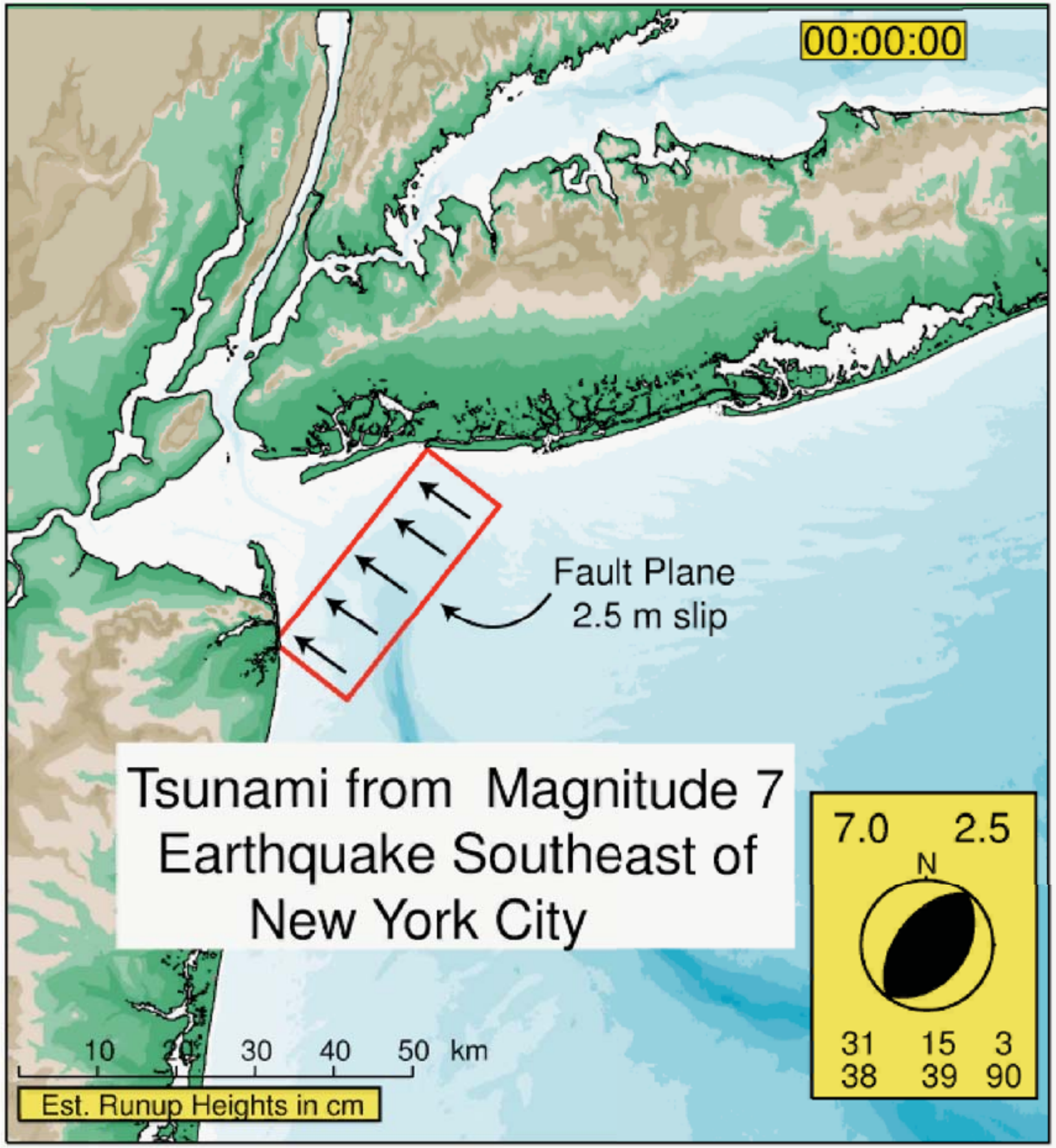

00:00:00
0 50 100 km
Krakatoa
Explosion
Tsunami
Simulation


00:00:00
0 50 100 km
Santorini
Explosion
Tsunami
Simulation
0.0E-01
2.0E+16

Dip Slip Fault in Uniform Depth Ocean



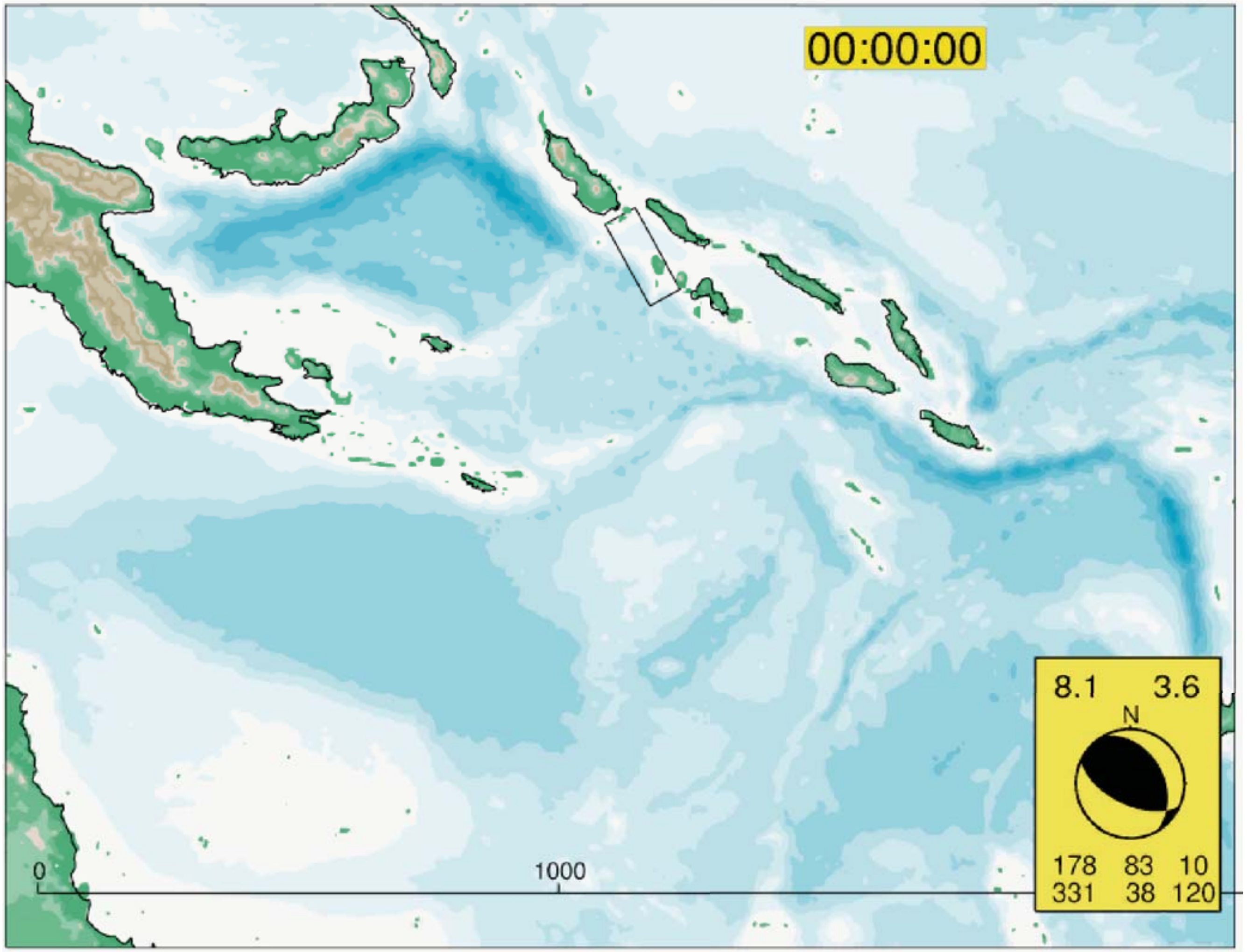
Dip Slip Fault in Variable Depth Ocean



Solomon Sea Earthquake, 4/1/2007 M8.1

Given estimate of earthquake parameters (and the quake is well behaved), I can provide a wave height estimate in about 1/2 hour on my laptop.

I suppose this counts as a *prediction* if you beat the wave to the beach!



Even this level of prediction would have helped the Australians make a decision.

9. Tsunami Energy/ Tsunami Efficiency

9.1 Tsunami Energy. Consider the kinetic energy of a single tsunami eigenmode (4.4.2) per unit area of ocean surface at x,y

$$\tau_n(x,y) = \frac{\rho_w \omega_n^2}{2} \int_0^H dz \left[\left| u_x(\omega_n, x, y, z) \right|^2 + \left| u_z(\omega_n, x, y, z) \right|^2 \right]$$

$$= \frac{\rho_w}{2} \left| u_z(\omega_n, x, y, 0) \right|^2 u_z^2(\omega_n, x, y, 0) \frac{k^2(\omega_n) g^2}{\omega_n^2 \cosh^2(k(\omega_n) H)} \int_0^H dz \left[\sinh^2(k(\omega_n)(H - z)) + \cosh^2(k(\omega_n)(H - z) \right.$$

$$= \frac{\rho_w}{2} \left| u_z(\omega_n, x, y, 0) \right|^2 \frac{k(\omega_n) g^2}{\omega_n^2} \tanh(k(\omega_n) H) = \frac{g \rho_w}{2} \left| u_z(\omega_n, x, y, 0) \right|^2$$

(9.1.1)

(9.1.1) says that the kinetic energy of the whole column of water is fixed by the amplitude of the mode's vertical displacement at the surface.

Because of the orthonormal properties of eigenmodes, the kinetic energy at any time t of the entire sum of modes can be written in terms of the Hilbert transform of the tsunami vertical displacement at the surface like

$$E_K(t) = \frac{g\rho_w}{2} \int_{\substack{whole\\ ocean}} dx\, dy\, H_z^2(x,y,0,t) \qquad (9.1.2)$$

The Hilbert transform of a function just adds an ex tra i to the spectrum of the function, effectively changing cos(ωt) to sin(ωt). For example, for the surface tsunami from 2-D bottom uplift of (5.3.1.)

$$u_z(x,0,t) = \mathrm{Re} \int_{-\infty}^{\infty} dk \frac{u_z^{bot}(k)\, e^{i(kx-\omega(k)t)}}{2\pi \cosh(kH)} \qquad (5.3.11)$$

The Hilbert transform would be

$$H_z(x,0,t) = \mathrm{Re} \int_{-\infty}^{\infty} dk\, i \frac{u_z^{bot}(k)\, e^{i(kx-\omega(k)t)}}{2\pi \cosh(kH)}$$

Generally where u_z is large, H_z is small and visa versa. You expect kinetic energy to be large where velocities are large, this is where displacements are small, hence the H_z in (9.1.2).

On t he other hand, you'd exp ect gravitational potential energy to be large where displacements are large,

$$E_G(t) = \frac{g\rho_w}{2} \int_{\substack{whole \\ ocean}} dx\, dy\, u_z^2(x,y,0,t)$$

so in fact, the total energy $E_T(t)$ of a tsunami is

$$E_T(t) = \frac{g\rho_w}{2} \int_{\substack{whole \\ ocean}} dx\, dy \left[u_z^2(x,y,0,t) + H_z^2(x,y,0,t) \right] \qquad (9.1.3)$$

When averaged over many wavelengths, both terms in (9.1.3) are equal.

Because I ignore dissipation, once the impact, quake or landslide is over, $E_T(t)$ should be constant so long as none of the waves have run all the way to shore.

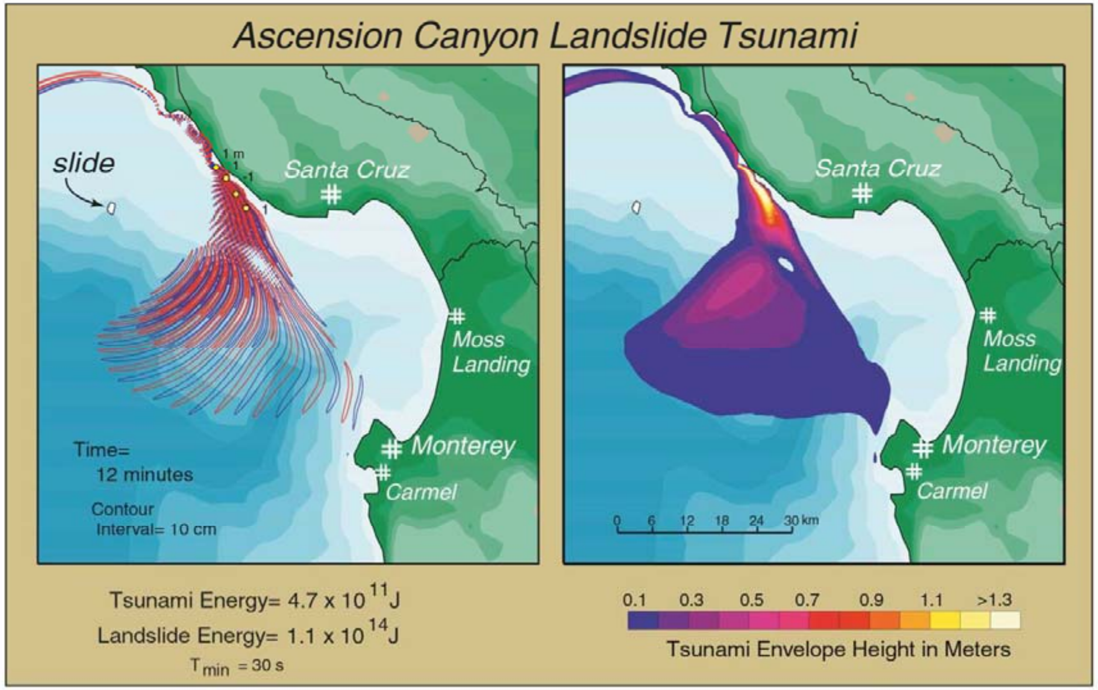


Figure 7. Example of the use of the tsunami envelope (9.1.4). To the left is the surface vertical motion of a tsunami from a small submarine landslide with red being up and blue down. The envelope to the right matches the same heights at the crests but varies smoothly elsewhere. See the shoaling of the waves near shore as the bright spot.

The function
$$E(t) = \left[u_Z^2(x, y, 0, t) + H_Z^2(x, y, 0, t)\right]^{1/2} \quad (9.1.4)$$
I call the tsunami "envelope". It is always positive, spatially smooth and it has units of meters. The envelope acts like the surface of a tight sheet placed over the tsunami wave train.

