The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1965-20

9th Workshop on Three-Dimensional Modelling of Seismic Waves Generation, Propagation and their Inversion

22 September - 4 October, 2008

Parameterization of seismic source.
Determination of source parameters from seismic records.

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I. Excitation of seismic waves

The description of seismic source we will consider is based on the formalism developed by Backus and Mulcahy, 1976.

Statement of the problem.

Motion equation

$$\rho \ddot{u}_i = \sigma_{ij,j} + f_i \tag{1.1}$$

Hook's law for isotropic medium

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij} \tag{1.2}$$

Initial conditions

$$\dot{\mathbf{u}} \equiv \mathbf{u} \equiv 0, t < 0 \tag{1.3}$$

Boundary conditions

$$\sigma_{ij} n_j |_{S_0} = 0 \tag{1.4}$$

Here $\mathbf{u}$ – displacement vector; σ_{ij} – elements of symmetric 3x3 stress tensor; i,j=1,2,3 and the summation convention for repeated subscripts is used; $\sigma_{ij,j} = \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j}$; ε_{ij} – elements of symmetric 3x3 strain tensor and $\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i})$; ρ - density; f_i – components of external force; n_j – components of the normal to the free surface S_0.

Solution of the problem (1.1)-(1.4) can be given by formula

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega G_{ij}(\mathbf{x},\mathbf{y},t-\tau) f_j(\mathbf{y},\tau) dV_y \tag{1.5}$$

or

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega H_{ij}(\mathbf{x},\mathbf{y},t-\tau) \dot{f}_j(\mathbf{y},\tau) dV_y \tag{1.6}$$

Here G_{ij} is the Green's function,

$$H_{ij}(\mathbf{x},\mathbf{y},t) = \int_0^t G_{ij}(\mathbf{x},\mathbf{y},\tau) d\tau, \tag{1.7}$$

$\mathbf{x} \in \Omega$ and $0 < t < T$ are the space region and time interval where $\dot{f}$ is not identically zero.

Internal sources of seismic disturbances

We will consider internal sources only (earthquakes). In this case any external forces are absent. We must then set $\mathbf{f} \equiv 0$ in equation (1.1), so that the only solution that satisfies the homogeneous initial (1.3) and boundary (1.4) conditions, as well as Hook's law (1.2) will be $\mathbf{u} \equiv 0$. Non-zero displacements cannot arise in the medium, unless at least one of the above conditions is not true.

Following Backus and Mulcahy, 1976, we assume seismic motion to be caused by a departure from ideal elasticity (from Hook's law) within some volume of the medium Ω at some time interval $0 < t < T$.

Let $\mathbf{u}(\mathbf{x},t)$ be the actual displacements, $\boldsymbol{\sigma}(\mathbf{x},t)$ - correspondent stresses, if Hook's law is valid, $\mathbf{s}(\mathbf{x},t)$ - actual stresses.

Let the difference

$$\Gamma(\mathbf{x},t) = \boldsymbol{\sigma}(\mathbf{x},t) - \mathbf{s}(\mathbf{x},t), \tag{1.8}$$

called the *stress glut tensor* or *moment tensor density*, is not identically zero for $0 < t < T$ and $\mathbf{x} \in \Omega$.
T we define as source duration, and Ω - source region. Within this region and time interval (and only there) the tensor $\dot{\mathbf{\Gamma}}(\mathbf{x},t)$ is not identically zero as well.

Replacing $\boldsymbol{\sigma}(\mathbf{x},t)$ by $\mathbf{s}(\mathbf{x},t)$ in equation (1.1), using definition (1.8) and the absence of external forces ($\mathbf{f} \equiv 0$) we can rewrite the motion equation (1.1) in form

$$\rho \ddot{u}_i = s_{ij,j}$$

or

$$\rho \ddot{u}_i = \sigma_{ij,j} + g_i \qquad (1.9)$$

where

$$g_i = -\Gamma_{ij,j} \; . \qquad (1.10)$$

Equation (1.10) defines the equivalent force $\mathbf{g}$. Using formula (1.6) with f_i replaced by g_i , definition (1.10) and Gauss theorem we have for displacements

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega H_{ij,k}(\mathbf{x},\mathbf{y},t-\tau)\dot{\Gamma}_{jk}(\mathbf{y},\tau)dV_y \, , \qquad (1.11)$$

where H_{ij} is differentiated with respect to y_k .
If the inelastic motions are concentrated at a surface Σ, then

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Sigma H_{ij,k}(\mathbf{x},\mathbf{y},t-\tau)\dot{\Gamma}_{jk}(\mathbf{y},\tau)d\Sigma_y \, . \qquad (1.12)$$

Relation of stress glut (moment tensor density) with classic definition of moment tensor M :

$$\mathbf{M} = \int_0^T dt \int_\Omega \dot{\mathbf{\Gamma}}(\mathbf{y},t)dV_y \quad . \qquad (1.13)$$

Normalizing moment tensor we define seismic moment M_0 :

$\mathbf{M}=M_0\mathbf{m}$, where tensor $\mathbf{m}$ is normalized by condition $\mathrm{tr}(\mathbf{m}^{\mathrm{T}}\mathbf{m}) = \sum_{i,j=1}^{3} m_{ij}^2 = 2$, $\mathbf{m}^{\mathrm{T}}$ is transposed tensor $\mathbf{m}$.

If the source is localized in a point $\mathbf{x}_0$ then $\Gamma_{ij}(\mathbf{x},t) = \Gamma_{ij}(t)\delta(\mathbf{x}-\mathbf{x}_0)$ and equivalent forces g_i take view of dipoles or couples:

$$g_i = -\Gamma_{ij}(t)\frac{\partial \delta(\mathbf{x}-\mathbf{x}_0)}{\partial x_j} \, .$$

Stress glut tensor for special types of seismic sources

1. Discontinuity of displacement $\Delta\mathbf{u}$ at a surface Σ in isotropic medium (stress is continuous):

$$\begin{aligned}\Gamma_{ij}(\mathbf{x},t) &= \lambda \Delta u_k(\mathbf{x},t) n_k(\mathbf{x}) \delta_{ij} \\ &+ \mu[n_i(\mathbf{x})\Delta u_j(\mathbf{x},t) + n_j(\mathbf{x})\Delta u_i(\mathbf{x},t)].\end{aligned} \qquad (1.14)$$

Here $\mathbf{n}(\mathbf{x})$ is the normal to the surface Σ, and seismic disturbances are given by formula (1.12).

2. In the case of tangential (shear) dislocation we have
$\Delta u_k n_k \equiv 0$ and formula (1.14) takes form

$$\Gamma_{ij}(\mathbf{x},t) = \mu[n_i(\mathbf{x})\Delta u_j(\mathbf{x},t) + n_j(\mathbf{x})\Delta u_i(\mathbf{x},t)]. \qquad (1.15)$$

3. Instant point centrum of expansion (ideal explosion)
located in $\mathbf{x} = \mathbf{0}$ at $t = 0$:

$\dot{\Gamma}_{ij}(\mathbf{x},t) = \sqrt{2/3}\, M_0 \delta_{ij} \delta(t) \delta(\mathbf{x})$, where M_0 is the seismic moment.

