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Non-uniqueness of moment tensor inversion from surface waves in case of shallow source

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1. Introduction

We consider surface waves radiated by an instant point source in horizontally homogeneous medium. For the spectrum of displacement in surface wave $\mathbf{u}(\mathbf{r},\omega)$ at a point $\mathbf{r}$ we have (Woodhouse, 1974; Babich *et al.*, 1976; Levshin *et al.*, 1989; Bukchin, 1990):

$$\mathbf{u}(\mathbf{r},\omega) = \mathbf{q}(\omega) P(\mathbf{M}, h, \omega, \varphi) \exp[-i\psi(\mathbf{r},\omega)], \quad (1)$$

where $\mathbf{q}(\omega)$ is a complex vector depending on Earth structure, $\mathbf{M}$ **-** moment tensor, h - source depth, φ - azimuth of surface wave radiation, $\psi(\mathbf{r},\omega)$ – propagation phase, and ω – circular frequency. The factor P determines the radiation pattern of the source (azimuth dependence of spectral amplitude) and the initial (source) phase.

For Love wave this factor is defined by formula

$$P(\mathbf{M}, h, \omega, \varphi) = \xi V^{(\tau)}(\omega, h)[0.5(M_{33} - M_{22})\sin 2\varphi + M_{23}\cos 2\varphi] + i\frac{\partial V^{(\tau)}(\omega, h)}{\partial z}(M_{12}\sin\varphi - M_{13}\cos\varphi), \quad (2)$$

where $V^{(\tau)}$ (z) – transverse eigenfunction, i is the imaginary unit, and the coordinate system is defined in the following way: 1 – vertical, 2 – north, and 3 – east.

For Rayleigh wave the function P is given by formula

$$P(\mathbf{M}, h, \omega, \varphi) = \frac{\partial V^{(z)}(\omega, h)}{\partial z} M_{11} - \xi V^{(r)}(\omega, h)(M_{22}\cos^2\varphi + M_{33}\sin^2\varphi + M_{23}\sin 2\varphi) + i[\xi V^{(z)}(\omega, h) + \frac{\partial V^{(r)}(\omega, h)}{\partial z}](M_{12}\cos\varphi + M_{13}\sin\varphi), \quad (3)$$

where $V^{(z)}$ (z) , $V^{(r)}$ (z) – vertical and radial components of eigenfunction.

If the source is rotated around vertical axis by 180° (strike axis is rotated by 180°) all moment tensor elements except M_{12} and M_{13} do not change, while elements M_{12} and M_{13} just change their sign to the opposite. As can be seen from formulas above as a result of such rotation the function $P(\mathbf{M},h,\omega,\varphi)$ is changing to complex conjugate.

The coefficients $\frac{\partial V^{(\tau)}(\omega, h)}{\partial z}$ for Love wave and $[\xi V^{(z)}(\omega, h) + \frac{\partial V^{(r)}(\omega, h)}{\partial z}]$ for Rayleigh wave are proportional to the force acting on a horizontal plane. But such a force is vanishing at the free surface ($h = 0$). As a consequence if the source depth h is much smaller than the wave length, the moment tensor elements M_{12} and M_{13} almost do not affect on the surface waves radiation pattern and source phase, and they can not be resolved from observed spectra. At the same time the imaginary part of function $P(\mathbf{M},h,\omega,\varphi)$ is small and the rotation of source around vertical axis by 180° doesn't change radiated surface waves. The last property of shallow source was studied by Henry *et al.*, 2002. They explained for the case of double-couple the two-fold rotational symmetry of the misfit function about the vertical axis of the moment tensor and demonstrated it for a set of earthquakes. Actually such symmetry is valid for general source.

The elements M_{12} and M_{13} do not affect on the surface waves radiation so long as they do not exceed significantly by absolute value other elements of moment tensor. But if these two elements are dominant then they do contribute in the surface wave radiation for any nonzero h. For example, a source with other components equal to zero will radiate surface waves for any nonzero depth.

Limiting ourself with the case of pure double-couple at the depth h which is much smaller than the wave length we describe completely the family of equivalent sources radiating practically the same surface waves in long period spectral band (see for the details Bukchin, 2006).

For simplicity we consider a double-couple with zero strike angle (nonzero strike angle results in rotation of radiation diagram around vertical axis by this angle). In this case moment tensor elements can be presented as following functions of two other angles defining the focal mechanism

$$M_{11} = M_0 \sin\lambda \sin 2\delta,\ \ M_{22} = 0,\ \ M_{33} = -M_0 \sin\lambda \sin 2\delta,$$
$$M_{23} = M_0 \cos\lambda \sin\delta,\ \ M_{12} = -M_0 \cos\lambda \cos\delta,\ \ M_{13} = M_0 \sin\lambda \cos 2\delta. \quad (4)$$

Here M_0 is the seismic moment, λ is the rake angle and δ is the dip angle of double-couple. As follows from formulas (4) the absolute values of moment tensor elements M_{12} and M_{13} start to exceed the absolute values of all other elements when δ is vanishing and nodal plane becomes sub-horizontal. Similar situation takes place when δ tends to a limiting value 90° together with λ tending to a limiting value ±90°, and as result all moment tensor elements apart from M_{13} are vanishing. Note that in this case the auxiliary nodal plane becomes sub-horizontal. So, moment tensor elements M_{12} and M_{13} can be resolved from observed spectra if and only if one of nodal planes of double-couple becomes sub-horizontal.

We are starting from consideration of two special cases: pure thrust (or normal) fault and pure strike-slip, and investigate the dependence of surface waves radiation on the value of dip angle δ. These cases were described by Kanamori and Given, 1982. After that we will consider general case of double-couple.

2. Two special types of double-couple.

2.1. *Pure thrust (or normal) fault.*

In the case of pure thrust (or normal) fault we have $\boldsymbol{\lambda} = 90°$ (or $\boldsymbol{\lambda} = -90°$), $\cos\lambda = 0$, and $\sin\lambda = 1$ ($\sin\lambda = -1$). As can be seen from formulas (4) in this case there are only three non-zero moment tensor elements: $M_{11} = -M_{33} = \pm M_0 \sin 2\delta$ and $M_{13} = \pm M_0 \cos 2\delta$. If the source depth is small and the value of δ at the same time is not too close to 0° or to 90° then the imaginary part of function $P(\mathbf{M},h,\omega,\varphi)$ is negligible. Substituting moment tensor elements as functions of M_0 and δ in real part of $P(\mathbf{M},h,\omega,\varphi)$ (see Appendix 1) it is easy to show (Bukchin, 2006) that the dependence of surface wave radiation pattern and source phase (which is close to 0 or π) on azimuth φ for different values of δ are similar to those for $\delta = 45°$ when $\cos 2\delta = 0$. Furthermore all double-couples with the same value of the product $M_0 \sin 2\delta$ radiate the same surface wave field. So, neither dip angle nor seismic moment (and as result - moment magnitude) can be uniquely determined from surface wave spectra. Note, that the family of double-couples under consideration consists of symmetric with respect to vertical axis pairs of double-couple.