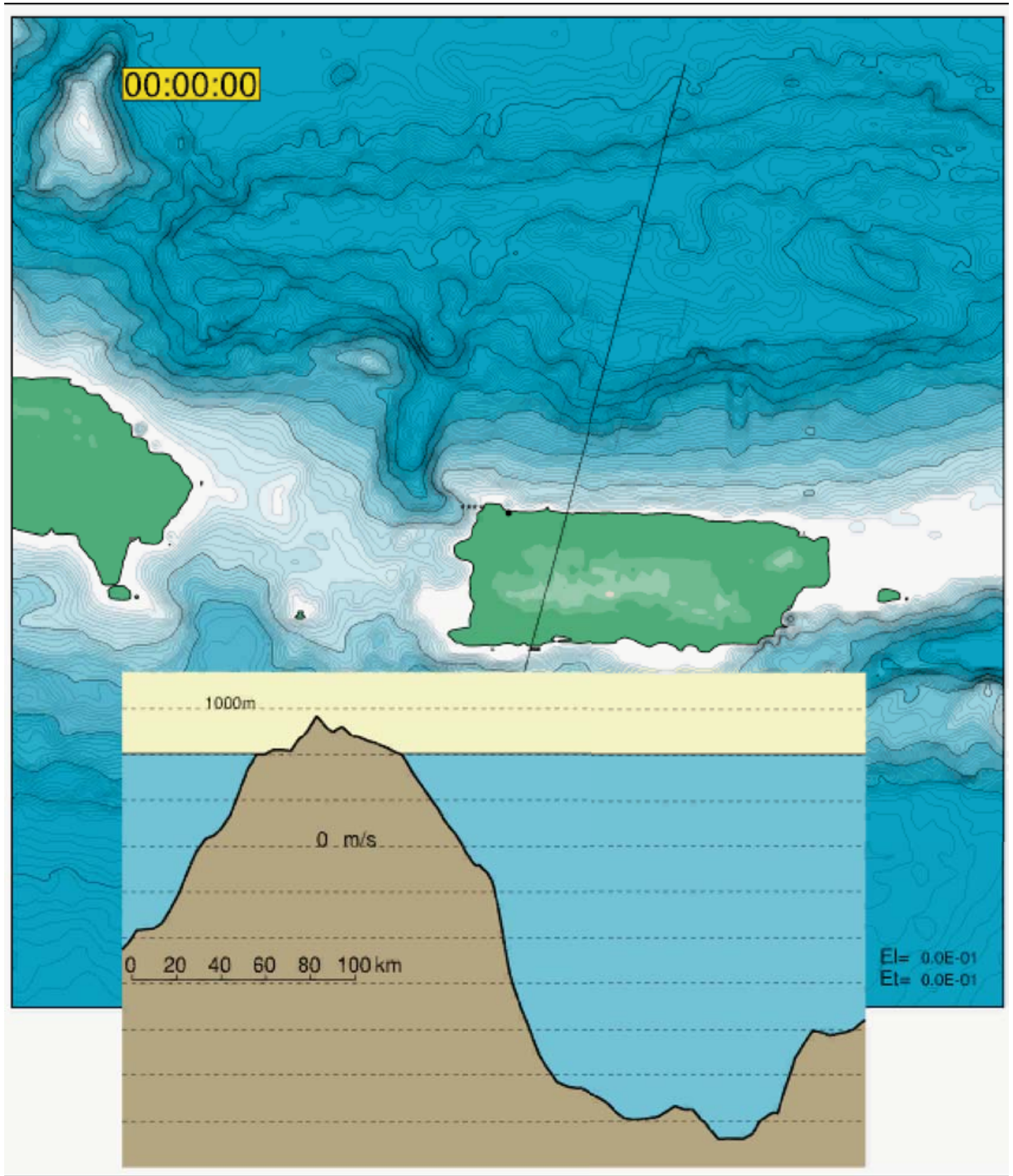
I often plot the tsunami envelope instead of the vertical displacement itself because E(t) shows well the size and shoaling of the waves with out all the distractions of sign changes.

In almost all of my simulations I will calculate and show a tsunami energy.

Typical Landslide Situation:

Slide material falls from high on a slope, possibly far under water.

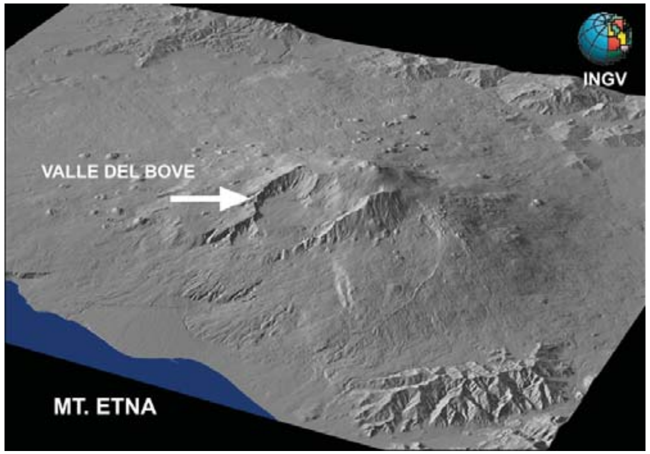
The process stirs tsunami waves on the surface of the ocean.

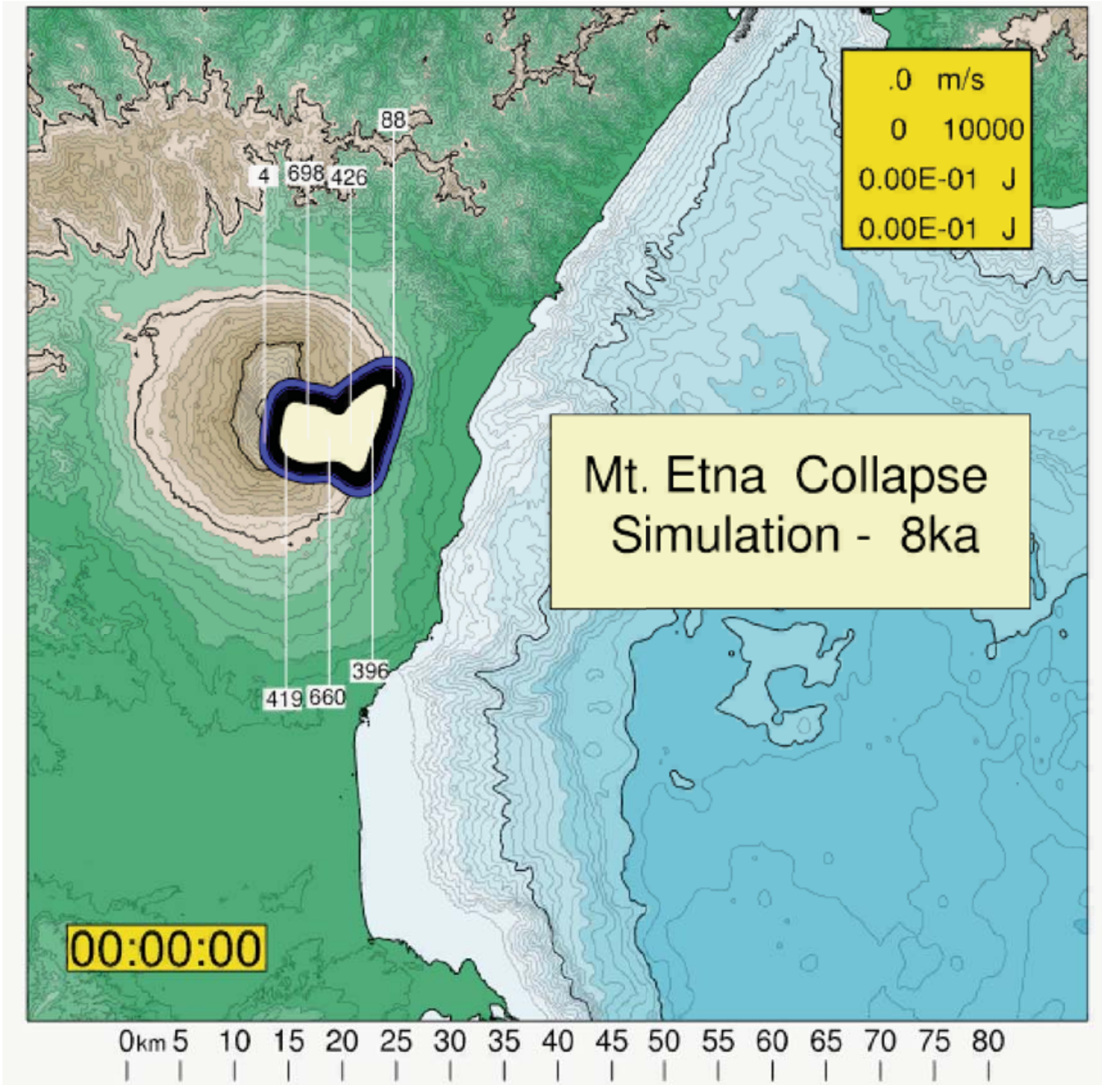


Landslide Simulation for 8ka event

Certainly for landslides, the tsunami energy at time t, must be less than the gravitational energy lost in the slide at that time.

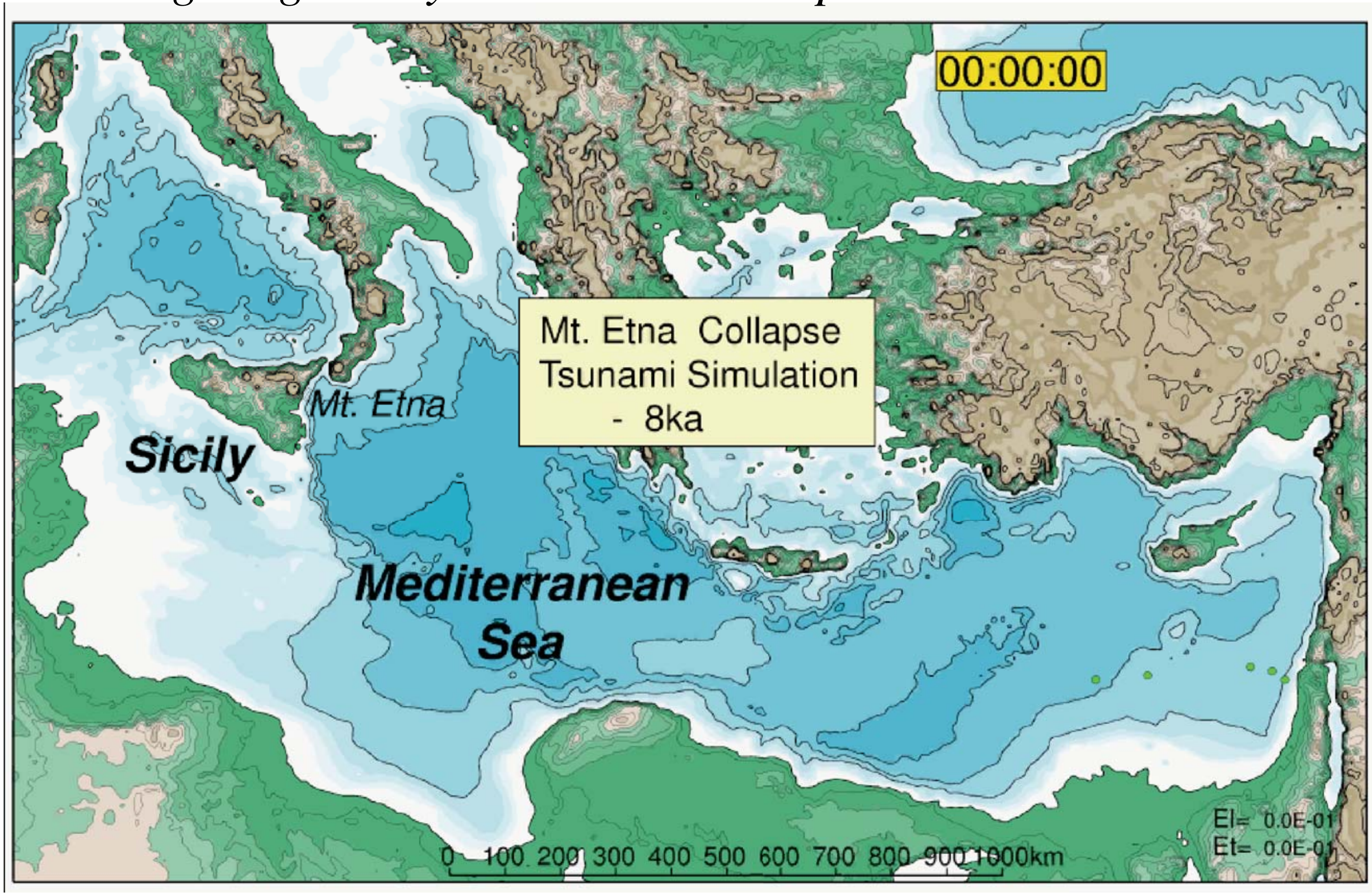
Italy's Mt. Etna (below).





Note multiple braided flow paths. If you observed this deposit years later, would you consider it a single event? Perhaps not.

A several km^3 slide can make waves of a few meters to tens of meters high regionally. Mt. Etna example.



The final stage of the tsunami is runup and inundation.

"Will I get Wet?"

is what everyone wants to know, but even if you saw the tsunami coming, the answer is complicated as it depends on:

Size of the Waves

Period of the Waves

Beach Slope

Offshore obstacles

Number of Waves

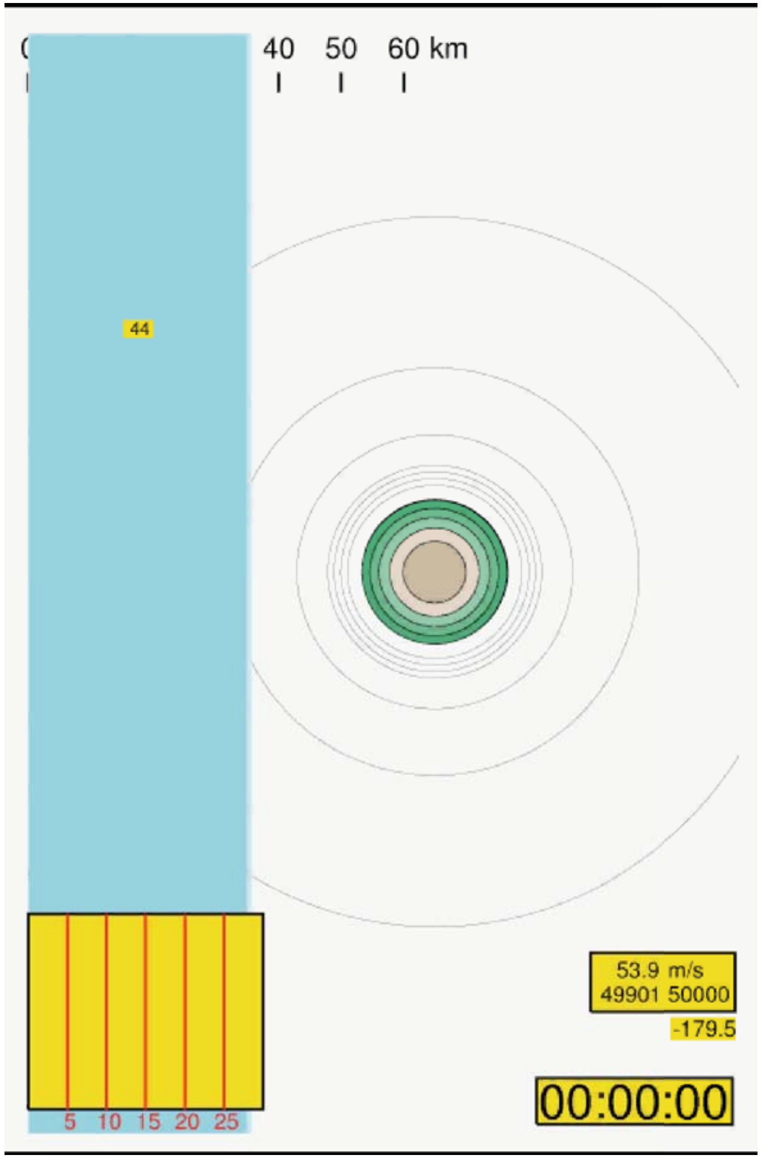
To run the “last mile”, non linear effects, multi-paths and reflections neglected so far do become important.

To handle runup, I use a new approach that I call “tsunami balls”.

Basically I treat the ocean as granular. I track the accelerations and positions of many “tsunami balls”, and compute water height from the density of the balls.