4. Instant point tangential dislocation (double-couple) occurred in the point $\mathbf{x}=\mathbf{0}$ at time $t=0$:

$$\dot{\Gamma}_{ij}(\mathbf{x},t) = M_0 m_{ij} \delta(t) \delta(\mathbf{x}), \qquad (1.16)$$

where $m_{ij} = n_i a_j + n_j a_i$, $\mathbf{a} = \Delta\mathbf{u}/|\Delta\mathbf{u}|$ and $M_0 = \mu\, |\Delta \mathbf{u}|$.

If $\Delta\mathbf{u}(\mathbf{x}, t)$ is not an instant slip localized in a point but it is distributed in a plane area and in time, then we have

$$M_0 = \int_\Sigma \mu\, |\Delta\mathbf{u}(\mathbf{x},T)|\, d\Sigma, \quad M_{ij} = M_0 m_{ij},$$

where $\Delta\mathbf{u}(\mathbf{x}, T)$ is the final distribution of slip, and T is the source duration.

As it follows from formula (1.12) an instant point double-couple excites a displacement field of the form

$$u_i(\mathbf{x},t) = M_0 H_{ik,l}(\mathbf{x},\mathbf{0},t) m_{kl}\,. \qquad (1.17)$$

We have for Fourier transforms $\mathbf{H}(\mathbf{x},\mathbf{y},\omega)$ and $\mathbf{G}(\mathbf{x},\mathbf{y},\omega)$ from equation (1.7):

$$\mathbf{H}(\mathbf{x},\mathbf{y},\omega) = \frac{1}{\mathrm{i}\omega} \mathbf{G}(\mathbf{x},\mathbf{y},\omega), \qquad (1.18)$$

where i is the imaginary unit, and ω is angular frequency.
As result the spectrum of displacements is given by formula

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\omega} M_0 m_{kl} G_{ik,\,l}(\mathbf{x},\mathbf{0},\omega). \qquad (1.19)$$

Main phenomena of double-couple.

- Matrix **m** has zero trace.
- The eigenvalues of matrix **m** are: 1, -1 and 0. The eigenvector correspondent to 1 defines the direction of maximum extension, and the eigenvector correspondent to -1 defines the direction of maximum compression.
- As far as radiation pattern of seismic waves is concerned and double-couple is considered to be an instant point source we can interchange vectors **n** and **a** (or equivalently, the fault plane and supplementary plane which is orthogonal to **a**).

Double couple specifications

- Double couple is completely defined by moment tensor’s elements.
- Double couple can be given by seismic moment and by Cartesian coordinates of two vectors: the normal to the fault plane **n** and a unit vector in the direction of slip **a**.
- We can also specify the double couple by seismic moment and focal mechanism defined by the fault strike ψ, dip δ and rake λ angles. Sometimes rake is called slip angle.

Definition of focal mechanism angles

Strike - the angle ($0° < \psi < 360°$) between North and the trace of the fault on a horizontal plane. It is measured positively clockwise from North with the fault dipping to the right.

Dip - the angle ($0° < \delta < 90°$) the fault plane makes with a horizontal plane.

Rake - the angle ($-180° < \lambda < 180°$) between the strike axis and slip vector (the shift of the hanging wall relative to the footwall). It is measured positively counterclockwise from the strike axis.

Relation between the displacement field and stress glut moments

We assume that following product can represent the time derivative of stress glut tensor:

$$\dot{\Gamma}(\mathbf{x},t) = f(\mathbf{x},t)\mathbf{m}\,, \qquad (1.20)$$

where $f(\mathbf{x},t)$ is non-negative function and $\mathbf{m}$ is a uniform normalized moment tensor.

The moment $f^{(l,n)}_{k_1 \dots k_l}(\mathbf{q},\tau)$ of spatial degree l and temporal degree n with respect to point $\mathbf{q}$ and instant of time τ is a tensor of order l and is given by formula

$$f^{(l,n)}_{k_1 \dots k_l}(\mathbf{q},\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_{k_1} - q_{k_1}) \cdots (x_{k_l} - q_{k_l})(t-\tau)^n \, dt\,, \qquad (1.21)$$

$k_1,\dots,k_l$=1,2,3.

Replacing $H_{ij}(\mathbf{x},\mathbf{y},t\text{-}\tau)$ in equation (1.11) by its Taylor series in powers of $\mathbf{y}$ and in powers of τ, we get:

$$u_i(\mathbf{x},t) = \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^n}{l!\,n!} m_{jk} f^{(l,n)}_{k_1 \dots k_l}(\mathbf{0},0) \frac{\partial^n}{\partial t^n} \frac{\partial}{\partial y_{k_1}} \cdots \frac{\partial}{\partial y_{k_l}} \frac{\partial}{\partial y_k} H_{ij}(\mathbf{x},\mathbf{y},t)\Big|_{\mathbf{y}=\mathbf{0}} \,. \qquad (1.22)$$

Using formulae (1.18) and (1.22) we have following equation for the spectrum of displacements:

$$u_i(\mathbf{x},\omega) = \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^n}{l!\,n!} m_{jk} f^{(l,n)}_{k_1 \dots k_l}(\mathbf{0},0)(\mathrm{i}\,\omega)^{n-1} \frac{\partial}{\partial y_{k_1}} \cdots \frac{\partial}{\partial y_{k_l}} \frac{\partial}{\partial y_k} G_{ij}(\mathbf{x},\mathbf{y},\omega)\Big|_{\mathbf{y}=\mathbf{0}} \,. \qquad (1.23)$$

Here we assume that the point $\mathbf{y} = \mathbf{0}$ and the instant $t = 0$ belong to the source region and the time of the source activity respectively.

When the spectra of displacements $u_i(\mathbf{x},\omega)$ and Green's function $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ have been low pass filtered, the terms in equation (1.23) start to decrease with l and n increasing at least as rapidly as $(\omega T)^{l+n}$ (T is the source duration, and ωT<1), and one might then restrict to considering finite sums only.

We will take into account in the following sections only the first terms in formula (1.23) for $l + n \le 2$.

II. Source inversion in moment tensor approximation

The first term in (1.23) corresponding to $l = 0$, $n = 0$, describes the spectra of displacements $u_i(\mathbf{x},\omega)$ excited by an instant point source (compare with formula (1.19) taking into account that seismic moment is equal to zero moment of function $f(\mathbf{x},t)$: $M_0 = f^{(0,0)}$). For a source with nonzero size and duration this term approximates $u_i(\mathbf{x},\omega)$ with high accuracy for periods much longer then source duration. Performing the inversion of long period seismic waves we describe the earthquake by an instant point source. As it was mentioned in previous section, an instant point source can be given by moment tensor - a symmetric 3x3 matrix $\mathbf{M}$. Seismic moment M_0 is defined by equation $M_0 = \sqrt{\tfrac{1}{2}\mathrm{tr}(\mathbf{M}^{\mathrm{T}}\mathbf{M})}$, where $\mathbf{M}^{\mathrm{T}}$ is transposed moment tensor $\mathbf{M}$, and $\mathrm{tr}(\mathbf{M}^{\mathrm{T}}\mathbf{M}) = \sum_{i,j=1}^{3} M_{ij}^{\;2}$. Moment tensor of any event can be presented in the form $\mathbf{M} = M_0\mathbf{m}$, where matrix $\mathbf{m}$ is normalized by condition $\mathrm{tr}(\mathbf{m}^{\mathrm{T}}\mathbf{m}) = 2$.

