The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1965-26

9th Workshop on Three-Dimensional Modelling of Seismic Waves Generation, Propagation and their Inversion

22 September - 4 October, 2008

Upper Mantle Anisotropy from Surface Wave Studies

Part I

Jean-Paul Montagner

Dept. Sismologie
I.P.G.
Paris
France

jpm@ipgp.jussieu.fr

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

SURFACE WAVES and UPPER MANTLE ANISOTROPY

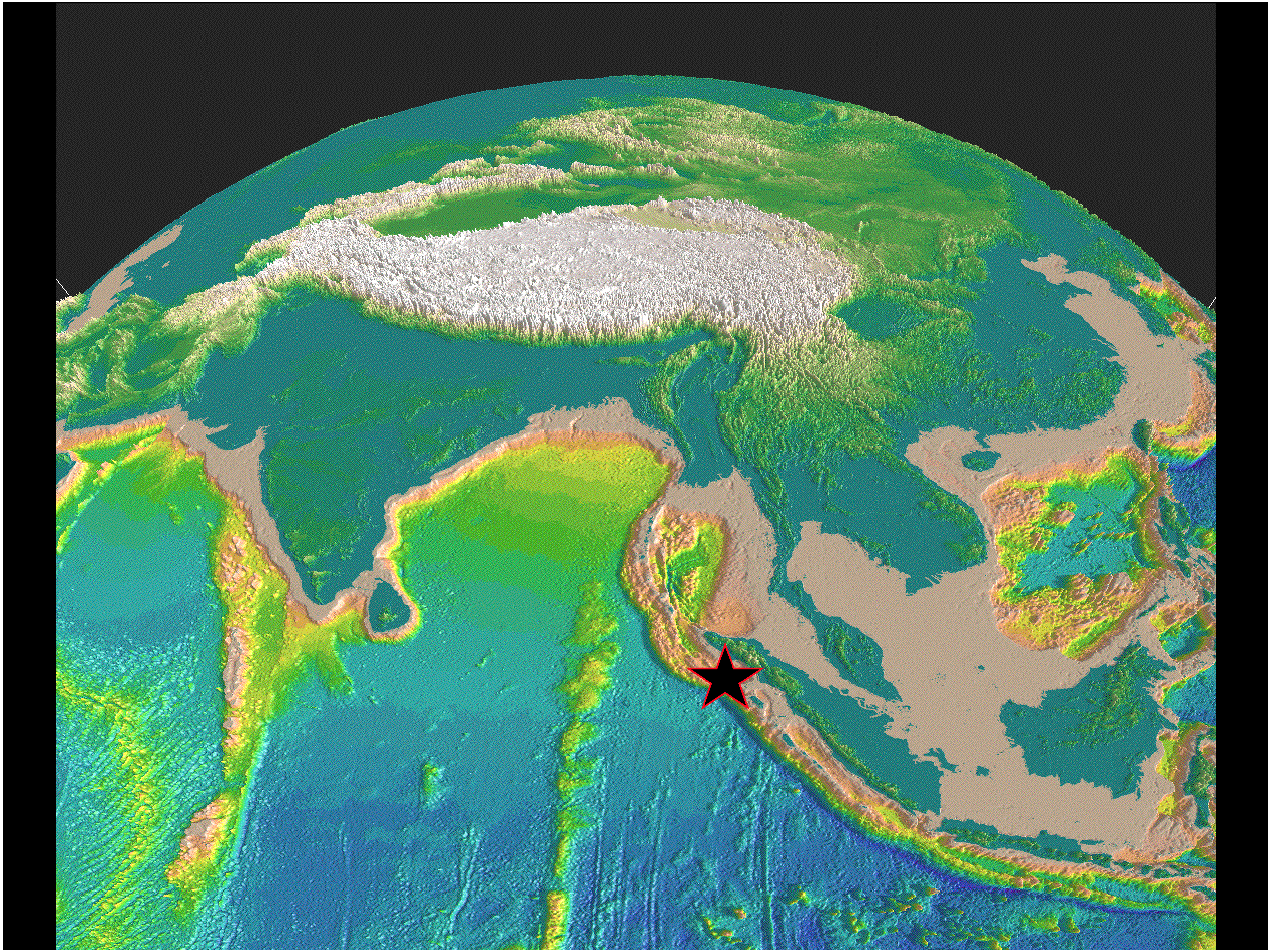
Jean-Paul Montagner

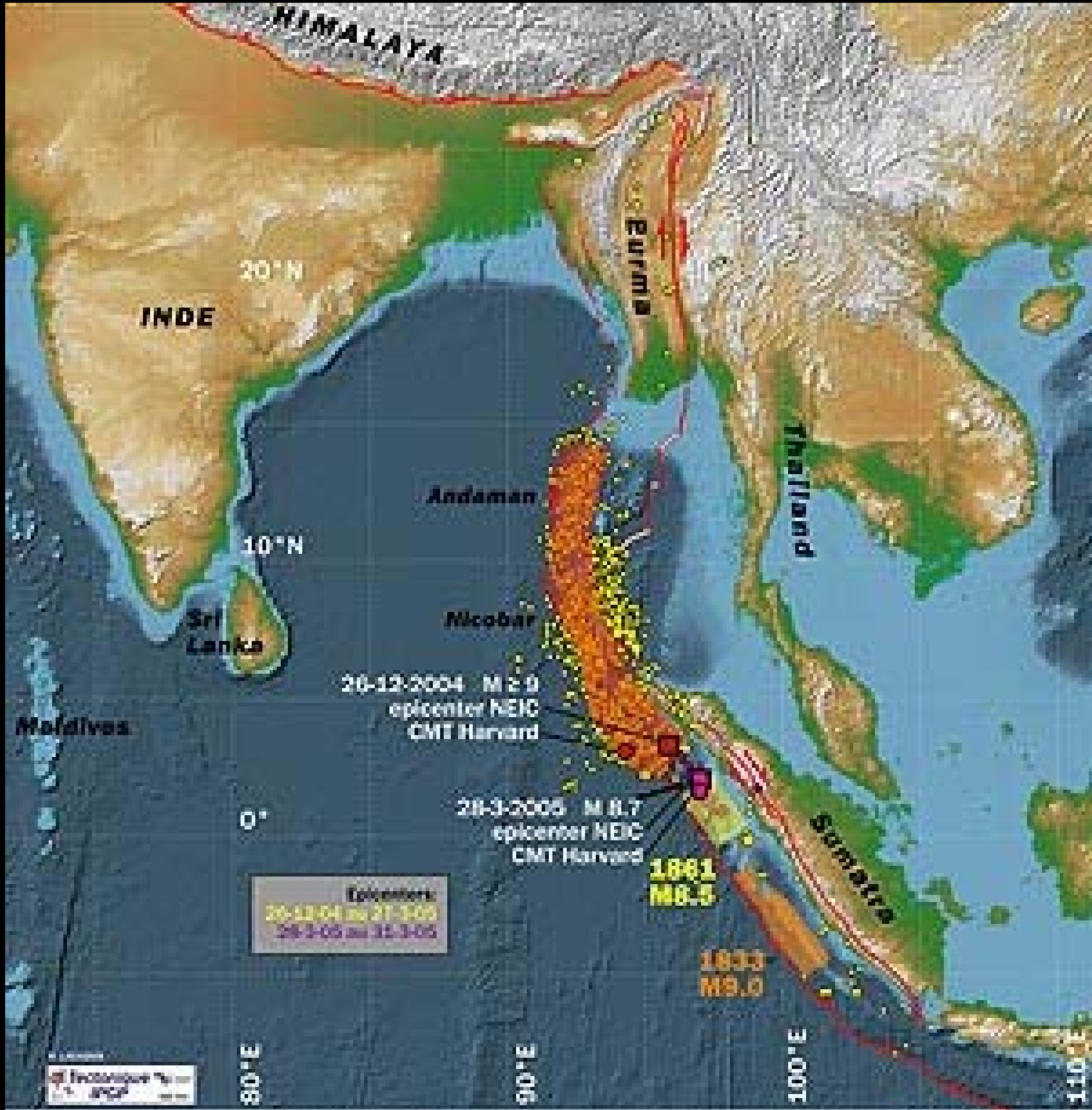
Dept. Sismologie, I.P.G., Paris; France

Overview
Large scale Seismology: an observational field

- Data (Seismic source) + Instrument (Seismometer) -> Observations (seismograms)
- Historical evolution: Ray theory, Normal mode theory, Numerical techniques (SEM, NM-SEM)
- Scientific Issues: Earthquakes (Sumatra-Andaman), Anisotropic structure of the Earth
- Tomographic Technique
- Geodynamic Applications

  Seismic Experiment: Plume detection
- Adjoint and time reversal methods





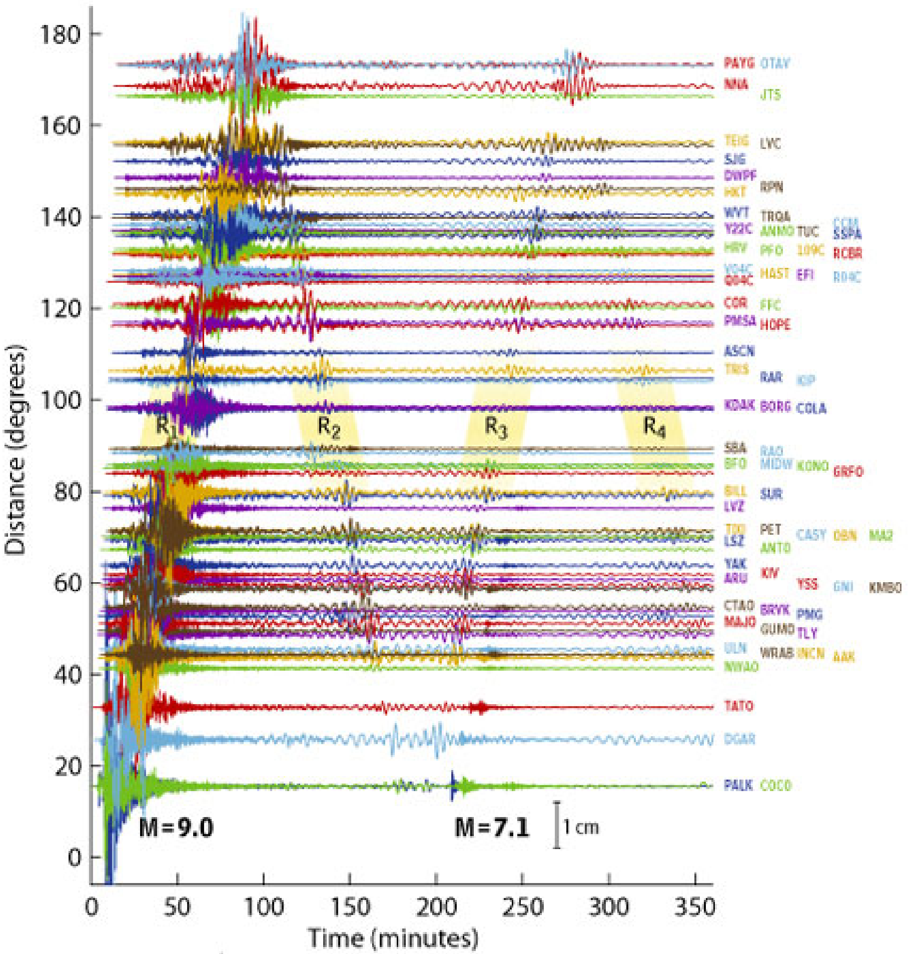
HIMALAYA
INDE
20°N
10°N
0°
Burma
Thailand
Andaman
Nicobar
Sri
Lanka
Maldives
26-12-2004 M ≥ 9
epicenter NEIC
CMT Harvard
28-3-2005 M 8.7
epicenter NEIC
CMT Harvard
1861
M8.5
1833
M9.0
Sumatra
Epicenters:
80°E
90°E
100°E
110°E

Banda Aceh

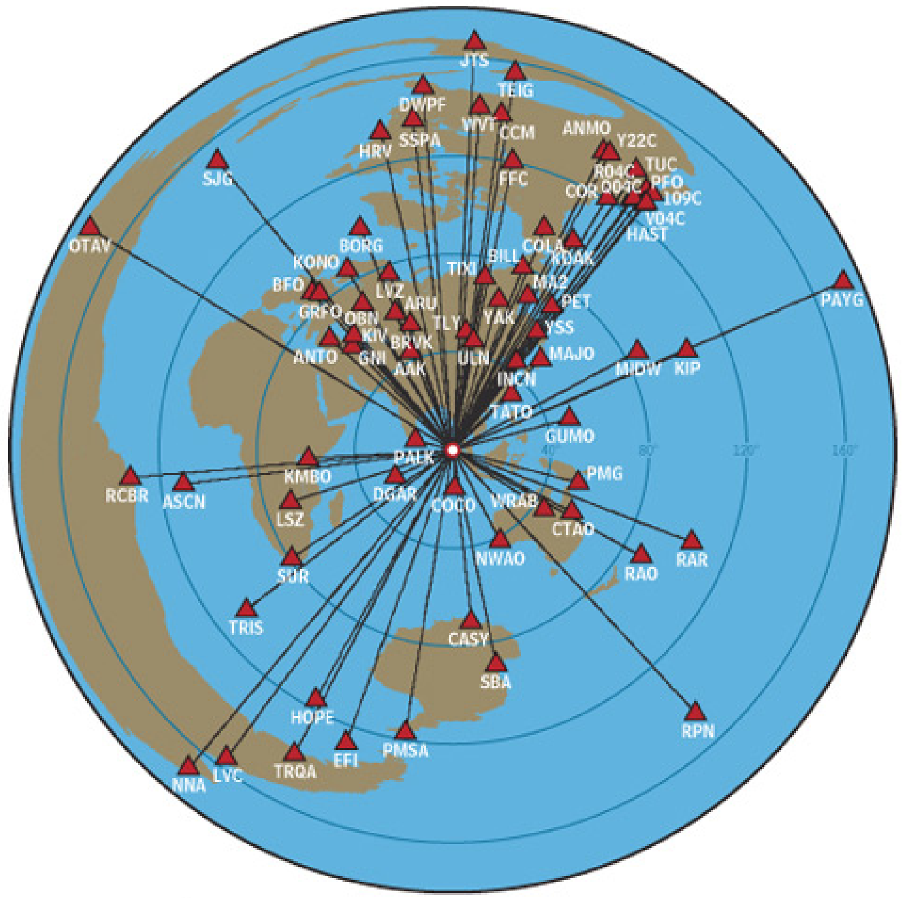
before

after

Sumatra - Andaman Islands Earthquake (M_W=9.0)
Global Displacement Wavefield from the Global Seismographic Network



Sumatra - Andaman Islands Earthquake
Global Seismographic Network Stations



Seismic Instruments

- Seismoscope

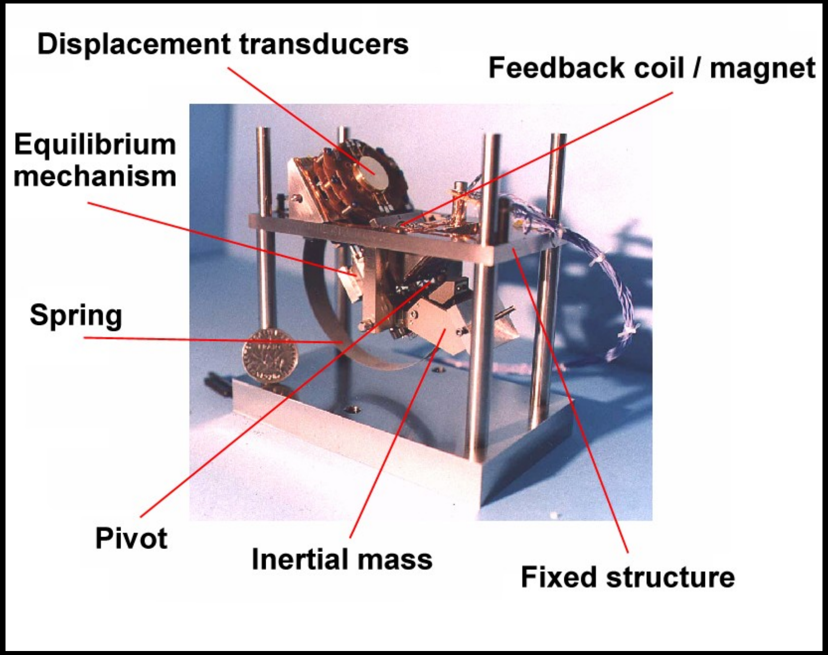
(China -100BC)

Broadband

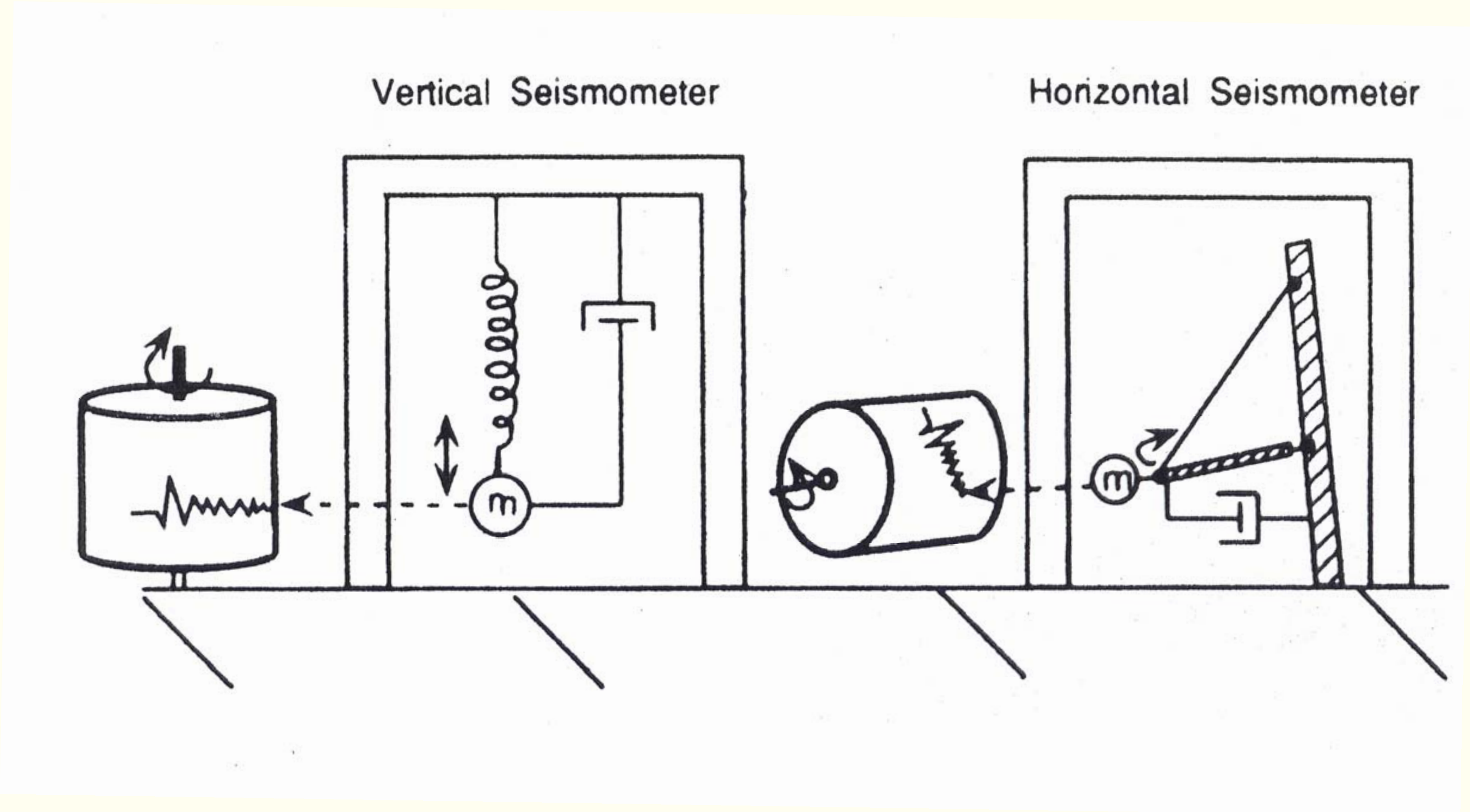
Seismometer

(1mHz-20Hz)

