The Abdus Salam
International Centre for Theoretical Physics

United Nations
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1965-27

9th Workshop on Three-Dimensional Modelling of Seismic Waves Generation, Propagation and their Inversion

22 September - 4 October, 2008

Upper Mantle Anisotropy from Surface Wave Studies

Part II

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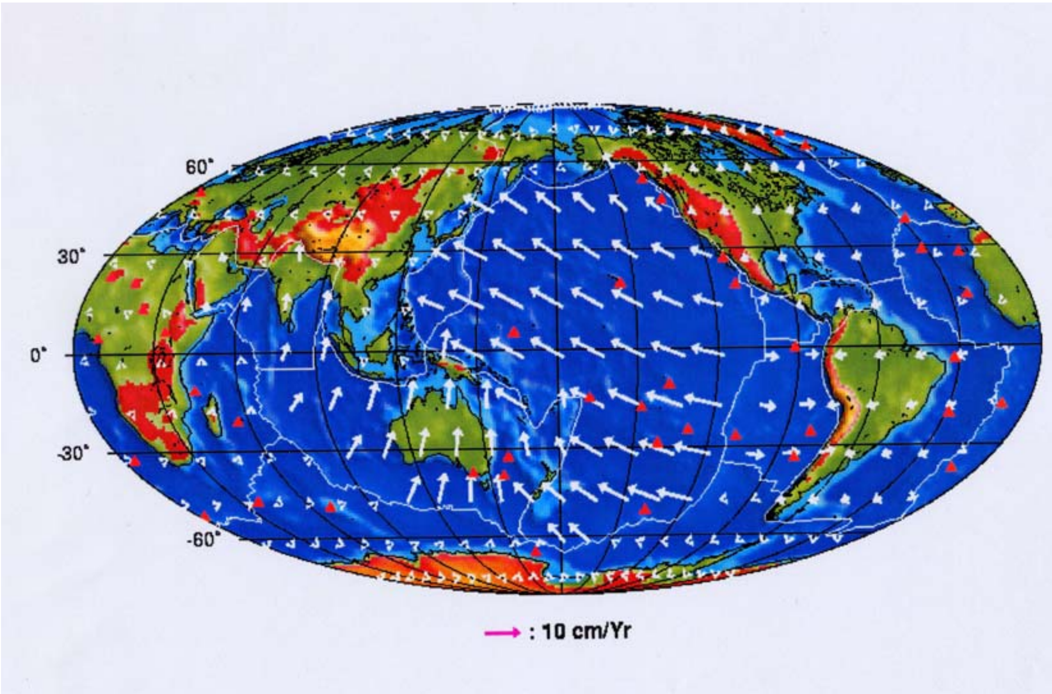
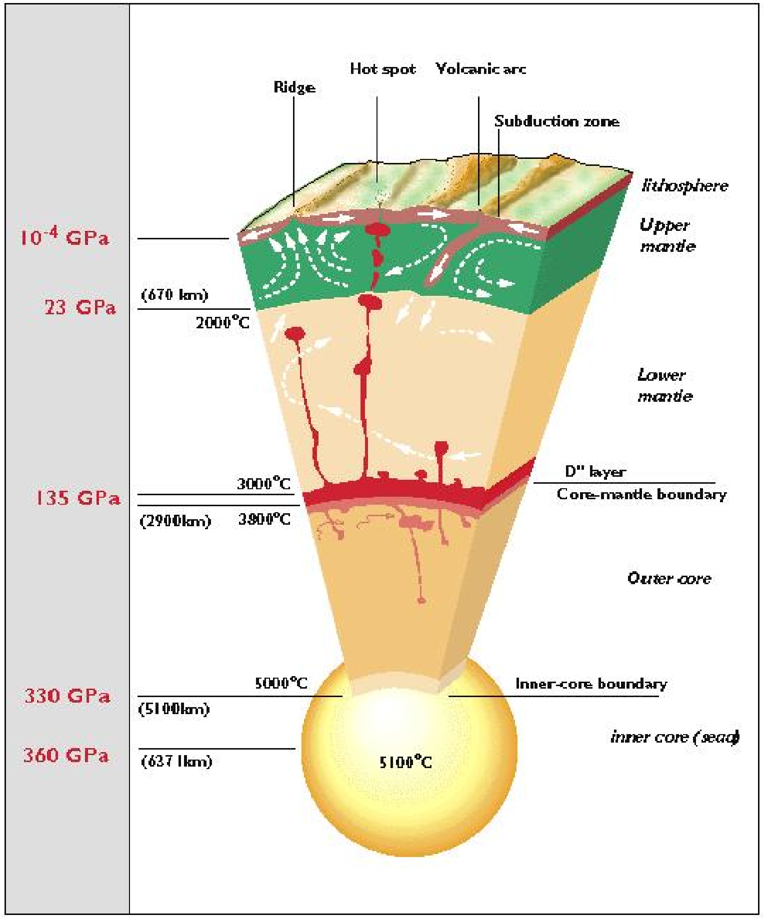
Structure of the Earth

Plate tectonics



Mantle Convection



Tomographic Technique

- ***Forward Problem***: Theory $\mathbf{d}=\mathbf{g}(\mathbf{p})$

d data space, **p** parameter space

- Reference Earth model p_0:

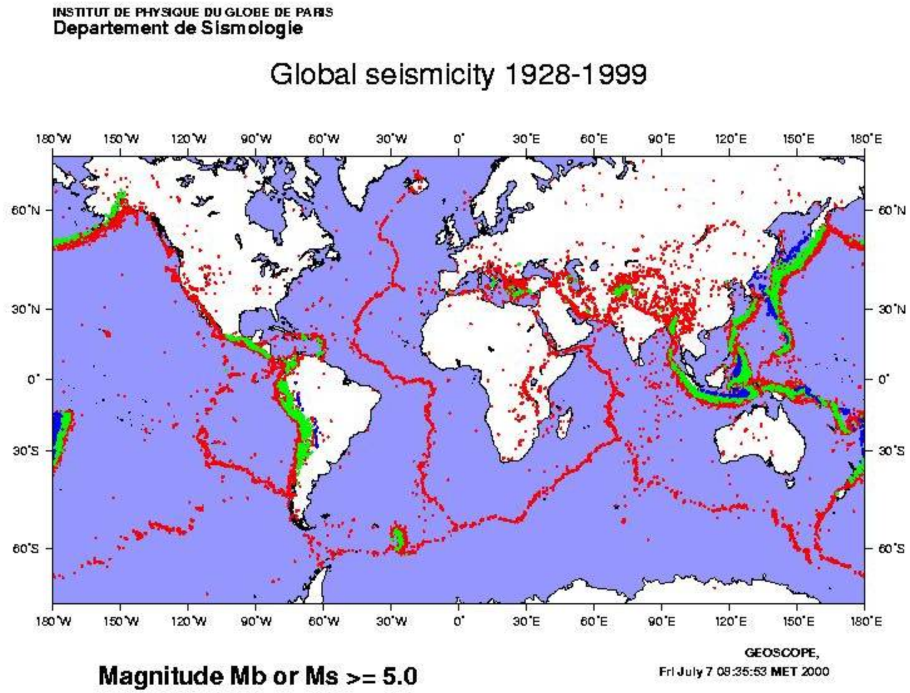
$d_0 = g(p_0)$

- Kernels $\partial g/\partial p$
- Cd function (or matrix) of covariance of data

- ***Inverse Problem***: $p-p_0 = g^{-1}(d-d_0)$

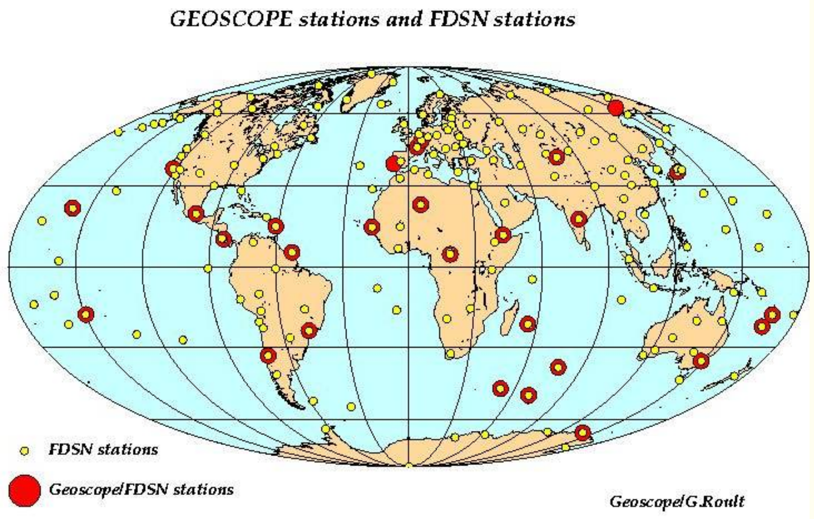
- C_{p0} a priori Covariance function of parameters
- C_{pf} a posteriori Covariance function of parameters
- R Resolution

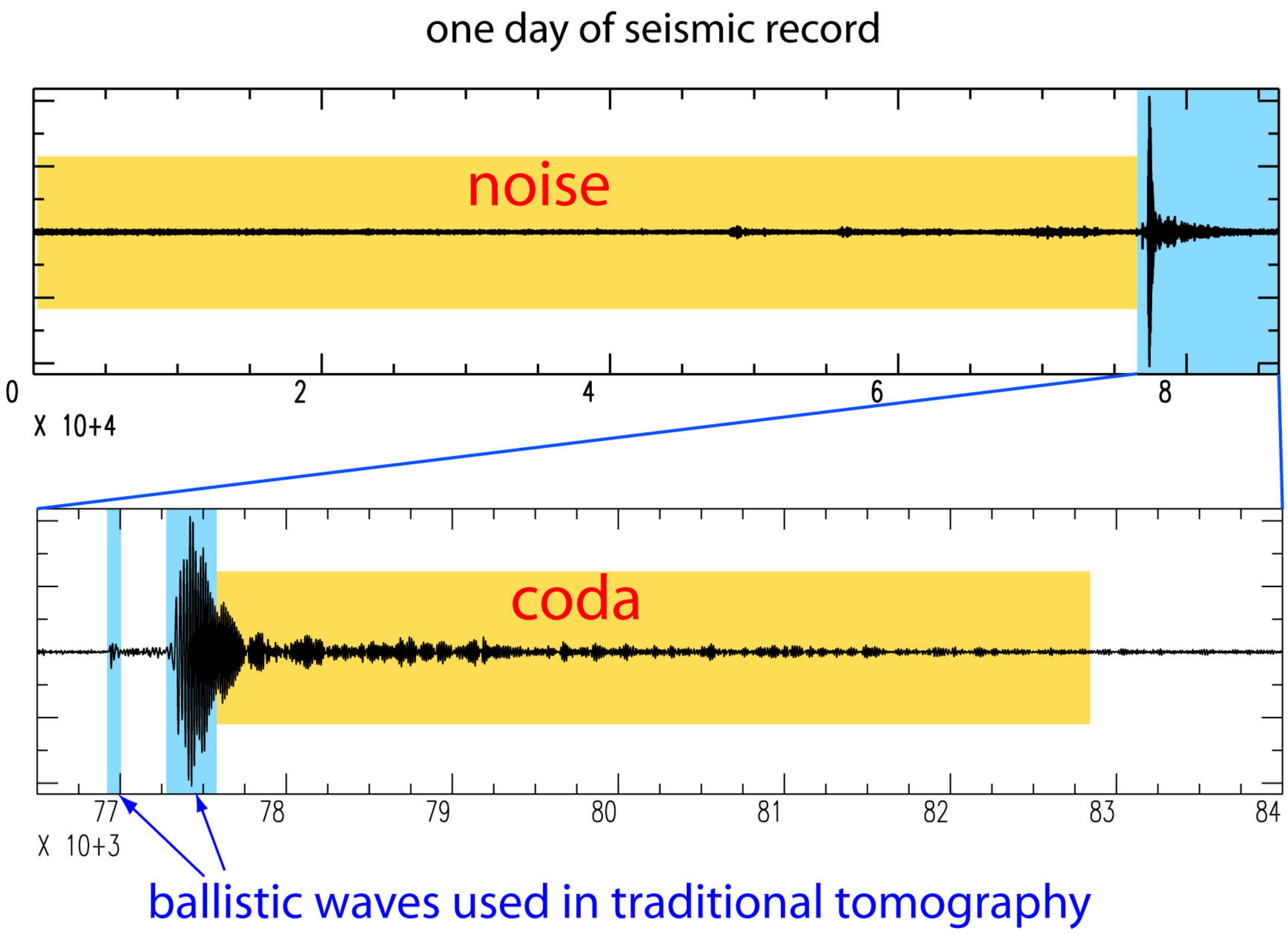
DATA



Seismic sources:
- earthquakes
- noise
(1-20s microseismic)
(100-400s seismic Hum)

Receivers





=> Nikolai Shapiro's Lecture

Importance of seismic anisotropy

ANISOTROPY is the Rule not the Exception

Seismic Anisotropy is present at all scales

Importance of seismic anisotropy
ANISOTROPY is the Rule not the Exception

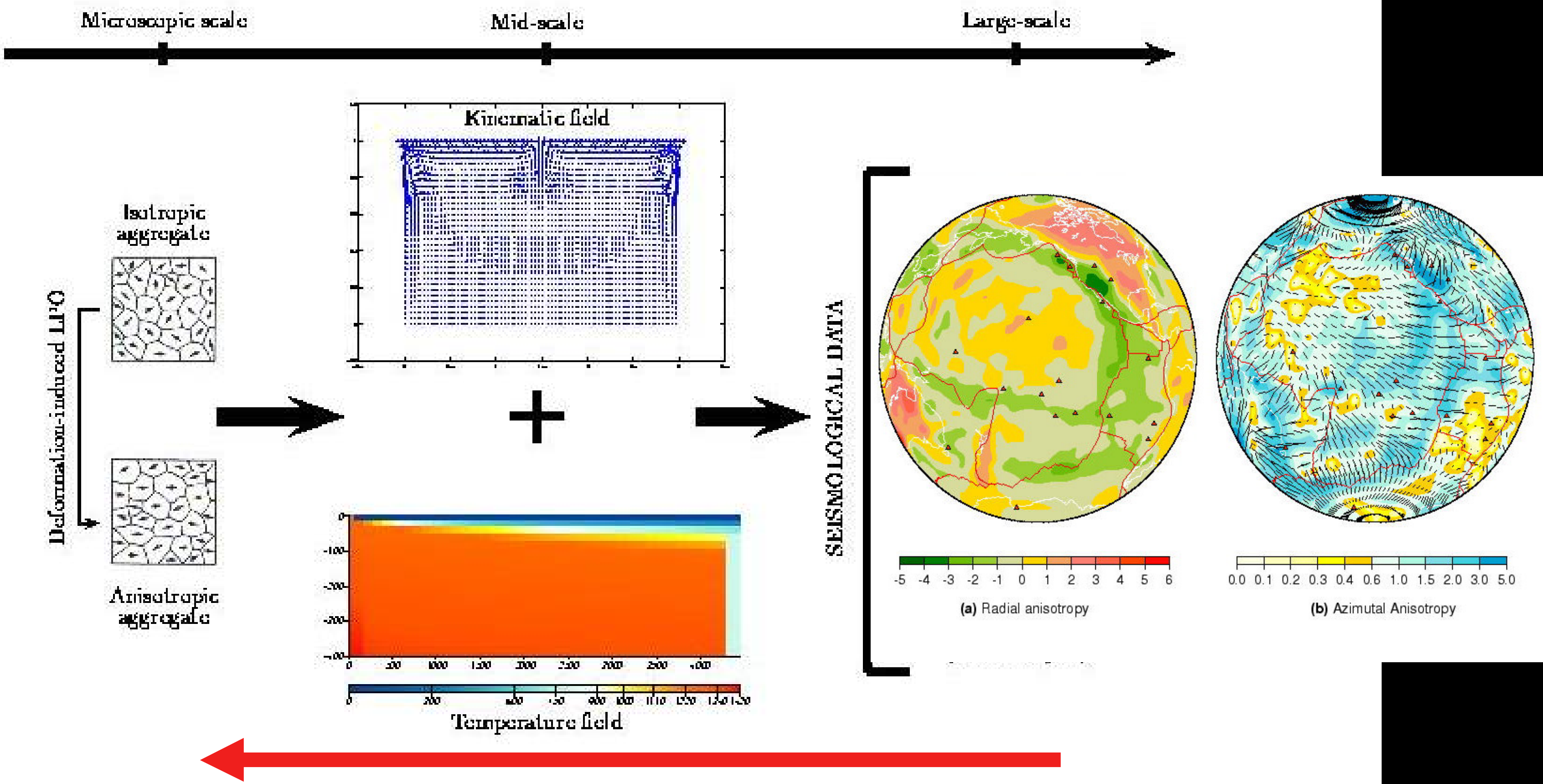
Anisotropy is present at all scales

-From microscopic scale up to macroscopic scale

-Efficient mechanisms of alignment in the upper mantle:
(L.P.O.: lattice preferred orientation;
S.P.O.: shape preferred orientation;
FINE LAYERING)