We'll consider a double-couple instant point source (a pure tangential dislocation) at a depth h. Such a source can be given by 5 parameters: double-couple depth, its focal mechanism which is characterizing by three angles: strike, dip and slip or by two unit vectors (direction of principal tension **T** and direction of principal compression **P**) and seismic moment M_0. Four of these parameters we determine by a systematic exploration of the four dimensional parametric space, and the 5-th parameter M_0 - solving the problem of minimization of the misfit between observed and calculated surface wave amplitude spectra for every current combination of all other parameters.

Under assumptions mentioned above the relation between the spectrum of displacements $u_i(\mathbf{x},\omega)$ and moment tensor $\mathbf{M}$ can be expressed by formula (1.19) rewritten below in slightly different form:

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\,\omega}[M_{jl}\frac{\partial}{\partial y_l}G_{ij}(\mathbf{x},\mathbf{y},\omega)] \tag{2.1}$$

i,j = 1,2,3 and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ in equation (2.1) is the spectrum of Green function for the chosen model of medium and wave type (see Levshin, 1985; Bukchin, 1990), $\mathbf{y}$ - source location. We will discuss the inversion of surface wave spectra, so $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ is the spectrum of surface wave Green function. We assume that the paths from the earthquake source to seismic stations are relatively simple and are well approximated by weak laterally inhomogeneous model (Woodhouse, 1974; Babich *et al.*, 1976). The surface wave Green function in this approximation is determined by the near source and near receiver velocity structure, by the mean phase velocity of wave, and by geometrical spreading. We assume that waves propagate from the source to station along great circles. Under these assumptions the amplitude spectrum $|u_i(\mathbf{x},\omega)|$ defined by formula (2.1) does not depend on the average phase velocity of the wave. In such a model the errors in source location do not affect the amplitude spectrum (Bukchin, 1990). The average phase velocities of surface waves are usually not well known. For this reason as a rule we use only amplitude spectra of surface waves for determining source parameters under consideration. We use observed surface wave phase spectra only for very long periods. Correcting the spectra for attenuation we use laterally homogeneous model for quality factor.

Surface wave amplitude spectra inversion

If all characteristics of the medium are known, the representation (2.1) gives us a system of equations for parameters defined above. Let us consider now a grid in the space of these 4 parameters. Let the models of the media be given. Using formula (2.1) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(\mathbf{x},\omega)$ be any observed value of the spectrum, i = 1,…,N; $\varepsilon^{(i)}_{\mathrm{amp}}$- corresponding residual of $|u^{(i)}(\mathbf{x},\omega)|$. We define the normalized amplitude residual by formula

$$\varepsilon_{\mathrm{amp}}(h,\mathbf{T},\mathbf{P}) = \left[\left(\sum_{i=1}^{\mathrm{N}} \varepsilon^{(i)\,2}_{\mathrm{amp}}\right)\Big/\left(\sum_{i=1}^{\mathrm{N}} |u^{(i)}(\mathbf{x},\omega\,|^2\right)\right]^{1/2}. \tag{2.2}$$

The optimal values of the parameters that minimize $\varepsilon_{\mathrm{amp}}$ we consider as estimates of these parameters. We search them by a systematic exploration of the four-dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions. Fixing the value of one of varying parameters we put in correspondence to it a minimal value of the residual $\varepsilon_{\mathrm{amp}}$ on the set of all possible values of the other parameters. In this way we define one residual function on scalar argument and two residual functions on vector argument corresponding to the scalar and two vector varying parameters: $\varepsilon_h(h)$, $\varepsilon_{\mathbf{T}}(\mathbf{T})$ and $\varepsilon_{\mathbf{P}}(\mathbf{P})$. The value of the parameter for which the

corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters. From geometrical point of view these functions describe the lower boundaries of projections of the 4-D surface of functional ε on the coordinate planes. A sketch illustrating the definition of partial residual functions is given in figure 1.
Here one of 4 parameters is picked out as 'parameter 1', and one of coordinate axis corresponds to this parameter. Another coordinate axis we consider formally as 3-D space of the rest 3 parameters. Plane Σ is orthogonal to the axis 'parameter 1' and cross it in a point p_0. Curve L is the intersection of the plane Σ and the surface of functional ε. As one can see from the figure the point $\varepsilon_1(p_0)$ belong to the boundary of projection of the surface of functional ε, and at the same time it corresponds to a minimal value of the residual ε on the set of all possible values of the other 3 parameters while 'parameter 1' is equal to the value p_0.
So, as it is accepted in engineering we characterize our surface by its 4 projections on coordinate planes.

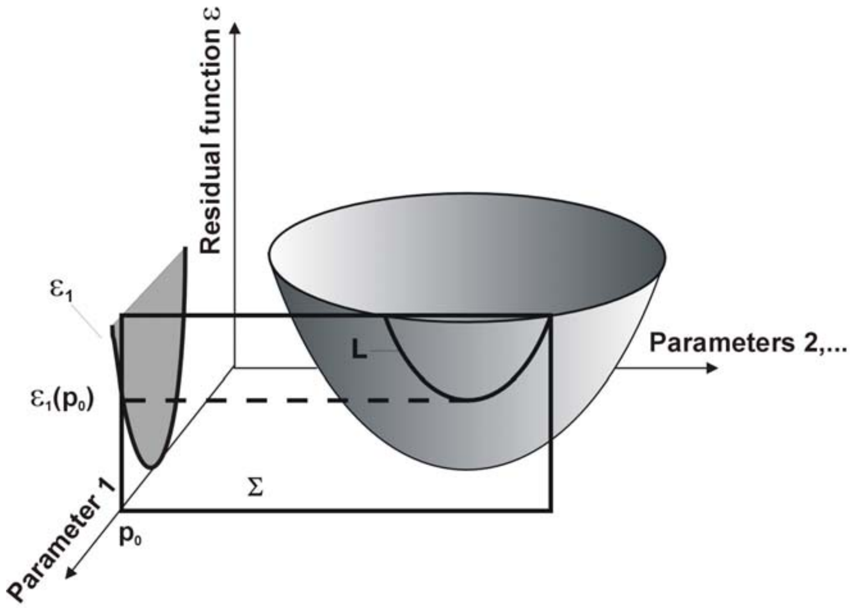

Figure 1. A sketch illustrating the definition of partial residual functions.

It is well known that the focal mechanism cannot be uniquely determined from surface wave amplitude spectra. There are four different focal mechanisms radiating the same surface wave amplitude spectra. These four equivalent solutions represent two pairs of mechanisms symmetric with respect to the vertical axis, and within the pair differ from each other by the opposite direction of slip.

To get a unique solution for the focal mechanism we have to use in the inversion additional observations. For these purpose we use very long period phase spectra of surface waves or polarities of P wave first arrivals.