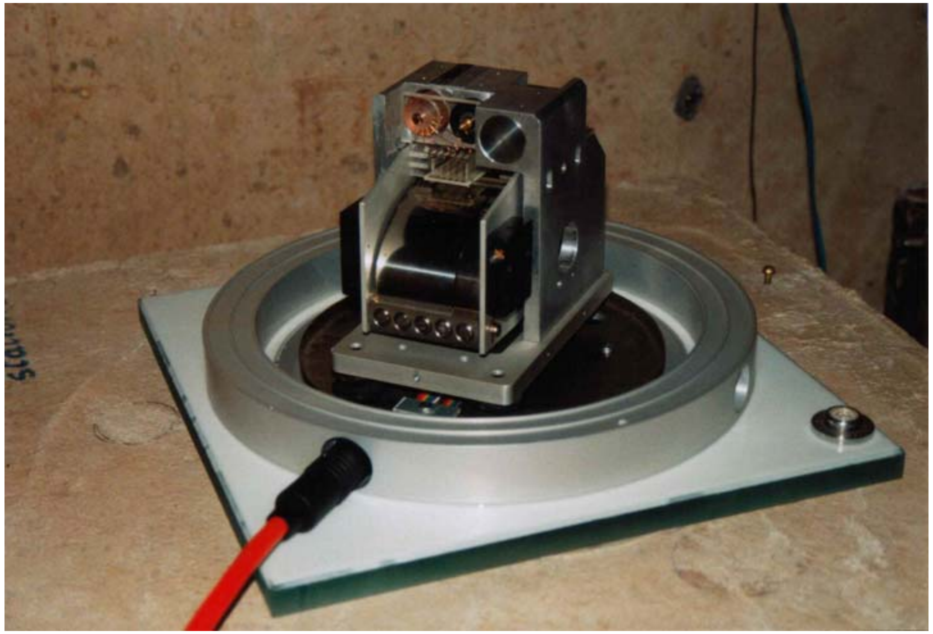
(Cacho, 1998)



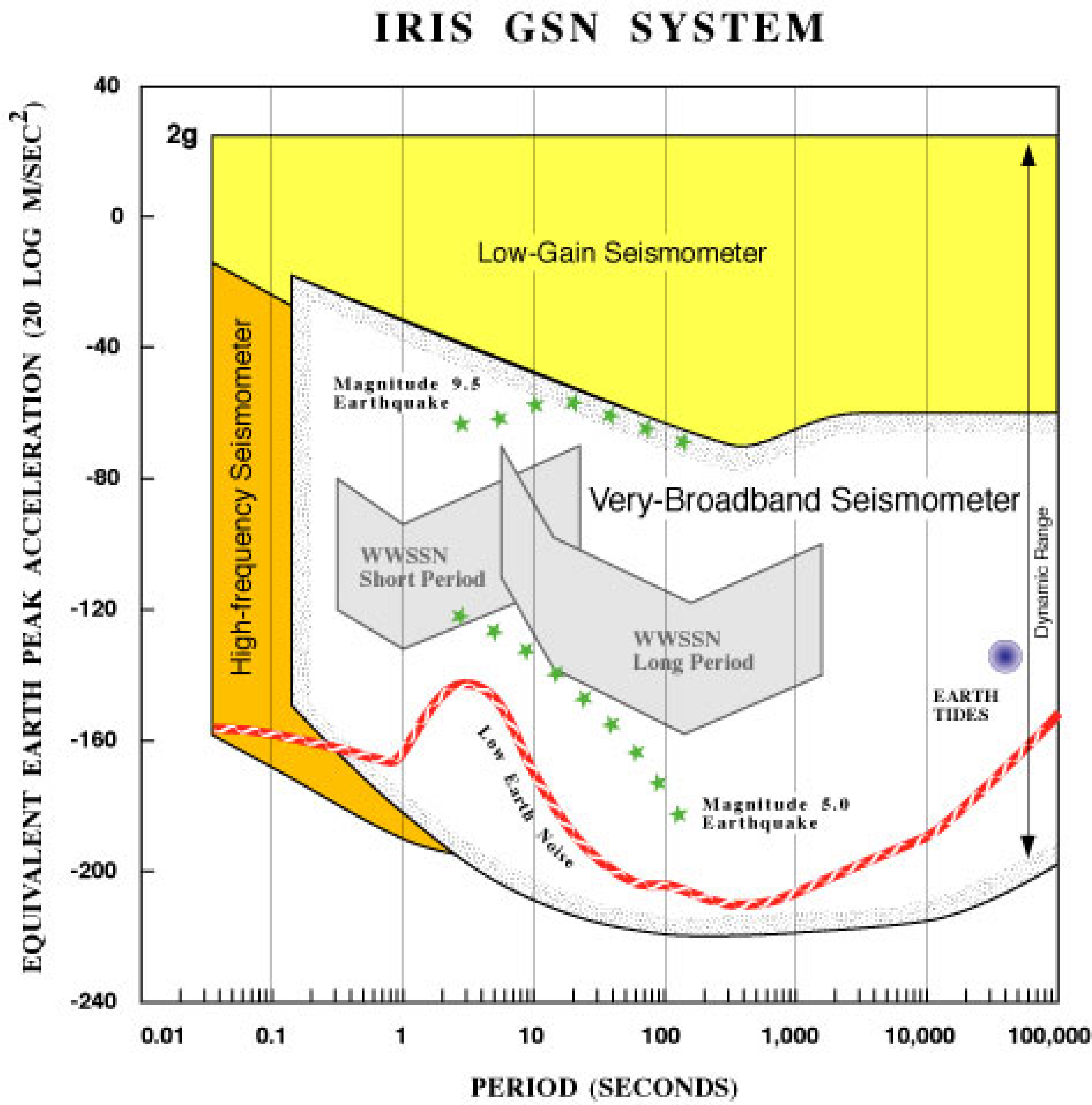
Principle of a Seismometer



Tiltmeter (1960)



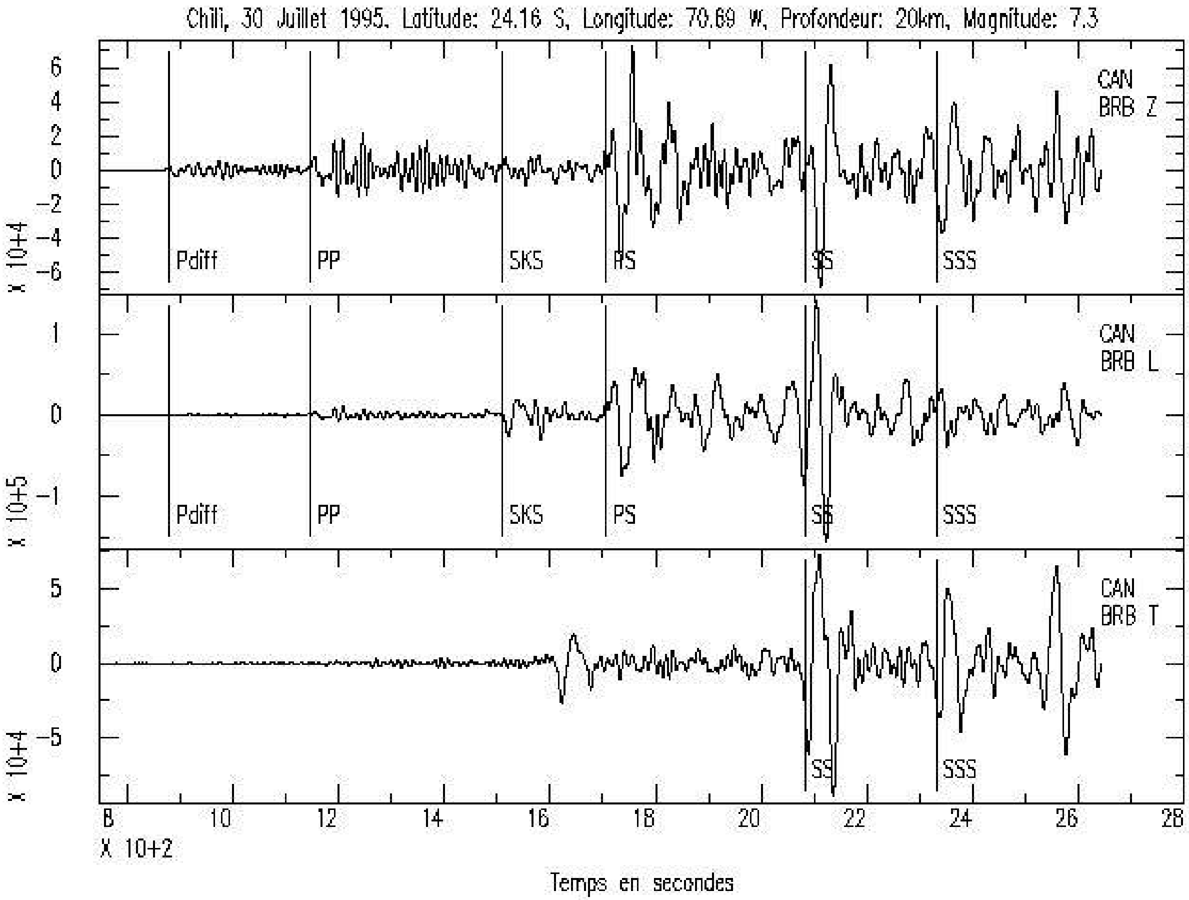
Broadband Seismometer (1982)
Streckeisen STS1: 0.05s< T< 5000s



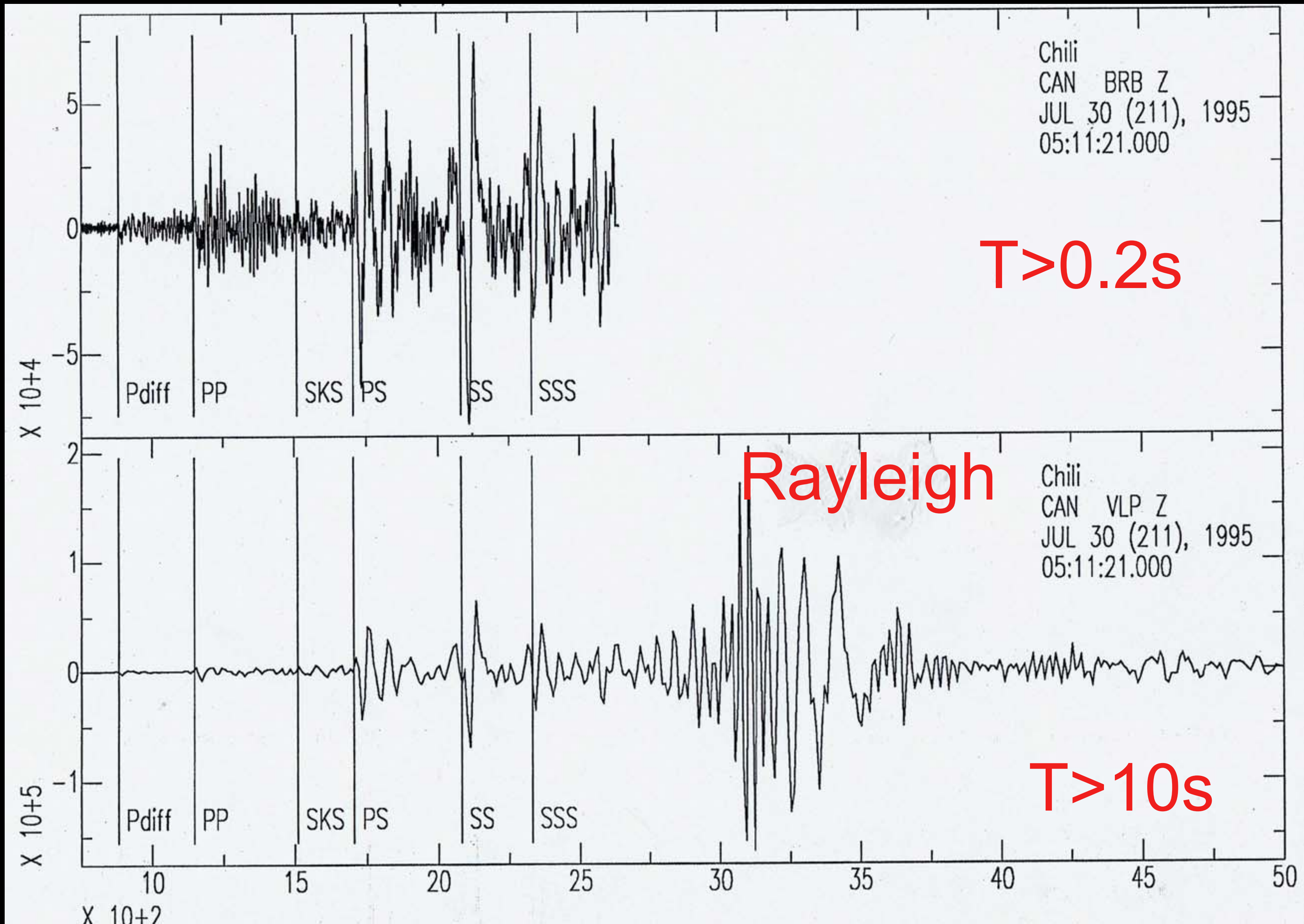
Butler et al., 2004

3 components
frequency range: 1mHz-20Hz
Period range: 0.05-1000s

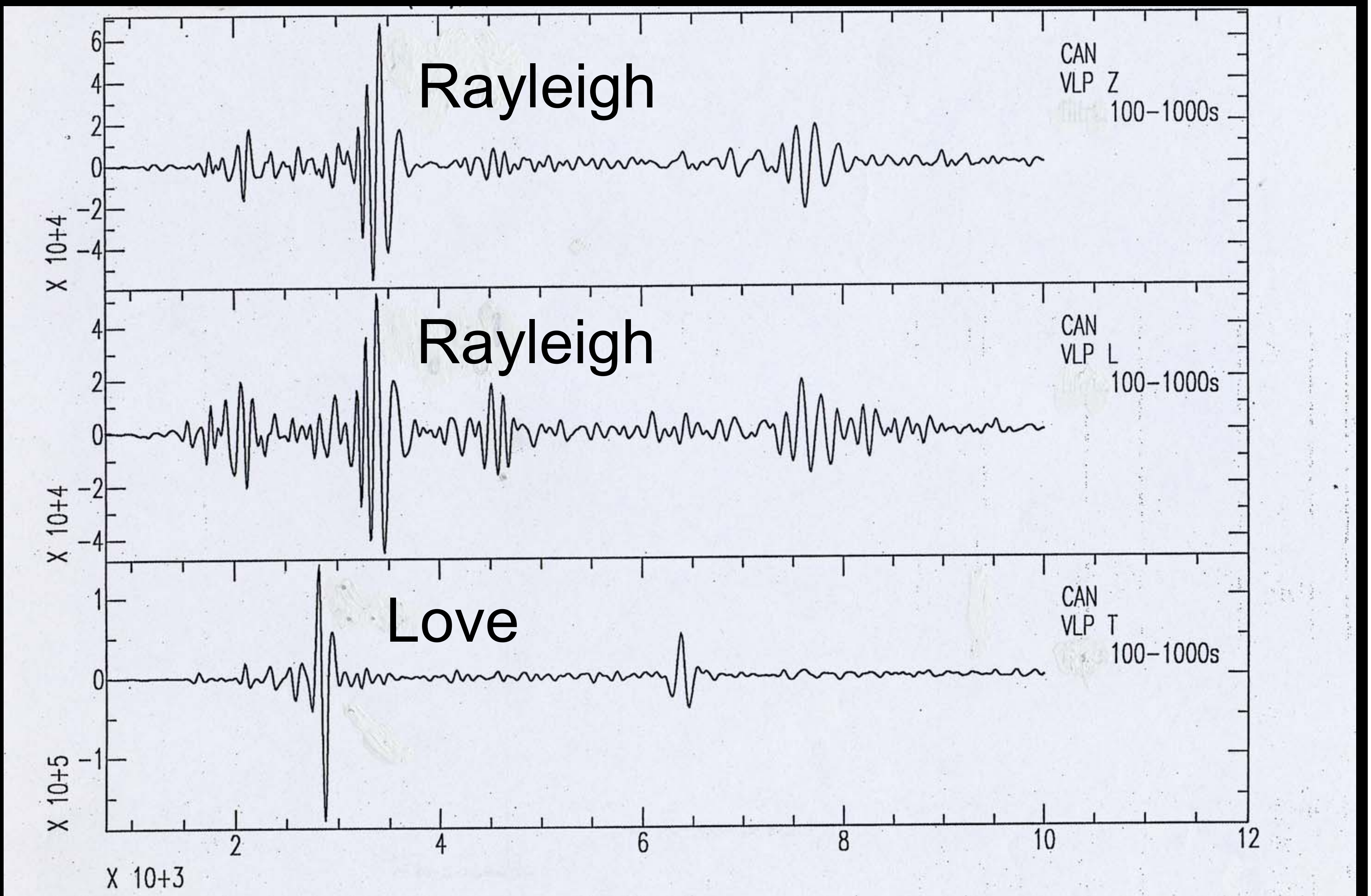
Chile July 30, 1995, Ms=7.3



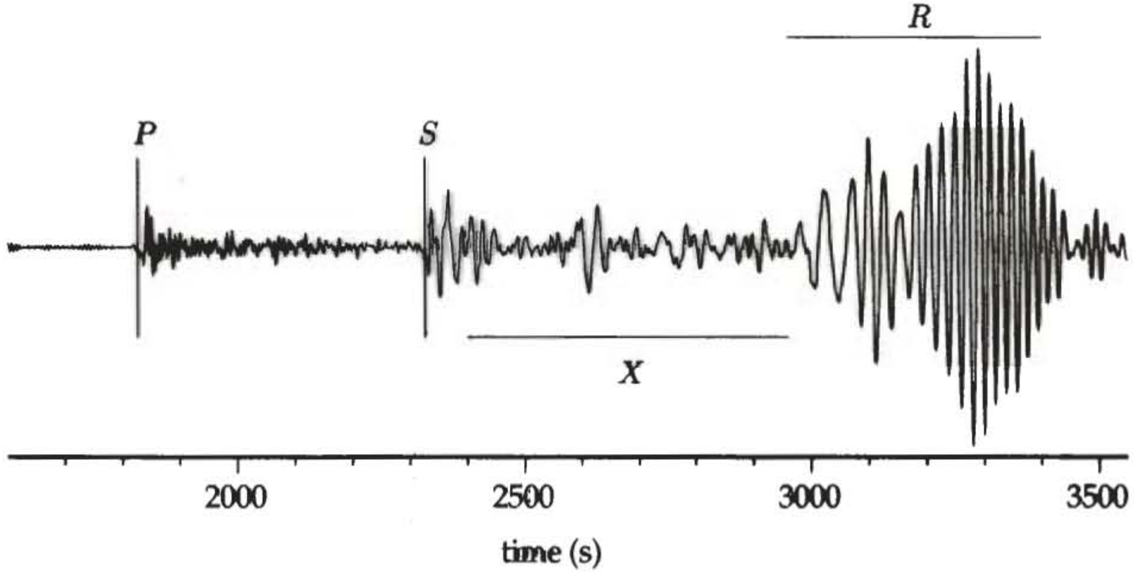
Chile earthquake magnitude= 7.3
Epicentral distance = 12,300km-depth 20km