NON UNIQUE INTERPRETATION

Micro Macro

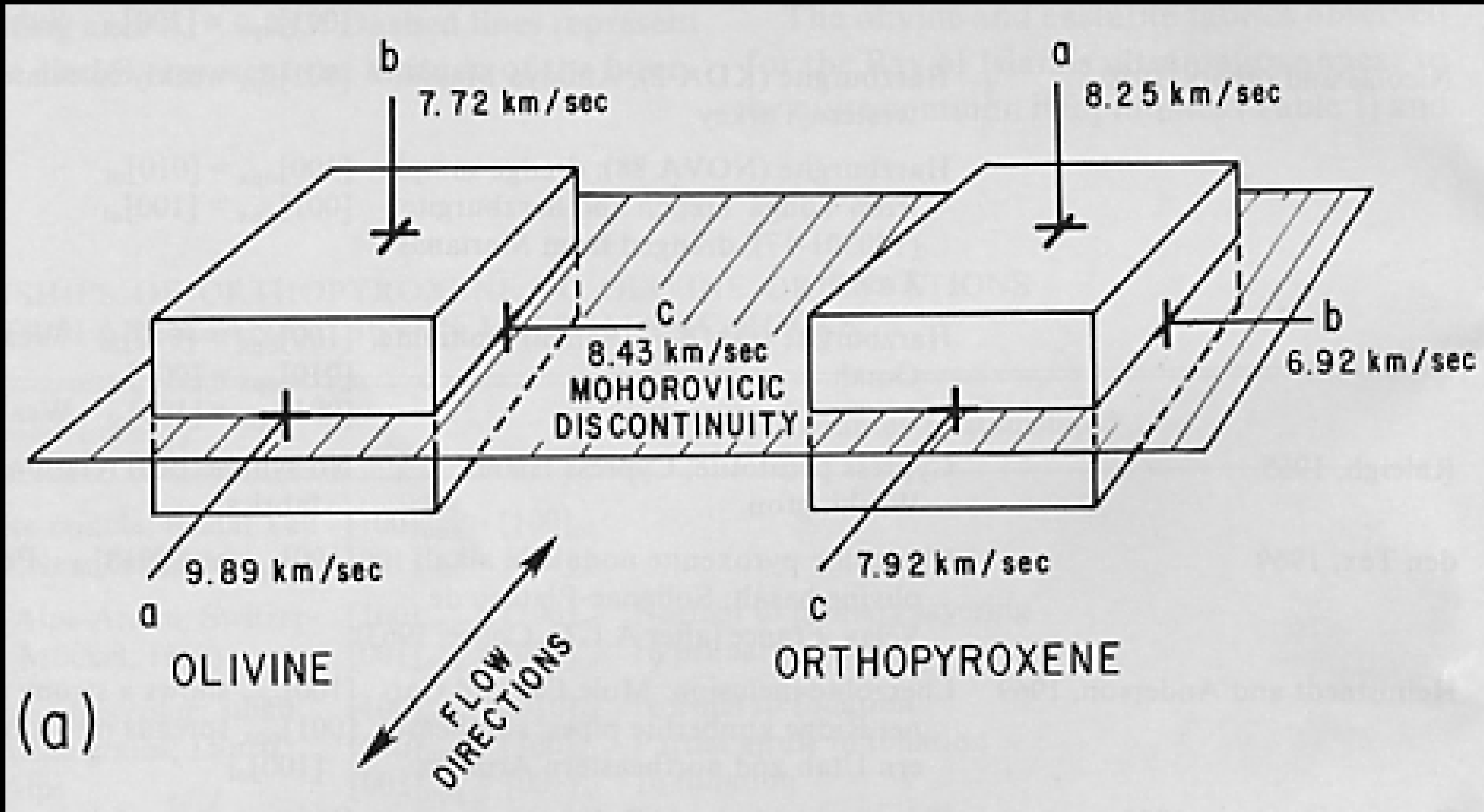


Mineralogical composition

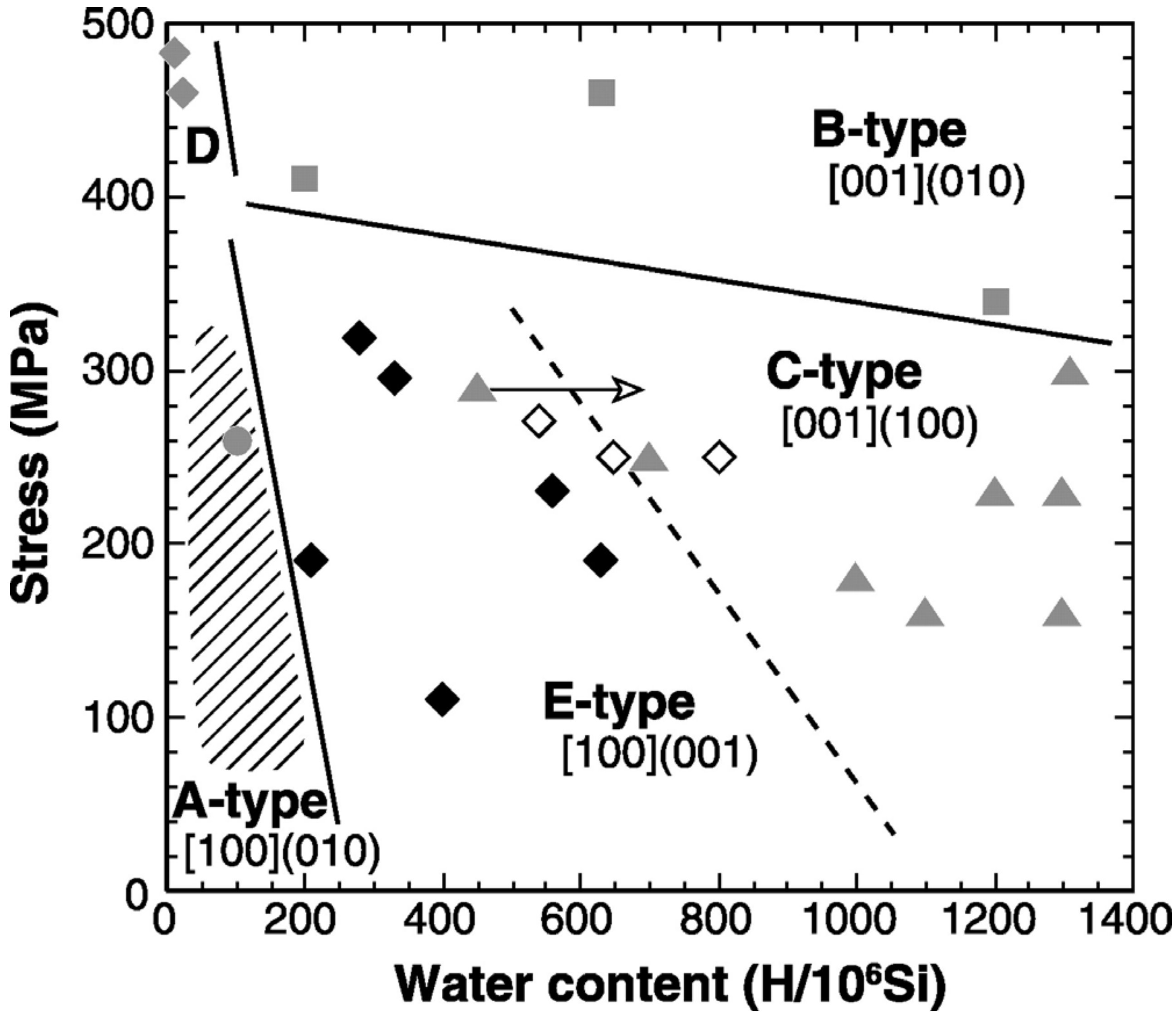
Mapping convection

(Montagner and Guillot, 2001)

L.P.O. : Lattice Preferred Orientation (strain field)



Christensen and Lundquist, 1982

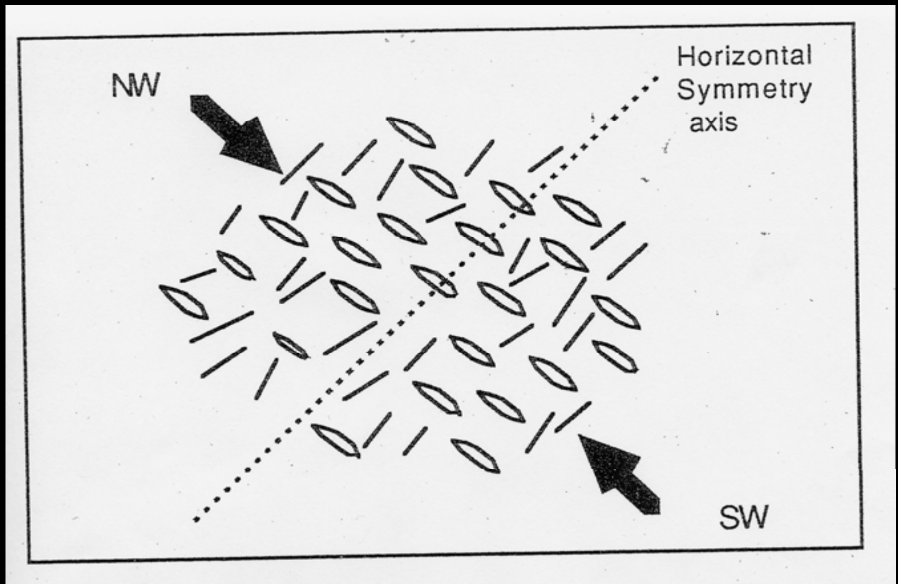


New Types of Olivine Fabric

Katayama et al., 2004

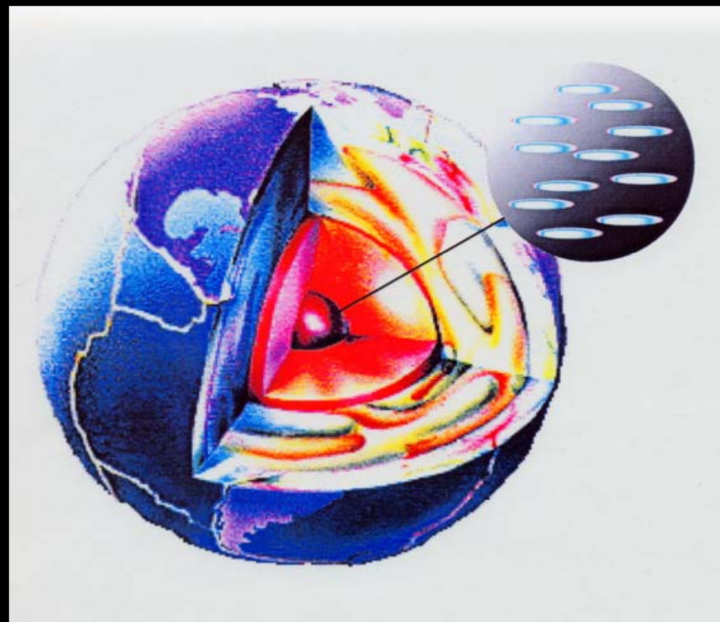
Cracks, fluid inclusions (stress field)

Crust



(Babuska and Cara, 1991)

Inner core



(Singh et al., 2001)

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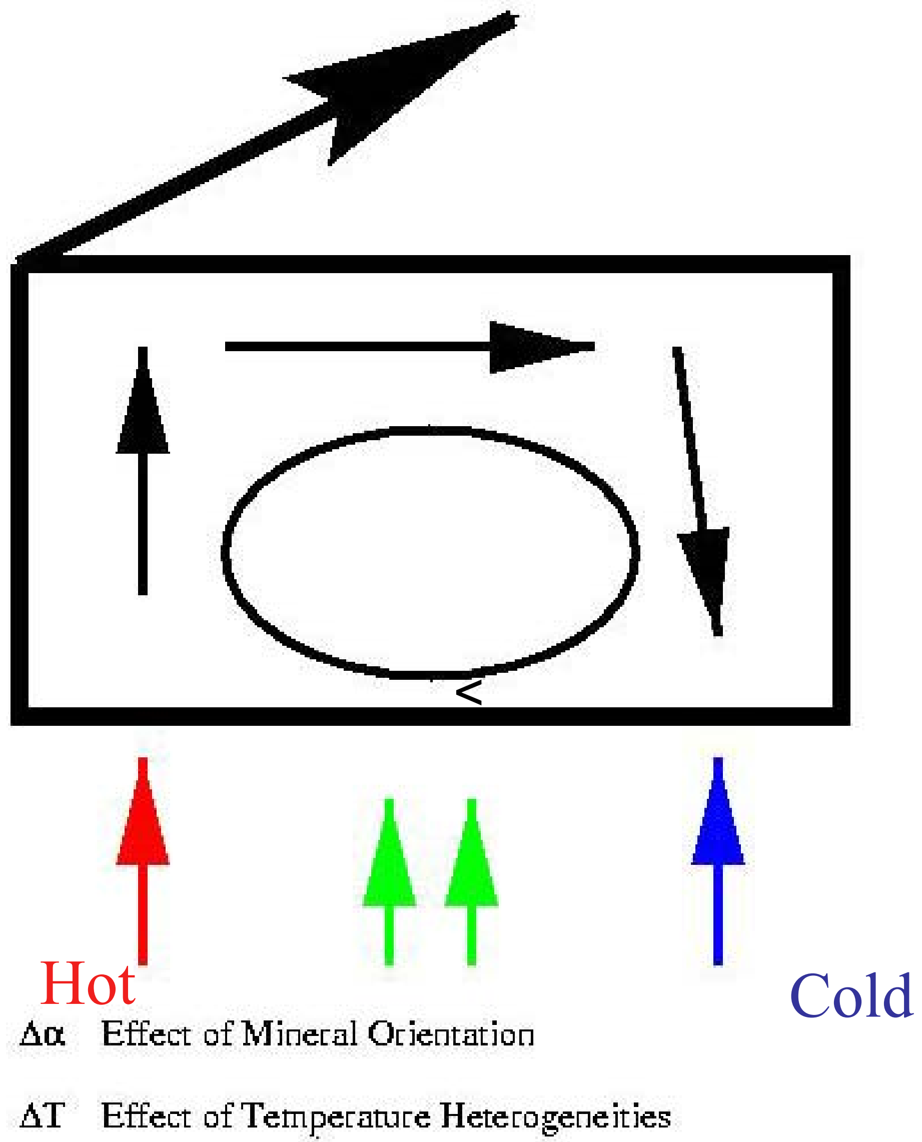
NON UNIQUE INTERPRETATION

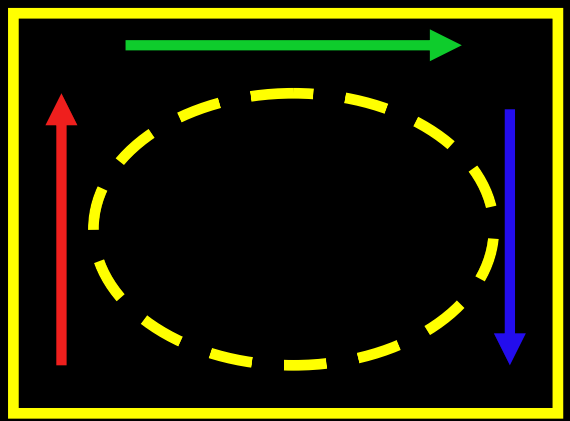


Montagner & Guillot, 2002

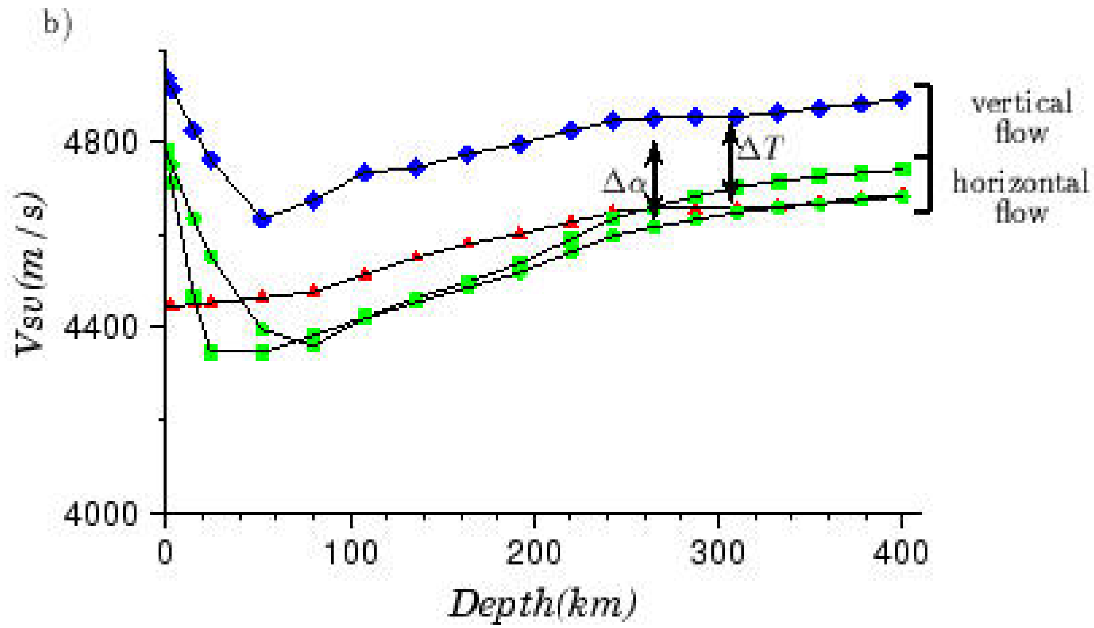
$\Delta\alpha$: Anisotropy Effect

ΔT: Temperature Effect

$$\Delta\alpha \approx \Delta T$$



Olivine (60%) +Opx (40%)