Joint inversion of surface wave amplitude and phase spectra

Using formula (2.1) we can calculate for chosen frequency range the phase spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed phase spectra give us a residual

$\varepsilon^{(i)}_{\mathrm{ph}}$ for every point of observation, every wave and every frequency ω. We define the normalized phase residual by formula

$$\varepsilon_{\mathrm{ph}}(h,\varphi,\mathbf{T},\mathbf{P}) = \frac{1}{\pi}\left[\left(\sum_{i=1}^{\mathrm{N}} \varepsilon^{(i)\,2}_{\mathrm{ph}}\right) / \mathrm{N}\right]^{1/2} . \qquad (2.3)$$

We determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_{\mathrm{ph}})(1 - \varepsilon_{\mathrm{amp}}) . \qquad (2.4)$$

To characterize the resolution of source characteristics we calculate partial residual functions in the same way as was described above.

Joint inversion of surface wave amplitude spectra and P wave polarities

Calculating radiation pattern of P waves for every current combination of parameters we compare it with observed polarities. The misfit obtained from this comparison we use to calculate a joint residual of surface wave amplitude spectra and polarities of P wave first arrivals. Let $\varepsilon_{\mathrm{amp}}$ be the residual of surface wave amplitude spectra, ε_{p} - the residual of P wave first arrival polarities (the number of wrong polarities divided by the full number of observed polarities), then we determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_{\mathrm{p}})(1 - \varepsilon_{\mathrm{amp}}) . \qquad (2.5)$$

For this type of inversion we calculate partial residual functions to characterize the resolution of parameters under determination in the same way as it was described for two first types.

Before inversion we apply to observed polarities a smoothing procedure (see Lasserre *et al.*, 2001), which we will describe here briefly.

Let us consider a group of observed polarities (+1 for compression and -1 for dilatation) radiated in directions deviating from any medium one by a small angle. This group is presented in the inversion procedure by one polarity prescribing to this medium direction. If the number of one of two types of polarities from this group is significantly larger then the number of opposite polarities, then we prescribe this polarity to this medium direction. If no one of two polarity types can be considered as preferable, then all these polarities will not be used in the inversion. To make a decision for any group of *n* observed polarities we calculate the sum $m = n_+ - n_-$, where n_+ is the number of compressions and $n_- = n - n_+$ is the number of dilatations. We consider one of polarity types as preferable if $|m|$ is larger then its standard deviation in the case when +1 and -1 appear randomly with this same probability 0.5. In this case n_+ is a random value distributed following the binomial low. For its average we have $M(n_+) = 0.5n$, and for dispersion $D(n_+) = 0.25n$. Random value m is a linear function of n_+ such that $m = 2n_+ - n$. So following equations are valid for the average, for the dispersion, and for the standard deviation σ of value m

$$M(m) = 2M(n_+) - n = n - n = 0, \quad D(m) = 4D(n_+) = n, \quad \text{and} \quad \sigma(m) = \sqrt{n} .$$

As a result, if the inequality $|m| \geq \sqrt{n}$ is valid then we prescribe +1 to the medium direction if $m > 0$, and -1 if $m < 0$.

Example of application

We illustrate the technique by results of its application for a study of large Tarapaca earthquakes in Chile, M_{w} = 7.8 occurred on 13 June 2005. To estimate the best double-couple we have used spectra of fundamental Love and Rayleigh modes in spectral range from 160 to 250 seconds. The records were processed by frequency-time and polarization analysis package. We selected 11 Love wave records and 14 Rayleigh wave records from IRIS and GEOSCOPE stations. Their distribution of stations is given in figure 2.

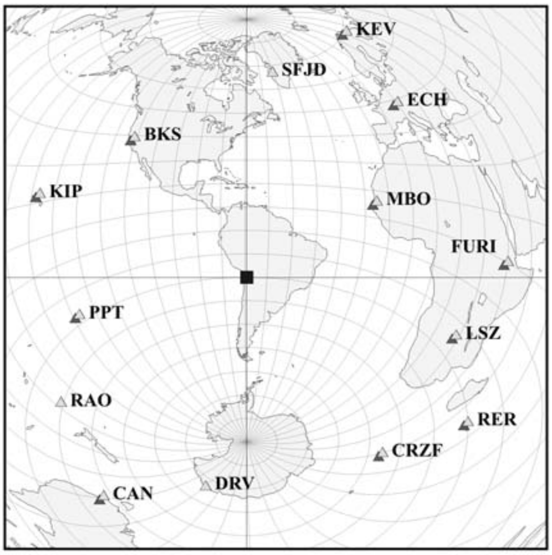

Fig.2. Distribution of stations used for moment tensor and source depth determination from Love and Rayleigh fundamental mode spectra in period band from 160 to 250 seconds. ▲ - Love waves used for the inversion, △ - Rayleigh waves used for the inversion.

The solution gives a mechanism described by the following values of strike, dip and slip for two nodal planes: P1: 192˚, 22˚, -64˚; P2: 345˚, 70˚, -100˚. The stereographic projection of this focal mechanism is given in figure 3 with first arrival polarities superimposed. The estimated value of seismic moment is $0.54 \cdot 10^{21}$ N·m. The estimate of source depth is equal to 80 km. The resolution of the source depth is illustrated by the residual curve given in figure 4.

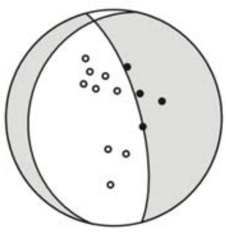

Fig. 3.The best solution for joint inversion of surface wave amplitude spectra and first arrival polarities. $M_0 = 0.54 \cdot 10^{21}$ N·m. P1: 192˚, 22˚, -64˚; P2: 345˚, 70˚, -100˚.

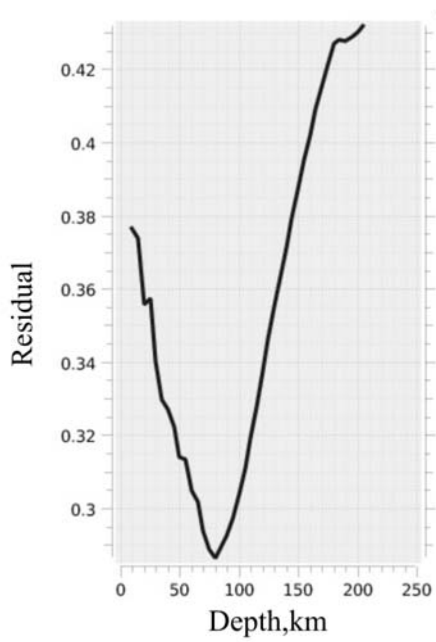

Fig. 4. Residual as function of source depth

The resolution of the focal mechanism is illustrated by the residual maps $\varepsilon_{\mathbf{T}}(\mathbf{T})$ and $\varepsilon_{\mathbf{P}}(\mathbf{P})$ given in figure 5.