Chile Earthquake Jul. 1995

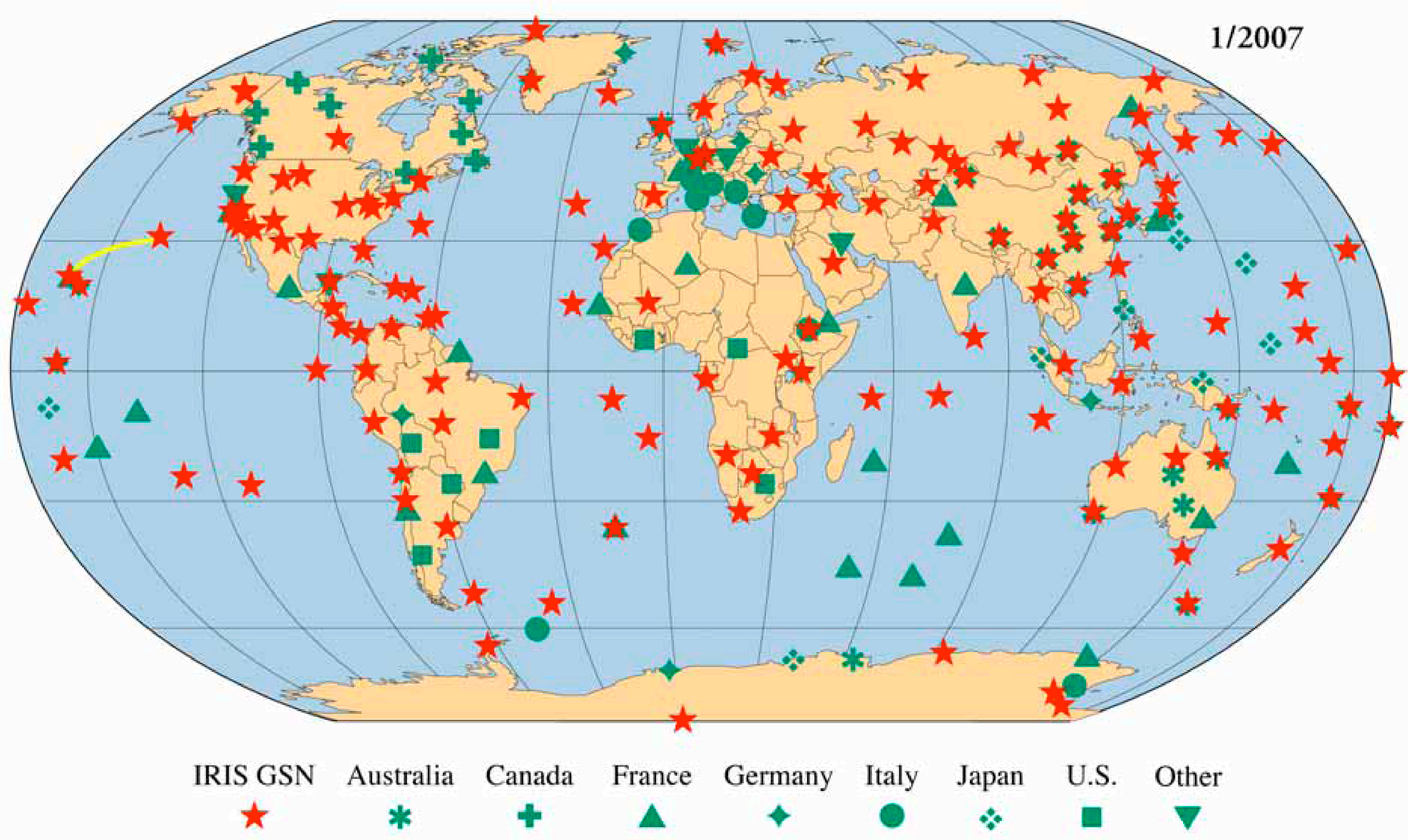


- Dispersive waves,
- Good global coverage,
- Large scale heterogeneities (min. 600 km).



Vertical component of displacement field recorded at DRV station corresponding to the New-Guinea 05/16/1999 earthquake.

F.D.S.N. (Federation of Digital Broadband Seismic Networks)



Ocean Bottom Observatories

=> International Ocean network (I.O.N.)

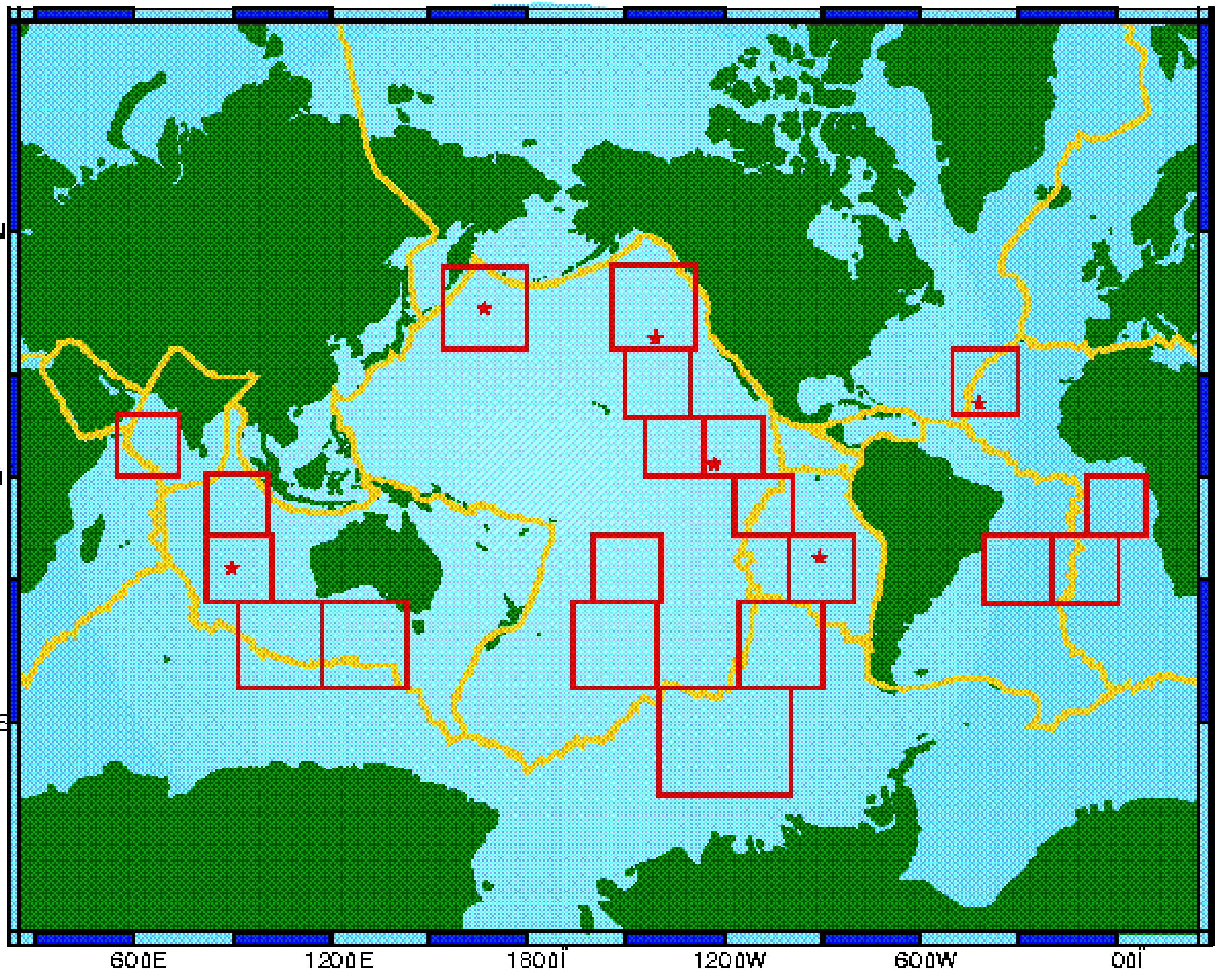
- 2/3 of the Earth are covered by water.
- seafloor seismometers enable:
  - – To investigate oceanic regions with a better resolution
  - – To fill gaps in the global coverage

NERO (joint French-Japanese Project)

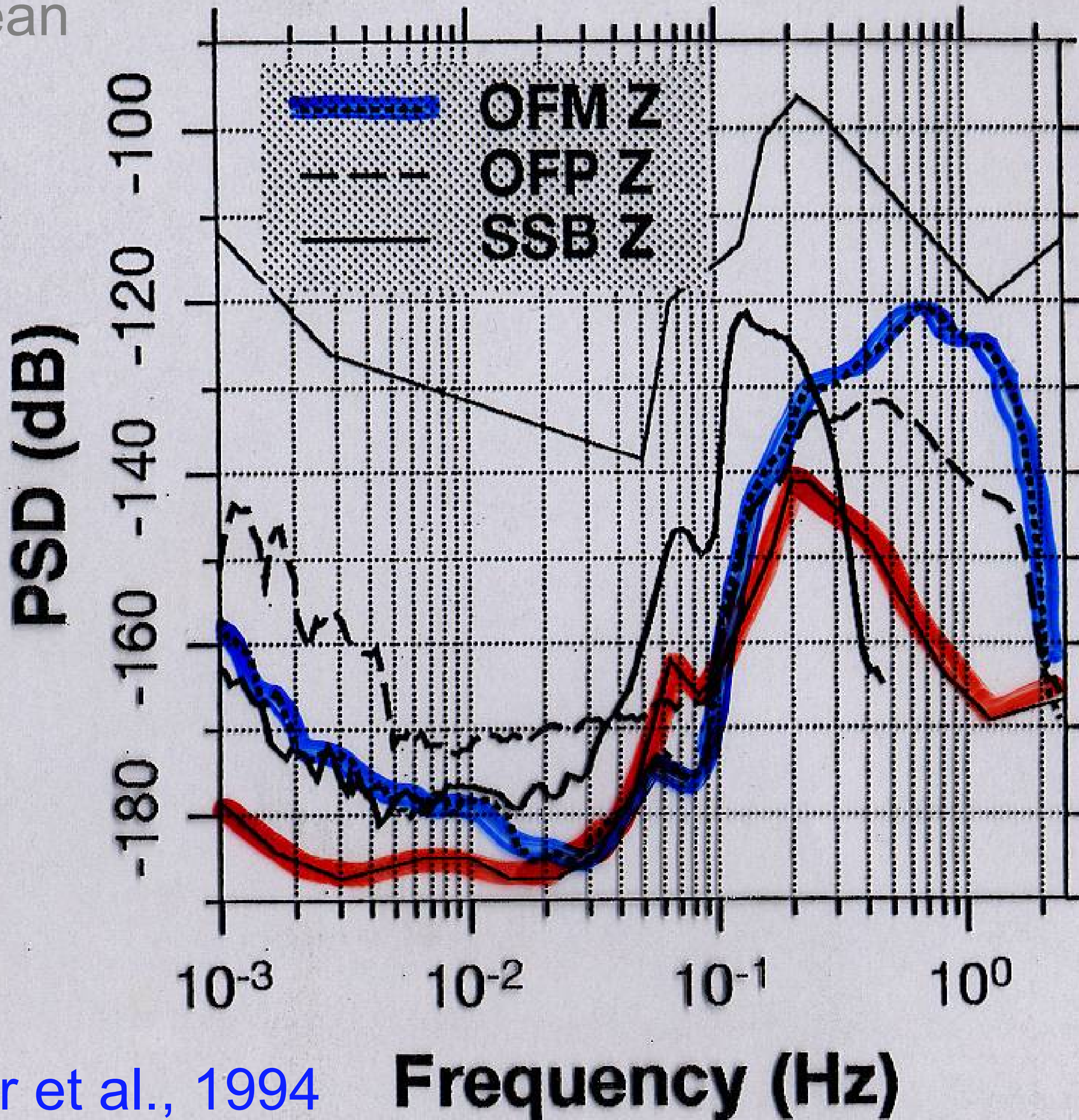
I.O.N.

International Ocean Network

ION (International Ocean network) France, Italy, Japan, UK, U.S.



OFM: Ocean bottom station



Montagner et al., 1994

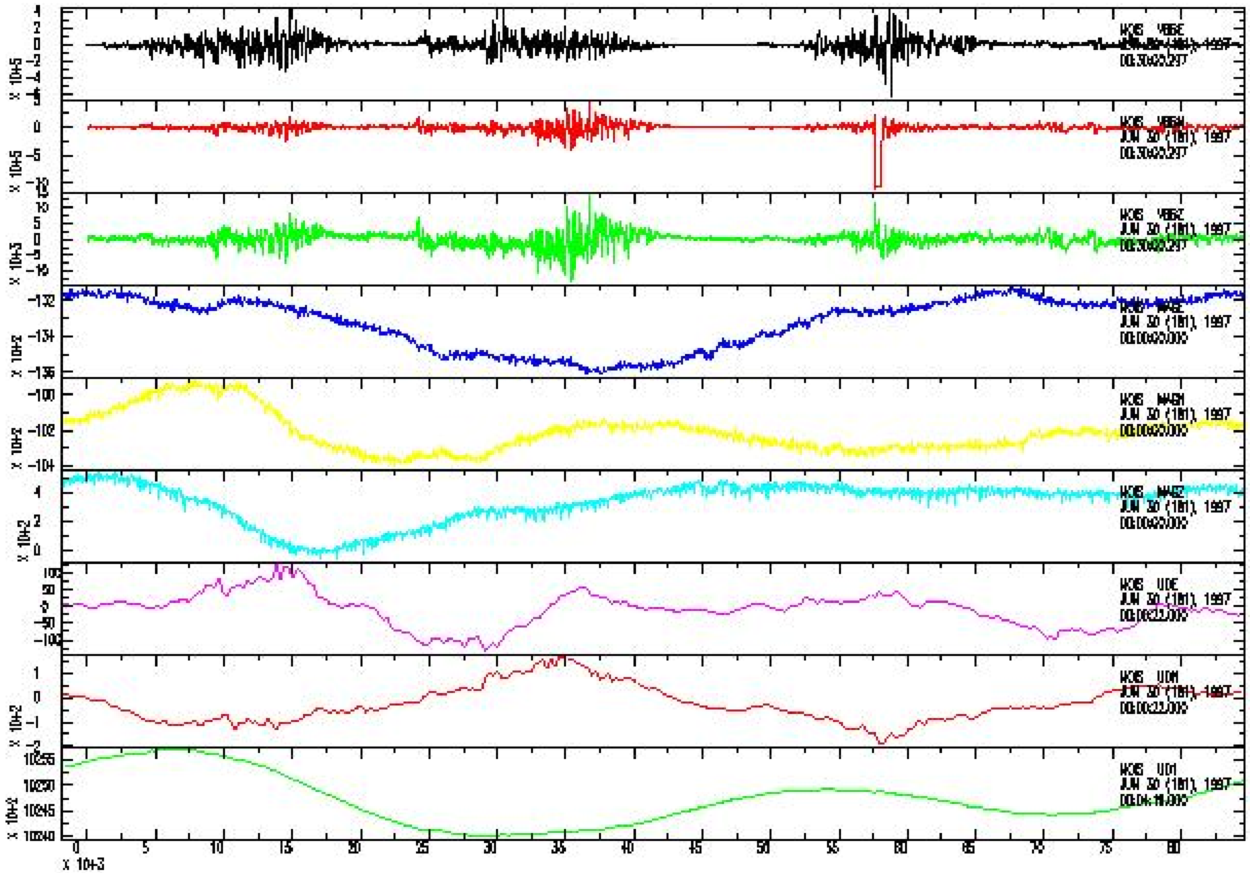
M.O.I.S.E (June-Sept. 1997)

(Monterey bay Ocean bottom International Experiment)

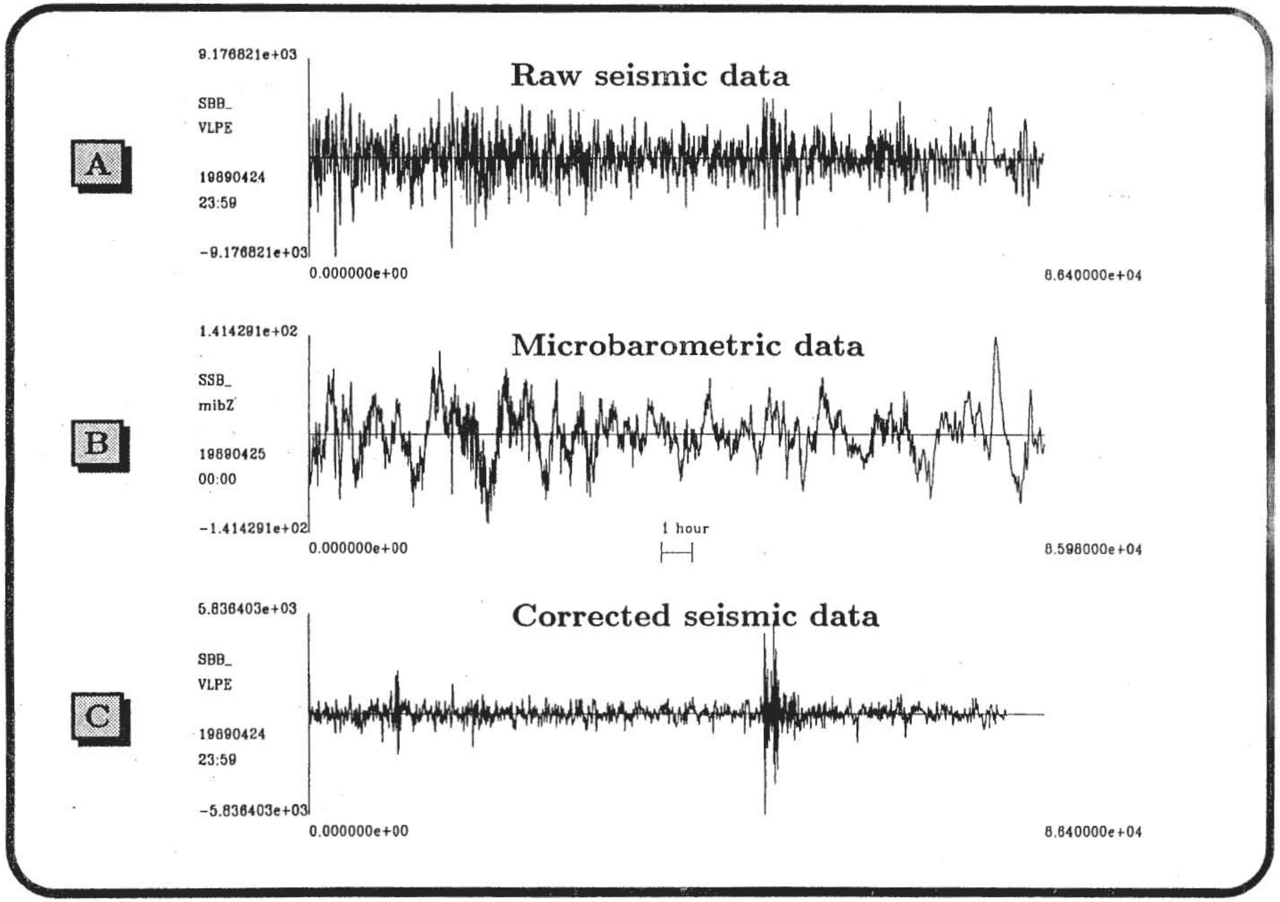
MBARI, UC Berkeley, IPG-Paris, UBO-Brest