But in the case when dip angle δ belongs to the vicinity of 0 $[0,\delta_0]$ or to the vicinity of 90° $[90°-\delta_0, 90°]$ the function $P(\mathbf{M},h,\omega,\varphi)$ essentially depends on the δ value. Consequently the dip angle and seismic moment can be uniquely determined from observed surface wave spectra. Here δ_0 is a small threshold value dependent on the structure, on the period and on the source depth.

This dependence of radiated surface waves on the value of dip angle is schematically illustrated in Fig. 1. The ‘integral change of spectrum’ denotes the integral over the radiation azimuth of the modulus of the difference between complex spectrum correspondent to the current value of δ and spectrum correspondent to $\delta = 45°$.

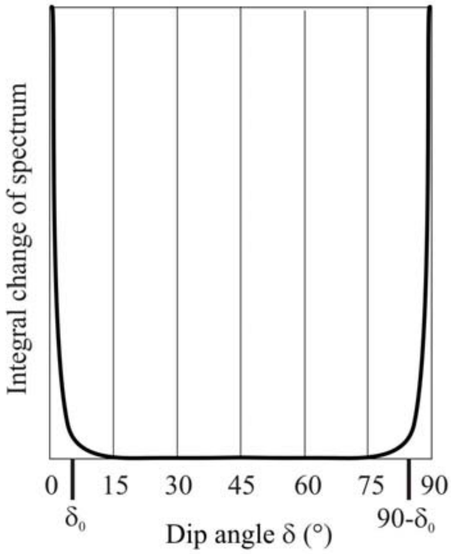

Fig.1. Dependence of radiated surface waves on the dip angle for a shallow trust (or normal) fault.

2.2. *Pure strike-slip fault.*

In the case of pure strike-slip fault the rake angle $\lambda = 0°$ for a left-lateral fault and $\lambda = 180°$ for a right lateral fault. Correspondingly $\sin\lambda = 0$, and $\cos\lambda = 1$ ($\cos\lambda = -1$). In this case there are only two non-zero moment tensor elements: $M_{23} = \pm M_0 \sin\delta$ and $M_{12} = \mp M_0 \cos\delta$. If the source depth is small and the value of δ at the same time is not too close to 0° then the imaginary part of function $P(\mathbf{M},h,\omega,\varphi)$ is negligible. Substituting moment tensor elements as functions of M_0 and δ in real part of $P(\mathbf{M},h,\omega,\varphi)$ (see Appendix 1) it is easy to show (Bukchin, 2006) that dependence of surface wave radiation pattern and source phase (which is close to 0 or π) on azimuth φ for different values of δ are similar to those for $\delta = 90°$ when $\cos\delta = 0$ (pure strike-slip on a vertical fault).

All double-couples with the same value of the product $M_0\sin\delta$ radiate the same surface wave field. So, as in the case of pure thrust or normal fault, neither dip angle nor seismic moment can be uniquely determined from surface wave spectra. Note, that double-couples symmetric with respect to vertical axis to those under consideration must be added to the family of equivalent sources.

But in the case when dip angle δ belongs to the vicinity of 0 $[0,\delta_0]$ the function $P(\mathbf{M},h,\omega,\varphi)$ essentially depends on the δ value. Consequently the dip angle and seismic moment can be uniquely determined from observed surface wave spectra. As before δ_0 is a small threshold value dependent on the structure, on the period and on the source depth.

This dependence of radiated surface waves on the value of dip angle is schematically illustrated in Fig. 2. The ‘integral change of spectrum’ denotes the integral over the radiation azimuth of the modulus of the difference between complex spectrum correspondent to the current value of δ and spectrum correspondent to $\delta = 90°$.

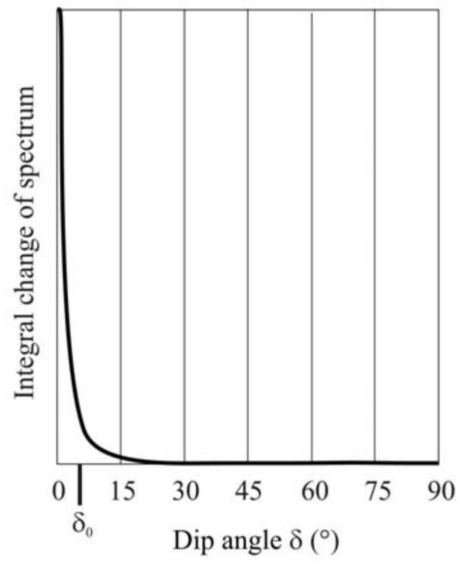

Fig.2. Dependence of radiated surface waves on the dip angle for a shallow strike-slip fault.

3. General case of double-couple.

A general double-couple with rake angle λ and dip angle δ can be presented as a sum of a thrust (or normal) fault and a strike-slip with weights sin λ and cos λ correspondingly and with the same dip angle δ.

When the value of λ is close to 0° or 180°, the value of sin λ is close to 0 and radiated surface waves are practically the same as in considered case of pure strike-slip. When the value of λ is close to 90° or -90°, the value of cos λ is close to 0 and radiated surface waves are practically the same as in considered case of pure thrust or normal fault. Let us consider now a double-couple with rake angle λ which is different from 0°, 180° and ±90°.

Let δ_0 be the larger of two threshold values estimated for thrust and for strike-slip components for given structure, source depth and period. Let δ be larger then δ_0. If the source depth is small then the imaginary part of function P(**M**,h,ω,φ) is negligible.