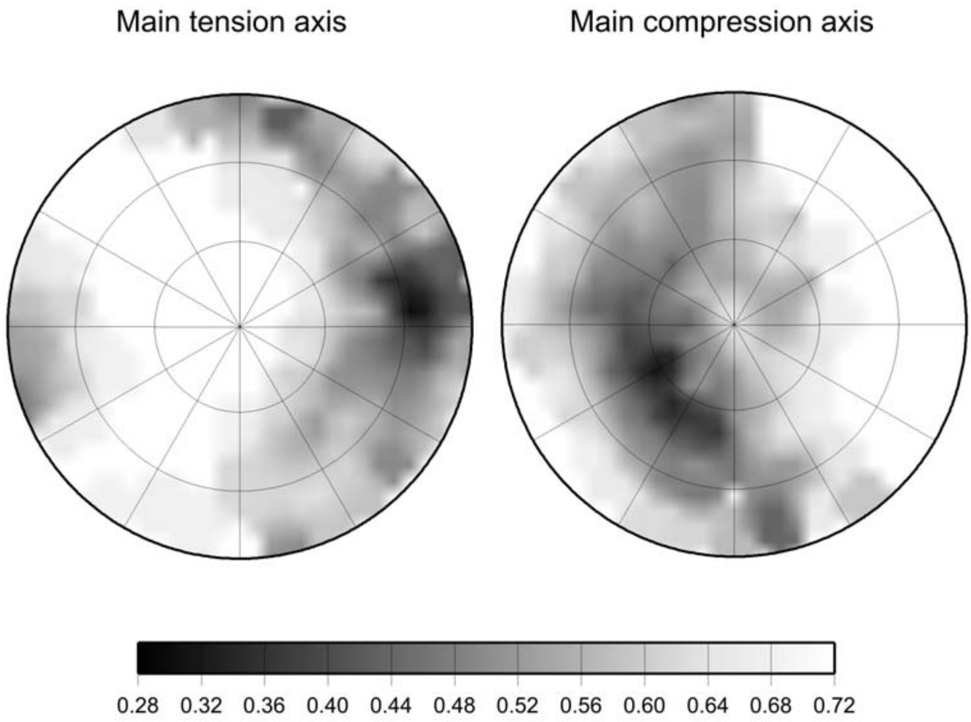

Fig.5. Joint residual of surface wave amplitude spectra and P wave polarities for main stress axes.

III. Second moments approximation. Characteristics of source shape and evolution in time.

We present here a technique based on the description of seismic source distribution in space and in time by integral moments (see Bukchin *et al.*, 1994; Bukchin, 1995; Gomez, 1997 a, b). We assume that the time derivative of stress glut tensor $\dot{\mathbf{\Gamma}}$ can be represented in form (1.20). Following Backus and Mulcahy, 1976 we will define the source region by the condition that function $f(\mathbf{x},t)$ is not identically zero and the source duration is the time during which nonelastic motion occurs at various points within the source region, i.e., $f(\mathbf{x},t)$ is different from zero.

Spatial and temporal integral characteristics of the source can be expressed by corresponding moments of the function $f(\mathbf{x},t)$ (Backus, 1977a; Bukchin *et al.*, 1994). These moments can be estimated from the seismic records using the relation between them and the displacements in seismic waves, which we will consider later. In general case stress glut rate moments of spatial degree 2 and higher are not uniquely determined by the displacement field (Pavlov, 1994; Das & Kostrov, 1997). But in the case when equation (1.20) is valid such uniqueness takes place (Backus, 1977b; Bukchin, 1995).

Following equations define the spatio-temporal moments of function $f(\mathbf{x},t)$ of total degree (both in space and time) 0, 1, and 2 with respect to point $\mathbf{q}$ and instant of time τ.

$$f^{(0,0)} = \int_V dV \int_0^\infty f(\mathbf{x},t)dt\,, \quad f_i^{(1,0)}(\mathbf{q}) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)dt\,,$$

$$f^{(0,1)}(\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(t-\tau)dt\,, \quad f^{(0,2)}(\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(t-\tau)^2 dt\,,$$

$$f_i^{(1,1)}(\mathbf{q},\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)(t-\tau)dt\,, \tag{3.1}$$

$$f_{ij}^{(2,0)}(\mathbf{q}) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)(x_j - q_j)dt$$

Using these moments we will define integral characteristics of the source. Source location is estimated by the spatial centroid $\mathbf{q}_\text{c}$ of the field $f(\mathbf{x},t)$ defined as

$$\mathbf{q}_\text{c} = \mathbf{f}^{(1,0)}(\mathbf{0}) / M_0\,, \tag{3.2}$$

where $M_0 = f^{(0,0)}$ is the scalar seismic moment.

Similarly, the temporal centroid τ_c is estimated by the formula

$$\tau_\text{c} = f^{(0,1)}(0) / M_0\,. \tag{3.3}$$

The source duration is Δt estimated by $2\,\Delta\,\tau$, where

$$(\Delta\tau)^2 = f^{(0,2)}(\tau_\text{c}) / M_0\,. \tag{3.4}$$

The spatial extent of the source is described by matrix **W**,

$$\mathbf{W} = \mathbf{f}^{(2,0)}(\mathbf{q}_\text{c}) / M_0\,. \tag{3.5}$$

The mean source size in the direction of unit vector **r** is estimated by value $2l_r$, defined by formula

$$l_r^2 = \mathbf{r}^\text{T}\mathbf{W}\mathbf{r}\,, \tag{3.6}$$

where $\mathbf{r}^\text{T}$ is the transposed vector. From (3.5) and (3.6) we can estimate the principal axes of the source. There directions are given by the eigenvectors of the matrix **W**, and the lengths are defined by correspondent eigenvalues: the length of the minor semi-axis is equal to the least eigenvalue, and the length of the major semi-axis is equal to the greatest eigenvalue.

In the same way, from the coupled space time moment of order (1,1) the mean velocity **v** of the instant spatial centroid (Bukchin, 1989) is estimated as

$$\mathbf{v} = \mathbf{w} / (\Delta\tau)^2\,, \tag{3.7}$$

where $\mathbf{w} = \mathbf{f}^{(1,1)}(\mathbf{q}_\text{c}, \tau_\text{c}) / M_0$.

The relation between integral estimates and real characteristics of source duration and spatial extent depends on the distribution of moment rate density in time and over the fault. Figure 6 illustrates this relation in the case of Gaussian distributions. In this case 99% confidence duration is 2.5 times larger then the integral estimate, and 99% confidence axis length is 3 times larger then correspondent integral estimate.

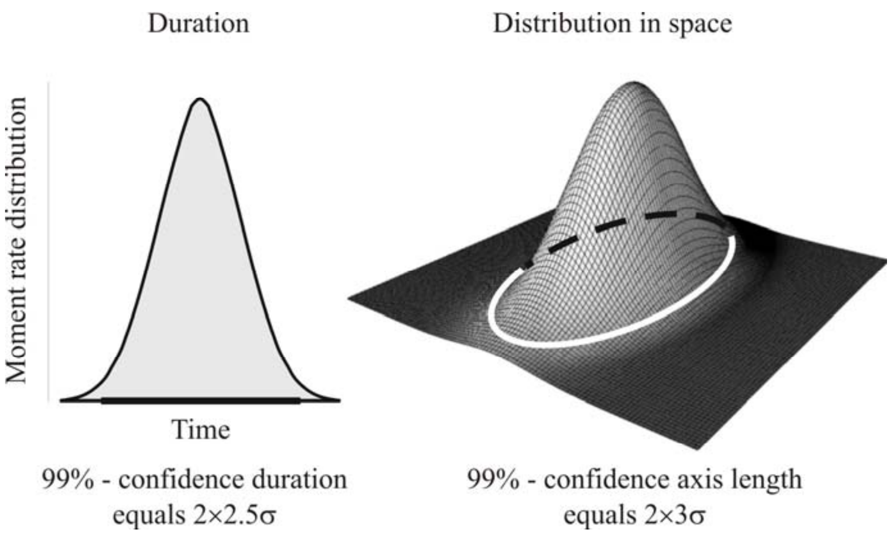

Fig. 6. Relation between integral estimates and real characteristics of source duration and spatial extent.