IPGP
INSTITUT DE PHYSIQUE
DU GLOBE DE PARIS

Multiparameter signals



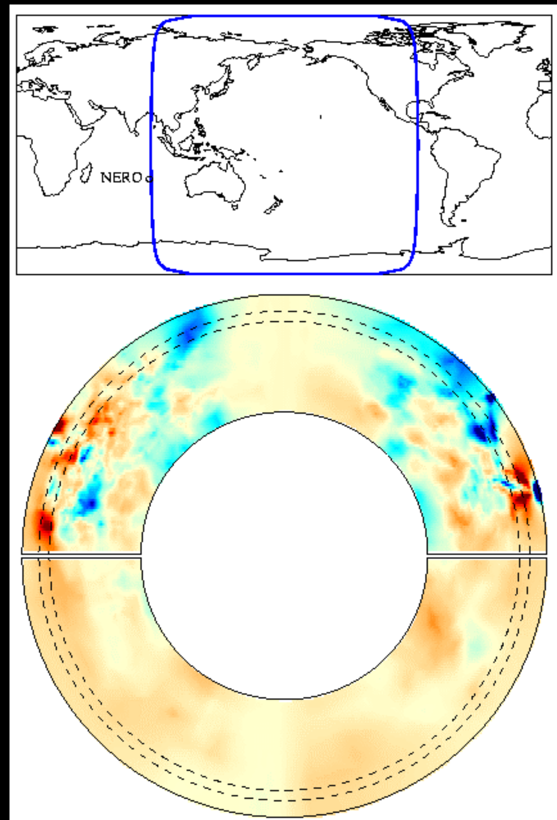
Deconvolution of the seismic signal from the pressure influence



Beauduin et al.,1996

INSTITUT DE PHYSIQUE DU GLOBE DE PARIS

NERO observatory
Scientific Interest
Global scale

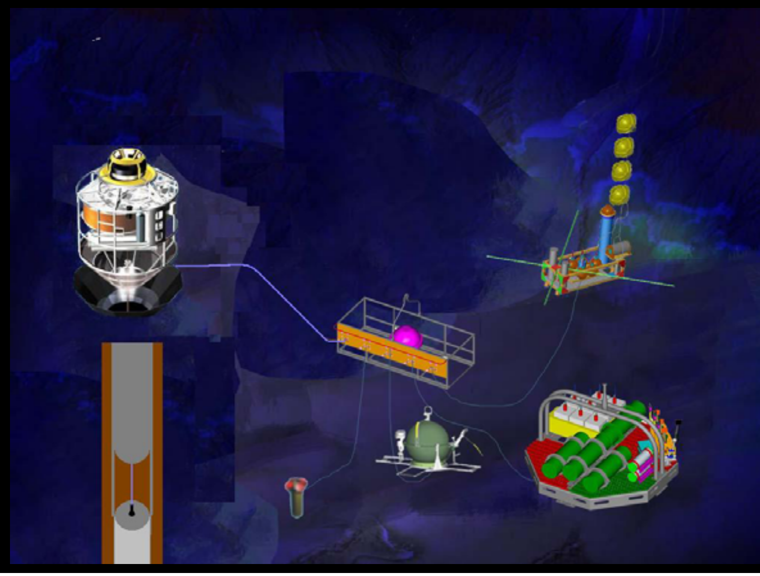


- To fill a gap in global station coverage

-To improve global tomographic model resolution

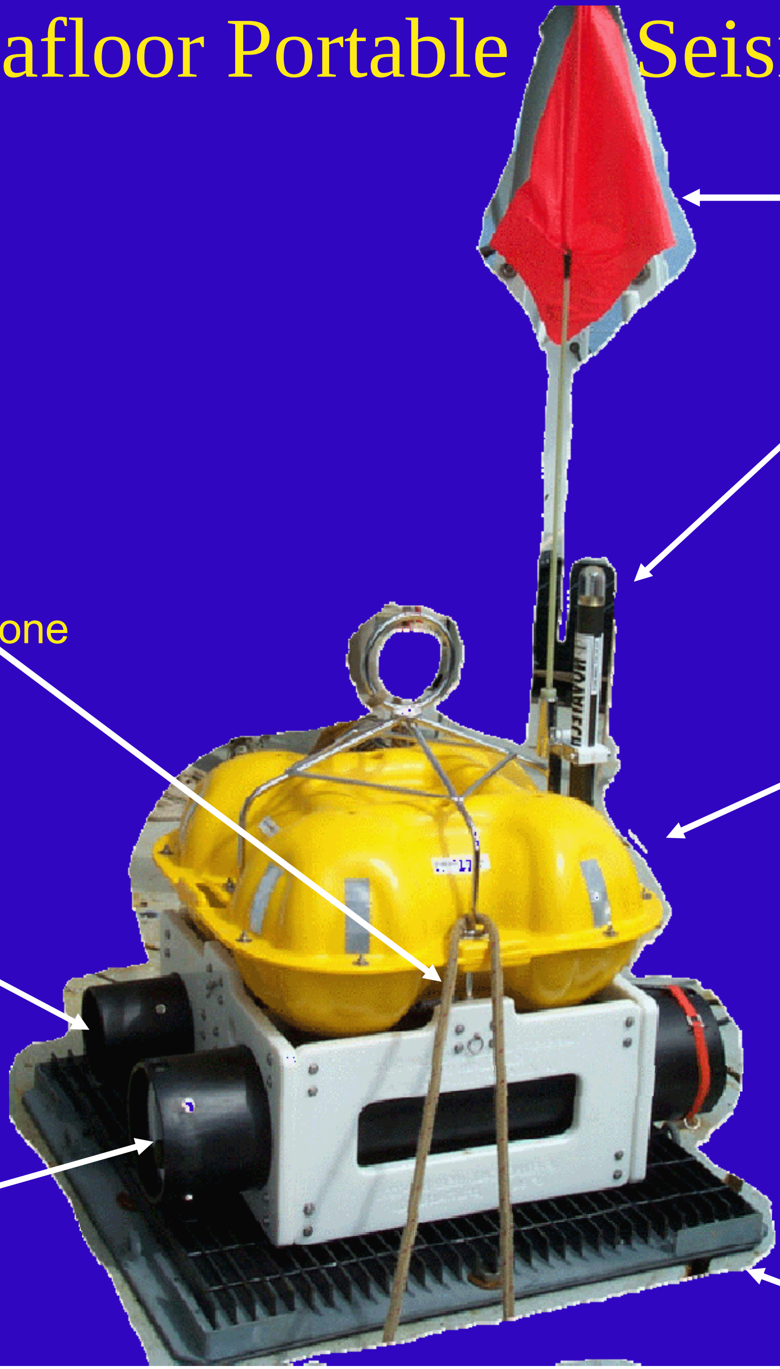
- To improve azimuthal distribution in determination of large earthquakes focal

NERO



Karason & van der Hilst, 2003

Project Seafloor Portable Seismometers



Crawford et al., 2002

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Propagation of seismic waves

Hypothesis: Elastic Medium : $\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$

Where ε_{kl} is the strain tensor, σ_{ij} the stress tensor

C_{ijkl} the elastic tensor: 81 elastic moduli

Symmetries of ε_{kl}, σ_{ij} and of the strain energy

$W = 1/2\ \sigma_{ij}\,\varepsilon_{ij}$ => **21** independent elements

Isotropic case => **2** independent elements

$C_{ijkl} = \lambda\,\delta_{ij}\,\delta_{kl} + \mu\,(\delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{jk})$

λ , μ are Lamé parameters

Elastodynamic equation of motion

$$\partial_j (C_{ijkl}\, \partial_k\, u_l) - \rho\, \partial_{tt}\, u_i = 0$$

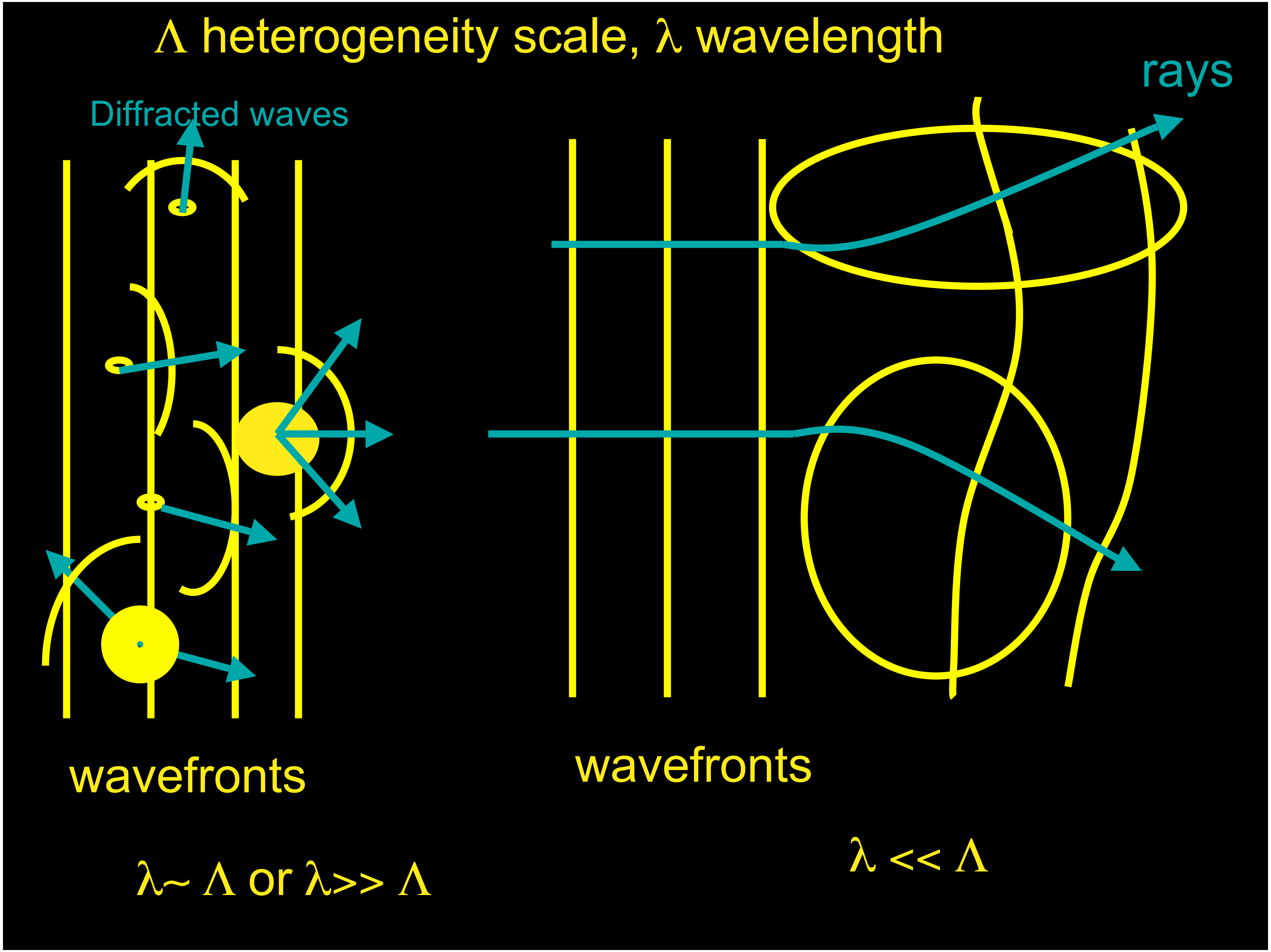
In the isotropic case, 2 solutions:

S-wave

P wave

In heterogeneous media, comparison between

Wavelength λ and scale of heterogeneity Λ


Λ heterogeneity scale, λ wavelength
Diffracted waves
rays
wavefronts
wavefronts
λ~ Λ or λ>> Λ
λ << Λ

Duality wave - particle:

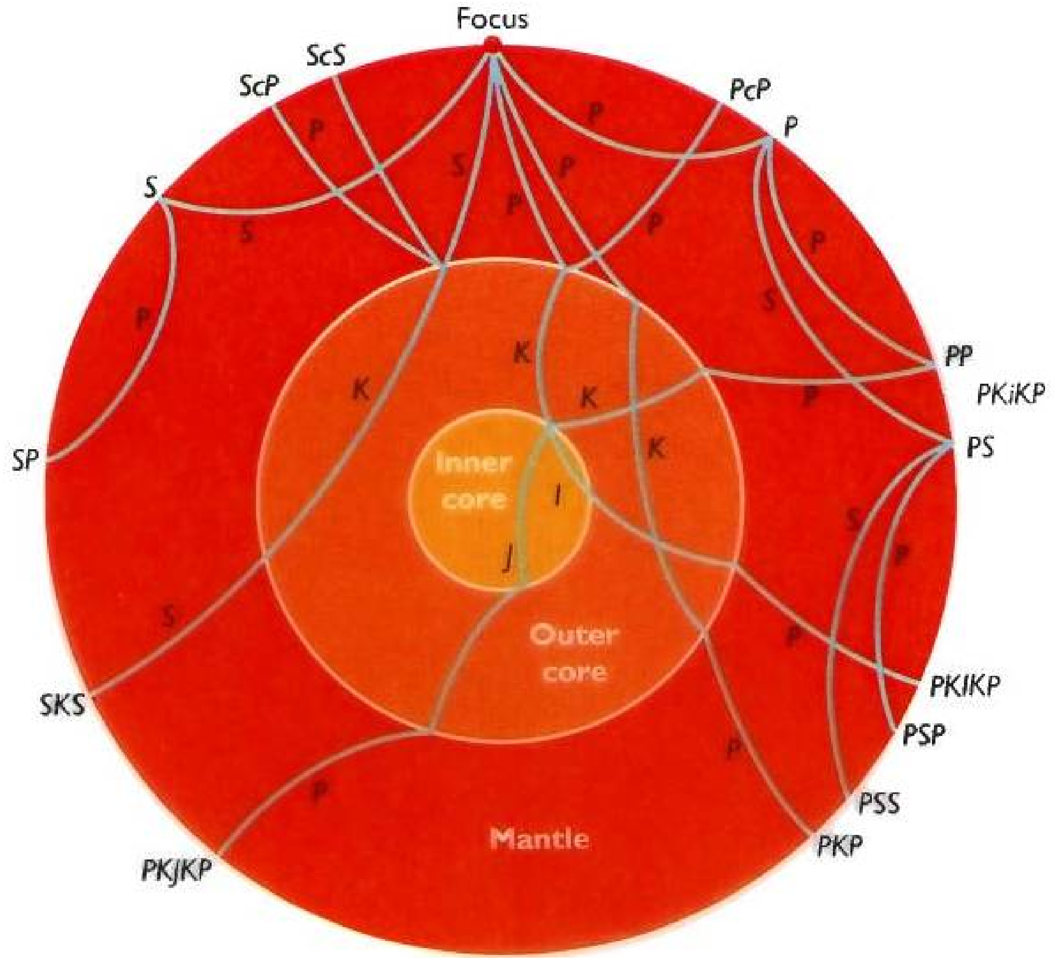
- λ seismic wavelength
- Λ scale heterogeneity

- Particle: Ray theory (XXth century)

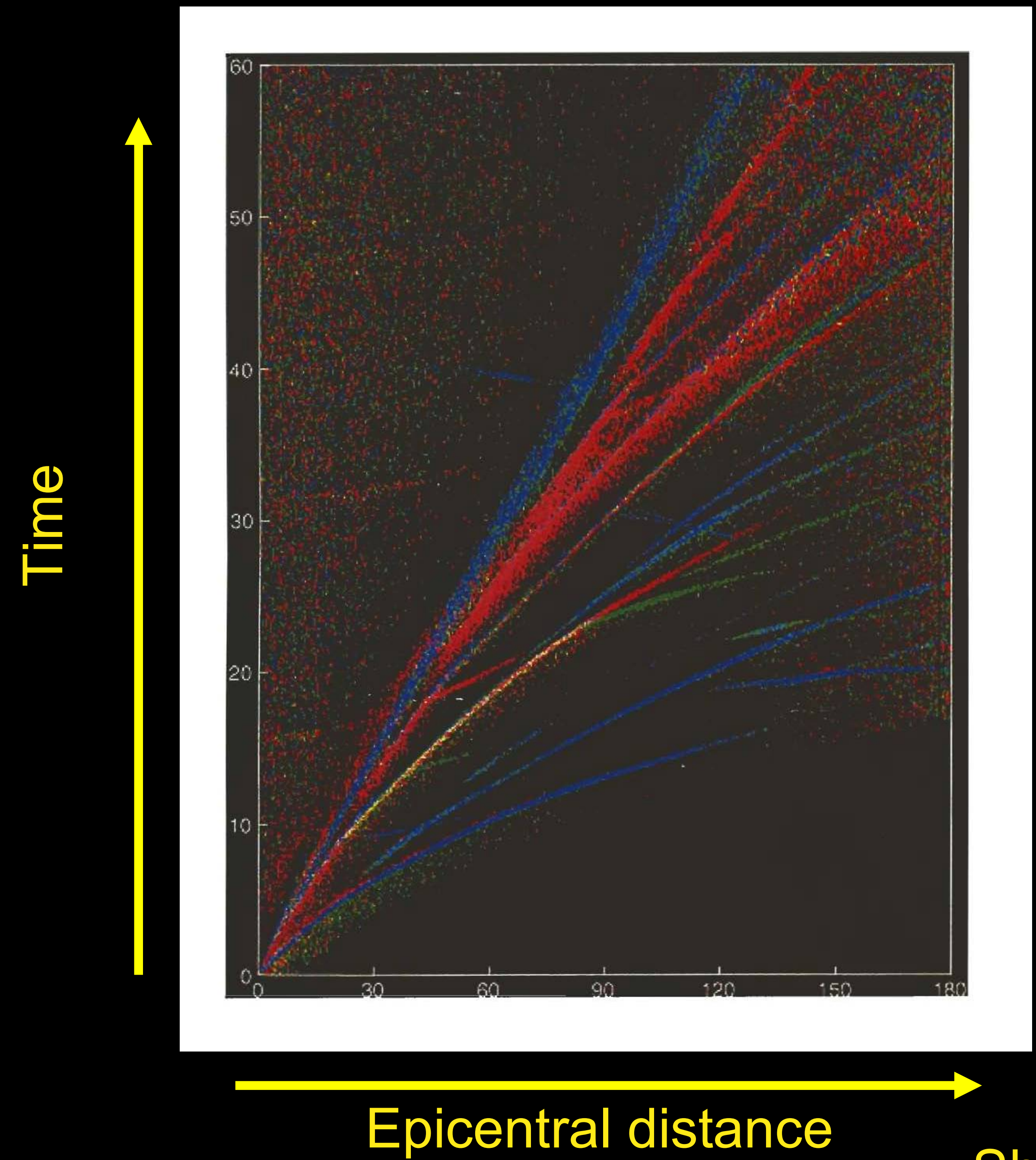
$\lambda << \Lambda$

- Wave: Normal mode theory (>1970)