Substituting moment tensor elements as functions of M_0 , λ and δ in the real part of $P(\mathbf{M},h,\omega,\varphi)$ (see formulas (A1.5) and (A1.6) in 1) it can be shown (Bukchin, 2006) that all double-couples with values of λ , δ and M_0 satisfying the identities

$$\tan\lambda\cos\delta \equiv C_1 \quad (5)$$

and

$$M_0 \sin\delta\cos\lambda \equiv C_2 \quad (6)$$

have the same surface wave radiation pattern and source phase. C_1 and C_2 in formulas (5) and (6) are constant values.

So, as before neither focal mechanism nor seismic moment for such a source can be uniquely determined from surface wave spectra. After we have found any of equivalent double-couple solutions we can calculate the constants in the right parts of identities (5) and (6) using obtained values of λ, δ and M_0. The identity (5) gives us the description of all equivalent solutions (pairs of dip and rake angle values), and using the identity (6) we calculate corresponding values of seismic moment. To decide between values of rake angle λ and λ+180° satisfying the identity (5) we use following condition: the value of cosλ must have the same sign as constant C_2. Adding to the pairs of dip and rake angle values the value obtained for strike angle we describe the first branch of equivalent double-couples. Substitution of the strike angle value ψ by value ψ+180° gives us the second branch of equivalent double-couples.

It is important to note that if we would use for calculation of the constants in the right parts of identities (5) and (6) the values of λ and δ correspondent to the second nodal plane of the same double-couple, the identities will describe the same family of equivalent sources. It follows from two formulas valid for any double-couple

$$\tan\lambda_1 \cos\delta_1 = -\tan\lambda_2 \cos\delta_2 \tag{7}$$

and

$$\sin\delta_1 \cos\lambda_1 = -\sin\delta_2 \cos\lambda_2 , \tag{8}$$

where λ_1, δ_1 and λ_2, δ_2 are rake and dip angles corresponding to two nodal planes of a double-couple. Formulas (7) and (8) can be obtained from main relations between angles corresponding to two double-couple nodal planes (Ben-Menahem and Singh, 1981).

Let in contrary the dip angle δ of one of nodal planes is so small that it belongs to the vicinity of 0 $[0,\delta_0]$, where δ_0 is the smaller of two threshold values estimated for thrust and for strike-slip components for given structure, source depth and period band. In this case the imaginary part of function $P(\mathbf{M},h,\omega,\varphi)$ becomes comparable to its real part and both the surface wave radiation pattern and source phase depend significantly on the dip and rake angles. As result if one of nodal planes is subhorizontal then the focal mechanism and seismic moment of a shallow source can be uniquely determined from the spectra of surface waves recorded at various points of the Earth's surface.

Let us consider equivalent solutions given by equation (5) and their behavior on proceeding to the limits $C_1 \to \pm\infty$ and $C_1 \to 0$. A contour plot of the left-hand part of identity (5) is shown in the figure 3.

Every isoline corresponds to a family of equivalent solutions. Gray strip along the axis $\delta = 0$ and gray sectors centred on points of intersection of the axis $\delta = 90°$ with axes $\lambda = -90°$, $\lambda = 90°$ and $\lambda = 270°$, show the regions where one of nodal planes is subhorizontal and every solution of inverse problem is unique. We show in the figure 3b beach balls correspondent to different points of axes $\lambda = -90°$, $\lambda = 0$, $\lambda = 90°$, $\lambda = 180°$, $\lambda = 270°$, and $\delta = 90°$ with 15° step. The value of strike angle for all double couples is fixed equal to 0. The beach balls at the axis $\lambda = 270°$ are repeating the beach balls at the axis $\lambda = -90°$.

Let us consider the behavior of isolines on proceeding to the limits $C_1 \to \pm\infty$ and $C_1 \to 0$.

(a) $C_1 \to \pm\infty$

As one can see from the figure 3 on proceeding to the limits $C_1 \to +\infty$ and $C_1 \to -\infty$ the isolines tend to coordinate axes $\lambda = 90°$, $\lambda = -90°$ and $\lambda = 270°$, and equivalent double-couples are pure dip-slips. Note, that for every double-couple with $\lambda = 90°$, $\lambda = -90°$ and $\lambda = 270°$ the symmetric with respect to vertical axis double-couple coincides with one of double couples of the same set shown in the figure. Proceeding to limits $\lambda \to 90°$, $\lambda \to -90°$ or $\lambda \to 270°$ in equations (5) and (6) one can get the equation $M_0 \sin 2\delta \equiv C$, which was obtained in paragraph (2.1) for pure dip-slip.