I can’t detail things here, but you can read about it here…

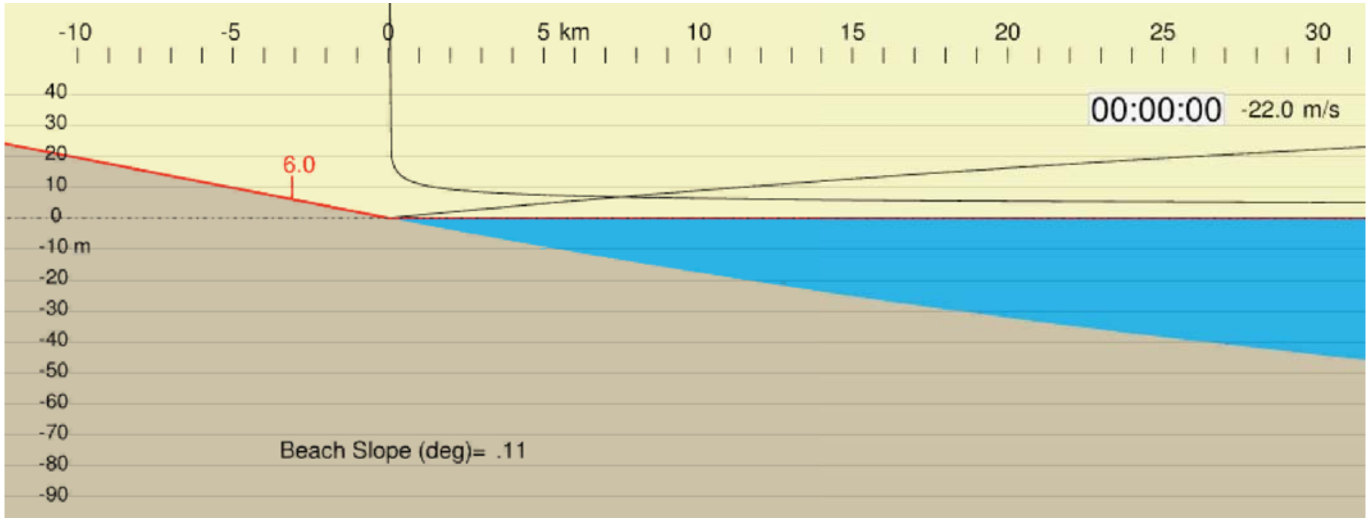
http://es.ucsc.edu/~ward/papers/Tsunami_Balls.pdf



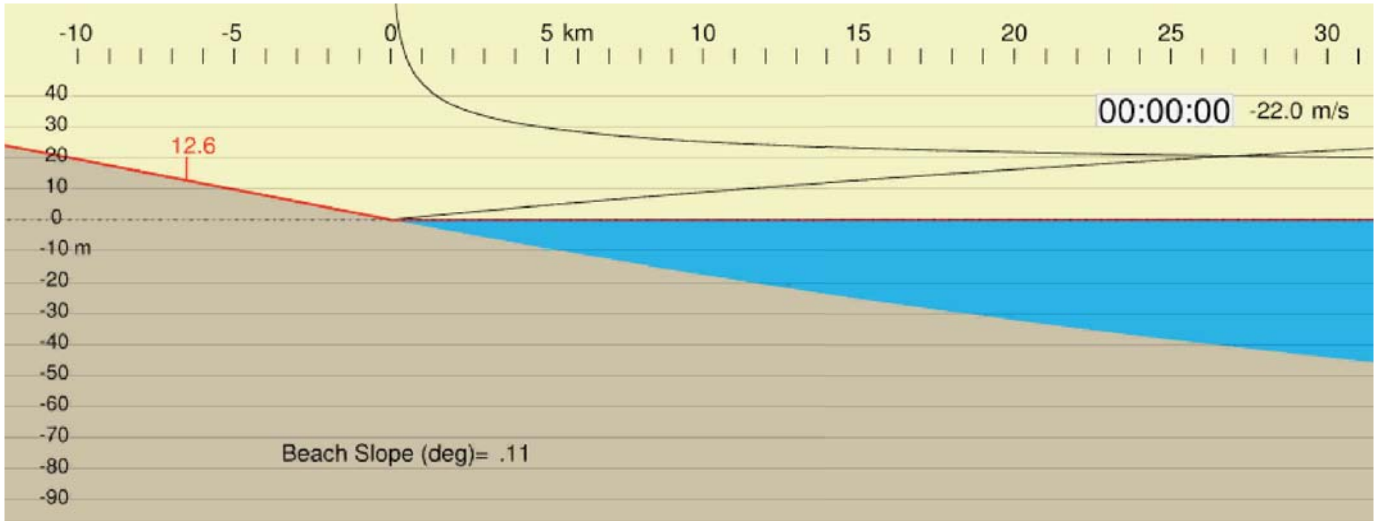
"Will I get Wet?"

Size Of Waves:

Smaller Wave



Larger Wave

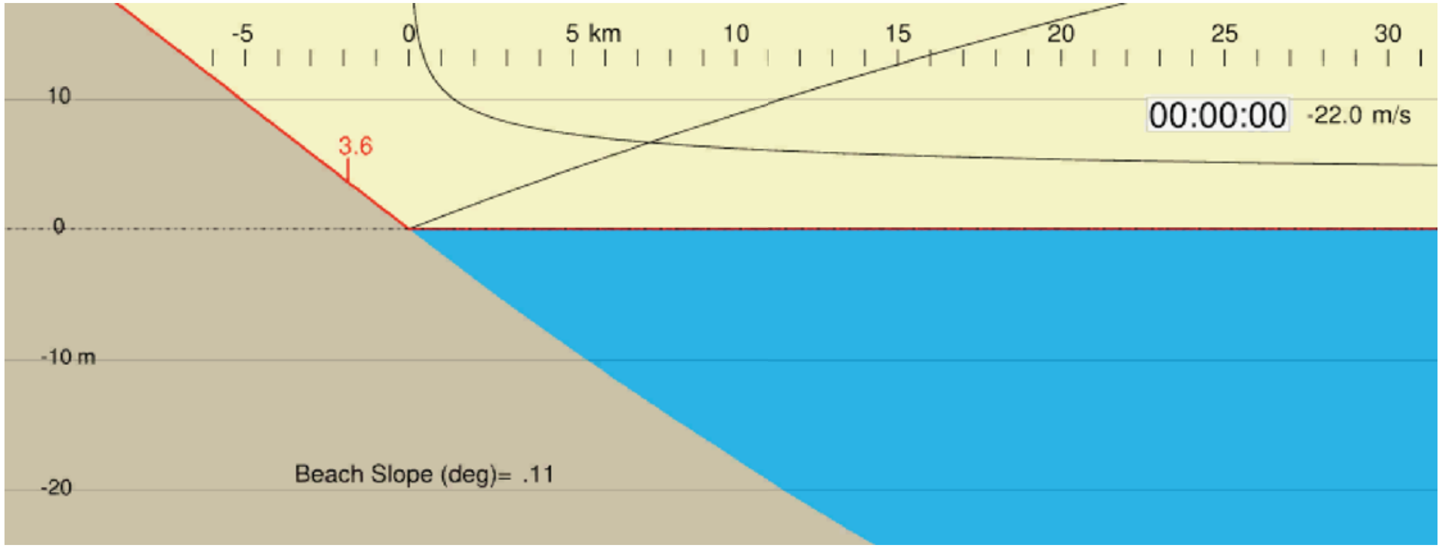


Generally, Bigger Waves = More Runup - duh

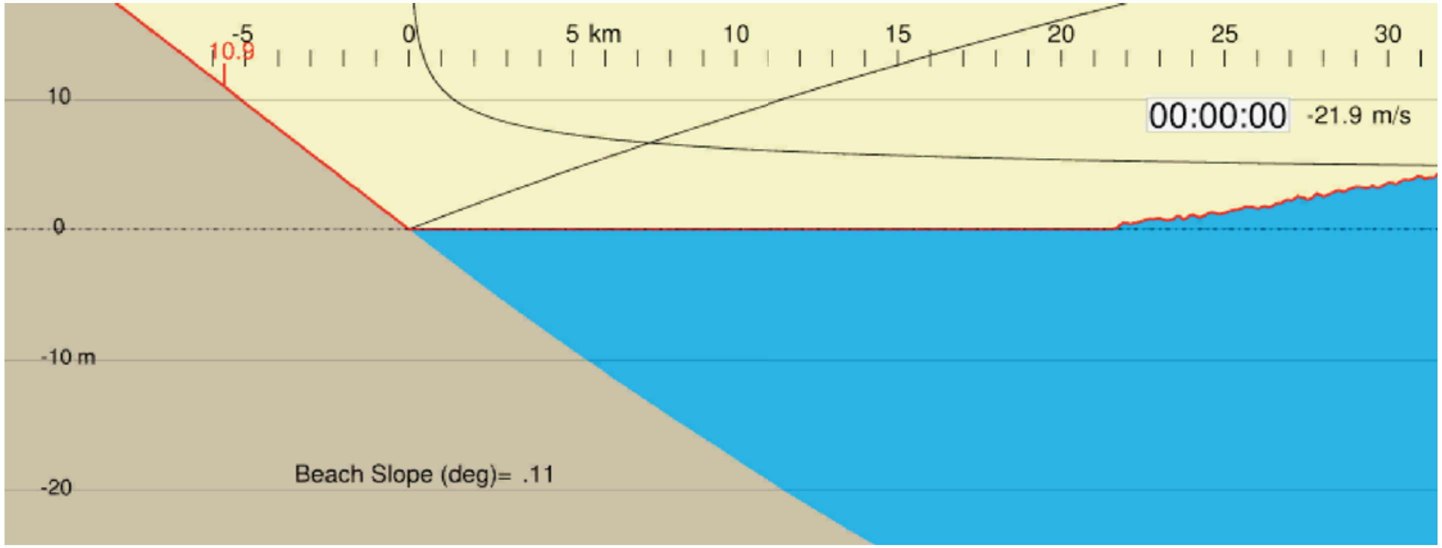
"Will I get Wet?"

Period Of Wave:

Shorter Period



Longer Period



Normal waves: 15s

Tsunami:

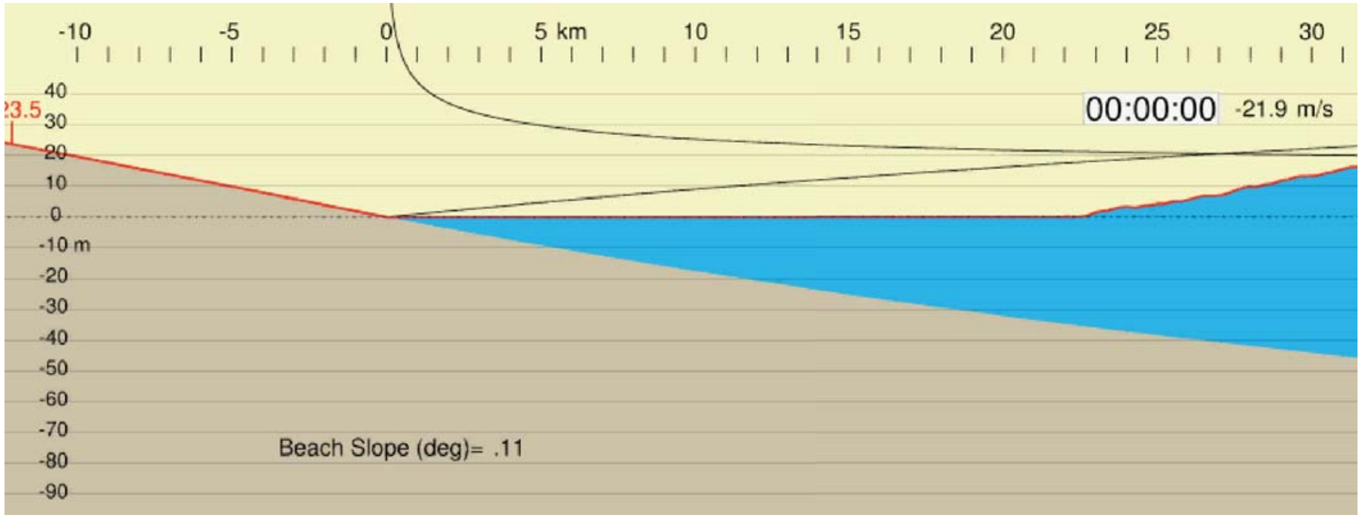
600s-1000s

Generally, Longer Period Waves = More Runup

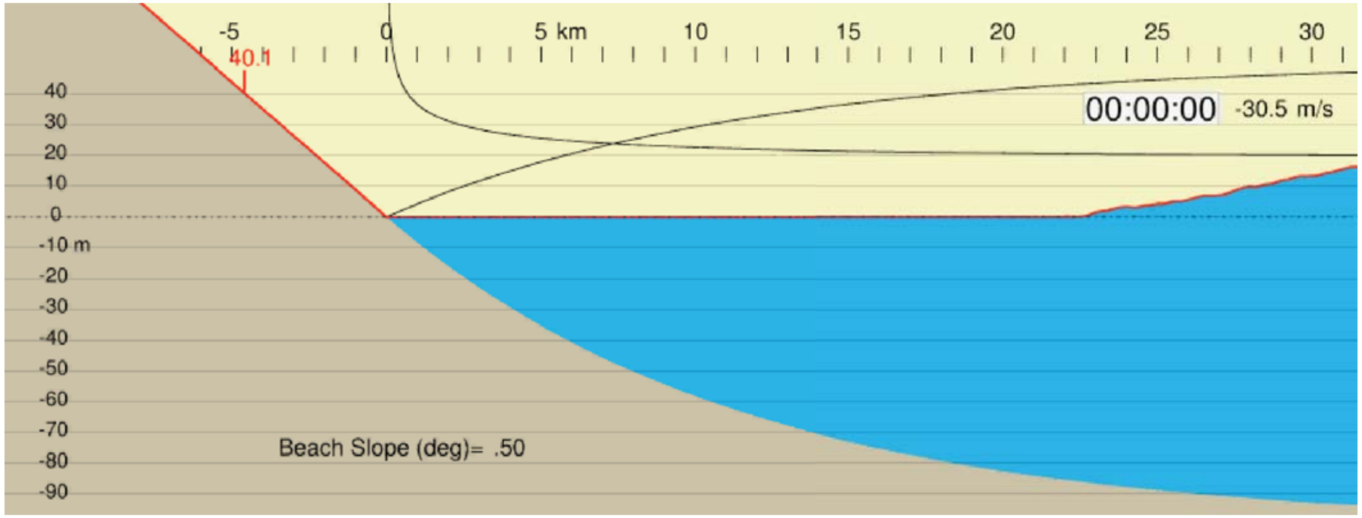
"Will I get Wet?"