RAY PATHS INSIDE THE EARTH

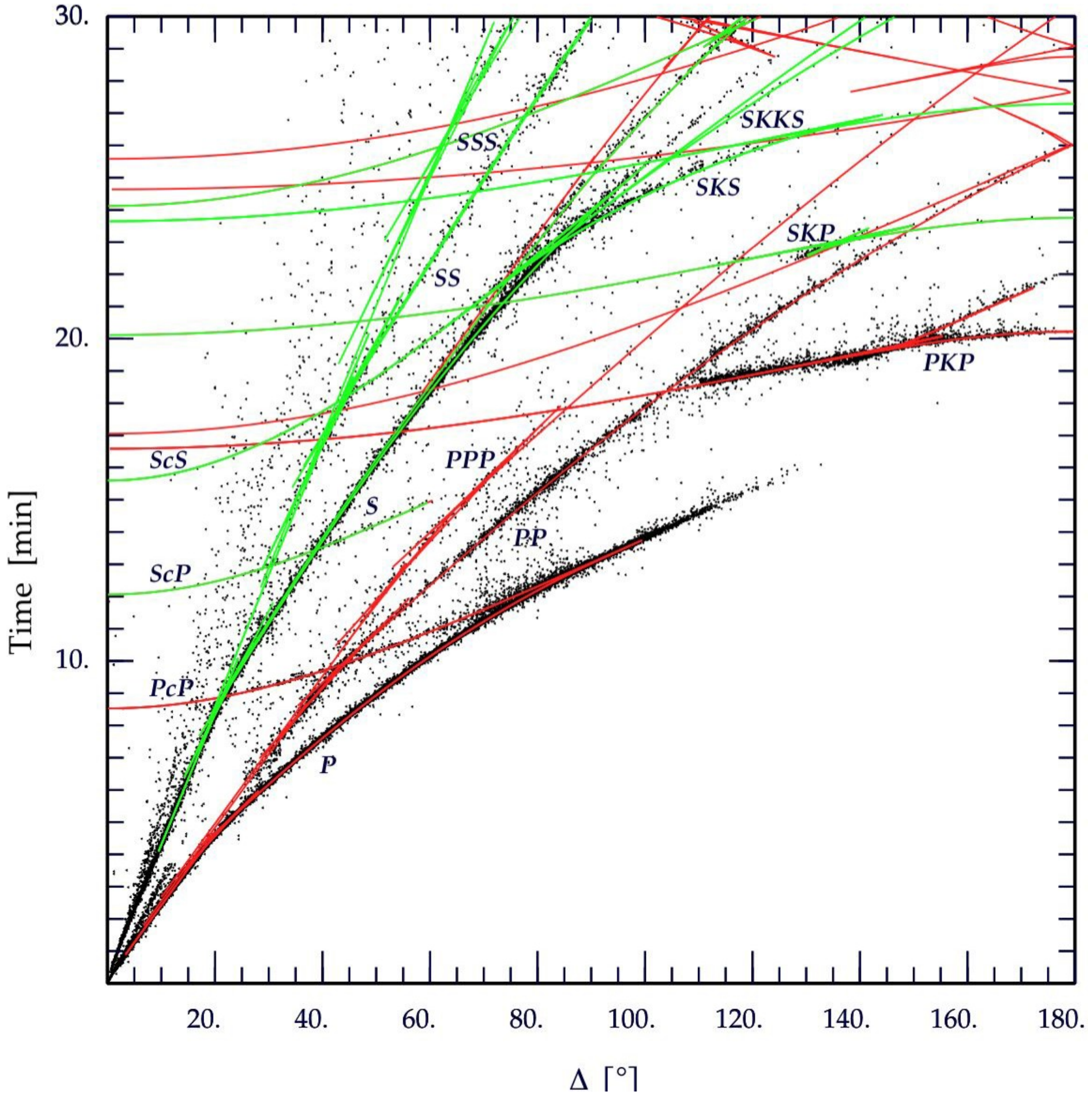


Bolt, 1993



Shearer, 1997

"Travel time" of certain seismic phases vs. epicentral distance



Duality wave - particle:

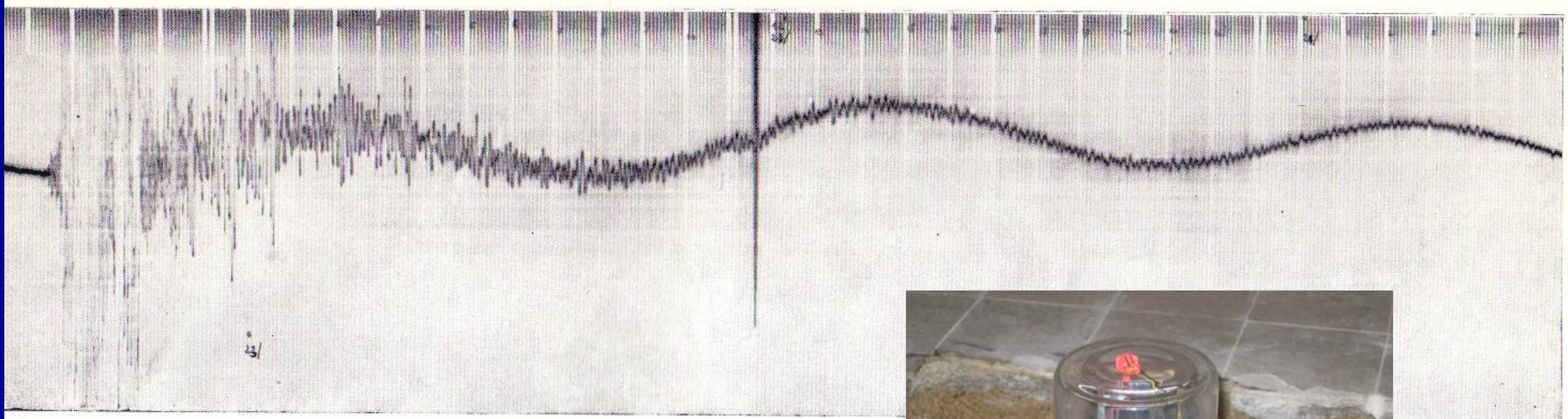
- λ seismic wavelength
- Λ scale heterogeneity

- Particle: Ray theory (XXth century)

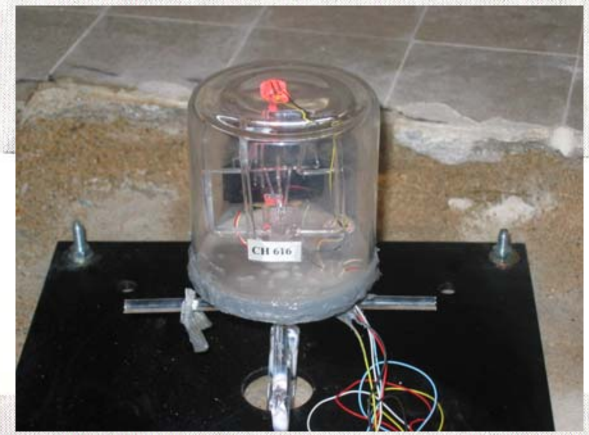
$\lambda \ll \Lambda$

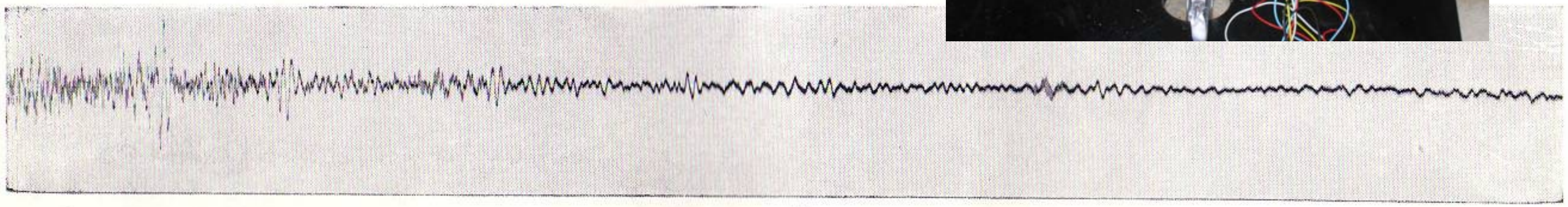
- Wave: Normal mode theory (>1970)

Chile Earthquake (22 may 1960) recorded at Paris (IPGP)



1*a*





1*b*

FIG. 1. — *a*) Enregistrement du pendule E, nº 1 (voir tableau I).
b) Enregistrement du pendule B, nº 4 (voir tableau I). (Un intervalle de 5 minutes est représenté sur cette figure par 1,45 mm.)

Chile earthquake (may 22 1960) recorded at Paris (IPGP)

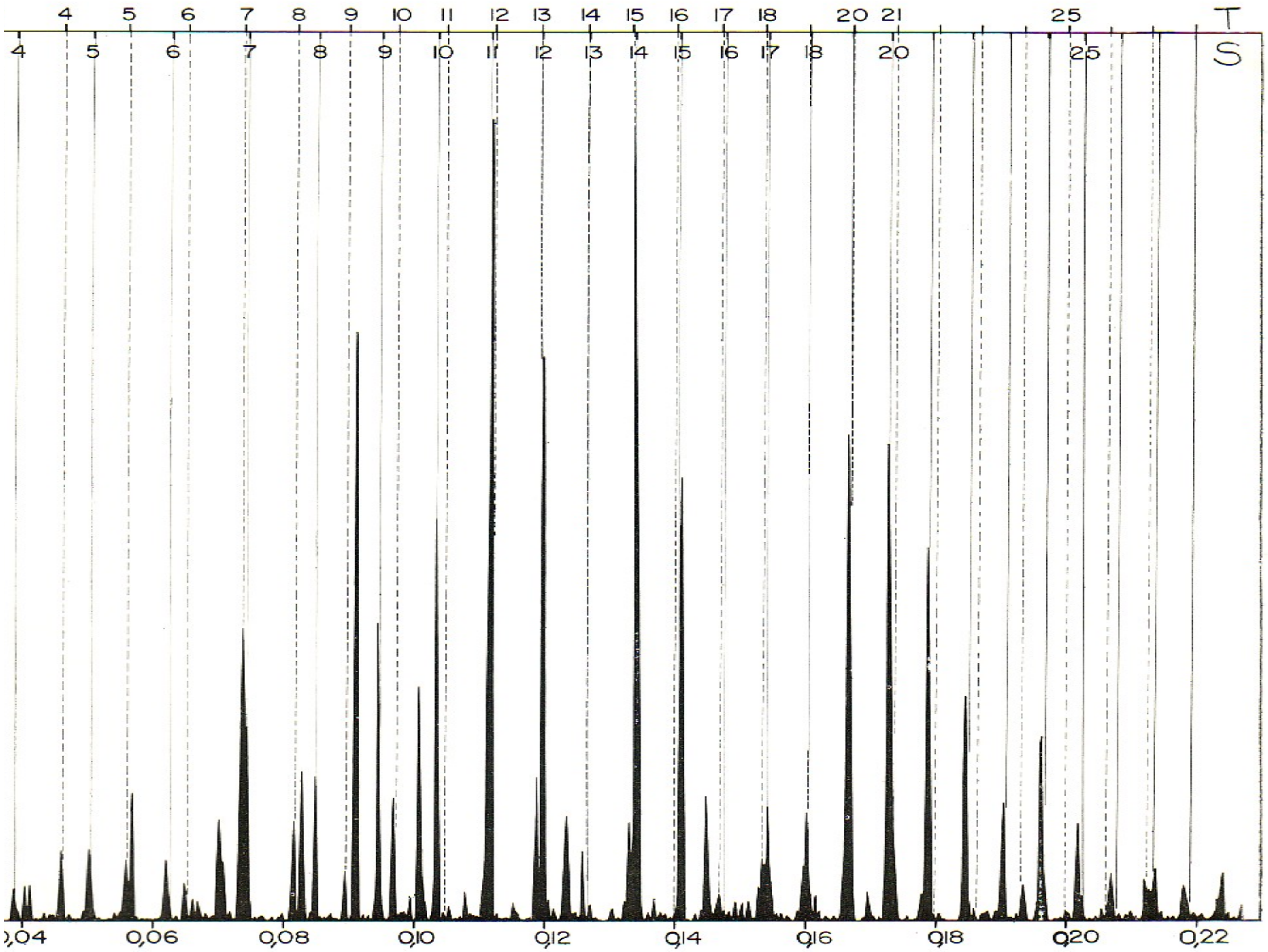


FIG. 3. — Spectre de l'enregistrement du pendule E (n° 1) — En haut : positions des pics théoriques pour les oscillations sphéroïdales S et les oscillations de torsion T du modèle de Gutenberg continental.

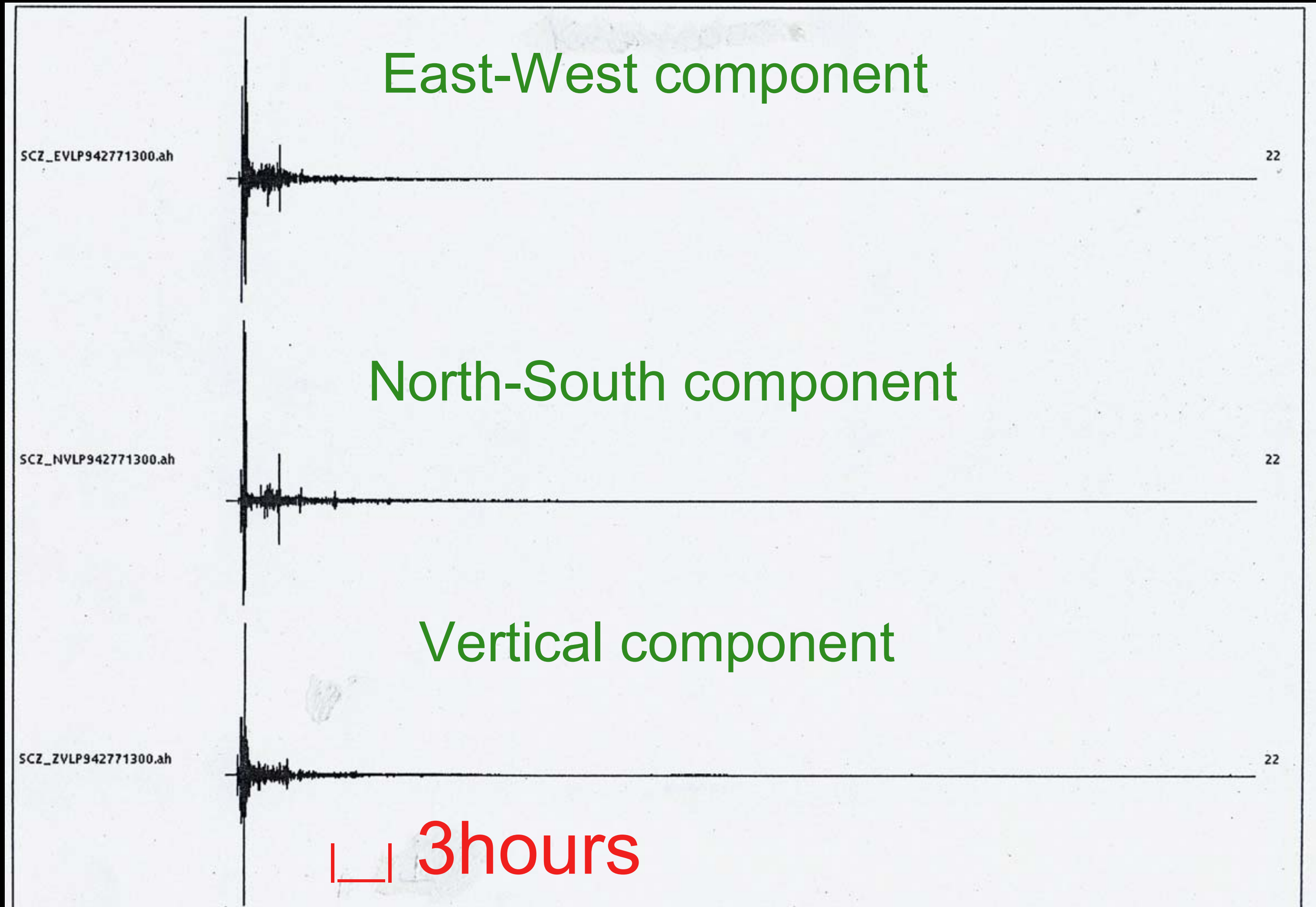
First observations of free oscillations of the Earth

1953? -> 1960 (Chile earthquake)

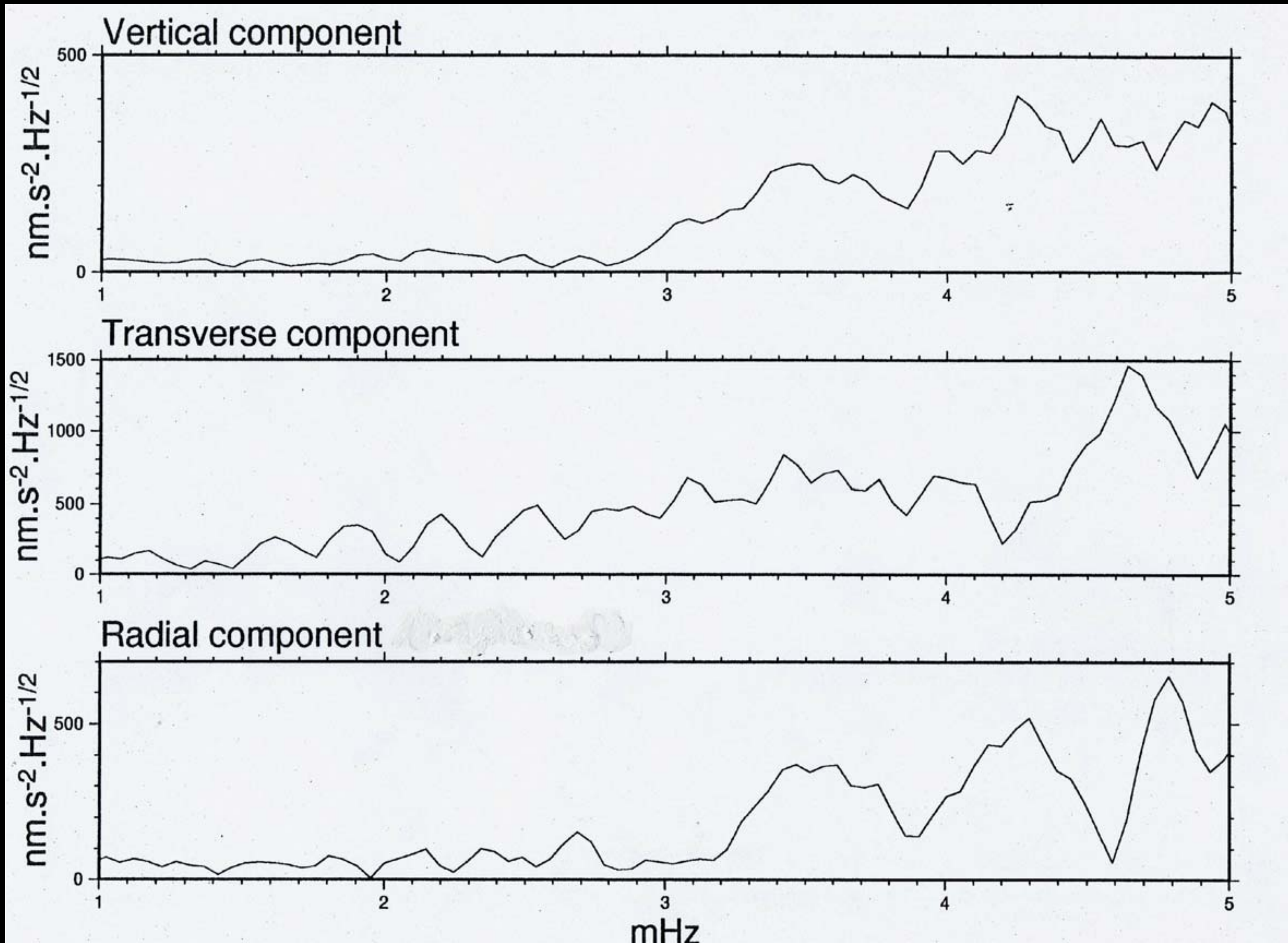
Frequency Peaks were not well understood