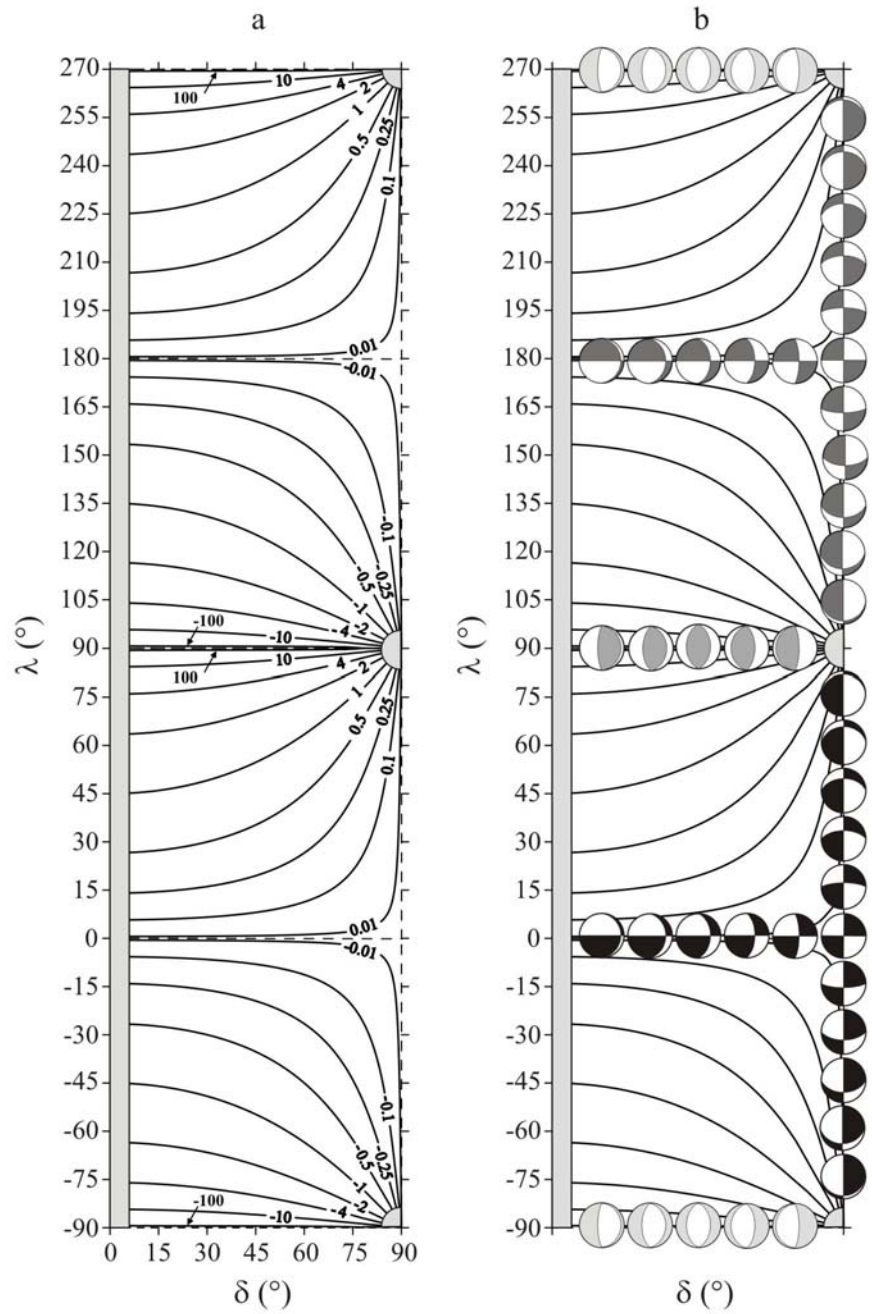

Fig. 3. (a) A contour plot of function $\tan\lambda\cos\delta$ and (b) the same contour plot with superimposed beach balls for different points of axes $\lambda = -90°$, $\lambda = 0$, $\lambda = 90°$, $\lambda = 180°$, $\lambda = 270°$, and $\delta = 90°$ with 15° step. Equivalent double-couples are given by beach balls filled by the same shades of gray. Contours are marked by correspondent value of constant C_1 in the equation (5).

(b) $C_1 \to 0$.

As one can see from the figure 3 on proceeding to the limit $C_1 \to 0$ the isolines tend to horizontal axes $\lambda = 0$ and $\lambda = 180°$, and to vertical axis $\delta = 90°$.

Double-couples correspondent to points at the axes $\lambda = 0$ and $\lambda = 180°$ are pure strike slips. Double-couples correspondent to points at the axis $\delta = 90°$ are slips on a vertical fault. Strike slips with $\lambda = 0$ together with slips on a vertical fault with λ belonging to the interval (-90°, 90°) form a family of equivalent double couples. Strike slips with $\lambda = 180°$ together with slips on a vertical fault with λ belonging to the interval (90°, 270°) form another family of equivalent double couples.

Note, that for every double-couple with $\delta = 90°$ the symmetric with respect to vertical axis double-couple coincides with one of double couples of the same set showing in the figure. But double-couples symmetric with respect to vertical axis to double-couples with $\lambda = 0$ or $\lambda = 180°$ (pure strike slips) don't belong to these sets and they must be added to the family of equivalent double-couples..

Proceeding to the limits $\lambda \to 0$ or $\lambda \to 180°$ we have for the right part of identity (5) $C_1 = 0$, and the identity (5) is valid for any value of δ. Identity (6) in this case transforms to the same identity $M_0 \sin\delta \equiv C$, which we obtained for seismic moment in paragraph 2.2. For $\lambda \to 0$ and $\lambda \to 180°$ we have $C = C_2$ and $C = - C_2$ correspondingly.

On proceeding to the limit $\delta \to 90°$ the identity (5) is valid for the same zero value of the constant C_1 for any value of λ, and identity (6) transforms to identity $M_0 \cos\lambda \equiv C_2$.

Application to March 25, 1998 Antarctic earthquake

We illustrate the adequacy of the description of equivalent focal mechanisms by application to March 25, 1998 Antarctic earthquake. A contour plot of the misfit function for this event given in the figure 4 was copied (with permission of S.Das) from Henry *et al.*, 2002. It is plotted for double-couple moment tensors at depth 10 km. Points to the left of the vertical dashed line at the center of the figure have strike 96° and rakes corresponding to the left ordinate. Points to the right of the line have strike 276° and rakes corresponding to the right ordinate. Crosses labeled with the letters C and D mark the optimal pure double-couple moment tensors.

The curves 1 and 2 superimposed on the contour plot are calculated using identity (5) with the value of constant in the right part corresponding to the optimal solution C. As one can see the curves are in a good agreement with the shape of the misfit contours.

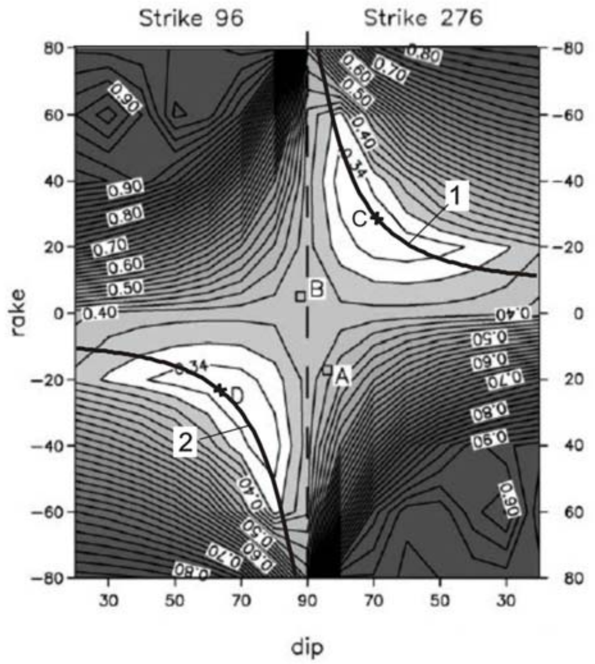

Fig.4. A contour plot of the misfit function for the March 25, 1998 Antarctic earthquake, copied from Henry *et al.*, 2002. The curves 1 and 2 superimposed on the contour plot correspond to solutions equivalent to solution labeled by letter C (strike 276°, dip 69°, rake -28°) for small source depth.