Now we will consider the low frequency part of the spectra of the i^{th} component of displacements in Love or Rayleigh wave $u_i(\mathbf{x},\omega)$. It is assumed that the frequency ω is small, so that the duration of the source is small in comparison with the period of the wave, and the source size is small as compared with the wavelength. It is assumed that the origin of coordinate system is located in the point of spatial centroid $\mathbf{q}_c$ (i.e. $\mathbf{q}_c = \mathbf{0}$) and that time is measured from the instant of temporal centroid, so that $\tau_c = 0$. With this choice the first degree moments with respect to the spatial origin $\mathbf{x} = \mathbf{0}$ and to the temporal origin t = 0 are zero, i.e. $\mathbf{f}^{(1,0)}(\mathbf{0}) = \mathbf{0}$ and $f^{(0,1)}(0) = 0$.

Under this assumptions, taking into account in formula (1.23) only the first terms for $l + n \le 2$ we can express the relation between the spectrum of displacements $u_i(\mathbf{x},\omega)$ and the spatio-temporal moments of the function $f(\mathbf{x},t)$ by following formula (Bukchin,1995)

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\omega} M_0 M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) + \frac{1}{2\mathrm{i}\omega} f_{mn}^{(2,0)}(\mathbf{0}) M_{jl} \frac{\partial}{\partial y_m}\frac{\partial}{\partial y_n}\frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega)$$

$$- f_m^{(1,1)}(\mathbf{0},0) M_{jl} \frac{\partial}{\partial y_m}\frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) + \frac{\mathrm{i}\omega}{2} f^{(0,2)}(0) M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega)\,, \qquad (3.8)$$

i,j,l,m,n = 1,2,3 and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ in equation (3.8) is the spectrum of Green function for the chosen model of medium and wave type. We assume that the paths from the earthquake source to seismic stations are well approximated by weak laterally inhomogeneous model. Under this assumption, as it was mentioned above, the amplitude spectrum $|u_i(\mathbf{x},\omega)|$ defined by formula (3.8) does not depend on the average phase velocity of the wave, and the errors in source location do not affect the amplitude spectrum.

If all characteristics of the medium, depth of the best point source and seismic moment tensor are known (determined, for example, using the spectral domain of longer periods) the representation (3.8) gives us a system of linear equations for moments of the function $f(\mathbf{x},t)$ of total degree 2. But as we mentioned considering moment tensor approximation the average phase velocities of surface waves are usually not well known. For this reason, we use only amplitude spectrum of surface waves for determining these moments, in spite of non-linear relation between them.

Let us consider a plane source. All moments of the function $f(\mathbf{x},t)$ of total degree 2 can be expressed in this case by formulas (3.2)-(3.7) in terms of 6 parameters: Δt - estimate of source duration, l_{max} - estimate of maximal mean size of the source, φ_l - estimate of the angle between the direction of maximal size and strike axis, l_{min} - estimate of minimal mean size of the source, v - estimate of the absolute value of instant centroid mean velocity $\mathbf{v}$ and φ_v - the angle between $\mathbf{v}$ and strike axis.

Using the Bessel inequality for the moments under discussion we can obtain the following constrain for the parameters considered above (Bukchin, 1995):

$$v^2 \Delta t^2 \left(\frac{\cos^2 \varphi}{l_{max}^2} + \frac{\sin^2 \varphi}{l_{min}^2} \right) \leq 1 , \qquad (3.9)$$

where φ is the angle between major axis of the source and direction of $\mathbf{v}$.

Assuming that the source is a plane fault and representation (1.20) is valid let us consider a rough grid in the space of 6 parameters defined above. These parameters have to follow inequality (3.9). Let models of the media be given and the moment tensor be fixed as well as the depth of the best point source. Let the fault plane (one of two nodal planes) be identified. Using formula (3.8) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(\mathbf{r},\omega)$ be any observed value of the spectrum, i = 1,…,N; $\varepsilon^{(i)}$ - corresponding residual of $|u^{(i)}(\mathbf{r},\omega)|$. We define the normalized amplitude residual by formula

$$\varepsilon(\Delta t, l_{max}, l_{min}, \varphi_l, v, \varphi_v) = \left[\left(\sum_{i=1}^{N} \varepsilon^{(i)2} \right) \Big/ \left(\sum_{i=1}^{N} | u^{(i)}(\mathbf{r}, \omega |^2 \right) \right]^{1/2} . \qquad (3.10)$$

The optimal values of the parameters that minimize ε we consider as estimates of these parameters. We search them by a systematic exploration of the six dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions in the same way as was described in previous section. We define 6 functions of the residual corresponding to the 6 varying parameters: $\varepsilon_{\Delta t}(\Delta t)$, $\varepsilon_{l_{max}}(l_{max})$, $\varepsilon_{l_{min}}(l_{min})$, $\varepsilon_{\varphi_l}(\varphi_l)$, $\varepsilon_v(v)$ and $\varepsilon_{\varphi_v}(\varphi_v)$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters.

Example of application

We illustrate the technique by results of its application for a study of largest earthquake in the last four decades: Sumatra-Andaman earthquake occurred on 26 December 2004.

To estimate the best double-couple, duration and geometry of the source we have used amplitude spectra of second and third orbits of fundamental Love and Rayleigh modes in spectral range from 500 to 650 seconds. The records were processed by frequency-time and polarization analysis package. We selected 24 Love wave records and 22 Rayleigh wave records from IRIS and GEOSCOPE stations. Their azimuthal distribution is given in figure 7.

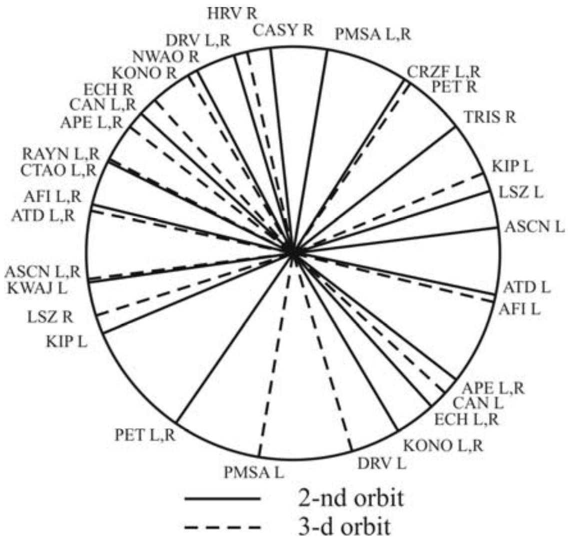

Fig. 7 Azimuthal distribution of radiation of waves used for inversion.
L and R after the name of station denotes Love and Rayleigh wave correspondingly.