Theory was incomplete

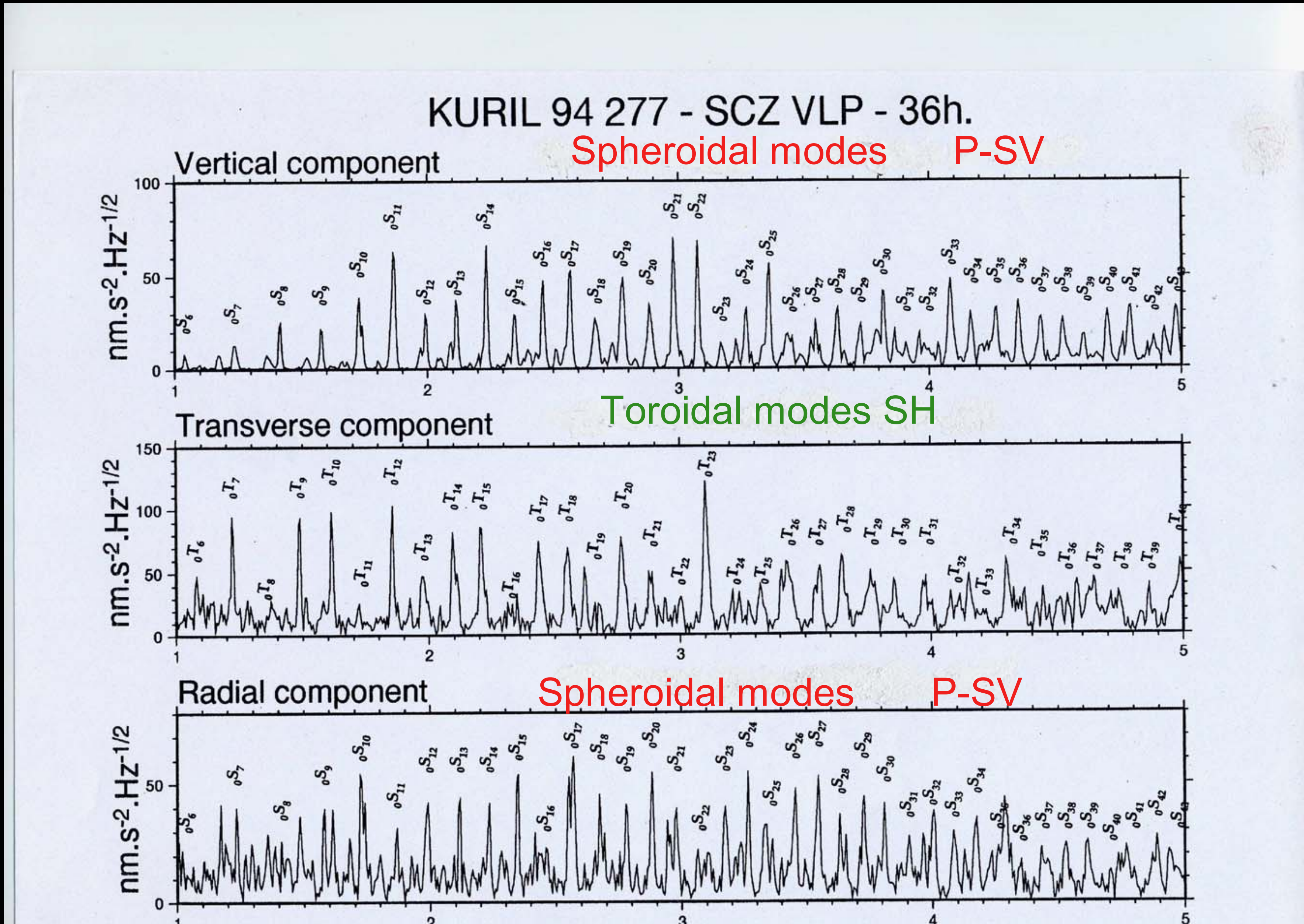
Kuril islands 1994-277 Ms=8.3



Kuril islands 1994-277 SCZ-VLP
Spectra 3 hours



1 Frequency (mHz) 5



Frequency (mHz)

Elastodynamic equation

$$\rho\partial_{tt}\mathbf{u}_{0i} = \partial_j\sigma_{\mathbf{ij}} + \rho\ \mathbf{g_i} + \mathbf{F_i}\ \mathbf{(+\ Fs_i\ +\ldots)}$$

Which can be rewritten:

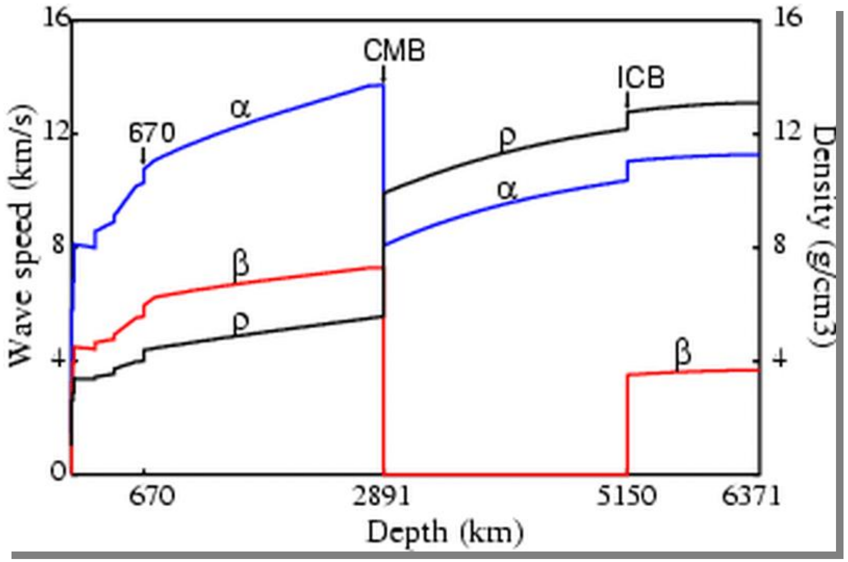
$$\rho\partial_{tt}\mathbf{u}_0 = \mathbf{H_0 u_0}\ \ \mathbf{(+\ Fs)}$$

H_0 is an integro-differential operator

1D-Reference Earth Model:

$M_0(r)$, $\rho(r\ )$, $V_P(r)$, $V_s(r)$

(PREM, Dziewonski and Anderson, 1981
or IASP91, Kennett and Engdahl, 1991)



⇨ John Woodhouse lecture

$$\rho\partial_{tt}\mathbf{u}_0 = \mathbf{H_0 u_0} \;\; (+\ \mathbf{Fs})$$

Eigenfrequencies: ${}_n\omega_l$

Eigenfunctions: ${}_nu_l^m\,(r,t) = |n,l,m>\ \exp(-i\,{}_n\omega_l t)$

3 quantum numbers ($k=\{n,l,m\}$) => $\mathbf{u}_k(\mathbf{r})\ \exp(-i\,{}_n\omega_l t)$

$$\int \rho\ \mathbf{u}_k{}^{*} \,.\, \mathbf{u}_k\ d^3x = \delta_{ij}$$

$$\mathbf{H_0}\,\mathbf{u}_k = \rho\ {}_n\omega_l{}^2\,\mathbf{u}_k$$

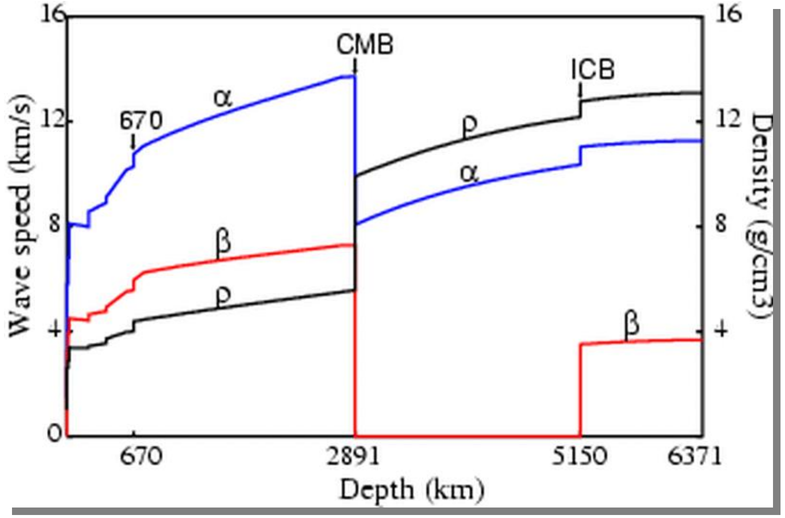
Displacement:

$$\mathbf{u}(\mathbf{r},t) = \Sigma_{n,l,m}\ \ {}_n\mathbf{a}_l^m\ |n,l,m>\ \exp(-i\,{}_n\omega_l t)$$

$$\mathbf{u}_k(\mathbf{r}) = \{U(r)\mathbf{e}_r + V(r)\mathbf{e}_\theta\,\partial_\theta + V(r)/\sin\theta\ \mathbf{e}_\phi\,\partial_\phi\}\ Y_l^m(\theta,\phi)$$
$$+ \{W(r)/\sin\theta\ \mathbf{e}_\theta\,\partial_\phi - W(r)\,\mathbf{e}_\phi\,\partial_\theta\}\ Y_l^m(\theta,\phi)$$

1D-Reference Earth Model:

$M_0(r)$, $\rho(r)$, $V_P(r)$, $V_s(r)$
(PREM, Dziewonski and Anderson, 1981)



$\rho \partial_{tt} \mathbf{u}_0 + \mathbf{H_0 u_0} = \mathbf{0}$

Eigenfrequencies: ${}_n\omega_l$

Eigenfunctions: ${}_nu_l^m\,(r,t) = |n,l,m\rangle \exp(-i\,{}_n\omega_l t)$

2 kinds of modes: Toroïdal ${}_nT_l$, Spheroïdal ${}_nS_l$

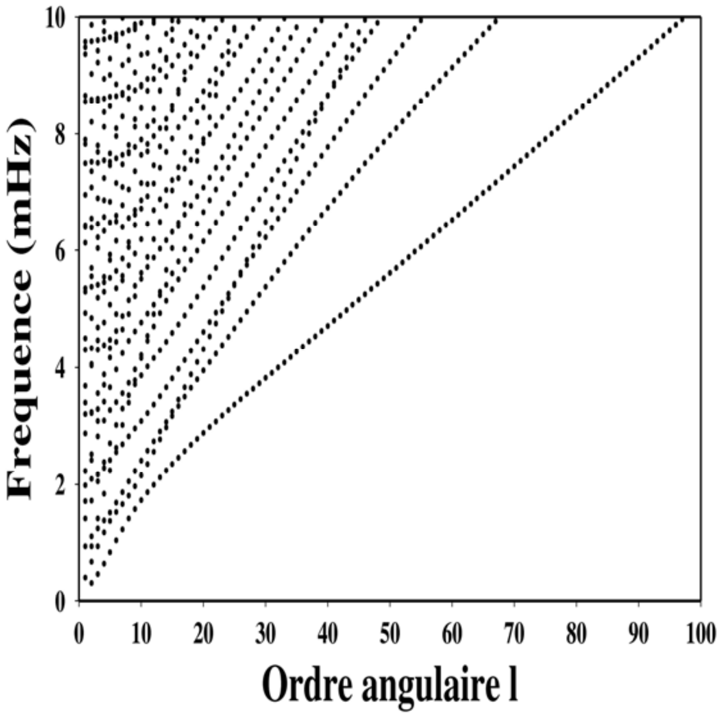
Degeneracy of eigenfrequencies ${}_n\omega_l$: 2 l +1

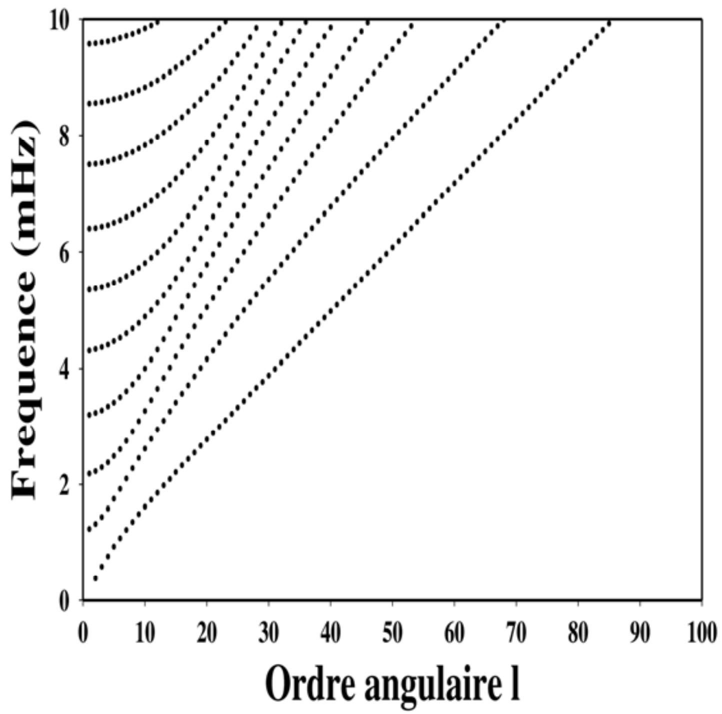
Spherical eigenfrequencies

Spheroïdal Modes $_{n}S_{l}$
(P-SV / Rayleigh)

Toroïdal modes $_{n}T_{l}$
(SH / Love)

Dispersion Branches





multiplet : (n,l) = 2l+1 singlets
singlet : (n,l,m)

n : radial order
l : angular order
m : azimuthal order

Spheroidal Modes

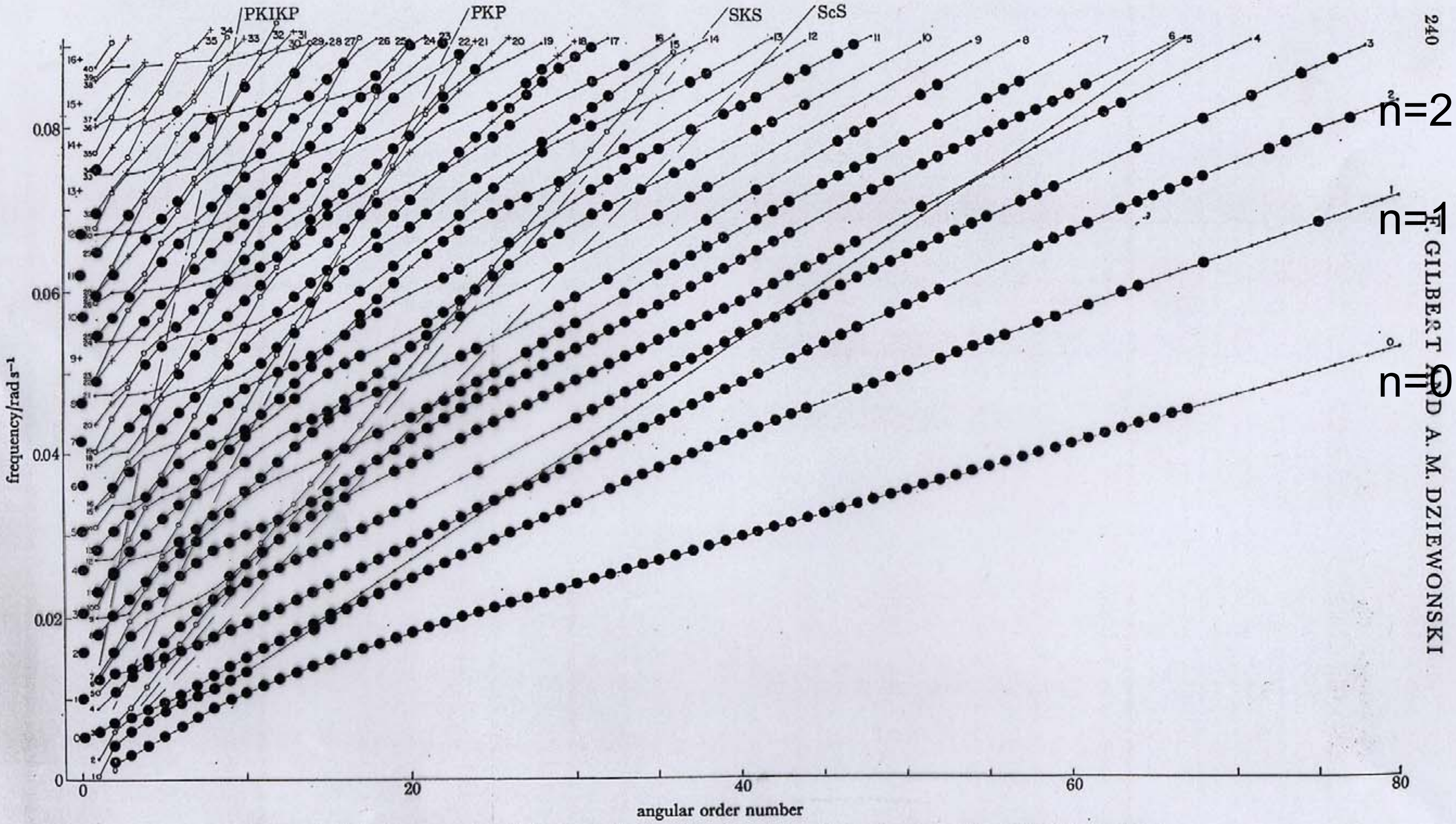
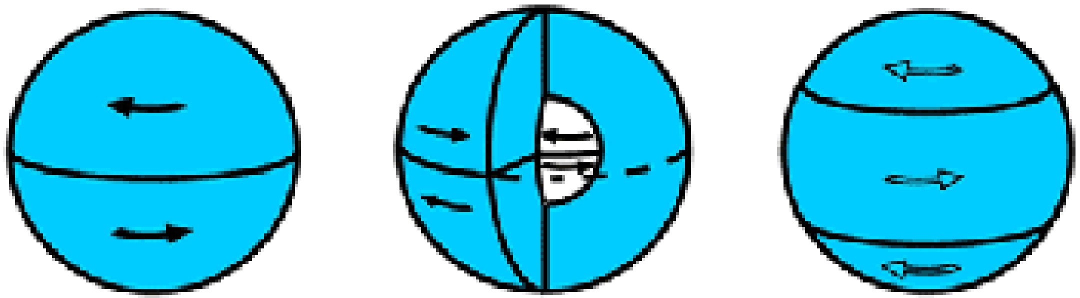
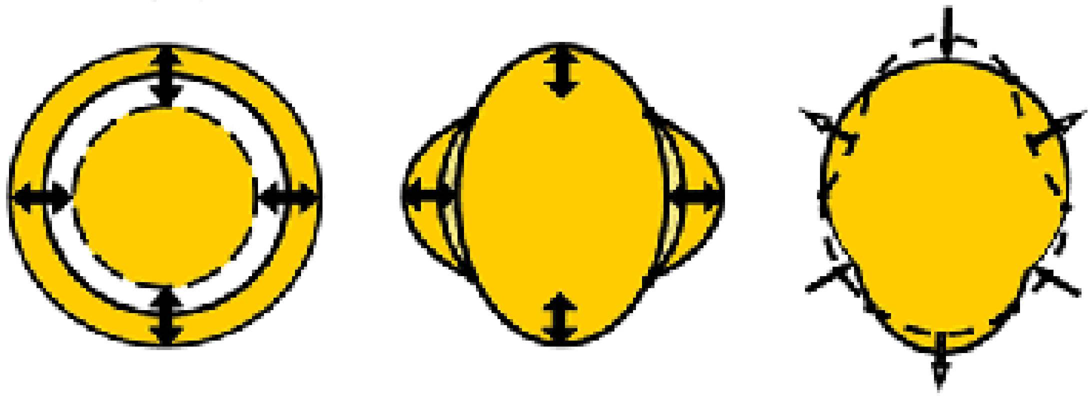


FIGURE 17. Spheroidal normal modes in the (ω, l) plane. The large dots indicate observed modes used in the inversions. For further details we refer the reader to §3 of Alaska II. •, $CE < 0.5$; +, $CE \geqslant 0.5$; ∘ core modes.