4. Solomon islands earthquake, 01.04.2007.

As a first step we determined the double-couple for this event by joint inversion of long period surface wave amplitude spectra and first arrival polarities (Lassere *et al.* 2001). We selected long period seismograms from 11 GEOSCOPE stations. The fundamental mode of Love and Raleigh waves are retrieved from observed surface wave trains using frequency-time analysis and floating filtering of signals, as described by Levshin *et al.* (1989). We selected records of a good quality in period range from 160 to 300 seconds. The distribution of stations is shown in the Fig. 5.

We calculate surface wave spectra assuming that the propagation medium has only weak lateral inhomogeneities. In this case, the surface wave part of the Green's function is determined by the structure near the source and the receiver, by the average phase velocity along the path, and by the geometrical spreading (Woodhouse, 1974; Babich *et al*., 1976; Levshin *et al*., 1989; Bukchin, 1990). For surface wave spectra calculation we used in the source region as well as under the receivers the 3SMAC model (Ricard *et al*. 1996) for the crust and the preliminary reference earth model (PREM) below.

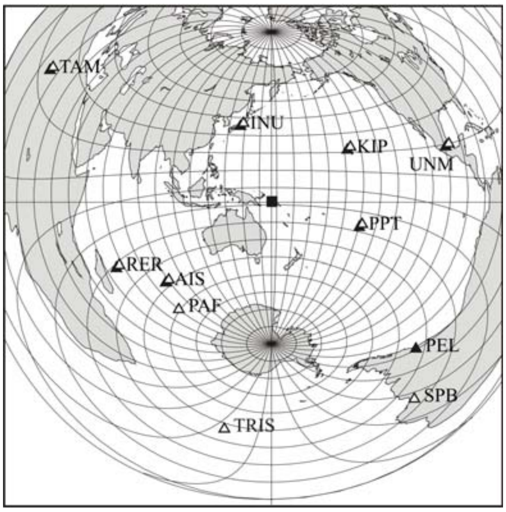

Fig. 5. Station distribution used for the double-couple and source depth inversion. The figure is centered on the Solomon islands earthquake epicenter. Filled and blank triangles mark the use of Love and Rayleigh wave records correspondingly.

Our best fitting solution gives a mechanism described by the following values of strike, dip and rake angles: 151°, 77°, 98° respectively. The best fitting source depth is around 10 km. The estimate of seismic moment is $0.27 \cdot 10^{22}$ N·m.

We compared this double-couple with best double-couples corresponding to global CMT solution and to GEOSCOPE solution. Both these solutions where obtained by long period surface waves inversion (T > 150 s). Our double-couple (solution 1) is significantly different from two other solutions (Fig. 6).

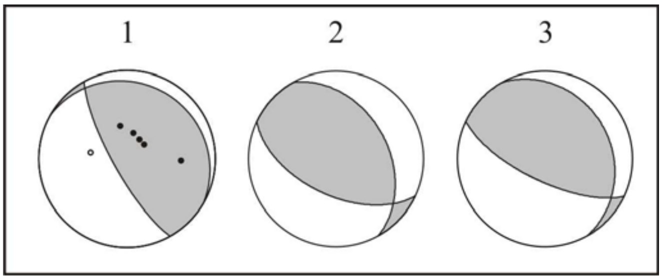

Fig. 6. Three double-couples obtained for Solomon Islands earthquake. 1 – result of this study by joint inversion of long period surface wave amplitude spectra and first arrival polarities (ψ = 151°, δ = 77°, λ = 98°), 2 - Global CMT solution (ψ = 331°, δ = 38°, λ = 120°), 3 - GEOSCOPE solution (ψ = 331°, δ = 25°, λ = 123°).

However we have found that solutions 2 and 3 are very similar to double-couples which are equivalent to solution 1 for vanishing value of source depth. This result is illustrated in the Fig.7.

This figure is constructed in the same way as Fig.4 for the 1998 Antarctic earthquake (Henry *et al.*, 2002) but instead of misfit between synthetic and observed spectra we consider here the misfit function between two synthetic spectra calculated for current double-couple (defined by focal mechanism angles at any point of the map) and for solution 1. Contour plot is given for period 200 s and for source depth fixed at 10 km. In the same figure we show by dashed curves the solutions equivalent to the solution 1 for vanishing depth. These curves are calculated using identity (5) with the value of constant in the right part corresponding to the solution 1. As one can see solutions 2 and 3 are very close to one of these curves. Note the growth of misfit for points in the equivalent solution curves verging towards vertical axes $\delta = 0$ and $\delta = 90°$. Double-couples correspondent to points in these curves in the vicinity of the vertical axis $\delta = 90°$ have rake angle close to 90°, and consequently the dip angle of correspondent auxiliary nodal planes have dip angle close to 0. So, in these both extreme cases one of nodal planes of double-couple is subhorizontal, and large misfit confirms our description of the family of shallow double-couples which can be uniquely determined from long period surface waves.