In the source region and under the receivers, we used the 3SMAC model (Ricard et al. 1996) for the crust and the PREM model below. We used the quality factor given by the PREM model for attenuation correction. The moment tensor describing the source in instant point source approximation is obtained by joint inversion of surface wave amplitude spectra and first arrival polarities at worldwide stations. The solution gives a mechanism described by the following values of strike, dip and slip: 330°, 8°, 105° respectively (see figure 8). The estimate of source depth is equal to 13 km. The estimated value of seismic moment is $0.52 \cdot 10^{23}$ N·m.

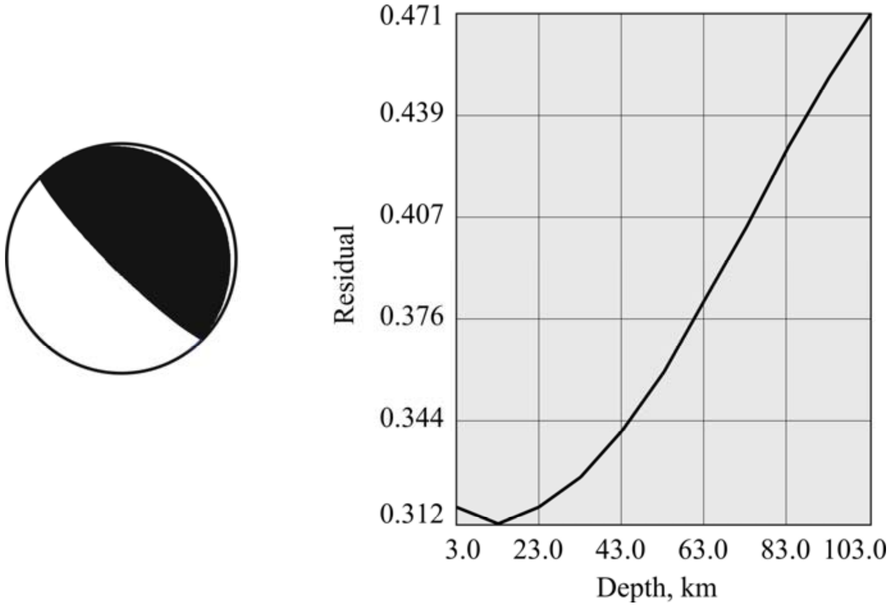

Fig. 8. Double-couple solution and source depth resolution curve.
P1: 330°, 8°, 105°; P2: 135°, 82°, 88°. $M_0 = 0.52 \cdot 10^{23}$ N·m

Determining 2-nd moments of moment tensor density we consider the nodal plane dipping to the northeast as a fault plane. We fixed source depth (13km) and focal mechanism obtained in instant point source approximation. Usually when double-couple parameters are obtained from periods long enough to consider the source as an instant and point, we fix seismic moment as well. But in this case the periods are not sufficiently long, so we recalculated seismic moment determining source 2nd moments. As it was mentioned above we estimate the duration and the geometry of the source from the same amplitude spectra of fundamental Love and Rayleigh modes in the same spectral band (from 500 to 650 seconds) that was used for inversion in instant point source approximation.

Our final estimate of seismic moment is equal to $0.84 \cdot 10^{23}$ Nm. The residual functions for integral estimates are given in figure 9. The inversion yields the integral estimate of duration being about 160 s, a characteristic source length (major axis length) of 300 - 400 km. The minor axis length is poorly resolved, lying between 0 and 200 km. The average instant centroid velocity estimate is about 2 km/s. The angles giving the major axis and velocity vector orientations are measured clockwise on the footwall starting from the strike axis. They are consistent with each other and residual functions attain their minimum values at 15°.

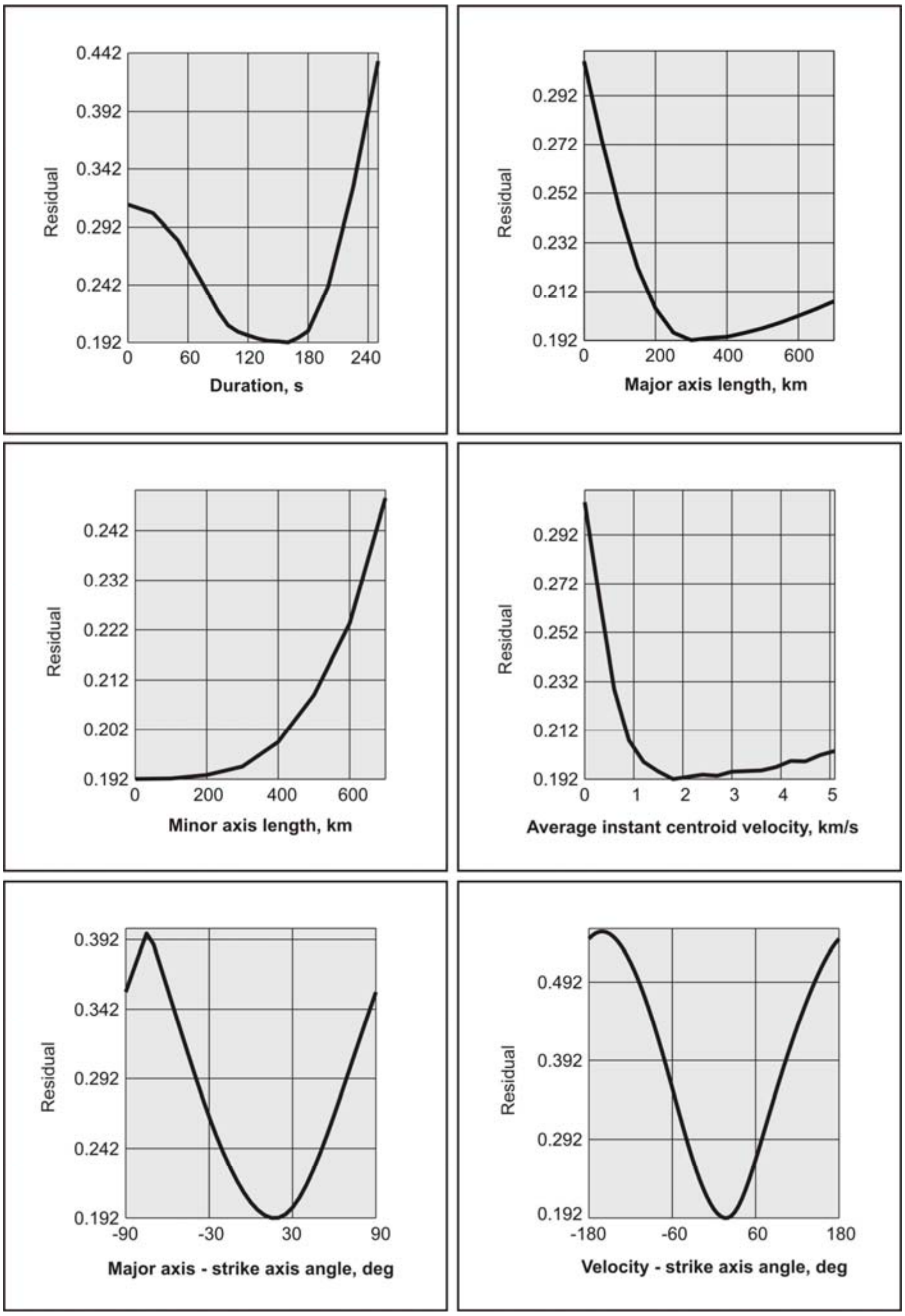

Fig. 9. Residual functions for source integral characteristics.