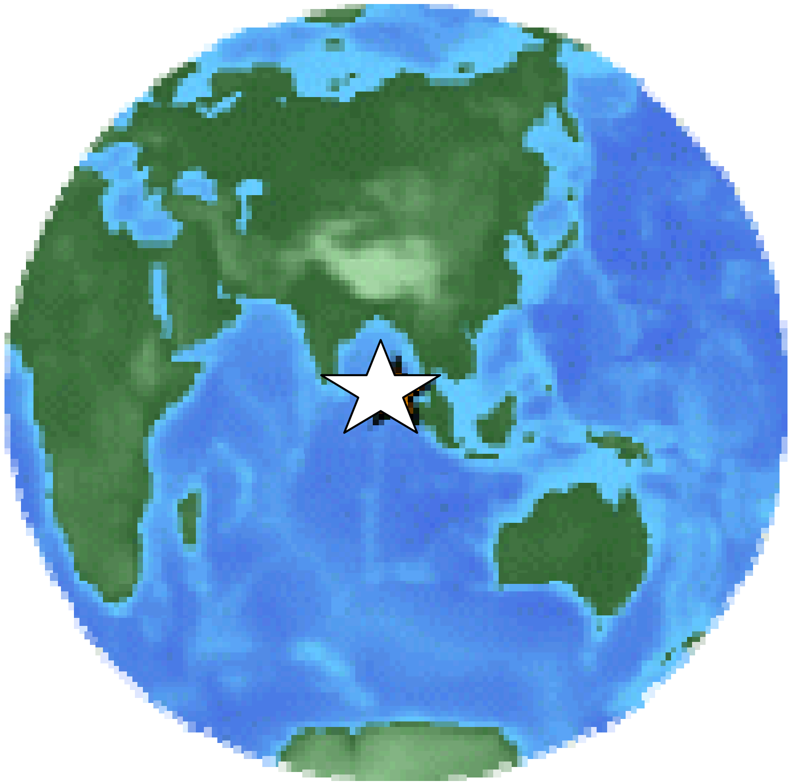


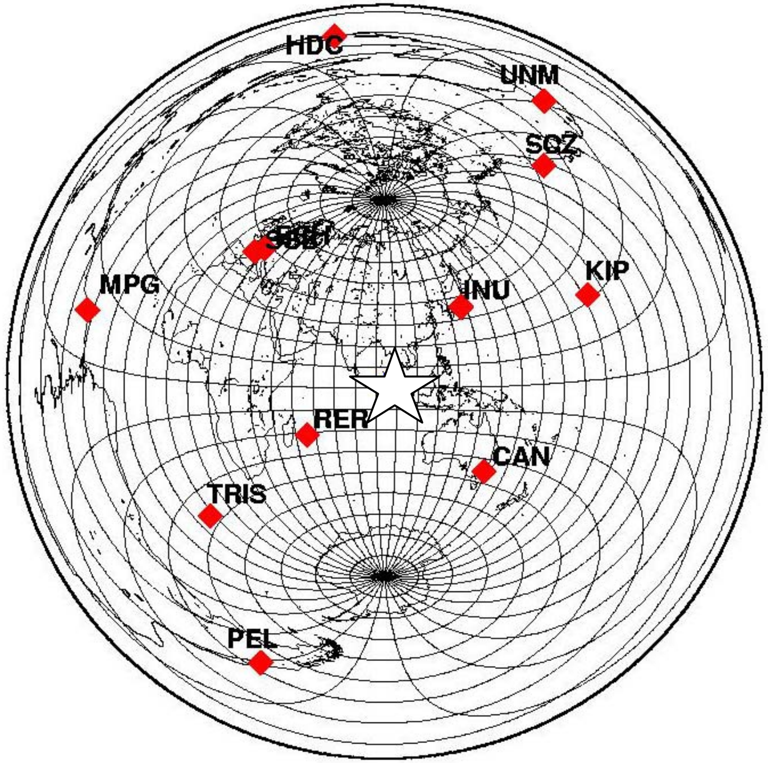
Toroidal modes ${}_0T_2$ (44.2 min), ${}_1T_2$ (12.6 min) and ${}_0T_3$ (28.4 min)



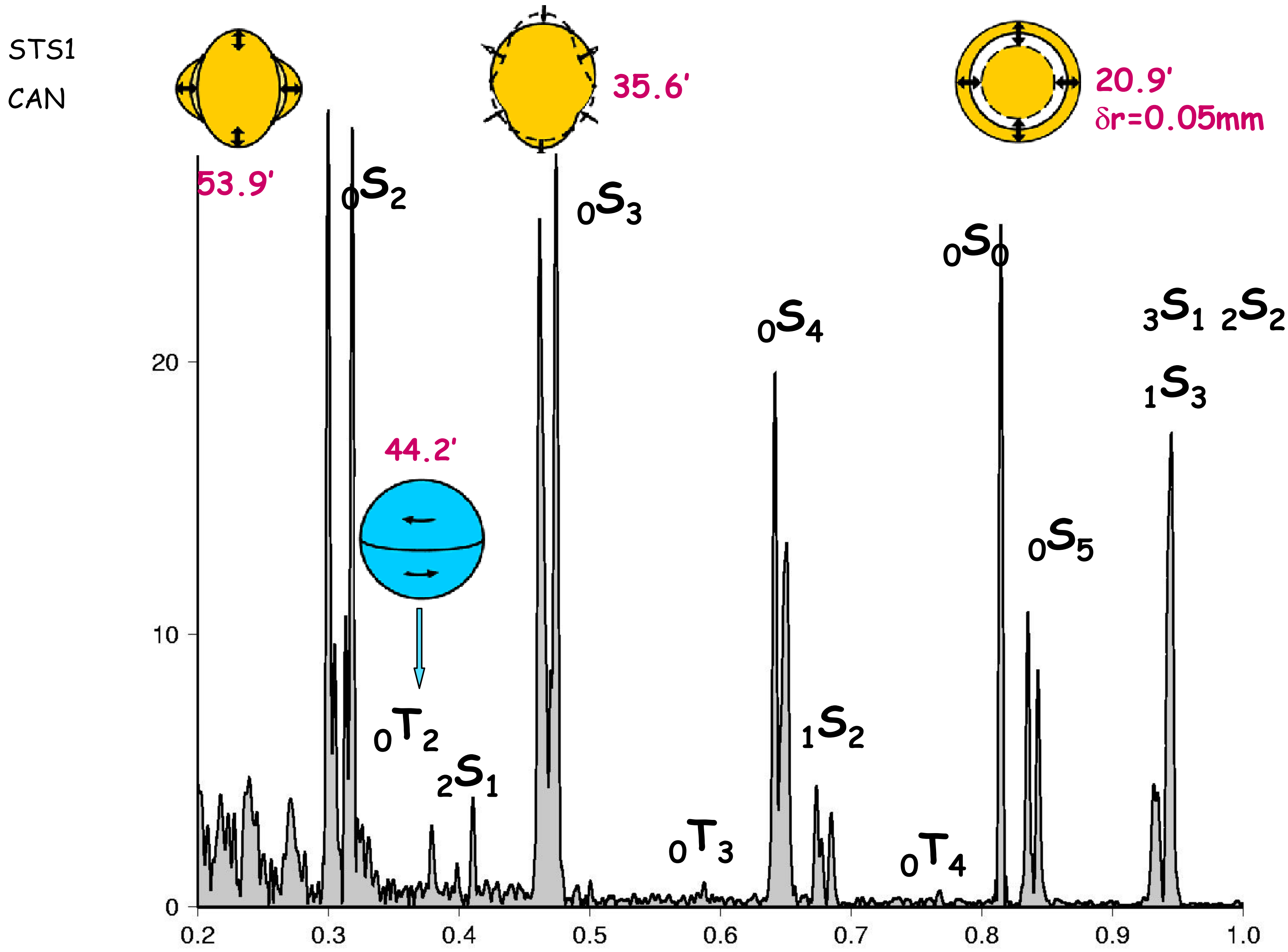
Spheroidal modes ${}_0S_0$ (20.5 min), ${}_0S_2$ (53.9 min) and ${}_0S_3$ ((35.6 min)

Study of Sumatra earthquake (26 december 2004)
With GEOSCOPE stations





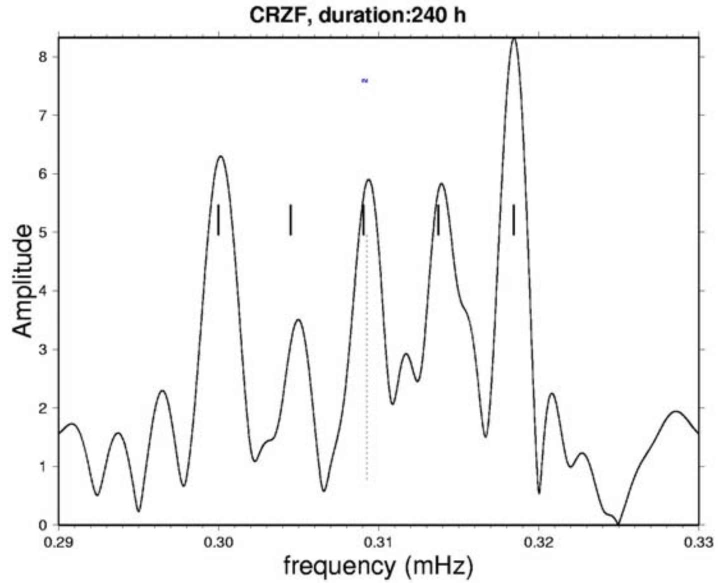
(Roult and Clévédé, 2005 ; Park et al., Science, 2005)

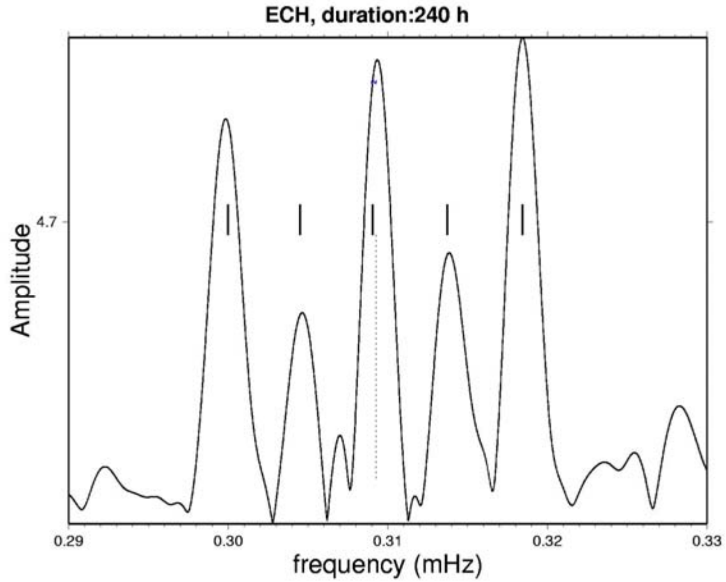


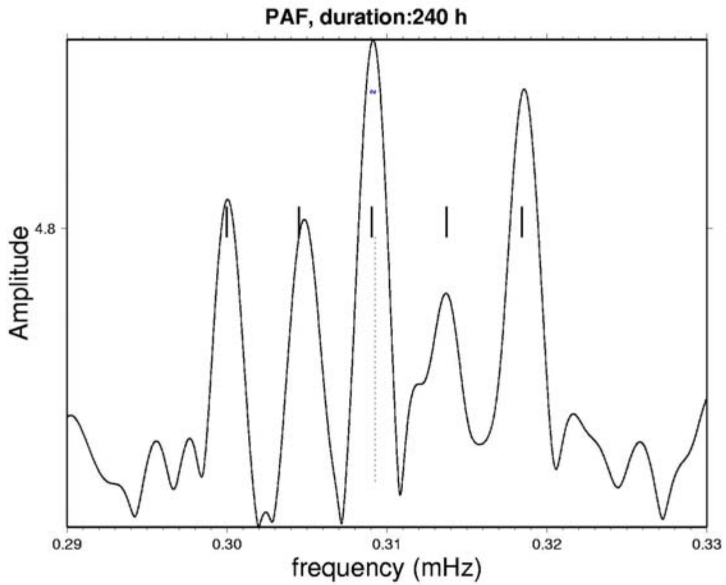
Sumatra-Andaman earthquake
26 December 2004

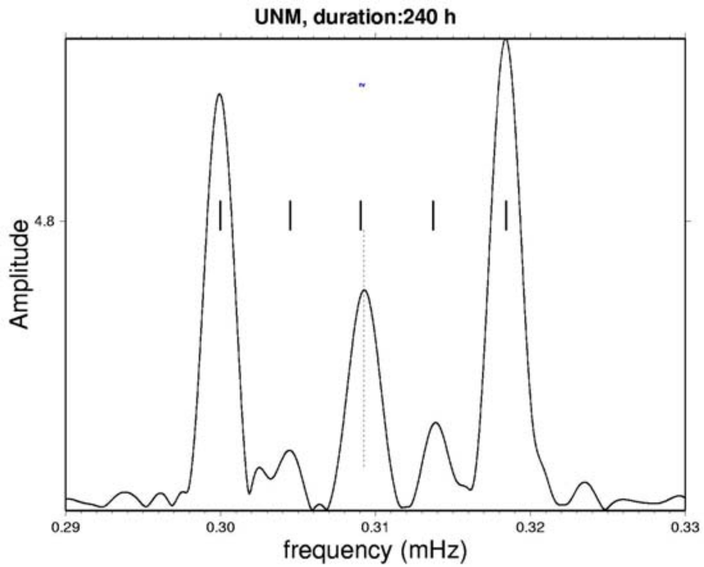
Roult and Clévédé, 2005

mode $_0S_2$ => splitting 5 singlets

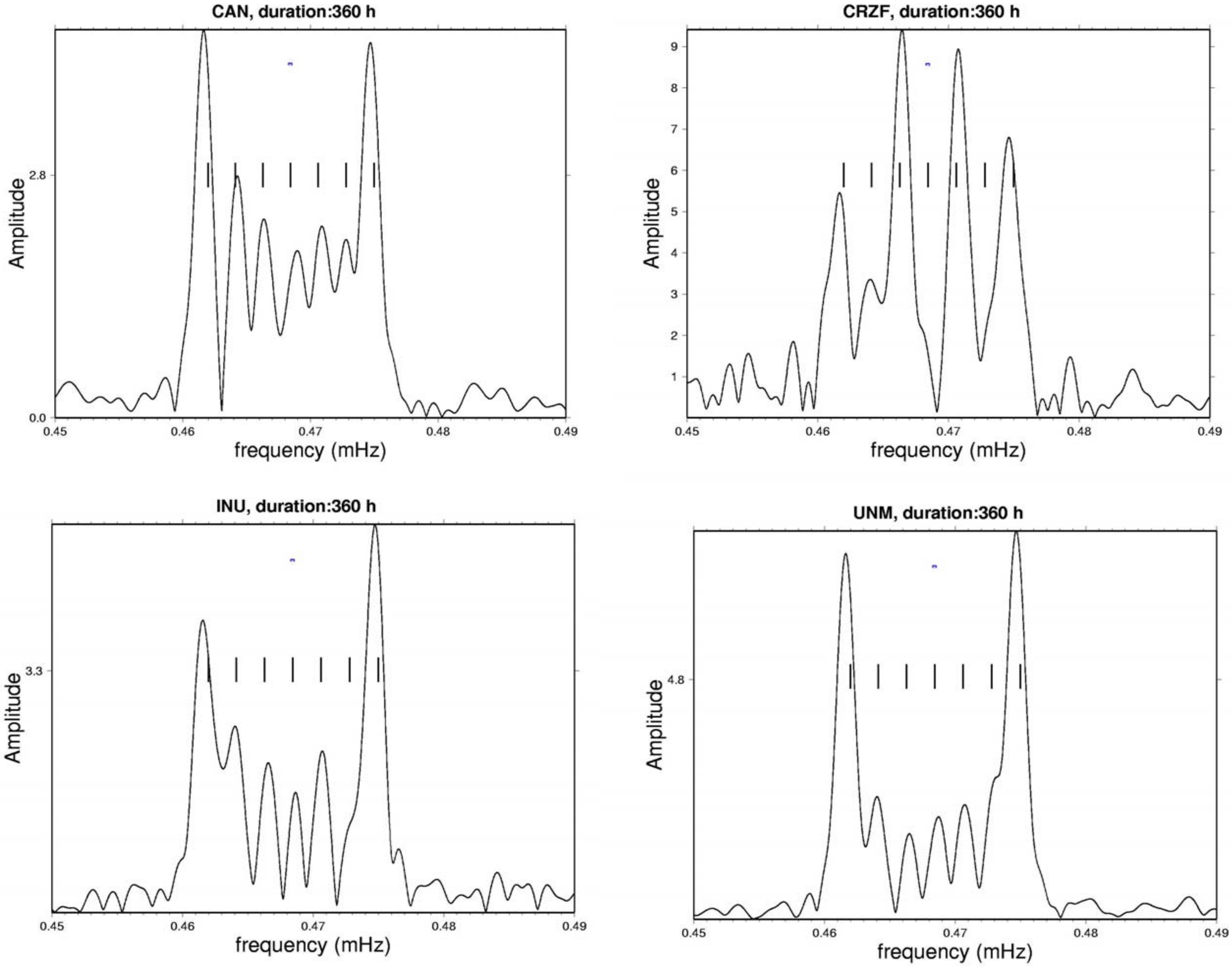




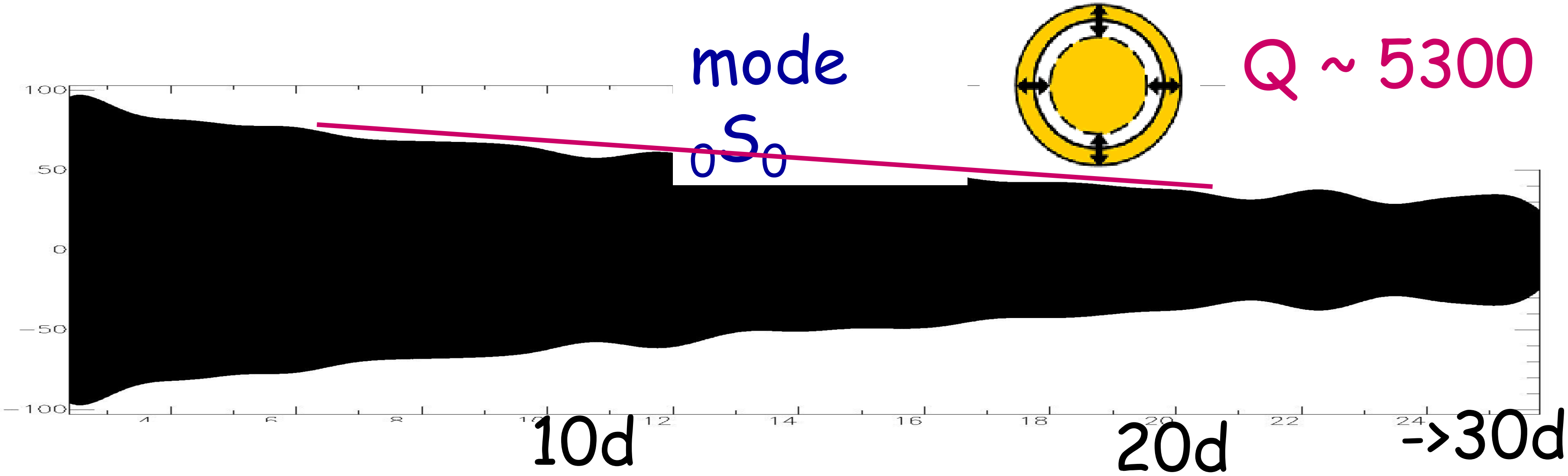




mode $_0S_3$

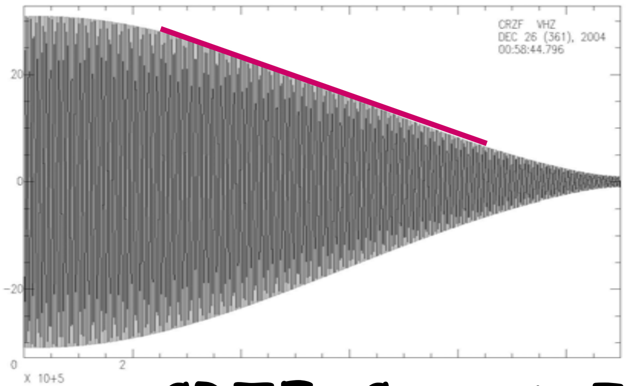

CAN, duration:360 h
Amplitude
2.8
0.0
0.45
0.46
0.47
0.48
0.49
frequency (mHz)
CRZF, duration:360 h
Amplitude
1
2
3
4
5
6
7
8
9
0.45
0.46
0.47
0.48
0.49
frequency (mHz)
INU, duration:360 h
Amplitude
3.3
0.45
0.46
0.47
0.48
0.49
frequency (mHz)
UNM, duration:360 h
Amplitude
4.8
0.45
0.46
0.47
0.48
0.49
frequency (mHz)