Beach Slope:

Shallow Slope



Steeper Slope

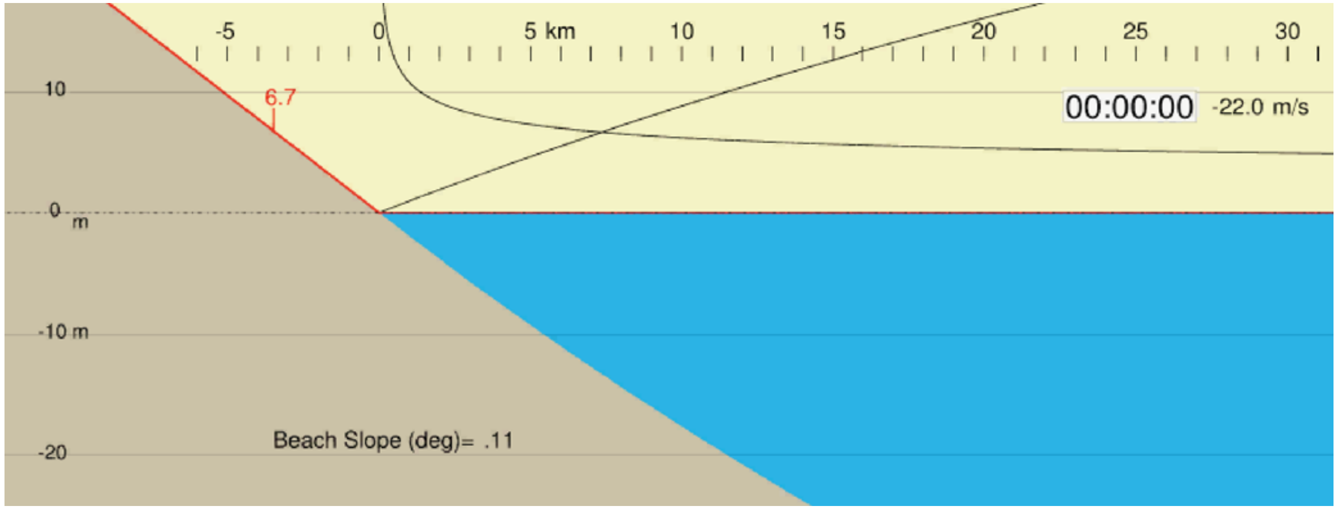


Generally, Waves on Steeper Beaches = More Runup

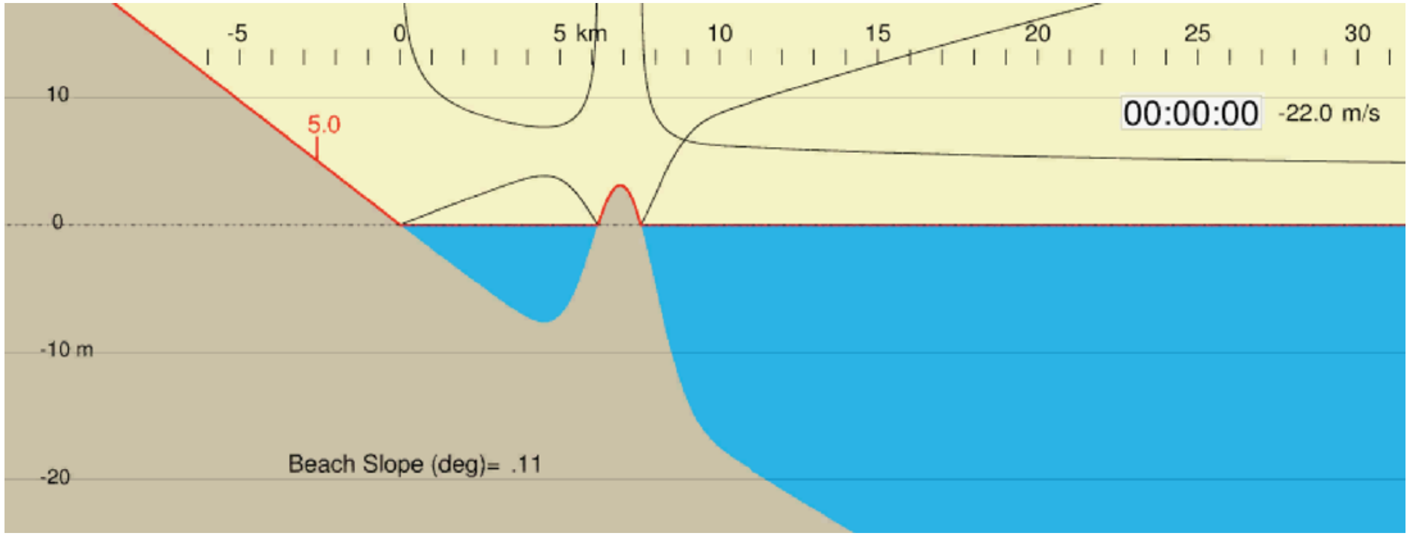
"Will I get Wet?"

Offshore Obstacles

No Protection



Some Protection

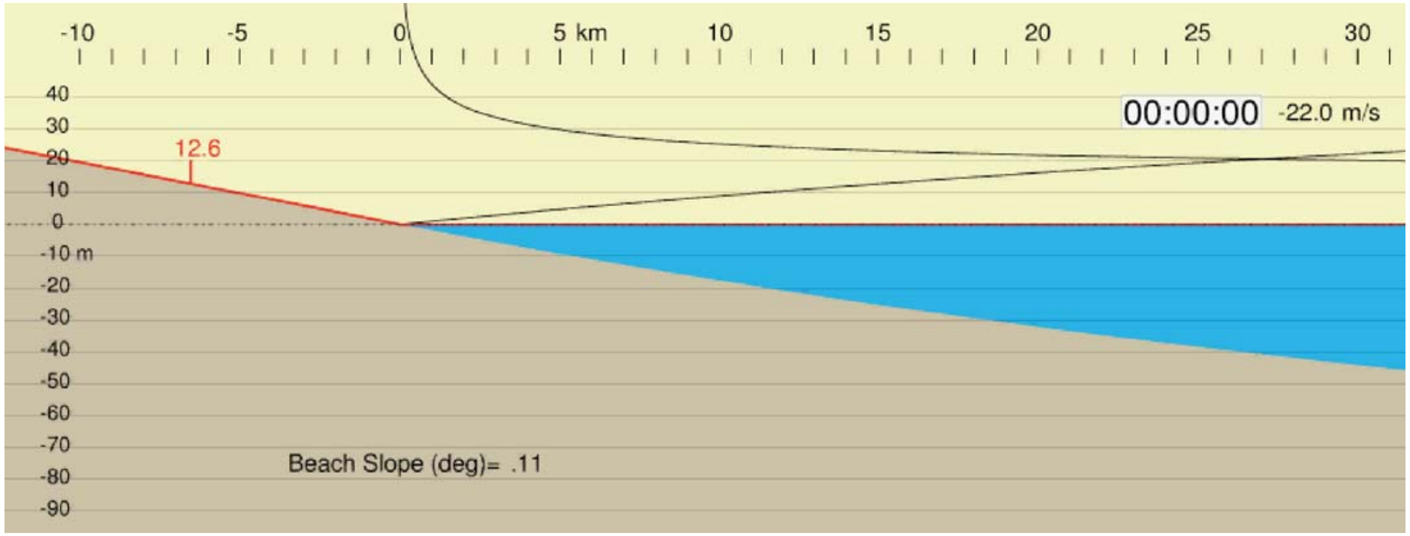


Generally, Waves on Protected Beaches =Less Runup

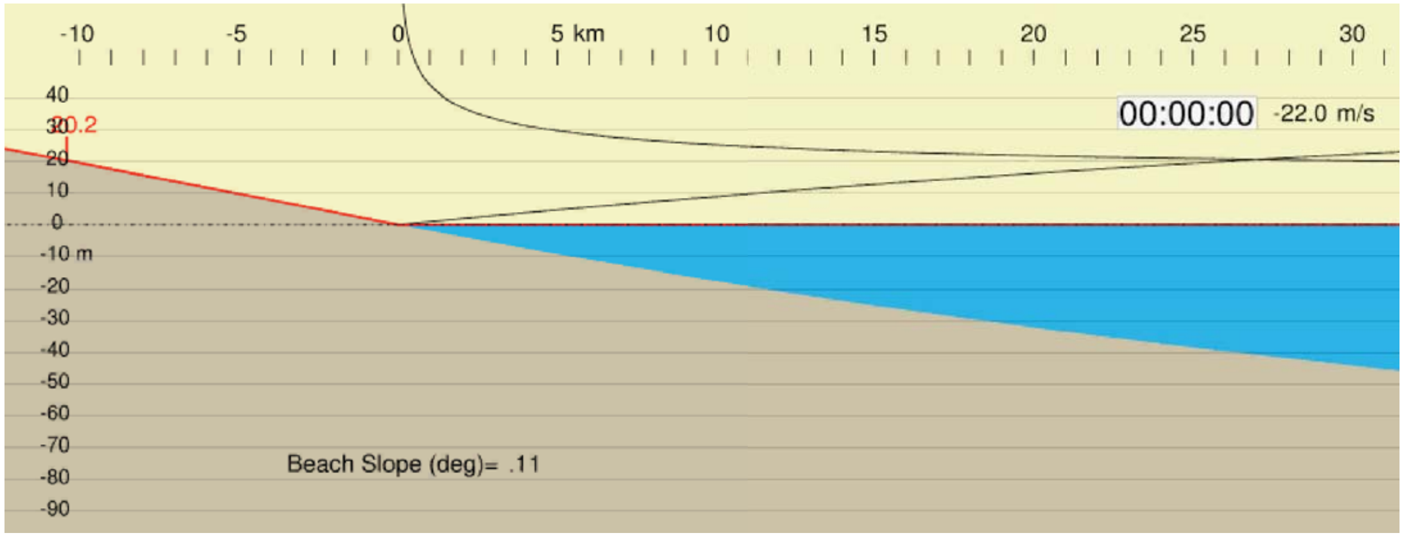
"Will I get Wet?"

Number of Waves

One Wave



Many Waves



Generally(?), Many Waves = More Runup

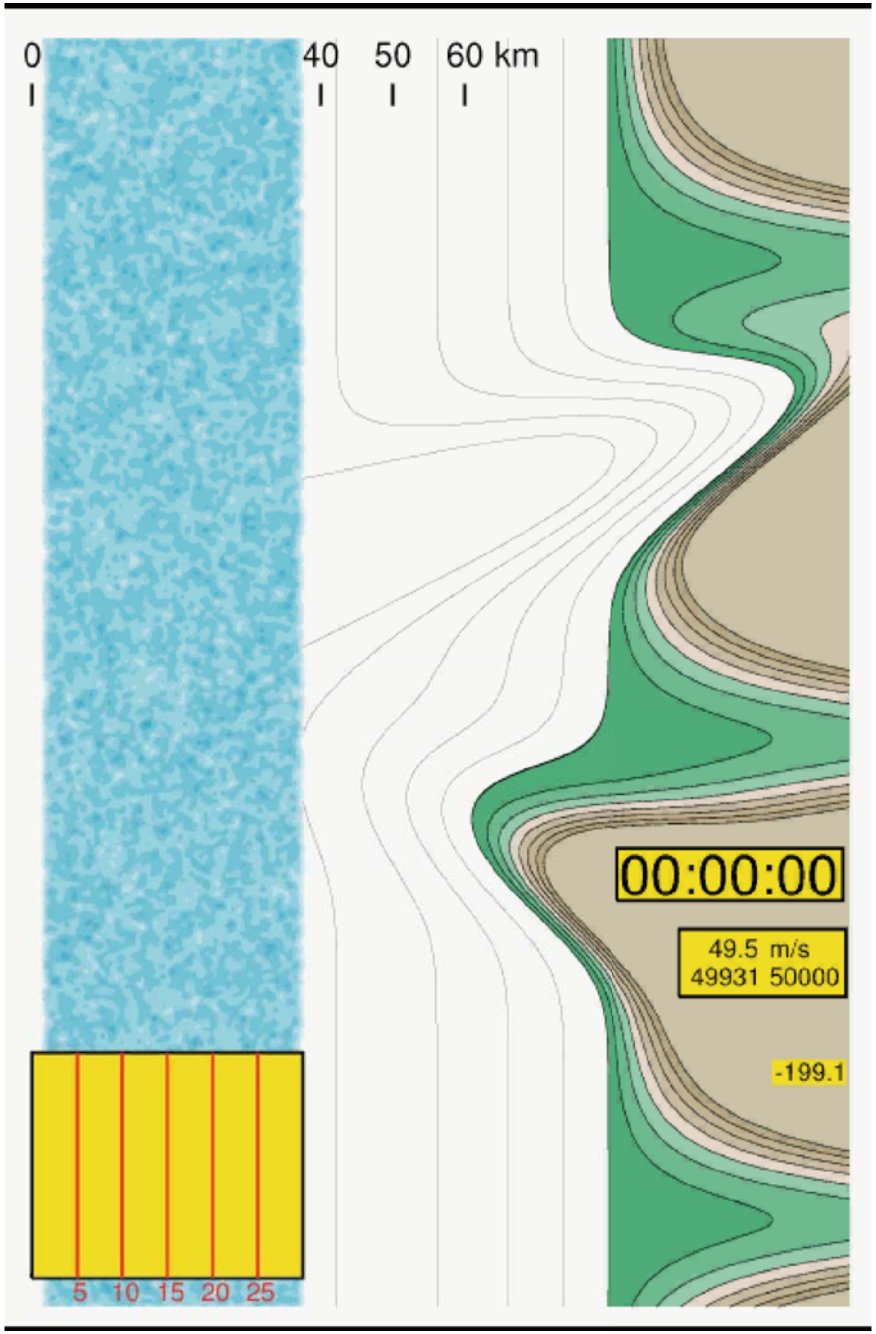
WELL,………

"Will I get Wet?"

True, scientists understand the general concepts of inundation, but in any given tsunami in a real 3-D world, don't expect miracles from us!

We really can only supply a "Best Guess"

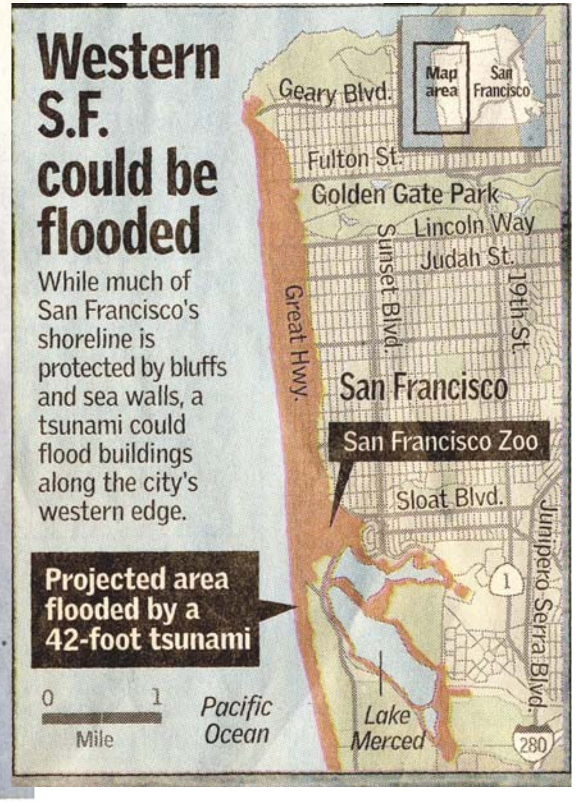
or "Vegas-Style" odds




WHAT IF A TSUNAMI HIT THE BAY AREA?

Little is known about damage giant wave could cause

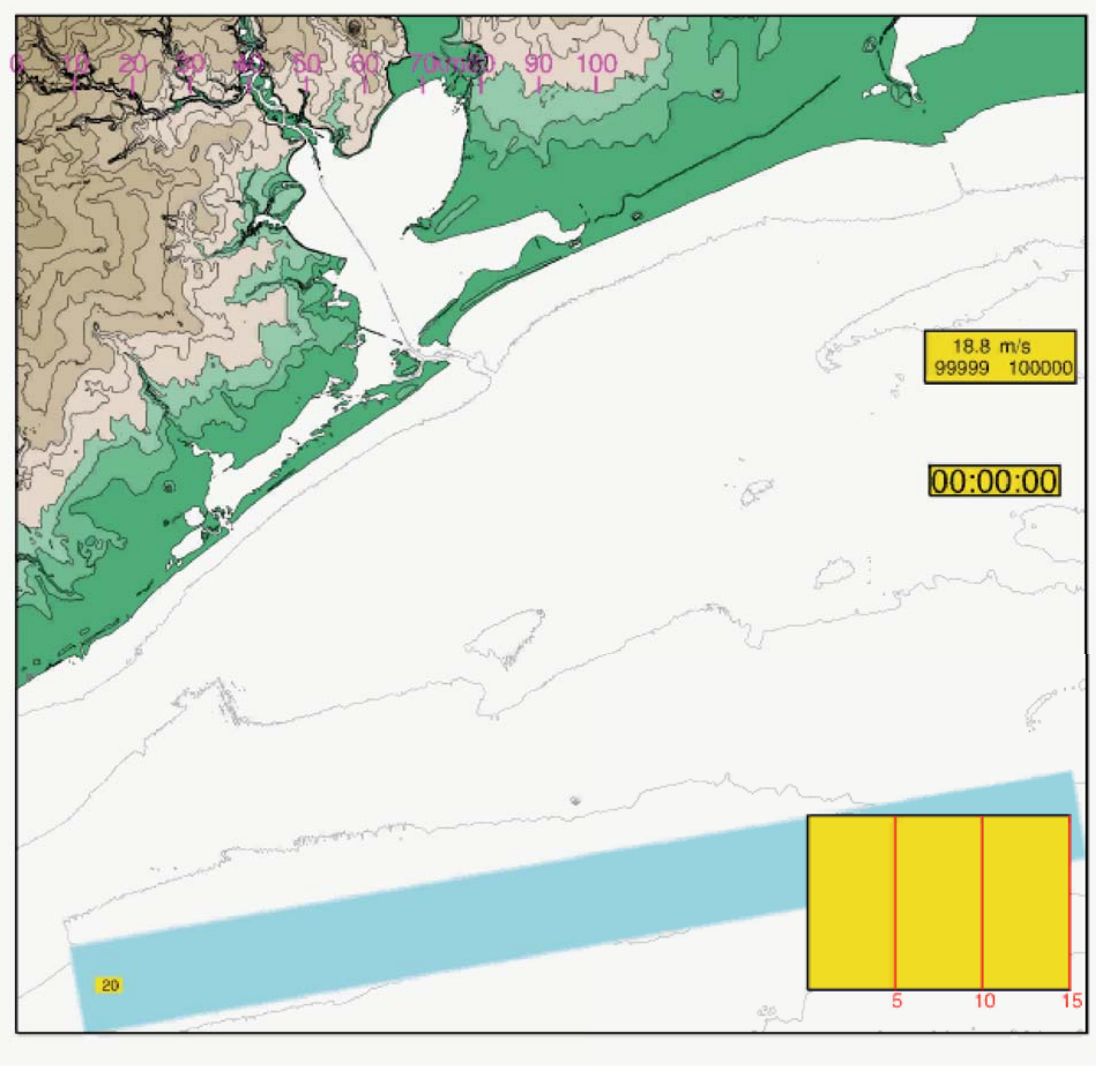
Santa Cruz County, home to the Santa Cruz Beach Boardwalk, estimates that a major tsunami would flood more than 5,000 buildings, nearly 400 roads and four fire stations.