Montagner & Guillot, 2002

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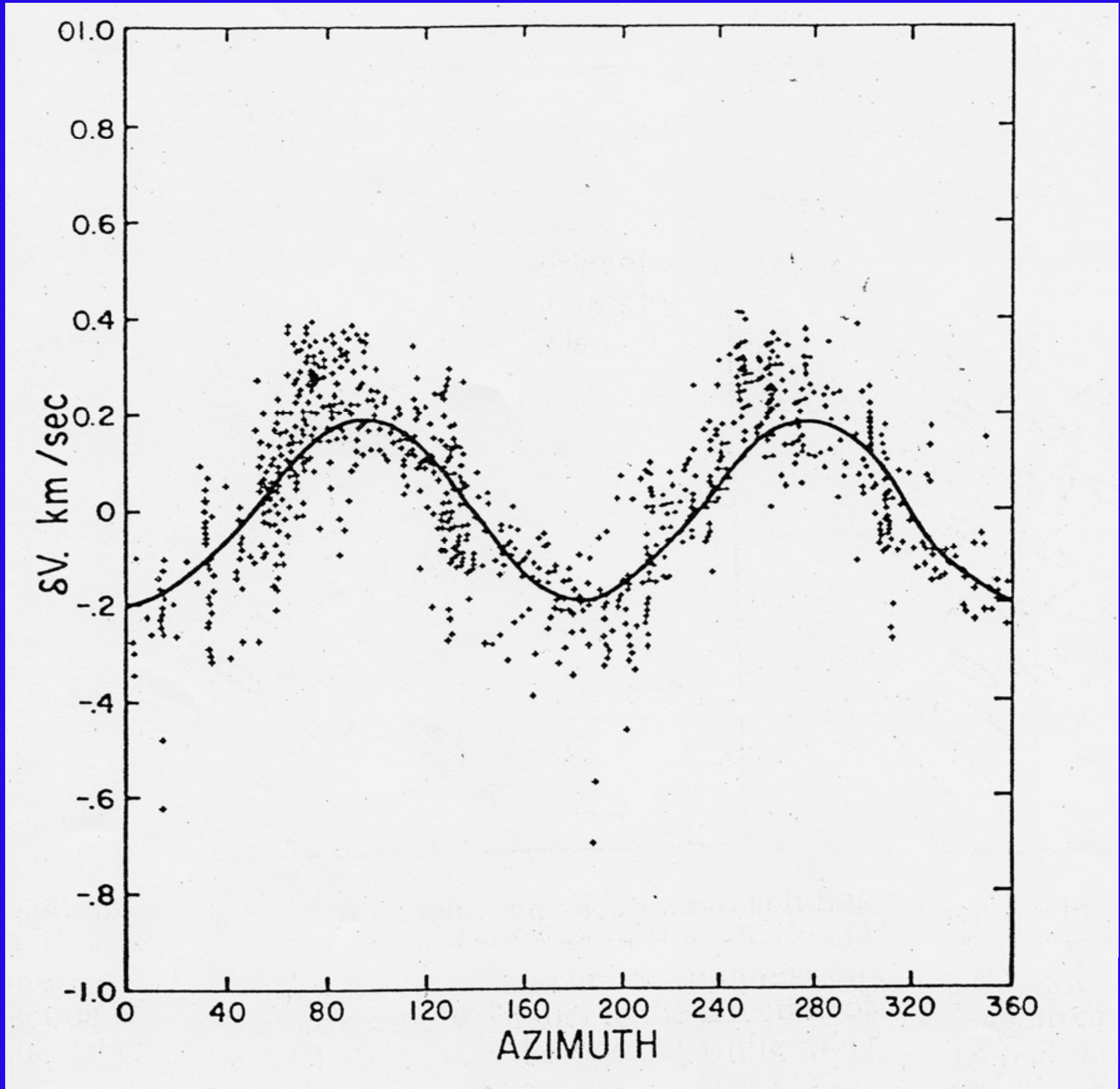
Anisotropy is observed on different kinds of seismic waves

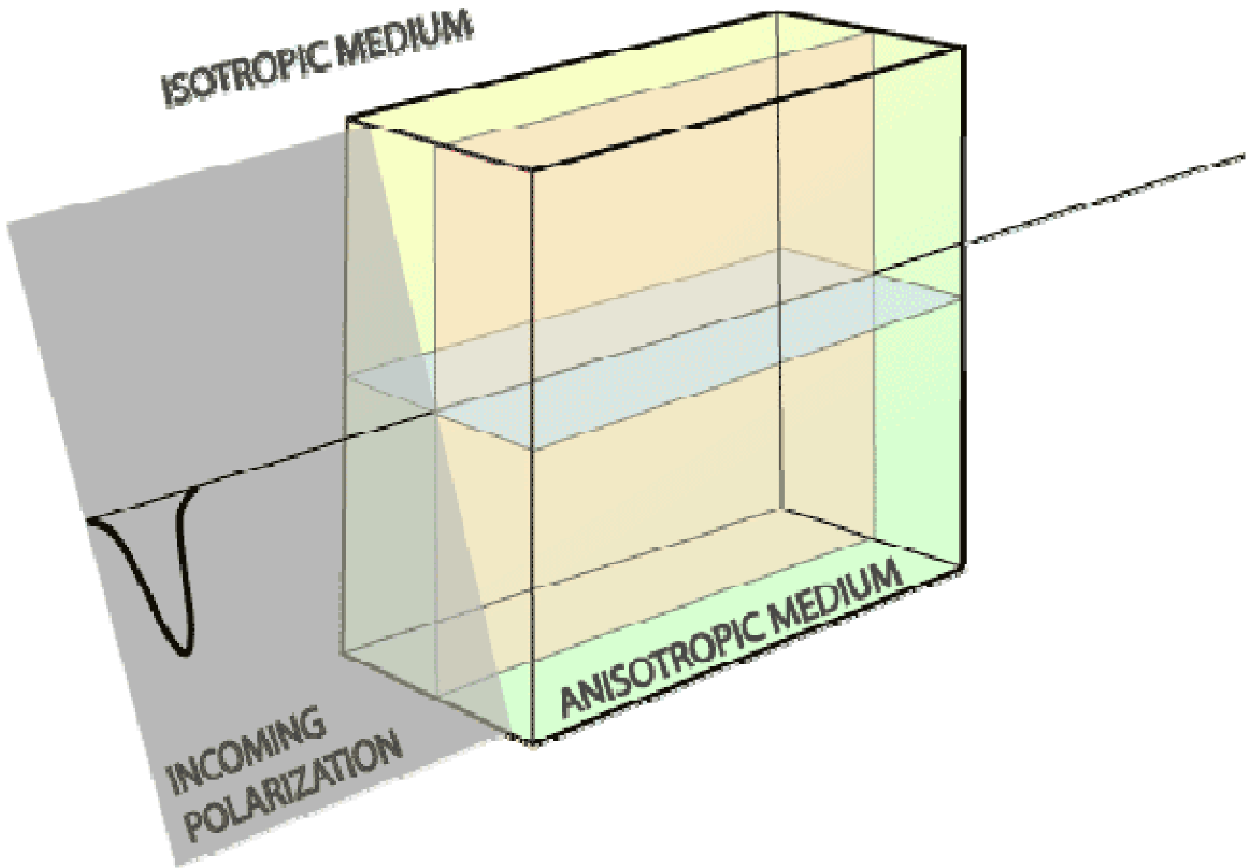
-Body waves (Pn; S-wave splitting)
-Surface waves (Rayleigh-Love discrepancy, azimuthal anisotropy)

Pn- velocities



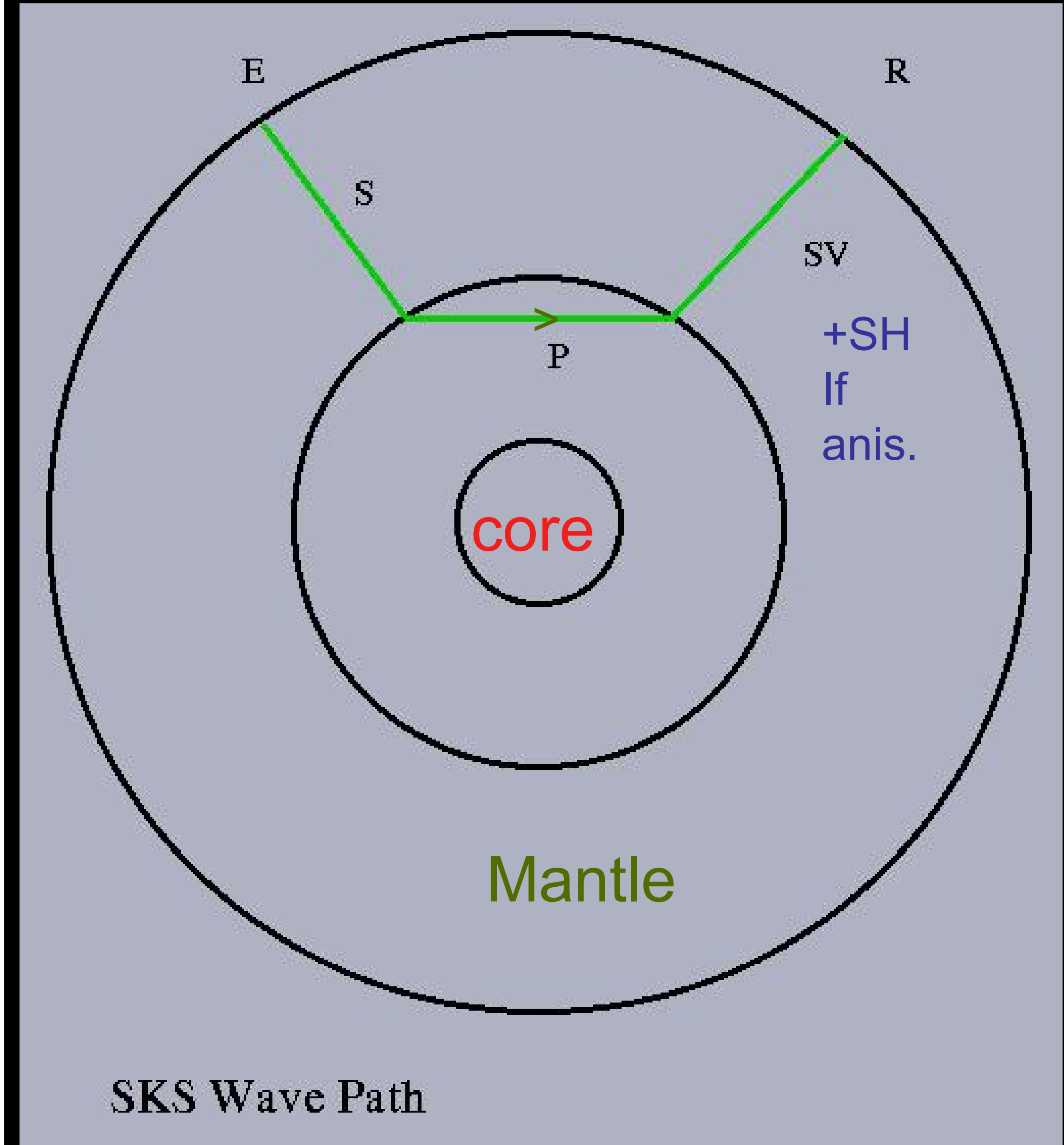
(Raitt et al., 1969)

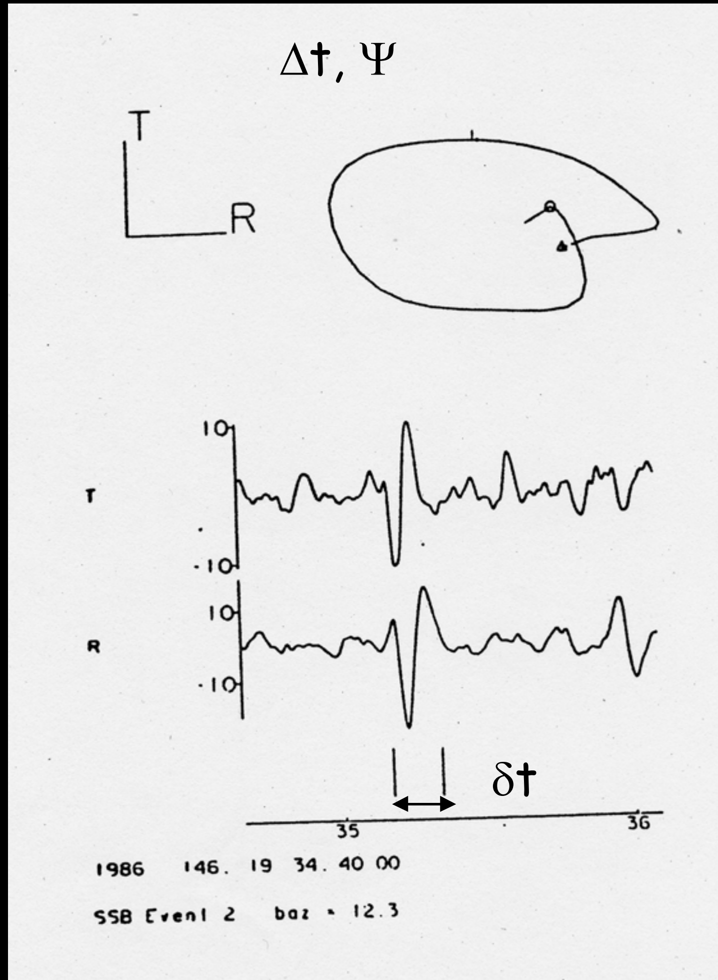
Shear Wave Splitting (Birefringence)



Animation courtesy of Ed Garnero

SKS- Splitting



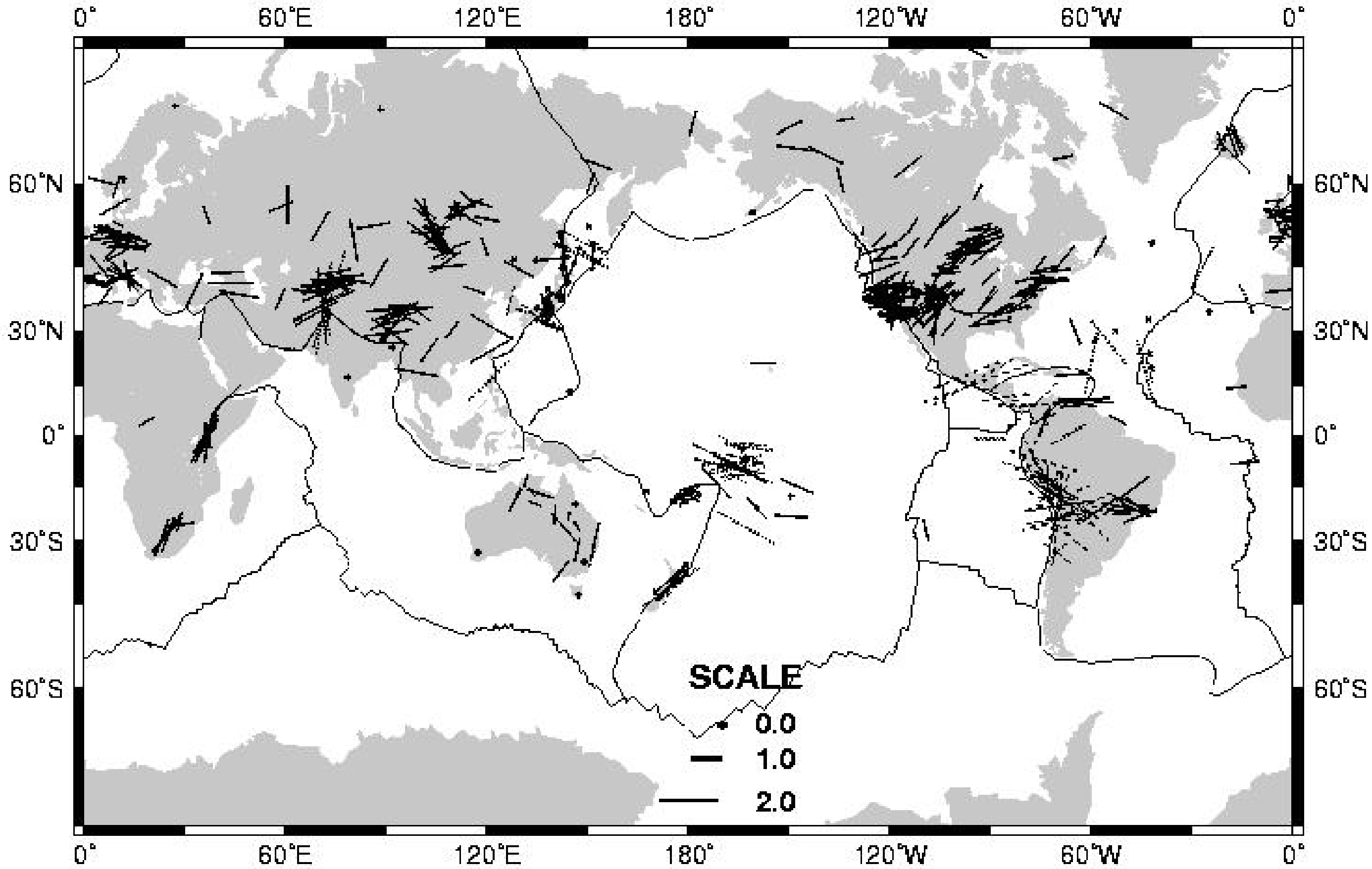


T

R

Vinnik et al., 1989

Compilation of S-wave splitting measurements



Savage, Rev. Geophys., 1999

SURFACE WAVES

Importance of seismic anisotropy
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(L.P.O.: lattice preferred orientation
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Anisotropy is observed on different kinds of seismic waves
-Body waves (Pn; S-wave splitting)
-Surface waves (Rayleigh-Love discrepancy, azimuthal anisotropy)