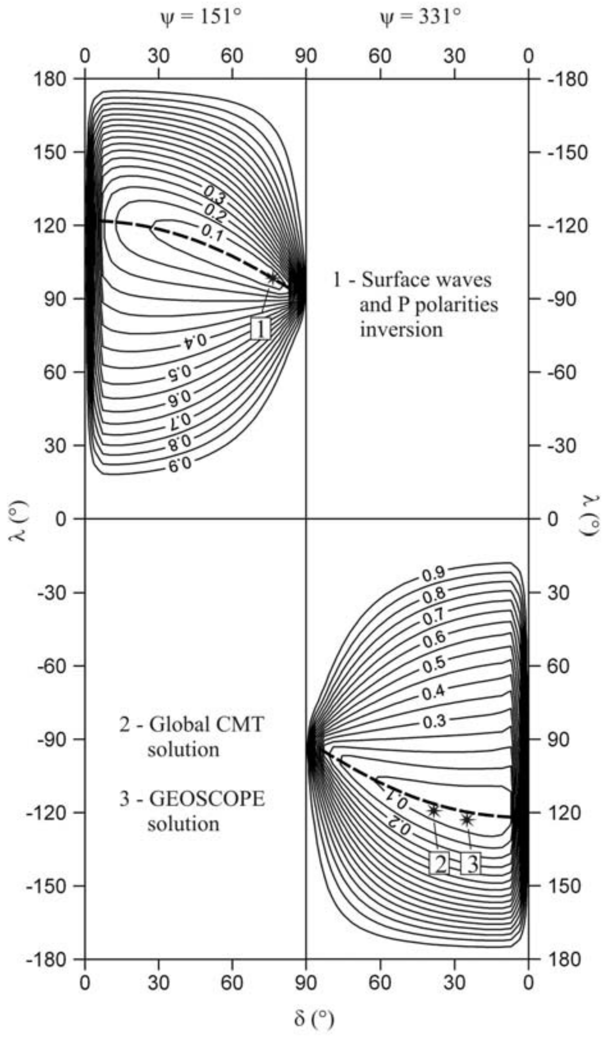

Fig. 7. Contour plot of the normalized misfit between spectra calculated for current double-couple and for double-couple obtained by joint inversion of surface wave amplitude spectra and first arrival polarities (marked by figure 1). The spectra of fundamental Love and Rayleigh modes are calculated for period 200 s for 72 points uniformly located around the source at a distance 9000 km. The source depth is fixed at 10 km. Points to the left of the vertical dashed line at the center of the figure have strike 151° and rakes corresponding to the left ordinate. Points to the right of the line have strike 331° and rakes corresponding to the right ordinate. Dashed lines correspond to

solutions equivalent to the solution 1 for vanishing depth. Global CMT and GEOSCOPE solutions are marked by figures 2 and 3 correspondingly.

To select from equivalent focal mechanisms the true one we have to use additional data. Note that our solution 1 in contrast to solutions 2 and 3 is well consistent with first arrival polarities observations. But this earthquake was rather long: CMT estimate of its duration was more than 50 seconds. Therefore it is not quite proper to use first arrival polarities – readings from short period seismograms. Instead of this we use as additional data the long period polarities of direct P-waves shown in Fig. 8. We select from equivalent focal mechanisms those

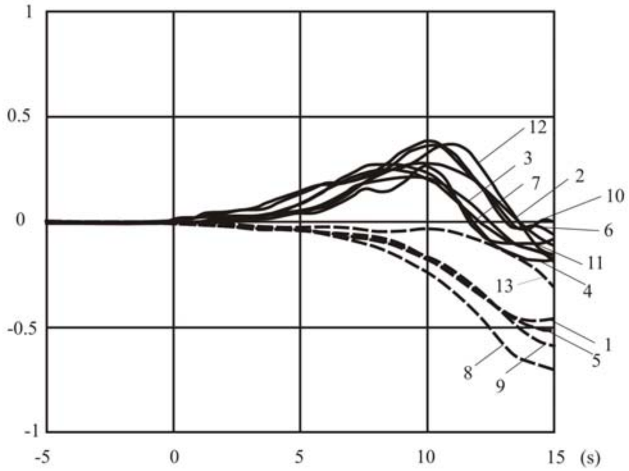

Fig. 8. Initial 15 seconds of normalized broadband records of displacements. The time origin corresponds to the first arrival time. Solid lines correspond to positive P-wave polarization, dashed lines correspond to negative P-wave polarization. 1 - DAV, 2 - ERM, 3 - INCN, 4 - INU, 5 - KAPI, 6 - KIP, 7 - MAJO, 8 - MBWA, 9 - NWAO, 10 - POHA, 11 - RAO, 12 - SNZO.

which are consistent with these observations. The results of such analysis are presented in the Fig.9. As one can see solution 1 and two nearest solutions (solutions marked out by rectangle) can be selected as optimal.

Thus, it is shown that using surface wave long period spectra, presented description of equivalent double-couples and as additional data - long period polarities of direct P-waves, it is possible to determine the optimal double-couple describing a shallow seismic source with sufficiently high accuracy.

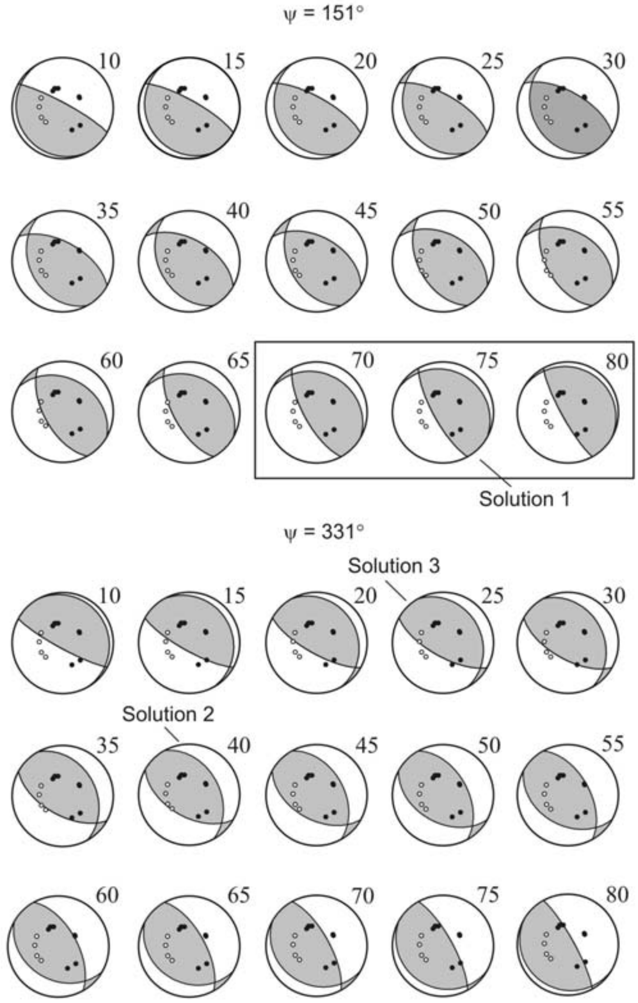

Fig. 9. Solutions equivalent to the solution 1 for vanishing depth. Double-couples in the upper part have strike equal to 151°, double-couples in the lower part have strike equal to 331°. Long period polarities of direct P-waves are superimposed. The values of dip angle d are given near every beach ball. Solutions fitting with the long period polarities of direct P-waves are marked out by rectangle.

5. Dependence of constants C_1 and C_2 conditioning the equivalent double-couples on moment tensor elements M_{11}, M_{22}, M_{33}, and M_{23}

Let elements M_{22}, M_{33} and M_{23} of deviatoric moment tensor **M** are given. Note that moment tensor element M_{11} is defined by zero-trace condition $M_{11} + M_{22} + M_{33} = 0$. Expressing M_{22}, M_{33} and M_{23} through seismic moment and focal mechanism angles we obtained existence condition for double-couples with given values of these moment tensor elements, and formula describing the set of such double-couples (see Appendix 2).