Some attempts have been made to map tsunami inundation zones down to the street level in the Bay Area. As we have said, 'Will I get wet?" is fairly uncertain business.

Computer simulations help somewhat in mapping inundation zones.

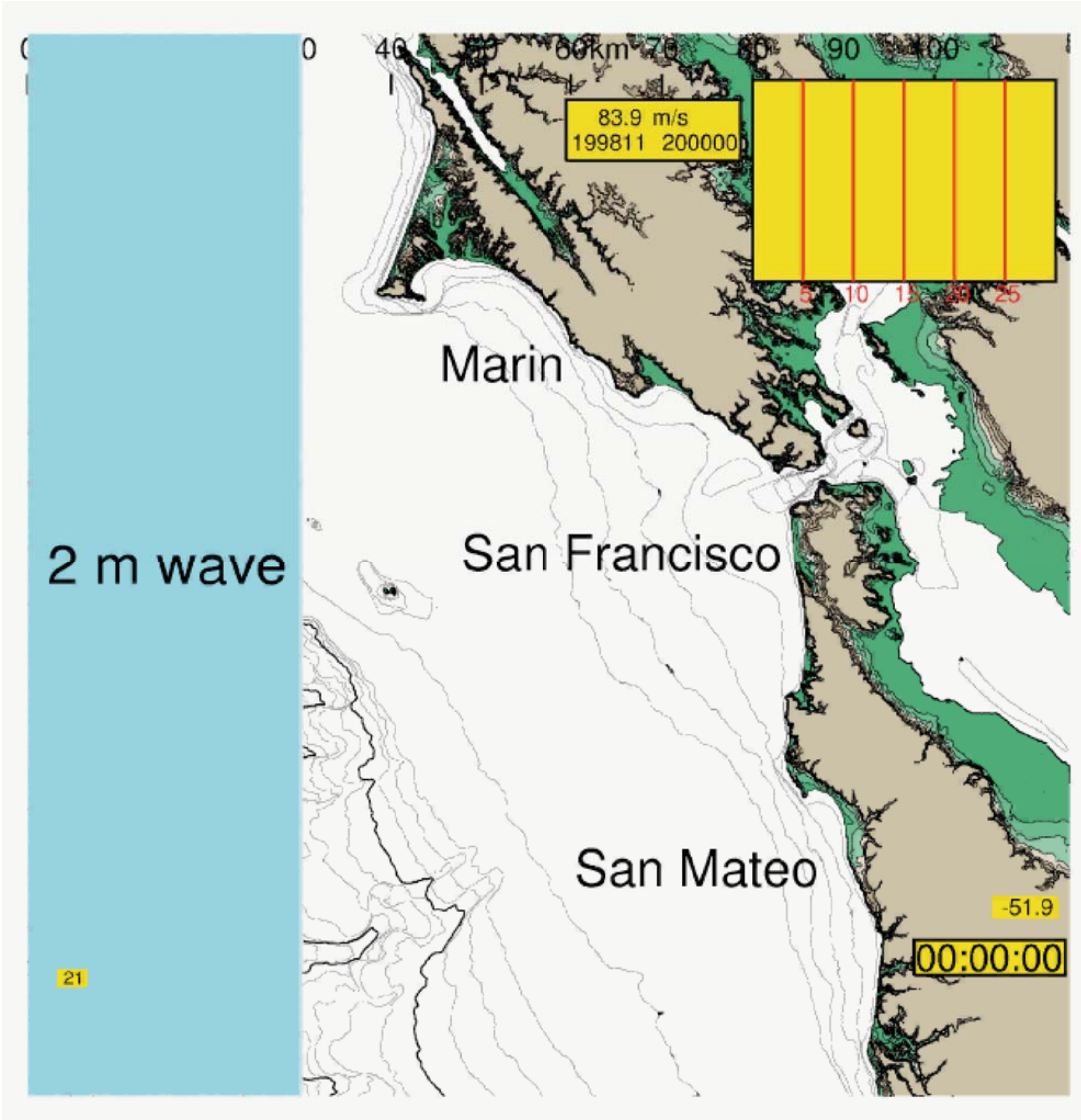
2m wave sent into sample coast.



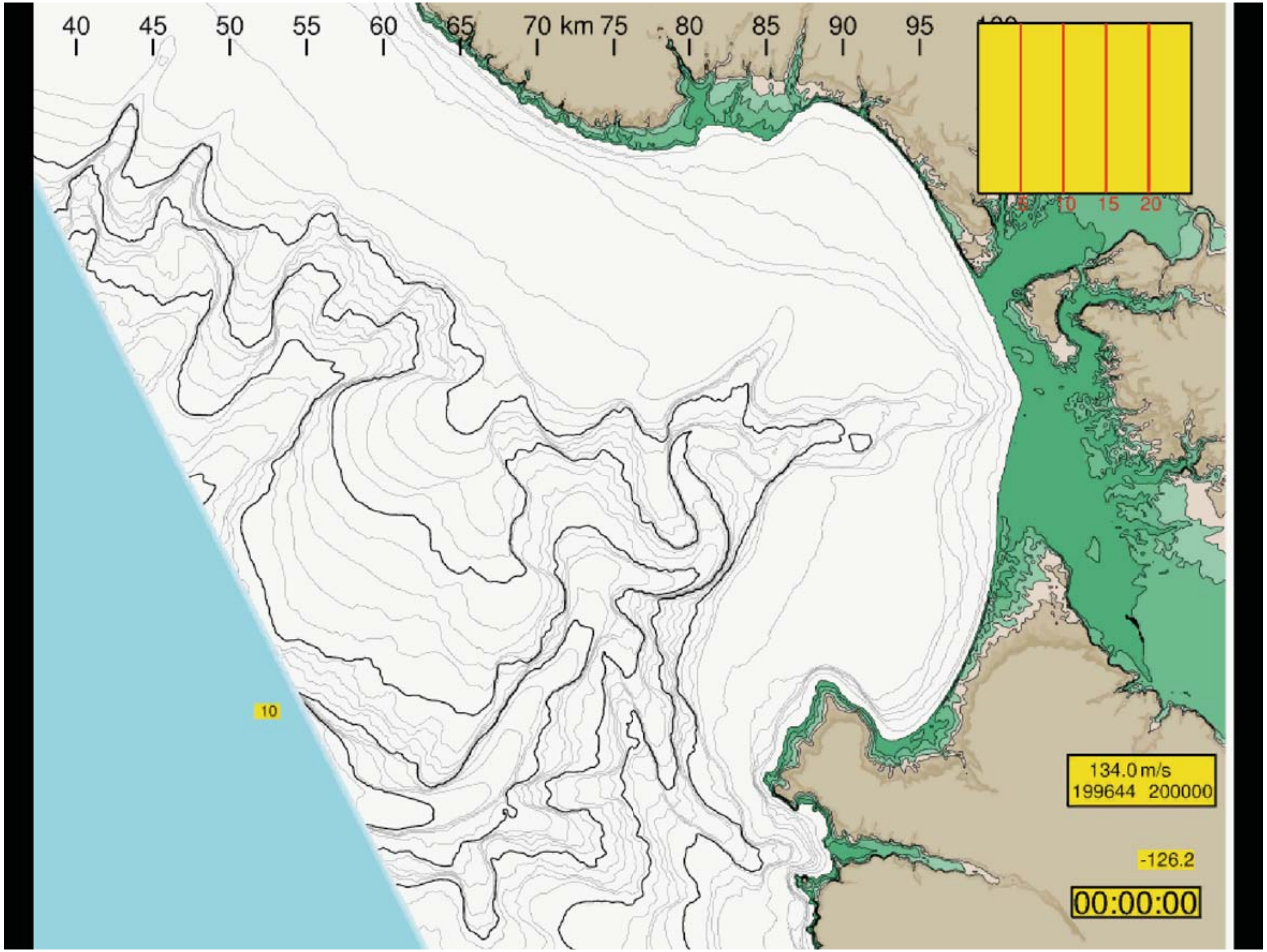
2m wave sent into the Marin/ San Mateo Coast.

Steep Beaches at Point Reyes take it hard.

Only a bit of water can push through the Golden Gate.



A bit closer -- 1 meter wave in



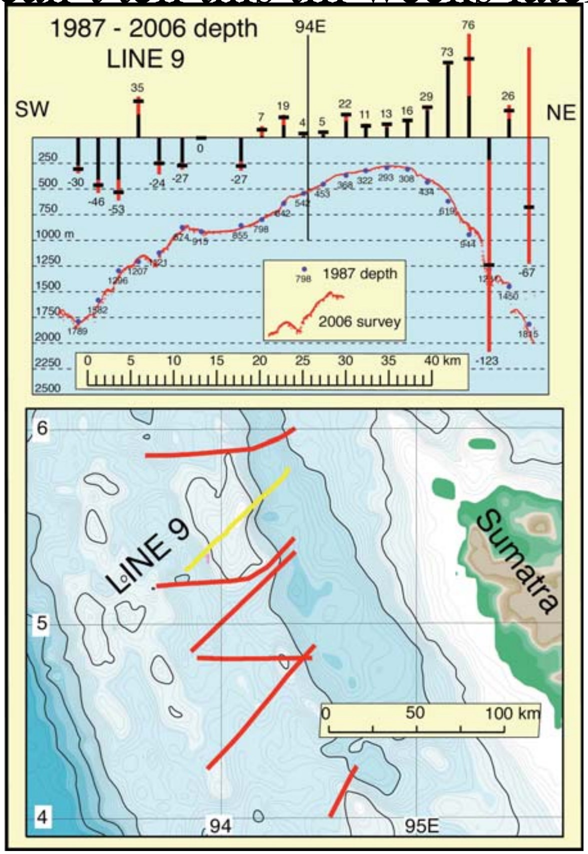
OK. At this point you should have a good idea how to do

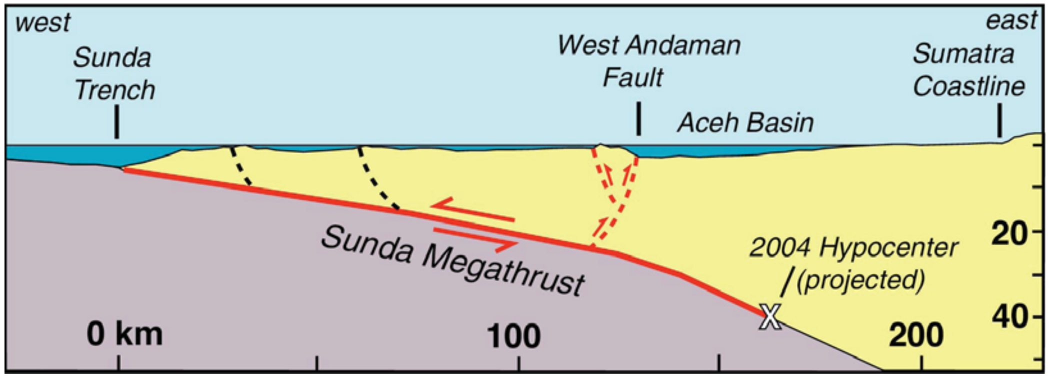
Tsunami a la Ward

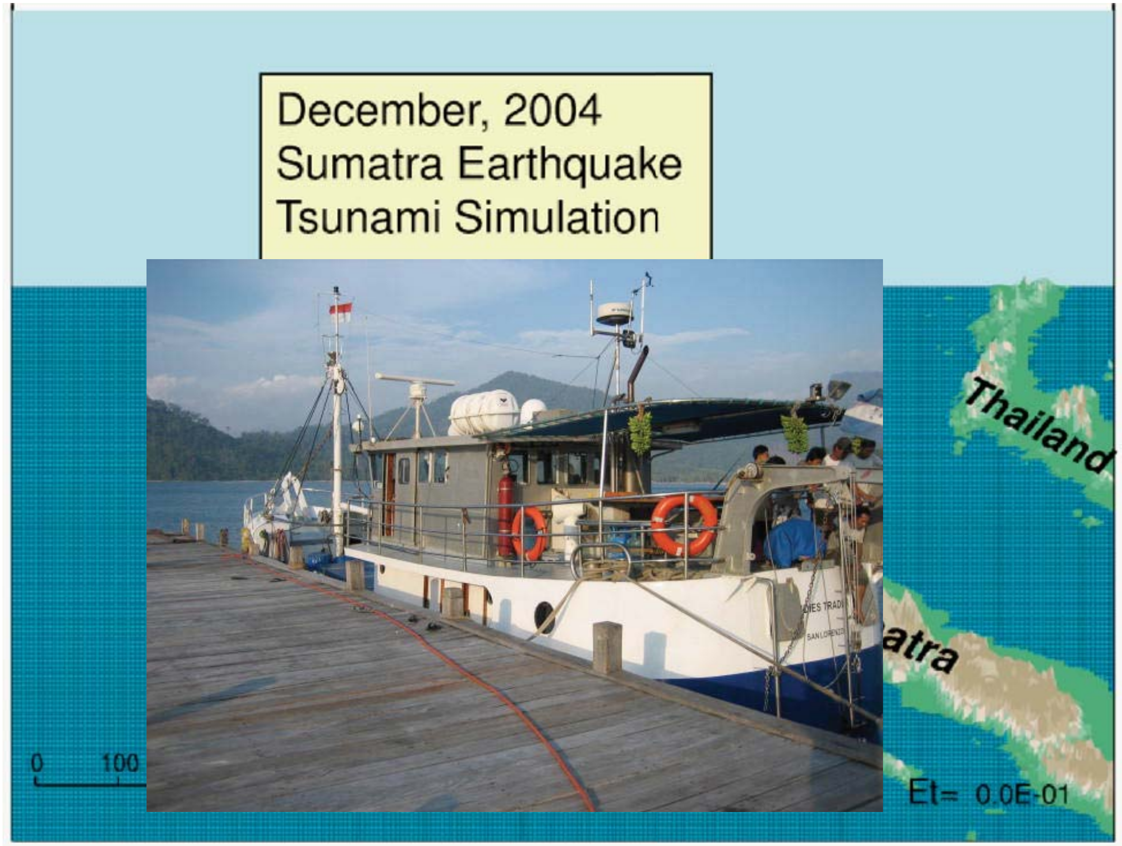
I think that I will leave you with a selection of tsunami simulations that have gotten my attention lately.

STILL--every earthquake has the potential to be “UNUSUAL”

Unusual= Extra Slow Rupture, or Geometrically complex. Often we can’t tell this till weeks later.