ANISOTROPY REFLECTS AN INNER ORGANIZATION

ANISOTROPY IS NOT A SECOND ORDER EFFECT

BUT NON UNIQUE INTERPRETATION

Tomographic Technique

- ***Forward Problem***: Theory **d**=**g**(**p**) ⟶ **1st order perturbation theory**

d data space

p parameter space

- Reference Earth model p_0:

$d_0 = g(p_0)$

- Kernels $\partial g/\partial p$
- Cd function (or matrix) of covariance of data

- ***Inverse Problem***: $p-p_0 = g^{-1}(d-d_0)$

- C_{p0} a priori Covariance function of parameters
- C_{pf} a posteriori Covariance function of parameters
- R Resolution

Effect of anisotropy on surface waves

Effect on eigenfrequency for multiplet k={n,l,m} (Rayleigh's principle)

$$\frac{\delta\omega_k}{\omega_k} = \frac{\int_\Omega \varepsilon_{ij}^* \, \delta C_{ijkl} \, \varepsilon_{kl} \, d\Omega}{\int_\Omega \rho_0 \, u_r^* \, u_r \, d\Omega} = \left.\frac{\delta V}{V}\right|_k$$

ε strain tensor, u displacement, δC_{ijkl} elastic tensor perturbation

Phase velocity pertubation $V(T,\theta,\phi,\Psi)$ at point r (θ,ϕ)
(Smith&Dahlen, 1973)

Ψ Azimuth (angle between North and wave vector)

$$\delta V(T,\theta,\phi,\Psi)/V = \alpha_0(T,\theta,\phi) + \alpha_1(T,\theta,\phi)\cos 2\Psi + \alpha_2(T,\theta,\phi)\sin 2\Psi + \alpha_3(T,\theta,\phi)\cos 4\Psi + \alpha_4(T,\theta,\phi)\sin 4\Psi$$

Table 1: Calculation of the various $c_{ij}\epsilon_i\epsilon_j$ for Love waves
$\alpha = cos\Psi$; $\beta = sin\Psi$

n	ij	$c_{ij}\epsilon_i\epsilon_j$
1	11	$c_{11}\alpha^2\beta^2.k^2W^2$
1	22	$c_{22}\alpha^2\beta^2.k^2W^2$
1	33	0
2	12	$-c_{12}\alpha^2\beta^2.k^2W^2$
2	13	0
2	23	0
2	24	
4	14	$c_{14}(-i\alpha^2\beta).\frac{kWW'}{2}$
4	15	$c_{15}(i\alpha^2\beta).\frac{kWW'}{2}$
4	16	$c_{16}(-\alpha\beta)(\alpha^2-\beta^2).\frac{k^2W^2}{2}$
4	24	$c_{24}(-i\alpha^2\beta).\frac{kWW'}{2}$
4	25	$c_{25}(-i\alpha\beta^2).\frac{kWW'}{2}$
4	26	$c_{26}(\alpha\beta)(\alpha^2-\beta^2).\frac{k^2W^2}{2}$
4	34	0
4	35	0
4	36	0
4	44	$c_{44}\alpha^2.\frac{W'^2}{4}$
8	45	$c_{45}(-\alpha\beta).\frac{W'^2}{4}$
8	46	$c_{46}(-i\alpha)(\alpha^2-\beta^2).\frac{kWW'}{2}$
4	55	$c_{55}\beta^2.\frac{W'^2}{4}$
8	56	$c_{56}(i\beta)(\alpha^2-\beta^2).\frac{kWW'}{2}$
4	66	$c_{66}(\alpha^2-\beta^2).\frac{k^2W^2}{4}$

$C_{mnpq} \longrightarrow c_{ij}$

Indices:

i= 9-m-n

j= 9-p-q

W(r) Love displacement

W ' = ($^{dW}/_{dr}$)

The first order perturbation in Love wave phase velocity $\delta C_L(k, \Psi)$ can be expressed as:

$$\delta C_L(k, \Psi) = \frac{1}{2C_{0L}(k)}[L_1(k) + L_2(k)cos2\Psi + L_3(k)sin2\Psi + L_4(k)cos4\Psi + L_5(k)sin4\Psi$$

where

$$\begin{array}{ll} & L_0(k) = \int_0^\infty \rho W^2 dz \\ 0\Psi \leftarrow & L_1(k) = \frac{1}{L_0} \int_0^\infty (W^2 dN + \frac{W'^2}{k^2} dL) dz \\ 2\Psi \leftarrow & \begin{cases} L_2(k) = \frac{1}{L_0} \int_0^\infty -G_c(\frac{W'^2}{k^2}) dz \\ L_3(k) = \frac{1}{L_0} \int_0^\infty -G_s(\frac{W'^2}{k^2}) dz \end{cases} \\ 4\Psi \leftarrow & \begin{cases} L_4(k) = \frac{1}{L_0} \int_0^\infty -E_c . W^2 dz \\ L_5(k) = \frac{1}{L_0} \int_0^\infty -E_s . W^2 dz \end{cases} \end{array}$$

The same procedure holds for Rayleigh waves, starting from the displacement given previously.

$$\delta C_R(k,\Psi) = \frac{1}{2C_{0R}(k)}[R_1(k) + R_2(k)cos2\Psi + R_3(k)sin2\Psi + R_4(k)cos4\Psi + R_5(k)sin4\Psi$$

where

$$R_0(k) = \int_0^\infty \rho(U^2 + V^2)dz$$

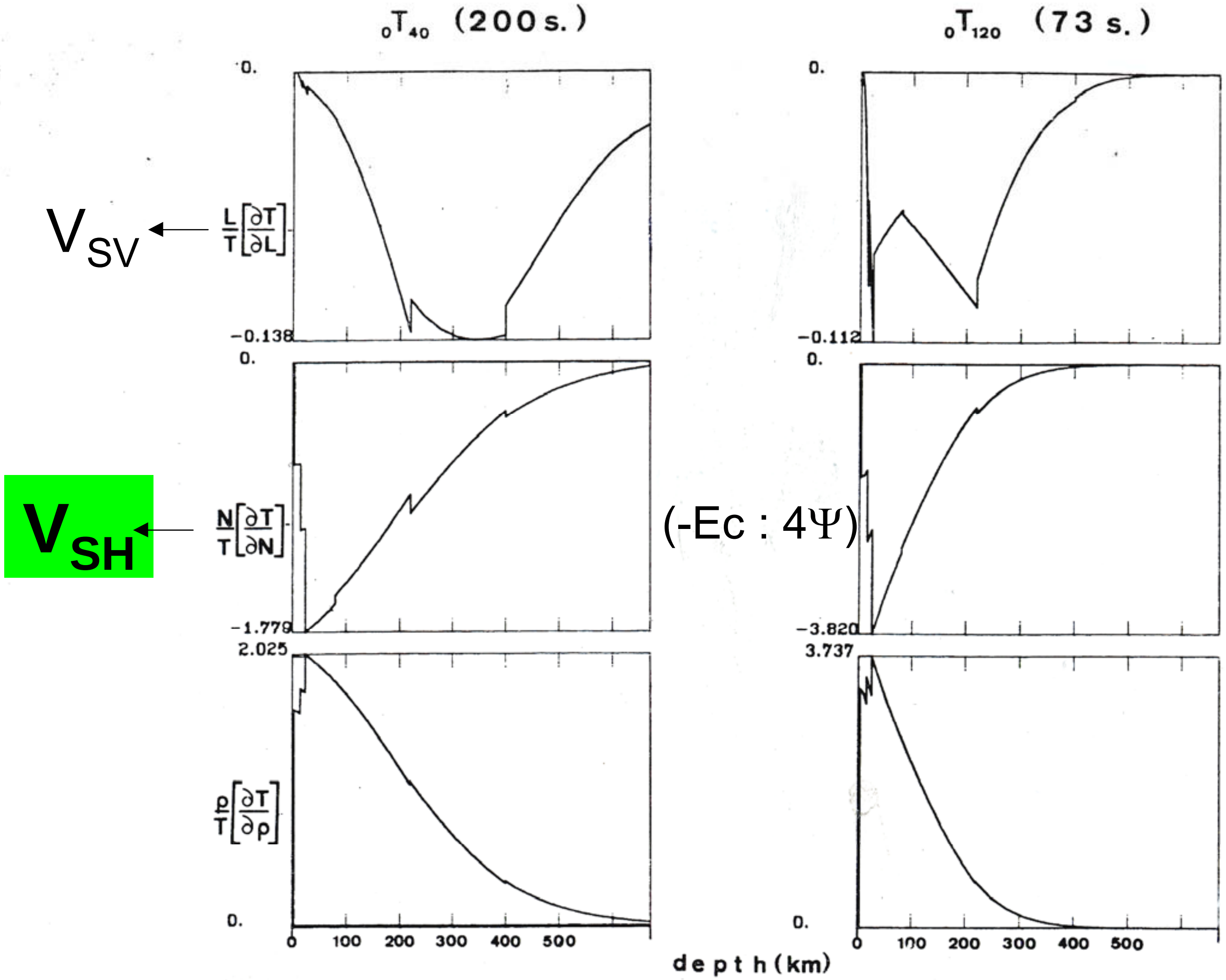
0-Ψ $$R_1(k) = \frac{1}{R_0}\int_0^\infty [V^2 dA + \frac{U'^2}{k^2}.dC + \frac{2U'V}{k}.dF + (\frac{V'}{k} - U)^2 dL]dz$$

2-Ψ $$\begin{cases} R_2(k) = \frac{1}{R_0}\int_0^\infty [V^2.B_c + \frac{2U'V}{k}.H_c + (\frac{V'}{k} - U)^2 G_c]dz \\ R_3(k) = \frac{1}{R_0}\int_0^\infty [V^2.B_s + \frac{2U'V}{k}.H_s + (\frac{V'}{k} - U)^2 G_s]dz \end{cases}$$

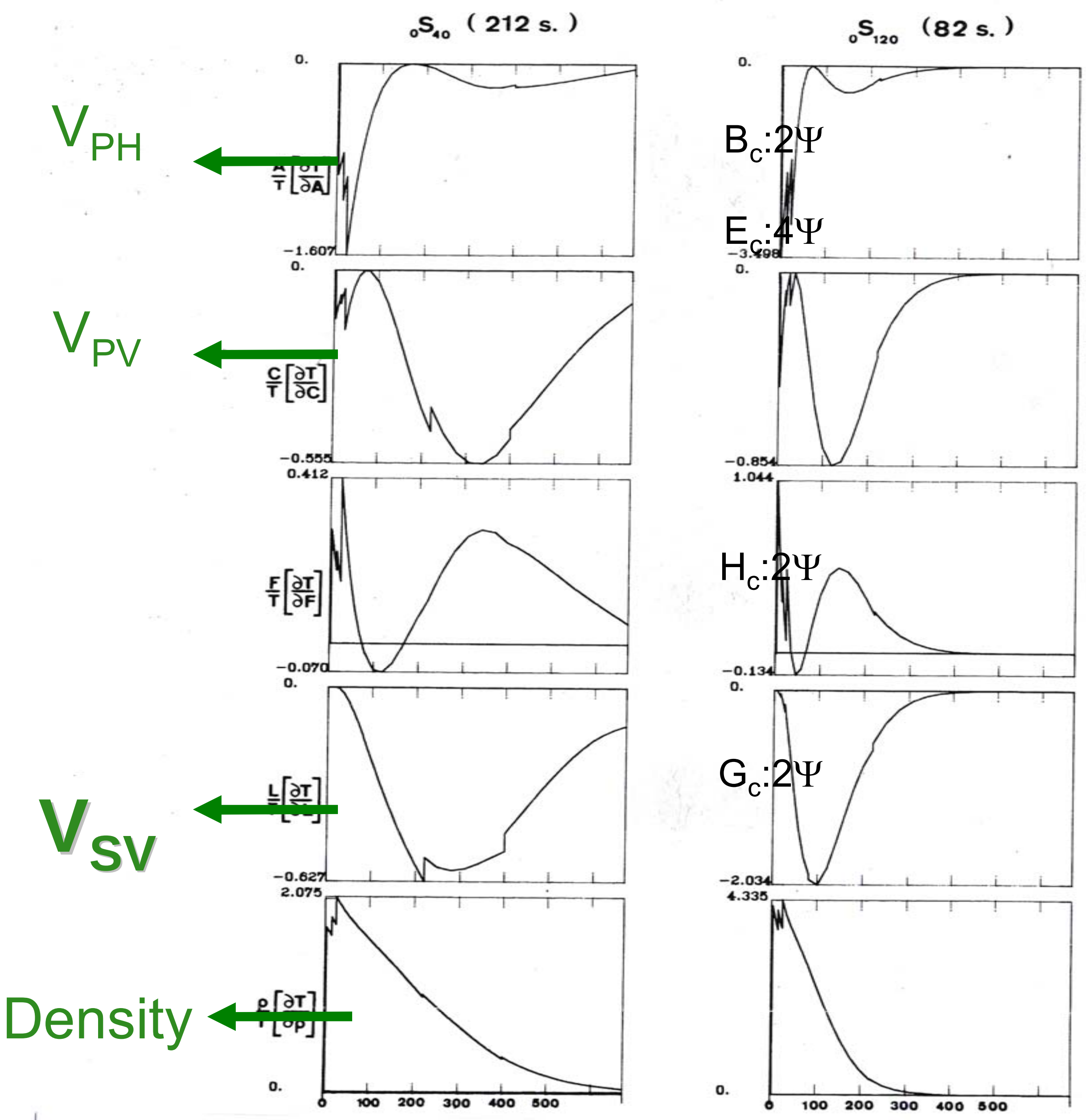
4-Ψ $$\begin{cases} R_4(k) = \frac{1}{R_0}\int_0^\infty E_c.V^2 dz \\ R_5(k) = \frac{1}{R_0}\int_0^\infty E_s.V^2 dz \end{cases}$$

The 13 depth-functions $A, C, F, L, N, B_c, B_s, H_c, H_s, G_c, G_s, E_c, E_s$ are linear combinations of the elastic coefficients C_{ij} and are explicitly given as follows:

Love wave partial derivatives



Rayleigh wave partial derivatives



Physical Meaning

13 parameters

0Ψ term	$A = \frac{3}{8}(C_{11}+C_{22}) + \frac{1}{4}C_{12} + \frac{1}{2}C_{66}$	→ V_{PH}
	$C = C_{33}$	→ V_{PV}
	$F = \frac{1}{2}(C_{13}+C_{23})$	
	$L = \frac{1}{2}(C_{44}+C_{55})$	→ V_{SV}
	$N = \frac{1}{8}(C_{11}+C_{22}) - \frac{1}{4}C_{12} + \frac{1}{2}C_{66}$	→ V_{SH}
	cos	sin
2Ψ term	$B_c = \frac{1}{2}(C_{11}-C_{22})$	$B_s = C_{16}+c_{26} \rightarrow B$
	$G_c = \frac{1}{2}(C_{55}-C_{44})$	$G_s = C_{54} \rightarrow G$
	$H_c = \frac{1}{2}(C_{13}-C_{23})$	$H_s = C_{36} \rightarrow H$
4Ψ term	$E_c = \frac{1}{8}(C_{11}+C_{22}) - \frac{1}{4}C_{12} - \frac{1}{2}C_{66}$	$E_s = \frac{1}{2}(C_{16}-C_{26}) \rightarrow E$

Transversely Isotropic Medium With Vertical Symmetry Axis (VTI)

Azimuthal variation of
V_{PH}
V_{SV}
F
V_{SH} (or V_{PH})

Isotropic medium: 2 parameters
VTI: 5 parameters (A,C,F,L,N)
General + 8 (from surface waves)

13 parameters

- Best Resolved parameters for Surface Waves

$\mathbf{L} = \rho V_{SV}^2$ Isotropic part of V_{SV}.