The existence condition for double-couples with given values of moment tensor elements M_{22}, M_{33} and M_{23} has form of inequality

$$M_{22}M_{33} \le M_{23}{}^2 . \tag{9}$$

If this condition is satisfied then such doubles exist and have strike angle given by formulas

$$\psi = 0.5(\pm \arccos \frac{A_1}{\sqrt{A_3{}^2 + A_2{}^2}} - \varphi) , \tag{10}$$

where φ, A_1, A_2, and A_3, are defined by following equations:

$$\sin\varphi = \frac{A_3}{\sqrt{A_3^{\ 2} + A_2^{\ 2}}}$$

$$\cos\varphi = \frac{A_2}{\sqrt{A_3^{\ 2} + A_2^{\ 2}}}$$

$$A_1 = M_{22} + M_{33},$$

$$A_2 = M_{33} - M_{22},$$

$$A_3 = 2M_{23}.$$

The constants C_1 and C_2 in formulas (5) and (6) describing the families of equivalent double-couples can be expressed through the same values defined above:

$$\begin{cases} C_1 = -\dfrac{A_1}{A_2 \sin 2\psi + A_3 \cos 2\psi} \\ C_2 = 0.5(A_2 \sin 2\psi + A_3 \cos 2\psi) \end{cases} \qquad (11)$$

So, if deviatoric moment tensor **M** in case of shallow source is obtained by inversion of long period surface waves then values of its four elements only (M_{11}, M_{22}, M_{33} and M_{23}) are reliable. If condition (9) is valid, then using formulas (10) – (11) one can obtain the values of constants C_1 and C_2. Formulas (5) and (6) with obtained constants C_1 and C_2 provides complete description of double-couples radiating the same long period surface waves as original deviator **M**.

If in contrary the condition is not valid, then there is no double-couples radiating the same long period surface waves as original deviator **M**.

Let us come back to the March 25, 1998 Antarctic earthquake. Harvard best double-couple solution is marked in the figure 4 by letter B. As one can see it is far enough from optimal double-couples C and D obtained by Henry *et al.*, 2002 and from our curves 1 and 2 correspondent to equivalent double-couples. Using values of moment tensor elements from Global CMT catalog we found that condition (9) is valid. We applied scheme described above. The difference between obtained family of equivalent focal mechanisms and family of focal mechanisms equivalent to the optimal double-couple C is very small. One of equivalent focal mechanisms is defined by following values of angles: $\psi = 275°$, $\delta = 69°$, $\lambda = -31°$. From the comparison of these values with corresponding values for the optimal double-couple C ($\psi = 276°$, $\delta = 69°$, $\lambda = -28°$) one can see that the difference between them is negligible. In spite of the fact that the difference between seismic moments of these two double-couples is significant ($1.87 \cdot 10^{28}$ dn·sm and $1.87 \cdot 10^{28}$ dn·sm correspondingly) one can see that these two double-couples are much closer to each other than solutions B and C in the figure4.

This example shows that for shallow deviatoric source inverted from long period surface waves and satisfying the condition (A2.6) it is more reasonable to consider the equivalent double-couples than the closest double-couple which depends on the values of elements M_{12} and M_{13} which are not estimated reliably.

Appendix 1. Radiation pattern dependence on focal mechanism angles.

For Love wave the function P is given by formula

$$P(\mathbf{M}, h, \omega, \varphi) =$$

$$-\xi V^{(\tau)}(\omega, h) M_0 [0.5 \sin 2\varphi \ \sin 2\delta \sin\lambda - \cos 2\varphi \ \sin\delta \cos\lambda] \qquad (A1.1)$$

$$-i \frac{\partial V^{(\tau)}(\omega, h)}{\partial z} M_0 (\cos\varphi \ \cos 2\delta \sin\lambda + \sin\varphi \ \cos\delta \cos\lambda).$$

For Rayleigh wave the function P is given by formula

$$P(\mathbf{M},h,\omega,\varphi)=[\frac{\partial V^{(z)}(\omega,h)}{\partial z}+\xi V^{(r)}(\omega,h)\sin^2\varphi]M_0\sin 2\delta\sin\lambda$$
$$-\xi V^{(r)}(\omega,h)M_0\sin 2\varphi\ \sin\delta\cos\lambda \qquad \text{(A1.2)}$$
$$+i[\xi V^{(z)}(\omega,h)+\frac{\partial V^{(r)}(\omega,h)}{\partial z}]M_0(\sin\varphi\ \cos 2\delta\sin\lambda-\cos\varphi\ \cos\delta\cos\lambda).$$

If neither of the two nodal planes is subhorizontal, i.e. δ is not infinitely near 0, and δ is not infinitely near 90° together with λ infinitely near ± 90°, then the imaginary part of function $P(\mathbf{M},h,\omega,\varphi)$ is negligible and formulae (A1.1, A1.2) take on form

$$P(\mathbf{M},h,\omega,\varphi)=-\xi V^{(\tau)}(\omega,h)M_0[0.5\sin 2\varphi\ \sin 2\delta\sin\lambda-\cos 2\varphi\ \sin\delta\cos\lambda] \qquad \text{(A1.3)}$$

and

$$P(\mathbf{M},h,\omega,\varphi)=[\frac{\partial V^{(z)}(\omega,h)}{\partial z}+\xi V^{(r)}(\omega,h)\sin^2\varphi]M_0\sin 2\delta\sin\lambda$$
$$-\xi V^{(r)}(\omega,h)M_0\sin 2\varphi\ \sin\delta\cos\lambda. \qquad \text{(A1.4)}$$

If λ is different from ± 90° and δ is different from 0 then formulae (A1.3, A1.4) can be rewritten in form

$$P(\mathbf{M},h,\omega,\varphi)=-\xi V^{(\tau)}(\omega,h)M_0\sin\delta\cos\lambda\ [\sin 2\varphi\ \cos\delta\tan\lambda-\cos 2\varphi] \qquad \text{(A1.5)}$$

and

$$P(\mathbf{M},h,\omega,\varphi)=M_0\sin\delta\cos\lambda\ \{2[\frac{\partial V^{(z)}(\omega,h)}{\partial z}+\xi V^{(r)}(\omega,h)\sin^2\varphi]\cos\delta\tan\lambda$$
$$-\xi V^{(r)}(\omega,h)\sin 2\varphi\} \qquad \text{(A1.6)}$$

for Love and Rayleigh correspondingly.