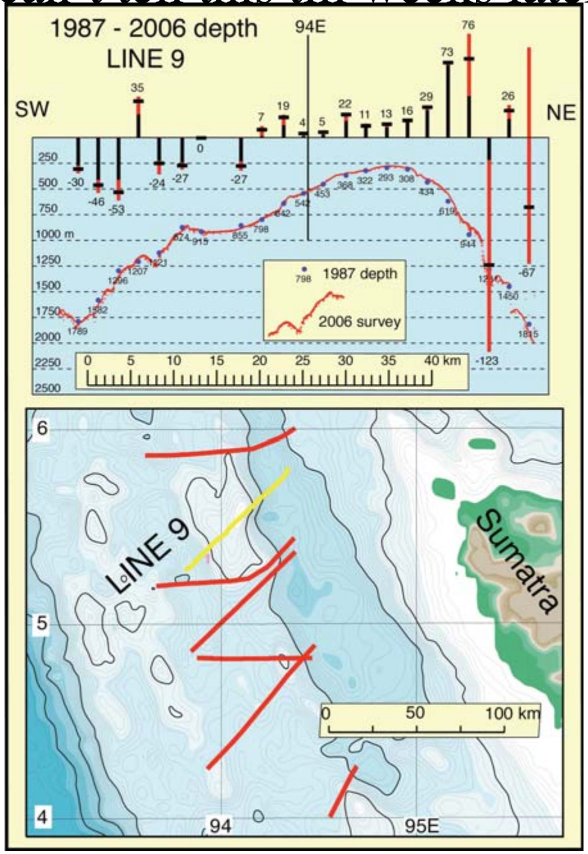


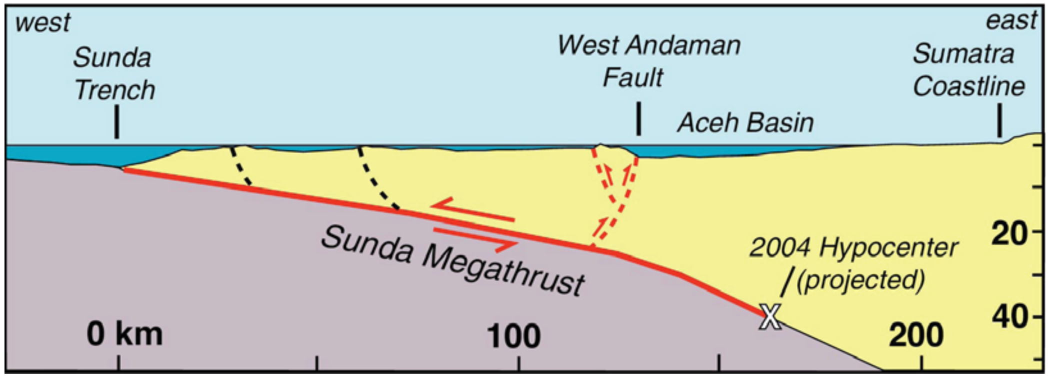


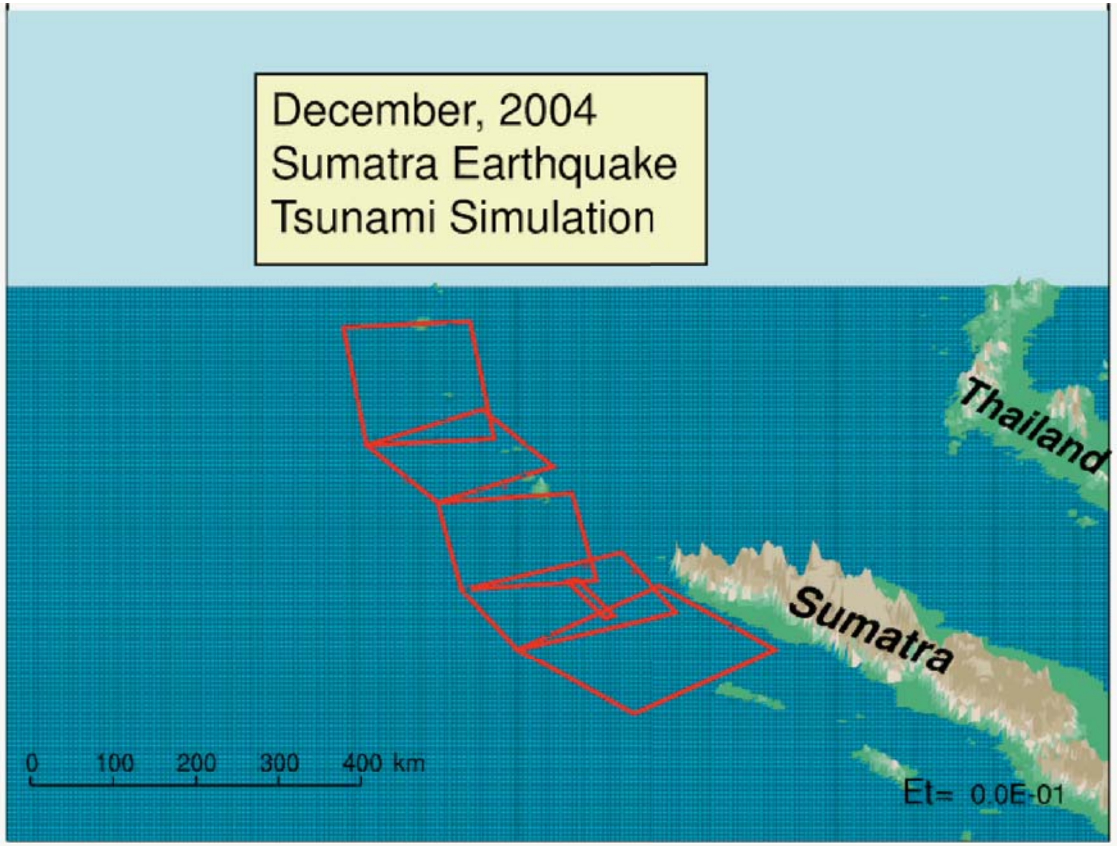


STILL--every earthquake has the potential to be "UNUSUAL"

Unusual= Extra Slow Rupture, or Geometrically complex. Often we can't tell this till weeks later.





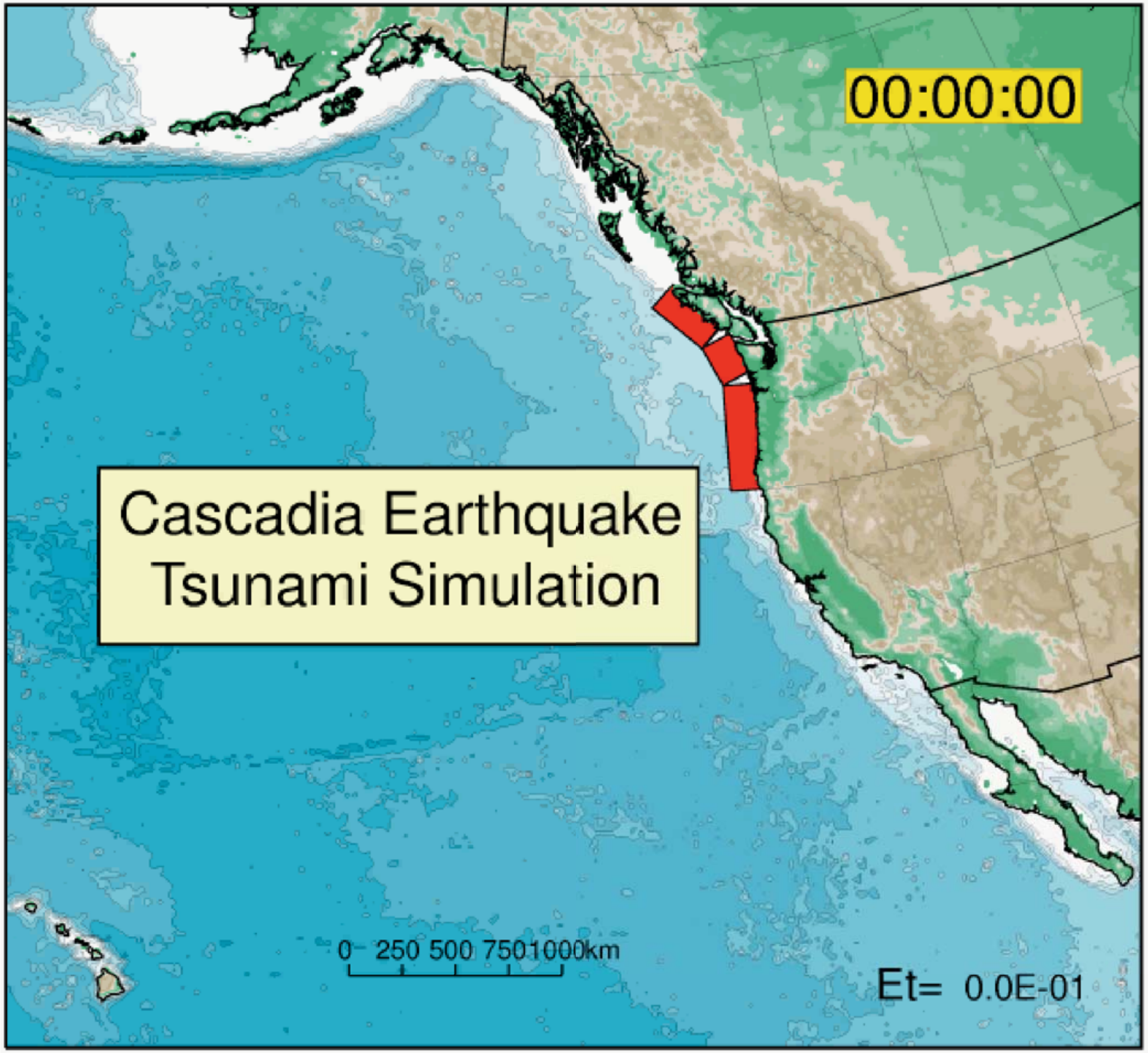


Where NEXT?

We expect the next Cascadia quake to be comparable to the Sumatra Event.

Here is what happened last time in 1700

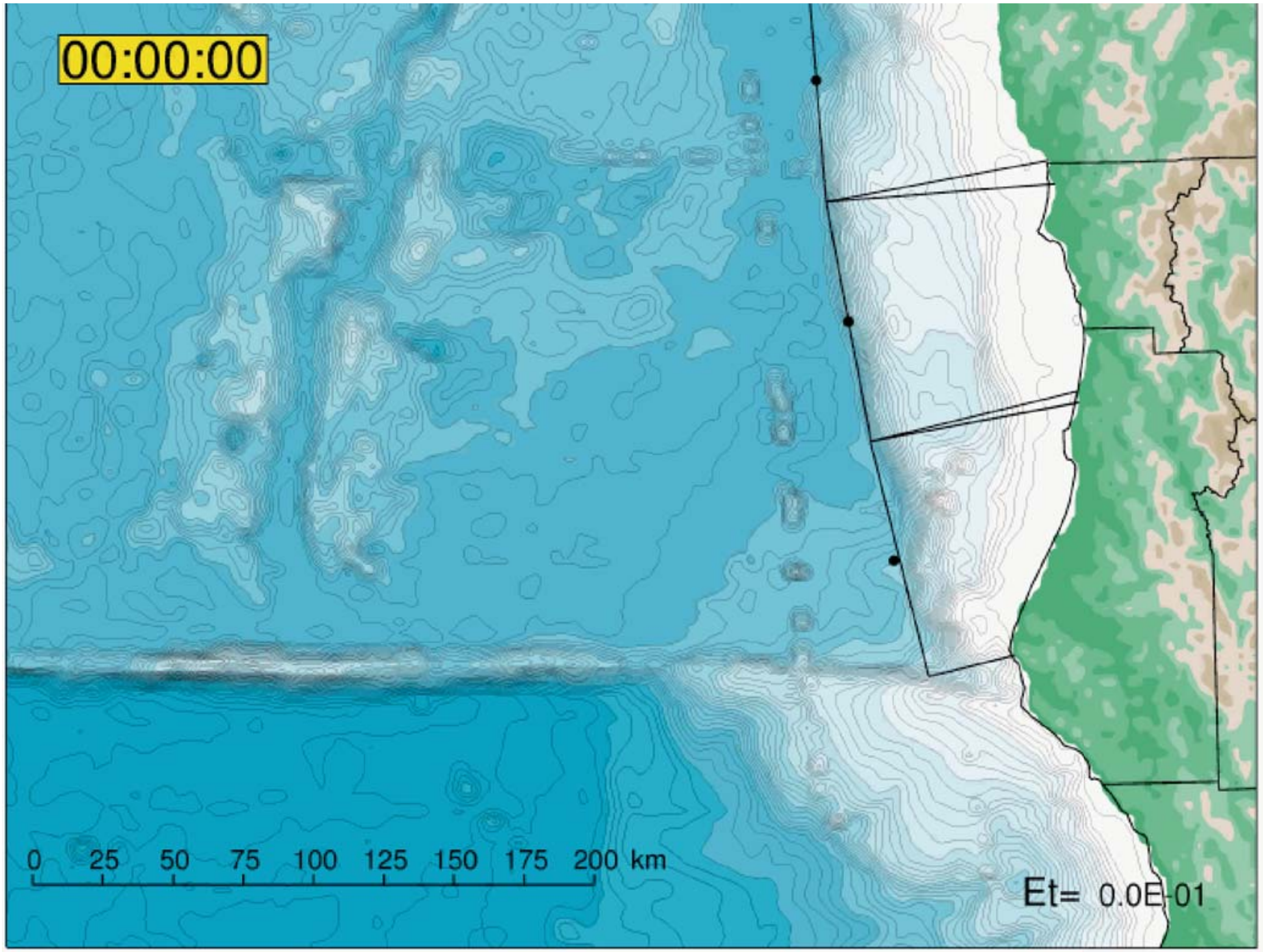
Likely we are getting toward the end of this cycle.



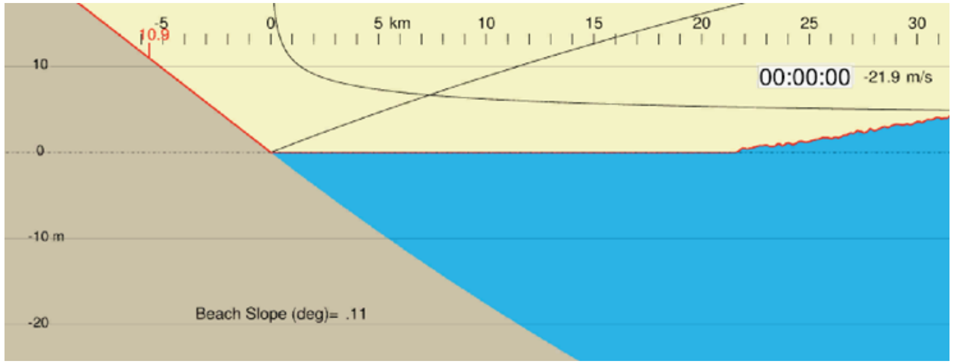
I guess this is another Prediction?

My concept of the next Cascadia Tsunami.