$\xi = \frac{N}{L} = \frac{V_{SH}^2}{V_{SV}^2}$ Radial Anisotropy.

G, Ψ_G Azimuthal Anisotropy of V_{SV}, also related to SKS splitting (when horizontal symmetry axis).

+ a priori information (from mineralogy, …)

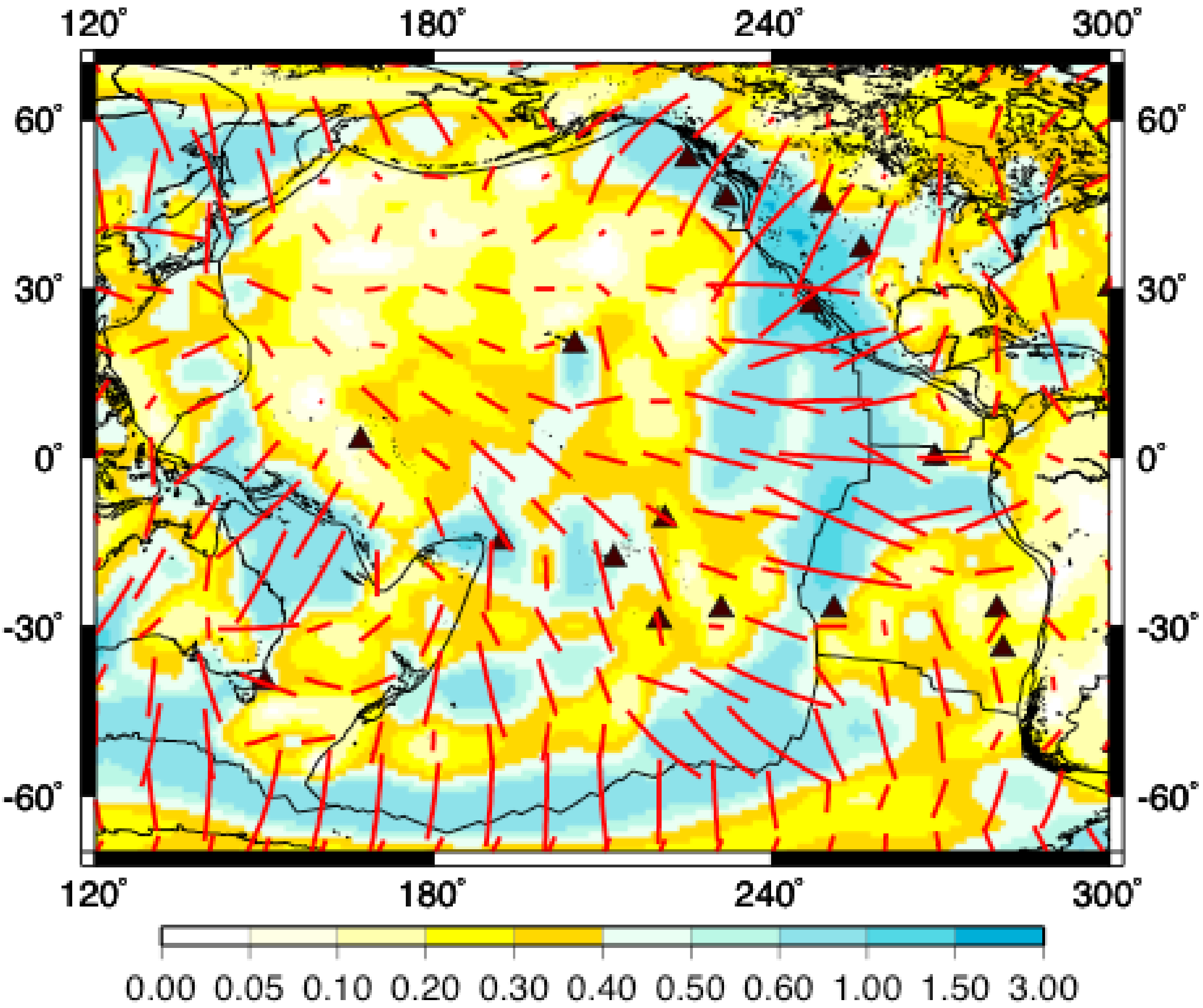
- Body Waves (*Crampin*, 1984)

$$\rho V_{qSV}^2 = L + G_c cos2\Psi + G_s sin2\Psi$$

$$\rho V_{qSH}^2 = N - E_c cos4\Psi - E_s sin4\Psi$$

Surface wave anisotropy: **L, G_c, G_s** parameters

Synthetic SKS-wave splitting



Geodynamic Interpretation

S- Velocity

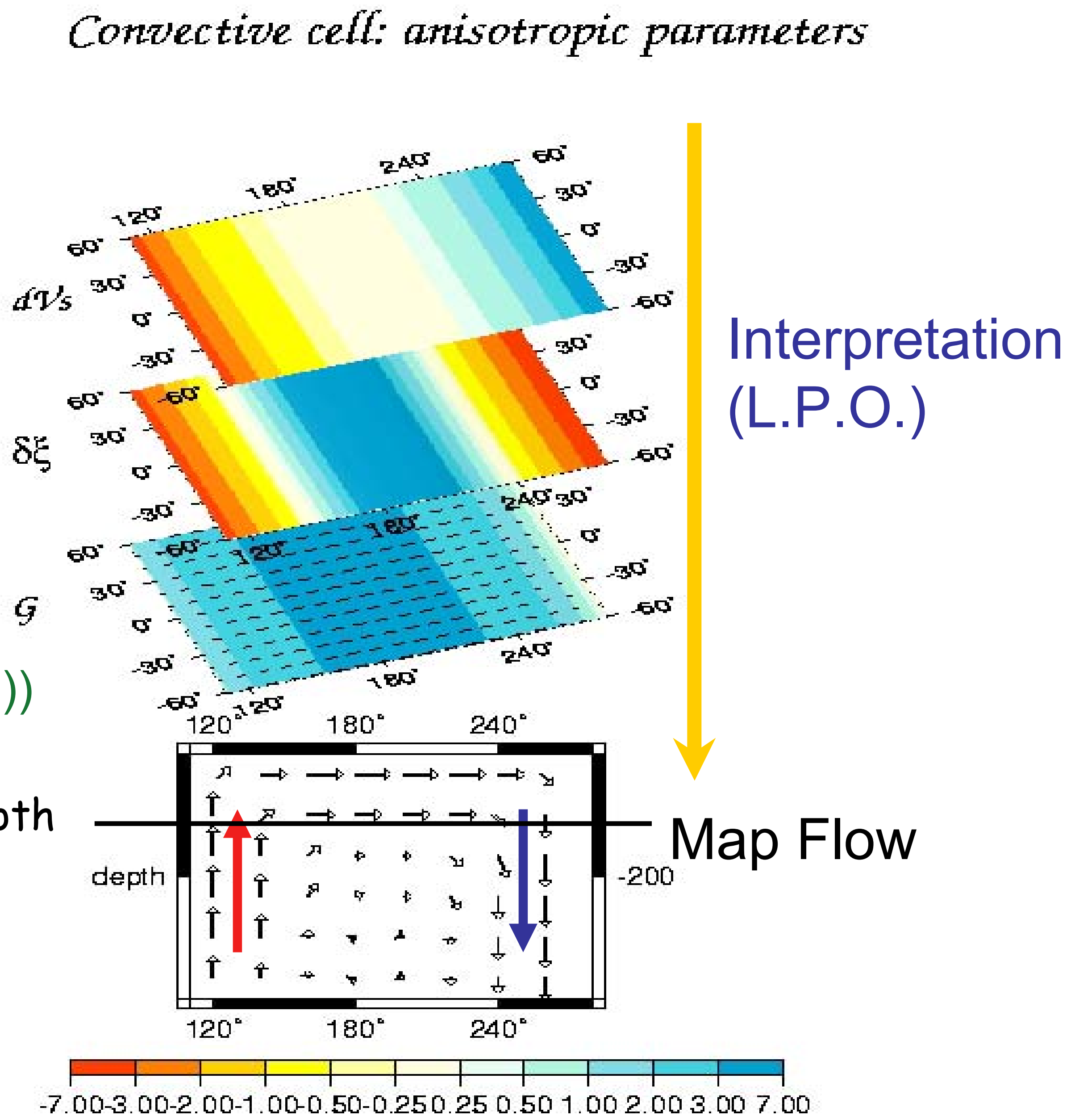
Radial Anisotropy

$\xi = (V_{SH}^2 - V_{SV}^2)/V_{SV}^2$

Azimuthal Anisotropy

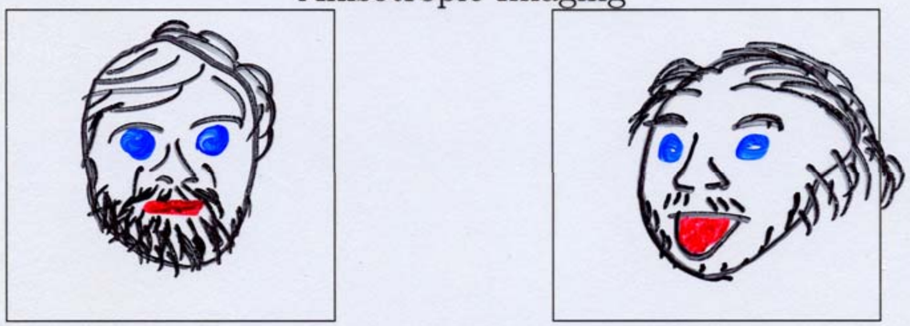
$V_{SV} \approx V_{SV0} + {}^1/_2 G\cos(2(\Psi - \Psi_G))$

At a given depth

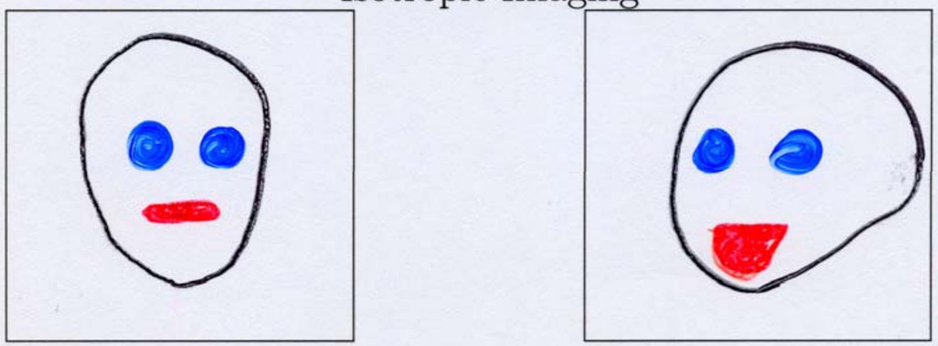


IMAGING OF FAMOUS SCIENTISTS

Anisotropic Imaging



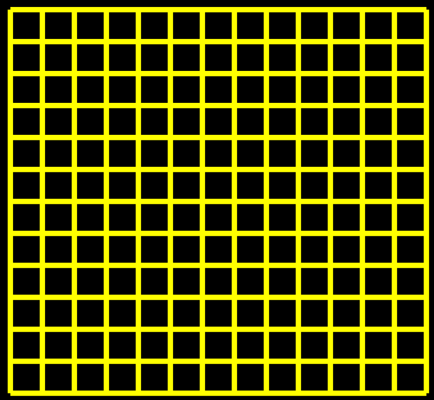
Isotropic Imaging



How can we know whether azimuthal anisotropy is significant or not?

Example of 2 D tomography (N cells)

Isotropic Inversion:

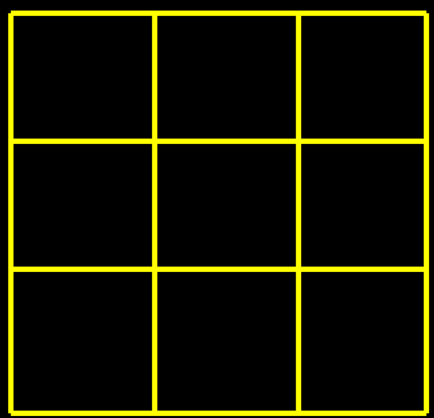


N independent parameters (1/cell)

0-Ψ term

Variance reduction: VR1

Anisotropic inversion



3N’ = N (3/cell)

(N’ cells)

0+2 Ψ terms

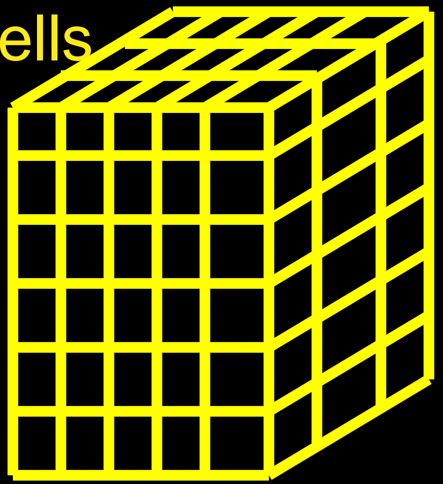
Variance reduction: VR2

VR2>VR1 =>the anisotropic model can be simpler than the isotropic model

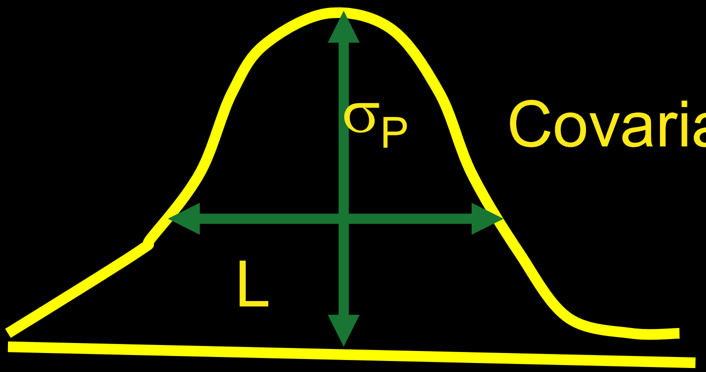
Parameter Space

Physical parameters: ρ + 13 physical parameters

Geographical parameterization: **p**(r,θ,ϕ)

- Cells 

Continuous parameterization



Covariance fonction

- Spherical harmonic expansion → Global scale

- Lateral resolution (global scale):

Hor 1000km, Rad 50km => 500*60*14 ≈ 420,000 parameters

Tomographic Technique

- ***Forward Problem***: Theory **d**=**g**(**p**)

d data space → **Phase or group velocities**

p parameter space

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$d_0 = g(p_0)$

- Kernels $\partial g/\partial p$
- Cd function (or matrix) of covariance of data