Appendix 2. Double-couples with given values of moment tensor elements $\mathbf{M_{22}}$, $\mathbf{M_{33}}$ and $\mathbf{M_{23}}$

Let elements M_{22}, M_{33} and M_{23} of deviatoric moment tensor **M** are given.

Assuming **M** to be a double-couple we express M_{22}, M_{33} and M_{23} through seismic moment and focal mechanism angles:

$$\begin{cases} M_{22}=-M_0(\cos\lambda\sin\delta\sin 2\psi+\sin\lambda\sin 2\delta\sin^2\psi) \\ M_{23}=M_0(\cos\lambda\sin\delta\cos 2\psi+0.5\sin\lambda\sin 2\delta\sin 2\psi) \\ M_{33}=M_0(\cos\lambda\sin\delta\sin 2\psi-\sin\lambda\sin 2\delta\cos^2\psi) \end{cases}$$

Transforming the system we have

$$\begin{cases} M_{22}+M_{33}=-M_0\sin\lambda\sin 2\delta \\ M_{33}-M_{22}=M_0(2\cos\lambda\sin\delta\sin 2\psi-\sin\lambda\sin 2\delta\cos 2\psi) \\ 2M_{23}=M_0(2\cos\lambda\sin\delta\cos 2\psi+\sin\lambda\sin 2\delta\sin 2\psi) \end{cases}$$

Let $M_{22}+M_{33}=A_1$, $M_{33}-M_{22}=A_2$, $2M_{23}=A_3$.

Then the system takes form

$$\begin{cases} -M_0\sin\lambda\sin 2\delta=A_1 \\ M_0(2\cos\lambda\sin\delta\sin 2\psi-\sin\lambda\sin 2\delta\cos 2\psi)=A_2 \\ M_0(2\cos\lambda\sin\delta\cos 2\psi+\sin\lambda\sin 2\delta\sin 2\psi)=A_3 \end{cases}$$

Combining the second and the third equations of the system we have finally

$$\begin{cases} -M_0 \sin\lambda \sin 2\delta = A_1 \\ M_0 \sin\lambda \sin 2\delta = A_3 \sin 2\psi - A_2 \cos 2\psi \\ 2M_0 \cos\lambda \sin\delta = A_2 \sin 2\psi + A_3 \cos 2\psi \end{cases} \quad \text{(A2.1)}$$

Summing the first and the second equations of the system (A2.1) we have equation for ψ

$$A_3 \sin 2\psi - A_2 \cos 2\psi = -A_1 . \quad \text{(A2.2)}$$

This equation divided by $\sqrt{A_3^{\ 2} + A_2^{\ 2}}$ takes form

$$\frac{A_3}{\sqrt{A_3^{\ 2} + A_2^{\ 2}}} \sin 2\psi - \frac{A_2}{\sqrt{A_3^{\ 2} + A_2^{\ 2}}} \cos 2\psi = -\frac{A_1}{\sqrt{A_3^{\ 2} + A_2^{\ 2}}} . \quad \text{(A2.3)}$$

We'll solve this equation using auxiliary argument technique. Let φ be an angle defined by following equalities

$$\frac{A_3}{\sqrt{A_3^{\ 2} + A_2^{\ 2}}} = \sin\varphi, \quad \frac{A_2}{\sqrt{A_3^{\ 2} + A_2^{\ 2}}} = \cos\varphi . \quad \text{(A2.4)}$$

Then equation (A2.3) takes form

$$\cos(2\psi + \varphi) = \frac{A_1}{\sqrt{A_3^{\ 2} + A_2^{\ 2}}} . \quad \text{(A2.5)}$$

Equation (A2.5) gives us the desired condition on given values of moment tensor elements:

$$\left| \frac{A_1}{\sqrt{A_3^{\ 2} + A_2^{\ 2}}} \right| \le 1 .$$

Using the definition of constants A_1, A_2, and A_3 we can rewrite the this condition in terms of moment tensor elements M_{22}, M_{33} and M_{23} :

$\dfrac{(M_{22} + M_{33})^2}{4M_{23}^{\ 2} + (M_{33} - M_{22})^2} \le 1$ or $(M_{22} + M_{33})^2 \le 4M_{23}^{\ 2} + (M_{33} - M_{22})^2$, and opening the brackets we have finally

$$M_{22} M_{33} \le M_{23}^{\ 2} . \quad \text{(A2.6))}$$

Value of φ is uniquely defined by formulas (A2.4), and if condition (A2.6) holds true, we get from formula (A2.5) following solutions for ψ

$$\psi = 0.5(\pm \arccos \frac{A_1}{\sqrt{A_3^{\ 2} + A_2^{\ 2}}} - \varphi) . \quad \text{(A2.7)}$$

Two solutions for ψ given by formula (A2.7) correspond to two nodal planes.
So, only in the case if given values of deviatoric moment tensor elements satisfy the condition (A2.6) there are double couples with the same values of three given elements. These double-couples are equivalent with respect to radiated surface wave fields if the depth of the source is much smaller than the wave length.
Considering the ratio of the first and the third equations of system (A2.1) and its third equation we have

$$\begin{cases} \tan\lambda \cos\delta = -\dfrac{A_1}{A_2 \sin 2\psi + A_3 \cos 2\psi} \\ M_0 \cos\lambda \sin\delta = 0.5(A_2 \sin 2\psi + A_3 \cos 2\psi) \end{cases} . \quad \text{(A2.8))}$$

The value of ψ in (A2.8) is defined by formula (A2.7)).
Formulas (A2.8) with computed value of ψ define the family of double couples equivalent to deviator **M** in the case if its elements M_{12} and M_{13} are not much larger by absolute value than all other elements.

Right parts of equalities (A2.8) define constants C_1 and C_2 in formulas (5) and (6) describing the families of equivalent double-couples:

$$\begin{cases} C_1 = -\dfrac{A_1}{A_2 \sin 2\psi + A_3 \cos 2\psi} \\ C_2 = 0.5(A_2 \sin 2\psi + A_3 \cos 2\psi) \end{cases} \tag{A2.9}$$

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