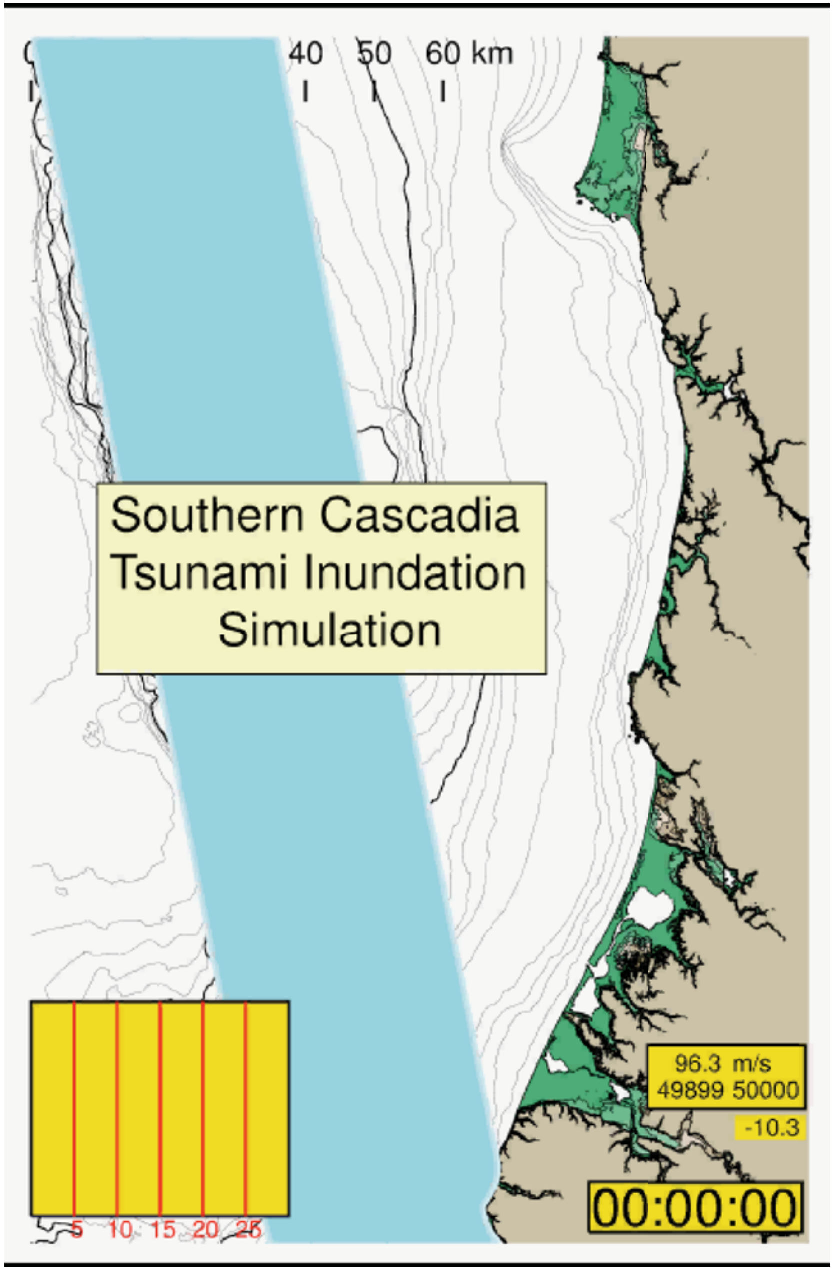
Humbolt and Del Norte counties are in the Red Zone



Runup/Inundation: *This is what you'd want to predict, but it is messy business even in simple 2D cases like below.*



Runup depends on wave height, wave period, beach slope, wave train duration and random interference. In real 3D cases, runup is nearly intractable. Best to consider it a random variable.



In terms of *"Predictability of Natural Disasters"* asteroid impacts can be one of the easiest to quantify -- Example: Asteroid 1950 DA has a 0.3% chance of Earth Impact on March 16, 2880.

No kidding.

WHERE NEXT?

Asteroid Apophis
Diameter: 350m
Discovered: 12/26/04

Passes <30,000km from earth in 2029!!

Projected Earth Impact probability: ~1/40000

On April 13, 2036

"Everyone talks about natural disaster, but no one does anything about it."

Asteroid Defense-

Time Before Impact and Controllability primary issues.

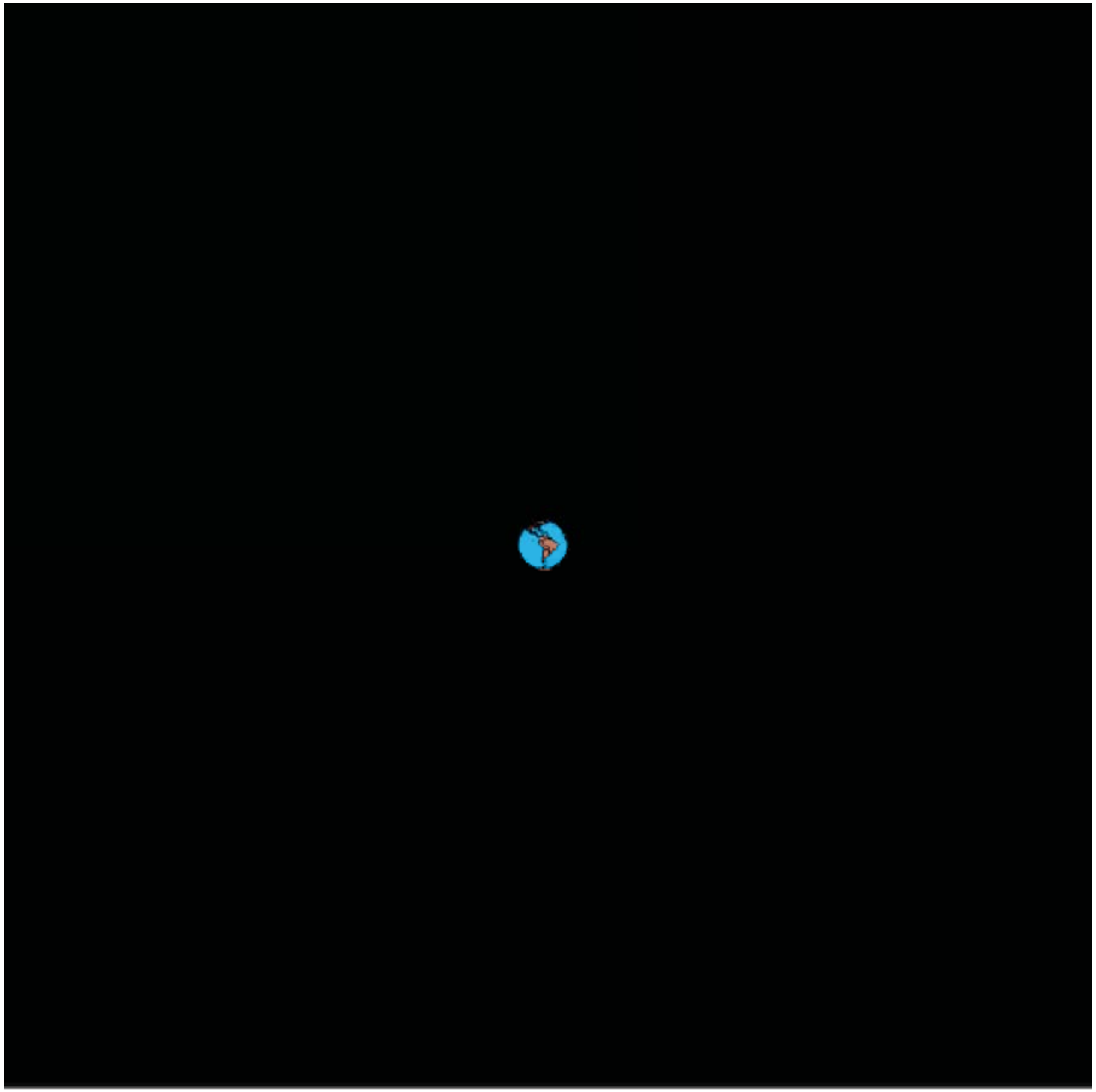
1) Gravity Tug

2) Kinetic Impact

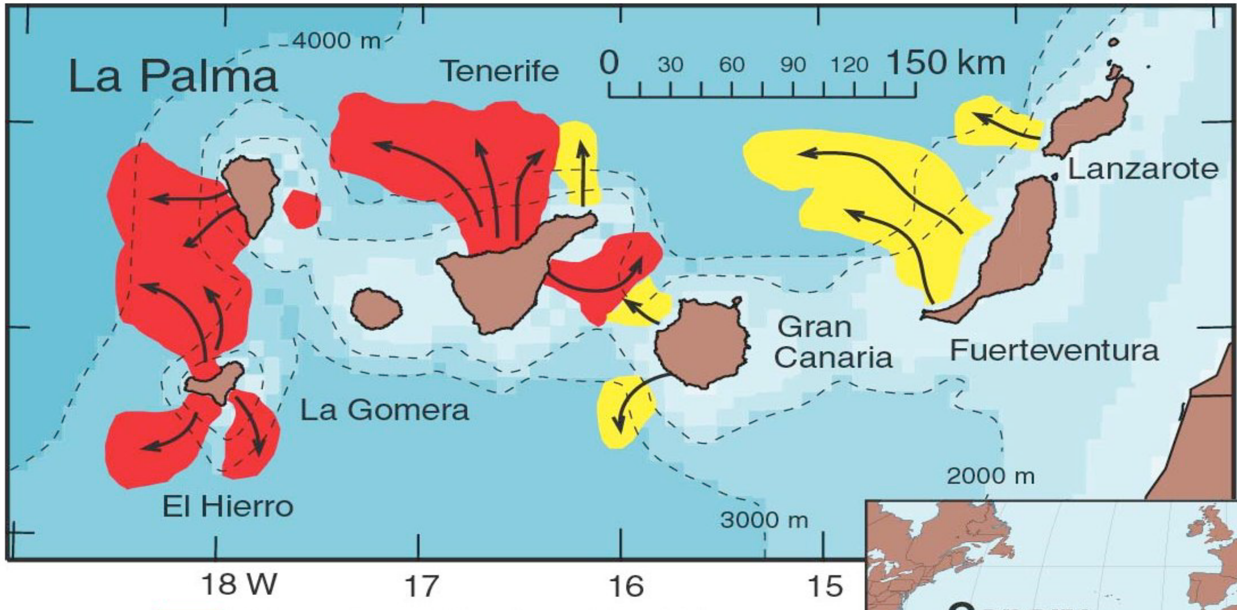
3) Standoff Explosion

4) Surface Explosion

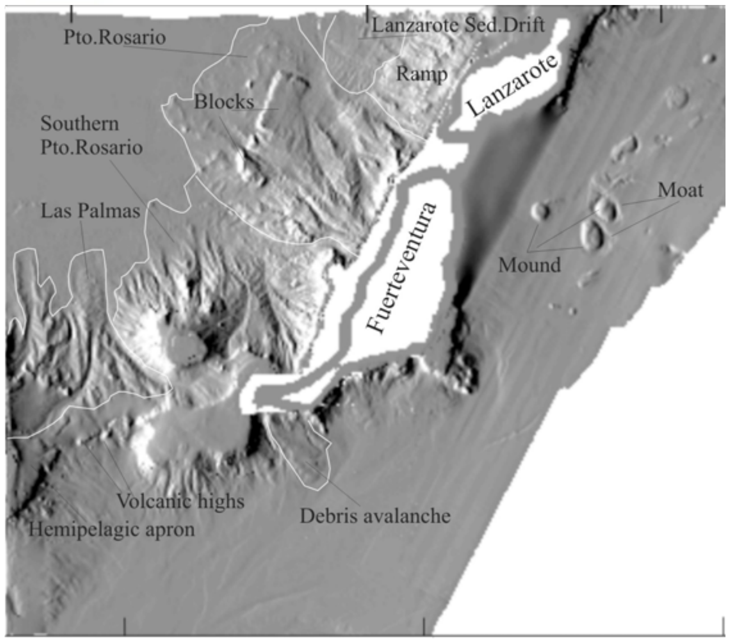
Apophis Impact Scenario: North of Panama Expect: 20m+ waves onshore Est. Infrastructure loss: 400 Billion US$



Where Next?



The Canary Islands have experienced about a dozen superslides in the past 2 million years.



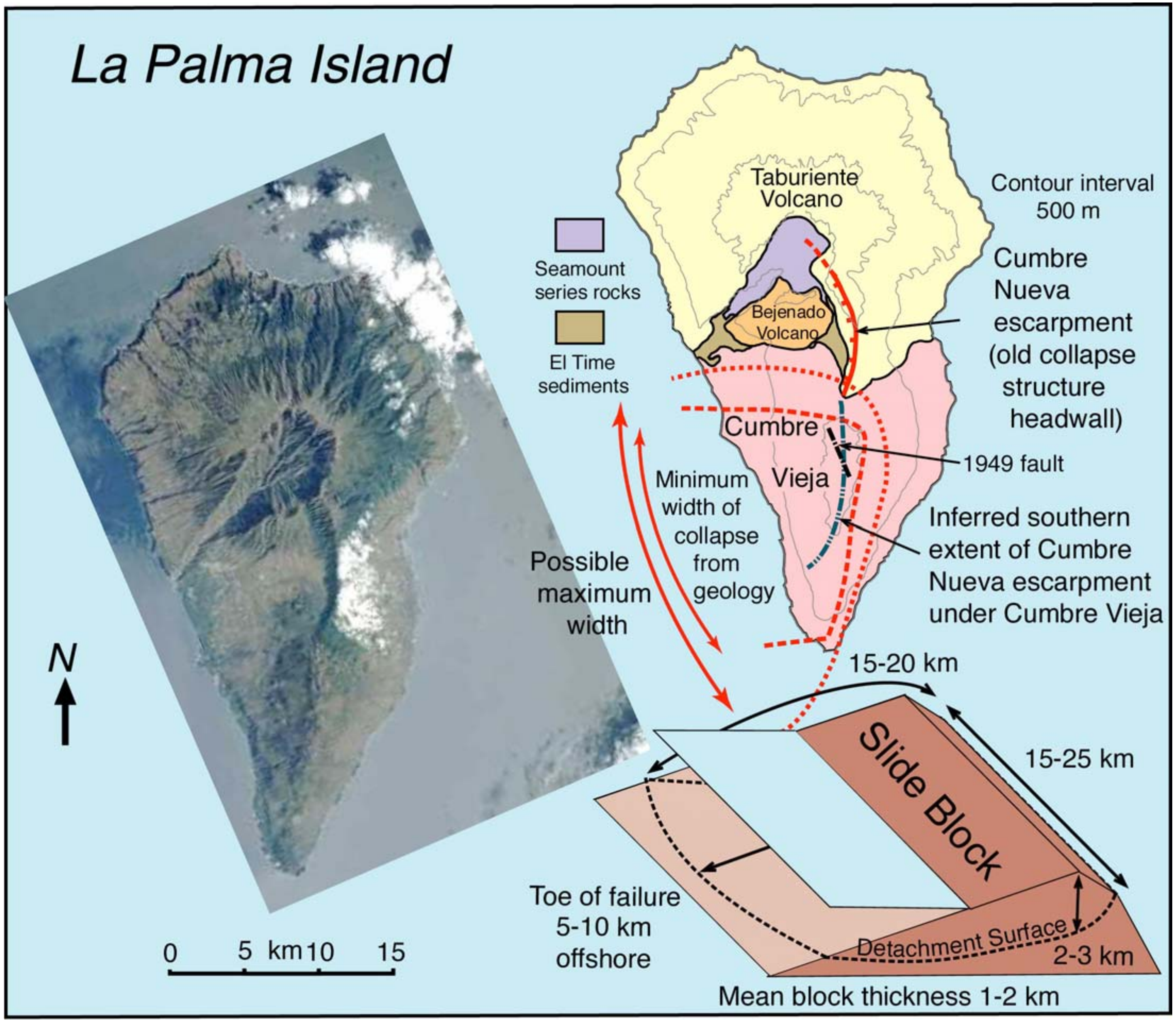
How do we know this?

The slide stuff is still lying there!

Volcano Cumbre Vieja on La Palma Island last collapsed 550,000 years ago.

According to Dr. Simon Day, the mountain is primed for another show.