- ***Inverse Problem***: $p - p_0 = g^{-1}(d - d_0)$

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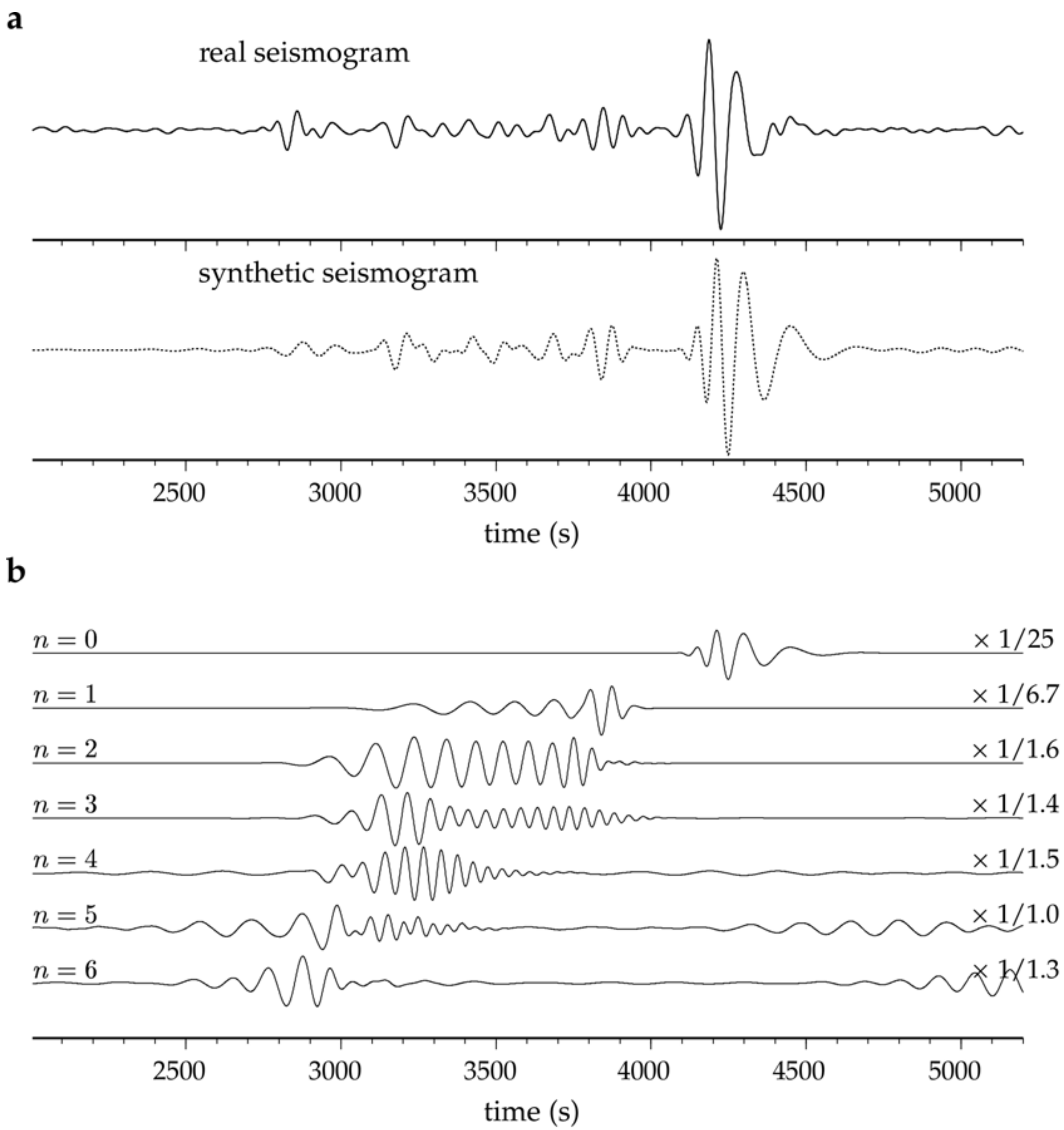
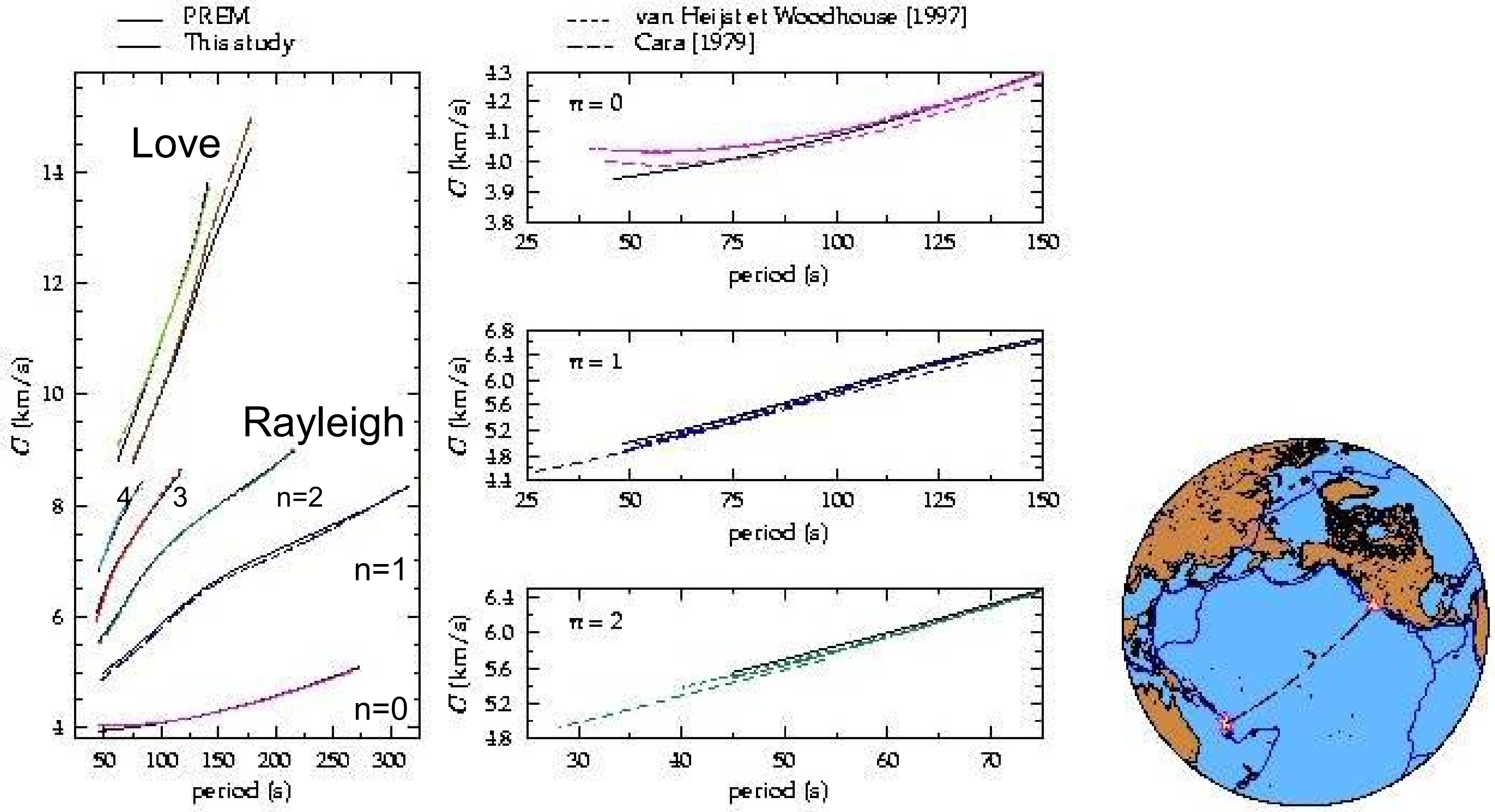
- R Resolution

Example of seismogram



Beucler, 2003

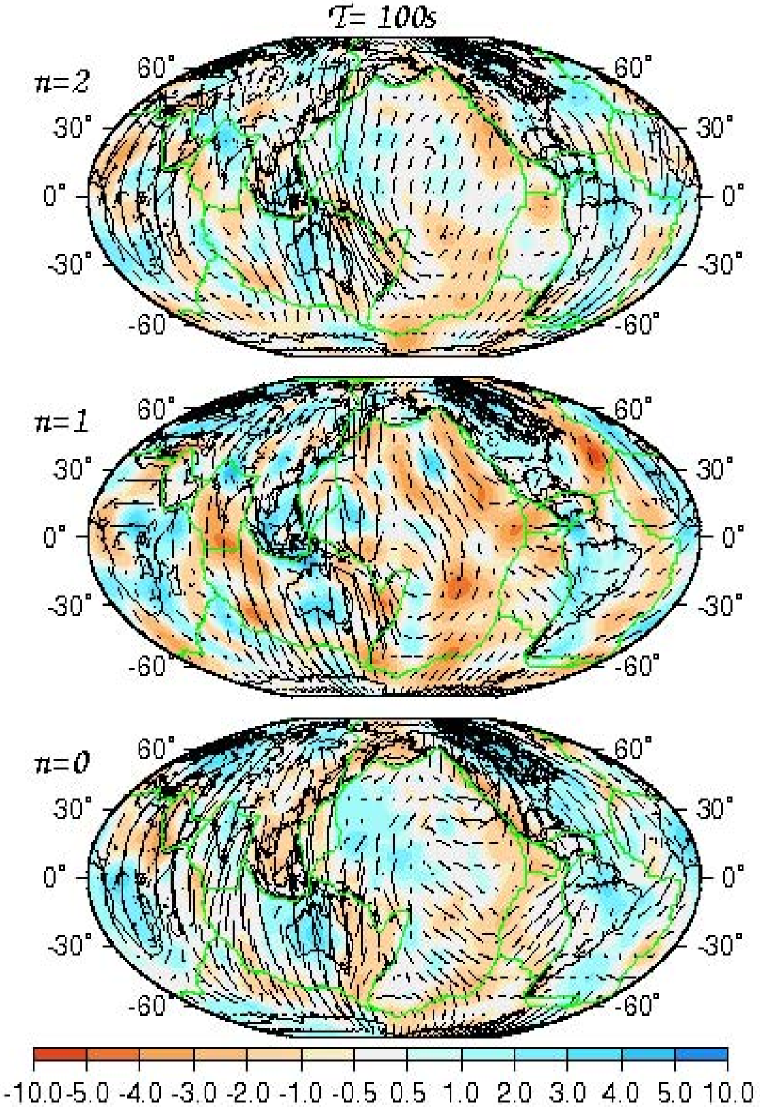
1st step: Calculation of dispersion curves:Fundamental modes and higher modes $c_n^R(T)$, $c_n^L(T)$ (Beucler et al., 2003)



Comparison with previous results along the Vanuatu-California path.

2nd step: Regionalization

Phase velocity maps
At 100s



2nd overtone

1st overtone

Fundamental mode

Beucler and Montagner, 2006

Global Tomography

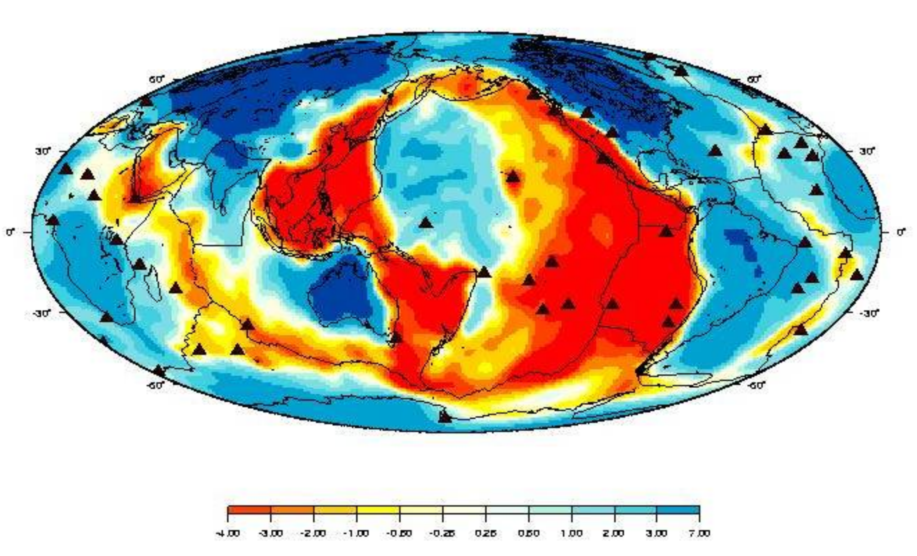
3rd step: inversion at depth

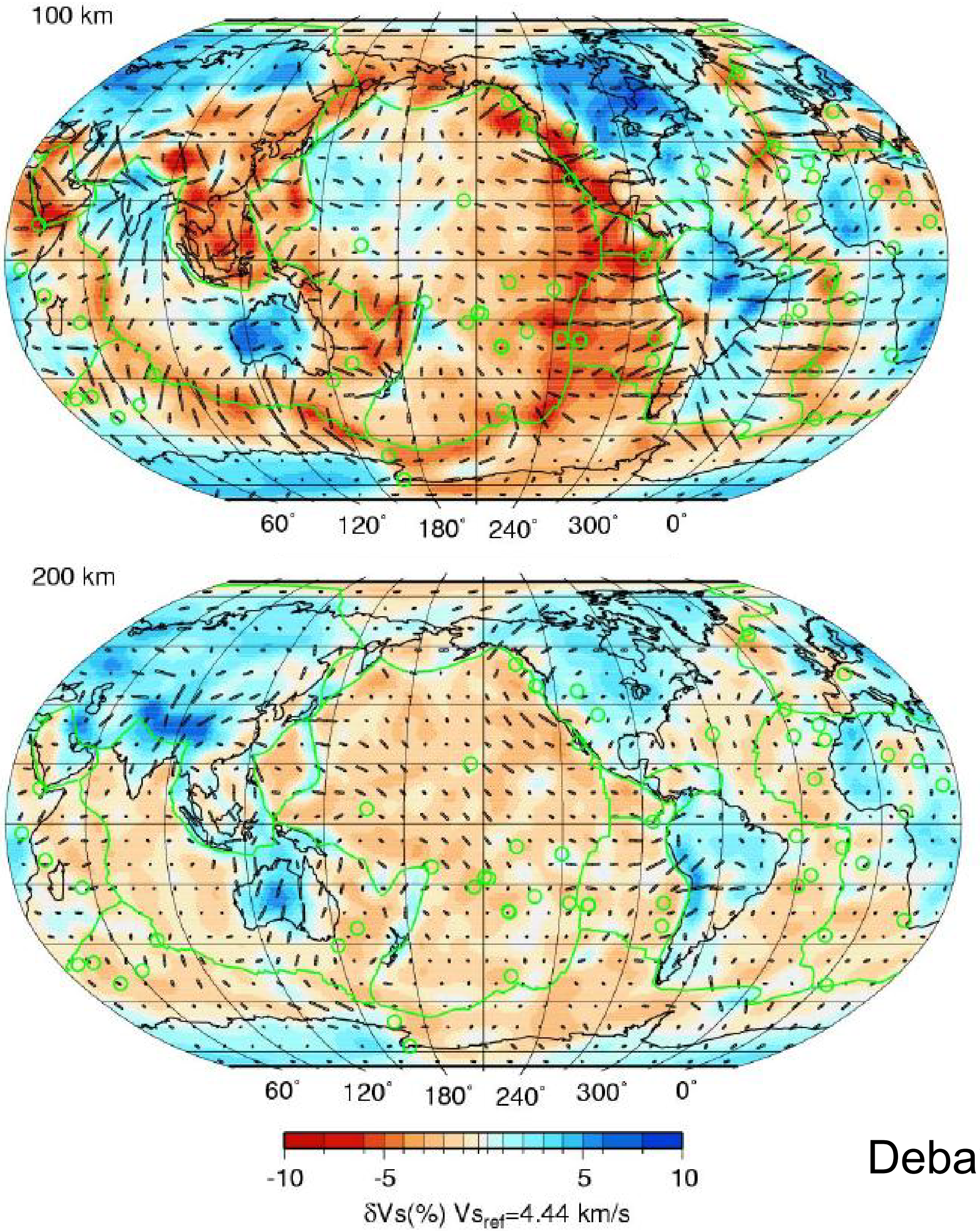
Scale $\Lambda \approx$ 2000km (degree 20)