IV. Comparison of large scale average characteristics of the Sumatra-Andaman earthquake models constructed from different observations.

The December 26, 2004, Sumatra-Andaman earthquake is not only one of the greatest earthquakes that occurred for the last decades - it is the first event of such a scale which was studied using a wide spectrum of observations with characteristic periods from fraction of seconds to months.

Direct comparison of resulting models shows their large difference. But it is more correct to compare their large scale characteristics. It was shown (McGuire *et al.* 2001; Clevede *et al.* 2004) that integral characteristics of rupture process estimated by second order moments of slip rate distribution over the fault related to source size, orientation, duration and rupture velocity vector, can be useful for
comparison of models obtained from different observations and their combinations. We compared second moments of coseismic slip distribution for models constructed from seismological, geodetic, altimetric, and tide gauge measurements. As a constrain in the comparison we use presented above estimates of the same integral moments retrieved from observed surface wave spectra in period band from 500 s to 650 s.

Models used in this study

Among the numerous studies of the Sumatra-Andaman rupture process we selected eight, considering different type of data either by inversion or as constrains.

- Lay *et al.* (2005) performed seismological analysis and tsunami modelling using altimetric and tide gauge data.
- Ammon *et al.* (2005) used very broadband seismological data (80-3000s) for the slip distribution model we use (refered as model II).
- Banerjee *et al.* (2007) used far-field and near field geodetic data (GPS).
- Pietrzak *et al.* (2007) used far-field and near field geodetic data (GPS).
- Rhie *et al.* (2007) used geodetic and long-period (100-500s) seismological data.
- Sladen and H´ebert (2008) used altimetric data.
- Ammon *et al.* (2005) used very broadband seismological data from 5s to 2000s (refered as model III)
- Ammon *et al.* (2005) 1D model used data from 80s to 500s (refered as IRT 1D model).

Integral estimates of models

In order to compare these models and our results of long period surface wave inversion, we compute the integral characteristics of these models corresponding to the stress glut rate moments of degree 0, 1 and 2 directly from their theoretical definitions given by formulas (3.1-3.6). Both distributions of moment rate (in space and in time) were available for model II (Ammon *et al.*, 2005) only. So, for this model we estimated spatial characteristics as well as duration. Only source time function was available for two last models in the list above, and only final moment distribution over the fault was available for the rest five models.

Spatial characteristics

For first six models we estimated the spatial centroid location and the ellipse characteristics (principal axes length and orientation). All these models are obtained using geodetic and/or very long period seismic or altimetric data.

In the case of Lay *et al.* (2005) the model includes two separate types of slip, one being fast, corresponding to a rise time of 50*s*, the other one being slow, corresponding to a rise time of 3500*s*. Thus we consider for Lay *et al.* (2005) two ‘sub-models’: one for total slip distribution

and another one for fast slip distribution only. These integral estimates for all seven models are summarized in the table 1.
The spatial integral characteristics of all the complete models are fairly compatible. The length of the minor axis ranges from 116 to 152km, the length of the major axis ranges from 523 to 708km. However, the estimates for length of major axis are not compatible with our estimate obtained from surface waves inversion (300 to 400 km). But, the estimate obtained for 'fast slip sub-model' from Lay *et al.* (2005) (327 km) fits our surface-wave estimate. In this model there is no fast slip to the North of Nicobar segment (about 8°N).
We decided to compare the lengths of major axis for all models being truncated, excluding slip to the North of 8°N. Results are presented in the last column in table 1. The estimates for all truncated models fit our surface wave estimate well.

Model	Azimuth of major axis, deg.	Length of minor axis, km	Length of major axis, km	Length of major axis for truncated model, km
Banerjee *et al.* Model C	347	127	708	361
Sladen model (altimetric data)	348	147	654	359
Pietrzak *et al.* Model asv8	336	104	550	402
Lay *et al.* total slip	342	131	523	328
Lay *et al.* fast slip only	328	116	327	-
Rhie *et al.* Model BJ	346	117	617	341
Ammon *et al.* Model II	343	152	648	403

Table 1. Spatial characteristics of models.

Integral estimate of duration
The moment rate dependence on time is given by Ammon *et al.* (2005) for three different models: The IRT 1D model is constructed using Rayleigh waves for periods from 80 s to 500 s by inverse Radon transform; the two other models use very long-period seismic waves, respectively 100 s to 3000 s for model II, and 250 s to 2000 s for model III.
We compute the integral estimate of duration for these three models (table 2). While the estimates for models using very long periods are much larger (241 s and 247 s) than our surface waves estimate (160 s), the estimate obtained for the IRT 1D model using long period waves (187 s) is close to our estimate.

Model	Period band of inverted seismic waves	Integral duration
IRT 1D model	80 to 500 s	187 s
Model II	100 to 3000 s	241 s
Model: III	250 to 2000 s	247 s

Table 2. Integral estimates for duration.

Conclusions

- Considering Long period Rayleigh wave directivity, Ammon *et al.* (2005) stated that modeling of simple propagating rupture suggests that Rayleigh waves observations for periods shorter than 600 s are compatible with north-northwest propagation of a rupture at about 2.5 to 3 km/s for 400 to 600 km from the southern end of the fault. But observations for periods longer than 600s are only partly accounted for by this model, suggesting that additional slip extended in either time, space, or both is required to explain the very long period surface wave data.
- Considering seismic and tsunami observations, Lay *et al.* (2005) suggest a composite slip model with fast slip in the southern portion of the rupture, and slow slip to the North of Nicobar segment (about 8°N).
- We have shown that integral estimates of the source length and duration for models obtained from geodetic and/or very long period seismic or altimetric data are larger than our long period (from 500 s to 650 s) surface wave estimates (tables 1 and 2). Integral estimates of size for the same models truncated to the North of 8° fit our long period surface wave estimate (table 1). Integral estimate of duration for the IRT 1D model (Ammon *et al.* 2005) using long period Rayleigh waves (up to 500 s) fit our long period surface wave estimate (table 2).

Summarizing these results, and relying on the fact that the size of the source is constrained by HF P-wave energy radiation (Gusev *et al.* 2007; Lomax 2005) and very long-period data (normal modes, tsunami) (Lay *et al.* 2005; Ammon *et al.* 2005; Park *et al.* 2005), we propose that the Northern part of the Sumatra-Adaman fault (to the North of 8°) radiated very long period seismic energy and did not radiate long period seismic energy at periods shorter than 650 s.

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