Expected to drop as much as 500 km^3 into the deep Atlantic Basin-- about the same as last time

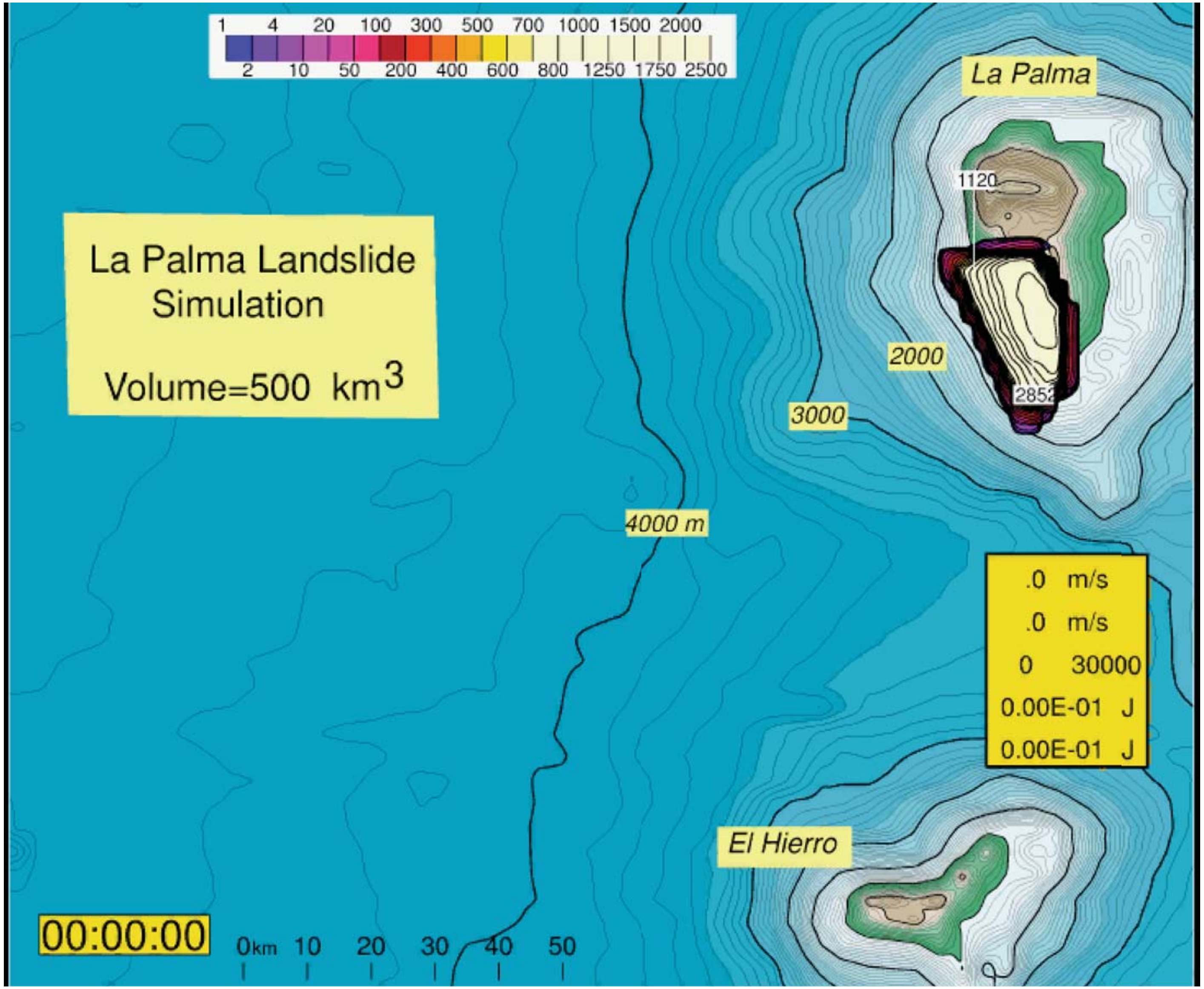


When this happens--

STAND BACK

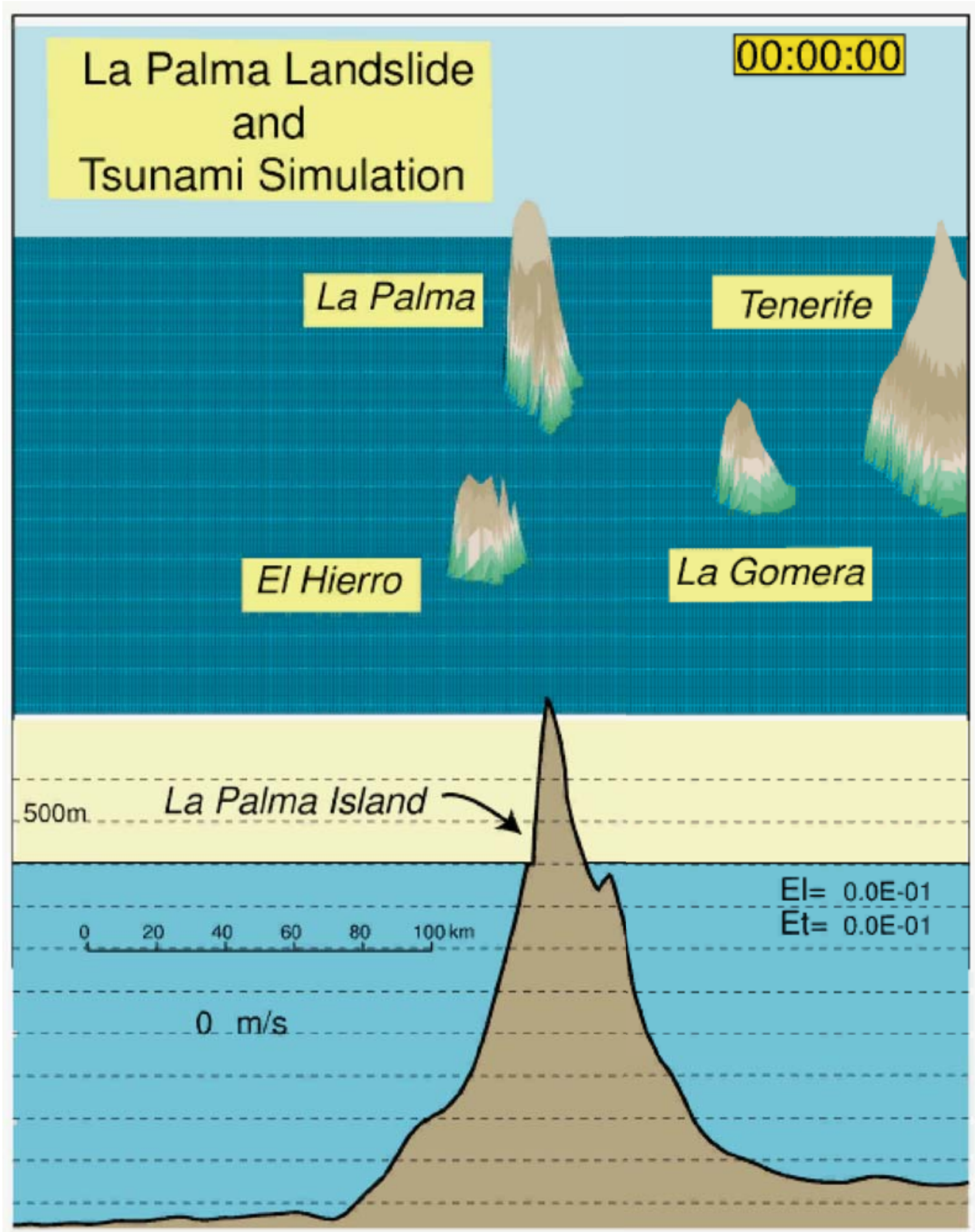
The slide will release about 3×10^{19} J of energy.

Roughly equivalent to detonating the entire world's nuclear weapons stockpile all at once. 8,000 Mt.



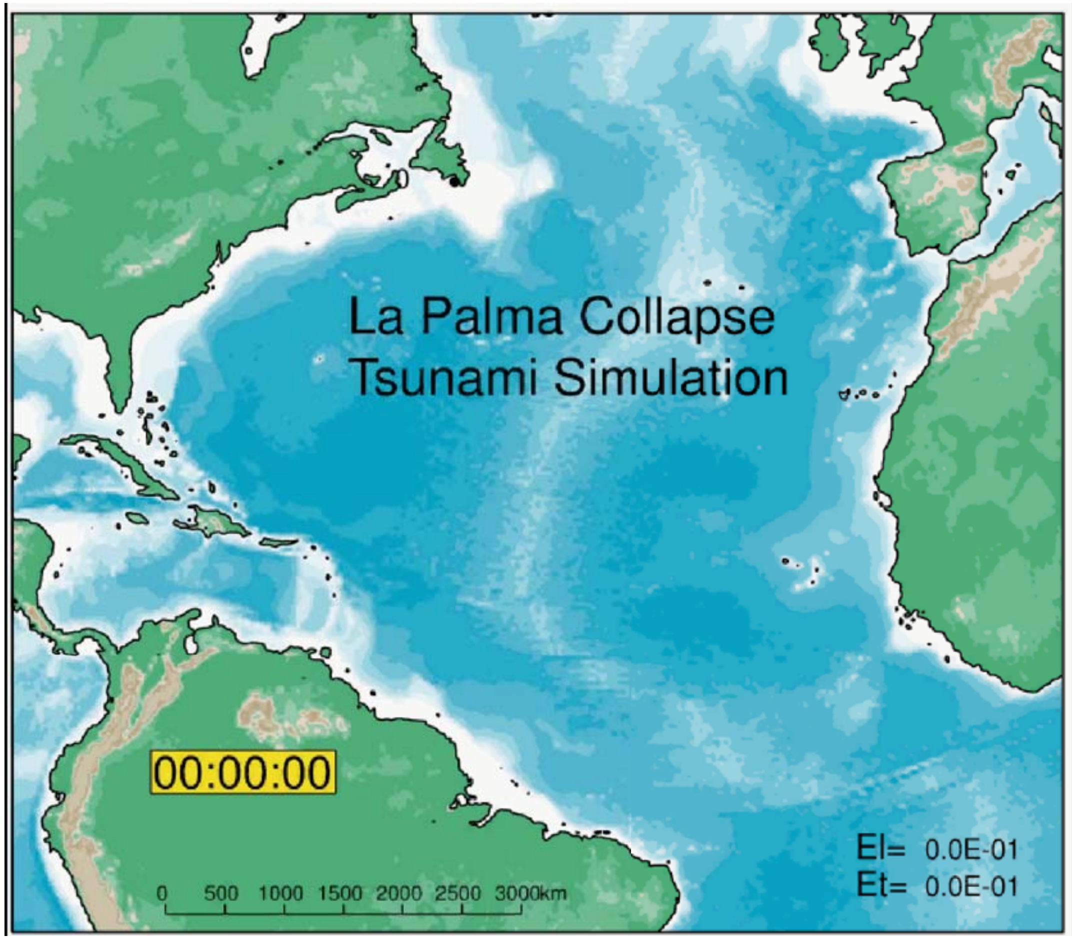
Only about 10-15% of slide energy goes into tsunami, but still...

Within 20 minutes, a wave will wet local islands to 150+ meters.

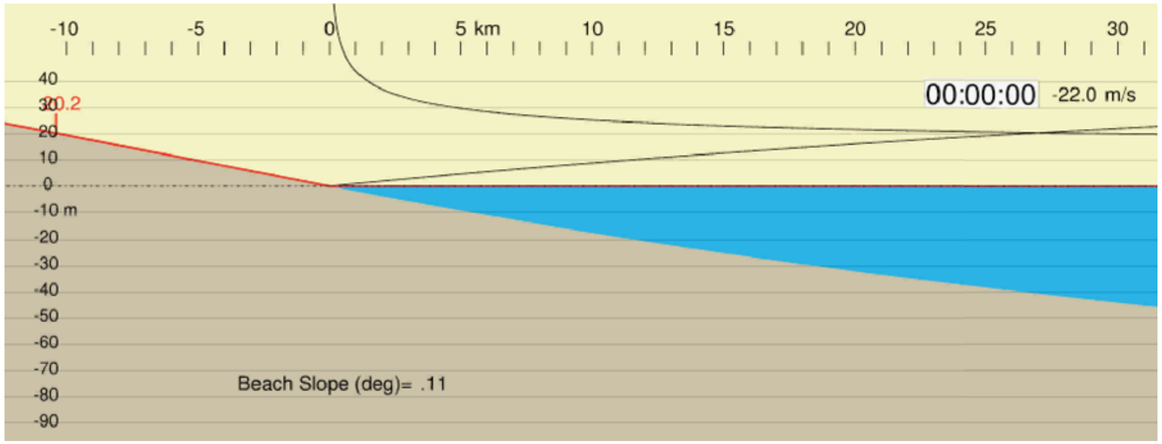


A La Palma Tsunami will affect coasts along entire Atlantic Basin.

Lisbon 40m   Ireland: 40m   Florida:30m NYC:25m Brazil:40m



Not just one wave. WAVE SET UP Storm surge-like

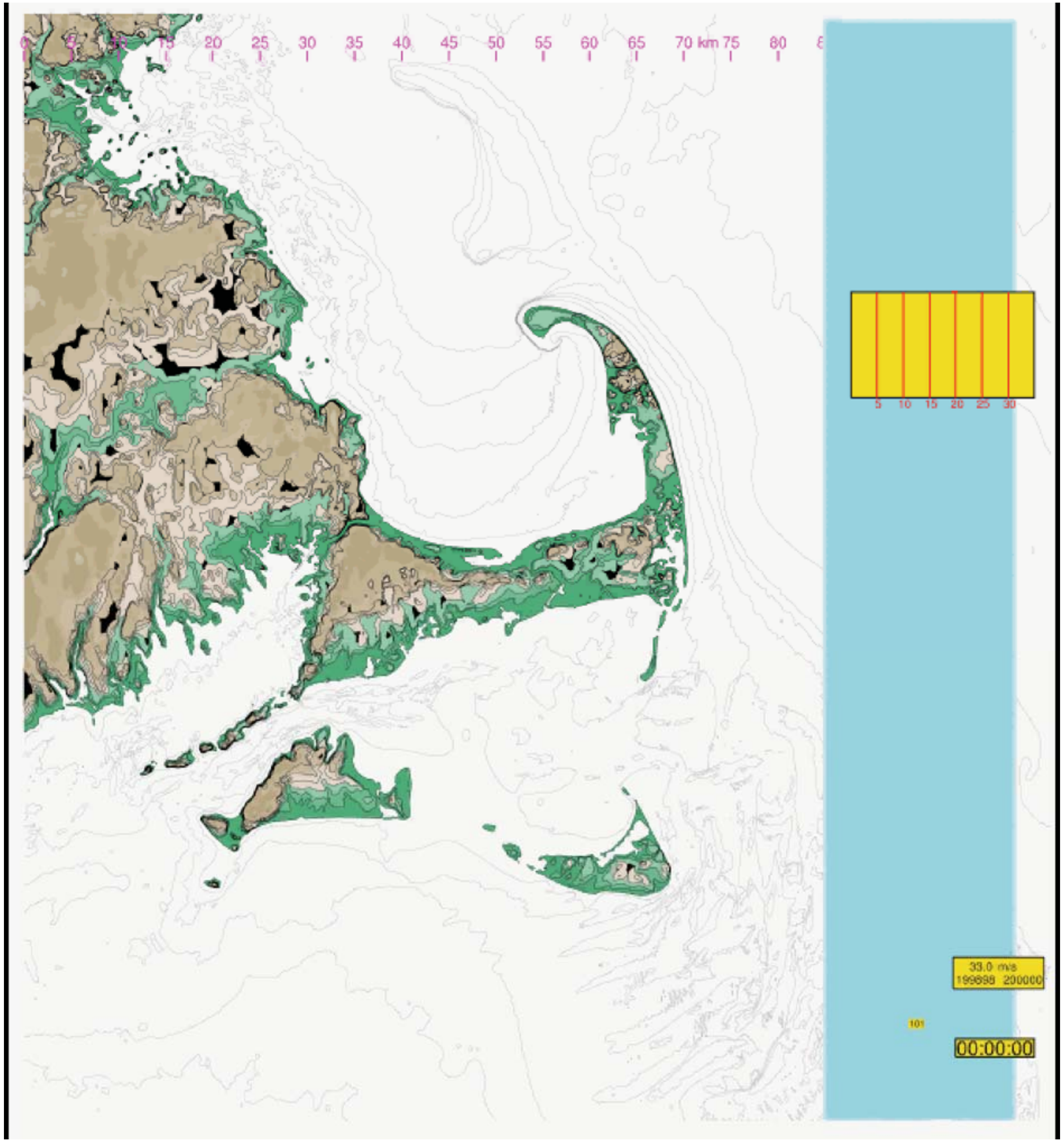


One 10m high wave - coming in to Cape Cod

Flooding several to many miles inland.

Nantucket folks will not be happy.

A LA PALMA TSUNAMI WON'T BE THE END OF THE WORLD, JUST PRETTY EXCITING.



Thanks for Attending!