Seismic wavelength $\lambda \leq$ 500km

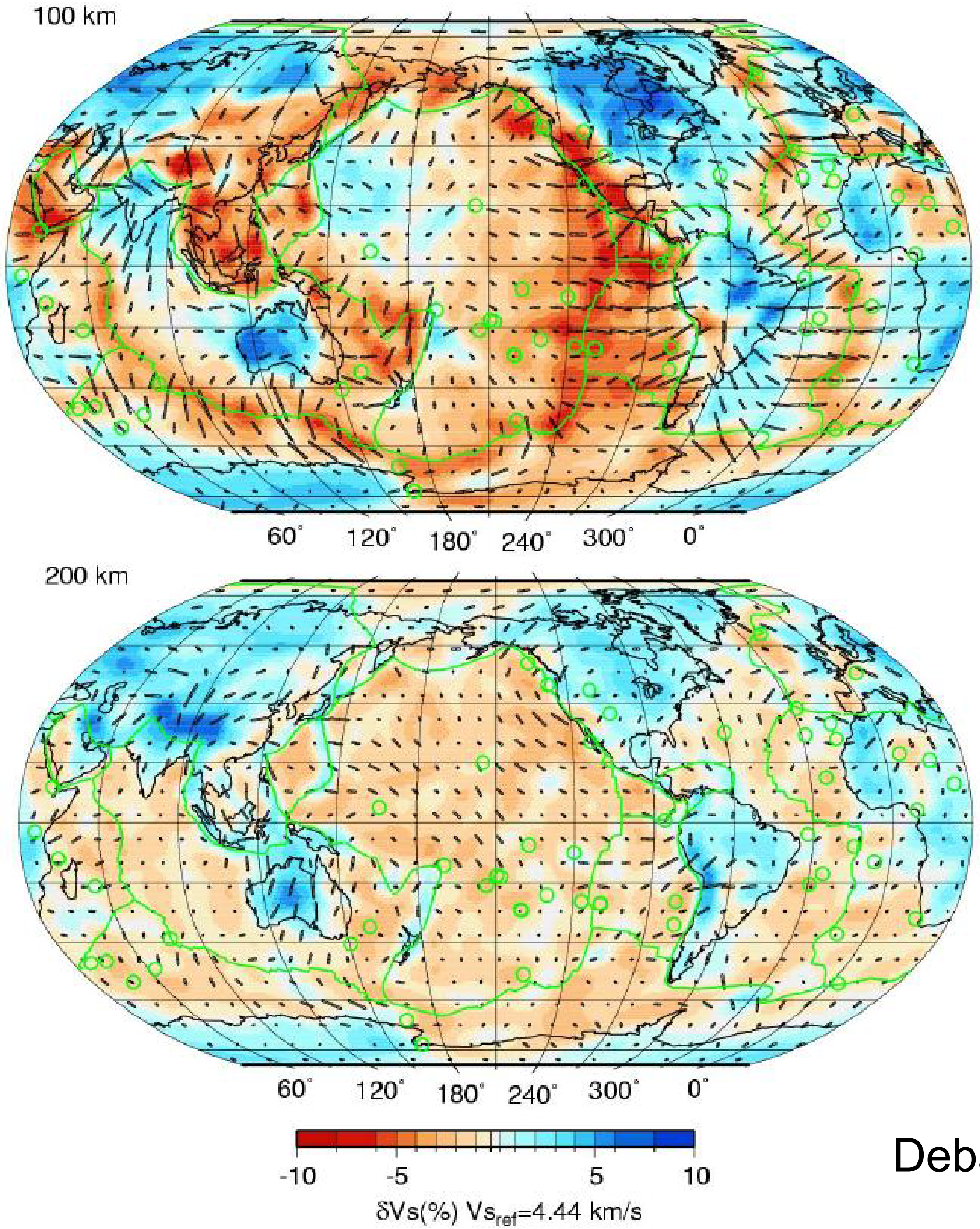
⇒ Ray theory applies

Shear wave velocities - depth = 100km





Debayle et al., 2005



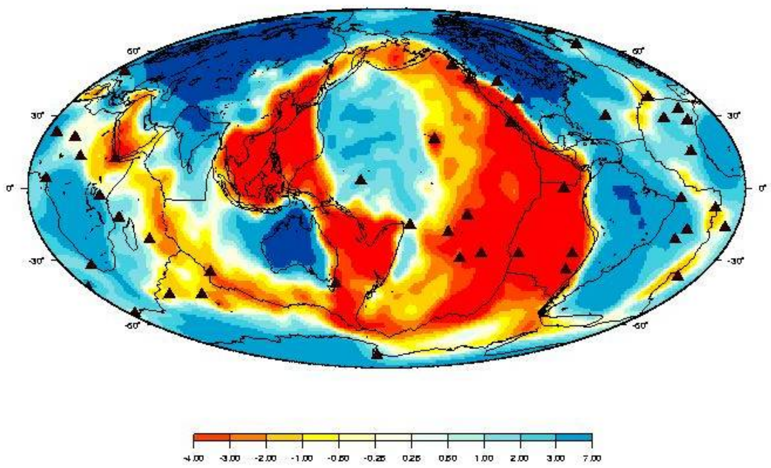
Debayle et al., 2005

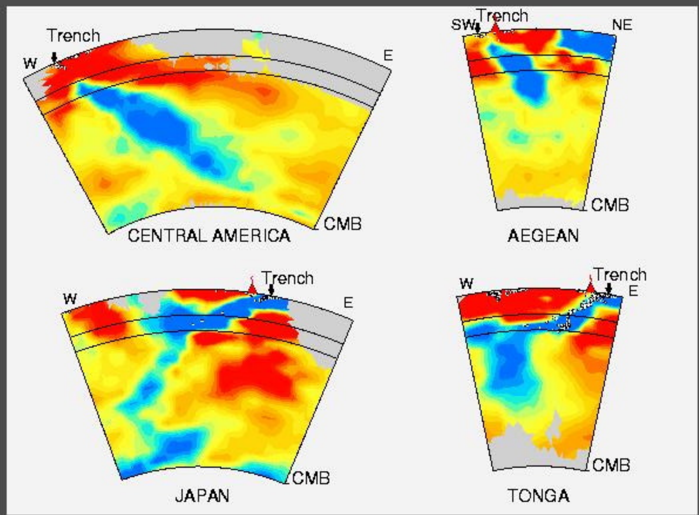
CAN WE NEGLECT SMALL-SCALE HETEROGENEITIES?

From Global scale	to Regional scale
Scale Λ >1000km	$\Lambda \approx$100km

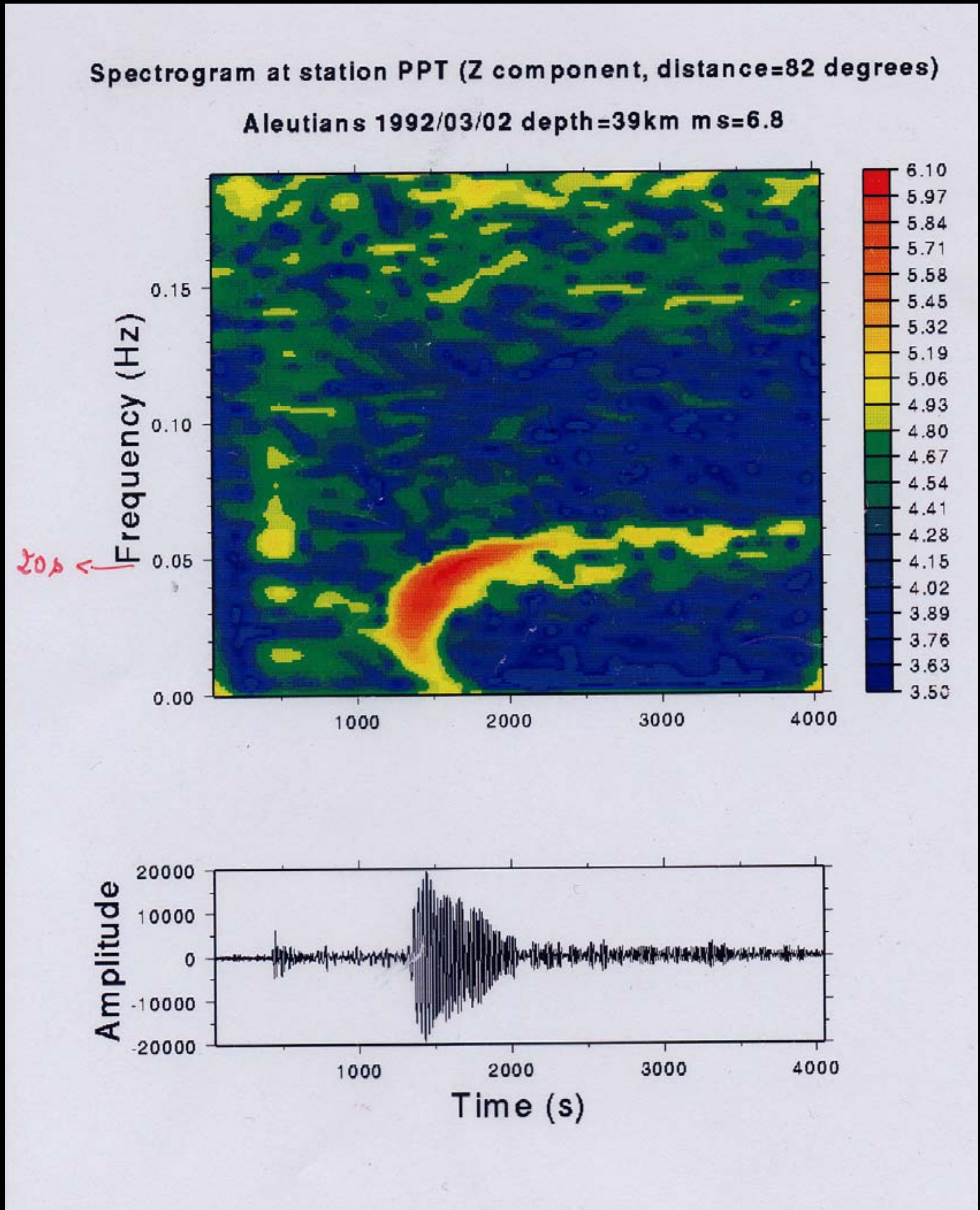
Seismic wavelength $20km \leq \lambda \leq 500km$