Attenuation of some modes

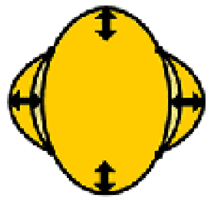


mode $_0S_0$

Q ~ 5300



mode $_0S_2$
singlet m=+2



Q ~ 500

CRZF, Crozet, Indian Ocean

Seismic Source

$$\rho\partial_{tt}\mathbf{u} + \mathbf{H_0 u} = \mathbf{F_s}$$

Displacement at point **r** and time t due to a force system $\mathbf{F_S}$ at point source $\mathbf{r_S}$

eigenfrequencies: ${}_n\omega_l$

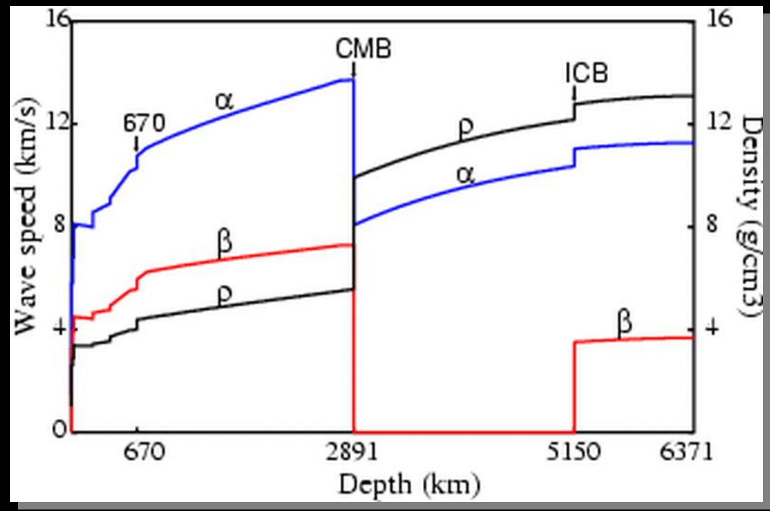
eigenfunctions: ${}_n u_l^m$ (r,t)= |n,l,m> $\exp(-i\,{}_n\omega_l t)$

$$\mathbf{u}(\mathbf{r},t) = \Sigma_{n,l,m}\ {}_n\mathbf{a}_l^m\ |n,l,m> \exp(-i\,{}_n\omega_l t)$$

Eigenfunction basis is a complete basis => any wave can be modelled by normal mode summation including surface waves and body waves.

1D- Reference Earth Model

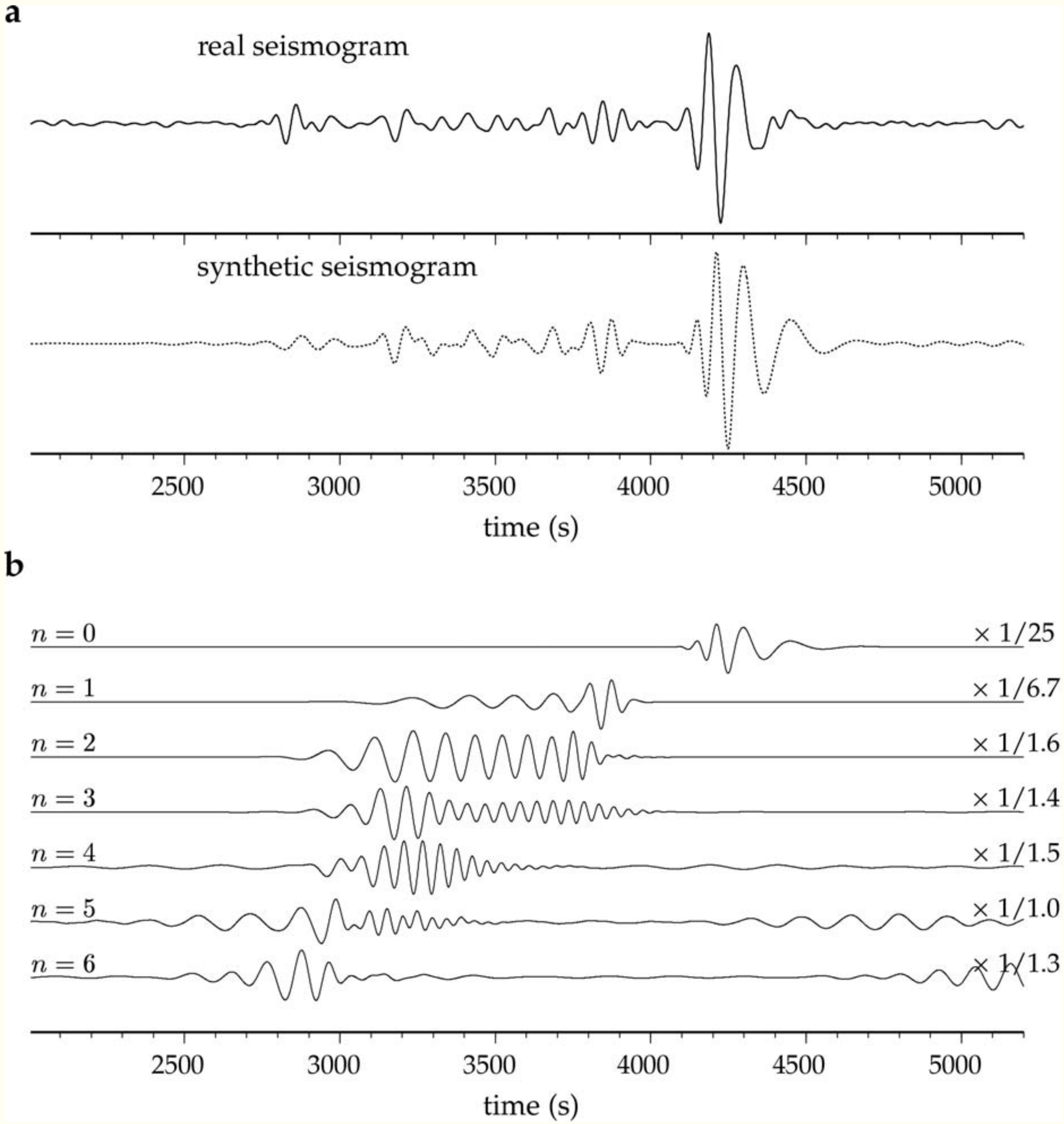
Synthetic Seismograms by normal mode summation $\mathbf{u_k}$(k={n,l,m}).



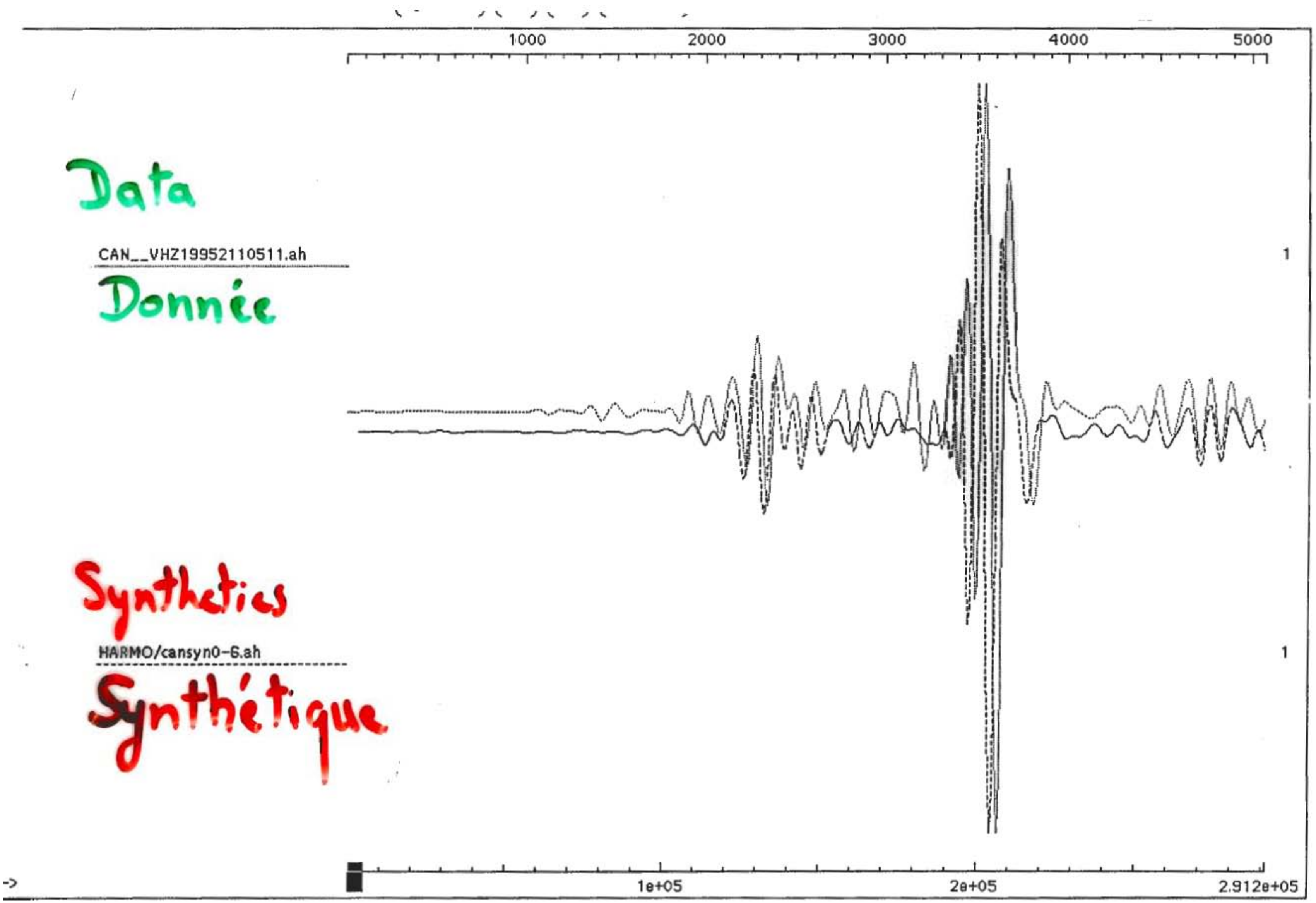
$$\mathbf{u}(\mathbf{r},t) = \Sigma_k\, u_k\,(\,r\,)\cos\omega_k t\,/\omega_k^{\,2}\,\exp(-\omega_k t/2Q_k)\,(\mathbf{u_k.F})_\mathbf{S}$$

Source Term $(\mathbf{u_k}\,\mathbf{.F})_\mathbf{S} = (\mathbf{M}{:}\varepsilon)_S$

M Seismic moment tensor, ε deformation tensor



Beucler et al., 2003



1000
2000
3000
4000
5000
Data
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Donnée
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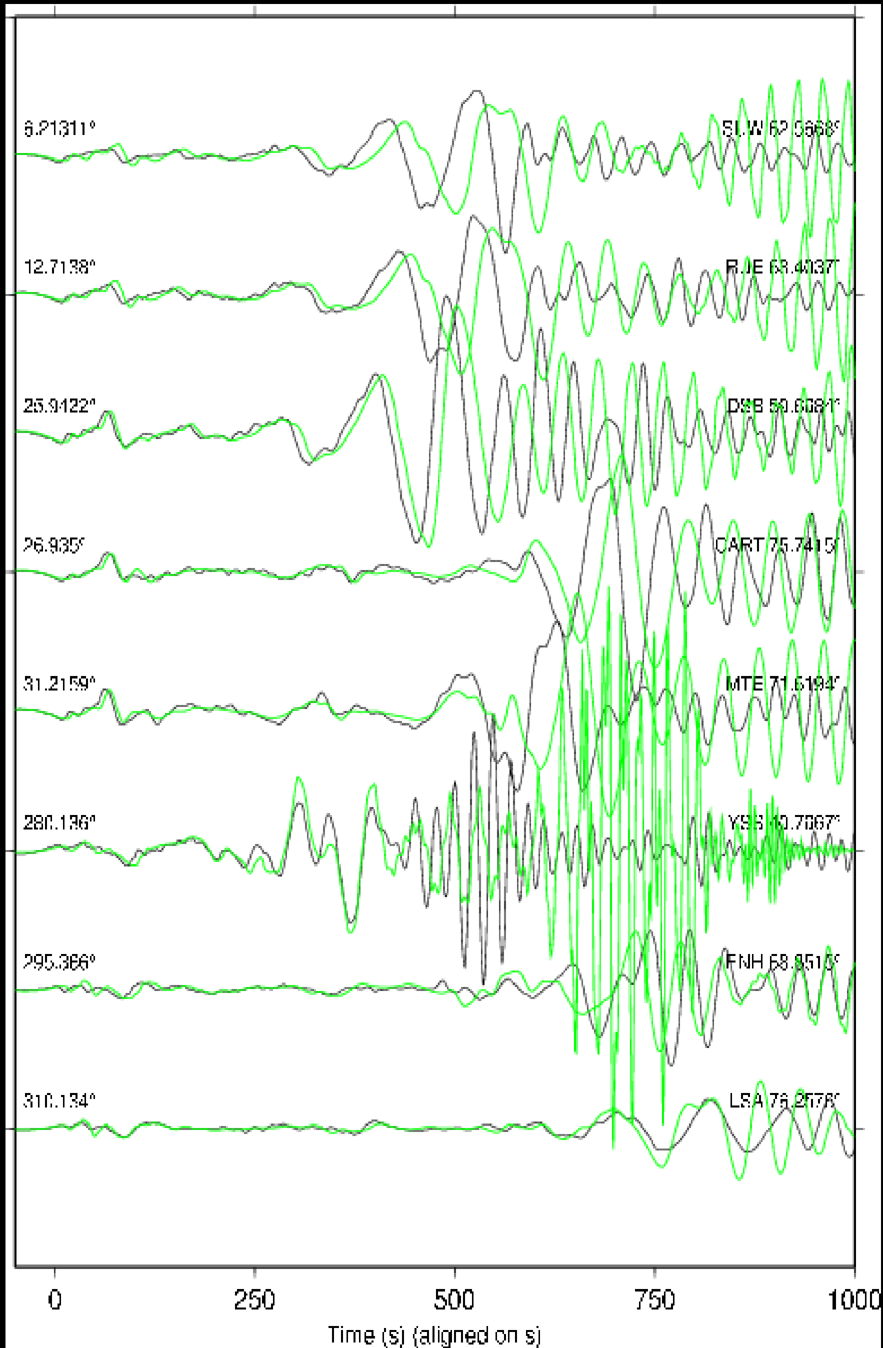
Synthetics
HARMO/cansyn0-6.ah
Synthétique
1
->
1e+05
2e+05
2.912e+05



Synthetic seismograms
By normal mode
summation

Denali-Alaska
earthquake (Nov. 2002)

Komatitsch and Tromp, 2003

Duality wave - particle:
λ seismic wavelength
Λ scale heterogeneity

Particle: **Ray** theory λ<< Λ
=>Finite frequency effects (**Guust Nolet**)
Wave: Normal **Mode** theory (NM) + Perturbation theories (small amplitude of 3D- heterogeneities) =>
(**John Woodhouse**)

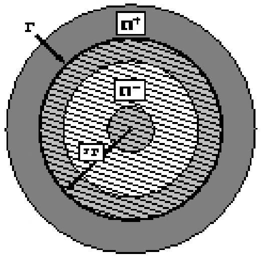
Numerical modelling of wave equation
Strong or weak forms: λ≈ Λ
-Spectral Element Method (SEM)
→ (**Dimitri Komatitsch**)
-Coupled SEM-NM method

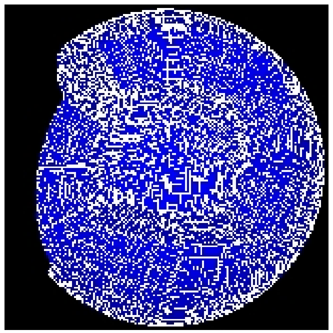
Spectral Element Method: D. Komatitsch (1999)

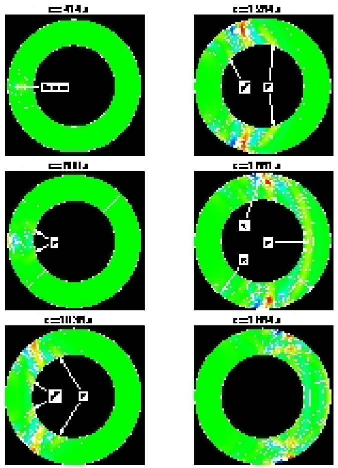
Coupled method of Spectral Elements and Modal Solution

Principle:

- Ω^+: Spectral Element area: 3D model
- Ω^-: Modal Solution area: 1D model







Capdeville et al., 2002

Overview
Large scale Seismology: an observational field

- Data (Seismic source) + Instrument (Seismometer)

-> Observations (seismograms)

- Historical evolution: Ray theory, Normal mode theory, Numerical techniques (SEM, NM-SEM)
- **Scientific Issues: Earthquakes (Sumatra), Anisotropic structure of the Earth**
- Tomographic Technique
- Geodynamic Applications.

  Seismic Experiment Plume detection
- NM-SEM and time reversal

Seismic Source Studies

$$\mathbf{u}(\mathbf{r},t) = S_k\, u_k(r) \cos \omega_k t / \omega_k^2 \exp(-\omega_k t/2Q)\, (\mathbf{u_k.F})_S$$

Source Term $(\mathbf{u_k.F})_S = (\mathbf{M}: \varepsilon)_S$

M Seismic moment tensor, ε deformation tensor