S-wave velocity - depth = 100km



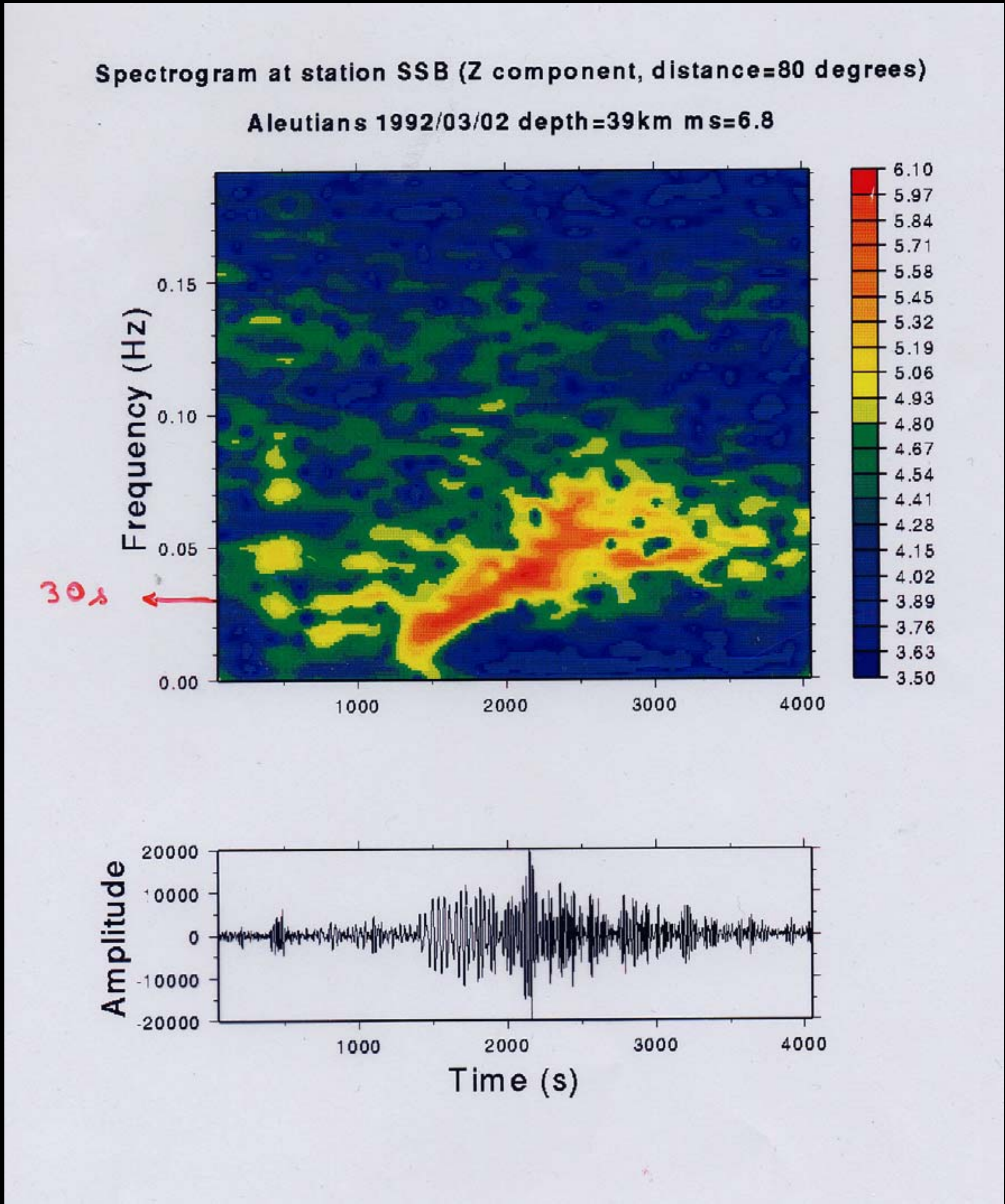


Van der Hilst et al., 1998



Oceanic
Path

simple
dispersion
Curve
Multiple scattering
For T<30s

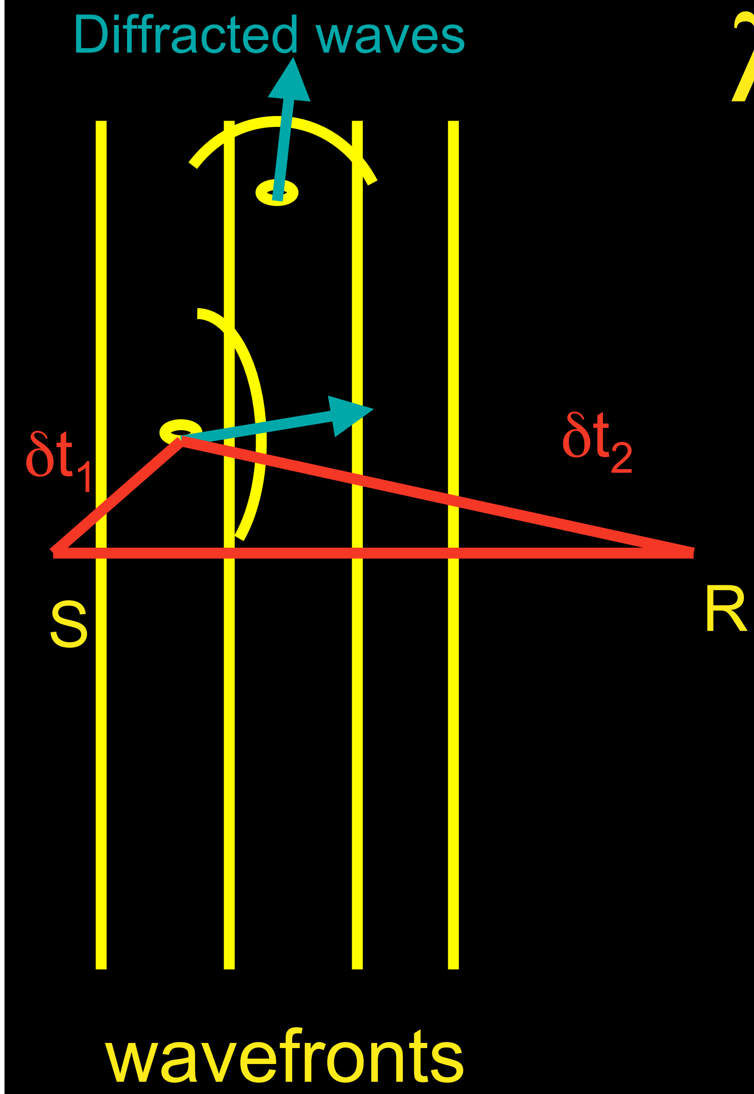


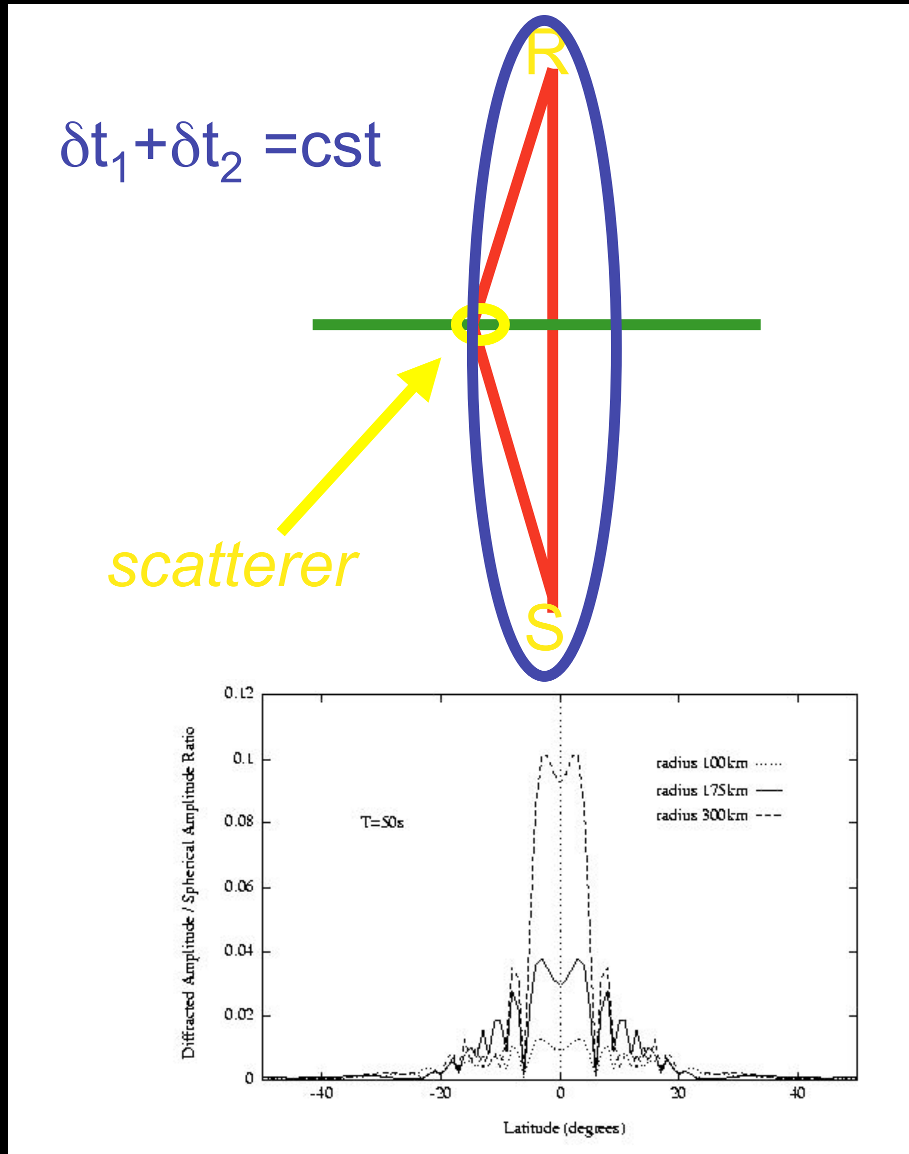
Continental
Path

complex
dispersion
Curve
Multiple scattering
For T<60s

Λ heterogeneity scale, λ wavelength

$\lambda \sim \Lambda$ or $\lambda >> \Lambda$

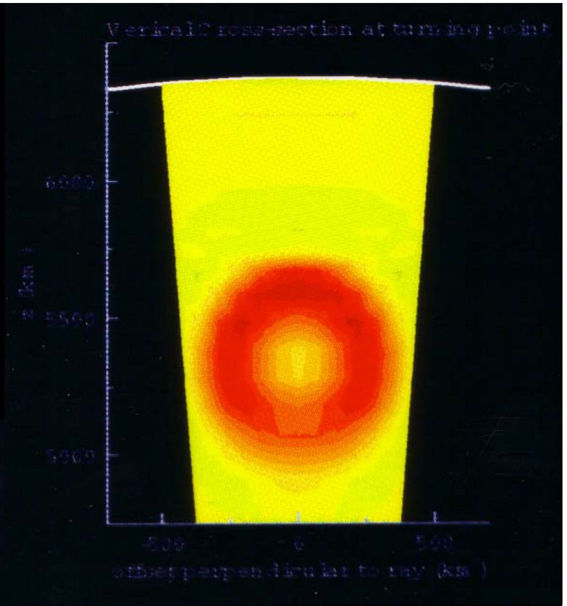


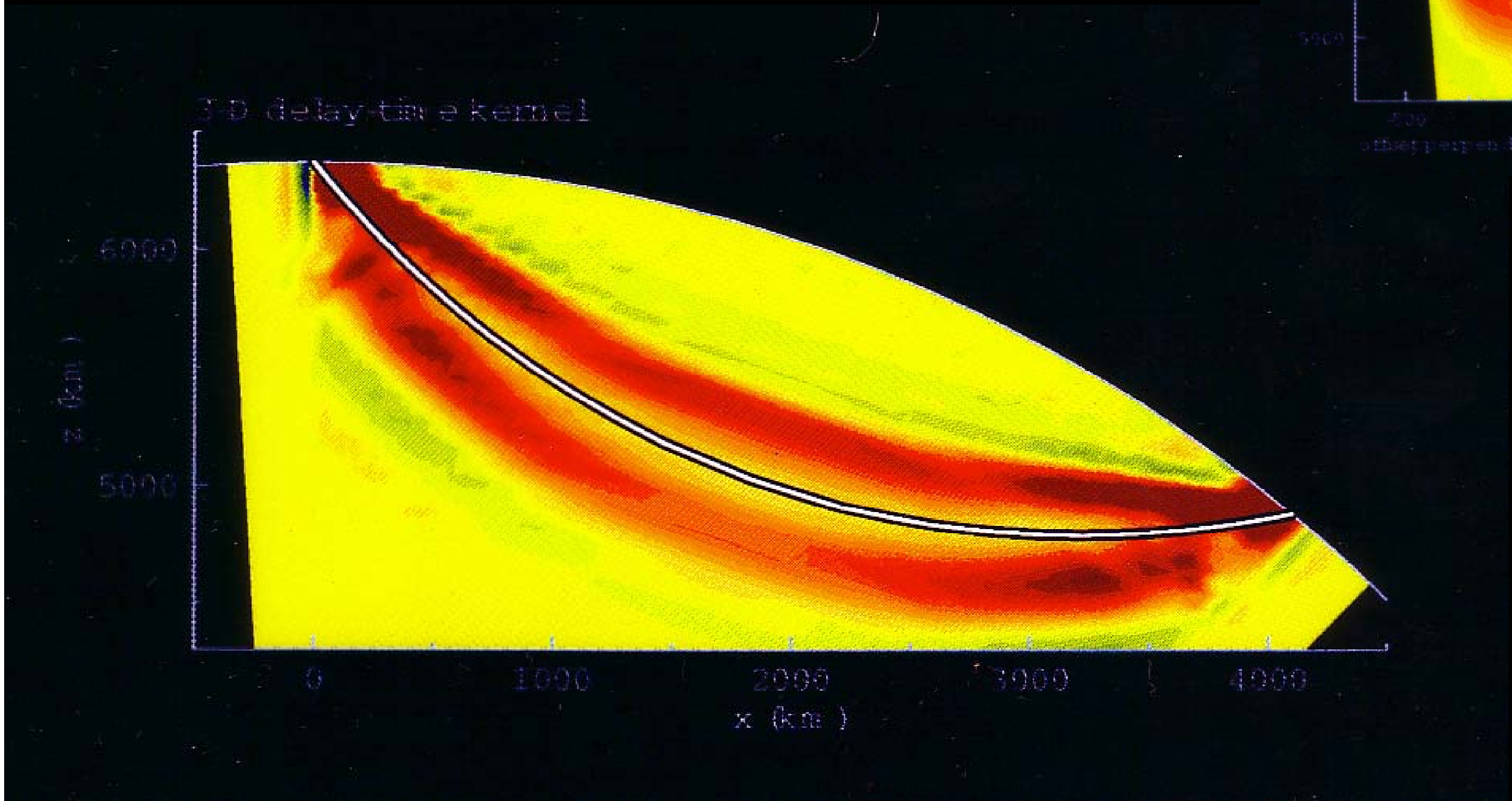


Capdeville et al., 2000

Sensitivity of the delay time to the local seismic velocity

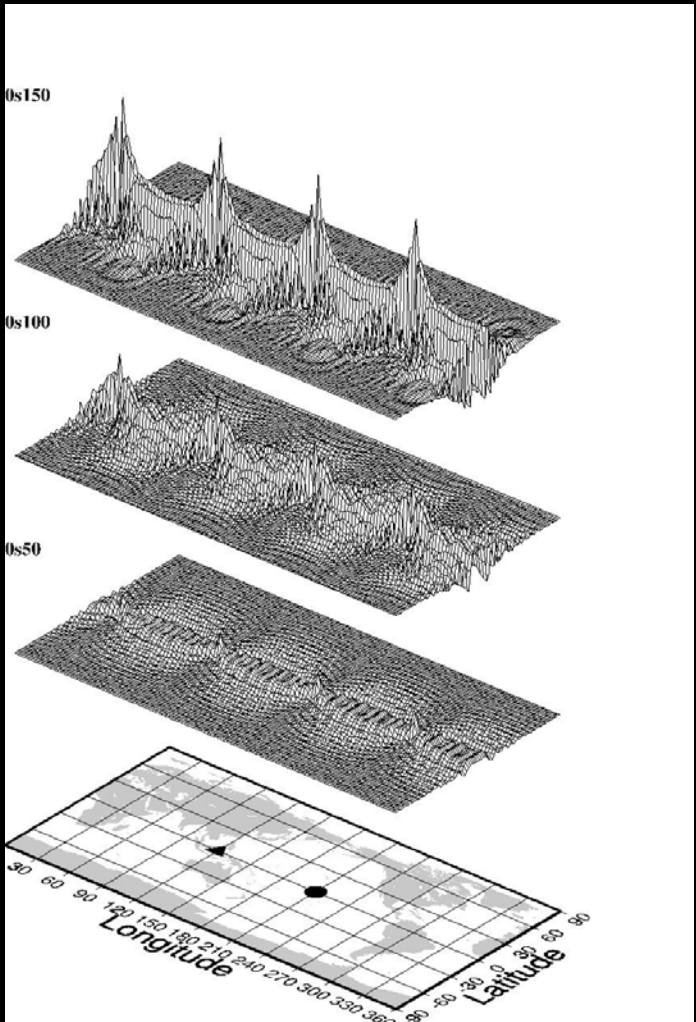
→ Princeton Group (G. Nolet)

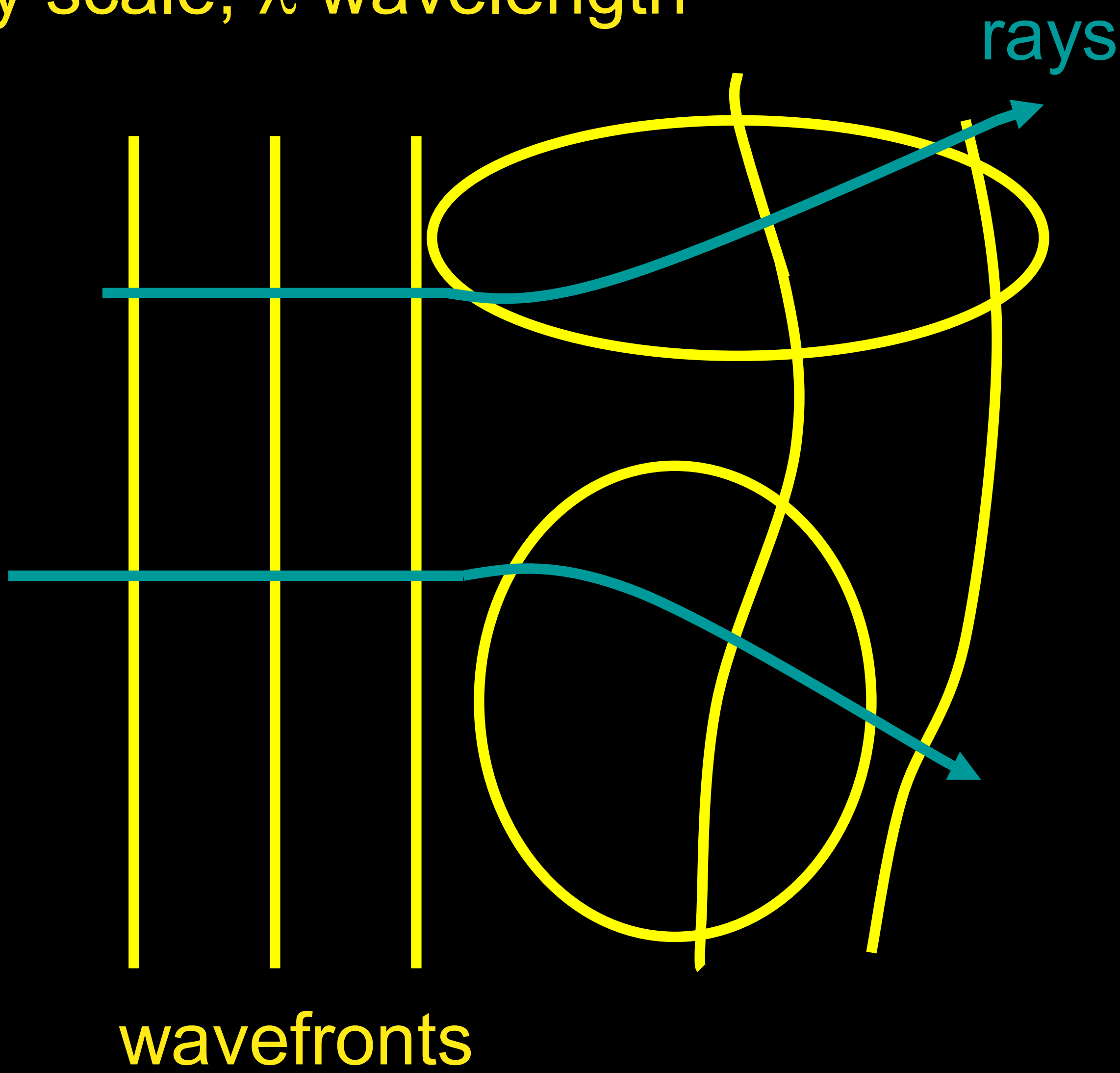




Marquering et al., GJI 1998

Λ heterogeneity scale, λ wavelength





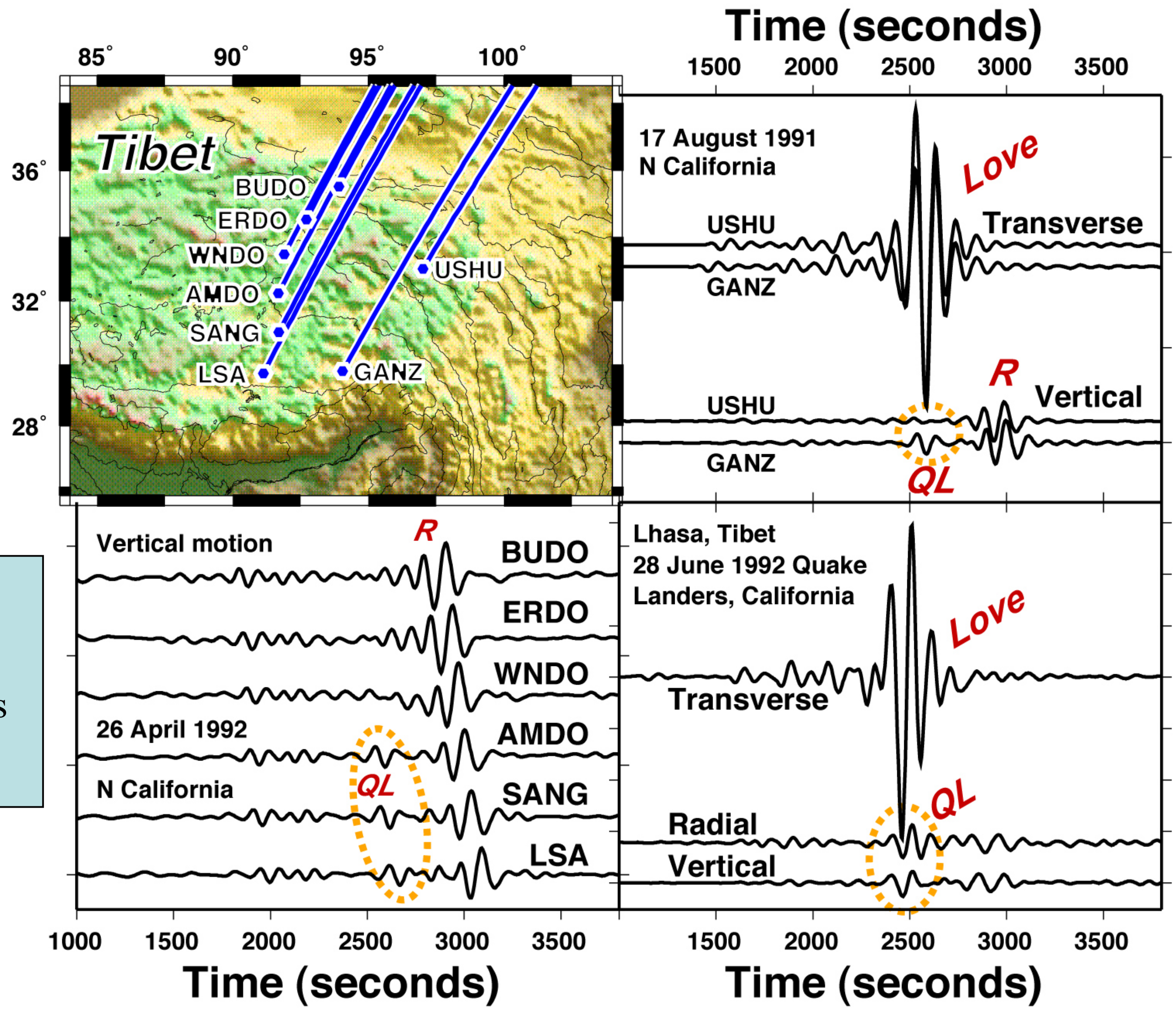
$\lambda << \Lambda$

Typical scales Λ≈2,000km, λ≈ 500km

Effects on amplitude?

- Surface wave tomography of mantle anisotropy is based on ray theory
- Finite frequency tomography has been tested to improve the imaging of the isotropic mantle
- How do finite frequency surface waves "sense" anisotropy ?
- Computational tool : Adjoint Spectral Element Method (ASEM) (Tromp et al., 2005)

An Example of
Using Quasi-Love waves
to infer anisotropic gradients
(Park, 2006)



Time Reversal- Adjoint Method

(Tarantola, 1984, Tromp et al., 2005):

Adjoint => Fréchet Derivatives